On the Non-Linearity of the Fiscal Multipliers

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Abstract

The literature on fiscal multipliers has been implicitly assuming the multipliers to be linear and symmetric. Using two independent datasets and two different empirical approaches, we present evidence that the fiscal multiplier is larger for increases in government spending than for contractions of the same absolute value. Using a third different dataset, which considers only consolidation episodes, we show that the larger the consolidation the smaller the multiplier. We make sense of this result using a neoclassical, life-cycle, incomplete markets model calibrated to match key features of the US economy, including the distribution of income and wealth, social security, taxes and debt. We find the multiplier to be monotonically increasing in the size of the shock and therefore asymmetric effects are only significant for large shocks. The relation between the multiplier and the size of the shock is explained by the impact of the shock on the percentage of constrained agents: increases (decreases) in government spending cause less (more) agents to be borrowing constrained. These agents have a smaller labor supply elasticity, so the smaller (larger) the share of constrained agents, the larger (smaller) the fiscal multiplier.

Keywords: Fiscal Multipliers, Non-linearity, Asymmetry
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1 Introduction

During the 2008-2009 financial crisis, expansionary fiscal policies were adopted across a range of OECD countries to stimulate the economic activity. This period of fiscal expansion was followed by austerity measures, with the objective of reducing the deficits created. The literature, inspired by this period of active fiscal policy, has been debating on the size of the multiplier and its determinants, such as the state of the economy, income and wealth inequality, demography, tax progressivity, stage of development of a country, among others (see for example Auerbach and Gorodnichenko (2012), Ramey and Zubairy (2014), Brinca et al. (2016), Brinca et al. (2017), Hagedorn et al. (2016), Krueger et al. (2016), Basso and Rachedi (2017), Ferriere and Navarro (2014), Ilzetzki et al. (2013)).

However, so far, the literature has been treating the effects of government interventions linearly: contractionary and expansionary fiscal policy are assumed to have the same (symmetric) effect, and small and large shocks are assumed to have the same (linear) effect. However, has wealth and income inequality have been shown to be important determinants of the effects of fiscal policy, different fiscal shocks, in terms of the sign and size, may cause the wealth and income distributions to change, generating non-linear effects of fiscal policy. In this paper we show that relaxing the assumption of linear fiscal multipliers is important in assessing the effects of fiscal shocks.

We begin by empirically documenting that fiscal multipliers are sign and size dependent: expansionary fiscal policy generates larger multipliers than austerity measures and, considering only contractionary fiscal policies, the fiscal multiplier is decreasing in the size of the consolidation. We document these size and sign dependency of the multipliers by using three recent empirical papers, covering different time periods and countries and using different methods: Ramey and Zubairy (2014), Ilzetzki et al. (2013) and Alesina et al. (2015a).

To rationalize the empirical facts, we make use of a neoclassical, life-cycle, heterogeneous
agents and incomplete markets model. The mechanism we propose is through the effects of the fiscal shocks on the left tail of the wealth distribution: increases (decreases) in government spending cause less (more) agents to be borrowing constrained. These agents have a smaller elasticity of labor supply, so the smaller (larger) the share of constrained agents the larger (smaller) the fiscal multiplier.

We start our empirical analysis by using the data and method proposed by Ramey and Zubairy (2014). This study creates a new quarterly dataset for the U.S. economy extending back to 1889. The identification hypothesis for government spending consists on a combination of the military news variable proposed by Ramey (2011), which captures news about forthcoming variations in military spending, and the Blanchard and Perotti (2002) identification hypothesis, which assumes government spending does not react within the same quarter to macroeconomic variables. Using the Jordà (2005)’s Projection Method and pooling observations across high and low unemployment periods, the authors find no evidence of state dependent fiscal multiplier. We instead pool observations across periods with negative and positive fiscal shocks, to assess the asymmetry of the multiplier. We find evidence that the fiscal multiplier is quantitatively and statistically different across negative and positive shocks. The 1 year multiplier for positive shocks is 0.56, while for negative shocks is 0.20.

Our second empirical exercise consists on replicating the paper by Ilzetzki et al. (2013). Using a time series for 44 OECD countries and the Blanchard and Perotti (2002) SVAR approach, the authors find the fiscal multipliers to depend on factors such as stage of development of the country, exchange rate regime and others. Pooling observations across positive and negative shocks once more, we find the multiplier associated with positive variations in government consumption to be larger than the multiplier for negative variations.

For our third empirical exercise we use the fiscal consolidation episodes dataset from Alesina et al. (2015a). The dataset comprises 16 OECD countries over the 1981-2014 period. Alesina et al. (2015a) expand the dataset created by Devries et al. (2011), who use Romer and Romer (2010) narrative approach to identify exogeneous fiscal consolidations. Here, as
the dataset consists only on consolidation episodes, we are not looking for asymmetry but rather for non-linear effects of fiscal consolidations. We find fiscal multipliers to be decreasing in the size of the consolidation.

To make sense of this empirical results we develop a neoclassical life-cycle model, with heterogeneous agents and uninsurable idiosyncratic risk, similar to Brinca et al. (2016) and Brinca et al. (2017). We calibrate the model to match key feature of the U.S. economy, such as the income and wealth distribution, hours worked, taxes, social security among others. We then evaluate how the economy responds to different variations of government spending, in terms of sign and size. The positive G variations are financed through deficits, while negative G variations originate a surplus used to pay off debt.

In both cases, the mechanism through which the non-linearities of the fiscal multiplier arise is the percentage of constrained agents. In the case of reduced government spending, as government debt is paid down, the capital stock and consequently the marginal productivity of labor (wage) will increase, leading to a higher expected lifetime income. This will cause agents to borrow against their future income, increasing the percentage of agents constrained in the period following the shock. If we consider an increase in government spending, the opposite will happen: larger debt stock lead to lower marginal productivity of labor, making agents save more as their lifetime income is reduced, decreasing the percentage of constrained agents. In both situations, the larger the shocks the stronger the variation in the percentage of agents constrained.

As constrained agents behave like hand-to-mouth, they will adjust their consumption by the same amount as the change in their income, keeping their labor supply practically unchanged. So, as constrained agents have a smaller elasticity of labor supply, the smaller the share of constrained agents in an economy the larger the labor supply response and the larger the multiplier. As the number of constrained agents is decreasing in the size of the government spending variation, the fiscal multiplier will be increasing in the size of the shock.
In the case of small shocks, the variation in agents’ life-time income will be small and the percentage of agents constrained will not change, causing the multiplier to be locally symmetric. Although, for large shocks, agents will change their savings/consumption decisions, leading to variations in the percentage of agents constrained. The larger the size of the shocks, the larger the variations in the percentage of agents constrained, causing the fiscal multiplier asymmetry to be stronger.

The rest of the paper is organized as follows: Section 2 presents the empirical results, Section 3 introduces the model and Section 4 the calibration strategy for the model. In Section 5 results from model simulations are presented and Section 6 concludes.

2 Empirical findings

In this section, using three different datasets and three different methodologies, we provide evidence that the sign and the size of the fiscal shocks are important determinants of the fiscal multipliers. We start by using Ramey and Zubairy (2014) historical dataset for the US and Jordà (2005) Projection Method to show that a positive government spending shock yields larger multipliers than negative shock, with the same absolute value. We then provide further evidence of larger multipliers for positive government spending variations using Ilzetzki et al. (2013) dataset and a SVAR. Lastly, using Alesina et al. (2015a) dataset, which only considers consolidation episodes, we show that larger consolidations yield smaller multipliers.

2.1 U.S. historical data

To compare the multipliers across positive and negative fiscal shocks, a sufficiently large span of observations for both shocks is needed. Using U.S. quarterly historical data addresses this problem, as it provides us with enough observations for both shocks\footnote{255 observations for positive fiscal shocks and 249 observations for negative ones.}. Historical data also includes periods of slack, zero lower bound, different ways of conducting monetary and fiscal policy, which lead us to believe multipliers from historical sample can be informative.
To exploit the historical sample information, we use the quarterly data from 1889 to 2015 for the U.S. economy, constructed by Ramey and Zubairy (2014). The dataset includes real GDP, GDP deflator, government purchases, federal government receipts, population, unemployment rate, interest rates and defense news.².

To identify exogenous government spending shocks Ramey and Zubairy (2014) use two different approaches: 1) the defense news series proposed by Ramey (2011), which consists on using the narrative approach to identify exogenous variations in government spending linked to political and military events, as they are probably independent of the state of the economy; 2) Blanchard and Perotti (2002) identification hypothesis that government spending does not react to variations in GDP within the same quarter. Ramey and Zubairy (2014) argue that using both instruments together can bring advantages, as the Blanchard-Perotti shock is highly relevant in the short run, since it is the part of government spending not explained by lagged control variables, while defense news is more relevant in the long run, as the news happen several quarters before the spending actually occurs.

In figure 1 we can see the combination of both shocks since 1889 Q1 until 2015 Q4. The larger variations in the late years of the 1910 and 1940 decades are the defense spending associated with the two world wars. The small variations during the entire sample are mostly associated with the Blanchard-Perotti shock. As we can see, we have in our sample several periods of negative and positive fiscal shocks, which allow us to compare the multiplier across the two shocks.

²For more details on the dataset see Ramey and Zubairy (2014)
To test the multiplier for both negative and positive fiscal shocks, we use the same methodology as Ramey and Zubairy (2014), who follow Jordà (2005) Local Projection Method. This method consists on estimating the following equation for different time horizons $h$

$$y_{t+h} = I_{t-1}[\alpha_h + \Psi_h(L)z_{t-1} + \beta_h shock_t] + (1-I_{t-1})[\alpha_h + \Psi_h(L)z_{t-1} + \beta_h shock_t] + \epsilon_{t+h}, \ h = 0, 1, 2, \ldots$$

(1)

where $y$ is the real GDP per capita divided by the trend GDP, $z$ is a vector of control variables, including real per capita GDP, government spending and tax revenues, divided by trend GDP. $z$ also includes the news variable to control for any serial correlation. $\Psi_h(L)$ is a polynomial of order 4 in the lag operator and $shock$ is the exogenous shock, which is composed by a combination of the defense news and the Blanchard-Perotti government spending shock. $I$ is a dummy variable indicating if the shock is positive or negative.

Ramey and Zubairy (2014) follow a literature (see Mountford and Uhlig (2009), Uhlig (2010) and Fisher and Peters (2010)) that highlights that in a dynamic environment, the multiplier should not be calculated as the peak of the output response to the initial government spending variation but rather as the integral of the output variation to the integral.
of the government spending variation. This method has the advantage of measuring all the GDP gains in response to government spending variations in a given period. To calculate the cumulative multiplier directly from the equation Ramey and Zubairy (2014) propose the following instrumental variable estimation

$$
\sum_{j=0}^{h} g_{t+j} = I_{t-1} [\delta_{A,h} + \phi_{A,h}(L) z_{t-1} + m_{A,h} \sum_{j=0}^{h} g_{t+j}] + (1 - I_{t-1}) [\delta_{B,h} + \phi_{B,h}(L) z_{t-1} + m_{B,h} \sum_{j=0}^{h} g_{t+j}] + \epsilon_{t+h}, \ h = 0, 1, 2, \ldots
$$

where $shock_t$ is used as an instrument to $\sum_{j=0}^{h} g_{t+j}$, which is the sum of the government spending from t to t+h. This way, $m_{A,h}$ and $m_{B,h}$ can be directly interpreted as the cumulative multiplier at horizon h for the two states.

Results, presented in table 1, show that the multipliers are quantitatively different, with the multiplier associated with positive fiscal shocks being larger than the multiplier for negative shocks. Ramey and Zubairy (2014) argue that as the instrument includes the Blanchard-Perotti shock, which may be anticipated, it may raise some questions on the instrument relevance. So, to test if the multipliers are also statistically different across positive and negative fiscal shocks, the authors use Anderson et al. (1949) (AR) statistics, which is robust to weak instruments. As it is possible to see in the last column in table 1 the instruments are not only quantitatively different but also statistically.

<table>
<thead>
<tr>
<th></th>
<th>Linear</th>
<th>Negative shocks</th>
<th>Positive shocks</th>
<th>AR P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year cumulative multiplier</td>
<td>0.27</td>
<td>0.20</td>
<td>0.56</td>
<td>AR = 0.13</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.26)</td>
<td>(0.17)</td>
<td></td>
</tr>
<tr>
<td>2 year cumulative multiplier</td>
<td>0.45</td>
<td>0.30</td>
<td>0.66</td>
<td>AR = 0.06</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.24)</td>
<td>(0.11)</td>
<td></td>
</tr>
<tr>
<td>3 year cumulative multiplier</td>
<td>0.56</td>
<td>0.55</td>
<td>0.72</td>
<td>AR = 0.05</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.16)</td>
<td>(0.09)</td>
<td></td>
</tr>
<tr>
<td>4 year cumulative multiplier</td>
<td>0.58</td>
<td>0.68</td>
<td>0.73</td>
<td>AR = 0.07</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.12)</td>
<td>(0.08)</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Cumulative multipliers for 1, 2, 3 and 4 year horizons for positive and negative fiscal shocks.
2.2 SVAR

In this section we use the dataset and methodology proposed by ? to provide further evidence on the non-linearities of the fiscal multiplier. We run VARs for two groups of observations pooled across negative and positive variations in government spending. We find the results to be consistent with the ones from section 2.1, with positive variations in government spending generating larger multipliers than negative ones.

The objective is to estimate the following system of equations

\[ AY_{nt} = \sum_{k=1}^{K} C_k Y_{n,t-k} + u_{n,t} \]  

with \( Y_{nt} \) being a vector containing government consumption, output, current account in percentage of GDP and the natural logarithm of the real effective exchange rate, which are the same variables as the ones considered in Ilzetzki et al. (2013), for country \( n \) in quarter \( t \). \( C_k \) is a matrix of coefficients capturing the lag own and cross effects of variables on their current observations. The problem is that as \( A \) is not observable we cannot estimate this regression directly. So, we pre-multiply everything by \( A^{-1} \) and now we estimate the matrix \( P = A^{-1}C_k \) and \( e_{n,t} = A^{-1}u_{n,t} \) using OLS. So we estimate the system

\[ Y_{nt} = \sum_{k=1}^{K} A^{-1}C_k Y_{n,t-k} + A^{-1}u_{n,t} \]  

We still need to make further assumptions on \( A \) in order to identify the innovations \( e_{n,t} = A^{-1}u_{n,t} \) and correctly estimate the effects of variations in government spending. We use the assumption introduced by Blanchard and Perotti (2002) and used by Ilzetzki et al. (2013), which is that government consumption expenditures cannot react to output shocks within the same quarter. The reason behind this assumption is that the government budget is defined in an yearly basis and can only react to shocks in output with a lag. The ordering of the remaining variables is as in Ilzetzki et al. (2013): current account follows output and the real exchange rate follows the current account. With these assumptions, we can now
identify the effects of a shock in government spending.

The impulse response functions shown in Figure 2 suggest that positive government spending variation yields a positive and statistically different from zero multiplier, while the multiplier for the negative government expenditure variations is not statistically different from zero. These results are in accordance with the ones in section 2.1, with positive variations in government spending generating larger multipliers than negative ones.

2.3 IMF shocks

In this section we provide evidence that the non-linearities of the fiscal multiplier are not only related to the sign of the shock but also to the absolute variation. Using the Alesina et al. (2015a) annual dataset of fiscal consolidation episodes since 1978 until 2013 for 12 European countries, we show that the larger the consolidation the smaller the multiplier.

Alesina et al. (2015a) expand the original dataset from Devries et al. (2011) with exogenous fiscal consolidations episodes, known as IMF shocks. Devries et al. (2011) use Romer and Romer (2010) narrative approach to identify exogeneous fiscal consolidations, this means, consolidations driven uniquely by the desire to reduce budget deficits. The use of the narrative approach guarantees the exogeneity of the consolidations identified as it allows

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3Austria, Belgium, Germany, Denmark, Spain, Finland, France, United Kingdom, Ireland, Italy, Portugal and Sweden.
to filter out all policy actions driven by the economic cycle.

Despite expanding the dataset of Devries et al. (2011), Alesina et al. (2015a) use the methodological innovation introduced by Alesina et al. (2015b), who alert for the fact that a fiscal adjustment is rather a multiyear plan than an isolated change and consequently it results in policies that are implemented unexpectedly and other that are known in advanced. Ignoring the link between both expected and unexpected policies may yield biased results.

The way fiscal consolidations are defined by Alesina et al. (2015a) is: expenditure deviations relative to expenditures if no policy had been adopted and expected revenues changes of tax code revisions. Moreover, fiscal consolidations that were not implemented are not included in the dataset and so fiscal consolidation episodes included are assumed to be fully credible.

The estimated equation is the following

$$\Delta y_{i,t} = \alpha_i + \beta_1 e^u_{i,t} + \beta_2 (e^u_{i,t})^2 + \beta_3 e^a_{i,t} + \beta_4 (e^a_{i,t})^2$$

(5)

where $\Delta y_{i,t}$ is the output growth rate in country $i$ in year $t$, $e^u_{i,t}$ is unanticipated fiscal consolidation shocks and $e^a_{i,t}$ is anticipated fiscal consolidation shocks. We included the squared terms to capture the non-linear effects of fiscal shocks. We follow Alesina et al. (2015a) and estimate the equation using Seemingly Unrelated Regressions (SUR), imposing cross-country restrictions on the $\beta$ coefficients.

Results are presented in table 2 and validate our hypothesis that the non-linear effects of fiscal shocks are not only related to the sign of the shock, but also to the size. The coefficients associated with the linear terms of both announced and unexpected fiscal consolidations are negative, indicating that fiscal consolidations lead to a decrease in output. However, the coefficients of interest, $\beta_2$ and $\beta_4$, have a positive sign, meaning that the larger the consolidation the smaller is going to be the fiscal multiplier, even though only the coefficient associated with the squared term of announced fiscal consolidations is statistically significant. This coefficient is not only statistically significant but also economically meaningful, as an
increase in one standard deviation of announced consolidations leads to a decrease of 80% of the fiscal multiplier.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_1 )</td>
<td>-0.004** ( (0.002) )</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>0.001 ( (0.001) )</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>-0.024*** ( (0.002) )</td>
</tr>
<tr>
<td>( \beta_4 )</td>
<td>0.007*** ( (0.001) )</td>
</tr>
</tbody>
</table>

** *** \( p < 0.01 \), ** \( p < 0.05 \), * \( p < 0.1 \)

Table 2: Non-linear effects of fiscal consolidation shocks.

3 Model

In this section, we describe the model we will use to study the non-linear effects of fiscal policy. Our model follows closely Brinca et al. (2016) and Brinca et al. (2017).

**Technology**

We assume a standard Neoclassical production sector, where there is a representative firm production function given by a Cobb-Douglas:

\[
Y_t(K_t, L_t) = K_t^\alpha [L_t]^{1-\alpha}
\]  

(6)

where \( L_t \) is the labor input, measured in efficiency units, and \( K_t \) is the capital input. Capital law of motion is defined as

\[
K_{t+1} = (1 - \delta)K_t + I_t
\]  

(7)

where \( \delta \) is the capital depreciation rate and \( I_t \) is the gross investment. Firms choose labor and capital inputs each period in order to maximize their profits:

\[
\Pi_t = Y_t - w_t L_t - (r_t + \delta)K_t
\]  

(8)
Under a competitive equilibrium, the factor prices will pay their marginal products given by:

\[ w_t = \frac{\partial Y_t}{\partial L_t} = (1 - \alpha) \left( \frac{K_t}{L_t} \right)^\alpha \]  
(9)

\[ r_t = \frac{\partial Y_t}{\partial K_t} - \delta = \alpha \left( \frac{L_t}{K_t} \right)^{1-\alpha} - \delta \]  
(10)

**Demographics**

Our economy is populated by \( J \) overlapping generations households. Peterman and Sager (2016) highlights the importance of having a life-cycle economic when accessing the effects of government debt. Life of the household starts at age 20 and after retiring at age 65 households face an age-dependent probability of dying, \( \pi(j) \), dying with certainty at age 100. \( j \) is the household’s age and varies between 1 (for age 20 households) to 81 (for age 100 households). We define a period in the model to correspond to 1 year, so an household works for 40 years. We assume no population growth in our economy and we normalize the size of each new cohort to 1. \( \omega(j) = 1 - \pi(j) \) defines the age-dependent probability of surviving, and so, at any given period, making use of the law of large numbers, the mass of retired agents is equal to \( \Omega_j = \prod_{q=65}^{J-1} \omega(q) \).

Households also differ across persistent idiosyncratic productivity shocks, asset holdings, a discount factor assuming three distinct values \( \beta \in \{ \beta_1, \beta_2, \beta_3 \} \), which are uniformly distributed across agents, and also in terms of ability, which is the initial productivity conditions assigned when agents are born. Working age agents have to choose how much to work, \( n \), how much to consume, \( c \), and how much to save, \( k \) to maximize their utility. Retired households have consumption and saving decisions and receive a retirement benefit, \( \Psi_t \).

The stochastic survivability after retiring will imply that a share of households leave unintended bequests \( \Gamma \). We assume these bequests to be uniformly redistributed across the living households. In order to match the relative wealth of retired households to the average wealth of a working age household we assume retired households’ gain utility from leaving bequest when they die to other generations.
Labor Income

Household’s wage will depend on three different characteristics, which dictate the number of labor efficiency units each household is endowed in each period: age, $j$, permanent ability, $a \sim N(0, \sigma^2_a)$, and idiosyncratic productivity shock, $u$, which we assume follows an AR(1) process:

$$u' = \rho u + \epsilon, \quad \epsilon \sim N(0, \sigma^2_\epsilon) \tag{11}$$

Household’s wage will depend equally on the wage per efficiency unit of labor $w$, which is equal to the marginal productivity of labor. So, the individual $i$'s wage is defined according to:

$$w_i(j, a, u) = we^{\gamma_1 j + \gamma_2 j^2 + \gamma_3 j^3 + a + u} \tag{12}$$

$\gamma_1$, $\gamma_2$, and $\gamma_3$ are calibrated directly from the data to capture the age profile of wages.

Preferences

The utility of the households, $U(c, n)$, is increasing on consumption and decreasing on work hours, $n \in (0, 1]$, and is defined as:

$$U(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - \chi \frac{n^{1+\eta}}{1 + \eta} \tag{13}$$

Retired households utility function has one extra term, as they gain utility from the bequest they leave to living generations:

$$D(k) = \varphi \log(k) \tag{14}$$

Government

The social security system is runned by the government and is characterized a balanced budget. The revenues are collected from taxes on employees and on the representative firm at rates $\tau_{ss}$ and $\tilde{\tau}_{ss}$ respectively, and are used to pay retirement benefits, $\Psi_t$.

The government then taxes consumption, $\tau_c$ and capital, $\tau_k$, at flat rates. Labor income
tax follows a non-linear functional form as in Benabou (2002):

\[ y_a = 1 - \theta_0 y^{\theta_1} \] (15)

where \( \theta_0 \) and \( \theta_1 \) define the level and progressivity of the tax schedule respectively, \( y \) is the pre-tax labor income and \( y_a \) is the after tax labor income.  

Tax revenues from consumption, income and capital taxes are used to finance public consumption of goods, \( G_t \), public debt interest expenses, \( r B_t \), and lump sum transfers, \( g_t \).

Denoting social security revenues by \( R^{ss} \) and the other tax revenues as \( R \), the government budget constraint is defined as

\[ g \left( 45 + \sum_{j \geq 65} \Omega_j \right) = R - G - rB, \] (16)

\[ \Psi \left( \sum_{j \geq 65} \Omega_j \right) = R^{ss}. \] (17)

**Recursive Formulation of the Household Problem**

In a given period, a household is defined by his age, \( j \), his asset position \( k \), the time discount factor \( \beta \in \beta_1, \beta_2, \beta_3 \), his permanent ability \( a \) and the persistent idiosyncratic productivity shock \( u \). Given this, a working-age household chooses consumption, \( c \), work hours, \( n \), and future asset holdings, \( k' \), to solve his optimization problem. The problem can be formulated

\^[4]See the appendix for a more detailed discussion of the properties of this tax function
recursively as:

\[ V(k, \beta, a, u, j) = \max_{c,k',n} \left[ U(c, n) + \beta E_u \left[ V(k', \beta, a, u, j + 1) \right] \right] \]

s.t.:

\[ c(1 + \tau_c) + k' = (k + \Gamma) (1 + r(1 - \tau_k)) + g + Y_L \]

\[ Y_L = \frac{nw(j, a, u)}{1 + \tau_{ss}} \left( 1 - \tau_{ss} - \frac{nw(j, a, u)}{1 + \tau_{ss}} \right) \]

\[ n \in [0, 1], \quad k' \geq -b, \quad c > 0 \] (18)

where \( Y_L \) is the household’s labor income after the social security taxes, both on the employee and on the employer, and labor income taxes. The problem of a retired household differs on three dimensions: age dependent probability of dying \( \pi(j) \), the bequest motive \( D(k') \), and the labor income is replaced by the retirement benefits. So, the retired household’s problem is defined as:

\[ V(k, \beta, j) = \max_{c,k'} \left[ U(c, n) + \beta (1 - \pi(j))V(k', \beta, j + 1) + \pi(j)D(k') \right] \]

s.t.:

\[ c(1 + \tau_c) + k' = (k + \Gamma) (1 + r(1 - \tau_k)) + g + \Psi, \]

\[ k' \geq 0, \quad c > 0 \] (19)

\textit{Stationary Recursive Competitive Equilibrium}

Defining the mass of households with the corresponding characteristics as \( \Phi(k, \beta, a, u, j) \), the stationary recursive competitive equilibrium is defined by:

1. Taking the factor prices and the initial conditions as given, the value function \( V(k, \beta, a, u, j) \) and the policy functions, \( c(k, \beta, a, u, j) \), \( k'(k, \beta, a, u, j) \), and \( n(k, \beta, a, u, j) \) solve the consumers’ optimization problem.
2. Markets clear:

\[\begin{align*}
K + B &= \int kd\Phi \\
L &= \int (n(k, \beta, a, u, j)) d\Phi \\
\int c d\Phi + \delta K + G &= K^\alpha L^{1-\alpha}
\end{align*}\]

3. The factor prices are paid their marginal productivity:

\[\begin{align*}
w &= (1 - \alpha) \left(\frac{K}{L}\right)^\alpha \\
r &= \alpha \left(\frac{K}{L}\right)^{\alpha-1} - \delta
\end{align*}\]

4. The government budget balances:

\[g \int d\Phi + G + rB = \int \left(\tau_k r (k + \Gamma) + \tau_c c + n\tau_l \left(\frac{nw(a, u, j)}{1 + \tilde{\tau}_{ss}}\right)\right) d\Phi\]

5. The social security system balances:

\[\Psi \int_{j \geq 65} d\Phi = \tilde{\tau}_{ss} + \tau_{ss} \left(\int_{j < 65} \frac{nw d\Phi}{1 + \tilde{\tau}_{ss}}\right)\]

6. The assets of the dead are uniformly distributed among the living:

\[\Gamma \int \omega(j) d\Phi = \int (1 - \omega(j)) kd\Phi\]

**Fiscal Experiment and Transition**

The fiscal experiment that we analyze in this paper is a variation of the government spending \((G)\), by a percentage of GDP during 10 periods to pay off debt, in case of a decrease, and financed by deficits, in case of an increase. After the 10 periods the government spending goes back to the initial level and we assume the economy takes additional 90 periods to
converge to the new steady state equilibrium, with a different debt to GDP ratio.

The recursive competitive equilibrium along the transition between steady states, in the context of our experiment, is defined as:

For a given level of initial capital stock, initial distribution of households and initial taxes, respectively $K_0$, $\Phi_0$ and $\{\tau_l, \tau_c, \tau_k, \tau_{ss}, \tilde{\tau}_{ss}\}_{t=1}^{t=\infty}$, a competitive equilibrium is a sequence of individual functions for the household, $\{V_t, c_t, k'_t, n_t\}_{t=1}^{t=\infty}$, of production plans for the firm, $\{K_t, L_t\}_{t=1}^{t=\infty}$, factor prices, $\{r_t, w_t\}_{t=1}^{t=\infty}$, government transfers $\{g_t, \Psi_t, G_t\}_{t=1}^{t=\infty}$, government debt, $\{B_t\}_{t=1}^{t=\infty}$, inheritance from the dead, $\{\Gamma_t\}_{t=1}^{t=\infty}$, and of measures $\{\Phi_t\}_{t=1}^{t=\infty}$, such that for all $t$:

1. For given factor prices and initial conditions, the value function $V(k, \beta, a, u, j)$ and the policy functions, $c(k, \beta, a, u, j)$, $k'(k, \beta, a, u, j)$, and $n(k, \beta, a, u, j)$ solve the consumers’ optimization problem.

2. Markets clear:

$$K_{t+1} + B_t = \int k_t d\Phi_t$$

$$L_t = \int (n_t(k_t, \beta, a, u, j)) d\Phi_t$$

$$\int c_t d\Phi_t + K_{t+1} + G_t = (1 - \delta)K_t + K^\alpha L^{1-\alpha}$$

3. The factor prices are paid their marginal productivity:

$$w_t = (1 - \alpha) \left( \frac{K_t}{L_t} \right)^\alpha$$

$$r_t = \alpha \left( \frac{K_t}{L_t} \right)^{\alpha-1} - \delta$$

4. The government budget balances:

$$g_t \int d\Phi_t + G_t + r_t B_t = \int \left( \tau_k r_t (k_t + \Gamma_t) + \tau_c c_t + n_t \tau_t \left( \frac{n_t w_t (a, u, j)}{1 + \tilde{\tau}_{ss}} \right) \right) d\Phi_t + (B_{t+1} - B_t)$$
5. The social security system balances:

\[ \Psi_t \int_{j \geq 65} d\Phi_t = \frac{\tau_{ss}}{1 + \tau_{ss}} \left( \int_{j < 65} n_t w_t d\Phi_t \right) \]

6. The assets of the dead are uniformly distributed among the living:

\[ \Gamma_t \int \omega(j) d\Phi_t = \int (1 - \omega(j)) k_t d\Phi_t \]

7. Aggregate law of motion:

\[ \Phi_{t+1} = \Upsilon_t(\Phi_t) \]

4 Calibration

We calibrate our model to the U.S. economy. Some parameters are calibrated directly from empirical counterparts, while others are calibrated using Simulated Method of Moments (SMM), so that the model matches key features of the U.S. economy.

Wages

The wage profile through the life cycle (see equation 12) is calibrated directly from the data. We run the following regression, using data from the Luxembourg Income and Wealth Study.

\[ \ln(w_i) = \ln(w) + \gamma_1 j + \gamma_2 j^2 + \gamma_3 j^3 + \varepsilon_i \]  \hspace{1cm} (20)

where \( j \) is the age of individual \( i \).

To estimate the parameters \( \rho \) and \( \sigma_\varepsilon \) we use PSID data and run equation 20. We then use the residuals of the equation to estimate both parameters. \( \sigma_a \) is among the parameters that are calibrated using SMM. The variance of \( \ln(w) \) is the corresponding moment.
Preferences

There has been a considerable debate in the literature on the value of the Frisch elasticity of labor supply, \( \eta \), with estimates ranging from 0.5 to 2 or higher. We set it to 1.0, which is the same value as in Brinca et al. (2016) and Brinca et al. (2017). The utility of leaving bequest, disutility of work and the discount factors, \( \varphi, \chi, \beta_1, \beta_2 \) and \( \beta_3 \) respectively are among the parameters calibrated to match key moments in the data. The corresponding moments are the ratio of wealth owned by households in the age cohort 75-80 years old relative to an average household, share of hours worked and the three quartiles of the wealth distribution respectively.

Taxes and Social Security

As described before, to capture the progressivity of both the tax schedule and government transfers, we use the same labor income tax function as Benabou (2002) (equation 15). To estimate the parameters \( \theta_0 \) and \( \theta_1 \) of the U.S. economy we use OECD data on labor income tax and we estimate the equation for different family types. Then, we weight the value of each parameter by the weights of each family type on the overall population, so that we have a single parameter for the individual household in our model.

For the social security rates we assume no progressivity. Both social security tax rates, on behalf of the employer and on behalf of the employee, are set to 7.65%, using the value from the bracket covering most incomes. Finally, following Trabandt and Uhlig (2011), the consumption and capital tax rates are set respectively to 23.3% and 1.55%.

Parameters Calibrated Endogenously

As mentioned before, some parameters that do not have any direct empirical counterparts are calibrated using SMM. These parameters are \( \varphi, \beta_1, \beta_2, \beta_3, b, \chi \) and \( \sigma_a \), bequest motive, discount factors, borrowing limit, disutility from working and variance of ability respectively. The SMM is set so that it minimizes the following loss function:

\[
L(\varphi, \beta_1, \beta_2, \beta_3, b, \chi, \sigma_a) = ||M_m - M_d||
\]  (21)
with $M_m$ and $M_d$ being the moments in the model and in the data respectively.

We have seven parameters to calibrate endogenously, so we need seven data moments to have a system that is exactly identified. The seven moments we select in the data are the ratio of wealth owned by households in the age cohort 75-80 years old relative to an average household, share of hours worked, the three quartiles of the wealth distribution, the variance of log wages and capital to output ratio. Table 4 presents the calibrated parameters and Table 3 presents the calibration fit.

| Table 3: Calibration Fit |
|--------------------------|-----------------|-----------------|-----------------|
| Data Moment             | Description                                              | Source | Data Value | Model Value |
| 75-80/all               | Share of wealth owned by households aged 75-80            | LWS    | 1.51        | 1.51         |
| K/Y                     | Capital-output ratio                                      | PWT    | 3.073       | 3.073        |
| Var(ln w)               | Variance of log wages                                     | LIS    | 0.509       | 0.509        |
| $\bar{n}$               | Fraction of hours worked                                  | OECD   | 0.248       | 0.248        |
| $Q_{25}, Q_{50}, Q_{75}$ | Wealth Quartiles                                         | LWS    | -0.014, 0.004, 0.120 | -0.010, 0.000, 0.121 |

<table>
<thead>
<tr>
<th>Table 4: Parameters Calibrated Endogenously</th>
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<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Preferences</td>
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<tr>
<td>$\varphi$</td>
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<tr>
<td>$\beta_1, \beta_2, \beta_3$</td>
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<td>$\chi$</td>
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<tr>
<td>$b$</td>
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<td>$\sigma_\epsilon$</td>
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5 Results

In the context of our experiment, the variations in public debt will affect the stock of productive capital in the economy. A decrease in public debt will shift resources to the productive side of the economy, driving marginal productivity of labor up, which will cause hours worked to drop, leading to a recession. If we consider an expansion of the public debt, resources will flow from the productive side of the economy, leading to a wage drop and consequently an increase in hours worked, generating an expansion.
What we show in this section is that the fiscal multiplier will change with the size and the sign of the variation in \( G \), with the multiplier increasing in the shock. To rationalize this first note that a negative variation of \( G \) will send agents to the borrowing constraint, as agents expect higher future wages, they will borrow more against their future income, causing more agents to be constrained. For a positive variation of \( G \) the opposite will happen. Agents will internalize the decrease in their life-time income, diminishing their borrowing and consequently taking agents away from the borrowing constraint. As the constrained agents are hand-to-mouth, they will adjust consumption by the variation of their current income and their labor supply elasticity is close to zero. So, the larger the share of constrained agents, the smaller the labor supply response, causing the multiplier to shrink.

In the case of small shocks, the variation in agents life-time income will be small and the percentage of agents constrained will not change, causing the multiplier to be locally symmetric. Although, for large shocks, agents will change their savings/consumption decisions, leading to variations in the percentage of agents constrained. The larger the size of the shocks, the larger the variations in the percentage of agents constrained, causing the fiscal multiplier asymmetry to be stronger.

In figure 3 we can see the fiscal multipliers that result from the simulations. In the Y-axis we have the fiscal multiplier and in the X-axis we have the variation in government consumption as percentage of GDP. As it is possible to see in the figure, the multiplier is monotonically increasing in the government spending variation. Notice that for government spending variations close to zero, the multiplier is going to be locally symmetric. Although, when the shock starts to increase asymmetry starts to arise.

The transmission mechanism is illustrated in figures 4 and 5. In the first figure it is possible to see that the percentage of agents constrained is decreasing in the shock. Positive variations of government spending are associated with increases in public debt, which will make resources to flee from the productive side of the economy, reducing wages. Agents internalize the decrease in their life-time income and will reduce the borrowing consequently
Figure 3: Government spending variation and fiscal multiplier: In the X-axis we have the variations in G in percentage of GDP and in the Y-axis we have the fiscal multiplier. Blue line represents G contractions and red G expansions. The multipliers is increasing in the shock.

decreasing the percentage of agents that are credit constrained in the economy. For negative G variations, the opposite will happen. Agents internalize the increase in their life-time income and will borrow against their future income, leading to an increase in the percentage of credit constrained individuals. These variations in the percentage of agents constrained are going to affect the fiscal multiplier as the constrained agents have a labor supply less elastic.

Figure 4: Government spending variation and percentage of constrained agents: In the X-axis we have the variation in G in percentage of GDP and in the Y-axis we have the percentage of credit constrained agents in the period following the shock. Blue line represents G contractions and red G expansions. The percentage of credit constrained agents is decreasing in the shock.

In figure 5 we have the labor supply response for constrained agents (blue line in both
Figure 5: Government spending variation and labor supply response: In the X-axis we have the $G$ variation in percentage of GDP and in the Y-axis we have labor supply response to the shock. In the left panel we have positive government spending variations and in the left panel negative ones. Blue lines in both panels represent the labor supply response of constrained agents. Dark lines represent the labor supply response of unconstrained agents, in both panels. The red line in the left panel represent the labor supply response of agents that were constrained before the shock but stop being constrained after the shock. In the right, the red line represents the labor supply response of agents that were not constrained before the shock and become constrained with the shock. Constrained agents are the ones with less elastic labor supply. As agents move to or away from the borrowing constrained, the labor supply response will change and the multiplier will change.

6 Conclusion

In this paper, we provide empirical evidence on the non-linear effects of fiscal policy, both in terms of sign and size, using three different datasets and methodologies. We start by using historical data for the U.S. and Jorda’s Projection Method to illustrate that positive variations of government spending yield larger multipliers than negative ones. We then use a dataset for 44 OECD countries and run VARs for two groups of observations polled across

panels), agents that are never constrained (dark line in both panels) and agents that change from constrained to unconstrained (red line left panel) or the other way around (red line right panel). It is possible to see that for positive $G$ variations (left panel) there are agents that stop being constrained, going from the blue to the red line, and their labor supply is now more elastic, causing the multiplier to be larger. For negative $G$ variations (right panel) there are agents becoming constrained, so moving from the dark line to the red line, and their labor supply is becoming less elastic to the shock, decreasing the multiplier.
positive and negative variations in government spending. The results are consistent with the results from the U.S. historical dataset. The last piece of empirical evidence consists on a dataset considering only fiscal consolidations exogenous to the economic cycle, and we show that the larger the consolidation the smaller the multiplier.

To make sense of the empirical results we develop a life-cycle, overlapping generations model with heterogeneous agents and uninsurable idiosyncratic risk. The model is calibrated to match key characteristics of the U.S. economy. Running simulations of positive and negative variations in government spending we match the empirical results. Moreover, we find that it is the impact that the fiscal shock has on the percentage of agents that are credit constrained that drives the result. Positive variations in government spending will result in a smaller share of constrained agents. As government spending is financed through deficit, lifetime income will decrease, making agents decrease their borrowing and consequently yielding a smaller share of constrained agents. On the opposite, negative variations in government spending will be used to pay off debt, increasing wages and resulting in more agents borrowing against their future income, yielding a larger share of constrained agents. The impact on the share of credit constrained agents is essential to determine the fiscal multiplier as these agents have a labor supply less elastic, so, the larger the share of constrained the smaller the labor supply response and consequently the smaller the multiplier.
References


