Abstract

There is a view that economic shocks and recessions may be beneficial for climate change in that reduced economic output reduces carbon emissions. However, it is possible that the transition away from carbon based energy infrastructure may require a healthy economy. In this paper, production involves energy from both fossil fuels and green technology. The efficacy of new investments in fossil fuel capital is decreasing over time while it is constant for green technology, reflecting the depletion of these fossil resources and the relative improvement in alternative technologies. The optimal path of investment therefore features a switch from investment in fossil fuel capital to alternative energy capital, when the productivity of the former drops below a certain level. First, we embed this framework into a Ramsey-style growth model, and show that consumption smoothing means that a sudden decrease in the availability of fossil fuel capital at a given productivity can delay the transition towards the alternative energy infrastructure. Second, we assume that investments require access to funding, and that funding renewable energy projects is deemed to be a riskier investment than the funding of conventional energy facilities. We show that a negative shock to borrower net worth can push credit towards the lower risk projects, further delaying the transition to the alternative energy infrastructure. This is the so-called Energy Trap and this is the first time it has been analysed formally in a theoretical model.

Keywords: energy trap, consumption smoothing, financial frictions, volatile depletion.

JEL Classification: Q43, E22.
1 Introduction

There is a view that economic shocks and recessions may be beneficial for climate change in that reduced economic output reduces carbon emissions. For example, according to The Global Carbon Atlas\textsuperscript{1}, global emissions of carbon fell from 32,844 MtCO\(_2\) in 2008, to 32,507 MtCO\(_2\) in 2009, as the financial crisis hit (before resuming their growth as the global economy recovered). However, it is possible that the transition away from carbon based energy infrastructure may require a healthy economy.

As the world economy grew and adapted to use more energy resources over the twentieth century, production of fossil fuels expanded. The easiest to extract resources were exploited first, and as time has gone on, more marginal resources have been brought into production. For example, oil used to feature an “energy-return-on-energy-invested” (EROI) of close to 100:1, but current oil production exhibits a much lower EROI, and fossil fuel resources that are uneconomic to extract given current technology have an EROI which is lower still. Assuming the trend to ever lower quality fossil fuels continues, it seems reasonable that at some stage, investment in other energy technologies comes to dominate fossil fuel investment. Such a transition is necessary if we are to see a zero carbon world.

However, fossil fuels have a number of desirable features relative to zero carbon alternatives. One such advantage is that the costs of alternatives tend to be up-front capital costs, whereas the costs of fossil fuel energy supply are spread more evenly over the lifetime of the asset due to much higher on-going fuel costs. This means that for two projects delivering the same energy supply and having the same net present value, the fossil fuel project will have a lower duration, lower initial costs, and higher internal rate of return.

Given this desirable feature of fossil fuels, an unforeseen shock to global output could induce agents to invest in more fossil fuels, even if in the absence of the shock they preferred to invest in alternative energy. This raises the spectre of what Murphy (2011) calls the “Energy Trap”.

In a first exercise, we model the Energy Trap in a modified Ramsey growth model, where a continuum of infinitely lived households choose levels of consumption and investment in fossil fuel and green-tech capital to maximize their lifetime utility. Utility comes from the consumption of a final good, which is produced by a continuum of competitive firms using a constant returns to scale technology which combines labour and energy. Energy is an aggregate input that can be produced using fossil fuel, and/or green-technology. In the deterministic version of the model, the solution path involves switching from investment in fossil fuel capacity to alternative green-tech when the productivity of the former drops below a threshold level. We use the deterministic framework to show the impact of different initial values for the state variables, which could represent the value of these variables “after a shock”. The arrival of a shock means that the household suddenly has fewer resources at a given productivity than they thought and we analyse how this affects the relative desirability of new fossil and alternative investments. Expected declines in productivity of the fossil fuel incentivise investment in initially lower productivity alternative infrastructure. However, a realised shock makes the representative agent poorer, partly discouraging these investments. This is the Energy Trap: realised shocks hurt the process of transitioning to alternative energy infrastructure, because the optimal transition point is a function of marginal utility levels.

Secondly, we incorporate the financial market imperfections of Matsuyama (2007) which compound

\textsuperscript{1}globalcarbonatlas.org
this story on the composition of investment: “A rise in borrower net worth not only eases the borrowing constraint, but it may also cause the composition of the credit to shift towards more productive projects”. In this context, a negative energy shock lowers borrower net worth, and causes a further shift towards lower productivity fossil fuel investments.

2 Investment in Energy Supply

The 2008 financial crisis hit net worth levels, and we see a fall in investment in renewables but a rise in investment in fossil fuels.

![Figure 1: Global investment in fossil fuel supply](https://www.nakedcapitalism.com/2014/07/will-fossil-fuel-subprime-cycle.html)

Obviously want to do a lot more on this, collect some proper data, and do a proper empirical analysis...

There could be other explanations for what we see here, e.g. cutbacks on government support for renewables in response to fiscal pressures; but in this paper we propose an explanation for this pattern whereby it is an optimal response given a particular feature of fossil fuel projects relative to alternative energy projects.

3 Model

We want to model the fact that fossil fuel projects involve lower up-front costs but higher on-going (fuel) costs than a zero-carbon energy project of identical productivity.

Suppose, given a discount rate \( r \), there are two projects, \( i \in \{ F, G \} \), where \( F \) denotes “Fossil” and \( G \) denotes “Green-tech”, which both create an asset with the same lifetime, \( T \), which yield the same income, \( \text{Inc} \), from the supply of energy sold, and which both have the the same net present value, \( \text{NPV} \). Then:
\[ NPV = NPV^i = \sum_{t=1}^{T} \frac{CF^i_t}{(1 + r)^t} \]
\[ CF^i_1 = -Capital\_Costs^i \]
\[ CF^i_{t>1} = Inc - OnGoing\_Costs^i \]
\[ Capital\_Costs^F < Capital\_Costs^G \]
\[ OnGoing\_Costs^F > OnGoing\_Costs^G \]

Then it is easy to show that the duration of the fossil fuel asset is less than the duration of the green-tech asset i.e.

\[ Dur^F = \sum_{t=1}^{T} \frac{tCF^F_t}{(1 + r)^t} < \sum_{t=1}^{T} \frac{tCF^G_t}{(1 + r)^t} = Dur^G \]

We would like to capture this feature of fossil fuel projects relative to alternative energy projects in a macroeconomic model. Here we typically we model capital accumulation using the formulation:

\[ K_{t+1} = I_t + (1 - \delta)K_t \]

Assuming a “price” of one unit of this capital stock (in terms of consumption goods forgone to create it) of \(1/D\), a constant rental rate of capital, \(r\), equal to the discount rate, then then value created by investing one unit of final goods in creating this capital stock is:

\[ D \frac{r}{1 + r} + D (1 - \delta) \frac{r}{(1 + r)^2} + D (1 - \delta)^2 \frac{r}{(1 + r)^3} + ... = \frac{rD}{1 + r} \sum_{t=1}^{\infty} \left( \frac{1 - \delta}{1 + r} \right)^{t-1} \]
Clearly, for a given value, the investment productivity, $D$ and the depreciation rate, $\delta$, are positively related. Therefore we can imagine two assets which, for a given interest rate, are valued the same, one with high $D$ and high $\delta$, the other with low $D$ and low $\delta$.

The duration of the asset is:

\[
D_t = \frac{r}{1 + r} + 2D_t (1 - \delta) \frac{r}{(1 + r)^2} + 3D_t (1 - \delta)^2 \frac{r}{(1 + r)^3} + \ldots = \frac{rD_t}{1 + r} \sum_{t=1}^{\infty} t \left( \frac{1 - \delta}{1 + r} \right)^{t-1}
\]

Again it is straightforward to show that if two assets are valued the same, one with high $D$ and high $\delta$, the other with low $D$ and low $\delta$, then the high $D$ and high $\delta$ asset will have lower duration than the low $D$ and low $\delta$ asset.

These then are the “microfoundations” of our model of the Energy Trap. We imagine time is discrete and indexed by $t = 0, 1, 2, \ldots, \infty$. The economy produces a single final good $Y$ through an identical constant return to scale (CRS) technology,

\[
Y_t = F(E_t, L_t) = E_t^\alpha L_t^{1-\alpha}
\]

where $E_t$ is energy and $L_t$ is labour. This production function satisfies the usual assumptions and we assume $\alpha \in (0, 1)$.

Energy is an aggregate input, which is assumed to be the sum of the energy produced using fossil fuel, $E^F_t$, and green-technology, $E^G_t$,

\[
E_t = E^F_t + E^G_t.
\]

Each type of energy is produced using a type of capital, $K^i_t$, according to $E^i_t = K^i_t$ for $i = \{F, G\}$. Both types of capital, $K^i_t$, in the economy at time $t$ depreciates at rate $\delta^i \in (0, 1)$, with $\delta^F \geq \delta^G$. This captures the shorter duration of fossil fuel assets compared to alternative energy assets.

The efficacy of new high carbon capital investment, $D^F_t$, is initially high, but is falling over time at rate $\Delta$,

\[
D^F_{t+1} = (1 - \Delta) D^F_t.
\]

Thus, the level of fossil fuel capital evolves according to

\[
K^F_{t+1} = D^F_t I^F_t + (1 - \delta^F) K^F_t
\]

where $I^F_t \geq 0$ is the investment in fossil fuel capital. On the contrary, the productivity of green-tech capital investment is assumed constant and is normalized to one, $D^G_t \equiv 1$ for all $t$, so that green-tech capital, $K^G_t$, evolves according to

\[
K^G_{t+1} = I^G_t + (1 - \delta^G) K^G_t.
\]

In each period, there is a continuum of constant mass one of households supplying their labour inelastically. This simplifying assumption allows us to disregard labour supply ($L_t = 1$) and rewrite the production function in (1) as

\[
F \left( K^F_t, K^G_t \right) = \left( K^F_t + K^G_t \right)^\alpha.
\]

Markets are competitive, and the factor rewards for energy and for labour are equal to their respective marginal products, $\rho^F_t = \rho^G_t = \alpha (K^F_t + K^G_t)^{\alpha-1} \equiv \rho_t$ and $w_t = (1 - \alpha)(K^F_t + K^G_t)^\alpha$. 
The total rate of return on capital is equal to the income it generates plus the change in value, net of depreciation, of the stock:

\[ r_{t+1}^i = \alpha (K_{t+1}^F + K_{t+1}^G)^{\alpha - 1}D_t^i + \left(1 - \delta^i\right) \frac{1}{D_{t+1}^i} \]

\[ = \rho_{t+1}D_t^i + \frac{1 - \delta^i}{1 - \Delta^i} \]

where \( \Delta^i = \begin{cases} \Delta & \text{if } i = F \\ 0 & \text{if } i = G \end{cases} \)

4 Model 1 - Consumption Smoothing

In this section, we model the Energy Trap in a modified Ramsey growth model, where a continuum of infinitely lived households choose levels of consumption and investment in fossil fuel and green-tech capital to maximize their lifetime utility. The solution path involves switching from investment in fossil fuel capacity to alternative green-tech when the productivity of the former drops below a threshold level. Unexpected decreases in the availability of fossil fuel capital make the household poorer, partly discouraging investment in alternative energy infrastructure.

There is a continuum of mass one of firms, each producing the single final good \( Y \). The firms buy inputs in competitive input markets, and sell their output in a competitive output market. There is a continuum of infinitely lived agents, of constant measure one, with constant relative risk aversion (CRRA) preferences. They supply labour inelastically, and own the firms. They choose investment levels, and consumption, \( \{I_{t+s}^F, I_{t+s}^G, C_{t+s}\}_{s=0}^\infty \), to maximise their lifetime utility

\[ U_t = E_t \left[ \sum_{s=0}^{\infty} \beta^s u(C_{t+s}) \right] = E_t \left[ \sum_{s=0}^{\infty} \beta^s \frac{C_{t+s}^{1-\theta} - 1}{1 - \theta} \right]. \] (7)

The coefficient of risk aversion, \( \theta > 0 \), and \( 0 < \beta < 1 \) is the discount factor.

Since the economy is closed, all output is devoted to either consumption or gross investment,

\[ Y_t = C_t + I_t^F + I_t^G. \] (8)

Since markets are competitive, firms make zero profit, and condition (8) also represents the budget constraint of the household.

We can combine (8) with the fossil fuel and green-tech capital accumulation equations in (5) and (4), to obtain a single capital accumulation equation,

\[ F(K_t^F, K_t^G) + \left(1 - \delta^F\right)K_t^F + \left(1 - \delta^G\right)K_t^G - C_t - \frac{K_{t+1}^F}{D_t^F} - K_{t+1}^G = 0. \] (9)

Formally, the problem for the household is to choose a sequence of consumption and investment\(^2\) to maximize lifetime consumption in (7) subject to the capital accumulation equation in (9), investments being non negative, \( I_t^F \geq 0 \) and \( I_t^G \geq 0 \), a set of initial conditions, \( \{K_0^F, K_0^G, D_0\} \in \mathbb{R}_+^3 \), and a transversality condition.

\(^2\)In either type of capital, the other amount of investment just follows from (8).
We can use equation (8) to re-write the problem by substituting consumption as a control variable with both levels of investment, \( C_t = F(K_t^F, K_t^G) - I_t^G - I_t^F \). The representative agent’s maximization problem then becomes

\[
V_t(D_t, K_t^F, K_t^G) = \max_{I_t^F, I_t^G} \left[ u(F(K_t^F, K_t^G) - I_t^G - I_t^F) + \beta V_{t+1}(D_{t+1}, K_{t+1}^F, K_{t+1}^G) \right]
\]

s.t. \( K_{t+1}^F = (1 - \delta^F) K_t^F + D_t^F I_t^F \)

\( K_{t+1}^G = (1 - \delta^G) K_t^G + I_t^G \)

\( D_{t+1} = (1 - \Delta) D_t \)

\( I_t^F \in [0, F(K_t^F, K_t^G)] \)

\( I_t^G \in [0, F(K_t^F, K_t^G)] \).

Standard dynamic programming arguments ensure that a solution exists, is unique and can be found by using first order conditions.³ Let \( -\mu_t \) and \( -\lambda_t \) be the Lagrangian multipliers associated with the fossil fuel and green-tech investment non-negativity conditions, respectively. By the First Order Conditions (FOCs)

\[
\mu_t = \frac{\partial u \left( F(K_t^F, K_t^G) - I_t^G - I_t^F \right)}{\partial I_t^F} + \beta E_t \left[ \frac{\partial V_{t+1}}{\partial D_{t+1}} \frac{\partial D_{t+1}}{\partial I_t^F} + \frac{\partial V_{t+1}}{\partial K_{t+1}^F} \frac{\partial K_{t+1}^F}{\partial I_t^F} + \frac{\partial V_{t+1}}{\partial K_{t+1}^G} \frac{\partial K_{t+1}^G}{\partial I_t^F} \right]
\]

\[
\lambda_t = \beta E_t \left[ \frac{\partial V_{t+1}}{\partial K_{t+1}^G} \right] - u'(C_t)
\]

and by the slackness conditions

\[
I_t^F > 0 \implies \mu_t = 0 \quad \text{or} \quad I_t^F = 0 \implies \mu_t < 0
\]

\[
I_t^G > 0 \implies \lambda_t = 0 \quad \text{or} \quad I_t^G = 0 \implies \lambda_t < 0.
\]

³Firstly, since the utility function is strictly increasing, strictly concave and continuous, the objective function is continuous. Secondly, the feasible set is closed, \( 0 \leq I^i \leq F(K_t^F, K_t^G) \) with \( i \in \{G, F\} \), and bounded (since the Inada conditions for \( K^i \to \infty \) with \( i \in \{G, F\} \) ensure that there are maximum attainable levels for both kinds of capital). Since any continuous function over a compact set has a well-defined maximum on that set, utility maximizing investment sequences exist. Secondly, the objective function is differentiable since the utility function is differentiable. Thirdly, since \( C_t \) can be neither zero (marginal utility shoots to infinity) nor \( Y_t \) (eventually \( C_\infty = 0 \) and marginal utility shoots to infinity) at the maximum, \( C_t \) is in the interior of the feasible set. So is \( I_t = I_t^G + I_t^F \). Together with a transversality condition, this ensures the necessity of the first order conditions. Finally, strict concavity of the objective function (through strict concavity of the utility function) means that, if the solution is in the interior, the first order conditions are sufficient.
The Benveniste-Scheinkman theorem gives us the following Envelope Theorem Conditions (ETCs):

\[
\frac{\partial V_t}{\partial K^F_t} = \frac{\partial u(F(K^F_t, K^G_t) - I^G_t - I^F_t)}{\partial K^F_t} + \beta E_t \left[ \frac{\partial V_{t+1}}{\partial D_{t+1}} \frac{\partial D_{t+1}}{\partial K^F_t} + \frac{\partial V_{t+1}}{\partial K^F_t} \frac{\partial K^F_t}{\partial K^G_t} + \frac{\partial V_{t+1}}{\partial K^G_t} \frac{\partial K^G_t}{\partial K^G_t} \right] \\
\frac{\partial V_t}{\partial K^G_t} = u'(C_t) \frac{\partial F(K^F_t, K^G_t)}{\partial K^G_t} + \beta (1 - \delta^F) E_t \left[ \frac{\partial V_{t+1}}{\partial K^G_t} \right] \\
\frac{\partial V_t}{\partial D_t} = u'(C_t) \frac{\partial F(K^F_t, K^G_t)}{\partial D_t} + \beta (1 - \Delta) E_t \left[ \frac{\partial V_{t+1}}{\partial D_{t+1}} \right]
\]

where we can ignore the last one, since the envelope condition for \( D^F_t \) does not feature in the FOCs. Rearrange the FOCs to obtain

\[
\frac{\mu_t + u'(C_t)}{\beta} = E_t \left[ \frac{\partial V_{t+1}}{\partial F^F_t} \frac{D^F_t}{D_t} \right] \\
\frac{\lambda_t + u'(C_t)}{\beta} = E_t \left[ \frac{\partial V_{t+1}}{\partial K^G_t} \right]
\]

and finally, by substituting these into the ETCs, we obtain the following Euler-like conditions:

\[
\frac{\mu_{t-1} + u'(C_{t-1})}{\beta D_{t-1}} = u'(C_t) \frac{\partial F(K^F_t, K^G_t)}{\partial K^F_t} + \beta (1 - \delta^F) \frac{\mu_t + u'(C_t)}{\beta D^F_t} \tag{10}
\]

\[
\frac{\lambda_{t-1} + u'(C_{t-1})}{\beta} = u'(C_t) \frac{\partial F(K^F_t, K^G_t)}{\partial K^G_t} + \beta (1 - \delta^G) \frac{\lambda_t + u'(C_t)}{\beta} \tag{11}
\]

The slackness conditions suggest that the optimization problem can be split up into 3 time periods:

1. A fossil regime, where investments are made only in fossil fuel capital,

\[
I^G_t = 0 \& \lambda_t < 0 \\
I^F_t > 0 \& \mu_t = 0.
\]

From (10), we can obtain an Euler equation of the form

\[
u'(C_t) \beta D^F_t = u'(C_{t+1}) \left[ \frac{\partial F(K^F_{t+1}, K^G_{t+1})}{\partial K^F_{t+1}} + \frac{(1 - \delta^F)}{D_{t+1}} \right]. \tag{12}
\]

2. A mixed regime, where households invest in both types of capital,

\[
I^G_t > 0 \& \lambda_t = 0 \\
I^F_t > 0 \& \mu_t = 0.
\]
From (10) and (11),
\[
\begin{align*}
\frac{u'(C_t)}{\beta D^F_t} &= u'(C_{t+1}) \left[ \frac{\partial F(K^F_{t+1}, K^G_{t+1})}{\partial K^F_{t+1}} + \frac{(1 - \delta^F)}{D_{t+1}} \right] \\
\frac{u'(C_t)}{\beta} &= u'(C_{t+1}) \left[ \frac{\partial F(K^F_{t+1}, K^G_{t+1})}{\partial K^G_{t+1}} + (1 - \delta^G) \right].
\end{align*}
\] (13) (14)

In this time-frame, both (13) and (14) must hold. This implies
\[
D^F_t \frac{\partial F(K^F_{t+1}, K^G_{t+1})}{\partial K^F_{t+1}} + \frac{1 - \delta^F}{1 - \Delta} = \frac{\partial F(K^F_{t+1}, K^G_{t+1})}{\partial K^G_{t+1}} + 1 - \delta^G
\]
i.e. the agents invest in both types of capital if and only if the returns are equal. The right-hand side is the return at \(t + 1\) of investing in one unit of green-tech capital in \(t\), and comprises the marginal product and the remaining amount of capital after depreciation. The left-hand side is the return of investing in one unit of fossil fuel capital, where both components are adjusted for the productivity of fossil fuel capital investment.

3. Finally, there is a green regime, where investments are made only in the alternative green-tech infrastructure,
\[
I^G_t > 0 \& \lambda_t = 0 \\
I^F_t = 0 \& \mu_t < 0.
\]

From (11), the relevant Euler equation is
\[
\frac{u'(C_t)}{\beta} = u'(C_{t+1}) \left[ \frac{\partial F(K^F_{t+1}, K^G_{t+1})}{\partial K^F_{t+1}} + (1 - \delta^G) \right].
\] (15)

In any time period, the Euler equation(s), together with the capital accumulation equation, represent (a) second order difference equation(s).

Using the functional forms specified above (CRRA utility and Cobb-Douglas for production), the Euler equations read as follows
\[
C^G_t - \theta = C^G_{t+1} D^F_t \left[ \alpha (K^F_{t+1} + K^G_{t+1})^{\alpha - 1} + \frac{1 - \delta^F}{D_{t+1}} \right] = C^G_{t+1} \left[ \alpha D^F_t (K^F_{t+1} + K^G_{t+1})^{\alpha - 1} + \frac{1 - \delta^F}{1 - \Delta} \right]
\]
or, equivalently,
\[
C_t = C_t \beta^\frac{1}{\theta} \times \max \left\{ \left[ D^F_t \alpha (K^F_{t+1} + K^G_{t+1})^{\alpha - 1} + \frac{1 - \delta^F}{1 - \Delta} \right]^\frac{1}{\theta}, \left[ \alpha (K^F_{t+1} + K^G_{t+1})^{\alpha - 1} + 1 - \delta^G \right]^\frac{1}{\theta} \right\}.
\]

As usual, this Euler equation tells us that along the optimal path the marginal impact of reducing consumption in \(t\), and investing the additional savings to finance consumption in \(t + 1\), must be zero, ceteris paribus.

We can also find the policy functions in terms of investment, as it is straightforward to use the
above equation to derive \( I_t = I_t^G + I_t^F \) from \( Y_t = C_t + I_t \). This is enough to find \( I_t^F \) and \( I_t^G \) in the fossil and green regime respectively. To disaggregate investment in the two components in the mixed regime,\(^4\) one can notice that (5) and (4) suggest

\[
D_t I_t^F + I_t^G = K_{t+1}^F + K_{t+1}^G - (1 - \delta^F)K_t^F - (1 - \delta^G)K_t^G. \tag{16}
\]

The agents invest in both types of capital only if the returns are equal, i.e.

\[
K_{t+1}^F + K_{t+1}^G = \left[ \frac{(D_t^F - 1) \alpha}{1 - \delta^G - \frac{\delta^F}{1 - \Delta}} \right] \frac{1}{\alpha} \tag{17}
\]

By using the result in (17) to substitute for the corresponding form in (16),

\[
D_t I_t^F + I_t^G = \left[ \frac{(D_t^F - 1) \alpha}{1 - \delta^G - \frac{\delta^F}{1 - \Delta}} \right] \frac{1}{\alpha} - (1 - \delta^F)K_t^F - (1 - \delta^G)K_t^G \equiv X_t.
\]

Therefore for the mixed regime, investment levels in each type of capital are given by

\[
D_t I_t^F = X_t - I_t^G = X_t - I_t + I_t^F - I_t^G \rightarrow I_t^F = \frac{X_t - I_t}{D_t^F - 1}, \quad I_t^G = I_t - I_t^F.
\]

Assume that the economy starts out with no green-tech capital, but a positive endowment of fossil fuel capital. The productivity of fossil fuel investment then starts decreasing according to the law of motion in (3), eventually the economy starts investing in the alternative productive capacity. Indeed, by keeping investing in fossil fuel capital only, agents get a return of

\[
D_t^F \alpha (K_{t+1}^F)^{\alpha^{-1}} + \frac{1 - \delta^F}{1 - \Delta} \tag{18}
\]

while by switching to alternative productive capacity, the return is

\[
\alpha ((1 - \delta^F)K_t^F + K_{t+1}^G)^{\alpha^{-1}} + 1 - \delta^G. \tag{19}
\]

It is intuitive, as \( D_t^F \) eventually approaches zero, that agents would basically waste resources by investing in fossil fuel capital, and thus that the economy inevitably moves to the green regime. Without shocks, we eventually reach a steady state where the economy is completely decarbonized, and \( K_{SS}^F = I_{SS}^F = 0 \). In this steady state,\(^5\)

\[
u'(C_{SS}) = \beta u'(C_{SS}) \left[ F\left(K_{SS}^G\right) + (1 - \delta^G) \right] \tag{20}
\]

\[
K_{SS}^G = (1 - \delta^G)K_{SS}^G - C_{SS} + F(K_{SS}^G) \tag{21}
\]

and thus

\[
F'(K_{SS}^G) = \frac{1 - \beta}{\beta} - (1 - \delta^G) = \frac{1 - \beta}{\beta} + \delta^G. \tag{22}
\]

\(^4\)Note that, in practice, there is no mixed regime

\(^5\)With a slight abuse of notation, \( F(K_{SS}^G) \equiv F(0, K_{SS}^G) \) and \( \partial F(0, K_{SS}^G) / \partial K_{SS}^G = F'(K_{SS}^G) \).
Since \( \beta^{-1} > 1 \) and the left-hand side of (22) is lower than 1 when \( K_G^{SS} \to \infty \) and goes to infinity when \( K_G^{SS} \to 0 \), there is exactly one \( K_G^{SS} \) that solves (20). To find \( C_{SS} \) just substitute that \( K_G^{SS} \) into (21).

We can find the level of \( C_t \) such that \( K_F^t + 1 = K_G^t \) for each level of \( K_G^t \),

\[
C_t = F(K_F^t, K_G^t) - \delta^G K_G^t. \tag{23}
\]

If \( C_t \) is below this line, then the green-tech capital stock is growing as there is output left over once consumption and depreciation have been taken into account. The Euler equation in (15) tells us that consumption is shrinking whenever the capital stock is lower than its steady state value. In particular, the qualitative dynamics of \( C_t \) are governed by

\[
\frac{\partial F(K_F^t, K_G^t)}{\partial K_G^t} = \beta^{-1} - (1 - \delta^G) = \frac{1 - \beta}{\beta} + \delta^G. \tag{24}
\]

For \( K_G^t \) below this level, \( C_t \) is increasing.\(^6\)

The steady state values for the decarbonized economy are

\[
\begin{align*}
D_{SS} &= K_F^{SS} = I_F^{SS} = 0 \\
K_G^{SS} &= \left(\frac{\alpha \beta}{1 - \beta (1 - \delta^G)}\right)^{1/\alpha} \\
Y_{SS} &= \left(\frac{\alpha \beta}{1 - \beta (1 - \delta^G)}\right)^{1/\alpha} \\
I_G^{SS} &= \delta^G K_G^{SS} \\
C_{SS} &= Y_{SS} - I_G^{SS} = \frac{1 - \beta + \beta \delta^G (1 - \alpha)}{\alpha \beta} \left(\frac{\alpha \beta}{1 - \beta (1 - \delta^G)}\right)^{1/\alpha}
\end{align*}
\]

We can now start tackling the spectre of the Energy Trap: we show that in our baseline model, it may happen that realized negative shocks to the amount of fossil fuel capital at certain productivity available to the agents hurt the transition towards a decarbonized energy infrastructure. This happens if the sudden fall in \( K_F^t \) boosts the return from investing in fossil fuel capital enough to satisfy, once again, the following inequality

\[
K_F^{t+1} + K_G^{t+1} < \left[\left(\frac{(1 - \Delta)D_0 - 1}{1 - \delta^G - \frac{1 - \delta^F}{1 - \Delta}}\right)^{1/\alpha}\right].
\]

The previous says that, given the current \( D_F^t \) and parameters, a sudden decrease in the availability of capital can push the economy back to the fossil fuel regime if the returns from investing in fossil fuel capital is now higher than the return from investing in green-tech energy.\(^7\) This is exemplified in Figure 3, where the lines represent values for fossil fuel capital along the path towards the decarbonized steady state, the only difference being the values at time 0 of this capital. This shows that, if \( K_F^t \) were to be hit by a negative shock such that the level moves from one of the three highest lines to one

\(^6\)Note that both (23) and (24) depend on the amount of fossil fuel capital still present in the economy and slowly depreciating, while in steady state production must depend only on \( K_G^{SS} \).

\(^7\)The same effect is given by an unexpected increase in \( D_F^t \). At the same time though, an increase in either type of capital stock (e.g. fracking) can help the transition towards the green regime.
of the three lowest lines, the transition towards the alternative productive capacity would be delayed by one period, everything else being equal.

![Diagram](image)

Figure 3: Paths of $K_{t}^{F}$ towards the decarbonized steady state

5 Model 2 - Credit Constraints

Another issue with the transition from fossil fuels to alternative energy is access to credit and the consequences that the potential presence of different financing arrangements has. The value of lower duration assets is less exposed to variations in interest rates and to variations in income flows, and therefore lower duration translates into lower risk.\(^8\)

Matsuyama (2007, Section III) argues that this can lead to trade-offs between productivity and pledgeability, where more productive investments appeal to borrowers (and future agents) while less productive but highly pledgeable projects may represent better alternatives for the lenders. Matsuyama (2007, page 504) shows that, if this is the case, “a rise in borrower net worth may cause the credit to switch toward more productive projects”. In this context, a negative energy shock lowers borrower net worth, and causes a shift towards more pledgeable fossil fuel investments.

We here consider a Diamond overlapping generations model with two-period lives. In each period, a unit measure of new agents is born. They stay active for two periods. In the first, they inelastically supply their endowment of labour for a wage $w_t$, which they save. They can choose to allocate their savings in one of two ways. They can become lenders and earn a return, so that they can consume

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\(^8\)As well as this direct map into the Energy Trap issue, it is easy to imagine that mundane projects, using well-established technologies as in the case of conventional energy facilities, could be viewed as less risky as compared with projects using still developing technologies. Further investor familiarity and the smaller presence of renewable energy technologies in the market, can also lead to perceptions of greater risks than for fossil energy investments.
The last term in (29) is the rate of return that a borrower running project $i$ offers to the lenders. If this is lower than the equilibrium return, project $i$ is not run: the credit only flows towards the project(s) with the highest return. It is important to notice that the rate of return depends on the
billion’s net worth, $w_t$. For sufficiently low net worth, $w_t < m^t(1 - \lambda^t)$, the last term becomes
\[\left(\rho_{t+1} D_{t+1}^F + \frac{1 - \delta^F}{\lambda^F} D_{t+1}^F\right) \lambda^t/(1 - w_t/m_t)\] which depends positively on $D_{t+1}^F\lambda^t$: when agents are poor, the credit goes to the project with the highest pledgeable rate of return, and the borrowing constraint is the relevant one. On the other hand, for sufficiently high net worth, $w_t > m^t(1 - \lambda^t)$, the last term becomes $\rho_{t+1} D_{t+1}^F + \frac{1 - \delta^F}{\lambda^F}$, and the credit goes to the most profitable project, as the profitability constraint is the relevant one.

For any initial values \(\{K_0^F, K_0^G, D_0^F\}\), an equilibrium is a sequence of capitals \(\{K_{t+s}^G, K_{t+s}^F\}_{s=0}^{\infty}\), given \(\{D_{t+s}^F\}_{s=0}^{\infty}\), satisfying the credit market equilibrium in (25), the capital accumulation equations in (4) and (5), and solving the equilibrium condition in (29).

Now consider aggregate equations of motion for an economy

If the economy starts with high $D_t^F$ and positive $K_t^F$, but no green-tech capital, agents will initially invest only in fossil fuel. Eventually, the productivity of new investment in fossil fuel will have decreased enough, and agents will start investing in green-tech. Eventually, the economy will be decarbonized and a steady state reached:

\[w_{SS} = I_{SS} = I_{SS}^G = (1 - \alpha) (K_{SS}^G)^\alpha\]
\[K_{SS}^G = I_{SS}^G D^G + (1 - \delta^G) K_{SS}^G = I_{SS}^G D^G / \delta^G = \frac{1 - \alpha}{\delta^G} D^G (K_{SS}^G)^\alpha = \left[\frac{1 - \alpha}{\delta^G} D^G\right]^{1 - \alpha}\]

Now we only consider the case in which $\lambda^G < \lambda^F$, i.e. we assume that the funding of green-tech projects are more credit constrained than fossil fuel projects. As described above, in any period $t$, the project selection relies on equation, which can be rewritten as

\[r_{t+1} = \max \left\{ \frac{\rho_{t+1} D_{t+1}^F + \frac{1 - \delta^F}{\lambda^F}}{\max \left\{ 1, \frac{1 - w_t/m_t}{\lambda^F} \right\}}, \frac{\rho_{t+1} + 1 - \delta^G}{\max \left\{ 1, \frac{1 - w_t/m_t}{\lambda^G} \right\}} \right\},\]

where, at the aggregate level,

\[\rho_{t+1} = \begin{cases} \alpha \left( (1 - \delta^F) K_t^F + (1 - \delta^G) K_t^G + w_t D_t^F \right)^{\alpha - 1} & \text{if investing in fossil} \\ \alpha \left( (1 - \delta^F) K_t^F + (1 - \delta^G) K_t^G + w_t \right)^{\alpha - 1} & \text{if investing in green-tech} \\ \alpha \left( (1 - \delta^F) K_t^F + (1 - \delta^G) K_t^G + w_t (\gamma D_t^F + 1 - \gamma) \right)^{\alpha - 1} & \text{if investing in both} \end{cases}\]

(where $\gamma$ is the share of investment in fossil projects). This equation gives us the ranking of the projects, considering both pledgeability and productivity. When the efficacy of new high carbon capital investment is high, funds flows towards the fossil fuel project, both more productive and more pledgeable. However, when the productivity of new high carbon investment drops below a certain level, there is a trade-off between productivity and pledgeability. Indeed, the ranking of the projects is a function of the agents’ net worth, $w_t$. If agents have low net worth, the credit goes to the more pledgeable fossil fuel projects; if, on the contrary, agents have high net worth, the credit flows to the more productive green project (as in Matsuyama (2007) we have a bang-bang solution because ranking is independent of the allocation of credit).

Therefore, a sudden decrease in the availability of fossil fuel capital, at a given productivity, makes the agents poorer by pushing $w_t$ down. This makes the borrowing constraint more relevant, and thus the ranking of the projects swings towards the more pledgeable one, thus pushing agents to invest in fossil fuel.
6 Conclusions

There is an idea that recessions, by reducing emissions, are good for climate change. The idea presented in this paper suggests that this is not true and poor economic performance may tip the composition of investment in global energy supplies back towards fossil fuels. The energy transition may require a healthy economy.
References
