Risk-Taking and Dynamic Prudential Regulation

Caterina Lepore*

ABSTRACT

This paper adopts a dynamic contracting framework to study the design of incentive-based regulation of a bank engaging in excessive risk-taking. The bank’s manager can enhance short term profits by either exerting effort or taking on excessive risk, which increases the bank’s exposure to tail risk. Without capital requirements, shareholders induce the manager to undertake excessive risk when the bank is undercapitalised and the regulator grants forbearance ex-post. I then show how the socially optimal regulation, internalizing the bank’s negative externalities on the rest of the economy, can guarantee its continuation under a safe management. The socially optimal regulation curbs excessive risks through a capital requirement, forcing shareholders to recapitalise the bank before entering financial distress. The capital requirement possesses bank’s specific attributes, being stricter for institutions with higher exposure to systemic risk and stronger agency problems. Alternatively, temporarily dismissing the manager allows the bank to recover from financial distress avoiding excessive risk-taking.

Keywords: Dynamic contracts, risk-taking, prudential regulation, capital requirements

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*Lepore (Caterina.Lepore@bankofengland.co.uk) is at Bank of England, Threadneedle St, London EC2R 8AH. The author would like to thank Xavier Freixas, Santiago Moreno-Bromberg, Amar Radia, Quynh-Anh Vo, Roman Inderst, Alexander Michaelides, Frederic Malherbe and participants of the GSE banking summer school, Imperial College Business School internal seminar, the EEA WinE Mentoring Retreat and the Bank of England internal seminar for helpful comments and suggestions.
I. Introduction

As shown by the recent financial crisis, banks’ excessive risk-taking can impose huge losses on the economy. The roots of the problem are multifaceted.

First, executives’ limited liability\(^1\) and a misalignment of risk and reward in managerial compensation structures, generated incentives to undertake excessive risks. Financial innovations, which led to higher degrees of sophistication and opaqueness of financial products, made those excessive risks hard to verify.

Second, shareholders’ interests could have been served by more risk-taking than is socially desirable. Empirical evidence, most notably Beltratti and Stulz (2012), Fahlenbrach and Stulz (2011), Laeven and Levine (2009), suggests that more powerful shareholders and better alignment of incentives between bank managers and shareholders implied worse performance during the crisis.

Finally, regulators failed to prevent risk-taking ex ante. Governments granted forbearance to undercapitalised banks and committed to bailout “too big to fail” financial institutions ex post\(^2\). Such regulatory failures provided banks with implicit protections from the consequences of their risky behaviour, which in turn exacerbated systemic risks. The impact on market discipline and moral hazard was significant.

In the crisis aftermath, preventing excessive risk-taking has been recognised as one of the priorities in the international regulators’ agenda\(^3\). New reforms have been developed to foster prudent behaviour and avoid costly government bailouts. In order to produce new effective regulatory policies, a full understanding of the strategic factors that led to excessive risks in the first place is needed.

To this end, I have developed a continuous-time contracting framework to study the optimal regulation of a bank, whose risky assets are exposed to rare shocks (such as financial crisis) generating unpredictable losses that have the potential to be very large.

The bank’s manager, who is protected by limited liability, can engage in two detrimental activities: shirking and risk-taking. In particular, the manager can increase the bank’s profitability by either exerting effort or taking large risks, which expose the bank to higher expected losses should a crisis occur. Furthermore, risk-taking produces large negative externalities which are not internalized by the bank. This social cost represents the impact of the bank’s risk-taking on other interconnected institutions or on

\(^{1}\)Limited liability allows executives to benefit from their risk-taking activities in the upside, while limiting their downside exposure.

\(^{2}\)Notable examples are Fannie Mae, Freddie Mac, and Bank of America in the US and RBS and Lloyds in the UK.

\(^{3}\)Most importantly: the Dodd-Frank Bill in the US, the Liikanen Commission in the EU and the Vickers Commission in the UK.
The banker’s actions are unobservable, hence a double moral hazard problem arises. Although effort and risk choices are independent, they are strictly linked via the incentive contract. In particular, because of the banker’s limited liability, when she does not have enough skin in the game it becomes impossible to incentivize effort and no risk-taking at the same time. As a result the optimal contract designs the manager’s incentives, striking a delicate balance between inducing effort and preventing risk-taking.

In the baseline model the optimal contract maximises the bank’s private value, accounting for the bank’s shareholders and manager’s payoffs. This contract can be seen as a private agreement between the manager and shareholders or alternatively as a regulation with no instruments to curb risk-taking ex-ante.

Under such contract, shareholders would like to induce high effort at all times and excessive risk-taking arises endogenously along the equilibrium path. The manager is incentivised to refrain from risk-taking by sharing parts of the losses upon the occurrence of a shock. However, when the bank is in financial distress, the manager does not have enough “skin in the game” and starts gambling. Even if risk-taking is detrimental, shareholders find it optimal to let the manager gamble in order to avoid costly liquidation. Furthermore, when incentivising prudent risk management is too costly, shareholders induce the manager to undertake risky strategies even when the bank has not yet entered financial distress. This is because, refraining risk-taking by punishing the manager after losses is costly, as it makes liquidation more likely. Hence the model can generate excessive risk-taking during good times, which then lead to worse outcomes during bad times.

The optimal baseline contract can be implemented through risk-based deposit insurance and regulatory forbearance. In particular, when the bank becomes undercapitalised, government forbearance emerges to avoid the bank’s closure, either as refraining from forcing banks to recognise their losses or as a public recapitalisation provision. Termination is then only used as a measure of last resort. In this set up, the absence of prompt corrective actions can be rationalised through the high cost of liquidation and the lack of transparency, which impedes verifiability of the bank’s risk-taking strategies.

The model is then extended to analyse how the socially optimal regulation, internalising the bank’s negative externalities, can guarantee its continuation under a safe management. The socially optimal regulation is...
contract can incentivize effort and prevent inefficient risky behaviours, by forcing shareholders to recapitalise the bank before reaching financial distress. The proposed regulation disciplines the bank, imposing recapitalisation’s costs to be shared internally, and in turn avoids the need for regulatory forbearance. In particular the capital requirement serves a double purpose. First, it provides the banker with enough “skin in the game” so as to steer her away from risky strategies. Moreover, it guarantees adequate loss absorbing capacity in the event of a crisis. As a result, the requirement possesses banks’ specific attributes; being stricter for institutions with higher exposure to systemic risk and stronger agency problems.

Alternatively, in order to prevent risk-taking, the regulator may employ a suspension phase during which the banker is allowed to shirk. During this phase, the manager is temporarily suspended, which in turn is sufficient to deter risk-taking. The bank keeps operating, running its core (deposit-oriented) functions, while speculative activities are interrupted. Despite the lower profitability, the bank is able to slowly recover from financial distress. Once the bank has overcome distress, management is reinstated and the bank resumes normal operation. Suspending risky activities after a period of poor performance prevents risk-taking and avoids the bank’s liquidation. The suspension provision could be implemented through a special administration of the bank, during which the regulator replaces the bank management and keeps operating the bank’s main functions.

II. Related literature

This paper contributes to two streams of literature: dynamic moral hazard and prudential regulation models. First, the paper is related to the fast-growing literature on dynamic agency problems in continuous-time settings, adopting the martingale techniques developed by Sannikov (2008). Most of the studies in this literature focus on frameworks characterized by Brownian uncertainty, such as for instance the study by DeMarzo and Sannikov (2006), or Poisson shocks of constant jump size, such as the works of Biais et al. (2010) and Myerson (2010), among others. This paper builds on that stream of literature by analysing a double moral hazard problem in a jump-diffusion model. In particular, similarly to Biffis and Lepore (2015), I consider Poisson risk with unpredictable jump size shaped by the agent’s actions.

Most of the existing studies on agency problems involving two hidden actions adopt static settings: most notably, Diamond (1998), Biais and Casamatta (1999), and Palomino and Prat (2003). Only recently have double moral hazard models been developed in dynamic frameworks. Among these contributions,
DeMarzo et al. (2013) study an agency problem in which a manager can divert funds and undertake risk-taking activities, which expose the firm to a disaster event. To deter the agent from stealing, his continuation utility must be reduced following poor performance. However, this provides poorly performing managers with incentives to gamble. Occurrence of a disaster loss is fully revealing of the agent’s risk-taking, leading to the immediate contract termination. Rochet and Roger (2015) adopt a similar setting to develop a theory of “risky utilities”, i.e. private firms managing services essential to the economy. Utilities can engage in risky activities that expose them to catastrophic losses, and are regulated by public authorities. They are interested in the optimal regulation contract minimising the social cost of restructuring and deterring cash flow diversion and speculation at all times.

In contrast, in this study the opacity of the manager’s strategies obstructs the principal from prompt intervention and excessive risks arise endogenously. The model is closest to that of Wong (2015), who analyses the case in which risk-taking increases the likelihood of negative shocks. In this paper, in contrast, the arrival of a shock is exogenous, while risk-taking exposes the bank to higher expected losses conditional on a crisis occurring. In my model I therefore focus on crisis events that are unavoidable, but whose consequences can be moderated through efficient risk management or amplified by imprudent risky investments.

Because risk-taking incentives are strictly connected to the agency cost of shirking, I then extend the analysis to explore contracts that deter risk-taking by implementing low levels of effort. Most of the agency literature on hidden actions focuses on optimal contracts inducing the first-best optimal action at all times. However, it is not a priori known that, even under the second-best contract, this action remains optimal. Indeed, as shown by Zhu (2012), shirking may constitute an integral part of the solution. In his setting, as in my model, the optimal contract can take different forms including phases when the agent shirks frequently.

The second strand of literature connected to this work is the one studying banking regulation and

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5 Even when the disaster state is contractible, paying a bonus contingent on the firm’s survival may not prevent risk-taking if the manager’s incentives are sufficiently weak.

6 Restructuring in their model stands for selling the firms to new shareholders in order to guarantee the continuation of the services.

7 Usually some necessary and sufficient conditions are imposed to ensure the optimality of high effort, as for instance in DeMarzo and Sannikov (2006).

8 In particular, Zhu shows that in addition to the baseline contract, inducing high effort at all times, and the static contract in which the agent shirks forever, two other forms of contracts may arise. He denotes these two new forms the “Quite-Life” contract and the “Renegotiating Baseline” contract. Under the former type of contract, shirking arises as a form of hidden compensation after a long enough period of good performance. On the other hand, in the Renegotiating Baseline model, after sustained poor performance, the principal punishes the agent inducing him to shirk.
resolution mechanisms. In a discrete-time dynamic agency model Shim (2011) analyses the optimality of the Federal Insurance Corporation Improvement Act (FDICIA) mandated prompt corrective action (PCA). Solving for the optimal contract between a regulator and a banker, he characterises the optimal stochastic liquidation procedure for undercapitalised banks, denoted as a form of “structured ambiguity”\(^9\) Vo (2009) has extended Shim’s work to include the possibility of recapitalisation in a continuous-time setting. A similar approach is followed in Section (V) when studying recapitalisation procedures under the socially optimal contract.

Conversely, Freixas and Rochet (2013) study the optimal regulation of systematically important financial institutions, whose liquidation is never possible. In order to guarantee the continuation of the bank and deter excessive risks, they propose the establishment of a systemic risk authority. The optimal regulation involves restructuring the bank after a crisis by firing the managers and expropriating the shareholders. In their framework, the bank is exposed to extremely large losses in the event of a crisis, and hence capital regulation has a very limited scope. In contrast, in my model, the bank cannot be wiped out by a single shock and capital requirements play a central role in the resolution of financial distress.

An extensive academic literature has studied bank capital regulation, leading to mixed predictions on the relationship between banks’ riskiness and capital standards\(^10\). My contribution adds to this literature and it specifically relates to theoretical models on bank behaviour under capital regulation in the face of moral hazard. In particular, my results are in line with Milne (2002), Santos (1999), and Morrison and White (2005) and more recently Klimenko (2014) and Moreno-Bromberg and Roger (2015) who advocate the positive incentives effect of capital requirements and their welfare-improving properties.

In particular Moreno-Bromberg and Roger (2015) have shown in a dynamic contracting framework that minimum capital requirements induce less risk-taking. In their setting breaching the leverage constraint triggers costly downsizing which in turn prevents risk-taking. Klimenko (2014) has shown, by adopting an incomplete contracting approach, how to design incentive-based capital requirements to prevent risk-taking. As in my model, what is crucial, in order to discourage gambling is making bank shareholders internalise the cost of recapitalisation.

\(^9\)Furthermore, he also studies the case in which low effort is first-best optimal. Under this assumption, low effort arises in the optimal contract when the bank’s capital level is high enough. This result is due to the assumption that high effort reduces the variance of realised income.

\(^10\)I refer to VanHoose (2007) for a survey of the literature on this topic.
The rest of the paper is organised as follows. In the next section I outline the model. Section 4 presents the baseline contract maximising the bank’s private value, under the outlined limited liability and incentive compatibility constraints. I then show how the contract can be replicated through capital regulation and deposit insurance. In Section 5 the model is extended to study socially optimal regulations deterring risk-taking at all times. The last section concludes the paper, proofs are relegated to the appendix.

### III. The model

There is a single representative financial institution, a bank, operated by a manager. The bank is owned by private investors, the shareholders, who need the manager to run it. Both the manager and the investors are risk-neutral and discount future cash flows at rate $\gamma$ and $\rho$ respectively, with $\gamma > \rho > 0$.

The bank’s investors “transfer” an amount $Q_0$ to a risk-neutral regulator, collect $D$ units of deposits which are insured by the regulator, and hire the manager to invest in a risky portfolio. The bank’s asset consists of a risky portfolio of financial products, such as derivatives and loans.

Define a probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P})$, where $\mathcal{F}$ is the completed natural filtration of the bank’s cash flows $Y := \{Y_t\}_{t \geq 0}$ and $\mathbb{P}$ is the reference probability measure. The cash flows generated by the risky investments $Y$ are exposed to volatility risk in financial markets, as represented by the standard Brownian motion $Z = \{Z_t\}_{t \geq 0}$. In addition, the bank is subject to tail risk, characterised by rare exogenous shocks inducing potentially large losses, such as financial crisis. The occurrence of these losses is modelled as a Poisson process $N = \{N_t\}_{t \geq 0}$ with constant intensity $\lambda$. Upon the occurrence of a shock at time $t$, the random loss generated $L^r_t$ can take two values: $\ell$ (small loss) or $\ell'$ (large loss, $\ell < \ell'$), with probability depending on the manager’s risk choice $r$ as specified below. The processes $Z$ and $N$ are independent.

The bank’s cash flows evolve as follows

$$dY_t = \delta(e_t, r_t)dt + \sigma dZ_t - L^r_t dN_t,$$

with positive drift $\delta(e_t, r_t) > 0$. I denote by $X := (X_t)_{t \geq 0}$ the diffusion component $dX_t = \delta(e_t, r_t)dt + \sigma dZ_t$. The bank’s cash flows are publicly observable, but the manager’s actions $a_t = (e_t, r_t)$ are not. Hence a double moral hazard problem arises.

Precisely at every time $t$ the banker decides to adopt a level of effort $e_t$, which takes value in $\{e^H, e^L\}$. When the manager chooses high effort $e^H$ (“working”), she raises the cash flows’ drift by $\Delta \delta > 0$, while
incurring a cost $B > 0$ expressed in units of compensation. The cost of low effort $e^L$ (“shirking”) is normalised to zero.

Moreover, the manager can engage in risky activities that shape the distribution of losses in the event of a sudden shock. In particular, the probability of observing a large loss depends on the manager’s risk choice $r_t \in \{r^H, r^L\}$. I denote by $q$ the probability of a large loss under $r^H$, and by $q - \Delta q$ (with $\Delta q > 0$) under low risk $r^L$. This in turn implies that the average loss size under high risk, denoted with $\mu(r^H)$, is greater than the one under low risk $\mu(r^L)$. Therefore, high risk increases the bank’s exposure to “crisis events” while increasing short-term profits by $\alpha > 0$.

Risk-taking is costless for the manager and can be interpreted as imprudent investments in risky assets, leading to faster returns at the cost of poorer risk management. If we consider a large part of the bank’s assets composed of loans, risk-taking corresponds to poor screening of loan quality and poor monitoring of loan performance. The following table summarises the combined effects of effort and risk choices.

<table>
<thead>
<tr>
<th>Effort and Risk Effects</th>
<th>drift $\delta(e, r)$</th>
<th>average loss size $\mu(r)$</th>
<th>manager’s cost $B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^S = (e^H, r^L)$</td>
<td>$\delta + \Delta \delta$</td>
<td>$\mu(r^L)$</td>
<td>$B$</td>
</tr>
<tr>
<td>$a^H = (e^H, r^H)$</td>
<td>$\delta + \Delta \delta + \alpha$</td>
<td>$\mu(r^H)$</td>
<td>$B$</td>
</tr>
<tr>
<td>$a^L = (e^L, r^L)$</td>
<td>$\delta$</td>
<td>$\mu(r^L)$</td>
<td>0</td>
</tr>
<tr>
<td>$a^I = (e^L, r^H)$</td>
<td>$\delta + \alpha$</td>
<td>$\mu(r^H)$</td>
<td>0</td>
</tr>
</tbody>
</table>

Although risk-taking raises short-term profits, it also increases the cost of crisis events and the resulting overall effect is negative. Risk-taking is detrimental in the sense that it decreases the bank’s expected returns:

$$\alpha - \lambda \Delta \mu(r) \leq 0,$$

where $\Delta \mu(r) = \mu(r^H) - \mu(r^L) \geq 0$ is the expected increase in losses induced by high-risk strategies. Nonetheless, even if the manager takes high risks, the bank remains profitable if she keeps working:

$$\delta + \Delta \delta + \alpha - \lambda \mu(r^H) - B > \rho R,$$

where $\geq 0$ is the bank’s liquidation value for shareholders. Hence liquidation is inefficient. On the
other hand, high effort is socially efficient:
\[ \Delta \delta > B, \]  
(iii.3)
i.e. the bank’s gains exceed the manager’s cost.

In the following, I first develop in Section (IV) a baseline model in which the optimal contract maximizes the bank’s private value, given by the manager’s and investors’ payoffs. In this baseline model there are no capital requirements or ex-ante regulations to curb risk-taking. Also I assume that shareholders are not willing to recapitalize the bank voluntarily. Then, when the bank becomes sufficiently undercapitalised, shareholders prefer inducing the manager to take excessive risks in order to avoid liquidation. However risk-taking is socially detrimental, as it generates large negative externalities.

In Section (V) I examine the socially optimal regulatory contract, which takes into account such negative externalities. The socially optimal regulation guarantees the bank’s continuation under safe management, imposing a minimum capital requirement which forces shareholders to recapitalise the bank while bearing the associated costs. Minimum capital requirements deter risk-taking by providing the manager with enough “skin in the game” at all times.

Alternatively, the regulator can prevent the bank from undertaking excessive risks through a suspension phase during which the manager is allowed to shirk after a long enough period of poor performance. Shirking guarantees the suspension of risky investments and, although the bank’s profitability is temporarily reduced, it allows the bank to recover from financial distress and avoid liquidation.

IV. Baseline contract

Based on the realised cash flows, shareholders create a contract specifying the bank’s liquidation time \( \tau \), which is an \( \mathbb{F} \)-stopping time, and the manager payments. I denote by \( T = \{ T_t \}_{0 \leq t \leq \tau} \) the cumulative payments, which is assumed to be an \( \mathbb{F} \)-adapted nonnegative and nondecreasing process. The agents fully commit to a long-term contract. During any time interval \([t, t + dt]\), with \( t < \tau \), the sequence of events can be described as follows:

1. Shareholders prescribe an action recommendation \( a_t = (e_t, r_t) \) to the bank’s manager.

\[ \delta + \Delta \delta - \lambda \mu(r^L) < \rho \]

12 This can be justified by assuming that the return from investing in the bank assets is less than the risk free rate:
2. The manager makes effort and risk choices.

3. The cash flow realisation $Y_t$ is publicly observed.

4. The manager receives a nonnegative transfer $dT_t \geq 0$ from the shareholders.

5. Shareholders decide whether or not liquidate the bank.

Given a contract $\Gamma = (\tau, T)$ and a prescribed action $a = (e, r)$, the manager’s initial expected payoff from future payments, net of the cost of effort, is:

$$W_0 = E^a \left[ \int_0^\tau \exp(-\gamma t) \left( dT_t - 1_{\{e_t = e_H\}} B dt \right) \right]. \quad (IV.1)$$

An action $a = (e, r)$ induces a probability measure $P^a$ equivalent to the reference probability $P$, and $E^a$ denotes the expectation under this measure. The investors’ initial expected payoff is given by the bank’s asset value, including the liquidation value, net of the manager’s payments:

$$S_0(W_0) = E^a \left[ \int_0^\tau \exp(-\rho t) \left( dY_t - dT_t \right) + \exp(-\rho \tau) R \right]. \quad (IV.2)$$

In this baseline model, the optimal contract maximizes the bank’s private value $V_0(W_0) = S_0(W_0) + W_0$, given by the manager’s and investors’ payoffs. In the absence of moral hazard, under the optimal contract, the manager would always exert high effort and no risk-taking. Given that the manager is more impatient than the shareholders, she would be paid immediately and the bank would operate forever. The first best bank’s private value is:

$$\frac{\delta + \Delta \delta - \lambda \mu(\rho^L) - B}{\rho}.$$ 

Observe that the first best value is positive and strictly greater than the liquidation value $R$, by assumptions (III.1) and (III.2). However, investors and outsiders in general can only observe the bank’s cash flows but not the manager’s hidden actions. Hence the first best contract is not attainable and the rest of the paper will focus on second-best contracts.
A. Incentive compatibility and limited liability

At every time $t \leq \tau$, for a given contract $(\tau, T)$, the manager’s discounted utility is given by:

$$W_t = E^a \left[ \int_t^\tau \exp(-\gamma s) \left( dT_s - \mathbb{1}_{\{e_s = e_H\}} B ds \right) \right].$$

(IV.3)

Following Sannikov (2008), I use martingale techniques to characterise the dynamic of the manager’s continuation utility as a function of the past cash flows:

$$dW_t = (\gamma W_t + 1_{\{e_t = e_H\}} B + \psi_t \alpha) dt - dT_t + \phi_t (dX_t - \delta(e_t, r_t) dt) - \psi_t L_t dN_t.$$  (IV.4)

where $(\phi_t)_{t \geq 0}$ and $(\psi_t)_{t \geq 0}$ are two $\mathbb{F}$-predictable processes. Then, the following proposition provides necessary and sufficient conditions for an action recommendation $a = (e, r)$ to be incentive compatible.

**Proposition IV.1. (Incentive compatibility)**

Define $\beta \equiv B / \Delta \delta \psi_t$ and $\theta \equiv \alpha / \lambda \Delta \mu(r)$. Given a contract $(\tau, T)$, the safe action $a^S = (e^H, r^L)$ is incentive compatible if and only if:

$$\phi_t \geq \beta \text{ and } \psi_t \geq \phi_t \theta, \quad \text{(IV.5)}$$

while the risky action $a^R = (e^H, r^H)$ is incentive compatible if and only if $\phi_t \geq \beta$ and $\psi_t \leq \phi_t \theta$.

Observe that $\phi_t$ and $\psi_t$ represent the banker’s sensitivities to volatility and tail risks, respectively. The incentive compatibility condition states that the manager prefers working to shirking if and only if the increase in her utility from choosing high effort $(\Delta \delta \phi_t)$ is greater than the cost of exerting effort $(B)$. Equivalently, her “skin in the game” must be at least equal to $\beta$, which represents the agency cost of inducing high effort.

At the same time, the manager chooses low-risk investments if and only if the short-term gain from risk-taking, $\phi_t \alpha$, is lower than its marginal cost represented by $\psi_t \lambda \Delta \mu(r)$, where $\Delta \mu(r)$ is the expected increase in losses induced by risk-taking. Observe that, although shirking and risk-taking are different activities, they are strictly linked via the incentive contract. Specifically, the incentives loading on volatility risk required to incentivise effort impose a lower bound on the loading on tail risk. As a consequence the more attractive shirking is, the larger the punishments needed to deter risk-taking.

In order to induce the manager to choose safe investments, she must be punished whenever downside
risk crystallizes. By incentive compatibility, given a realised loss of size $l \in \{L, I\}$ the manager’s continuation utility must be reduced by at least $\beta \theta l$. However, the banker is protected by limited liability, which limits the amount of feasible punishments. Specifically limited liability imposes that the manager’s continuation utility must be non-negative at all times, and in particular after the realisation of a shock:

$$W_t - \psi_t L_t^r \geq 0.$$  \hspace{1cm} (IV.6)

As a result in the region $[0, \hat{W})$, with $\hat{W} \equiv \beta \theta I$, inducing high effort and low-risk strategies is not possible and the manager starts to gamble. By assumption (III.2), shareholders prefer to allow for risk-taking rather than liquidate the bank. Furthermore, as I will show in the next section, under the optimal contract risk-taking may arise even when the limited liability constraint is not binding.

B. Derivation of the optimal contract

In this section I characterise the optimal incentive compatible contract. In particular, I focus on contracts inducing high effort $e^H$ at all times\(^\text{13}\) while optimising among the different risk strategies (safe $r^L$ or risky $r^H$ investments). In the extensions presented in Section [V], I show how the optimal contract changes when allowing for low effort.

Denote by $V(W) = S(W) + W$ the bank’s private value function, given by the shareholders’ payoff and the manager’s continuation utility. In the following heuristic derivation $V$ is assumed to be concave. A formal proof is provided in the appendix.

First I analyse the optimal transfers to the manager. Consider the strategy of making a lump sum payment of $dT > 0$ immediately, decreasing the manager’s continuation utility to the value $W - dT$. Then, the shareholders’ payoff satisfies $S(W) \geq S(W - dT) - dT$, implying that $S'(W) \geq -1$ for every $W$. The left-hand side can be interpreted as the marginal benefit from promising future compensation, while the right-hand side represents the shareholder’s marginal cost of an immediate payment. The optimal contract delays payments to the manager when $V'(W) = S'(W) + 1 > 0$ that is, as long as

\[ s^L \leq \min_{w \geq 0} S(w) - S'(w) \frac{\gamma w}{\rho} \]  \hspace{1cm} (IV.7)

where $s^L \equiv \frac{\delta - \lambda_\theta (e^L)}{\rho}$ is the shareholders’ payoff if the shirking strategy $a^L = (e^L, r^L)$ is implemented forever. Note that condition [IV.7] is also sufficient to guarantee the sub-optimality of the strategy $a^L = (e^L, r^H)$.

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\(^\text{13}\)In line with DeMarzo and Sannikov (2006) a necessary and sufficient condition for the optimality of high effort is that:

\[ s^L \leq \min_{w \geq 0} S(w) - S'(w) \frac{\gamma w}{\rho} \]  \hspace{1cm} (IV.7)
they are more costly than utility promises. Denote by \( W = \min \{ W \geq 0 \mid V'(W) = 0 \} \) the payment threshold. Then, by concavity of \( V \), payments are postponed whenever \( W < W \). On the other hand, when the manager continuation utility reaches the threshold \( W \), continuous payments are transferred to the manager. Finally, if \( W \) starts from a value above \( W \), a lump payment of size \( W - W \) immediately brings the manager’s continuation utility down to the payment threshold. Then the bank’s private value function can be set at \( V(W) = V(W) \) for \( W \geq W \).

I will now analyse the bank’s private value function for \( W \in [0, W] \). First I characterise \( V \) when the risky action \( a^R = (e^H, r^H) \) is implemented. Using the dynamic programming approach it is known that \( V(W) \) must satisfy the following equation:

\[
\rho V(W) = \delta + \Delta \delta + \alpha - \lambda \mu(r^H) - B + \max_{\phi \geq \beta, \psi \leq \phi \theta} \mathcal{L}_R V(W)
\]

where the “risky action’s operator” \( \mathcal{L}_R \) is defined as:

\[
\mathcal{L}_R V(W) = (\rho - \gamma)W + V'(W)(\gamma W + B + \lambda \psi \mu(r^H)) + \frac{1}{2} V''(W) \phi^2 \sigma^2 + \lambda e^{\phi R} [V(W - \psi l)] - \lambda V(W).
\]

By concavity of \( V(W) \), it is optimal to set the manager’s sensitivities to the minimum feasible values implementing the risky action. From the incentive compatibility constraint, these values are equal to \( \phi_t = \beta \) and \( \psi_t = 0 \). Note that, when the risky strategy is implemented, the manager is fully insured against tail risk. It follows that the manager’s continuation utility evolves as

\[
dW_t = (\gamma W_t + B) dt - dT_t + \beta \sigma dZ_t,
\]

and the bank’s private value function solves the following equation:

\[
\rho V(W) = \delta + \Delta \delta + \alpha - \lambda \mu(r^H) - B + (\rho - \gamma)W + V'(W)(\gamma W + B) + \frac{1}{2} V''(W) \beta^2 \sigma^2.
\]

Now I consider a regulatory contract implementing the safe action \( a^S = (e^H, r^L) \). Under the safe strategy, the bank’s private value function satisfies:

\[
\rho V(W) = \delta + \Delta \delta - \lambda \mu(r^L) - B + \max_{\phi \geq \beta, \psi \geq \phi \theta} \mathcal{L}_S V(W)
\]
where the “safe action’s operator” $L_S$ is defined as:

$$L_S V(W) = (\rho - \gamma) W + V'(W) (\gamma W + B + \lambda \psi (r^L)) + \frac{1}{2} V''(W) \phi^2 \sigma^2 + \lambda E^{a^S} [V(W - \psi l)] - \lambda V(W). \quad (IV.10)$$

By concavity of $V(W)$, the optimal manager’s sensitivities are fixed at the lower bounds given by incentive compatibility: that is $\phi_t = \beta$ and $\psi_t = \beta \theta$. In order to incentivise the safe strategy, the agent must bear part of the losses at every shock occurrence. Then, the solution to the following equation represents the bank’s private value function when the safe strategy is implemented:

$$\rho V(W) = \delta + \Delta \delta - \lambda \mu (r^L) - B + (\rho - \gamma) W + V'(W) (\gamma W + B + \lambda \beta \theta \mu (r^L)) + \frac{1}{2} V''(W) \beta^2 \sigma^2 + \lambda E^{a^S} [V(W - \beta \theta l)] - \lambda V(W). \quad (IV.11)$$

Given the value from implementing the safe and the risky strategies, (IV.11) and (IV.9) respectively, the regulator’s value function can be characterised as the solution to the following HJB equation:

$$\rho V(W) = \max \left\{ \delta + \Delta \delta - \lambda \mu (r^L) - B + L_S V(W), \delta + \Delta \delta + \alpha - \lambda \mu (r^H) - B + L_R V(W) \right\}. \quad (IV.12)$$

The shareholders find it optimal to prescribe the safe strategy if the expected value from $a^S$ is greater than the one from the risky strategy $a^R$. That is, if and only if:

$$\lambda \Delta \mu (r) - \alpha \geq A(W) \equiv \lambda E^{a^S} [V(W) - V(W - \psi l) - V'(W) \psi l]. \quad (IV.13)$$

The left-hand side represents the bank’s expected monetary cost associated with risky investments. Since risk-taking is harmful by assumption (III.1), this term is positive. By concavity of $V$, the operator $A(W)$ on the right-hand side is positive as well. This term can be interpreted as the expected cost of inducing the safe action against the risky action. Inducing the safe strategy is costly because shareholders must promise punishments to the manager upon the arrival of a loss of size $l$, decreasing her continuation utility to $W - \psi l$. Punishments in turn decrease the bank’s private value function from $V(W)$ to $V(W - \psi l)$, while increasing the manager promised utility by $\psi l$. As a result, the overall cost is given by $A(W)$.

The shareholders thus face a trade-off when optimising over the two strategies. Condition (IV.13) states that inducing the safe action is optimal as long as the agency cost of refraining excessive risks
does not exceed the monetary cost of risk-taking. I denote by \( W \) the threshold at which the two costs offset each other, i.e. where condition (IV.13) holds as an equality. Then for values of \( W \) above \( W \), the shareholders implement the safe action \( a^S \), while for values below \( W \) the regulator prefers inducing the risky action \( a^R \). By limited liability, the gambling threshold \( W \) must lie in the region \([\tilde{W}, W]\).

However, it is not a priori obvious that a unique gambling threshold exists. Uniqueness is guaranteed under the assumption specified in the following proposition, which is assumed to hold throughout this section.

**Proposition IV.2.** Assume \( 0 \leq V'(0) \leq 1 \). Then a unique gambling threshold \( W \) exists:

i) If \( A(W) < \lambda \Delta \mu(r) - \alpha < A(\tilde{W}) \), then \( W > W > \beta \theta I \);

ii) If \( A(W) \geq \lambda \Delta \mu(r) - \alpha \), then \( W = W \);

iii) If \( A(\tilde{W}) \leq \lambda \Delta \mu(r) - \alpha \), then \( W = \tilde{W} \).

Lastly, because liquidation is costly, the shareholders avoid terminating the bank unless strictly necessary, which occurs when the manager continuation utility reaches the lower bound \( W = 0 \). The following proposition provides a complete characterisation of the bank’s private value function and the optimal contract in terms of the manager’s continuation utility.

**Proposition IV.3.** The bank’s private value function is given by the following concave function:

\[
V(W) = \begin{cases} 
V_R(W), & W \in [0, W), \\
V_S(W), & W \in [W, \tilde{W}], \\
V_S(\tilde{W}), & W > \tilde{W}.
\end{cases}
\]  

(IV.14)

where \( V_R(W) \) and \( V_S(W) \) respectively solve equations (IV.9) and (IV.11), with boundary conditions:

\[
V(0) = R, \quad V'(\tilde{W}) = 0, \quad V''(\tilde{W}) = 0.
\]

The optimal contract implements high effort all the time. Define the gambling threshold \( W \in [\tilde{W}, W] \) such that \( A(W) = \lambda \Delta \mu(r) - \alpha \). Then, under the optimal contract, shareholders recommend that the bank’s manager chooses the safe action \( a^S \) when \( W \geq W \) and to switch to the risky action \( a^R \) when \( W < W \).
The optimal sensitivity to volatility risk is $\phi = \beta$ for every $W$, while the sensitivity to tail risk is given by $\psi = \beta \theta, W \geq W$, and $\psi = 0$ when $W < W$. The manager is compensated upon reaching the payment threshold $\overline{W} = \min \{W \geq 0 \mid V'(W) = 0\}$. The optimal amount transferred by the shareholders to the manager is equal to:

$$dT = 1\{W = \overline{W}\} (\gamma \overline{W} + B + \lambda \beta \theta \mu(r^L)) dt + \max \{W - \overline{W}, 0\}$$

The bank is liquidated the first time $W = 0$. Under the optimal contract the banker’s continuation utility evolves as:

$$dW_t = (\gamma W_t + B) dt - dT_t + \beta \sigma dZ_t + \beta \theta (\lambda \mu(r^L)) dt - L_t^r dN_t 1\{W_t \geq \overline{W}\}.$$ 

C. Discussion

The optimal contract incentivises the manager to work all the time by providing her with enough “skin in the game” and promising future payments. After a long enough period of good performance, when $W \geq \overline{W}$, the manager is compensated with a positive transfer.

On the other hand, in order to stop the manager from taking large risks, punishments must be imposed after every loss. Since the manager is protected by limited liability, however, when her continuation utility is too low ($W < \tilde{W}$) punishments become unfeasible. The bank enters in financial distress and the shareholders allow risk-taking, providing full insurance to the manager against realised losses ($\psi = 0$). Even if risk-taking is detrimental, in order to avoid an inefficient liquidation shareholders prefer allowing the manager to gamble for resurrection. Only when the bank is no longer viable do the shareholders mandate its closure.

In general, the optimal contract involves risk-taking even when the bank has not yet entered financial distress and the limited liability constraint is not binding. In fact, unless the monetary cost of risk-taking is extremely large, shareholders induce the manager to take excessive risks even in normal times, exposing the bank to higher losses (on average) should tail risk materialise. This result is in line with Beltratti and Stulz (2012), Fahlenbrach and Stulz (2011), Laeven and Levine (2009), who find that banks with more shareholders-friendly boards or better alignment of incentives between CEOs and shareholders performed significantly worse during the crisis than other banks. Thus policy suggestions aimed at aligning managers and shareholders incentives cannot eliminate excessive risk taking and could actually make the problem
It is important to notice that the optimality of risk-taking is strictly linked to the opacity of the bank’s strategy. If risk-taking were fully revealing, as for instance in DeMarzo et al. (2013) and Rochet and Roger (2015), the optimal contract would prescribe to immediately close the bank at the arrival of the first shock. Fully revealing risk-taking can be obtained as a special case of my model, setting the loss size \( L_t = r_t l \), where \( r_t \in \{ r^L = 0, r^H = 1 \} \) and \( l \) is a positive constant. Under these assumptions, the bank is exposed to tail risk only under high-risk strategies. Hence, the realisation of a loss is fully revealing of the manager’s risk-taking and it is therefore optimal to immediately terminate the contract.

In the full generality of my framework, on the other hand, it is not possible to immediately detect the managerial risk choices from the observation of a single loss. The opacity of the banker’s actions obstructs the outsiders, such as investors and financial authorities, from properly assessing the bank’s risks. The lack of transparency allows to forgo prompt interventions and the bank to capitalise on excessive risk-taking.

D. Implementation

The contract derived in the previous section can also be interpreted as the regulatory contract optimizing the bank’s private value in the absence of any ex-ante mechanism to prevent risk-taking. Thus this section shows how the optimal baseline contract can be implemented in terms of capital regulation and deposit insurance. In line with Shim (2011), I adopt the bank’s book-value capital \( Q_t = (Q_t)_{t \in [0, \tau]} \) as record-keeping device to keep track of the banker’s continuation utility \( W_t \). In particular, the bank capital level \( Q_t \) is required to map the ratio \( \frac{W_t}{\beta_\theta} \), measuring the bank’s financial slackness.

**Book-value capital regulation.** The regulator requires investors to hold an initial amount of capital \( Q_0 \) in order to set up the bank. Investors raise \( D \) units of deposits, which are invested in risky financial products. The initial capital is kept as cash in a reserve account and grows at the risk-free rate \( \rho \). The account is used to meet possible losses due to the arrival of a crisis, to pay a deposit insurance premium, and to distribute dividends. Based on the amount of capital available in the account, which measures the bank’s financial slack, the regulator prescribes different sets of actions and restrictions on the bank’s activities. The scheme is reminiscent of the US banking regulation system, as introduced by

\footnote{Consistent with this result, Gallemore (2013) empirically documents that a bank’s opacity is positively associated with forbearance and negatively associated with the probability of closure.}

\footnote{Note that under the optimal contract discussed in the previous section, the bank is closed down when the banker’s continuation utility reaches the zero lower bound. Thus the banker’s expected payoff indicates the distance to default and can be interpreted as the bank’s financial slackness.}
the FDICIA\textsuperscript{16} In particular banks are classified into four different categories:

- well capitalised, if \(Q_t > Q \equiv W/\beta \theta\);
- adequately capitalised, if \(Q \leq Q_t \leq \bar{Q}\) where the threshold \(Q \equiv W/\beta \theta\);
- undercapitalised, if \(1 \leq Q_t < \bar{Q}\);
- significantly undercapitalised, if \(Q_t < 1\), i.e. only one large loss away from default.

Under the optimal regulatory contract, dividends may be distributed only when the bank is well capitalised, while they are suspended for levels of capital lower than \(\bar{Q}\). In order to align the manager’s incentives with those of shareholders, the manager is granted a fraction \(\beta \theta\) of equity. The fraction \(\beta\) is necessary to incentivise the manager to exert effort, whereas the fraction \(\theta\) is introduced to deter risk-taking. Note that, since risk-taking is detrimental (\(\theta < 1\)), the banker’s equity stake is lower than in the absence of risk-taking incentives.

\textit{Regulatory forbearance.} When the bank capital falls below the threshold \(Q\), the bank becomes undercapitalised. In this region the regulator exerts forbearance and refrains from forcing banks to recognize large losses. That is even if downside risk materialises the book-value capital would not reflect the amount lost. This and more general practices of forbearance to keep distressed banks operating have been often observed in the past as well as during the recent financial crisis. Some examples are Japan’s banking crisis of the 1990s and the US savings and loan crisis in the 1980s, as documented by Nelson and Tanaka \textsuperscript{2014} and White \textsuperscript{1991}, respectively.

Alternatively the absence of losses in the book-value capital of undercapitalised banks can be interpreted as a public recapitalisation provision, i.e. the regulator makes up for the shortfall by injecting public funds. The bank is only terminated when it entirely depletes its capital due to market volatility. Figure (1) illustrates a sample path of the bank’s capital under this regulation.

\textsuperscript{16}See Table 2 in Shim \textsuperscript{2011} for the FDICIA classification of banks as in PCA.
**Deposit insurance premium.** In exchange for deposit insurance, the banker transfers a flow of payments \(dP_t\) to the regulator equal to:

\[
dP_t = \begin{cases} 
(p - \gamma)Q_t dt + (\frac{\delta + \Delta\delta}{\theta} - \frac{E}{\theta^2} - \lambda\mu(r)) dt + (1 - \frac{1}{\theta}) dX_t, & \text{if } Q_t \geq Q, \\
(p - \gamma)Q_t dt + (\frac{\delta + \Delta\delta + \alpha}{\theta} - \frac{E}{\theta^2}) dt + (1 - \frac{1}{\theta}) dX_t, & \text{if } Q_t < Q 
\end{cases}
\]

This specific implementation involves a strongly risk-based insurance premium. In fact, \(dP_t\) is decreasing both in the bank’s level of capital and cash flows. The latter property is due to the assumption that risk-taking is detrimental for the bank’s value\(^ {\text{17}}\). Moreover, the deposit insurance premium increases when the bank is significantly undercapitalised and regulatory forbearance is exercised. Nonetheless, even a risk-based insurance premium is not able to curb ex-ante excessive risk-taking.

**V. The socially optimal contract**

The baseline model has highlighted that the contract optimizing the bank’s private value leads to excessive risk-taking. This is because the bank’s investors and manager take into account only their private benefits, and not the social costs of risk-taking. Moreover, weak supervisory powers and no ex-ante regulatory instruments, leads to socially inefficient forbearance of excessive risks. In this section I show how the socially optimal regulation, internalizing the bank’s negative externalities, can guarantee its continuation under a safe management.

Such negative externalities can be interpreted as the impact of bank’s risk-taking on other institutions, due to their interconnections and financial linkages, or on the real economy\(^ {\text{18}}\). Specifically, I denote with \(C > 0\) the social costs of risk-taking, which materialises every time \(t\) a sudden shock arrives \(dN_t = 1\) and the bank’s manager has chosen a high-risk strategy \(r^H\). In the remainder of this section \(C\) is assumed to be large enough so that it is always socially optimal to prevent risk-taking.

Furthermore, the socially optimal regulator never liquidates the bank, that is the bank is ‘too big to fail’. This is possibly because of adverse spillover effects stemming from contagion risks with others institutions and the economy as a whole. Additionally, the regulator might be concerned with maintaining the bank’s critical economic functions. However, I abstain from modelling these channels explicitly.

\(^{17}\)Observe that \(1 - \frac{1}{\theta} < 0\), since \(\theta = \frac{\alpha}{\lambda\mu(r)} < 1\) by assumption\(^ {\text{III.1}}\).

\(^{18}\)For instance, during the 2008 subprime mortgages crisis banks’ interconnections through securitisations of badly screened loans led to large credit losses and contributed to amplifying the crisis.
A. Recapitalisation and optimal capital requirements

Assume that the regulator can now force the bank to recapitalise by raising equity through the issuance of new shares\textsuperscript{19}. Equity injections are lumpy and involve proportional costs. I denote by $\xi_0 > 1$, the cost of equity injections for outside investors. The manager, representing the bank’s internal shareholders, also has to bear a proportional cost $\xi_1 > 0$. Formally, define equity injections as the $\mathbb{F}$-adapted non-negative non-decreasing process $I = \{I_t\}_{t \geq 0}$:

$$I_t = \sum_{n \geq 1} i_n \mathbb{1}_{\{t \geq \tau_n\}},$$

where $(i)_{n \geq 1}$ is the sequence of nonnegative random variables representing the amount of capital injected, and $(\tau_n)_{n \geq 1}$ is the sequence of stopping times representing the equity issuance dates. Accounting for recapitalisation’ costs, the manager’s and shareholders’ payoffs become:

$$W_t = E^a \left[ \int_t^{\infty} \exp(-\gamma s) (dT_s - 1_{\{e_s = e_H\}} B ds - \xi_1 dI_t) \right],$$

and

$$S(W_t) = E^a \left[ \int_t^{\infty} \exp(-\rho s) (dY_s + (1 - \xi_0) dI_t - dT_s) \right],$$

respectively. The regulator’s value function, which incorporates the bank’s private value and the social costs of risk-taking, is given by:

$$F(W_t) = S(W_t) + W_t - E^a \left[ \int_t^{\infty} \exp(-\rho s) C 1_{\{r_s = r_H\}} dN_s \right].$$

The socially optimal regulation maximises social welfare $F(W_t)$ over the injections $I_t$ and transfers $T_t$ strategies, while forcing the manager to implement the safe action $a^S = (e^H, r^L)$ at all times. By incentive compatibility and limited liability constraints [IV.5] and [IV.6], the manager’s continuation utility must always be $W_t \geq \bar{W} \equiv \beta \theta L$. Otherwise, any further penalty prescribed to preserve incentive compatibility

\footnote{Alternatively, recapitalisation could be realised through the conversion of hybrid debt instruments such as contingent convertible bonds. Recapitalisation could also be interpreted as restructuring the bank, by expropriating shareholders and management and selling the shares of the bank to new investors, as suggested by Freixas and Rochet (2013). However, I will focus on the simple case of new equity issuance.}

\footnote{The manager’s cost of equity issuance can be interpreted in several ways. First it could comprise transaction and asset restructuring costs, or reputational costs. Moreover, in order to ensure incentive compatibility, it is important that the issuance of new shares does not change the proportion of equity held by the manager. Hence this cost could be interpreted as the cost of buying some of the new shares.}
would violate limited liability. In order to avoid terminating the bank, for values of \( W_t < \tilde{W} \) it becomes necessary to require the injection of additional equity. However, as I will analyse in the following, it might be optimal to force recapitalisation even before this is strictly necessary.

Let \( \epsilon > 0 \) and consider the strategy that, for a given initial continuation utility \( W - \epsilon \), makes a capital injection immediately to bring the manager’s continuation utility up to the value \( \tilde{W} \). Then it follows that \( F(W - \epsilon) \geq F(W) - \frac{\epsilon}{\xi_1} (\xi_0 + \xi_1 - 1) \), which implies that \( 1 + \xi_1 F'(\tilde{W}) \leq \xi_0 + \xi_1 \). The left-hand side represents the benefits of recapitalisation, including the marginal benefit from the manager’s threat of capital adjustment costs, while the right-hand side is the social cost of an equity injection. The bank will not be recapitalised as long as the social benefits from a capital injection are lower than its costs.

Define the recapitalisation threshold \( \tilde{W} \equiv \min \{ W \geq \tilde{W} | F'(\tilde{W}) = \frac{\xi_0 + \xi_1 - 1}{\xi_1} \} \). Then the optimal injections can be characterized by \( i_n = \frac{(\tilde{W} - W_n)}{\xi_1} \) and \( \tau_n = \inf \{ t > \tau_{n-1} | W_n < \tilde{W} \} \) for all \( n \geq 1 \). After a long period of poor performance, when the manager’s continuation utility drops below \( \tilde{W} \), the bank is recapitalised through a lumpy injection of capital, which brings \( W \) back to the value \( \tilde{W} \).

The recapitalisation threshold depends on the cost of injections, balancing two opposite effects generated by internal and external costs. In particular, if the manager faces a high cost of equity issuance, increasing the benefits from the threat of mandatory recapitalisation, the requirement becomes tighter. On the other hand, the boundary is decreasing in the cost of investors’ capital. When external financing is expensive, the regulator relaxes the recapitalisation requirement. In particular, if the social cost of equity injections is especially large relative to the manager’s cost, recapitalisation is enforced only if strictly necessary to guarantee the continuation of the bank under a safe management. That is the two boundaries coincide \( \hat{W} = \tilde{W} \).

Finally, it should be observed that, as in the baseline model, the regulatory contract allows for transfers to the manager only at or above the payment threshold \( \overline{W} \equiv \min \{ W \geq 0 | F'(W) = 0 \} \). The above heuristic derivation of the socially optimal regulatory contract is formalised in the next proposition.

\[ \text{A sufficient condition for this is } F'(\tilde{W}) < \frac{\xi_0 + \xi_1 - 1}{\xi_1}, \text{ due to the concavity of the social value function.} \]
Proposition V.1. The socially optimal regulatory value function is given by the following concave function:

\[
F(W) = \begin{cases} 
F(\hat{W}) + \frac{1-\xi_0-\xi_1}{\xi_1}(\hat{W} - W), & W < \hat{W}, \\
F(W), & W \in [\hat{W}, W], \\
F(W), & W > W, 
\end{cases} 
\]  

(V.1)

where \( F(W) \) solves equation (IV.11) with boundary conditions:

\[
F'(\hat{W}) = \frac{\xi_0 + \xi_1 - 1}{\xi_1}, F''(\hat{W}) = 0, F'(W) = 0, F''(W) = 0. 
\]

The socially optimal contract implements the safe action \( a^S = (e^L, r^L) \) at all the times and the optimal sensitivities are \( \phi = \beta \) and \( \psi = \beta \theta \) for every \( W \). Recapitalisation is required when \( W \) falls below the threshold \( \hat{W} \), and the optimal capital injection strategy is

\[
dI = \frac{\hat{W} - W^+}{\xi_1}.
\]

The manager is compensated upon reaching the payment threshold \( W \), with a transfer equal to:

\[
dT = 1_{\{W=W\}}(\gamma W + B + \lambda \beta \theta \mu(r^L))dt + \max\{W-W, 0\}.
\]

Under the socially optimal contract, the manager’s continuation utility evolves as:

\[
dW_t = (\gamma W_t + B)dt - dT_t + \beta \sigma dZ_t + \beta \theta(\lambda \mu(r^L)dt - L_t^L dN_t) + \xi_1 dU_t
\]

and the bank is never terminated.

I will now discuss how the socially optimal contract differs from the baseline model presented in Section 4 in terms of capital implementation. As under the baseline regulatory contract, dividends’ distribution is allowed only when the bank is well capitalised i.e. the level of capital exceeds the boundary \( \bar{Q} \equiv W/\beta \theta \). What differs from the previous implementation is the resolution of financial distress.

Whenever the bank’s capital \( Q_t \equiv \frac{W_t}{\beta \theta} \) reaches \( \hat{Q} \equiv \hat{W}/\beta \theta \), the socially optimal regulation forces shareholders to recapitalise the bank. An amount of capital equal to \( (\hat{Q} - Q_t) \) is injected, bringing the
level of capital back to the boundary. Hence $\hat{Q}$ represents an hard minimum requirement that must be met at all times. By construction $\hat{Q} \geq \bar{l}$, therefore under this regulation the bank never becomes significantly undercapitalised. The bank’s capital is restored to a sufficiently high level before capital is completely depleted, as shown in figure (2).

<Figure 2 about here>

The capital requirement $\hat{Q}$ ensures that the manager always has enough “skin in the game” and hence the right incentives to refrain from taking excessive risks. Moreover, the requirement guarantees that there is enough capital to absorb subsequent losses, and in turn eliminates the need for public bailouts. It is interesting to analyse the determinants of the magnitude of the requirement. In particular the requirement is increasing in the likelihood of arrival of a shock. Most importantly, the optimal capital requirement contains some elements that are bank’ specific.

Intuitively, the magnitude of the requirement depends on the bank’s loss size distribution. The larger the losses to which the bank is exposed, the higher the capital requirement is. Furthermore, the capital requirement increases with the severity of the agency problem. Banks that are more prone to moral hazard, face stricter capital requirements. Finally, the magnitude of the requirement depends on the cost of capital, being increasing in the cost of internal capital and decreasing in cost of external capital\footnote{The intuition behind this result is that internal capital acts as a discipline device to incentivize the bank’s manager, thus the higher its cost the higher the benefits of the requirement. On the other hand, expensive external capital has only the effect of increasing social costs. Thus, as the cost of external capital increases, it is optimal to loosen the capital requirement (which could be therefore interpreted as being counter-cyclical).}

Because\footnote{Indeed $F'(\tilde{W}) > 0$ and using the concavity of the social value function, it follows that $\tilde{W} > \hat{W}$.} $\bar{Q} > \hat{Q}$, dividends are never distributed in the face of a recapitalisation provision. The difference between the two thresholds $\bar{Q} - \hat{Q}$ can be interpreted as a buffer of capital that can be used to absorb losses. While the buffer is depleted the bank faces restrictions on divided distributions but does not need to immediately recapitalise.

The results of the model provides a theoretical underpinning for some of the new features of the Basel III agreement on capital regulation. In the crisis aftermath, regulatory reforms have been carried out, in an effort to strengthen the effectiveness of capital requirements and improve the resilience of the banking system. As specified in\cite{BIS2011}, the overall set of measures produced aim to reduce moral hazard and the negative externalities associated with it. Specifically, as stylised in this model, Basel III prescribes banks to hold a capital conservation buffer in addition to an increased minimum capital requirement,
in order to endure future periods of financial and economic distress (BIS (2010)). Furthermore, capital
buffers should be calibrated taking into account firm-specific characteristics, and include surcharges for
global systemically important banks.

B. Shirking and the suspension phase

So far the paper has focused on contracts, inducing the manager to exert high effort at all times. However, in some cases inducing shirking, which can be considered equivalent to suspending the manager from working, can constitute part of an optimal strategy.

This section proposes a heuristic description of an alternative socially optimal regulation, considering
the implementation of low levels of effort. In order to prevent risk-taking the regulator can employ, as an
alternative device to costly recapitalisation, a suspension phase in which the manager is allowed to shirk.
Throughout the section it is assumed that the bank remains profitable when the manager shirks if there
is no risk-taking:

$$\delta + \alpha - \lambda \mu(r_H) < \rho R < \delta - \lambda \mu(r_L).$$  \hfill (V.2)

This assumption implies that the action $a^I = (e_L, r_H)$ is always inefficient. Analogously to proposition
(IN.5), it is possible to show that the shirking action $a^L = (e_L, r_L)$, involving low levels of effort and risk,
is incentive compatible if and only if:

$$\phi_t \leq \beta \text{ and } \psi_t \geq \phi_t \theta.$$  \hfill (V.3)

Notice that the incentives needed to induce low-risk strategies are strictly connected to the agency cost
of shirking. This link plays a crucial role in deriving the optimal strategy, as I will show below. Under
the shirking action $a^L = (e_L, r_L)$, the regulator’s value function satisfies the following equation:

$$\rho F(W) = \delta - \lambda \mu(r_L) + \max_{\phi \leq \beta, \psi \geq \phi \theta} \mathcal{L}_L F(W),$$

where the “shirking operator” $\mathcal{L}_L$ is defined as:

$$\mathcal{L}_L F(W) = (\rho - \gamma)W + F'(W)(\gamma W + \lambda \psi \mu(r_L)) + \frac{1}{2} F''(W)\phi^2 \sigma^2 + \lambda E^{a_L}[F(W - \psi l)] - \lambda F(W).$$

Since it is costly to expose the manager to risks, the regulator prefers to minimise the volatility of the

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24The optimality of high effort was insured under the condition (IV.7).
agent’s continuation utility, setting $\phi_t = 0$, and in turn reducing the probability of the bank’s distress. Indeed, when the agent is allowed to shirk, no sensitivity to asset performance is required$^{25}$ It follows that it is optimal to shut down the sensitivity to tail risk as well, setting $\psi_t = \phi_t \theta = 0$. When the manager is allowed to exert low effort, restraining her from risk-taking does not require any further punishments. Then, the solution to the following equation represents the social value function implementing the shirking strategy $a^L$:

$$\rho F(W) = \delta - \lambda \mu(r^L) + (\rho - \gamma)W + F'(W)\gamma W. \quad (V.4)$$

Analogously to the baseline contract, the optimal policy can be characterised comparing the regulator’s value function under the two strategies: the safe and the shirking actions in this case. In order to preserve limited liability, it is not possible to incentivise the safe action when the manager’s continuation utility becomes lower than $\bar{W}$. Then, after a long enough period of poor performance, when $W < \bar{W}$, implementing shirking becomes necessary in order to deter risk-taking and avoid costly liquidation$^{26}$ For values of $W \geq \bar{W}$, the regulator finds it optimal to incentivise high effort when the social value from the safe action $a^S = (e^H, r^L)$, given by equation (IV.11), is greater than the value from shirking $a^L = (e^L, r^L)$, as in equation (V.4): that is, if and only if:

$$\Delta \delta - B \geq B(W) \equiv A(W) - \frac{1}{2}F''(W)\beta^2 \sigma^2 - BF'(W) \quad (V.5)$$

The right-hand side represents the agency cost of incentivising the safe action against the shirking action. As in the baseline model, $A(W)$ is the regulatory cost of incentivising low risk. The second term, which is positive by concavity of $F(W)$, represents the cost of incentivising high effort; this is the cost of exposing the manager to volatility risk, in order to provide her with enough skin in the game. The remaining term is the marginal benefit, as an increase in the manager’s continuation utility, from implementing costly effort. The left-hand side represents the bank’s monetary benefit from high effort, net of the manager’s cost of effort, and it is positive by assumption (III.3). Condition (V.5) specifies that as long as this benefit exceeds the incentives’ cost, the safe action is optimal.

Define the shirking threshold $W^* \equiv \min \{W \geq \bar{W} | B(W^*) = \Delta \delta - B\}$. Then for values of $W \geq W^*$,

---

$^{25}$ In terms of implementation, this could be achieved by divesting the manager of her shares.

$^{26}$ Note that by assumption (V.2) the bank remains profitable under low effort if there is no risk taking.
the safe action $a^S$ is implemented and the manager’s continuation utility dynamic is:

$$dW_t = (\gamma W_t + B)dt - dT_t + \beta \sigma dZ_t + \beta \theta (\lambda \mu(r^L) - L_t^L) dN_t.$$ 

For values of $W < W^*$, the shirking action $a^L$ is socially optimal and $W$ evolves deterministically as:

$$dW_t = \gamma W_t dt,$$

with a positive drift. The evolution of the manager’s continuation utility is depicted in figure (3).

If $W$ hits the boundary $W^*$ exactly, as long as no shocks occur, the agent’s continuation utility sticks around it for a positive period of time.\textsuperscript{27} If instead $W$ crosses the shirking boundary with an overshoot, it drifts upwards until the boundary $W^*$ is reached. Once $W$ overcomes the shirking threshold, high effort is implemented again. Notice that, only in the special case $W$ is exactly at the boundary $W^*$, and a large loss brings the manager’s continuation utility down to zero, shirking is implemented forever. In general, the shirking period represents a suspension phase, during which risky investments are interrupted and the bank operates at a lower profitability rate. This phase allows for a slow recovery from financial distress, curbing excessive risks and avoiding the bank’s termination.

In terms of implementation, the suspension period could be achieved through a special compulsory administration\textsuperscript{28} in which the regulator limits the manager’s duties or runs the bank’s main functions, replacing the manager. During this period, speculative activities would be prohibited, yielding in turn lower but risk-free returns. Regulatory interventions, acting as an early measure, would avoid costly bailouts while protecting the bank’s critical functions.

\textsuperscript{27}In the absence of jumps, this dynamic is known as Sticky Brownian Motion, see Harrison and Lemoine (1981). This motion of the agent’s continuation utility appears under the optimal Quiet-Life and Renegotiating Baseline contracts in Zhu (2012).

\textsuperscript{28}Banks’ administration procedures constitute part of the regulatory tools of special resolution regimes, as established in the EU bank recovery and resolution directive (BRRD (2014)).
C. Extensions

To conclude, I will discuss some possible extensions of the model.

First, when addressing the socially optimal regulatory problem in my model, the social cost of risk-taking has been assumed to be large enough so that risk-taking is always socially inefficient. Instead, under the unrestricted class of contracts, high-risk strategies could still arise under the equilibrium path. In such a model, it would be interesting to analyse the interplay between gambling and recapitalisation strategies and how the optimal capital requirement would depend on the negative externality of risk taking.

More generally, one could study the optimal regulatory contract optimising the safe, risky and shirking actions contemporaneously. This framework would generate a larger set of feasible contracts, depending on the values of the shirking payoff and risk-taking cost.

Furthermore, the model could accommodate a broader class of banks’ cash flow loss distributions with bounded support. However, the present framework could not include any possible large loss, since it would not be feasible to guarantee the compatibility between the limited liability and incentive compatibility constraints. This setting would depict the case in which disaster losses could eliminate any plausible buffer and capital regulation would have only a limited scope. These analyses are involving and therefore left for future research.

VI. Conclusion

This paper proposes an incentive-based approach to prudential regulation. In a principal agent model I investigated the design of the optimal regulation of banks whose management can simultaneously engage in inefficient activities: shirking and risk-taking. In particular, shirking reduces the bank’s profitability while risk-taking raises short-term profits but increases the bank’s exposure to tail risk. The manager’s actions depend on the compensation scheme offered by the bank’s shareholders, which in turn is shaped by the regulations designed by financial authorities.

In the baseline model, the optimal contract maximises the bank’s private value and there are no ex-ante regulatory instruments to curb risk-taking. In this framework, excessive financial risks arise due to several factors. The manager’s limited liability, the opacity of the bank’s strategies, and the high cost

29The case of losses with unbounded support has been analysed by Hugonnier and Morellec (2015) in a model to assess the effects of prudential regulation on banks’ insolvency risk.
of liquidation make “gambling for resurrection” optimal. Furthermore, because of the agency costs of high-risk prevention, risk-taking occurs when the bank is undercapitalised, even if it has not yet reached financial distress. The implementation of the contract features regulatory forbearance when the bank becomes undercapitalised.

The model has then been extended to analyse socially optimal regulatory contracts, internalising the social cost of risk-taking and the negative externalities from the bank’s closure. In this setting, the regulator is able to curb excessive risks by either forcing the shareholders to recapitalise the bank or suspending the manager’s speculative activities after a period of poor performance. In the first case, the optimal capital requirement guarantees enough loss absorbing capacity and shareholders’ “skin in the game” to avoid liquidating the bank and to curtail risk-taking incentives. Alternatively the suspension phase, allowing the agent to shirk, guarantees the bank’s continuation under a safe management. Even though profitability is reduced, the bank eventually overcomes financial distress, thereby avoiding liquidation.
References


BIS (2010). Group of governors and heads of supervision announces higher global minimum capital standards.


A. Figures

**Figure 1. Bank’s capital under the baseline contract.**
The figure shows a sample path of the bank’s capital under the baseline contract. When the bank is overcapitalized, dividends are distributed keeping the capital value at the boundary $Q$. When the bank becomes undercapitalized, public capital is injected after every loss (or the regulator forbear). The bank is liquidated when capital is completely exhausted.

**Figure 2. Bank’s capital under the socially optimal contract.**
The figure shows a sample path of the bank’s capital under the socially optimal regulatory contract. When the minimum capital requirement $\hat{Q}$ is breached, shareholder’s equity injections replenish the bank’s capital, avoiding liquidation.
Figure 3. Manager’s continuation utility under shirking.
The figure shows a sample path of the manager’s continuation utility under the socially optimal regulatory contract allowing for shirking.

B. Proofs

A. Proof of Proposition (IV.1)

The manager’s expected payoff conditional on \( F_t \),

\[
H_t = \int_{0}^{\tau} \exp(-\gamma s) \left( dT_s - 1_{\{\hat{e}_s = e_H\}} Bds \right) + \exp(-\gamma t) W_{t \wedge \tau},
\]

is an \( F \)-martingale under \( P^a \). By the Martingale Representation Theorem for jump-diffusion processes (see Theorem 12.11, Hanson (2007)), two \( F \)-predictable processes \( (\phi_t)_{t \geq 0} \) and \( (\psi_t)_{t \geq 0} \) exist such that at any time \( dH_t = -\exp(-\gamma t)\phi_t dZ^a_t - \exp(-\gamma t)\psi_t dM^a_t \) where \( dZ^a_t = dX_t - \delta(e_t, r_t) dt \) and \( dM^a_t = L_t^r dN_t - \lambda \mu(r_t) dt \) are \( (F, P^a) \)-martingales. Differentiating \( H_t \) and rearranging the terms yields that the manager’s continuation utility evolves following equation (IV.4). Suppose that the manager deviates from action \( a = (e, r) \) switching to action \( \hat{a} = (\hat{e}, \hat{r}) \) during the time interval \([0, t]\). The manager’s discounted cumulated payoff is given by

\[
H_t = \int_{0}^{t} e^{-\gamma s} \left( dT_s - 1_{\{\hat{e}_s = e_H\}} Bds \right) + e^{-\gamma t} W^a_t.
\]

By differentiation, it follows that

\[
dH_t = e^{-\gamma t} (dT_t - 1_{\{\hat{e}_t = e_H\}} Bdt) - \gamma e^{-\gamma t} W^a_t dt + e^{-\gamma t} dW^a_t.
\]
\[
e^{-\gamma t} (dT_t - 1_{\{e_t = e_H\}} B dt) - \gamma e^{-\gamma t} W^a_t dt \\
+ e^{-\gamma t} \left( (\gamma W^a_t + \psi_t \lambda (r_t)) dt + 1_{\{e_t = e_H\}} B dt - dT_t \right) \\
+ \phi_t (dX_t - \delta (e_t, r_t) dt - \psi_t L_t dN_t) \\
= e^{-\gamma t} \left( 1_{\{e_t = e_H\}} - 1_{\{e_t = e_H\}} \right) B dt + \phi_t (dX_t - \delta (e_t, r_t) dt - \psi_t M^a_t dt). \\
\]

Therefore, if the manager deviates from the safe strategy \( a = a^S \equiv (e^H, r^L) \) to shirking \( \hat{a} = a^L \equiv (e^L, r^L) \), the above expression becomes

\[
dH_t = e^{-\gamma t} (B - \phi_t \Delta \delta) dt + e^{-\gamma t} (\phi_t dZ^\hat{a} - \psi_t dM^a_t). \\
\]

If \( \phi_t \geq \frac{B}{\Delta \delta} \) the drift of \( H_t \) is non-positive. If instead the manager deviates from the safe strategy \( a = a^S \equiv (e^H, r^L) \) to the risky strategy \( \hat{a} = a^R \equiv (e^H, r^H) \), it follows that

\[
dH_t = e^{-\gamma t} (\phi_t \alpha - \psi_t \lambda (r)) dt + e^{-\gamma t} (\phi_t dX_t - (\delta + \alpha) dt - \psi_t M^\hat{a}_t). \\
\]

When \( \psi_t \geq \frac{\phi_t \alpha}{\Delta \mu (r)} \) the drift is again non-positive. Therefore the manager’s cumulative discounted payoff is a supermartingale, and deviating from action \( a^S \) is always sub-optimal. If on the other hand the incentive compatibility constraint (IV.5) is violated, that is either \( \phi_t < \frac{B}{\Delta \delta} \) or \( \psi_t < \frac{\phi_t \alpha}{\Delta \mu (r)} \), then \( H_t \) would be a submartingale contradicting the optimality of action \( a^S \). Analogous arguments lead to the incentive compatibility condition for action \( a = a^R \equiv (e^H, r^H) \).

**B. Proof of Proposition (IV.2)**

Differentiating (IV.9) follows that the bank’s private value function implementing the risky strategy satisfies

\[
\frac{1}{2} V''(W) \beta^2 \sigma^2 = (\gamma - \rho) (1 - V'(W)) - V''(W) (\gamma W + B). \tag{B.1}
\]

Evaluating the above expression in \( W = 0 \) yields

\[
\frac{1}{2} V''(0) \beta^2 \sigma^2 = (\gamma - \rho) (1 - V'(0)) - V''(0) B. \\
\]
From the concavity of $V$ and the assumption that $V'(0) \leq 1$, it follows that $V'''(0) > 0$. Moreover, $V'(0) \leq 1$ implies that $V'(W) \leq 1$ for every $W$. Hence from (IV.11), I obtain $V'''(W) > 0$ for $W \in [0, W]$. For $W \geq W$, the contract implements the safe strategy. Differentiating (IV.11) yields that

$$\frac{1}{2}V''(W)\beta^2 \sigma^2 = (\gamma - \rho)(1 - V'(W)) - V''(W)(\gamma W + B + \lambda \theta \mu(r^L)) + \lambda E^a[V'(W) - V'(W - \theta l)].$$

(B.2)

Now suppose that $V'''(W)$ is not always positive. Then there exists a point $W_l = \inf \{W \in (W, \hat{W}) : V'''(W) \leq 0\}$, and by continuity $V'''(W_l) = 0$. Differentiating (B.2) in $W_l$ and using that $V'''(W_l) = 0$ leads to

$$\frac{1}{2}V^{(4)}(W_l)\beta^2 \sigma^2 = -(2\gamma - \rho)V''(W_l) + \lambda E^a[V''(W_l) - V''(W_l - \theta l)].$$

From the concavity of $V$, and that $V'''(W) > 0$ for $W \in [0, W_l]$, I obtain $V^{(4)}(W_l) > 0$ contradicting the definition of $W_l$. Therefore I have proved that $V'''(W) > 0$ for every $W$. Differentiating $A(W)$ gives

$$A'(W) = \lambda E^a[V'(W) - V'(W - \psi l) - V''(W)\psi l].$$

The convexity of the first order derivative together with the concavity of $V$ imply that $A'(W) < 0$. Hence $A(W)$ is strictly decreasing in $W$. Then $A(W) = \lambda \Delta \mu(r) - \alpha$ holds at most for one value of $W$, denoted as $W$. If $A(W) < \lambda \Delta \mu(r) - \alpha < A(\hat{W})$, by continuity of $A$, $W > W > \hat{W}$. If $A(W) \geq \lambda \Delta \mu(r) - \alpha$, then risk-taking is always optimal. That is $a^* = (e^H, r^H)$ for every $W \in [0, W]$, with $W = \hat{W}$. Finally if $A(\theta \hat{l}) \leq \lambda \Delta \mu(r) - \alpha$, by limited liability, I must set $W = \theta \hat{l}$.

C. Proof of Proposition (IV.3)

First I prove that $V(W)$ is strictly concave on $[0, \hat{W}]$. Notice that evaluating (IV.9) at $W = 0$ yields

$$\rho V(0) = \delta + \Delta \delta + \alpha - \lambda \mu(r^H) - B + V'(0)B + \frac{1}{2}V''(0)\beta^2 \sigma^2.$$

Using that $V(0) = R$, I obtain

$$\frac{1}{2}V''(0)\beta^2 \sigma^2 = \rho R - \delta + \Delta \delta + \alpha - \lambda \mu(r^H) - B - V'(0)B,$$

which is negative by assumption (III.2). By contradiction, assume that $V$ is not concave on $(0, \hat{W})$. Then, there exists a point $W_c = \inf \{W \in (0, \hat{W}) : V''(W) \geq 0\}$. By continuity of $V$, it follows that $V''(W_c) = 0$. There are two possible cases:

i) $W_c < W$

ii) $W_c \geq W$

---

Take an $\epsilon > 0$, then $V'''(W_l - \epsilon) > V'''(W_l) = 0$, which contradicts the definition of $W_l$. 

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I start by analysing the first case. Taking the derivative of the bank’s private value function implementing the risky action \((IV.9)\) for \(W = W_c\) I obtain

\[
\frac{1}{2} V'''(W_c) \beta^2 \sigma^2 = (\gamma - \rho)(1 - V'(W_c)).
\]  

(E.3)

Evaluating \((IV.9)\) at \(W_c\) yields

\[
V'(W_c) = 1 + \frac{\rho(V(W_c) - W_c) - (\delta + \Delta \delta + \alpha - \lambda \mu(\sigma^H))}{\gamma W_c + B}.
\]

Hence, if (and only if) \(V(W_c) - W_c = S(W_c) > \frac{\delta + \Delta \delta + \alpha - \lambda \mu(\sigma^H)}{\rho} \) it holds that \(V'(W_c) > 1\). Then, from \((B.3)\) follows that \(V'''(W_c) < 0\), contradicting the definition of \(W_c\). Conversely, if \(S(W_c) \leq \frac{\delta + \Delta \delta + \alpha - \lambda \mu(\sigma^H)}{\rho}\) then \(V'(W_c) \leq 1\), implying that \(V'''(W_c) \geq 0\). Then, for a small enough \(\epsilon > 0\), \(V''(W_c + \epsilon) > V''(W_c)\) and \(V'(W_c + \epsilon) > V'(W_c)\). From \((IV.9)\), it follows that \(V(W_c + \epsilon) + (W_c + \epsilon) = S(W_c + \epsilon) > V(W_c) + W_c = S(W_c)\) contradicting \(V'(W_c) \leq 1\).

Now I analyse the second case \(W_c \geq W\). Differentiating the bank’s private value function implementing the safe action \((IV.11)\) in \(W_c\) leads to

\[
\frac{1}{2} V'''(W_c) \beta^2 \sigma^2 = (\gamma - \rho)(1 - V'(W_c)) + \lambda E^{aS} [V'(W_c) - V'(W_c - \beta \theta l)].
\]  

(B.4)

The second term on the right-hand side is negative because \(V''(W) \leq 0\) for \(W \in [0, W_c]\). Then, if \(V'(W_c) \geq 1\), it follows that \(V'''(W_c) < 0\), contradicting the definition of \(W_c\). Assume instead that \(V'(W_c) < 1\). By definition of \(W_c\), \(V'''(W_c) \geq 0\) and, for a small enough \(\epsilon > 0\), \(V'(W_c + \epsilon) \geq V'(W_c)\) and \(V''(W_c + \epsilon) \geq 0\). Then, by concavity of \(V\) on \([0, W_c]\), it follows that

\[
A'V(W_c) = -\lambda E^{aS} \left[ V''(W_c) \beta \theta l + V'(W_c - \beta \theta l) - V'(W_c) \right] \leq 0.
\]

Therefore \(A V(W_c + \epsilon) \leq A V(W_c)\). This implies, by \((IV.11)\), that \(V(W_c + \epsilon) - (W_c + \epsilon) = S(W_c + \epsilon) \geq V(W_c) - W_c = S(W_c)\) contradicting \(S'(W_c) = V'(W_c) - 1 < 0\). Thus the concavity of \(V(W)\) follows. Consequently, by definition of \(W = \min \{ W \geq 0 \mid V'(W) = 0 \}\), \(V'(W) > 0\) for every \(W < W\).
I will now show that, under any incentive compatible contract, the bank’s private value can achieve at most the value $V(W)$ given by (IV.14):

$$V(W_0) \geq E^a \left[ \int_0^T e^{-\rho t} (dY_t - dT_t) + e^{-\rho T} R \right] + E^a \left[ \int_0^T e^{-\gamma t} (dT_t - 1_{e_t = e^H}) B dt \right].$$

Further, I will prove that the bank’s private value attains the upper bound $V(W_0)$ only under the contract described in proposition (IV.3), which is therefore optimal. Define the shareholders’ and manager’s discounted cumulated payoffs respectively as

$$G_t = \int_0^t e^{-\mu s} (dY_s - dT_s) + e^{-\rho t} (V(W_t) - W_t),$$

and

$$H_t = \int_0^t e^{-\gamma s} (dT_s - 1_{e_s = e^H}) B ds + e^{-\gamma t} W_t.$$  

Notice that, by definition, $G_0 = V(W_0) - W_0$ and $H_0 = W_0$. As I showed in the proof of proposition (IV.5), under any incentive compatible contract the process $H_t$ is a supermartingale, and it is a martingale under the optimal contract. Under any incentive compatible contract, the manager’s continuation utility evolves as follows

$$dW_t = (\gamma W_t + B + \psi_t \mu(r_t)) dt - dT_t + \phi_t (dX_t - \delta(e_t, r_t) dt) - \psi_t L_t^x dN_t.$$  

Applying Itô’s Lemma gives

$$e^{\rho t} dG_t = \left( -\rho V(W_t) + \delta(e_t, r_t) - \mu(r_t) - B + (\rho - \gamma) W_t dt + V'(W_t) (\gamma W_t + B + \lambda \psi(r_t)) 
+ \frac{1}{2} V''(W_t) \phi^2 + \lambda E^a[V(W_t - \psi l) - V(W_t)] \right) dt - V'(W_t) dT_t$$

$$+ \sigma dZ_t^a ((1 - \phi) + \phi V'(W_t)) - (1 + \psi) dM_t^a + \Delta J_t^a,$$

where $dM_t^a = L_t^t dN_t - \lambda \mu(r_t) dt$ and $\Delta J_t^a = [V(W_t - \psi l) - V(W_t)] dN_t - \lambda E^a[V(W_t - \psi l) - V(W_t)] dt$ are $(\mathbb{F}, \mathbb{P}^a)$-martingales. Observe that the drift is non-positive by the HJB equation (IV.12). Moreover, $V'(W_t) dT_t \geq 0$ because the manager’s transfers are non-negative and $V'(W_t) \geq 0$. Therefore, under any incentive compatible contract, the process $G_t$ is a supermartingale. Under the optimal contract of
proposition [IV.3], \( V'(W_t)dT_t = 0 \) and the drift of \( G_t \) is equal to
\[
-\rho V(W) + \delta + \Delta \delta + \alpha - \lambda \mu(r^H) - B + (\rho - \gamma)W + V'(W)\gamma W + \frac{1}{2}V''(W)\beta^2 \sigma^2
\]
for \( W_t \in [0, W) \) and to
\[
-\rho V(W) + \delta + \Delta \delta - \lambda \mu(r^L) - B + (\rho - \gamma)W + V'(W)(\gamma W + \lambda \beta \theta \mu(r^L)) + \frac{1}{2}V''(W)\beta^2 \sigma^2 + \lambda E^S[V(W - \beta \theta t)] - \lambda V(W)
\]
for \( W_t \in [W, \bar{W}] \). In both cases the drift is zero, from [IV.9] and [IV.11] respectively, implying that the process \( G_t \) is a martingale. Now I will consider the bank’s private value from any incentive compatible contract
\[
E^a \left[ \int_0^\tau e^{-\rho t}(dY_t - dT_t) + e^{-\rho \tau} R \right] + E^a \left[ \int_0^\tau e^{-\gamma t}(dT_t - 1_{\{e = e_H \}} Bdt) \right]
\]
\[
= E^a \left[ G_{t \wedge \tau} + H_{t \wedge \tau} \right] + E^a \left[ 1_{\{t \leq \tau\}} \left( \int_t^\tau e^{-\rho s}(dY_s - dT_s) + e^{-\rho \tau} R - e^{-\rho t}S(W_t) \right) \right]
\]
\[
+ E^a \left[ 1_{\{t \leq \tau\}} \left( \int_t^\tau e^{-\gamma s}(dT_s - 1_{\{e = e_H \}} Bds) - e^{-\gamma t}W_t \right) \right]
\]
\[
= E^a \left[ G_{t \wedge \tau} + H_{t \wedge \tau} \right] + e^{-\rho t} E^a \left[ 1_{\{t \leq \tau\}} \left( E^a \left[ \int_t^\tau e^{-\rho(s-t)}(dY_s - dT_s) + e^{-\rho \tau} R \right] - S(W_t) \right) \right]
\]
\[
+ e^{-\gamma t} E^a \left[ 1_{\{t \leq \tau\}} \left( E^a \left[ \int_t^\tau e^{-\gamma(s-t)}(dT_s - 1_{\{e = e_H \}} Bds) \right] - W_t \right) \right].
\]
Letting \( t \to \infty \) the second and third terms vanish, while the first term tends to:
\[
G_\tau + H_\tau \leq G_0 + H_0 = V(W_0),
\]
where the inequality follows from \((G_t)_{t \geq 0}\) and \((H_t)_{t \geq 0}\) being supermartingales. Analogously, since under the optimal contract \((G_t)_{t \geq 0}\) and \((H_t)_{t \geq 0}\) are martingales, the upper bound \(V(W_0)\) is achieved with equality. Thus the result holds.

D. Proof of Proposition [V.1]

The social value function \( F(W) \) satisfies the following HJB equation
\[
\max \left\{ \delta + \Delta \delta - \lambda \mu(r^L) - B + \mathcal{L}_S F(W) - \rho F(W), \xi_1 F'(W) + (1 - \xi_0 - \xi_1) \right\} = 0 \quad (B.5)
\]
Applying Ito’s Lemma gives

Under any incentive compatible contract the manager’s continuation utility evolves as follows

Define the investors’ and the manager’s discounted cumulated payoffs respectively as

Analogously to the proof of proposition \(\text{IV.3}\), I will now show that, under any incentive compatible contract, the regulator’s utility can achieve at most the value \(F(W)\) given by \(\text{V.1}\), that is

Define the investors’ and the manager’s discounted cumulated payoffs respectively as

Under any incentive compatible contract the manager’s continuation utility evolves as follows

Applying Ito’s Lemma gives

where \(\mathcal{L}_S\) is the “safe operator” defined as \(\text{IV.10}\). For \(W < \hat{W}\), \(F\) is linear and satisfies the second part of the equation. For \(W \geq \hat{W}\), the social value function solves the first part of the variational inequality \(\text{B.5}\). As shown in the proof of proposition \(\text{IV.3}\), the solution of this equation is concave, given a concave initial history function.

Analogously to the proof of proposition \(\text{IV.3}\), I will now show that, under any incentive compatible contract, the regulator’s utility can achieve at most the value \(F(W)\) given by \(\text{V.1}\), that is

Define the investors’ and the manager’s discounted cumulated payoffs respectively as

Under any incentive compatible contract the manager’s continuation utility evolves as follows

Applying Ito’s Lemma gives

where \(dM_t^\alpha = L_t^\alpha dN_t - \lambda \mu(r_t) dt\) and \(\Delta J_t^\alpha = [V(W_t - \psi l) - V(W_t)] dN_t - \lambda E^\alpha [V(W_t - \psi l) - V(W_t)] dt\) are \((\mathbb{F}, \mathbb{P}^\alpha)\)-martingales. Observe that the drift is non-positive by the HJB equation \(\text{B.5}\), \(F'(W_t) dT_t \geq 0\) and \(dI_t(\xi_1 F'(W) + 1 - \xi_0 - \xi_1) \geq 0\). The remaining terms are \((\mathbb{F}, \mathbb{P}^\alpha)\)-martingales. Hence the process \(D_t\) is a supermartingale. Under the optimal contract defined in proposition \(\text{V.1}\), \(F'(W_t) dT_t = 0\) and
\(dI_t(\xi_1 F'(W) + 1 - \xi_0 - \xi_1) = 0\). Then, for \(W_t \in [\hat{W}, \tilde{W}]\), the drift of \(D_t\) is equal to

\[-\rho F(W) + \delta + \Delta \delta - \lambda \mu(r^L) - B + (\rho - \gamma) W + F'(W)(\gamma W + B + \lambda \theta \mu(r^L)) + \frac{1}{2} F''(W) \beta^2 \sigma^2
\]

\[+ \lambda E^a [F(W - \beta \theta l)] - \lambda F(W) = 0,\]

by (IV.11). Therefore, under the optimal contract, the process \(D_t\) is a martingale. Now I will consider the regulator’s utility from any incentive compatible contract

\[E^a \left[ \int_0^\infty e^{-\rho t} (dY_t + (1 - \xi_0) dI_t - dT_t) \right] + E^a \left[ \int_0^\infty e^{-\gamma t} (dT_t - 1_{\{\epsilon = e^H\}} B d\tau - \xi_1 dI_t) \right]
\]

\[- E^a \left[ \int_0^\infty e^{-\rho t} C 1_{\{r_t = r^H\}} dN_t \right]
\]

\[= E^a \left[ D_t + E_t \right] + E^a \left[ \left( \int_t^\infty e^{-\rho s} (dY_s + (1 - \xi_0) dI_s - dT_s) - e^{-\rho t} S(W_t) \right) \right]
\]

\[+ E^a \left[ \left( \int_t^\infty e^{-\gamma s} (dT_s - 1_{\{\epsilon = e^H\}} B d\tau - \xi_1 dI_t) - e^{-\gamma t} W_t \right) \right] - E^a \left[ \int_0^\infty e^{-\rho t} C 1_{\{r_t = r^H\}} dN_t \right]
\]

\[\leq E^a \left[ D_t + E_t \right] + e^{-\rho t} E^a \left[ \left( E^a_t \left[ \int_t^\infty e^{-\rho(s-t)} (dY_s - dT_s - (1 - \xi_0) dI_s) + e^{-\rho t} R \right) - S(W_t) \right) \right]
\]

\[+ e^{-\gamma t} E^a \left[ \left( E^a_t \left[ \int_t^\infty e^{-\gamma(s-t)} (dT_s - 1_{\{\epsilon = e^H\}} B d\tau - \xi_1 dI_s) \right] - W_t \right) \right].\]

When taking the limit for \(t\) that goes to \(\infty\), the second and term terms vanish, leaving the regulator’s utility bounded above by the value \(D_0 + E_0 = F(W_0)\), using that \((D_t)_{t \geq 0}\) and \((E_t)_{t \geq 0}\) are supermartingales. Under the optimal contract instead \((D_t)_{t \geq 0}\) and \((E_t)_{t \geq 0}\) are martingales, hence the upper bound \(F(W_0)\) is achieved with equality. Therefore the contract described in proposition (V.1) is optimal.