Bank Liquidity and the Cost of Funding

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Abstract

Since the crisis, tougher bank liquidity regulation has been imposed which aims to ensure banks can survive a severe funding stress. Critics of this regulation suggest that it raises the cost of maturity transformation and reduces beneficial lending. In this paper we build a bank run model, based on the seminal work by Rochet and Vives (2004), which has a unique equilibrium where solvent banks can fail due to illiquidity. We endogenise banks’ funding costs and show how they are negatively related to liquidity, therefore offsetting some of the costs from tougher liquidity requirements. We find evidence for this relationship using post-crisis data for US banks, implying that liquidity requirements may be less costly than previously thought.

JEL codes: G21, G28

Keywords: bank runs; global games; liquidity

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1 Introduction

During the global financial crisis many banks which had adequate capital failed or experienced financial distress as they were not liquid. Previously bank regulation had focused on capital but the crisis showed the importance of liquidity and the potential for solvent banks to experience runs. The introduction of global liquidity standards - the Net Stable Funding Ratio and the Liquidity Coverage Ratio - in the Basel III accord are aimed at ensuring banks could survive these runs. These regulations have been criticised for forcing banks to hold more liquid assets reduces their ability to perform maturity and liquidity transformation, reducing lending in productive assets, and likely imposes costs, because yields on liquid assets are generally below the yield on illiquid assets. But on the other hand if liquidity requirements increase banks’ resilience to runs, then liquidity requirements should make failure less likely and therefore reduce banks’ cost of funding, offsetting some of the initial cost.

In this paper we use a simple framework to endogenise the probability of a bank run as a function of the liquid assets a bank has. We use this framework to examine the effect of higher liquid holdings on a bank’s probability of a run and solvency; and then given these probabilities we use the investors’ incentive to invest to endogenously derive the cost of funding, the bank’s choice of liquid assets and bank profitability. A few papers in the literature examine the cost of liquidity requirements, usually - like us - in the context of banks having to forego investment in the more profitable long-term asset. When the cost of liquidity holdings is mitigated this is due to liquid assets being low or zero risk weighted. This means as liquid assets increase, risk-weighted assets decrease, and so required capital is lower. This is the channel in both Roger and Vlcek (2011) and Boissay and Collard (2016). However, to our knowledge no paper has formally modeled lower funding costs as a result of increased resilience to runs.

We employ a simple setup, based on Rochet and Vives (2004), to show how holding more liquid assets can reduce the probability of a bank run. We endogenise the probability of a run by using Global Games as an equilibrium selection technique. While most global games models focus on the investors’ decision to run, our focus is on the initial period and the return the investor demands which depends on the bank’s resilience to runs and initial choice of liquidity. By endogenising funding costs to account for liquidity risk, we show that as a solvent bank holds more cash then the probability of a run decreases (up to the point when it becomes run-proof). The bank can therefore pay less for its funding and that this can offset some of the costs of liquidity regulation. We test our model’s prediction that banks with more liquidity should have lower funding costs using post-crisis data for large US banks. We find a negative association between asset liquidity and credit-default swap (CDS) spreads, indicating that investors may now be more conscious of liquidity risk and are pricing it into firms’ funding costs. In turn this implies that the cost of liquidity requirements could be lower than previously thought.

The rest of this paper proceeds as follows. Section 2 reviews the relevant literature. Section 3 provides our theoretical model while section provides empirical support. Finally
section 5 concludes and offers some policy considerations.

2 Literature review

Our paper is most relevant to the literature on bank runs, cost of liquidity requirements and estimating Modigliani-Miller ”offsets” for banks’ capital requirements.

There is a long literature in banking such as [Diamond and Dybvig (1983)] which shows that banks are susceptible to runs due to coordination failures amongst depositors. In many of these models there are multiple equilibria as depositors coordinate following a sunspot, and the probability of investors seeing this sunspot is exogenous. [Diamond and Kashyap (2016)] modify this framework to study the effect of liquidity regulation by allowing for partial runs in which some depositors do not run following the sunspot. In the model depositors are uncertain as to whether the bank’s liquidity holdings are sufficient to allow it to survive a run. The bank then faces a tradeoff between being more robust to a run against the foregone profits form investing in a risky asset. They find that the additional liquidity needed to survive a run will turn out to be excessive whenever a run is avoided.

This is not the approach that we take as endogenise the effect that holdings of liquid assets have on the probability of a run. In the seminal work of [Morris and Shin (2000)] they use Global Games techniques to solve for the unique equilibrium in a bank run game, which is the method that we employ. Our model is closely related to [Rochet and Vives (2004)] who build a model of bank runs where a solvent bank can fail due to a coordination failure and creditor run, and they use global games to solve for a unique equilibrium. In their model banks have to resort to the repo market whereby it can raise funds only at a discount. We deviate from this by employing a Lender of Last Resort (as they do in their final section) who also receives a signal about the outcome of the project but still demands a haircut. This avoids the slightly awkward problem of the repo market perfectly revealing information about the bank and thus invalidating the global games equilibrium. In [Rochet and Vives (2004)] they explore a LOLR and show that the LOLR can only be fully effective when the interest rate it charges is arbitrarily close to zero. We go beyond their model by solving for the bank’s optimal ex ante liquidity choice, as in [Ahnert (2016)], and endogenising their funding costs. We are unaware of any papers that have taken the latter step.

The second strand of the literature is the cost of liquidity regulation. [Bruno, Onali, and Schaeck (2016)] examine market reactions to announcements about liquidity regulation, which were made as the basel framework was negotiated and find that liquidity regulation announcements are associated with negative abnormal returns. But these are mainly driven by announcements which also tighten capital regulation. The authors interpret this as suggesting that markets do not consider liquidity regulation to be binding. [Boissay and Collard (2016)] examine the problem of a social planner which sets capital and liquidity
requirements. Liquidity requirements are costly as they reduce investment in the risky asset, but some of the cost is offset as increasing liquidity holdings de facto reduces risky assets, making capital requirements less binding. Roger and Vlcek (2011) build a DSGE model with a banking sector and introduce liquidity requirements as a requirement to hold a share of their assets in liquid government securities. Liquid assets have a cost as they have a lower yield meaning that bank revenues decline as they increase holdings of liquid assets. However, as holdings of liquid assets increase, banks risk-weighted assets decline, allowing them to reduce capital.

A number of papers use a similar approach to our econometric specification when examining the cost of higher capital requirements. Miles, Yang, and Marcheggiano (2013), Yang and Tsatsaronis (2012) and Hanson, Kashyap, and Stein (2011) all use a CAPM framework to examine the cost of higher capital requirement on the cost of bank equity. All of them find that there is a Modigliani-Miller 'offset': as a bank’s capital increases the volatility of its equity falls and that this decreases the cost of holding extra equity, which in turn implies higher optimal capital requirements. Our econometric framework is similar but we instead focus on bank liquidity, for which there is very little empirical literature thus far.

3 The model

Consider a three-period economy $t = 0, 1, 2$ in which there are two types of agent. The first is a bank whose size is normalised to 1. The liability side of the bank’s balance sheet is composed of uninsured short-term debt (D) and equity (E). The bank optimises over its assets consisting of cash (c) and loans (L). The return on cash is 1 and the return on loans $R_L$ is random with density $f(L)$ and distribution $F(L)$, which is common knowledge.

The second type of agent is a continuum of investors of size 1 that each provide D units of funding to the bank in period 0. The investors have preferences given by $u()$ and an outside option utility of $U > 1$. The bank offers investors a contract in period 0 that follows Table 1. If the investors withdraw in period 1, they receive 1 with certainty. If they wait until period 2, they receive $r_D$ (which is endogenous) if the bank succeeds but 0 if it fails.

The bank’s profit is any surplus remaining after paying investors at time 2 if it succeeds. In period 0 the bank chooses cash and $r_D$ to maximise expected profit, subject to the investors’ participation constraint of $u(\text{invest}) \geq U$.

<table>
<thead>
<tr>
<th>Action</th>
<th>Bank fails</th>
<th>Bank Survives</th>
</tr>
</thead>
<tbody>
<tr>
<td>Withdraw in period 1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Don’t withdraw</td>
<td>0</td>
<td>$r_D$</td>
</tr>
</tbody>
</table>

Table 1: Period 1 payoffs for investors

In period 1 a fraction of investors \( w \in [0, 1] \) investors decide to withdraw based on a private signal over the return on the risky asset \( x_i = R + e_i \), where \( e_i \) is independently and identically distributed \( N(0, \sigma^2) \) and is independent of \( R \). The bank can pay withdrawing investors using cash or via interest-free secured borrowing from the central bank. We assume that there are no other available forms of funding.\(^1\)

The central bank is essentially passive: they run a committed facility and lend at a haircut \( \theta \ast R \), where \( \theta \in (0, 1) \). The bank can therefore borrow up to \( \theta R (1 - c) \) and will receive more central bank funding if its assets are high quality. The central bank only lends to the bank if \( R \) is high enough such that the bank will be solvent. In equilibrium the bank’s preferred strategy will be to not borrow from the central bank if it is insolvent. For simplicity we assume the central bank knows the firm’s solvency position with a high degree of precision - due to its supervision of banks - but cannot reveal it to the market.

The central bank lends with a haircut of \( \theta \) despite knowing the value of the asset with certainty. In practice central banks do have superior information to the market but also charge haircuts in their liquidity facilities to protect themselves against possible further falls in the value of collateral in the period between the default and them being able to sell the collateral. An alternative justification for the central bank charging a haircut \( \theta \) could come from the fact that the return of \( R \) is only realised if the bank itself is the owner of the project; if the bank failed (for any reason) the assets would either have to be sold or the central bank would have to manage the bank with inferior technology.

We solve the model first by solving for the investor’s decision in the middle period, given the signal that he receives. Given this decision we then solve backwards to the interest rate the investor demands to invest in the bank. And then given this interest rate the bank’s optimal cash holdings. As with [Rochet and Vives (2004)](#rochet_vives_2004) the bank is a corporation which acts in the interest of its equity holders, not the investors.

### 3.1 Critical thresholds

The bank will fail in period 1 if withdrawals exceed available funds i.e.

\[
wD > \theta R (1 - c) + c.
\]

If the bank survives then it repays the central bank in period 2 along with any remaining depositors. Withdrawing investors receive 0 and other investors receive \( r_D \). The firm will fail from insolvency in period 2 if its remaining deposit liabilities exceeds its assets i.e.

\[
R < \frac{(1 - w) r_d D + wD - c}{1 - c} = R_s
\]

\(^1\)We could, as [Rochet and Vives (2004)](#rochet_vives_2004) add a repo market, but we wanted to reflect the increased committment to public liquidity support in the post-crisis era.
Note that the total value of its assets is invariant to period 1 withdrawals, because the central bank does not charge interest. Therefore runs will not harm the bank’s solvency position. In fact solvency can improve if some depositors withdraw early, because they receive 1 in period 1 but would have received \( r_D \) if they had waited for period 2. To isolate the impact on solvency risk, assume that no depositors withdraw i.e. \( w = 0 \). Evaluating the partial derivative:

\[
\frac{\partial R_s}{\partial c} = \frac{Dr_D - 1}{(1-c)^2}
\]

Holding cash can both reduce or increase the bank’s solvency risk, depending on the values of \( D \) and \( r_D \). The bank makes a maximum loss on holding cash of \( Dr_D - 1 \), which is the negative interest margin from holding cash to pay depositors. Equation 3 shows that if \( Dr_D - 1 > 0 \) then holding more cash will make the bank less likely to be solvent (higher \( R_s \)), because they do not have enough equity to absorb the negative margin from holding cash. In the limit \( c \to 1 \) the bank is always insolvent, because their assets yield 1 with certainty which is not enough to pay depositors.

However if \( Dr_D - 1 < 0 \) then holding more cash makes the bank more solvent (lower \( R_s \)), because they have enough equity to absorb the negative margin from holding cash. In the limit \( c \to 1 \) the bank is always solvent, because their assets yield 1 with certainty which is enough to pay depositors.

An implication is, given that \( r_D > 1 \), if the bank has no equity then holding cash will always raise their solvency risk. We explore this dynamic further in section 3.5.

### 3.2 Investors’ withdrawing decisions

As stated before, the bank fails from illiquidity at \( t = 1 \) if:

\[
wD > \theta R(1-c) + c
\]

Therefore, investors will decide to wait until the end of the contract if:

\[
\Delta u \equiv u(\text{wait}, w, R) - u(\text{run}, w, R) \geq 0
\]

where \( u(a, w, R) \) is the investor’s payoff when the investor takes action \( a \in \{\text{wait, run}\} \) and \( w \) is the proportion of other investors that run. Therefore we have that:

\[
\Delta u = \begin{cases} 
  r_D - 1 & \text{if } wD < \theta R(1-c) + c \text{ and } R(1-c) + c \geq r_D(1-w)D + wD \\
  -1 & \text{otherwise}
\end{cases}
\]

\[2\]This was for simplicity but will hold so long as \( r_{cb} < r_d \).
The first condition in equation (5) is that the bank is not illiquid in period 1. The second is that it is solvent in the final period. We will show that a solvent bank can fail due to illiquidity, but an insolvent bank will never survive period 1.

3.3 Period 1 equilibrium run decision

We use techniques from the Global Games literature to solve for the period 1 equilibrium decision for investors. We will show that there is a unique equilibrium asset return, \( R^* \), for which the bank fails and a unique equilibrium run frequency, \( F(R^*) \). The equilibrium is fully determined by:

- Exogenous parameters - \( F(R), U, \theta \) and \( D \).
- The bank’s period 0 choices of \( c \) and \( r_D \), which are taken as given in period 1.

This differs from the classic Diamond-Dybvig setup, where there are multiple equilibria and we cannot determine ex ante which will occur.

**Proposition 1:** There exists a threshold strategy equilibrium where all investors stay if they receive a signal above some value \( x \), and run if they receive a signal below \( x \):

\[
    s_i(x_i) = \begin{cases} 
    \text{stay if } x_i \geq x \\
    \text{run if } x_i < x 
    \end{cases} \tag{6}
\]

At this equilibrium:

\[
    R^* = \frac{1}{\theta(1-c)} \left( \frac{D}{r_D} - c \right) \tag{7}
\]

Proof: See Annex A.1. An interesting property of the threshold equilibrium is that it exists for low values of \( \sigma \), which is counter-intuitive. Suppose \( \sigma \) is just above zero. I could receive some \( x \), just below \( R^* \), but well above the insolvency threshold. I also know that other investors will have received a signal well above the insolvency threshold, given that \( \sigma \) is near zero, but in equilibrium we will all run anyway. We unpack the intuition further in Appendix A2.

**Proposition 2:** The threshold strategy equilibrium given by \( R^* \) is the only strategy surviving iterated deletion of dominated strategies. It is therefore the unique equilibrium for the investor decision in period 1. Proof: See Annex A.2. The uniqueness property is crucial for analysing comparative statics - we can see that the equilibrium run threshold is:

- Falling in \( c \) - As cash increases then the bank can survive a higher level of depositors withdrawing.
• Falling in $\theta$ - the proportion it can raise from the central bank. If the bank can generate more liquidity from the central bank from their risky assets then it is more able to survive a run.

• Falling in $r_D$ - if the payoff to rolling over is greater then investors are less incentivised to run following a particular signal.

• Increasing in $D$ - banks that have a higher proportion of their funds in deposits have less equity, meaning a lower proportion have to run before the bank fails.

The first two illustrate the substitutability between cash and obtaining funding from the central bank. The risky asset provides two functions: it returns a high return if the project succeeds but it is also collateral which can be used to raise cash from the central bank.

Proposition 3: If the firm holds sufficient cash $c \geq c^*$ then there will be no liquidity risk in the model. Only insolvent firms will fail in period 1. Proof: see Annex ???. The bank could choose to eliminate its liquidity risk but will never do so in equilibrium. We discuss this further in section 4.5.

Figure 1 shows how the failure point depends on cash choice. The return at which the bank is insolvent is always below the return at which the bank is potentially illiquid i.e. a solvent firm sometimes fails due to illiquidity and an insolvent firm will always be illiquid. The left diagram shows that, for a firm with high equity, failure always becomes less likely as cash increases. This is because both liquidity risk and solvency risk are reduced; so even after the point where there is no liquidity risk cash improves its chance of survival. However for a poorly capitalised firm, shown in the right diagram, cash makes it more likely to fail after liquidity risk is eliminated, due to the lower return on cash as discussed in section 3.1.

Knowing that the unique period 1 equilibrium is failure for $R$ below $R^*$, we can now solve backwards for the period 0 equilibrium. We do this by first solving for the rate that investors demand $r_d$ for a given amount of cash held by the firm. Then taking this $r_d$ as given the firm maximises expected profits to solve for the optimal amount of cash.

### 3.4 Period 0 equilibrium - parameter restrictions

From Proposition 3, the firm can choose in period 0 to eliminate its liquidity risk by holding $c = c^*$, or it can hold $c \in [0, c^*)$ and suffer some runs while solvent. Below we derive their optimal choice of $\{c, r_D\}$.

For tractability going forwards we assume that $R$ is distributed uniformly on $[0, \bar{R}]$ and investors are risk neutral. This immediately provides us with some parameter restrictions:
The first restriction is from the bank’s need to make a positive expected profits - it is sufficient to assume that the expected loan return exceeds the expected payout to depositors. The second restriction is from the Global Games framework. There needs to be a state of the world for which it is strictly dominant not to run, because the bank has access to sufficient liquidity to survive a run from all depositors.

### 3.5 Period 0 equilibrium - deposit rate and cash choice

Equation 7 pins down the highest value of $R$ for which the firm will fail, $R^*$, for a given cash choice and deposit rate. Depositors will run for signals below $R^*$ and receive 1. They will stay for signals above $R^*$ and receive $r_D$. As noted earlier investors have an outside option of utility $U > 1$. For simplicity we assume that investors are risk neutral.

In our model, as in Rochet and Vives (2004) the bank is a profit maximising bank. It does not exist to act as liquidity insurance for depositors and therefore depositors’ participation constraint strictly binds. The participation constraint is therefore:
Equation 10 follows from applying the quadratic formula to 9. The deposit rate is falling in the firm’s cash choice, up until the point that the firm is run-proof. Whether it falls after that will depend on how much equity the firm has. If they have sufficient equity then holding more cash reduces their asset risk, so the deposit rate will continue to fall. However if they have little equity then more cash raises their solvency risk, because they cannot absorb the loss of $r_D - 1$ from holding cash. Figure 2 shows the relationship graphically for both a firm with high equity and a firm with low equity.

The bank then maximises profits by choosing cash, subject to the participation constraint (equation 10).

$$c^{**} = \arg\max_c \left( \frac{1}{R} \int_{R}^{R} R(1 - c) + c - Dr^{**}_D \right)$$

We know that $c^{**} \in [0, c^*]$ where $c^*$ is the cash choice that eliminates liquidity risk. If there is an interior solution i.e. $c^{**} \in (0, c^*)$ then it will satisfy the following first-order condition:
The first term is the cost of insuring against runs, given by the expected return on foregone loans. The second term is the positive effect from less frequent runs. The final term is the **funding cost offset** from investors knowing the bank is safer. The optimal cash choice will trade off these 3 effects.

Both interior and exterior \((c = 0)\) solutions are numerically possible. Figure 4 shows an example of each. Cash choice will generally be higher when:

- \(R\) is low, because the opportunity cost of holding cash will be low.
- \(U\) is low, because this will relax the participation constraint and reduce \(r_D\), therefore reducing the loss from holding cash.
- \(D\) is low, so that the firm has more ”skin in the game” and is able to absorb the loss of \(r_D - 1\) from holding cash.

**Proposition 4:** If the bank has no equity then it will never choose to hold \(c > 0\), regardless of the other parameters. Proof: See Appendix B. This is an interesting result that is driven by two dynamics:

- The bank has no ”skin in the game” and therefore little incentive to insure against illiquidity.
- Holding cash would make the bank less solvent because they have no equity to absorb the loss from holding cash.

### 3.6 The funding cost offset

If the firm chooses \(c = c^*\) there will only be solvency risk, and investors will run only if they believe the firm is insolvent. Let \(R_s\) denote the loan return for which the firm is just solvent.

\[
(1 - c^*)R_s + c^* - Dr_D = 0 \quad (13)
\]

\[
R_s = R^* = \frac{Dr_D - c^*}{1 - c^*} \quad (14)
\]
This outcome is never supported in equilibrium. The bank has no incentive to prevent runs for states s.t. \( R = R_* \), because their equity will be wiped out in period 2. Therefore they will be indifferent over reaching period 2 and have no incentive to insure against runs in the state of the world where they are just solvent. In equilibrium they will therefore always choose some liquidity risk.

Bank runs may be socially costly. There are many ways we could justify this, such as externalities from fire sales in [Ahnert (2016)](#), but it is not the focus of the paper. Instead we assert that under some circumstances a social planner would want the bank to hold \( c = c^* \) such that they only fail when insolvent.

By definition this will reduce the bank’s profits. However this reduction will be somewhat offset by the falling deposit rate. Ignoring this offset would lead us to overestimate the negative impact of liquidity requirements on firm profitability. Let profits under the assumption of an exogenous deposit rate be ”naive profits”. We define the offset below and show it graphically in figure 4.

\[
\text{offset} = \text{profit}_{|c=c^*} - "\text{naive \ profit}_{|c=c^*}\] (15)

The offset will be larger when the opportunity cost of holding cash is low, and when the deposit rate is more elastic with respect to cash.
4 Empirics

4.1 Empirical specification

We want to test our model’s prediction that funding costs are negatively related to banks’ liquidity positions. If so then there is evidence that liquidity requirements may be less costly than previously thought. Our empirical specification is as follows:

\[
\text{cost of funding}_{it} = \beta_1 \text{LAR}_{it} + \beta_2 \text{LEV}_{it} + \beta_3 \text{STD}_{it} + \text{VIX}_t + \text{UST}_t + \alpha_i + \epsilon_{it} \tag{16}
\]

- cost of funding, $\text{cost of funding}_{it}$, is given by the 5 year senior USD credit default swap (CDS) spread for firm $i$ in period $t$. Although CDS spreads are not actually a funding cost, they serve as a good proxy for wholesale funding costs (Beau, Hill, Hussain, and Nixon (2014)). We use them instead of direct measures of funding cost, such as secondary market bond yields, because the CDS market is very deep and liquid, whereas any given funding instrument may have periods of non-trading.

- LAR is the ratio of liquid assets to total assets. $\beta_1$ is our main coefficient of interest, because it measures the association between a firm’s asset-side liquidity position and cost of funding. Our model predicts a negative relationship.
• STD is the ratio of short term deposits to total assets, which is a measure of funding fragility. $\beta_3$ is therefore a secondary coefficient of interest. Funding fragility is also an important control because a firm may have more liquid assets because of an increased reliance on short term funding, which could be correlated with CDS spreads.

• LEV is the ratio of equity to total assets i.e. their leverage ratio. Augustin, Subrahmanyan, Tang, and Wang (2014) shows these are correlated to firms’ CDS spreads. Leverage could also be correlated to liquidity (a more prudent firm may have higher capital and higher liquidity), so omitting it could bias our results.

• $VIX_t$ is a control for S&P 500 volatility in period $t$, which has been found to be a significant driver of CDS spreads in previous studies (Fama and French (1989)). The VIX also serves as proxy for investor sentiment.

• $UST_t$ is a control for the average yield on 5 year US treasuries in period $t$, which we use as a proxy for the risk-free rate. Risk free rates should be negatively related to CDS yields for two reasons. Firstly Longstaff and Schwartz (1995) finds that higher risk-free rates are generally associated with better macroeconomic conditions. Second Annaert, Ceuster, Roy, and Vespro (2009) argues that higher risk-free rates reduce default probabilities.

• $\alpha_i$ are firm-level fixed effects. These control for time invariant firm-level unobservables, such as their business model, which may be correlated with both their balance sheet variables and CDS spreads.

We run our specification in logs because we expect there to be diminishing returns from holding more liquidity. The coefficients therefore estimate the percentage change in CDS spreads associated with a marginal percentage change in each variable. Logs have the further advantage of being invariant to whether total assets are on the numerator or denominator, whereas for levels this would matter. For example if our independent variable were liquid assets \( \frac{\text{liquid assets}}{\text{total assets}} \), the specification would be linear in liquid assets and non-linear in total assets. But if our independent variable were \( \frac{\text{total assets}}{\text{liquid assets}} \) then it would be non-linear in liquid assets and linear in total assets.

4.2 Data

We use the Federal Reserve’s Financial Reports (form FRY9-C) to obtain data for the balance sheet variables:

• Liquid assets are the sum of cash, withdrawable reserves and US treasury securities. Our liquid asset measure is quite narrow: for example it excludes demand deposits at other banks and non-US government securities, both of which may be reliable sources of liquidity.
• Capital is defined as the ratio of Core Equity Tier 1 capital to total assets. Again this is a relatively narrow definition as it excludes other forms of Tier 1 capital and loss-absorbing capacity.

• Short term debt refers to time deposits with remaining maturity of less than a year. This covers both retail and wholesale deposits.

The FRY9-C Reports are publically available so it is plausible that investors may use them directly when making decisions about banks. They date back to 1986 at quarterly frequency for bank holding companies. However Goldman Sachs and Morgan Stanley were purely investment firms, not banks, until Q4 2008. Therefore they were not regulated by the Federal Reserve and their first Call Report submission is Q4 2008 and the time period for our sample is Q4 2008 - Q1 2017. We have a balanced panel of 198 firm-quarter observations.

We use Bloomberg data on senior credit default swap (CDS) spreads as a proxy for banks’ funding costs. These are available daily over the whole sample for 6 of the largest US firms: Bank of America, Citigroup, Goldman Sachs, JPMorgan Chase, Morgan Stanley and Wells Fargo. Bloomberg is also the source of the VIX and US treasury yield data.

The balance sheet variables are reported for quarter-end dates and we aggregate the daily CDS data in order to match the immediate period following that reporting date. For example: the call report for Q1 2009 would refer to the firm’s balance sheet on 31st March 2009, which we would match to the average of the daily CDS spreads from 1st April to 30th June 2009. Therefore our balance sheet variables are "lagged" by a quarter. We think this deals with potential reverse causality issues e.g. higher CDS spreads may trigger a run, causing the firm’s liquidity to fall.

Figure 5 shows how our variables have evolved over time. Annex C provides a fuller breakdown of descriptive statistics and how these variables evolve over time for each firm.

CDS spreads spiked during the financial crisis, when investors suddenly became aware of risks that had built up in the banking system. There was another spike in 2012 during the Eurozone crisis, as banks were very exposed to Eurozone sovereign debt which was looking less sustainable. Since then spreads have been more stable, although higher than the pre-crisis period. This could reflect permanently higher awareness of financial risks, or a more credible resolution regime reducing the likelihood of future public bailouts.

Liquidity positions were very poor in the pre-crisis period. There was a high reliance on short-term funding and banks held insufficient liquid assets to cover this risk. There was a general belief that financial markets had become so efficient that a solvent firm should always find liquidity. Indeed we would not expect a funding cost offset pre-crisis, because there was little belief in liquidity risk. Banks built up their liquidity in the aftermath of the crisis and sharply reduced their reliance on short-term funding. They
Figure 5: Variables over time

(a) CDS spreads
(b) Liquid asset ratio
(c) Leverage
(d) Short term debt ratio
have continued to build liquidity, likely in response to more stringent regulation (the US implemented the LCR at the end of 2014).

Capital positions follow a similar path to liquidity positions. They were relatively poor pre-crisis and firms took significant losses during the crisis. However firms were forced to re-capitalise quickly in the aftermath of the crisis and have continued to improve since then, largely due to more stringent regulation.

Figure 6 shows the within-firm correlations between liquid assets and CDS spreads. These are generally negative, although the strength of the relationship varies.

4.3 Initial results

Table 2 presents the results of our regressions as we build up the specification. Column 1 includes only the liquid asset ratio and firm fixed effects. We see that there is a significant negative association between liquidity and CDS spreads. Column 2 adds controls for the firm’s leverage and reliance on short term funding. The estimated magnitude of the association between liquidity and funding costs falls, but it gains significance. Column 3 adds firm-invariant controls for stock market volatility and the risk-free rate. The magnitude of the liquidity coefficient falls but it remains highly significant at the 1% level, because these controls capture a lot of variation.
Table 2: Regression results

<table>
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<th>(2)</th>
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<td></td>
<td>FE only</td>
<td>FE + BS Variables</td>
<td>FE + BS Variables + Controls</td>
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<td>-0.243***</td>
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<td>(0.609)</td>
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<td>0.301</td>
<td>0.706</td>
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Robust t-statistics in parentheses
*** p<0.01, ** p<0.05, * p<0.1

The coefficient in column 3 implies that a 1% rise in a firm’s liquid asset ratio is associated with a -0.24% decline in their CDS spreads. Note that this is not a percentage point change i.e. a firm that raised its liquid asset ratio from 2% to 4% would have raised their ratio by 100%, but only 2 percentage points. We also find evidence of a negative association between capital positions and CDS spreads, consistent with previous research on the link between leverage and funding costs.

We do not find evidence that liability-side funding fragility, given by the variable STD, is associated with funding costs in any of our specifications. This may be because due to collinearity with the leverage variable, because debt funding must be negatively correlated with equity funding. Alteratively, investors are perhaps less informed on funding risks than liquidity risks.

4.4 Robustness to outliers

We perform two basic outlier robustness checks of our specification. The first is to drop out each firm individually and re-estimate without that firm. The second test is similar; we re-estimate without each of the years in our sample. If the association between liquidity and funding costs is relatively stable then we can conclude that our results are not being driven by any given firm or year.
Figure 7: Robustness checks

(a) Dropping each firm out
(b) Dropping each year out

Figure 7 shows the results of these checks. The liquid asset ratio coefficient remains fairly stable in both cases. Dropping out each year yields a range of coefficients from -0.18 to -0.30. Dropping out each firm yields a slightly wider range from -0.14 to -0.35, although that may be due to each firm being more of the sample than each year. Therefore we can conclude that our results are not being solely driven by a given firm or year.

4.5 Robustness to specification changes

We also test the robustness of our results to changes in specification. If changes to the specification were to dramatically change our results, this would suggest the underlying relationship is not very robust.

Table 3 presents the results of these specification changes. The significance of the liquidity coefficient varies but is significant to at least the 10% level in all specifications. Column 1 broadens the liquid asset measure to include other US government securities, such as local governments, and interest-bearing demand deposits at other banks. The significance of the relationship falls but the point estimate is fairly similar to our baseline result of -0.24. Columns 2-4 deepen the lag of the independent variables by 1, 2 and 3 periods respectively. The coefficient is slightly smaller but still significant at the 5% level. Finally column 5 re-estimates with a linear specification, rather than logs, therefore we cannot directly compare coefficient size. Reduced significance and $R^2$ suggest this is a poorer fit, but the relationship still exists.
Table 3: Robustness - different specifications

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<td>-19.06**</td>
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<td>0.698</td>
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Robust t-statistics in parentheses
*** p<0.01, ** p<0.05, * p<0.1

5 Conclusion

In this paper, we have built a model of bank runs with a unique equilibrium where solvent banks may fail due to illiquidity. We go beyond the existing literature by endogenising the firm’s funding costs to take account of this risk, and solve for the bank’s optimal choice of liquidity. While forcing the bank to hold more liquidity may impact the bank’s profits, we have shown that it can reduce their funding costs, because they are less likely to fail, thereby offsetting some of the fall in profits.

We test our model’s prediction that banks with stronger liquidity positions have lower funding costs. Using post-crisis data for US banks, we find evidence of such an association. This result is robust to removing any individual firm or year from the sample. Our baseline estimate suggests doubling a bank’s liquid asset ratio would be associated with a 24.4% decline in their CDS spreads. However we find no evidence for a relationship between funding fragility and CDS spreads.

Our results show that liquidity requirements may be less costly than previously thought. This has a clear policy implication: any analysis of optimal liquidity requirements should account for the beneficial effect on funding costs. We believe this could have a similar role to the "Modigliani-Miller" offset for bank capital requirements.
References


URL https://ideas.repec.org/p/cwl/cwldpp/1275r.html

URL https://ideas.repec.org/a/tpr/jeurec/v2y2004i6p1116-1147.html

URL https://ideas.repec.org/p/imf/imfwpa/11-103.html

URL https://ideas.repec.org/a/bis/bisqtr/1203g.html
A Proofs

We will first show that a threshold strategy equilibrium exists, where all investors stay if their signal exceeds a certain value but run if the signal drops below that value.

We then show that this threshold strategy equilibrium is the unique equilibrium surviving iterated deletion of dominated strategies.

A.1 Proof of existence of a threshold strategy equilibrium

Proposition 1: There exists a threshold strategy equilibrium where all investors stay if they receive a signal above some value $x$, and run if they receive a signal below $x$:

We begin by assuming that each investor is using a threshold strategy around some signal $x$.

$$s_i(x_i) = \begin{cases} 
\text{stay if } x_i \geq x \\
\text{run if } x_i < x 
\end{cases}$$  \hspace{1cm} (17)

The key question for investors in the intermediate period: given that my signal $x_i$ is the threshold signal, what is the probability that enough investors run for the bank to become illiquid? Remember that if $W > \frac{\theta R (1-c) + c}{D} = z$ investors run then the bank will become illiquid. Investors want to know $Pr(W > z|x_i)$ i.e. the bank dies.

Given that $x$ is the threshold signal, the bank will die if fewer than $z$ investors get a signal of at least $x$. We want to know the maximum value of $R$ that would cause less than $z$ investors to get a signal of at least $x$. Given that we have a continuum of investors and $x_i = R + e_i$, the fraction of investors with a signal higher than $x$ will be $1 - \Phi(\frac{x-R}{\sigma})$. 

$$1 - \Phi(\frac{x-R}{\sigma}) \leq z$$  \hspace{1cm} (18)

$$R \leq x - \sigma \Phi^{-1}(1-z) = R^*$$  \hspace{1cm} (19)

Therefore the bank will fail due to illiquidity if $R < R^*$, given that depositors withdraw when they receive a signal lower than $x$. To find the probability that the bank will become illiquid, given I have received the threshold signal $x$, we simply need to find the probability that $R$ is below $R^*$. 

23
\[ P(R < R^* | x_i = x) = \Phi \left( \frac{R^* - x}{\sigma} \right) \] (20)
\[ = 1 - \Phi \left( \frac{x - R^*}{\sigma} \right) \] (21)
\[ = 1 - \Phi \left( \frac{x - (x - \sigma\Phi^{-1}(1 - z))}{\sigma} \right) = z \] (22)

So if I receive the threshold signal then I believe the bank dies with probability z, where z is the critical number of people needed to run i.e. \( Pr(W \geq z) = z \). This is the \( U[0,1] \) distribution.

Now that we know beliefs over the distribution of investors that will run to be \( U[0,1] \), we can determine the threshold equilibrium loan return \( R^* \) that will cause the bank to fail. At equilibrium, investors must be indifferent staying (LHS) and running (RHS).

\[ \int_{W=0}^{\theta R^*(1-c)+c} r_D dl + \int_{W=0}^{\theta R^*(1-c)+c} 0 dl \]
\[ = \int_{W=0}^{1} 1 dl R^* = \frac{1}{\theta(1-c)} \left( \frac{D}{r_D} - c \right) \] (23)

So a threshold equilibrium is that depositors will only stay if they receive \( x_i \geq \frac{1}{\theta(1-c)} \left( \frac{1}{r_D} - c \right) \). We know this is a stable equilibrium because the expected staying payoff (LHS) is strictly increasing in \( R^* \), so investors receiving a signal above \( R^* \) would strictly prefer to stay and those receiving a signal below \( R^* \) would strictly prefer to run.

### A.2 Proof of Uniqueness

**Proposition 2:** The threshold strategy equilibrium given by \( R^* \) is the only strategy surviving iterated deletion of dominated strategies. It is therefore the unique equilibrium for the investor decision in period 1.

The key to understanding the threshold equilibrium is iterated deletion of dominated strategies. Let’s continue with the assumption that \( \sigma \) is very small, such that investors disregard their prior in forming an expectation over \( R \). Let \( Pr(\text{bank fails}) = P \). Investors will run if \( P > (1 - P) r_D \) i.e. \( P > 1 - \frac{1}{r_D} = \gamma \).

Let \( R_0 \) be the lowest return at which the firm is solvent i.e. \( R_0 * (1-c) + c = d \). The probability that the firm is insolvent, given signal \( x \), is \( Pr(R < R_0 | x) = \Phi \left( \frac{R_0 - x}{\sigma} \right) \). Therefore
it is strictly dominant to run if investors observe any signal $x$ such that $\Phi(R_0-x) > \gamma$. Denote $x_0$ the highest signal for which it is strictly dominant to run s.t. $x_0 = R_0 - \sigma \Phi^{-1}(\gamma)$. We can delete all strategies that involve rolling over at $x < x_0$.

Let $R_1 > R_0$ be the highest return for which a firm will fail due to illiquidity given that all investors with a signal below $x_0$ run, because the proportion of investors that run will exceed available liquidity i.e. $\Phi(R_0-x) > \theta R_1(1-c) + c$. Therefore the probability that the firm fails due to illiquidity because $R \leq R_1$, given signal $x$, is at least $\Phi(R_0-x)$. Given the previous round of deletion, it is now strictly dominant to run if investors receive signal $x$ such that $\Phi(R_0-x) > \gamma$. Investors will run below any signal $x_1 = R_0 - \sigma \Phi^{-1}(\gamma) > x_0$ and we can delete all strategies that rollover with signal $x < x_1$.

We can iterate this deletion until we reach some pair $x_k = R_k - \sigma \Phi^{-1}(\gamma)$ s.t. $\#R_{k+1} > R_k$ where $\Phi(R_0-x) > \theta R_{k+1}(1-c) + c$. In English, the firm will hold enough liquidity to survive a run at any return $R_{k+1} > R_k$, where the proportion of runners is given by the threshold signal $x_k$ from the previous round of deletion. We have therefore deleted all strategies that involve rolling over with signals $x < x_k$.

Now denote $R_0$ as the lowest loan return that the bank is immune to runs i.e. $\theta R_0(1-c) + c = 1$. The probability that the bank is immune to runs, given signal $x$, is $Pr(R > R_0|X) = 1 - \Phi(R_0-x)$. Therefore $Pr(fail) \leq \Phi(R_0-x)$. Denote $x_0$ as the lowest signal for which it is strictly dominant to roll over because $\Phi(R_0-x_0) = \gamma$. Then $x_0 = R_0 - \sigma \Phi^{-1}(\gamma)$. We can delete all strategies that run with signals $x \geq x_0$, because any investor expect the bank to survive with high enough probability even if all other investors run.

Let $R_1 < R_0$ be the smallest return for which a firm cannot fail due to illiquidity given that investors with a signal above $x_0$ roll over i.e. $\Phi(R_0-x) < \theta R_1(1-c) + c$. The probability that a firm cannot fail due to illiquidity because $R \geq R_1$, given signal $x$, is at least $1 - \Phi(R_0-x)$. Denote $x_1$ as the lowest signal for which it is dominant to roll over because $\Phi(R_0-x) = \gamma$. Then $x_1 = R_1 - \sigma \Phi^{-1}(\gamma)$.

We can iterate this deletion until we reach some pair $x_k = R_k - \sigma \Phi^{-1}(\gamma)$ s.t. $\#R_{k+1} < R_k$ where $\Phi(R_0-x) < \theta R_{k+1}(1-c) + c$. In other words, we eventually we reach some $R_{k+1}$ s.t. it is no longer strictly dominant to roll over, where the proportion of investors definitely rolling over is given by the threshold signal $x_k$ from the previous round of deletion. We have therefore deleted all strategies that involve running with signals $x > x_k$.

Given continuity of the distributions and payoff functions, the limits of these two sequences will converge i.e. $x_k = x_k = x^*$ and $R_k = R_k = R^*$. Therefore there will be a unique equilibrium where investors roll over if they observe signals above $x^*$, and run if they receive signals below. Moreover $\lim_{x \to x^*} x^* = R^*$. Our model satisfies the general conditions laid out in Morris & Shin (2000) for existence of a unique equilibrium.
A.3 Proof of existence of $c^*$ that eliminates liquidity risk

Proposition 3: If a firm holds sufficient liquidity $c^*$ s.t. $\theta R_0(1 - c^*) + c^* \geq D - \frac{1}{r_D} = \gamma$, there will be no liquidity risk as investors will only run if they observe $x < x_0$.

We prove this by contradiction. Recall that the point at which it is strictly dominant to run, due to solvency concerns, is $x_0 = R_0 - \sigma \Phi^{-1}(\gamma)$. Suppose $c \geq c^*$ and $R^* > R_0$ i.e. there is some liquidity risk because solvent banks can fail. There must exist at least 1 possible value $R_1$ s.t. $R_0 < R_1 < R^*$ where it is still strictly dominant to run. At signal $R_1$, the firm has liquidity of $\theta R_1(1 - c^*) + c^* > \gamma$ therefore it can no longer be strictly dominant to run, so $\nexists R^* > R_0$. There is no liquidity risk if $c \geq c^*$ - only insolvent banks will fail.

B Proof that interior solutions exist only if the bank has equity

We show that the bank will never choose $c > 0$ in equilibrium unless they have some equity. It’s sufficient to show that $\frac{\delta \pi}{\delta c|_{c=0,D=1}} < 0$.

\[
\frac{\delta \pi}{\delta c|_{c=0}} = -(\bar{R} - R^*)(\frac{1}{2}(\bar{R} + R^*) - 1) - \frac{dR^*}{dc}(R^* - Dr^*_D) - \frac{Dr^*_D}{dc}(\bar{R} - R^*) \tag{25}
\]

We evaluate each of these terms individually at $c = 0$. The firm is most likely to hold cash if it less able to raise liquidity from its loans e.g. $\theta \bar{R} = D$. Evaluating $r_D|c = 0$ and its derivative:

\[
r_D|_{c=0,\theta \bar{R} = D} = \frac{1}{2}(1 + U + \sqrt{(U + 3)(U - 1))} \tag{26}
\]

\[
\frac{\delta r_D}{\delta c|_{c=0,\theta \bar{R} = D}} = \frac{1}{2D}[1 - UD - 2(D(U + 2) - 1)\sqrt{\frac{U - 1}{U + 3}} - (1 - D)(1 + U + \sqrt{(U + 3)(U - 1))}] \tag{27}
\]

Evaluate the failure point $R^*$ and its derivative:

\[
R^*|_{c=0,\theta \bar{R} = D} = \frac{R}{r_D} \tag{28}
\]

\[
\frac{\delta R^*}{\delta c|_{c=0,\theta \bar{R} = D}} = -\bar{R}[\frac{1}{2}(1 + U + \sqrt{(U + 3)(U - 1))} - D] \tag{29}
\]
We can make one final simplification by evaluating at $\lim_{U \to 1}$, because lower reserve utilities reduce the loss from holding cash.

$$\frac{\delta \pi}{\delta c \mid_{c=0, \bar{\rho}, \bar{R} = D, U \to 1}} = \bar{R}(1 - D)(\bar{R} - D) > 0 \text{ if } D < 1 \quad (30)$$

If the bank has no equity, it will never hold any cash because the profit function is downward sloping at $c = 0$, even with the parameter choices that most incentivise holding cash. The intuition is that without equity, the bank has no “skin in the game” and therefore little incentive to insure against runs.

However if the bank has equity then there will be parameters for which this derivative is positive, therefore the bank may hold cash.
## Descriptive Statistics

Table 4: Summary statistics

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Figure 8: CDS spread by firm
Figure 9: Liquid asset ratio by firm

![Liquid asset ratio by firm](image)
Figure 10: Leverage ratio by firm

```
capital ratio

- Bank of America Corporation
- Citigroup Inc.
- JPMorgan Chase & Co.
- Morgan Stanley
- The Goldman Sachs Group, Inc.
- Wells Fargo & Company
```

5 6 7 8 9 10 11
Figure 11: Short term funding ratio by firm

short term funding ratio

- Bank of America Corporation
- Citigroup Inc.
- JPMorgan Chase & Co.
- Morgan Stanley
- The Goldman Sachs Group, Inc.
- Wells Fargo & Company

0 1 2 3 4 5 6 7 8