UNDERUTILIZED CAPACITY
AND INADEQUATE DEMAND FOR LABOR

By

Christodoulos Stefanadis*

December 2017

ABSTRACT: In monopolistic competition a possible cause of inadequate demand for labor and underutilized production capacity is the presence of a small number of firms in the market in the short run. Because of the possession of market power by firms, total equilibrium output may be insufficient to utilize all the available labor, driving equilibrium wages to the reservation wage. The economy inefficiently operates inside its production possibilities frontier. In the long run a self-balancing mechanism is the free entry of firms. In the short run a government-imposed increase in real wages and expansionary fiscal policies may cause a weak Pareto-improvement.

JEL Classification Codes: E32, D43, L13.

Key Words: Monopolistic Competition, Aggregate Labor Demand, Production Capacity, Unemployment, Number of Firms, Short Run.

This work has been partly supported by the University of Piraeus Research Center.
* University of Piraeus, Department of Banking and Financial Management, 80 Karaoli & Dimitriou, Piraeus, 18534, Greece, cstefana@unipi.gr.
1. INTRODUCTION

The notion of underutilized production capacity, which originates in Malthus (1836) and mainly in Keynes (1936), tends to be controversial in economics. It implies that an economy’s aggregate production of goods may sometimes fail to take full advantage of all available factors of production, such as labor. The presence of unused factors of production does not necessarily self-correct through the creation of corresponding demand in the factor market (or in the goods market). Thus occasional shortfalls in the aggregate demand for labor (or for other factors) may lead to recessions and to unemployment. Such an idea often comes under attack, especially from the classical school; several critics, starting with Say (1821), question the idea’s rigor, logic and validity. Despite, however, the fierce ongoing debate, the microfoundations of underutilized production capacity and of inadequate aggregate demand for labor, especially when they are not driven by nominal rigidities, have received insufficient attention in formal economic theory. We still lack a complete understanding of the specific mechanisms through which such notions may possibly manifest themselves.

In this paper I provide a general equilibrium industrial organization model of a supply-driven mechanism of underutilized production capacity and of inadequate aggregate demand for labor. Such a mechanism, which is based on the possession of market power by firms, may accommodate both the possibility of economic downturns in the short run and of self-balancing in the long run. As in the standard framework of monopolistic competition (Dixit and Stiglitz (1977), Krugman (1979)), an agent faces diminishing returns to the consumption of each individual good, but enjoys unlimited potential utility gains from an increase in product variety. Our model has two basic ingredients. First, similar to Krugman (1979), the utility function of agents implies a price elasticity of demand for each good that is strictly decreasing in the good’s quantity. Second, as in basic microeconomic theory (e.g., Varian (1992), Mankiw and Taylor (2014)), we make a distinction between the long run and the short run. In a long-run equilibrium there is free entry of firms into the market so that the profit of each firm is driven to zero. In a short-run equilibrium, on the other hand, the number of firms in the market is temporarily considered fixed.
In the base model agents have identical preferences and reservation wages. The reservation wage is the lowest wage for which agents would be willing to work; an agent is strictly willing to work for a pay at least as high as the reservation wage, while he is unwilling to work for strictly less than such a wage. Our analysis shows that when the number of firms in the market is sufficiently small, there can be short-run equilibria in which workers earn the reservation wage. Since such a wage is low, each worker consumes a rather small amount of each product. Entrepreneurs (or firm owners), on the other hand, have a larger income than workers since they also share firm profits. However, although each firm faces low marginal costs in its sales to entrepreneurs as it pays workers only the reservation wage (which can even be zero in a simple textbook setup), the firm may take full advantage of its market power by selling each entrepreneur a rather limited amount of its good. Then, there is inadequate aggregate demand for labor in that the total equilibrium output is insufficient to warrant the employment of the entire labor force despite the strict willingness of labor to work for the equilibrium wage; unemployment drives wages to their lower bound (the reservation wage). For a small number of firms in the market, such a corner solution constitutes a unique equilibrium.

Thus a standard short-run supply irregularity in the product market, — i.e., a sufficiently small number of firms in the market, — leads (often necessarily) to inadequate aggregate demand for labor and to unemployment.¹ Although such unemployment is only marginally (or only infinitesimally) involuntary, it corresponds to disproportionately substantial social costs since it entails underutilized production capacity. Firms fail to take full advantage of idle workers that would be able to produce strictly (and substantially) more than the opportunity cost of labor, or than the reservation wage; the economy inefficiently operates inside its production possibilities frontier. Such inadequate demand is exacerbated if firm ownership (and the distribution of firm profits) is more concentrated since the group of agents that consume more in equilibrium, which

¹ Although changes in the extensive margin, or in the number of firms, constitute a driving force in our analysis, they may also impact the economy indirectly by affecting the strategies (e.g., production, hiring) of continuing firms in the intensive margin. Thus depending on the specific parameter values, even a small reduction in the number of firms in the market may sometimes have a substantial economy-wide impact by causing employment changes in the intensive margin (through the strategies of continuing firms) that can be substantially larger than employment changes in the extensive market. It is noteworthy that according to some empirical research, most of the net reduction in economy-wide employment in downturns occurs in the intensive margin (e.g., Moscarini and Postel-Vinay (2012), Lee and Mukuyama (2015)).
is the group of entrepreneurs, becomes a smaller proportion of the population. Overall, a self-balancing mechanism for such shortfalls in labor demand is the free entry of firms, which, however, constitutes a long-run feature.

In a reservation-wage short-run equilibrium a government may reduce unemployment and generate a weak Pareto-improvement by imposing an increase in the real wage (the ratio of wages to prices) as long as such an increase is not excessive. For example, the government may introduce minimum wage laws or strengthen the bargaining power of unions. A higher real wage boosts workers’ demand for goods and encourages firms to produce more, shifting the economy closer to its production possibilities frontier. After such a beneficial government intervention, unemployment (in case it is not fully eliminated) becomes considerably (rather than only marginally) involuntary. Even when a government-imposed increase in the real wage fails to restore full employment, the government can cause a further weak Pareto-improvement by hiring, for example, unemployed workers to perform a useless task, while financing such hiring through a lump-sum tax on firms (maintaining a balanced government budget). Then, payments to unemployed workers increase total spending and production, while firm profits are unchanged; the lump-sum tax and firm gains from selling to newly hired workers (by firms or by the government) cancel each other out. Such a fiscal mechanism has a positive impact on social welfare.

In practice, economies exhibit continuous entry and exit of firms as new products are expected to fill the void of obsolete ones. However, as table 1 shows, the entry process tends to be quite long, and the time-to-market of new products often takes several years or, at best, several months (Griffin (2002)). Such substantial time lags imply that if

---

2 The result that a government-imposed increase in real wages leads to a reduction in unemployment is in marked contrast with mainstream economics, where minimum wages may cause unemployment (Mankiw and Taylor (2014)). A useful historical example of a government-imposed increase in real wages is the National Industry Recover Act (NIRA) of 1933, which, as a New Deal policy, set minimum wages and increased the bargaining power of unions (although it also increased the monopoly power of firms); NIRA appears to have increased real wages in the U.S. (Weinstein (1980)). NIRA is widely criticized as contractionary by several economists, including Keynes (1933). However, according to our analysis, the impact of NIRA may have actually been expansionary. Eggertsson (2012) discusses a different reason NIRA may have been expansionary; increased inflation expectations may have decreased real interest rates.

3 A controversial Keynesian argument is that in economic downturns it behooves the government to hire agents to dig holes and then fill them, i.e., to employ agents just to perform a useless task. Our analysis combines such a policy with a balanced government budget to demonstrate its feasibility clearly.
the exit rate of firms (or of products) suddenly accelerates, there may be a temporary vacuum in the number of firms (or of products) in the market since, given the long entry process, new products may only gradually, rather than immediately, fill the void of obsolete ones. For example, an abrupt increase in the exit rate may occur if there is an unexpected change in customer profiles (which is to the detriment of certain existing products). In addition, if there is an unexpected change in the profiles of targeted customers, the entry process itself (inflexible as it is) may slow down or be delayed temporarily because of the need for unplanned modifications to the type of entering products (so that they can fit suddenly changed customer profiles). For example, there may be abrupt changes in customer profiles if a Minsky moment occurs, i.e., if customers suddenly realize that they have run up excessive debt (as in Eggertsson and Krugman (2012)). After realizing that they are less wealthy than they believed, such customers may abruptly turn to a different product mix (e.g., shunning luxury goods).

Furthermore, as is well-known, firm entry is especially dependent on ready access to financing (Klapper, Laeven, and Rajan (2006)). Then, the entry process may temporarily be delayed, creating a vacuum in the number of firms in the market, if access to financing is suddenly impeded, as in times of a financial crisis or of financial fragility. Overall, in our analysis as long as the number of firms or of products in the market decreases (or as long as net entry (entry minus exit) decreases), it is immaterial whether it decreases through reduced entry or increased exit (although some empirical research finds that downturns tend to affect entry more than exit (Broda and Weinstein (2010), Lee and Mukuyama (2015)).

Our analysis is consistent with the well-known empirical observation that firms’ price-to-marginal-cost markups tend to be countercyclical, i.e., markups (which, among other things, may be an indication of market power) tend to be higher in economic downturns (e.g., Chevalier and Scharfstein (1996), Chevalier, Kashyap and Rossi

---

4 For example, in the U.S. Great Recession many mortgage loans to households suddenly became non-viable as land prices declined rather unexpectedly (Mankiw and Taylor (2014)). Then, the customer profile of affected households changed abruptly. Similarly, in the Great Recession in southern Europe customers suddenly realized that their prior (and largely debt-financed) consumption patterns were non-viable.

5 For example, in the U.S. Great Recession mortgage loans to households (see note 4) were financed through securitization; several financial institutions made substantial investments in mortgage-backed securities and collateralized debt obligations. Thus when household loans suddenly became non-viable, a
In particular, it is well-known that the degree of concentration of firm ownership is different across countries (e.g., La Porta, Lopez-de-Silanes, and Shleifer (1999)). Then, our model implies that countries with more concentrated firm ownership may be more vulnerable to recessions. Such a conclusion has not been explored in the empirical literature yet. Furthermore, in our argument the substantial time-to-market for new products (and the ensuing inflexibility of the entry process) may play an important role in making net entry (entry minus exit) prone to temporary slowdowns. Then, given that the average time-to-market for new services is substantially shorter than the average time-to-market for new goods (Griffin (2002)), our analysis may imply that a services-dominated economy may be more stable in downturns than a goods-dominated economy. Although there is some casual evidence supporting such a view (Schnorbus and Watson (2012)), more rigorous empirical analysis may be warranted.

1.1. Review of the Theoretical Literature

In the new-Keynesian theoretical literature several articles discuss the concepts of inadequate aggregate demand for labor and of underutilized production capacity, showing how they can be caused by the presence of nominal rigidities, such as sticky prices (e.g., Mankiw (1983), Akerlof and Yellen (1985), Mankiw (1985), Blanchard and Kiyotaki (1987), Eggertsson (2012), Eggertsson and Krugman (2012)), or sticky wages (e.g., Akerlof and Yellen (1985), Mankiw and Taylor (2014)). Sticky information, i.e., the infrequent acquisition of information by firms, may also lead to inadequate demand (e.g., Mankiw and Reis (2006, 2007)). Some work emphasizes the role of debt, showing that deleveraging combined with price stickiness may cause a downturn (e.g., Eggertsson and

---

(2003)). Furthermore, our model is consistent with the empirical finding that the number of products in the market tends to be smaller in economic downturns (Broda and Weinstein (2010)). Our analysis also brings out novel empirical implications that may encourage new empirical research.

full-blown financial crisis erupted (Mankiw and Taylor (2014)). Similarly, the Great Recession in southern Europe involved a collapse of the financial system (e.g., in Greece).

6 Krugman (2016), among others, also points to the possibility of downturns being linked to larger market power by firms. In addition, it appears that in several European countries firm markups were counter-cyclical in the recent Great Recession. As the European Commission (2013), p. 26-27, points out, “overall,
Krugman (2012)). We supplement this literature by bringing out a different mechanism of inadequate aggregate demand and of underutilized production capacity that is driven by real variables and does not entail price, wage or information stickiness.\(^7\) Although our mechanism is supply-driven, it leads to Keynesian-type conclusions; downturns constitute market failures that can be ameliorated by active government policy.\(^8\) Our analysis also implies that possible nominal rigidities, which prevent the real wage from falling too much in economic downturns, may actually be Pareto-improving.

The link between the multiplicity of sunspot equilibria and business cycles is the focus of a large literature (originating in Azariadis (1981), among others). Monopolistic competition (with fully flexible wages) may generate multiple equilibria that are driven by self-fulfilling agent expectations and can often be Pareto-ranked (e.g., Kiyotaki (1988), Benhabib and Farmer (1994), Gali (1994)); an inferior outcome occurs because agents actually expect it.\(^9\) Unlike this literature, in our analysis labor demand shortfalls are supply-driven (although they may be exacerbated by pessimistic expectations) and can be caused by a small number of firms in the market even in the optimistic equilibrium. Our model also emphasizes the distinction between the short run (fixed number of firms) and the long run (free entry of firms), while the literature adopts the long-run framework. In this way, our model may better account for the short-run nature of economic downturns, i.e., for both the possibility of a downturn in the short run and of self-balancing in the long run. Finally, in our analysis we focus on different issues, such as the possibility of inadequate demand for labor and involuntary unemployment (either marginally or substantially involuntary depending on the degree of government intervention); the policy implications are different than in the literature.

---

\(^7\) In a different vein, Mortensen and Pissarides (1994), among others, show how search and matching costs in the labor market may lead to unemployment. Unlike this literature, in our model unemployment stems from the market power of firms in the product market, rather than from frictions in the labor market.

\(^8\) As is well-known, imperfect competition does not always generate first-best outcomes. In this paper we focus on the specific inefficiencies that stem from the presence of a small number of firms in the market in the short run and on possible remedies, rather than on the general inefficiencies of imperfect competition.

\(^9\) Weitzman (1982) and Solow (1998) incorporate sticky nominal wages into a monopolistic competition framework with sunspot equilibria. Then, Pareto-inferior equilibria that are generated by self-fulfilling pessimistic expectations (and coexist with superior outcomes) entail involuntary unemployment.
Some research combines standard RBC (Real Business Cycle) theory with monopolistic competition (e.g., Devereux, Head, and Lapham (1996), Comin and Gertler (2006), Bilbiie, Ghironi, and Melitz (2012)). It shows that the free entry of firms in monopolistic competition may be a propagation mechanism for RBC-type exogenous aggregate productivity shocks. A negative aggregate productivity shock may discourage the entry of firms into the market, aggravating the initial shock. In our model, on the other hand, temporary delays in the entry process constitute a fundamental cause of downturns, rather than a mere propagation mechanism. We incorporate a different type of supply irregularity, — i.e., a small number of firms in the market, — which stems from standard microeconomic theory (Varian (1992), Mankiw and Taylor (2014)), instead of an RBC-type negative aggregate productivity shock.\footnote{Summers (1986) questions the relevance of RBC-type aggregate productivity shocks. On the other hand, the supply irregularity in our model is standard in microeconomics (Varian (1992), Mankiw and Taylor (2014)).} Furthermore, our model examines different issues than this literature since it explores the possibility of inadequate aggregate demand for labor, which is not explored in the RBC literature. In addition, the RBC literature adopts the long run framework, while our analysis makes a distinction between the short run (fixed number of firms) and the long run (free entry of firms).

In a different vein, a line of research shows that overly generous wages that may stem from the presence of powerful worker unions (e.g., Hart (1982)) or from the implementation of efficient-wage policies by firms (e.g., Yellen (1984)) can lead to unemployment. The mechanism in our paper is quite distinct from this research since it focuses on the number and the market power of firms in the product market, rather than on frictions in the labor market. Furthermore, some of our basic conclusions are the opposite of this literature since we show that a government-imposed increase in real wages may reduce unemployment and generate a weak Pareto-improvement.

2. THE MODEL

The economy is populated by a unit mass of agents. There is a continuum of differentiated goods produced, although smaller than the potential range of products. As in the standard model of monopolistic competition (Dixit and Stiglitz (1977)), all
potential goods enter symmetrically into an agent’s utility function. Furthermore, similar to Krugman (1979), all agents have the same utility function

\[ V = \int_0^n u(c(i))di , \quad (1) \]

where \( c(i) \) is consumption of good \( i \), \( n \) is the number, or the mass, of available product varieties, and \( u(c) \) is a strictly increasing and strictly concave function of \( c \), i.e., \( \partial u(c) / \partial c > 0 \) and \( \partial^2 u(c) / \partial c^2 < 0 \).

We define a variable \( \varepsilon(i) \) for each good \( i \), where

\[ \varepsilon(i) = -\frac{\partial u(c(i)) / \partial c(i)}{[\partial^2 u(c(i)) / \partial c(i)^2]c(i)} . \quad (2) \]

Variable \( \varepsilon(i) \) will be useful in the equilibrium analysis in section 3; as we will see, it is the price elasticity of demand that each firm faces. As in Krugman (1979), we assume that

\[ \varepsilon(i) > 0 , \forall c(i) < \hat{c} \quad \text{and} \quad \varepsilon(i) < 1 , \forall c(i) > \hat{c} . \quad (3) \]

To ensure the existence of both an elastic and an inelastic range in each firm’s demand, we assume that there is a \( \hat{c} > 0 \) for which \( \varepsilon(i) = 1 \), which (given condition (3)) implies that \( \varepsilon(i) > 1 , \forall c(i) < \hat{c} \) and \( \varepsilon(i) < 1 , \forall c(i) > \hat{c} . \)

For simplicity, we assume that the only factor of production is labor. An agent is endowed by nature with one unit of labor and sells his labor to a firm in exchange for a

\[ \hat{c} = \frac{1}{c} . \]

\[ Several standard demand functions in partial equilibrium analyses exhibit a price elasticity of demand that is strictly decreasing in quantity. An example of a utility function \( u(c) \) that meets condition (3) is the negative exponential function, \( -e^{-bc} \) (where \( b > 0 \)), for which \( \varepsilon = 1/(bc) \) and \( \hat{c} = 1/b \).
wage. There is a common economy-wide wage rate, \( w \), that will be determined in equilibrium. Each firm \( i \) produces a differentiated good and has a production function

\[
h(i) = \alpha + \beta x(i),
\]

where \( h(i) \) is the amount of labor that is used in producing good \( i \), \( x(i) \) is the output of good \( i \) and \( \alpha, \beta > 0 \). Production thus entails both a fixed and a variable cost.

As is standard in labor economics, each agent has a reservation wage, which is the lowest pay for which the agent would be willing to work (i.e., to sell his unit labor endowment to firms); an agent would be unwilling to work for a pay strictly lower than the reservation wage, while he would be strictly willing to work for a pay at least as high as the reservation wage (Mankiw and Taylor (2014)).\(^{12}\) Since agents have identical preferences, they also have an identical reservation wage. For simplicity, as in the standard framework of monopolistic competition (e.g., Dixit and Stiglitz (1977), Krugman (1979)), it is initially assumed that the reservation wage is zero, which allows us to bring out clearly the possibility of inadequate demand. Later on we will explain how our results may carry through to strictly positive reservation wages (section 4). Furthermore, in section 5.2 we will discuss how our results may carry through to an upward-sloping non-vertical labor supply curve (i.e., to an economy where reservation wages vary among agents).

In the model each firm is owned by \( \omega > 0 \) entrepreneurs, where \( \omega \) is an exogenous parameter. An entrepreneur is an agent that collects a share of a firm’s profit in addition to his wage, i.e., in addition to selling his one unit of labor to firms in exchange for a wage.\(^{13}\) A firm’s profit is equally divided among its \( \omega \) entrepreneurs or owners. Each entrepreneur has an ownership share in only one firm. The labor force that

\(^{12}\) The definition of the reservation wage incorporates the tie-breaking convention that if an agent is indifferent between working and not, he always chooses to work; the agent is thus effectively considered strictly (albeit marginally or infinitesimally) worse off if he does not work than if he works for the reservation wage. Our results would be similar if the tie-breaking convention was reversed, and each agent chose not to work if he was offered the reservation wage. Then, firms would offer a pay strictly (albeit infinitesimally) higher than the reservation wage to convince agents to accept employment; unemployed agents would still be strictly worse off than employed agents.
is employed by a firm that operates in the market is always strictly larger than the firm’s entrepreneurs, i.e., $\omega < \alpha$. For simplicity, it is also assumed that in case there is excess supply of labor, a firm gives priority to hiring its own entrepreneurs. Overall, a smaller $\omega$ signifies a larger concentration of firm ownership in the economy.

We examine two basic types of market equilibria, namely, long-run and short-run equilibria (e.g., Varian (1992), Mankiw and Taylor (2014)). As is well-known in microeconomic theory, in the long run there is free entry of firms into the market so that the profit of each firm is driven to zero. The long-run equilibrium, — in which there is an endogenous mass $n$ of firms, or of varieties, — is the focus of the standard model of monopolistic competition, such as Dixit and Stiglitz (1977) and Krugman (1979). Four conditions are met in such an equilibrium. First, each firm chooses its price to maximize its profits. Second, each agent chooses the amounts of goods he buys to maximize his utility. Third, the economy-wide wage ensures that the supply of labor is equal to the demand for labor (unless there is a corner solution in which there is excess labor supply, and the equilibrium wage is equal to the reservation wage, i.e., the equilibrium wage has fallen to the lower bound). And fourth, the mass of firms ensures that the profit of each firm is zero (so that there is no further firm entry or exit). In the short run, on the other hand, the mass, $n$, of firms in the market is exogenously given since (as we explain in the introduction) the free entry process may sometimes temporarily freeze or be temporarily held back (e.g., Varian (1992), Mankiw and Taylor (2014)). Then, only the first three of the above four conditions are met, while $n$ is exogenous.

3. LONG-RUN AND SHORT-RUN EQUILIBRIA

The long-run equilibrium of the model, which is similar to Krugman (1979), is described in detail in the appendix (proof of lemma 1). In equilibrium all firms $i \in [0, n]$ charge the same price and sell the same output to each agent, worker or entrepreneur, i.e., $p(i) = p$ and $c(i) = c$, $\forall i \in [0, n]$. Given the underlying utility-maximizing decisions of

---

13 The assumption that firm owners or entrepreneurs also earn a wage by selling their labor to firms is standard in basic general equilibrium theory (Varian (1992)). Our results, however, would be similar if entrepreneurs did not earn a wage, but only collected the profits of their firms.
agents, we can see in figure 1 that profit maximization by firms (i.e., $p/w = \beta \epsilon / (\epsilon - 1)$) and the free-entry zero-profit condition (i.e., $p/w = \beta + \alpha/c$) imply that each firm $i$ in the market sells each agent a unique amount $c_{LR}^*$ in equilibrium. Then, according to the full-employment condition, the equilibrium mass of firms is:

$$n_{LR}^* = \frac{1}{\alpha + \beta c^*}.$$  

(5)

We summarize in lemma 1.

**Lemma 1**: There exists a unique long-run equilibrium, in which each firm in the market sells each agent an output $c_{LR}^*$, and the mass of firms in the market is $n_{LR}^* = 1/(\alpha + \beta c_{LR}^*)$.

**Proof**: The proof is in the appendix.

The long-run equilibrium in lemma 1 would remain identical if instead of practicing uniform pricing, firms had the ability to practice third-degree price discrimination (or group pricing), i.e., to charge active entrepreneurs (owners of firms that operate in the market) and workers different prices for their products. Since firms earn zero profits, the income of a worker is equal to the income of an entrepreneur, i.e., of an agent that has an ownership share in a firm in addition to earning a wage. Thus the ability of firms to price discriminate would be irrelevant in the long-run equilibrium; firms would charge the same price to an agent regardless of his type.

**3.1. Interior Short-Run Equilibrium**

We will now examine the interior short-run equilibrium in which the equilibrium wage is strictly larger than the lower bound (or the reservation wage) of zero, constituting an interior solution. For simplicity, in short-run equilibria we assume that firms practice third-degree price discrimination in that they are able to charge active entrepreneurs and
workers different prices for their products. Price discrimination allows us to bring out our argument in a clear and straightforward manner by demonstrating that under certain conditions neither sales to entrepreneurs nor sales to workers constitute a self-balancing mechanism for the economy. It also allows us to simplify our derivation of the equilibrium by studying markets with homogeneous customers. As is well-known, price discrimination is a common strategy in practice (e.g., Varian (1992), Mankiw and Taylor (2014)). In any case, as section 5.1 will explain, our findings largely carry through to uniform pricing.

Since we focus on the short run, the mass, $n$, of firms that operate in the market is exogenously fixed. Given its ability to price discriminate, a firm $i$ ($i \in [0,n]$) charges workers and entrepreneurs a price $p^W(i)$ and $p^E(i)$, respectively, for its good. Then, as we explain in the appendix (proof of lemma 2), in equilibrium all firms $i \in [0,n]$ charge the same price and sell the same output to each worker, i.e., $p^W(i) = p^W$ and $c^W(i) = c^W$, $\forall i \in [0,n]$. We can see in figure 2 that on the basis of the underlying utility-maximizing decisions of workers, profit maximization by firms (i.e., $p^W/w = \beta E^W/(c^W - 1)$) and a worker’s budget constraint (i.e., $p^W/w = 1/(nc^W)$) imply that each firm $i$ sells each worker a unique amount $c^W(n)$ in equilibrium. Each worker has a unique real wage $(w/p^W)*(n)$.

Furthermore, in equilibrium all firms $i \in [0,n]$ charge the same price and sell the same output to each entrepreneur, i.e., $p^E(i) = p^E$ and $c^E(i) = c^E$, $\forall i \in [0,n]$. However, we can see that on the basis of the underlying utility-maximizing decisions of entrepreneurs, profit maximization by firms (i.e., $p^E/w = \beta E^E/(c^E - 1)$) and an entrepreneur’s budget constraint (i.e., $p^E/(w+\Pi/\omega) = 1/(nc^E)$) imply that there may be multiple equilibria regarding firms’ sales to entrepreneurs, i.e., $c^E \in [c^W*(n),\tilde{c}^E]$, where $\tilde{c}^E = \min\{\hat{c}, c^W*(n) + (1-n\beta c^W*(n) - n\alpha)/(n\omega\beta)\}$. Thus profit maximization by firms and an entrepreneur’s budget constraint cannot pin down a unique amount that an entrepreneur consumes (unlike equilibrium sales to workers that are unique). As $p^E/w$
increases, a profit-maximizing firm is encouraged to sell an entrepreneur a larger amount according to \( p^E / w = \beta e^E / (e^E - 1) \) (similar to \( p^W / w = \beta e^W / (e^W - 1) \) in figure 2). However, at the same time a firm’s profit \( \Pi \), and thus an entrepreneur’s income \( w + \Pi / \omega \), also increases so that the entrepreneur can afford to purchase such a larger amount, which leads to multiple possible equilibria.

There is a lower and an upper bound to the equilibrium purchases by entrepreneurs that profit-maximization by firms and an entrepreneur’s budget constraint can justify. First, since an entrepreneur also earns a wage, and firms cannot earn a negative profit (i.e., they would rather refrain from production instead), the entrepreneur cannot consume less than a worker (i.e., \( c^E \geq c^W * (n) \)). Second, even if in its sales to entrepreneurs an individual firm encounters a price-to-marginal-cost markup, \( p^E / (\beta w) \), that approaches infinity, it optimally decides to sell an amount, \( \hat{c} \) (where \( \varepsilon = 1 \)), of its good (i.e., \( c^E \leq \hat{c} \)). Furthermore, the upper bound must not violate capacity constraints, i.e., \( c^E \leq c^W * (n) + (1 - n\beta c^W * (n) - n\alpha) / (n\omega\beta) \); the unit mass of agents must be sufficient for each firm to produce an amount \( c^E \) for each entrepreneur plus an amount \( c^W * (n) \) for each worker. Thus \( c^E \in [c^W * (n), \tilde{c}^E] \), where \( \tilde{c}^E = \min \{ \hat{c}, c^W * (n) + (1 - n\beta c^W * (n) - n\alpha) / (n\omega\beta) \} \).

However, in addition to profit maximization and entrepreneurs’ budget constraints, a necessary condition for an interior short-run equilibrium is the presence of full employment; otherwise, a strictly positive level of unemployment would drive the wage to a corner solution, i.e., to the lower bound (or the reservation wage) of zero. Thus the full-employment condition specifies a unique amount (in the feasible range \( c^E \in [c^W * (n), \tilde{c}^E] \)) that each entrepreneur consumes. In particular, each entrepreneur purchases a unique amount \( c^E * (n) = c^W * (n) + (1 - n\beta c^W * (n) - n\alpha) / (n\omega\beta) \) from each firm. Furthermore, when the mass of firms in the market is sufficiently small, — when \( n < n_0 \), where \( n \in (0, n_{LR}) \), — the full-employment condition is violated even if \( c^E \) takes the largest feasible value, i.e., even if \( c^E = \tilde{c}^E = \hat{c} \) (where \( \varepsilon = 1 \)). It follows that an
interior short-run equilibrium exists if and only if \( n \in [n, \infty) \). We can also see that since entrepreneurs consume more than workers \( c^E(n) \geq c^W(n) \) in equilibrium, less concentrated firm ownership expands the range in which an interior short-run equilibrium exists by facilitating the attainment of full-employment, i.e., \( \frac{\partial n}{\partial \omega} < 0 \). We summarize in lemma 2.

**Lemma 2:** There exists an interior short-run equilibrium if and only if the mass of firms in the market is sufficiently large (\( n \in [n, \infty) \), where \( n \in (0, n_{LR}^*) \)). In this equilibrium:

(i) Each worker has a unique real wage \( (w/p^W)^*(n) \), buying a unique amount \( c^W(n) \) from each firm, and the level of unemployment is zero, i.e., \( U^*(n) = 0 \).

(ii) Each entrepreneur buys a unique amount \( c^E(n) = c^W(n) + (1 - n\beta c^W(n) - n\alpha) / (n\rho\beta) \) from each firm.

(iii) Less concentrated firm ownership expands the range in which an interior short-run equilibrium exists, i.e., \( \frac{\partial n}{\partial \omega} < 0 \).

**Proof:** The proof is in the appendix.

### 3.2. Zero-Wage Short-Run Equilibrium

In this section we will examine the zero-wage short-run equilibrium, which allows us to demonstrate clearly our argument. Suppose that the wage is zero (or equal to the zero reservation wage), i.e., \( w = 0 \). Then, each worker has zero income and consumption, i.e., \( c^W(i) = 0 \), \( \forall i \in [0, n] \). Only entrepreneurs are in the market for goods. In such an equilibrium, all firms charge entrepreneurs the same price, i.e., \( p^E(i) = p^E \), \( \forall i \in [0, n] \). Furthermore, each firm \( i \) (\( \forall i \in [0, n] \)) sells each utility-maximizing

---

14 If \( n > n_{LR}^* \) (i.e., if the number of firms is strictly larger than in the long-run equilibrium), firms would earn strictly negative profits if they all operated. Thus \( n - n_{LR}^* \) firms would totally refrain from production, or would immediately exit, leaving \( n_{LR}^* \) active firms in the market. Then, \( (w/p^W)^*(n) = (w/p^W)^*(n_{LR}^*), \forall n \in [n_{LR}^*, \infty) \)

15 For simplicity, it is assumed that even if the wage is zero, a firm does not hire unnecessary workers of no use. For example, hiring a worker could entail a strictly positive infinitesimal cost for a firm.
entrepreneur the same equilibrium amount, \( \hat{c} \) (for which \( \varepsilon = 1 \)) of output, i.e., \( \hat{c}^x = \hat{c} \).

As we can see in the appendix (proof of proposition 1), there cannot exist multiple zero-wage short-run equilibria; if a zero-wage equilibrium exists, it is the unique short-run equilibrium subject to \( w = 0 \).

As we explain in the appendix, if and only if the mass of firms in the market is sufficiently small, — i.e., if and only if \( n \in (0, \bar{n}) \), — there exists a zero-wage equilibrium, which entails a strictly positive level of unemployment, \( \overline{U}^*(n) > 0 \). In such an equilibrium the excess supply of labor, — since labor supply minus labor demand is \( \overline{U}^*(n) > 0 \), — leads to a corner solution in the labor market in which the equilibrium wage is driven to the lower feasible bound, i.e., to the zero reservation wage. We have:

\[
\overline{U}^*(n) = 1 - (\alpha n + \beta n^2 \omega \hat{c}),
\]

\[
- \frac{n}{\omega} = -\alpha + (\alpha^2 + 4\beta \omega \hat{c})^{1/2},
\]

\( 2\beta \omega \hat{c} \) (6)

In the above corner solution unemployment \( \overline{U}^*(n) > 0 \) is (marginally) involuntary and corresponds to excess supply of labor; unemployed agents are denied employment although they would be strictly (albeit only marginally) willing to work for the equilibrium zero wage, which is equal to agents’ zero reservation wage.\(^{16}\) Although unemployment \( \overline{U}^*(n) > 0 \) is only marginally involuntary, it corresponds to disproportionately large social costs since it constitutes underutilized production capacity; firms fail to employ factors of production (i.e., unemployed labor) that are available for free in equilibrium and have a strictly positive marginal product \( 1/\beta \). Such inadequate aggregate demand for labor causes the economy to operate inside its production possibilities frontier. We can also see that \( \text{ceteris paribus} \) (for a given mass, \( n \), of firms) the equilibrium level of unemployment is larger when firm ownership is more concentrated, i.e., \( \partial \overline{U}^*(n) / \partial \omega < 0 \), \( \forall n \in (0, \bar{n}) \). In addition, greater ownership

\(^{16}\) See also note 12.
concentration exacerbates unemployment by increasing the unemployment threshold, $n$, and thus the range $[0, \overline{n})$ of the mass of firms that corresponds to strictly positive unemployment, i.e., $\partial \overline{n} / \partial \omega < 0$. We summarize in proposition 1.

**Proposition 1:** If and only if the mass of firms in the market is sufficiently small $(n \in (0, \overline{n}))$, there exists a zero-wage equilibrium (which is a unique equilibrium subject to $w = 0$) so that:

(i) The level of unemployment is strictly positive ($U^*(n) > 0$), the wage rate is zero ($\overline{w}^* = 0$), and each firm in the market sells each entrepreneur an output $c^*$.

(ii) More concentrated firm ownership exacerbates unemployment, i.e., $\partial U^*(n) / \partial \omega < 0$, $\forall n \in (0, \overline{n})$ and $\partial \overline{n} / \partial \omega < 0$.

**Proof:** The proof is in the appendix.

Intuitively, each individual firm has market power because of the presence of product differentiation. Then, if condition (3) is met, each firm may choose to produce a limited amount of its good even if its cost is zero; in this way, a firm takes full advantage of its market power. When the mass, $n$, of firms in the market is sufficiently small, which is a standard short-run irregularity on the supply side of the product market, the total output that is sold by firms to entrepreneurs is insufficient to warrant the employment of the entire unit mass of agents even if the wage is zero, and thus workers are available to firms for free. In equilibrium we have a corner solution in which there is excess supply of labor and strictly positive (marginally involuntary) unemployment, driving workers’ wage to its lower bound of zero; only entrepreneurs have strictly positive consumption. Such unemployment corresponds to disproportionately substantial

---

17 A good practical (partial equilibrium) example of firm behavior under such rather extreme conditions is the high-technology sector. As is well-known (e.g., Varian, Farrell and Shapiro (2004)), many technology-related companies have very small, or even zero, marginal costs at the production stage (although they have incurred large upfront fixed costs at the product development stage). However, despite their near-zero marginal costs, such companies often produce limited amounts of their products to take full advantage of their market power and maximize their profits.
social costs since it constitutes underutilized production capacity. Firms fail to employ workers whose marginal product, $1/\beta$, is strictly larger than the zero opportunity cost of labor (or the zero reservation wage), and the economy inefficiently operates inside its production possibilities frontier.

Furthermore, unemployment is exacerbated when firm ownership is more concentrated in that the group of entrepreneurs, who have strictly positive consumption in equilibrium, is smaller. Overall, the free entry of firms (as in lemma 1) is a necessary feature to ensure full employment by preventing shortfalls in the aggregate demand for labor.\textsuperscript{18} However, since the free entry of firms, which is a self-balancing mechanism for the economy, is a long-run feature, it cannot avert occasional occurrences of inadequate aggregate demand for labor and of underutilized production capacity in the short run (as in proposition 1).

\textit{Lemma 3:} In the short run:

(i) When $n \in (0, \bar{n})$, there exists a unique equilibrium, i.e., the zero-wage equilibrium.

(ii) When $n \in [\bar{n}, \bar{n})$, there exist two equilibria, i.e., the zero-wage equilibrium and the interior equilibrium

(iii) When $n \in [\bar{n}, \infty)$, there exists a unique equilibrium, i.e., is the interior equilibrium.

\textit{Proof:} The proof is in the appendix.

\textsuperscript{18} For example, the zero-wage equilibrium of proposition 1 cannot be a long-run equilibrium because the zero-profit condition is violated; the presence of strictly positive firm profits would encourage the entry of new firms into the market in the long run. The unique long-run equilibrium is described in lemma 1.
4. STRICTLY POSITIVE RESERVATION WAGE

Our results on the zero-wage short-run equilibrium carry through when the reservation wage is strictly positive, rather than zero, as long as it is not excessively high. For example, the real reservation wage may take the form \( w / \left[ \int_0^n p^W (i) di / n \right] \geq w / p^W \geq 0; \) a worker would be strictly willing to work if his average real wage, i.e., the ratio of his wage, \( w \), to the average price, \( \int_0^n p^W (i) di / n \), in the economy, is at least as large as \( w / p^W \). A worker would be unwilling to work for any real wage strictly lower than \( w / p^W \) (see note 12). \(^{19}\) We assume that when there is excess supply in the labor market (i.e., in corner solutions), workers, who compete for a limited number of job openings, commit that they will refund to their employer firm any savings they possibly attain if they purchase a good at a price strictly lower than the implicit reservation price, \( w / (w / p^W) \) (i.e., the highest acceptable average price stemming from the reservation wage, \( w / p^W \)). \(^{20}\) For simplicity, we also assume for the moment that \( w / p^W < (w / p^W)^*(n) \), i.e., the reservation wage is lower than the smallest possible real wage in the interior short-run equilibria of section 3. \(^{21}\) Later on in this section we will discuss the role of the size of the reservation wage. A strictly positive reservation wage

\(^{19}\) Similarly, an entrepreneur may be unwilling to work if his real overall pay \( (w + \Pi / \omega) / \left[ \int_0^n p^E (i) di / n \right] \) is strictly lower than \( w / p^E \), although the specific form of an entrepreneur’s willingness to work is immaterial to our results.

\(^{20}\) Such a commitment on the part of the workers is a simple device that allows us to incorporate into the model the downward pressure on real wages that excess labor supply exerts. We can thus derive a reservation-wage equilibrium in a straightforward manner. For example, according to such a commitment, if an agent purchases a quantity \( q_i \) of a good \( i \) at a price \( p_i < w / (w / p^W) \), he refunds to his employer firm an amount \( \left[ w / (w / p^W) - p_i \right] q_i \).

\(^{21}\) Since \( \partial (w / p^W)^*(n) / \partial n > 0 \) (proof of lemma 2), and an interior equilibrium exists for \( n \geq n \) (lemma 3), the smallest possible real wage in an interior equilibrium is \( (w / p^W)^*(n) \).
implies that equilibria with non-zero economy-wide production, which can support the reservation wage, exists only when \( n \geq n^0 \), where \( n^0 \in (0, n) \).

Suppose that the real wage is equal to the reservation wage, i.e.,
\[
\frac{w}{p^W} = \left[ \int_0^n p^W(i)di / n \right] = \frac{w}{p^W}.
\]
Then, as we can see in the appendix (proof of proposition 2), there is a corner solution in the workers’ market in which each firm charges workers a price \( \frac{w}{p^W} \), while each worker consumes an amount \( \left( \frac{w}{p^W} \right) / n \) of each good, i.e., \( p^* = \frac{w}{(w/p^W)} \) and \( c^*(n) = \left( \frac{w}{p^W} \right) / n \). Furthermore, similar to section 3.1, in the entrepreneurs’ market profit maximization by firms and entrepreneurs’ budget constraints do not specify a unique amount that an entrepreneur consumes; there exist multiple equilibria. In addition, unlike section 3.1, the full-employment condition, which can pin down a unique equilibrium in the entrepreneurs’ market, does not hold in the reservation-wage equilibrium (which constitutes a corner solution in the labor market). In any case, among the multiple equilibria in the entrepreneurs’ market the Pareto-superior equilibrium is the optimistic equilibrium where \( \hat{c^*} = \hat{c} \).

In the optimistic reservation-wage equilibrium a firm \( i \) sells each entrepreneur the maximum possible profit-maximizing amount, \( \hat{c} \), of output (for which \( \varepsilon = 1 \)), which is larger than in other reservation-wage equilibria. At the same time a unique amount, \( \hat{c^*(n)} = \left( \frac{w}{p^W} \right) / n \), is sold to each employed worker in all reservation-wage equilibria. Then, total economy-wide production and total employment are larger in the optimistic equilibrium than in other reservation-wage equilibria. It follows that in the optimistic equilibrium the utility of each agent is weakly, and possibly strictly, higher than in any other short-run reservation-wage equilibrium. We will focus on such an optimistic equilibrium (among the multiple reservation-wage equilibria) to bring out our

---

22 Since \( \hat{c}(w/p^W)^*(n)/\hat{c}n > 0 \) (see note 21), if \( n < n^0 \), the reservation wage cannot be supported in equilibrium. In particular, given that \( (w/p^W)^*(n) < \hat{c}w/p^W, \forall n \in [0, n^0] \), a real wage \( w/p^W \) is unattainable; each firm has an incentive to produce an amount strictly smaller than \( (w/p^W) / n \) (see figure 2). Thus workers are unwilling to work, and total production is zero.
results clearly and to show that our argument about the importance of the mass, \( n \), of active firms does not depend on agent expectations, i.e., it carries through even when agents are optimistic. Our results would be even stronger in more pessimistic reservation-wage equilibria (where \( c^* < \hat{c} \)). In any case, research in game theory indicates that in the presence of multiple Pareto-ranked equilibria, agents are often able to coordinate and reach the Pareto-superior equilibrium (e.g., Harsanyi and Selten (1988)).

As we explain in the appendix, if and only if the mass of firms in the market is viable \( (n \geq n^0) \) and sufficiently small \( (n < \bar{n}) \), i.e., if and only if \( n \in [n^0, \bar{n}) \), there exists an optimistic reservation-wage short-run equilibrium, which entails a strictly positive level of unemployment, \( U^*(n) > 0 \). In such an equilibrium the excess supply of labor, since labor supply minus labor demand is \( U^*(n) > 0 \), leads to a corner solution in the labor market in which the equilibrium real wage is driven to the lower feasible bound, i.e., to the reservation wage \( w/p^w \).23

Similar to section 3.2., unemployment, \( U^*(n) > 0 \), is (marginally) involuntary; unemployed agents are denied employment although they would be strictly (albeit only marginally) willing to work for the equilibrium real wage \( w/p^w \). However, although such unemployment is only marginally involuntary, it entails disproportionately substantial social costs. As is well-known, in monopolistic competition a worker’s equilibrium real wage is smaller than his marginal product since firms constrain their production to take full advantage of their market power in the product market; as, for example, condition (A16) implies, in the optimistic reservation-wage equilibrium, we have \( w/p^w < 1/\beta \). Thus equilibrium unemployment \( U^*(n) > 0 \) translates into underutilized production capacity since firms fail to employ idle labor that is able to produce strictly more than its reservation pay (or strictly more than the opportunity cost of labor). Such inadequate aggregate demand for labor implies that the economy operates inside its production possibilities frontier. Similar to section 3.2, we can also see that
greater ownership concentration exacerbates unemployment, i.e., \( \frac{dU^*(n)}{dn} > 0 \), \( \forall n \in [n^0, n^1] \), and \( \frac{dU^*(n)}{d\omega} < 0 \). We summarize in proposition 2. \(^{24}\)

**Proposition 2:** If and only if the mass of firms in the market is viable but sufficiently small \( (n \in [n^0, n^1]) \), there exists an optimistic reservation-wage equilibrium so that:

(i) A worker has a unique real wage \( \frac{w}{p^w} \), buying a unique amount \( c^w(n) = (w/p^w)/n \) from each firm, while an entrepreneur buys a unique amount \( \hat{c} \) from each firm.

(ii) The level of unemployment is strictly positive, i.e., \( U^*(n) > 0 \).

(iii) More concentrated firm ownership exacerbates unemployment, i.e., \( \frac{dU^*(n)}{dn} > 0 \), \( \forall n \in [n^0, n^1] \) and \( \frac{dU^*(n)}{d\omega} < 0 \).

**Proof:** The proof is in the appendix.

The intuition is similar to proposition 1. Each worker consumes a rather small amount, \( \frac{(w/p^w)}{n} \), of each good since his real wage has fallen to the reservation wage. Furthermore, to take full advantage of its market power, each firm chooses to sell entrepreneurs a limited amount of its good although production costs (i.e., real wages) are rather low. Thus when the mass, \( n \), of firms in the market is sufficiently small, the total output that is sold by firms is insufficient to warrant the employment of the entire unit mass of agents. In equilibrium we have a corner solution in which there is inadequate demand for labor and strictly positive (marginally involuntary) unemployment, driving workers’ real wage to its lower bound of the reservation wage. Such unemployment

\(^{23}\) Our results are even stronger if reservation-wage equilibria are more pessimistic. More pessimistic reservation-wage equilibria may exist when \( n < n' \), where \( n' > n \).

\(^{24}\) When the reservation wage is strictly positive, there also exists an extreme Pareto-inferior zero-production equilibrium in the short run and the long run. In particular, if each firm expects that no other firm will engage in strictly positive production, leading to a real wage strictly smaller than the reservation wage (since \( n^0 > 0 \)) and to workers that are unwilling to work, no firm will produce, and equilibrium total
entails disproportionately substantial social costs since it constitutes underutilized production capacity. Firms fail to employ workers that are able to produce strictly more than the opportunity cost of labor (the reservation wage), i.e., \( \frac{1}{\beta} > \frac{w}{p^w} \), and the economy inefficiently operates inside its production possibilities frontier. Overall, the free entry of firms in the long run (as in lemma 1) is a necessary feature to ensure full employment by preventing temporary shortfalls in the aggregate demand for labor.

We can also see that similar to lemma 3, when \( n \in [n^0, n] \), there exist only reservation-wage equilibria (such as the optimistic equilibrium). When \( n \in [n, \infty) \), there exist both the interior equilibrium of lemma 2 and the optimistic reservation-wage equilibrium (as well as other more pessimistic reservation-wage equilibria). When \( n \in (n, \infty) \), an interior equilibrium exists, while an optimistic reservation-wage equilibrium does not exist.

As we explained in section 3.1 (see figure 2), in an interior solution profit maximization by firms and a worker’s budget constraint lead to a unique real wage \((w/p^w)^*(n)\) for any \( n > 0 \). When the reservation wage, \( w/p^w \), is strictly larger than \((w/p^w)^*(n)\), production is not viable; as we can see in figure 2, each individual firm has an incentive to produce an amount of its good that is strictly smaller than \((w/p^w)/n\), which implies that a real wage \( w/p^w > (w/p^w)^*(n) \) is unattainable. Thus since the reservation wage cannot be supported in equilibrium, workers are unwilling to work, economy-wide production is zero, and unemployment is one. However, in an optimistic reservation-wage equilibrium if the reservation wage is in its viable range, it reduces (and often strictly reduces) unemployment as it increases. Since we will
examine the impact of changes in the reservation wage, we momentarily (for the purposes of proposition 3) relax the assumption that \( \frac{w}{p^W} < \left( \frac{w}{p^W} \right)^*(n) \).

When an interior short-run equilibrium does not exist (i.e., \( n \in (0, n) \)), an increase in the reservation wage in its viable range (\( \frac{w}{p^W} \in [0, (w/p^W)^*(n)] \)) always leads to strictly less unemployment, i.e., \( \frac{\partial U^*}{\partial \left( \frac{w}{p^W} \right)} < 0 \) (see figure 4). If, on the other hand, an interior short-run equilibrium coexists with the optimistic reservation wage-equilibrium (i.e., \( n \in [\underline{n}, \bar{n}) \)), an increase in the reservation wage leads to strictly less unemployment, i.e., \( \frac{\partial U^*}{\partial \left( \frac{w}{p^W} \right)} < 0 \), as long as \( \frac{w}{p^W} \in [0, (w/p^W)^*(n)] \), where \( (w/p^W)^*(n) \in (0, (w/p^W)^*(n)) \).

In the rest of the viable range (\( \frac{w}{p^W} \in ([w/p^W)^*(n), (w/p^W)^*(n)] \)), unemployment is always zero, and thus changes in the reservation wage do not affect unemployment, i.e., \( \frac{\partial U^*}{\partial \left( \frac{w}{p^W} \right)} = 0 \); in this case, the level of the reservation wage only affects the distribution of output between workers and entrepreneurs. We summarize in proposition 3.

**Proposition 3:** In an optimistic reservation-wage equilibrium unemployment is decreasing in the reservation wage as long as the latter is in the viable range, i.e., we have

\[
\frac{\partial U^*}{\partial \left( \frac{w}{p^W} \right)} < 0, \quad \text{if} \quad \frac{w}{p^W} \in [0, (w/p^W)^*(n)], \quad \forall n \in (0, n), \quad \text{or if} \quad \frac{w}{p^W} \in [0, (w/p^W)^*(n)], \quad \forall n \in [\underline{n}, \bar{n}) \quad \text{(where} \quad (w/p^W)^*(n) \in (0, (w/p^W)^*(n))) \quad \text{), and we have} \quad \frac{\partial U^*}{\partial \left( \frac{w}{p^W} \right)} = 0, \quad \text{if} \quad \frac{w}{p^W} \in ([w/p^W)^*(n), (w/p^W)^*(n)], \quad \forall n \in [\underline{n}, \bar{n}).
\]

**Proof:** The proof is in the appendix.

Intuitively, in a reservation-wage equilibrium a worker consumes a rather small amount of each good since his real wage has fallen to the lower bound; there is economy-wide unemployment and underutilized production capacity. Then, an increase in the reservation wage (in the viable range) boosts equilibrium consumption by workers,
increasing total production and aggregate demand for labor and decreasing equilibrium unemployment.

4.1. Extensive and Intensive Margin

Our analysis suggests that when there is a shift from the long-run equilibrium of section 3 to a reservation-wage equilibrium (or the reverse), there may sometimes be a substantial increase (or decrease in the reverse scenario) in unemployment even if the change in the mass, $n$, of firms in the market is only minor. An extreme example may illustrate our argument. In particular, suppose that $\omega \to 0$ so that $n \to n_{LR}^*$ (as condition (A11) implies). Then, given that no interior equilibrium exists for $n < n$, even an infinitesimal decrease in the mass of active firms when $n = n_{LR}^*$ necessarily leads to a reservation-wage equilibrium with substantial unemployment since $\overline{U}^*(n_{LR}^*) > 0$ (condition (A16)); the ratio of the percentage change in employment to the percentage change in the mass of active firms approaches infinity. As the real wage falls from $(w/p^w)^*(n_{LR}^*)$ (where equilibrium unemployment is zero) to its lower bound of the reservation wage, $w/p^w$, — or as consumption per worker falls from $c_{LR}^*$ to $(w/p^w)/n$, — unemployment increases abruptly although the change in $n$ is only infinitesimal ($\Delta n \to 0$, or $n \to n_{LR}^*$).

Overall, temporary changes in firm entry or exit constitute a driving force in our analysis, impacting economy-wide unemployment both directly through hiring by entering firms or firing by exiting firms and also indirectly by affecting the hiring (and production) decisions of continuing firms. The relative importance of the two channels depends on the specific parameter values. In any case, as we showed above, depending on the parameter values, the role of the latter (indirect) channel can sometimes be especially pronounced. Employment changes in the intensive margin can sometimes be substantially larger than (or even dwarf) employment changes in the extensive market (although the extensive margin is the underlying cause of all such changes in the strategies of firms in the intensive margin).
4.2. Social Welfare

Compared with the long-run equilibrium of lemma 1, when \( n < n_{LR}^* \), any short-run equilibrium strictly reduces the welfare of an employed worker. The real wage becomes strictly smaller since \( \partial (w / p'^w)(n) / \partial n > 0 \) (see proof of lemma 2) in interior short-run equilibria, and \( w / p'^w < (w / p'^w)(n) < (w / p'^w)(n_{LR}^*) \) in reservation-wage equilibria. Employed workers also have access to a strictly smaller range of available varieties \( (n < n_{LR}^*) \), which further decreases their welfare.\(^{29}\) In addition, in some equilibria, there is strictly positive unemployment, and unemployed workers have an equilibrium utility of zero. Moreover, the equilibrium utility of exiting entrepreneurs, or owners of firms that are no longer in the market (since \( n < n_{LR}^* \)), is strictly smaller. On the other hand, compared with the long-run equilibrium, the impact of an optimistic reservation-wage equilibrium on continuing entrepreneurs, or owners of firms that remain in the market, is ambiguous and depends on the specific parameter values. There are two opposing effects, namely, a reduction in available varieties (negative effect), and an increase in the consumption of each individual good, i.e., \( \hat{c} > c_{LR}^* \) (positive effect).

For example, we can see that there is a unique \( n' \in (0, n_{LR}^*) \) so that
\[
n'\hat{c} = n_{LR}^* c_{LR}^* \quad \text{(since } \partial (n\hat{c}) / \partial n = \hat{c} > 0, \text{ while } 0 < n_{LR}^* c_{LR}^* \text{ and } n_{LR}^* \hat{c} > n_{LR}^* c_{LR}^* \text{).}
\]
Then, a continuing entrepreneur is strictly worse off (better off) in an optimistic reservation-wage equilibrium than in a long-run equilibrium if the mass of firms in the market is \( n < n' \) (\( n > n' \)).\(^{30}\) Furthermore, independently of the specific parameter values, a continuing entrepreneur is always worse off (compared with the long-run equilibrium) in more pessimistic reservation-wage equilibria in which his consumption of each good is sufficiently close to the lower bound of the viable range \( c^E \in \left[ c^* = \left( w / p'^w \right) / \hat{c} \right] \), i.e., sufficiently close to \( (w / p'^w) / n \).

\(^{29}\) The welfare effects on workers may be more ambiguous in reservation-wage equilibria with \( n > n_{LR}^* \) (if such equilibria exist). However, even if such equilibria technically exist, they are rather unlikely to occur in practice. The temporary breakdown of the free-entry process, which caused the switch from the long run to the short run in the first place, seems likely to reduce, rather than increase, the number of firms.

\(^{30}\) Krugman (2016), among others, points to the possibility that recessions may sometimes be beneficial to owners of firms that operate in the market, while they are detrimental to labor.
5. GOVERNMENT POLICIES

In this section we will examine some possible government policies to combat downturns (although other policies in addition to the ones we discuss may also be relevant). We can see that if in the short run the economy reaches an optimistic reservation-wage equilibrium, a social planner can rather easily attain an outcome that is strictly Pareto-superior to the market outcome by simply increasing each firm’s output (so that idle production capacity is utilized). For example, a social planner may assign to each of the \( n \) firms an output target of \( (1-an)/(n\beta) \), which leads to the employment of the entire unit mass of agents. Such an output target implies that each firm produces an extra amount \( \bar{U}^*(n)/(n\beta) > 0 \) compared to its output in the optimistic reservation-wage equilibrium. If the social planner distributes this extra output among all agents, he is able to readily achieve a strict Pareto improvement compared with the market outcome. The argument is even stronger in more pessimistic reservation-wage equilibria.

Furthermore, in an optimistic reservation-wage equilibrium (or in any reservation-wage equilibria) the government may cause a weak Pareto-improvement by imposing an increase in the equilibrium real wage. For example, in practice, the government may introduce minimum wage laws or strengthen the bargaining power of unions. As far as the impact on unemployment is concerned, a government-imposed real wage floor is effectively identical to an increase in the reservation wage. It then follows from proposition 3 that the government is able to strictly increase the welfare of each employed worker by setting the minimum real wage equal to \( (w/\bar{p}^w)^*(n) \) if \( n \in (0,\hat{n}) \), or equal to \( (w/\bar{p}^w)^*(n) \) if \( n \in [\hat{n},\bar{n}) \); each employed worker now earns a strictly larger real wage. Since unemployment also strictly decreases, the mass of workers who enjoy the (now larger) gains from employment becomes strictly larger. Unemployment is even driven to zero if \( n \in [\hat{n},\bar{n}) \).

On the other hand, the welfare of entrepreneurs and of (the now smaller mass of) unemployed workers is unchanged since the former still consume an amount \( \hat{c} \), while the latter still consume a zero amount, of each good. Thus overall, the introduction of such a real wage floor by the government causes a weak Pareto-improvement. We can also see
that after the introduction of such government-imposed Pareto-improving real wage floors, unemployment is substantially, rather than only marginally, involuntary since the equilibrium pay of employed workers is substantially larger than the reservation wage.\footnote{Our analysis also implies that possible nominal rigidities that prevent the real wage from falling too much in economic downturns (Mankiw and Taylor (2014)) may be welfare-enhancing or Pareto-improving.}

As proposition 3 implies, if \( n \in (0, \underline{n}) \), although a government-imposed increase in the real wage reduces unemployment, it is not sufficient to drive unemployment to zero. However, even in this case, if the real wage floor is combined with appropriate fiscal policies, a further weak Pareto improvement can be attained; all workers can be allowed to derive the full benefits from employment (although they may not all be meaningfully employed). For example, there is a simple fiscal instrument that the government can utilize while operating on a balanced budget. In particular, the government may pay an unemployed worker the market wage \( w \) (which corresponds to a real wage \((w/ p^w)^* (n)\)) to perform a useless task and at the same time recoup such an expenditure by imposing a lump-sum tax \( w/n \) on each firm. The rather extreme assumption that unemployed workers are paid to perform useless tasks allows us to examine the controversial Keynesian argument that the government can cause an improvement in the economy even by paying workers to dig holes in the ground and then fill them, or, in general, by paying workers to perform useless tasks. Our argument would be even stronger if workers were engaged in useful projects.

We can see that such a government payment to each unemployed worker generates a mass \( \beta nc^w (n) /[1 – \beta nc^w (n)] \) of extra employment (see the proof of proposition 4 in the appendix). The government may maximize the impact of the policy by hiring a unique proportion \( s \) \( (s \in (0, \underline{U}^*(n)) \) of agents so that all agents are either employed by firms or (uselessly) by the government in equilibrium. At the same time, the lump-sum tax, \( sw/n \), on each firm, does not affect firm profit-maximizing behavior; profit-maximizing decisions based on marginal relationships, rather than on lump-sum parameters. Furthermore, newly hired agents (who now work either in firms or for the government) spend their income buying goods from firms. The new profit that each firm generates from newly hired agents is equal to the lump-sum tax that is pays (and which in
turn is used by the government to recoup the pay of government workers); firm profits are unchanged.

Overall, in an optimistic reservation-wage equilibrium compared with a simple government-imposed increase in the real wage to \((w/ p^W)^*(n)\) (if \(n \in (0, n)\)), the above balanced-budget fiscal policy (which is combined with a real wage increase) does not affect the welfare of entrepreneurs and of previously employed workers, who still consume an amount \(\hat{c}\) and \(c^W*(n)\), respectively, of each of the \(n\) goods. However, it strictly increases the welfare of previously unemployed workers, who now consume an amount \(c^W*(n)\) of each good (regardless of whether they are employed by firms or by the government). It thus causes a weak Pareto-improvement. The result also carries through to other more pessimistic equilibria. We summarize in proposition 4.

**Proposition 4:** In an optimistic reservation-wage equilibrium:

(i) A social planner can attain full employment and generate a strict Pareto-improvement on the market outcome by assigning to each firm an output target of \((1−an)/(n\beta)\).

(ii) A government can cause a weak Pareto-improvement by setting the minimum real wage equal to \((w/ p^W)^*(n)\) if \(n \in (0, n)\), or equal to \((w/ p^W)^*(n)\) if \(n \in [n, \bar{n})\).

(iii) When \(n \in (0, n)\), a government can cause a further weak Pareto-improvement if in addition to 4(ii), it pays the market wage to a unique proportion \(\hat{s} (\hat{s} \in (0, U^*(n))\) of agents (so that all agents are employed either by firms or by the government) to perform a useless task, while simultaneously imposing a lump-sum tax \(\hat{sw}/n\) on each firm.

**Proof:** It directly follows from the discussion above.

Intuitively, each firm produces a rather small amount of output to take full advantage of its market power, which may lead to inadequate demand for labor and

\[\text{---32---}\]

However, although such a fiscal policy (in proposition 4(iii)) leads to a weak Pareto-improvement on the market outcome, it is still strictly Pareto-inferior to the social planner’s strategy in proposition 4(i). Unlike proposition 4(i) that entails full employment, in proposition 4(iii) there is still underutilization of the economy’s production capacity (or a proportion \(\hat{s}\) of agents that are not employed meaningfully).
underutilized production capacity (propositions 1 and 2). Then, a government can mitigate such market failure and cause a weak Pareto-improvement by imposing, for example, an increase in the real wage. A higher real wage strictly reduces unemployment by boosting workers’ demand for goods and encouraging firms to produce more. Even when a government-imposed increase in the real wage fails to restore full employment, the government can boost demand for goods and total production further if in addition to the real wage increase, it hires, for example, unemployed workers to perform a useless task, while financing such hiring through a lump-sum tax on firms (maintaining a balanced government budget). Payments to unemployed workers raise total spending and production, while firm profits are unchanged; firm gains from selling to newly hired workers (by firms or by the government) exactly neutralize the lump-sum tax. Such a fiscal policy thus causes an overall weak Pareto-improvement.

In the discussion of fiscal policy in proposition 4(iii) we have made the most unfavorable assumption about the government’s ability to produce output and to employ workers meaningfully; the government simply pays agents to perform useless tasks. Such an assumption allows us to focus on the controversial Keynesian argument that in economic downturns it behooves the government to pay agents simply to dig holes and then fill them. We have seen than even in such an unfavorable setup government fiscal policy may cause an improvement in the economy. Of course, the case for active fiscal policy becomes stronger once we allow for productive government projects as, for example, in infrastructure.

5.1. Uniform Pricing

In the base model firms practice third-degree price discrimination (or group pricing) in the short run, setting different product prices for workers and entrepreneurs. Such an assumption allows us to reach a simple analytical solution and to present our argument in a straightforward manner since we study markets with homogeneous customers; we show that under certain circumstances neither sales to entrepreneurs nor sales to workers can be a self-balancing mechanism for the economy. Furthermore, in practice, price discrimination is often a standard strategy (e.g., Varian (1992), Mankiw and Taylor (2014)). In any case, our argument carries through to uniform pricing.
For example, the zero-wage equilibrium of section 3.2, which demonstrates clearly the logic of our argument, would remain identical if firms practiced uniform pricing, rather than third-degree price discrimination. Firms would charge all agents the price they charge entrepreneurs in section 3.2, and the equilibrium would be exactly the same; entrepreneurs and workers would still buy an amount \( \hat{c} \) and zero, respectively, of each good. Furthermore, in case the reservation wage is strictly positive, a possible equilibrium with uniform pricing entails workers earning the reservation wage \( \frac{w}{p^w} \) and buying an amount \( \frac{w}{p^w} / n \) of each good (in a corner solution) as in section 4. It also entails entrepreneurs buying an amount \( c^E \) of each good at the same price \( p = p^w \) as workers so that \( p / w = \beta \varepsilon / (\varepsilon - 1) \) (in an interior solution that meets profit-maximizing condition (A8b)). Since such conditions imply that \( c^E < \hat{c} \), this uniform-pricing equilibrium leads to less total production and larger unemployment than the optimistic reservation-wage equilibrium of section 4, making our argument even more pronounced.

Then, propositions 1 to 4 directly carry through to the above uniform-pricing equilibria. Furthermore, even in case other possible uniform-pricing reservation-wage equilibria also exist, the logic of our argument still applies. If the price elasticity of each firm’s market demand (which now includes both workers and entrepreneurs) is sufficiently decreasing in quantity, each firm may produce a small amount of its good even when the wage, or the cost of production, is low; total economy-wide output is then insufficient to warrant full employment if the number of active firms is small.

5.2. Non-Vertical Upward-Sloping Labor Supply Curve

In the base model agents have identical preferences for work (as well as for products), which is a standard assumption in monopolistic competition. Thus the labor supply curve is vertical (and equal to a unit mass of agents) when the real wage is weakly larger than \( \frac{w}{p^w} \) so that all agents are willing to work. There is zero labor supply when the real wage is strictly smaller than \( \frac{w}{p^w} \) so that the supply curve is discontinuous
(between a labor supply of zero and one) at \( w/p^W = w/p^W \). It follows that in corner solutions in which the demand for labor is in the range \((0,1)\), the economy effectively operates along the truncated horizontal bottom of the labor supply curve where \( w/p^W = w/p^W \) (exhibiting excess supply in equilibrium). Such a setup allows us to bring out clearly the mechanics of underutilized capacity and inadequate demand. Furthermore, in practice, the vertical upper part of the supply curve may be justified by the obvious physical constraints on the supply of labor, such as the size of the population and the bounded amount of time in a day. In addition, in microeconomics the supply curves of various goods are often truncated at the bottom; for example, the presence of a homogeneous group of agents (with identical reservation wages) at the bottom of the labor supply curve may lead to corner equilibria similar to our base model.

In any case, most of our results carry through if the labor supply curve is upward-sloping and non-vertical with respect to the real wage, i.e., if agents have varying, rather than identical, reservation wages. In particular, as in the base model, if the number of firms in the market is not too much smaller than in the long-run equilibrium, an interior short-run equilibrium exists with an interior real wage \((w/p^W)^*(n)\) (similar to lemma 2). If, on the other hand, the number of firms in the market is sufficiently small, downward pressure on wages leads to a corner solution where the equilibrium real wage, \((w/p^W)^U(n)\), is strictly smaller than the interior real wage, \((w/p^W)^*(n)\), even in the optimistic equilibrium (similar to the reservation-wage equilibrium of proposition 2). Then, the only difference from the base setup of section 4 is that in equilibrium the supply of labor is equal to the demand for labor (instead of having excess labor supply as in base setup); unemployment is voluntary, rather than marginally involuntary.

However, apart from the voluntary nature of unemployment, all the other results carry through. In particular, in the optimistic equilibrium (as well as in more pessimistic equilibria) there is underutilized capacity from a social standpoint since (as in models of monopolistic competition) the marginal product of labor, \(1/\beta\), is strictly larger than the real wage, \((w/p^W)^U(n)\). Given that there is available labor along the upward-sloping
supply curve, some workers that are unemployed in equilibrium would be willing to work for a real wage between \((w / p^W)^U(n)\) and \(1/\beta\), which is strictly smaller than workers’ marginal contribution, \(1/\beta\), to production. It follows that proposition 4 carries through; setting a minimum real wage higher than \((w / p^W)^U(n)\) (but not excessively high) and adopting as fiscal policy as in proposition 4(ii) causes a weak Pareto improvement. Similar to the base model, after such beneficial government interventions, some unemployment can be substantially involuntary.\(^{34}\) Furthermore, the virtues of less concentration in firm ownership carry through from the base model.\(^{35}\)

6. CONCLUSION

Despite their empirical importance, the notions of inadequate aggregate demand for labor and of underutilized production capacity, especially when they are not driven by nominal rigidities, have received relatively little attention in formal economic theory. I provide a simple general equilibrium industrial organization model of monopolistic competition to show that a possible cause of inadequate labor demand may be a standard short-run supply irregularity, namely, the temporary presence of a small number of firms in the market in the short run. The possession of market power by firms implies that if in the short run the number of firms is sufficiently small, total equilibrium output may be insufficient to warrant the employment of the economy’s entire labor force.

In such short-run corner equilibria, which are often unique, the economy underutilizes its production capacity and inefficiently operates inside its production possibilities frontier. Firms fail to utilize unemployed workers that would be able to produce strictly more than the opportunity cost of labor. Under those circumstances a natural self-balancing mechanism for the economy is the free entry of firms, which, however, occurs only in the long run. In the short run a government may reduce unemployment and generate a weak Pareto-improvement by imposing an increase in the

---

\(^{33}\) The labor supply curve may still become vertical above a certain real wage because of physical constraints (population size, bounded amount of time).

\(^{34}\) Furthermore, as in note 31, possible nominal rigidities may be welfare-enhancing or Pareto-improving.

\(^{35}\) If the labor supply curve is upward-sloping and non-vertical, it may sometimes be possible to have multiple long-run equilibria. This feature is not surprising since, as is well-known, imperfect competition may not always lead to first-best outcomes. In this paper we focus on the specific inefficiencies that stem
real wage (as long as the increase is not excessive) and by adopting demand-side fiscal policies that are effective even if the government operates on a balanced budget. Overall, our analysis makes a clear distinction between the long run and the short run that is consistent with basic microeconomic theory. At a macroeconomic level, those rudimentary microeconomic features may accommodate both the possibility of inadequate aggregate demand in the short run and of self-balancing in the long run.

from the presence of a small number of firms in the market in the short run (or on fluctuations around a long-run equilibrium), rather than on the general inefficiencies of imperfect competition.
REFERENCES


APPENDIX

**Proof of Lemma 1**

Each agent maximizes his utility function (1) subject to this budget constraint, which leads to the following first-order conditions:

\[
\frac{\partial u(c(i))}{\partial c(i)} = \lambda p(i), \ \forall i \in [0, n],
\]  

(A1)

where \( c(i) \) is the agent’s consumption of good \( i \), \( p(i) \) is the price of good \( i \), \( n \) is the mass of available varieties, and \( \lambda \) is the agent’s marginal utility of income (or his Lagrange multiplier).

Total production \( x(i) \) by a firm \( i \) is equal to the sum of the amounts of good \( i \) that the entire unit mass of individual agents consume, i.e., \( x(i) \) corresponds to a mass \( \int_0^1 c(i) zdz = c(i) \) of good \( i \). Then, condition (A1) implies that the demand curve of an individual firm \( i \) is

\[
p(i) = \lambda^{-1} \frac{\partial u(c(i))}{\partial c(i)}. \]

(A2)

Since there is a continuum of firms in the market, each firm’s pricing decision has a negligible effect on an agent’s marginal utility, \( \lambda \), of income. Then, taking \( \lambda \) as given in condition (A2), the price elasticity of demand, \( \frac{\partial(p(i)/c(i))}{\partial p(i)} \), that firm \( i \) faces is equal to \( \varepsilon(i) \), where \( \varepsilon(i) \) is defined in condition (2).

Each firm \( i \) aims to maximize its profit

\[
\max_{p(i)} \Pi(i) = p(i)c(i) - [\alpha + \beta c(i)]w. \]

(A3)

Thus, as is standard in microeconomic theory (e.g., Krugman (1979), Varian (1992)), profit-maximization implies that

\[
p(i) = \frac{\varepsilon(i)}{\varepsilon(i)-1} \beta w \Rightarrow \frac{p(i)}{w} = \frac{\beta \varepsilon(i)}{\varepsilon(i)-1}. \]

(A4)

Since \( \partial \varepsilon(i)/\partial c(i) < 0 \) (condition (3)), we have

\[
\frac{\partial[\varepsilon(i)/(\varepsilon(i)-1)]}{\partial c(i)} = -\frac{1}{(\varepsilon(i)-1)^2} \frac{\partial \varepsilon(i)}{\partial c(i)} > 0. \]

(A5)

We can see that in equilibrium all firms charge the same price and produce the same output, i.e., \( p(i) = p \) and \( c(i) = c \), \( \forall i \in [0, n] \). In particular, suppose that firm \( i \)
charges agents a strictly larger price than firm \( j \) \((i \neq j)\), i.e., \( p(i) > p(j) \). Then, profit maximization by the two firms (conditions (A4), (A5)) implies that \( c(i) > c(j) \). However, utility maximization by each agent (condition (A1)), implies that \( c(i) < c(j) \). It follows that there can be an equilibrium in which \( c(i) \) and \( c(j) \) meet both the profit-maximization and the utility-maximization conditions only when \( p(i) = p(j) = p \), \( c(i) = c(j) = c \) and \( \varepsilon(i) = \varepsilon(j) = \varepsilon \).

Furthermore, in the long-run equilibrium, another condition (in addition to (A4)) that is met is that each firm’s profit is zero, which implies that

\[
PC - (\alpha + \beta c)w = 0 \Rightarrow P = \beta + \frac{\alpha}{c}.
\]  

(A6)

Overall, in the long-run equilibrium, both conditions (A4) and (A6) are met (see figure 1), i.e., subtracting condition (A6) from condition (A4) is equal to zero. It follows that

\[
\frac{\beta \varepsilon}{\varepsilon - 1} - \beta - \frac{\alpha}{c} = 0.
\]  

(A7)

We can see that condition (A7) is a continuous and strictly increasing function of \( c \) in the range \( c \in [0, c^\ast] \). Furthermore, when \( c = 0 \), condition (A7) approaches \(-\infty\). When, on the other hand, when \( c = c^\ast \) (i.e., \( \varepsilon = 1 \)), condition (A7) approaches \( \infty \). It follows that there exists a unique \( c_{LR}^\ast \in (0, c^\ast) \) for which condition (A7) is equal to zero. In addition, the full-employment condition for the unit mass of agents leads to the equilibrium mass of firms in the market in condition (5). Lemma 1 directly follows.

**Proof of Lemma 2**

If a firm \( i \) earned a strictly larger profit than firm \( j \) \((i \neq j)\), firm \( j \) could strictly increase its profit by imitating the profit-maximizing strategy of firm \( i \), charging the same prices in the workers’ and the entrepreneurs’ market, i.e., \( p^W(j) = p^W(i) \) and \( p^E(j) = p^E(i) \). It follows that in equilibrium all firms earn the same profit, and thus each entrepreneur has the same income and the same marginal utility of income. By replacing \( \lambda \) with \( \lambda^W \), a worker’s utility-maximizing problem is described by condition (A1), while a worker’s demand curve is described by condition (A2). Furthermore, by replacing \( \lambda \) with \( \lambda^E \), an entrepreneur’s utility-maximizing problem is described by condition (A1), while an entrepreneur’s demand curve is described by condition (A2). Similar to the proof of lemma 1, in equilibrium all firms charge the same price and produce the same output in the workers’ market, i.e., \( p^W(i) = p^W \) and \( c^W(i) = c^W \), \( \forall i \in [0, n] \), as well as in the entrepreneurs’ market, i.e., \( p^E(i) = p^E \) and \( c^E(i) = c^E \), \( \forall i \in [0, n] \).

Then, similar to the proof of lemma 1, profit-maximization by firms in the workers’ and the entrepreneurs’ market implies that
\[
\begin{align*}
p^W_w &= \frac{\beta e^W_w}{e^W_w - 1}, \quad (A8a) \\
p^E_w &= \frac{\beta e^E_w}{e^E_w - 1}. \quad (A8b)
\end{align*}
\]

In addition, in equilibrium each worker and each entrepreneur meet their budget constraints:

\[
\begin{align*}
w = \int c^w w^w dz \Rightarrow p^w_w &= \frac{1}{nc^w_w}, \quad (A9a) \\
w + \frac{\Pi}{\omega} = \int c^E p^E_dz \Rightarrow p^E_w &= \frac{1}{nc^E_5}. \quad (A9b)
\end{align*}
\]

In the workers’ market equilibrium both conditions (A8a) and (A9a) are met (see figure 2), i.e., subtracting condition (A9a) from condition (A8a) is equal to zero. It follows that

\[
\frac{\beta e^W_w}{e^W_w - 1} - \frac{1}{nc^w_w} = 0. \quad (A10)
\]

We can see that condition (A10) is a continuous and strictly increasing function of \(c^W\) in the range \(c^W \in [0, \hat{c})\). Furthermore, when \(c^W = 0\), condition (A10) approaches \(-\infty\). When, on the other hand, when \(c^W = \hat{c}\) (i.e., \(c = 1\)), condition (A10) approaches \(+\infty\). It follows that there exists a unique \(c^W(n) \in (0, \hat{c})\) for which condition (A10) is equal to zero and thus a unique equilibrium real wage, \(w^r(n)\), which is defined by condition (A8a) or by condition (A9a) (by plugging in \(c^W(n)\)).

Suppose that a mass \(l\) of agents are employed by all firms. Then, when each firm sells each employed worker an amount \(c^W(n)\) and each entrepreneur an amount \(c^E\), it employs \(\alpha + \beta(1 - \omega)c^W(n) + \beta n\omega c^E\) agents, i.e., the mass of employed agents (by all firms) in the economy is \(n[\alpha + \beta(1 - \omega)c^W(n) + \beta n\omega c^E]\). Since the condition \(l = n[\alpha + \beta(1 - \omega)c^W(n) + \beta n\omega c^E]\) must be met (given the assumption that a mass \(l\) of agents are employed), we have a mass, \(l(n, c^E)\), of employed agents that is:

\[
l(n, c^E) = \frac{n[\alpha + \beta n\omega(c^E - c^W(n))] + 1 - n\beta c^W(n)}{1 - n\beta c^W(n)}. \quad (A11)
\]

It follows that a firm’s profit, \(\Pi(n, c^E)\), is:
\[ \Pi(n, c^E) = p^w c^w * (n) [l(n, c^E) - n\omega] + p^E c^E n\omega - \{\alpha + \beta c^w * (n) [l(n, c^E) - n\omega] + \beta c^E n\omega\} w \\
= p^w c^w * (n) [l(n, c^E) - n\omega] + p^E c^E n\omega - w l(n, c^E) / n \\
= [w / (n c^w * (n))] c^w * (n) [l(n, c^E) - n\omega] + p^E c^E n\omega - w l(n, c^E) / n \\
= -\omega + p^E c^E n\omega. \quad (A12) \]

Condition (A12) implies that an entrepreneur’s budget constraint is met for any feasible \( c^E \). In particular, the left-hand side, \( p^E / (w + \Pi / \omega) \), of condition (A9b) is equal to \( 1/(n c^E) \), which is thus always equal to the right-hand side, \( 1/(n c^E) \), for all feasible \( c^E \).

In conditions (A8b) and (A9b), the range of feasible \( c^E \) is \( c^E \in [c^w * (n), \tilde{c}^E] \), where \( \tilde{c}^E = \min \{c, c^w * (n) + (1 - n\beta c^w * (n) - n\alpha) / (n\omega\beta)\} \). Specifically, an entrepreneur also earns a wage and thus consumes weakly more than a worker (i.e., \( c^E \geq c^w * (n) \)). Furthermore, even if the price-to-marginal-cost markup that an individual firm encounters approaches infinity, i.e., \( p^E / w \rightarrow \infty \), the firm optimally sells an amount, \( \hat{c} \) (where \( \varepsilon = 1 \)), of its good, i.e., \( c^E \leq \hat{c} \). Finally, since the economy is populated by a unit mass of agents, it has a capacity constraint \( l(n, c^E) \leq 1 \), which implies that \( c^E \leq c^w * (n) + (1 - n\beta c^w * (n) - n\alpha) / (n\omega\beta) \). In addition, a necessary condition for an interior short-run equilibrium to exist is the presence of full employment, i.e., \( l(n, c^E) = 1 \); otherwise, a strictly positive level of unemployment would drive the wage to a corner solution, i.e., to the lower bound of zero. Thus the full-employment condition implies in an interior short-run equilibrium the unique amount (in the feasible range \( c^E \in [c^w * (n), \tilde{c}^E] \)) that each entrepreneur purchases from each firm is \( c^E * (n) = c^w * (n) + (1 - n\beta c^w * (n) - n\alpha) / (n\omega\beta) \).

Condition (A10) implies that \( \partial \hat{c}^w * (n) / \partial n < 0 \). Then, according to conditions (A8a) and (A5), the (unique) real wage rate is strictly increasing in \( n \), i.e., \( \partial (w / p^w) * (n) / \partial n > 0 \). Differentiating condition (A11) when \( c^E = \hat{c} \) leads to

\[
\frac{\partial l(n, \hat{c})}{\partial n} = \frac{\alpha + 2\beta n\omega (\hat{c} - c^w * (n)) - \beta n^2 \omega c^w * (n)}{1 - n\beta c^w * (n)} \frac{\hat{c} c^w * (n)}{l(n, \hat{c})} + n [\alpha + \beta n\omega (\hat{c} - c^w * (n))] \beta \frac{\partial (w / p^w) * (n)}{\partial n} > 0. \quad (A13) \]

Thus \( l(n, \hat{c}) \) is a continuous and strictly increasing function of \( n \). Furthermore, when \( n = 0 \), \( l(n, \hat{c}) \) is equal to zero (condition (A11)). When, on the other hand \( n = n_{LR} * \), \( l(n, \hat{c}) \) is strictly larger than 1 (since \( l(n_{LR} *, c^w * (n_{LR} *)) = 1 \) and, as the proof of lemma 1

---

36 See note 14.
explains, \( c^W \ast (n_{LR}) < \hat{c} \). It follows that there exists a unique \( n \in (0, n_{LR}) \) for which condition (A11) is equal to 1 when \( c^E = \hat{c} \). Thus \( \forall n \in [0, n) \), we have \( l(n, \hat{c}) < 1 \), and a interior short-run equilibrium does not exist; the full-employment condition is violated even if \( c^E \) takes the largest feasible value, i.e., even if \( c^E = \hat{c}^E = \hat{c} \). If, on the other hand, \( n \geq n \), we have \( l(n, c^E \ast (n)) = 1 \), and a unique interior short-run equilibrium always exists. As a result, an interior short-run equilibrium exists if and only if \( n \geq n \).

We can also see that

\[
\frac{\partial l(n, \hat{c})}{\partial \omega} = \beta n^2 (c - c^W \ast (n)) \frac{1}{1 - n \beta c^W \ast (n)} > 0.
\]  

(A14)

It directly follows that \( \frac{\partial n}{\partial \omega} < 0 \).

**Proof of Proposition 1**

Suppose that the wage rate is zero, i.e., \( w = 0 \), which implies that each worker has zero income and consumption, i.e., \( c^w (i) = 0 \), \( \forall i \in [0, n] \). Then, only entrepreneurs are in the market for goods.\(^{37}\) If a firm \( i \) earned a strictly larger profit than firm \( j \) \((i \neq j)\), firm \( j \) could strictly increase its profit by imitating the profit-maximizing strategy of firm \( i \), charging a price \( p(j) = p(i) \). It follows that in a zero-wage equilibrium all firms earn the same profit, and thus each entrepreneur has the same income and the same marginal utility of income. Each entrepreneur maximizes his utility function (1) subject to his budget constraint as in condition (A1). The demand that each firm \( i \) faces is defined by condition (A2), and total production by firm \( i \) is

\[
x(i) = \int_{0}^{n} c^E (i) dz = n \omega c^E (i).
\]

The profit maximization problem of each firm \( i \) is

\[
\max_{p^E(i)} \Pi(i) = n \omega p^E(i) c^E(i), \forall i \in [0, n].
\]  

(A15)

As is standard in microeconomic theory (e.g., Varian (1992)), \( p(i)c(i) \) is maximized when \( \varepsilon(i) = 1 \) (i.e., when \( c^E(i) = \hat{c} \)).

Thus if a zero-wage equilibrium exists, it is the unique equilibrium subject to \( w = 0 \); there cannot exist multiple zero-wage equilibria. In such an equilibrium, each firm \( i \) \((\forall i \in [0, n])\) sells each entrepreneur an output \( \hat{c} \), i.e., \( c^E(i) = \hat{c}^E = \hat{c} \) and produces a total output \( x(i) = \hat{x}^* = \int_{0}^{n} c d z = n \omega \hat{c} \), \( \forall i \in [0, n] \). Equilibrium employment is

\(^{37}\) An entrepreneur always has strictly positive income since a firm would sell its product to its own entrepreneurs (generating strictly positive profits for the firm and strictly positive income for the firm’s entrepreneurs) even if all the remaining firms made zero sales to entrepreneurs and workers.
\[ \bar{l}^*(n) = \int_0^n (\alpha + \beta n \omega) dz = \alpha n + \beta n^2 \omega, \] and equilibrium unemployment is

\[ \bar{U}^*(n) = 1 - \bar{l}^*(n) = 1 - (\alpha n + \beta n^2 \omega). \] There is strictly positive equilibrium unemployment, i.e., \( \bar{U}^*(n) > 0 \), if and only if \( n < \bar{n} = \left[ -\alpha + (\alpha^2 + 4\beta \omega)^{1/2} \right] / (2\beta \omega) \). It follows that if and only if \( n < \bar{n} \), the equilibrium wage is indeed zero, i.e., \( \bar{w}^* = 0 \); because of the excess supply of labor (i.e., labor supply minus labor demand is \( \bar{U}^*(n) > 0 \)), the labor market reaches a corner solution in which the wage is drive to its lower feasible bound (i.e., to the zero reservation wage). Thus if and only if \( n < \bar{n} \), the zero-wage equilibrium exists. We can also see that \( \partial \bar{U}^*(n) / \partial \omega = -\beta n^2 \omega < 0 \), which directly implies that \( \partial \bar{n} / \partial \omega < 0 \). Proposition 1 follows.

**Proof of Lemma 3**

We have

\[ \bar{l}^*(n) - l(n, \hat{c}) = -[n^2 \beta c^w * (n) (\alpha - \omega) + n^3 \beta^2 \omega c^w * (n)^2] / (1 - n \beta c^w * (n)) < 0, \] which implies that \( n < \bar{n} \). Then, lemma 3 follows directly from lemma 2 and proposition 1.

**Proof of Proposition 2**

Suppose that there is excess supply of labor in the economy, i.e., \( l < 1 \). Then, each worker commits that he will refund to his employer firm any savings he possibly attains if he purchases a good at a price strictly lower than the implicit reservation price, \( w / (w / p^w) \). Thus a firm \( i \ (i \in [0, n]) \) never sets its price \( p^w (i) \) strictly lower than \( w / (w / p^w) \) since the benefits from such a low price would not be passed on to workers (who would continue to face an actual price \( w / (w / p^w) \)), and workers would still purchase the same quantity of the good \( i \) (while firm \( i \) would obtain a smaller payment), i.e., we always have \( p^w (i) \geq w / (w / p^w) \). Furthermore, a firm \( i \) never sets its price \( p^w (i) \) strictly higher than \( w / (w / p^w) \) since the real wage would become strictly smaller than the reservation wage (given that \( p^w (j) \geq w / (w / p^w) \), \( \forall j \in [0, n] \)), making workers unwilling to work and leading to zero demand for good \( i \) (or any other good). In addition, such an excessively small real wage cannot be an equilibrium because the unwillingness of workers to work would put an upward pressure on the wage, \( w \). It follows that if \( l < 1 \), there is a unique (corner) equilibrium in the workers’ market in which \( p^w (i) = p^* = w / (w / p^w) \) and (following from the symmetric prices and condition (A1)) \( c^w (i) = c^* (n) = (w / p^w) / n \), \( \forall i \in [0, n] \).

Similar to the proof of lemma 2 there may be multiple equilibria in the entrepreneurs’ market (conditions (A8b), (A9b)). Furthermore, as in the proof of lemma 2, the maximum profit-maximizing amount that a firm can sell an entrepreneur is \( \hat{c} \).
Thus in the optimistic reservation-wage equilibrium each entrepreneur purchases a unique amount $c^w(n)$ from each firm.

Similar to condition (A11), when each firm sells each employed worker an amount $c^w(n)$ and each entrepreneur an amount $c^w(n)$, the mass of employed agents in equilibrium is:

$$l^w(n) = n[\alpha + \beta n\omega \hat{c} - c^w(n)] - n(\alpha + \beta n\omega \hat{c} - \beta \omega - p^w) \frac{1 - n\beta c^w(n)}{1 - \beta w/p^w}.$$  \hspace{1cm} (A16)

We can see that

$$l^w(n) - l^w(n) = -[n^2 \beta c^w(n)(\alpha - \omega) + n^2 \beta^2 \omega c^w(n)] / (1 - n\beta c^w(n)) < 0.$$  According to condition (A11), $\partial l(n, c) / \partial c^w(n) = n\beta(l - n\omega) / (1 - n\beta c^w(n)) > 0$. Since $c^w(n) = (w/p^w)/n < c^w(n) = (w/p^w)*(n)/n$, we have $l(n, c) - l^w(n) > 0$. Then, given that $l(n, c) = 1$ and $l^w(n) = 1$, there exists a unique $\bar{n} \in (n, \bar{n})$ so that $l^w(n) = 1$.

Since $\partial l^w(n) / \partial n > 0$ (as follows directly from condition (A16)), we can see that if and only if $n \in [n^0, \bar{n})$, the equilibrium real wage is the reservation wage, i.e., $w/p^w = w/p^w$; because of the excess supply of labor (i.e., labor supply minus labor demand is $U^*(n) = 1 - l^w(n) > 0$), the labor market reaches a corner solution in which the wage is drive to its lower feasible bound (i.e., to the reservation wage). Thus if and only if $n \in [n^0, \bar{n})$, the optimistic reservation-wage equilibrium indeed exists. We can also see that $\partial U^*(n) / \partial \omega = -\beta n^2(\hat{c} - c^w(n)) / (1 - n\beta c^w(n)) < 0$, which directly implies that $\partial n / \partial \omega < 0$. Proposition 2 follows.

**Proof of Proposition 3**

It follows from condition (A16) that $\partial U^*(n) / \partial (w/p^w) = -\beta l^w(n) - n\omega) / (1 - \beta w/p^w) < 0$, $\forall w/p^w \in [0, (w/p^w)^*(n)]$, $\forall n \in (0, \bar{n})$. When $n < n$, full employment is not attained even if $w/p^w = (w/p^w)^*(n)$ (since an interior short-run equilibrium does not exist). Suppose now that $n \in [n, \bar{n})$. Then, in an interior equilibrium full employment is attained with a real wage that is $(w/p^w)^*(n)$, and a level of consumption of each good by entrepreneurs that is $c^e(n) < c$ (lemma 2). Furthermore, since $n < \bar{n}$, in a zero-wage equilibrium we have $\bar{U}^*(n) > 0$. There thus exists a unique $(w/p^w)^*(n) \in (0, (w/p^w)^*(n))$ so that full employment is attained when the real wage is $(w/p^w)^*(n)$, and the level of consumption
of each good by entrepreneurs that is \( \hat{c} \). It then follows from condition (A16) that
\[
\frac{\partial U^*(n)}{\partial (w/p^w)} = -\beta(n^*(n) - n\omega)/(1 - \beta w/p^w) < 0, \quad \forall w/p^w \in [0, (w/p^w)^*(n)],
\]
\( \forall n \in [n, \bar{n}] \). Furthermore,
\[
\frac{\partial U^*(n)}{\partial (w/p^w)} = 0, \quad \forall w/p^w \in ((w/p^w)^*(n), (w/p^w)^*(n)) \cap [0, (w/p^w)^*(n)], \quad \forall n \in [n, \bar{n}]
\]
since unemployment is zero anyway in this range (only the distribution of output between workers and entrepreneurs changes as entrepreneurs consume less than \( \hat{c} \)). Proposition 3 follows.

**Proof of Proposition 4**

Proposition 4(i) and 4(ii) follow directly from the discussion in the main text (as well as from proposition 3). Suppose now that the government pays a mass \( s \) of agents the market wage to perform a useless task as in proposition 4(iii). By following the same procedure as in the derivation of condition (A11), we can see that
\[
\Pi(n, c^E) = \frac{n[\alpha + \beta n\omega(c^E - c^w*(n)) + \beta sc^w*(n)]}{1 - n\beta c^w*(n)}. \quad (A17)
\]
Thus because of such government payments to a mass \( s \) of agents, firms hire a mass \( s\beta n c^w*(n)/[1 - \beta n c^w*(n)] \) of previously unemployment workers. There is a unique level, \( \tilde{s} = [1 - l(n, \hat{c})][1 - \beta n c^w*(n)] \), of \( s \), for which each agent is employed either by firms or by the government, i.e., for which \( l(n, c^E, s) + s = 1 \).

Suppose now that the government imposes a lump-sum tax \( T = ws/n \) on each of the \( n \) firms to balance the government budget (which entails an amount \( ws \) of expenditures). Using the same procedure as in condition (A12) and substituting \( l(n, c^E, \tilde{s}) \) for \( l(n, c^E) \) we can see that
\[
\Pi(n, c^E) = -\omega w + p^E c^E n\omega + ws/n - T = -\omega w + p^E c^E n\omega \quad (as \ in \ condition \ (A12)).
\]
Thus the implementation of such a policy by the government does not affect firm profits, or an entrepreneur’s budget constraint in condition (A9b). Furthermore, since the policy does not impact conditions (A8a) and (A9a), the level of \( c^w*(n) \) remains the same. It follows that the fiscal policy of proposition 4(iii) leads to a weak Pareto improvement.
Table 1: Average Product Development Cycle Time (in Months) for Consumer Products. Source: Griffin (2002).

<table>
<thead>
<tr>
<th>Type</th>
<th>Time (in Months)</th>
</tr>
</thead>
<tbody>
<tr>
<td>New-to-the-World</td>
<td>29</td>
</tr>
<tr>
<td>New Product Line</td>
<td>21</td>
</tr>
<tr>
<td>Next Generation Improvements</td>
<td>15</td>
</tr>
<tr>
<td>Incremental Improvements</td>
<td>8</td>
</tr>
</tbody>
</table>

Figure 1: Long-Run Equilibrium.
\[ \frac{p^w}{w} = \frac{1}{nc^n} \quad \text{and} \quad \frac{p^w}{w} = \frac{\beta c^w}{c^n - 1} \]

**Figure 2:** Interior Short-Run Equilibrium for Workers.

Only the Zero-Wage Equilibrium Exists

Both Equilibria Exist

Only the Interior Equilibrium Exists

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \bar{n} )</th>
<th>( n )</th>
</tr>
</thead>
</table>

**Figure 3:** Existence of Short-Run Equilibria
Figure 4: Unemployment in the Optimistic Reservation-Wage Equilibrium for $n \in (0, n)$