The asymmetric fundamental transformation*

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February 17, 2017

Abstract
This paper examines how asymmetries in partners’ ability to appropriate the returns from collaboration affect investment patterns and governance choices. We consider multiple pairs of upstream and downstream firms, where the hold-up problem in each pair depends on the investment decisions of others. Initial asymmetries in appropriability and large market size can cause all unstructured collaboration to break down, leading some pairs to formalize their relationship or integrate, to restore investment incentives. The relationship between organizational forms is symbiotic: different forms may coexist in equilibrium, where the presence of hierarchical forms allows other seemingly more efficient forms to be sustained.

Keywords: Power asymmetries, incomplete contracting, collaboration, organizational forms, hold-up.


*We would like to thank Ashish Arora, Sharon Belenzon, Matthew Ellman, David Pérez-Castrillo and seminar participants at the University of Edinburgh, Fuqua School of Business, UEA, University of Copenhagen and Universitat Autònoma de Barcelona for helpful discussions and remarks. All remaining errors are ours.

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1 Introduction

How the availability of potential trade partners affects the nature and stability of collaboration has been the subject of much research in economics. Transaction-cost economics (TCE) holds that firms will often integrate or choose more ‘hierarchical’ forms of collaboration when potential trade partners are hard to come by, because competition will then be unable to effectively restrain opportunism (Williamson, 1975, 1985; Klein et al., 1978; McLaren, 2000). Relationship-specific investment is often seen as the root of this problem, for it tends to transform an ex ante competitive situation into a bilateral monopoly. Williamson (1985) refers to this change as the ‘fundamental transformation’.

Others have pointed out that the availability of many potential trade partners may also create problems. A concern is that, when it is easy to find a new partner, the incentives to perform well in any given partnership may be reduced. The threat of termination as a disciplining device then becomes moot (Shapiro and Stiglitz, 1984; Kranton, 1996). As a consequence, partners may seek to limit their options, for instance by exchanging hostages, sinking relationship-specific investments, or choosing to integrate (Williamson, 1983; Ramey and Watson, 2001).

In this paper we examine a third type of bargaining problem. These problems arise when, instead of transforming an ex ante competitive situation into a bilateral monopoly, a partially relationship-specific investment creates an asymmetry in the external opportunities firms face. For instance, collaboration between a biotechnology firm and a pharmaceutical company may generate research results that are more easily appropriated by one partner than the other. Also, collaboration between buyers and suppliers may lead to product component improvements that have different values outside the relationship.

Consider two firms, a car manufacturer and a supplier. Through co-design and information sharing, the car manufacturer may develop a better chassis, and the supplier may develop a better car body. Together, they could create a new car; however, they could also use their components with other partners. The extent to which either the chassis or the car body can easily be redeployed outside the relationship affects the partners’ relative bargaining power. We show that if collab-
orative investment induces large shifts in bargaining positions, then the partner whose position weakens may refuse to collaborate, even if collaboration is socially efficient (an underinvestment problem à la Grossman and Hart, 1986, and Hart and Moore, 1990). We embed this problem in a setting with many identical upstream and downstream firms. Prior to investment, each initial upstream-downstream pair can choose its organizational form (unstructured or structured partnerships, and integration). The main contribution of the paper is to characterize aggregate patterns of governance and investment in this economy.¹

The analysis yields a number of insights. We begin by assuming that only unstructured partnerships can be formed. If collaboration is unstructured, there is no contract specifying how the returns from joint investment are divided. Multiple equilibria can then arise because of complementarities in investment choices. Investment tends to make upstream-downstream pairs more stable because it is partly relationship-specific (a chassis and a car body that are jointly designed fit together very well). The greater the number of pairs that jointly invest and remain together, the smaller the chance that another firm will find a suitable partner in the ‘rematching market’. This lack of outside options mitigates the “weaker” partner’s concern of being held up at the renegotiation stage. Thus, the incentives to collaborate (which are driven by the incentives of the weaker partner) increase with the number of investing pairs.² There are two pure-strategy, Pareto-ranked equilibria: one where all pairs invest and go to the product market together, and one where no pair invests.

The Pareto-inferior equilibrium emerges when firms have pessimistic expectations about the probability that other pairs will invest. This equilibrium is especially plausible when market size (i.e., the number of pairs) is large, since not investing then becomes the “safer” option. We make this argument precise using the concept of $p$-dominance (Morris et al., 1995). We highlight three

¹Throughout this paper, we use the terms “collaborative investment”, “joint investment” and sometimes simply “investment” interchangeably. The crucial aspect of the partners’ investment decisions is that they are highly complementary—activities such as knowledge sharing and co-design are not likely to produce significant value unless both partners exert effort.

²Put another way, the severity of the hold-up problem in our setting is endogenous and depends negatively on the total number of pairs that invest.
factors that are necessary to bring about inefficiencies: (i) significant initial asymmetries in the appropriability of investment between upstream and downstream firms, (ii) the non-specificity of investment, and (iii) a market size large enough that initial asymmetries can effectively be exploited.

In order to avoid the Pareto-inferior outcome, pairs may turn to more hierarchical forms of collaboration: structured partnerships and integration. Structured partnerships help because they contractually allow partners to flexibly distribute the surplus from collaboration. The firm experiencing an adverse shift in bargaining position can be compensated with a greater share of the joint surplus. However, structured partnerships cannot in general fully restore efficiency, since shifts in bargaining positions can be large relative to the joint surplus. Integration solves the problem of shifting bargaining positions, because shifts are internalized within a merged firm (a ‘consolidated balance sheet effect’). However, integration may involve costs associated with tendering or the loss of high-powered incentives.

We show that all organizational forms can co-exist in equilibrium, despite all pairs being identical ex ante and facing the same environment. More hierarchical modes of governance exert a positive externality over less hierarchical modes because integrated pairs always bring their products to the market together and hence restrict the size of the rematching market. In equilibrium, just enough pairs form structured partnerships or merge to make the rematching market small enough for pairs in unstructured partnerships to invest. The relationship is symbiotic: without some degree of integration in the industry, unstructured collaboration between independent firms would not be feasible.

Different organizational forms also result in different equilibrium payoffs. ‘Hierarchy’ is associated with lower performance because of integration costs, while ‘hybrids’ (structured partnerships) and the ‘market’ (unstructured partnerships) largely free-ride on these. To our knowledge, Legros and Newman (2013) is the only other existing model that also obtains co-existence of organizational forms and heterogeneity of performance. In their model, however, integration produces superior outcomes, while in our setting the opposite is true.

The organization of the automotive supply chain (both in Japan and the U.S.) provides some support for the core ideas of our model. Cooperation between manufacturers and their suppliers
is extensive despite the notable lack of detailed contracts or formal partnership agreements and
the fact that manufacturers often rely on few outside suppliers for a particular part (Helper et al.,
2000; Ahmadjian and Oxley, 2006). Thus, ‘bilateral monopoly’, even when contracts are incomplete,
does not seem to significantly hinder cooperation. A more serious problem appears to be power
imbalances. Manufacturers are often much larger and with deeper pockets than their suppliers.
Ahmadjian and Oxley (2006) find that, when supplier dependence is more pronounced, governance
mechanisms such as partial integration (minority equity stakes) and ‘volume-based dependence
balancing’ more often occur. Credible commitments or “mutual hostages” (Williamson, 1983) have
also been observed in a variety of other contexts and industries (Fein and Anderson, 1997; Jap and
Anderson, 2003; Kim and Mahoney, 2006; Somaya et al., 2011) and have been associated with joint
design and improved operational performance (Heide and John, 1990).3 Our paper is the first to
examine how the severity of power imbalances between particular firms can depend on the strategic
behavior of other firms in the market.

The paper is organized as follows. The remainder of this section discusses related literature.
The baseline model is presented in Section 2. Sections 3, 4 and 5 study unstructured partnerships,
integration, and structured partnerships, respectively. Section 6 deals with a number of extensions,
including asymmetric numbers of upstream and downstream firms, ‘large’ firms with both upstream
and downstream capabilities, and contractual solutions. Section 7 briefly concludes. All proofs can
be found in the appendix.

1.1 Related literature

By emphasizing incomplete contracts, bargaining, and underinvestment, our model builds on the
property-rights view of the firm (e.g., Grossman and Hart, 1986; Hart and Moore, 1990; Halonen,
2002; Cai, 2003; Gans, 2005). This literature generally obtains underinvestment as the outcome of

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3A non-obvious dependence balancing mechanism is discussed by Heide and John (1988) in the context of marketing
channels. Typically agents are much smaller than the manufacturers that they represent and must make significant
transaction-specific investments. How can they safeguard their investments? Heide and John show that one way is
through “offsetting investments” in relationships with their customers, which restore at least in part the balance of
power with manufacturers.
a hold-up problem—the unwillingness of parties to privately contribute to creating value that will be captured by their partners. We obtain underinvestment as a result of shifts in outside options (or bargaining positions), the extent of which depend on the investment decisions of others in the market.

A few papers examine how collaboration is affected by changes in outside options. Caballero and Hammour (1996a, 1996b) study an economy where joint production makes capital more vulnerable to appropriation than labor because of technological specificity. Their focus is on the macroeconomic implications of this ‘fundamental transformation’ (e.g., depressed creation, excessive destruction, labor market segmentation). Nicita and Sepe (2012) show that when investment affects outside options, overinvestment as well as underinvestment can occur. In contrast to these papers, we focus on the formation and stability of pairs in a market-for-partnerships setting, where outside options are endogenous. Hellmann and Thiele (forthcoming) also consider the issue of the stability of partnerships when bargaining with other potential partners is possible. Our approach is complementary to theirs: while they focus on imperfect information and liquidity constraints as the source of inefficiency, we focus on market thickness.4

An extensive literature has examined the robustness of the hold-up problem to contractual solutions (e.g., Edlin and Reichelstein, 1996; Che and Hausch, 1999; Segal and Whinston, 2000; Matouschek and Ramezzana, 2007). We consider two possible solutions to the problem of shifting bargaining positions: structured partnerships (which can be viewed as equity joint ventures where participation is voluntary) and termination-fee contracts. We show that these commonly observed contractual solutions can be of some, but limited, use in our setting.

The papers most closely related to the present work are McLaren (2000), Ramey and Watson (2001), Gibbons et al. (2012) and Legros and Newman (2013).

McLaren (2000) examines vertical integration in a setting where small-numbers bargaining problems are present. In his model, when transaction partners are few, upstream firms can be held up by downstream firms. Vertical integration thins the market, thus exacerbating the opportunism

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4The idea that bargaining with other potential partners determines each firm’s outside option also features in de Fontenay and Gans (2005, 2014) and Inderst and Montez (2015), in an IO context.
problem. This induces multiple equilibria in integration decisions: either all firms integrate, or all firms contract at arm's-length. In our model, vertical integration also thins the market, but this mitigates rather than exacerbate the opportunism problem, because shifts in bargaining positions are reduced. Different organizational forms do not crowd each other out, but rather co-exist with one another.

Ramey and Watson (2001) examine large-numbers bargaining problems. In their model, a thick rematching market reduces partners’ incentives to exert effort by lowering the cost of break up. To counteract this adverse effect, firms can sink relationship-specific investments or integrate, thus strengthening their bilateral ties. We focus on asymmetries between partners rather than on large-numbers bargaining problems. Large markets do play an important role in our analysis, but only because they magnify initial asymmetries. Most importantly, Ramey and Watson are not concerned with externalities in investment and governance choices, and restrict attention to symmetric equilibria. Unlike in our setting, their model exhibits no multiplicity of equilibria, no co-existence of organizational forms, and no heterogeneity in performance among ex ante identical pairs.

Gibbons et al. (2012) develop a property-rights model where prices convey information. The greater the number of firms that focus on information gathering, the more informative the price mechanism becomes. In equilibrium, only some firms gather information, the rest infer it from prices. There is co-existence of organizational forms, but no heterogeneity in performance.

Legros and Newman (2013) also develop a model where prices matter. Integration generates private costs for managers, but is effective at coordinating production decisions. When the price is high, efficient production becomes more important, and upstream-downstream pairs integrate. There is a price and a generic set of demand functions at which different organizational forms co-exist and performance is heterogeneous. Legros and Newman find that integrated firms perform better because they better coordinate production, to the detriment of managers’ private benefit. By contrast, we assume that less hierarchical modes of governance perform better than others.

More broadly, our mechanism driving the coexistence of organizational forms relies on the idea of coordination failure, where each pair’s negative expectations about the investment choices of others
in the market can drive down aggregate investment. Coordination failure has also featured in a variety of earlier work, in different strands of literature, that looks at inefficient equilibria in settings with equilibrium multiplicity and strategic complementaries. For instance, work in finance has suggested that coordination failure can help shed light on bank runs, liquidity dry-ups and currency attacks.\(^5\) In macroeconomics, coordination failure can help explain inefficiently low production, trade and investment.\(^6\) Our work examines how, through decentralized governance choices, the adverse consequences of coordination failure in a market for partnerships can be mitigated.

2 Model

We consider a setting where firms come together to form productive partnerships. The model has the following four main features. First, a partnership consists of one upstream firm and one downstream firm, each of which produces a component for a product. Both components are essential, in the sense that both are necessary to bring the product to the market. For example, in the case of car manufacturing, GM may build the chassis and Fisher Body may create the closed car body. Second, collaboration improves the quality of both components, which increases product value, but also takes time and effort, and as such is costly to both firms. We assume that both partners must invest in collaborative activities such as knowledge sharing and product co-design for investment to bear any fruit. For this reason, we will sometimes call it collaborative investment (or investment in collaboration). Third, components are to some extent tailored to the characteristics of the collaborating firms. Thus, investment is partly relationship-specific. Fourth, the firms that take a product to the market together need not necessarily be the same firms that initially collaborated with one another, potentially leading to a hold-up problem.

Activity in this setting takes place in three periods. In period 1, \(N\) upstream and \(N\) downstream firms are matched into \(N\) pairs, and the firms in each pair agree on an organizational form. In period 2, the firms in each pair decide whether or not to invest. In period 3, the firms bargain over

\(^5\)For bank runs, see the seminal contribution of Diamond and Dybvig (1983). For the relationship between coordination failure and liquidity dry-ups and currency attacks, see Morris and Shin (1998) and Mahlerbe (2014).
how to split the value they would create by taking their product to the market, and payoffs accrue.

We now provide more details about the choices firms face in each period.

In period 1, firms in each pair can choose between three organizational forms: unstructured partnership, integration, or structured partnership. For a given pair \( i = 1, \ldots, N \), denote the upstream firm by \( U_i \) and the downstream firm by \( D_i \). If a \((U_i, D_i)\) pair forms an unstructured partnership, then both \( U_i \) and \( D_i \) maximize their individual payoffs in all later periods. That is, an unstructured partnership provides no constraints to the firms’ subsequent interactions. Integration means that \( U_i \) and \( D_i \) merge their operations and are only concerned with the position of their joint balance sheet. Thus, merged firms maximize their joint payoff in later periods. A structured partnership means that the firms maximize their individual payoffs in later periods, but first sign a contract stipulating how to divide the value of the final product, should they agree to bring it to the market together. In the rest of this section and in the next, we restrict attention to unstructured partnerships (which we will often simply call partnerships). Sections 4 and 5 provide more details about integration and structured partnerships, and explore whether firms may want to choose these alternative organizational forms.

In period 2, firms \( U_i \) and \( D_i \) in each partnership \( i \) simultaneously and independently choose whether or not to invest. A firm that invests must pay cost \( I/2 > 0 \). Collaboration occurs if and only if both \( U_i \) and \( D_i \) invest. If collaboration occurs, then the joint value produced by the partners is high, \( \Delta > 0 \), and otherwise the value produced is low, where we normalize this low value to zero. Thus, the parameter \( \Delta \) can be seen as the added value generated by collaboration. For example, \( \Delta \) could be the difference in value between a car with two high-quality components (good body and good chassis) and a car with two lower-quality components (average body and average chassis).

In period 3, firms engage in multiple stages of Nash bargaining. At \( s = 1 \), \( U_i \) and \( D_i \) bargain over how to split the value generated in period 2, with bargaining weights equal to one half. If the firms come to an agreement, then they go to the product market and realize the product’s value (either \( \Delta \) or 0). Payoffs are realized pursuant to the bargaining agreement and \( U_i \) and \( D_i \) then leave the game. If \( U_i \) and \( D_i \) fail to agree, then the firms separate, where we assume separations are for good. Both firms enter the rematching market and can potentially meet other firms that
also separated from their initial partners.

At \( s = 2 \), all firms in the rematching market are randomly rematched into new pairs, \((U_i, D_j) \neq (U_i, D_i)\).\(^7\) Firms \( U_i \) and \( D_j \) then bargain over how to split the value they would generate by going to the product market. If neither pair \( i \) nor pair \( j \) collaborated in period 1, then this value is equal to zero. If both pairs collaborated, then this value is equal to \( \Delta \in (0, \bar{\Delta}) \), where \( \bar{\Delta} - \Delta \) measures the extent to which the period-1-investment was relationship specific.\(^8\) If pair \( i \) collaborated but pair \( j \) did not, then this value is equal to \( \alpha \Delta \). If pair \( j \) collaborated but pair \( i \) did not, then this value is equal to \( (1 - \alpha)\Delta \). Thus, \( \alpha \Delta \) can be interpreted as the added value of a good car body when the chassis is average, and \( (1 - \alpha)\Delta \) as the added value of a good chassis when the body is average. We assume \( \alpha \in (1/2, 1] \), so the car body is relatively more important (or more easily redeployable) than the chassis. If \( U_i \) and \( D_j \) come to an agreement, then they go to the product market and realize the product’s value. Payoffs are realized pursuant to the bargaining agreement and the two firms then leave the game. If \( U_i \) and \( D_j \) do not agree, then they separate, and both firms reenter the rematching market.

At \( s = 3 \), all firms in the rematching market are again matched into new pairs, and bargaining takes place exactly as at \( s = 2 \). This process continues in all subsequent stages until either all firms have left the game or no further rematching is possible. Any firm remaining in the game when the process terminates earns a payoff of zero.\(^9\)

Conditional on all firms being in partnerships, a strategy consists of a decision whether to invest in period 2, along with a decision whether to go to the product market at each stage \( s \) of period 3, for any given history. All actions are observable, but investment decisions are not verifiable and hence not contractible. We look for a Subgame Perfect Nash Equilibrium in pure strategies where each firm’s strategy maximizes its expected payoff, after any history, given the strategies of others.

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\(^7\)The only structure we place on the rematching mechanism is anonymity with respect to organizational form, which is relevant when we consider integration and structured partnerships.

\(^8\)That is, \( \bar{\Delta} - \Delta \) measures the extent of misfit between the two components. These components may not be perfectly tailored to one another, given that they are the fruit of collaboration that occurred between other firms.

\(^9\)Our results would remain unchanged if the terminal payoff was strictly negative, which would reflect the fact that a product with average components (whose value is normalized to zero) is still better than no product at all.
We also require that in a candidate equilibrium where neither \( U_i \) nor \( D_i \) invests, both firms cannot strictly increase their payoffs by both investing. This stability requirement effectively allows firms within a partnership to jointly deviate in their investment decision, and thereby rules out coordination problems within pairs. The intuition is that coordination should be relatively straightforward within a partnership, but not generally between different pairs across the market.\(^{10}\) Finally, if firms are indifferent at any given stage between going to the product market or separating, then we assume they will separate. Section 6 discusses the implication of relaxing this tie-breaking rule.

### 3 Baseline Analysis

Throughout this section, we assume that all firms choose partnerships, and we concentrate on their incentives for investment. For firms \( U_i \) and \( D_j \) matched at some stage \( s \) of period 3, let \( V_{ij} \) denote the value this pair can create by going to the product market.\(^{11}\) Let \( \pi_{U_i} \) and \( \pi_{D_i} \) denote the expected value of \( U_i \) and \( D_i \)’s outside options, which the firms expect to earn if they separate. These firms will come to an agreement if the surplus they bargain over is strictly positive, \( V_{ij} - (\pi_{U_i} + \pi_{D_j}) > 0 \).

The way they then split \( V_{ij} \) depends on the relative strength of their outside options, where Nash bargaining implies

\[
\Pi_{U_i} = \pi_{U_i} + \frac{1}{2}(V_{ij} - \pi_{U_i} - \pi_{D_j}),
\]

\[
\Pi_{D_j} = \pi_{D_j} + \frac{1}{2}(V_{ij} - \pi_{U_i} - \pi_{D_j}).
\]

We first present two lemmas showing that an initial pair that collaborates will immediately go to the product market, and showing how the firms in this pair will split the value created.

**Lemma 1.** At stage \( s = 1 \) of period 3, all collaborating pairs will go to the product market, and all other pairs will separate. At any stage \( s \geq 2 \), firms \((U_j, D_k)\) that are currently matched will separate if neither \( U_j \) nor \( D_k \) was part of an initial pair that collaborated.

\(^{10}\)Moreover, our interest is not in low-investment equilibria caused by miscoordination within pairs, but instead in possible externalities between pairs and the impact of different organizational forms.

\(^{11}\)Specifically, if \( s = 1 \), then \( V_{ij} = \Delta \) if the pair collaborated in period 2 and \( V_{ij} = 0 \) otherwise. If \( s \geq 2 \), then \( V_{ij} \) either equals \( \Delta, \alpha \Delta, (1 - \alpha) \Delta, \) or 0, depending on the history.
Firms that collaborate will immediately take their product to the market because investment is partly specific, $\Delta > \Delta$. These firms can create more value together than they could in any other pair to which either might later belong. In contrast, pairs where neither firm was involved in collaboration have no surplus to bargain over. These firms can go to the product market and earn a payoff of zero, but they can also guarantee themselves at least as much by separating, and possibly rematching with a firm that collaborated.

**Lemma 2.** Suppose that pair $i$ collaborates and $m - 1 \leq N - 1$ pairs do not. Then gross of investment costs, firms in pair $i$ earn

$$\Pi_{U_i} = \frac{\Delta}{2} + \left(\alpha - \frac{1}{2}\right) \left[1 - \left(\frac{1}{2}\right)^{m-1}\right] \Delta,$$

$$\Pi_{D_j} = \frac{\Delta}{2} - \left(\alpha - \frac{1}{2}\right) \left[1 - \left(\frac{1}{2}\right)^{m-1}\right] \Delta.$$ 

Lemma 2 shows that an upstream firm captures at least half the value generated by collaboration, and strictly more than half this value if at least one pair did not collaborate. In this latter case, the upstream firm earns strictly more than its downstream partner.\(^{12}\) The upstream firm’s payoff is increasing in the relative importance of its component, $\alpha$, the non-specificity of investment, $\Delta$, and the number of non-collaborating pairs, $m - 1$, whereas the downstream firm’s payoff is decreasing in all these parameters.

By Lemma 1, the number of non-collaborating pairs determines the size of the rematching market, which affects collaborating pairs by changing the relative strength of firms’ outside options. If the rematching market is large, then these outside options can be quite valuable. Firms in a pair that collaborate and separate then have a strong bargaining position relative to their new partners. This is both because a large rematching market provides many potential partners, and because any new partner realizes that it will never rematch with another firm that collaborated (by Lemma 1). The upstream firm benefits more than the downstream firm from this strong position because its component is more important, $\alpha > 1/2$. A larger rematching market magnifies this asymmetry.

\(^{12}\)This follows from $\alpha > 1/2$, $\Delta > 0$, and $m - 1 > 0$. 

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in favor of upstream firms, and generates a shift in bargaining positions that hurts their initial downstream partners.\footnote{Notice that increase in rematching market size will reduce a downstream firm’s payoff, despite increasing the value of its outside option. This is because the upstream firm’s outside option increases in value by even more.}

This ex post shift of bargaining positions can generate a hold-up problem and affect ex ante incentives for investment. Collaboration is efficient, $\Delta > I$, but it will only occur if the private benefits to both firms exceed their investment costs. By Lemma 2, the relevant incentive constraint involves the downstream firm, $\Pi_D \geq I/2$, or equivalently

$$\frac{\Delta}{2} - \left( \frac{\alpha - 1}{2} \right) \left[ 1 - \left( \frac{1}{m} \right)^{-1} \right] \Delta \geq \frac{I}{2}. \quad (3)$$

From (3), it is clear that a downstream firm will not invest if it expects the rematching market $m - 1$ to be “sufficiently large”. Using (3), we obtain the following definition.

**Definition 1.** The critical market size, $N^*$, is defined as follows. If

$$\left( \frac{\alpha - 1}{2} \right) > \frac{1}{\Delta} \left( \frac{\Delta - I}{2} \right), \quad (4)$$

then $N^*$ is the value of $m \in \mathbb{R}^+$ for which (3) holds with equality. If (4) is violated, then $N^* \equiv +\infty$.

Downstream firms will not invest if they expect the rematching market to exceed the critical size $N^* - 1$. In particular, this size depends on $\alpha$, the asymmetry in importance of the two firms’ components. If this asymmetry is small, $\alpha \approx 1/2$, then the hold-up problem is relatively mild, and $N^*$ is infinite. But if this asymmetry is large, $\alpha \gg 1/2$, and also $\Delta > \Delta - I$, then $N^*$ is finite, so that a large rematching market will generate sufficiently large shifts in bargaining positions to change investment behavior. We comment more on this issue below after presenting our results in Proposition 1.

Figure 1 illustrates for specific parameter values how investment incentives depend on the size of the rematching market. Specifically, it plots the payoffs for firms that collaborate as a function of how many pairs do not. The upstream firm’s payoff is given by the dashed curve, whereas the downstream firm’s payoff is given by the dotted curve.
Proposition 1. If \( N < N^* \), then a unique equilibrium exists, where all firms invest. If \( N \geq N^* \), then two equilibria exist and they are Pareto-ranked: one equilibrium where all firms invest, and another where no firm invests.

Proposition 1 shows that small markets and markets with small asymmetries will have high aggregate investment. Collaborating strictly benefits all firms in such markets, regardless of how any other pairs also collaborate, as the rematching market always remains below the critical size. In
contrast, large markets with relatively large asymmetries may exhibit low- and high-trust equilibria. The high-trust equilibrium is characterized by optimism and by high investment. Firms in each pair expects all others to collaborate and immediately go to the product market, which gives them an incentive to collaborate themselves. This situation contrasts with the inefficient low-trust equilibrium, where pessimism about aggregate investment is self-confirming. Multiple equilibria exist because the severity of each pair’s hold-up problem is endogenous and depends on other pairs’ investment decisions. Specifically, externalities between pairs mean that downstream firms view investment decisions as strategic complements: the more pairs that collaborate, the smaller the hold-up problem, so the more downstream firms benefit from collaborating.\footnote{Upstream firms instead view investment decisions as strategic substitutes. However, since only the downstream firm incentive constraint can bind, aggregate outcomes are shaped by complementarities.}

Inefficiently low investment is only possible when $N \geq N^*$. From this inequality and Definition 1, Corollary 1 immediately follows.

**Corollary 1.** Inefficiently low investment is more likely when (i) the initial asymmetry between upstream and downstream firms is large ($\alpha$ close to 1), (ii) components are easily redeployable ($\Delta$ close to $\bar{\Delta}$), and (iii) the market size, $N$, is large.

Corollary 1 has a distinctly anti-TCE flavour. Transaction-cost economics emphasizes asset specificity and small-numbers bargaining problems. In our model, bargaining frictions are enhanced when components are easily redeployable and markets are large. However, the problem is not just the availability of many potential trading partners (i.e., a large-numbers bargaining problem). Asymmetries between partners are also crucial.

To see this, note that sufficiently high values of each of the three key parameters $\alpha$, $\Delta$ and $N$ are necessary for inefficiencies to emerge. Consider for instance the case with a large market ($N$ large) and almost perfectly redeployable components ($\Delta \approx \bar{\Delta}$). The potential for opportunism is then high because a firm that separates from its initial partner can create relatively high value and rematch with one of many possible partners. Nonetheless, the unique equilibrium will involve high investment as long as $\alpha$ is sufficiently small.\footnote{This claim follows from the fact that $N^* \equiv +\infty$ when $\alpha$ is sufficiently close to $1/2$. Using Definition 1, a necessary
stream firm will still come to agreement after collaborating, despite their attractive outside options, since immediately going to the product market still generates the most value. The potential for opportunism is equally balanced between upstream and downstream firms and therefore does not generate any shift in bargaining positions. As a result, it does not affect ex ante incentives for investment.

**Equilibrium selection.** In the rest of the analysis, we will assume that firms will play the low-trust equilibrium whenever it exists. That is, hold-up problems in large markets with large asymmetries will destroy investment incentives. We make this assumption for two main reasons.

First, there is nothing interesting we can say about the choice of organizational forms if the high-trust equilibrium is always played. Unstructured partnerships are the least costly governance form and in the high-trust equilibrium firms in unstructured partnerships always invest. This is clearly the best possible outcome for each pair. Thus, unless we introduce some form of ‘failure’ in the market for partnerships, no pair would have an incentive to deviate from unstructured partnerships. Coordination failure as in the low-trust equilibrium is one type of failure. Our paper can be seen as an investigation of how, through decentralized governance choices, the adverse consequences of coordination failure in markets can be mitigated.

Second, failure to coordinate to a superior equilibrium is plausible if the inferior equilibrium is considerably “less risky”. In the supplementary appendix, we make this argument precise by using the concept of \( p \)-dominance (Morris et al., 1995).\(^{16}\) \( p \)-dominance for \( p < 1/2 \) generalizes the notion of risk dominance in symmetric games to settings with more than two players. There is considerable experimental evidence that coordination failure, in the sense of playing equilibria that are Pareto inefficient, tends to occur when the efficient equilibria are relatively risky, i.e. if they give a very low payoff when others do not coordinate.\(^{17}\) Theoretical work has also shown that the

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\(^{16}\) An equilibrium is \( p \)-dominant, for a particular value of \( p \in [0, 1] \), if each player’s strategy is a best reply to any conjecture that places probability at least \( p \) on all other players choosing their equilibrium strategies. Thus, \( p \)-dominance captures the idea that no player should have an incentive to deviate, as long as they believe there is sufficiently low risk that others will play something unexpected.

\(^{17}\) See Devetag and Ortmann (2007) for a review of the experimental work on coordination failure.
equilibrium selected by $p$- or risk dominance has attractive robustness properties. This equilibrium is robust to the introduction of incomplete information (Carlsson and Van Damme, 1993; Kajii and Morris, 1997) and is selected by a variety of evolutionary approaches (Young, 1993; Kandori et al., 1993; Maruta, 1997; Ellison, 2000; Oyama, 2002), even if other equilibria are more efficient.

In our setting, $p$-dominance for $p < 1/2$ selects the low-trust equilibrium if and only if the market is sufficiently large, that is, if $N$ exceeds a critical value $N^{*}_{p=1/2}$. This critical value itself exceeds $N^{*}$ and depends on parameter values in precisely the same way that $N^{*}$ does, as described in Corollary 1. Thus, Proposition 1 and Corollary 1 still qualitatively hold—one simply has to replace the condition $N \geq N^{*}$ with the more restrictive condition $N \geq N^{*}_{p=1/2}$. In what follows, instead of assuming that firms play the low-trust equilibrium whenever it exists, one could assume that firms play the low-trust equilibrium only when $N \geq N^{*}_{p=1/2}$.

4 Integration

Having shown how investment incentives can break down in partnerships, we explore the impact of allowing firms to choose between different organizational forms. Specifically, for each initial pair $i$, we now assume that firms $U_{i}$ and $D_{i}$ can form a partnership, as in Section 3, or they can instead choose to integrate. Integration carries an explicit cost $L \in (0, \Delta - I)$, to cover any tendering, overhead, dilution of incentives, or related expenses.\(^{18}\) The advantage of integration is that it changes $U_{i}$ and $D_{i}$’s incentives.

Specifically, we assume that integrated firms merge their operations and are only concerned with the position of their joint balance sheet. This means that if $U_{i}$ and $D_{i}$ integrate, they choose their subsequent actions so as to maximize their joint payoff. Thus, our approach to modelling integration echoes that of Hart and Holmstrom (2010), in the sense of assuming that integration solves the hold-up problem. We take this reduced-form approach to maintain focus on our issue

\(^{18}\)We assume that the explicit cost of forming a partnership is equal to zero. Thus, $L$ can be interpreted as the extra cost associated with integration, over and above any cost associated with a partnership. The assumption that $L < \Delta - I$ ensures that integration can create positive value. If $L > \Delta - I$, then integration would be too costly and would never be selected.
of interest: externalities in firms’ choice of organizational forms, in particular how the decision of certain pairs to integrate affects the incentives of other pairs in the market.\textsuperscript{19}

This way, integration changes firms’ incentives, but it does not affect the set of actions available to \(U_i\) and \(D_i\) in later periods.\textsuperscript{20} Both firms must still decide whether to invest in period 2, where mutual investment is necessary for collaboration. They can agree to take their product to the market in period 3 and realize its value, or instead separate for good, in which case they both enter the rematching market. No bargaining takes place in period 3 under integration, unlike in a partnership, since integrated firms make the decision that maximizes their joint payoff.

In equilibrium, we require that no pair in a partnership can strictly increase their joint payoff by integrating. Moreover, no integrated pair can strictly increase their joint payoff by instead forming a partnership. These requirements fit naturally with our approach that coordination should be relatively simple within pairs, and that firms will integrate if doing so is in their joint interest. As in Section 3, all actions are observable, including each pair’s choice of organizational form.

Finally, we assume that conditional on any given choice of organizational forms, firms will play the equilibrium of the subsequent subgame with the lowest level of aggregate investment. This amounts in practice to saying that whenever the number of pairs in partnerships exceeds the critical value \(N^*\), from Definition 1, these pairs will play the low-trust equilibrium where none invest, rather than the high-trust equilibrium where all invest. Our qualitative results are unchanged if we assumed that these pairs only play the low-trust equilibrium if it is less risky than the high-trust

\textsuperscript{19}Our reduced-form approach to integration allows for a variety of interpretations. Integration could give an outsider formal authority over unit managers, as in Hart and Holmstrom (2010), allowing the former to instruct the latter to take particular decisions. Costs would then correspond to possible effort shading under integration or to the outsider’s inability to internalize the managers’ private benefit. Alternatively, integration might allow one firm to acquire all the relevant expertise necessary for production from the other, thus eliminating the potential for opportunism but at some explicit cost. In both cases, it is the new organizational structure, rather than any explicit contract, that eliminates the risk of hold-up. A potential limitation of contractual solutions is that some information may be verifiable, as we discuss in Sections 5 and 6.

\textsuperscript{20}An alternative approach would be to assume that separation is prohibitively costly for integrated firms, which would effectively prevent \(U_i\) and \(D_i\) from entering the rematching market. This alternative approach to modelling integration would give identical results.
equilibrium, for the reasons described at the end of Section 3.

Our next result shows that integration can dramatically affect the level of aggregate investment. In particular, it not only affects the investment decisions of firms that integrate, but also the investment decisions of firms that do not.

**Proposition 2.** Suppose that firms can either form partnerships or integrate. Then in equilibrium, all firms will invest. No pair integrates if \( N < N^* \), whereas \( K \) pairs integrate if \( N \geq N^* \), with \( N - N^* < K \leq N - N^* + 1 \).

Allowing firms to integrate rules out the inefficient low-trust equilibrium from Proposition 1 and guarantees high investment. Both firms in a pair that integrates will choose to invest, since integration effectively solves their hold-up problem. Integrated firms are only concerned with maximizing their joint payoff and can therefore ignore any shifts of bargaining positions that would affect investment incentives under partnerships. Because of specificity, the way to maximize their joint payoff is to collaborate and immediately go to the product market.

More subtly, the behavior of firms that integrate also encourages investment from firms that do not. By immediately going to the product market, integrated firms reduce the potential size of the rematching market for firms in partnerships. The equilibrium number of partnerships drops below the critical size, limiting the possible shift in bargaining positions away from downstream firms, and increasing their incentives for investment. Effectively, the strategic complementary in investment that led to multiple equilibria in Proposition 1 generates a strategic substitutability between different organizational forms. When a sufficiently high number of firms integrate, downstream firms in partnerships become willing to invest, which makes partnership more profitable.

One consequence of strategic substitutability is that different organizational forms can coexist alongside one another. In large markets with relatively large asymmetries, integrated firms exist alongside firms in partnerships, even though all firms are ex-ante identical. Firms in partnerships earn the higher joint payoff since they save on the cost of integration. And yet each integrated pair realizes that deviating to a partnership would reduce their joint payoff, since the rematching market would become large enough to destroy incentives for investment. Just enough pairs integrate to
guarantee that both firms in each partnership can recover their investment costs, regardless of how other non-integrated pairs behave.

Figure 2 depicts the distribution of organizational forms for different parameter values. The vertical axis shows the total market size, $N$, and the horizontal axis shows the relative importance of an upstream firm’s component, $\alpha$. The dashed vertical line denotes the value of $\alpha$ for which (4) holds with equality. To the left of this line, the critical market size is infinite, so that all firms form partnerships. To the right of this line, the critical size is finite, and different organizational forms co-exist whenever the market exceeds this size.

Fixing a value of $\alpha$ to the right of the dashed line in Figure 2 and allowing $N$ to vary illustrates that the amount of integration is increasing in market size. As the market becomes arbitrarily large, the fraction of integrating firms tends to one, since the number of partnerships cannot exceed the critical size. Fixing a value of $N$ and allowing $\alpha$ to vary illustrates that the amount of integration is also increasing in the asymmetry between upstream and downstream firms. The intuition is
that large asymmetries lead to large shifts in bargaining positions, which hurt downstream firms in partnerships. More integration is then required to reduce the hold-up problem in non-integrated pairs and support investment.

The notion that large markets and large asymmetries encourage integration holds more generally, as can be seen in the following corollary to Proposition 2.

**Corollary 2.** The number of integrated firms is increasing in (i) the initial asymmetry between upstream and downstream firms, $\alpha > 1/2$, (iii) the non-specificity of investment, $\Delta$, and (iii) the market size, $N$.

These results (which parallel those of Corollary 1) follow immediately from Proposition 2 and the definition of $N^\ast$. As well as confirming the intuition from Figure 2, the corollary shows that integration is more common in markets where investment is less specific. Non-specific investment makes firms more tempted to use the rematching market, which increases the hold-up problem for firms in partnerships, and allows integration to play a greater role.

### 5 Structured Partnerships

We now explore whether structured partnerships can also play a role in supporting high levels of aggregate investment. Proposition 2 shows that allowing firms to integrate can restore incentives that may collapse in unstructured partnerships, but a potential problem with integration is its cost.\footnote{Throughout this section, we will explicitly refer to unstructured partnerships, rather than simply partnerships, in order to clearly distinguish them from structured partnerships.} If $L$ is relatively close to $\Delta - I$, then the cost of integration takes up a significant portion of the net value created by investment, suggesting firms might seek alternative ways to structure their relationships.

In our setting, a structured partnership for pair $i$ consists of a contract between $U_i$ and $D_i$ that specifies each firm’s payoff if they collaborate in period 2 and go to the product market together in period 3. The contract does not oblige the firms to take their product to the market. Instead, $U_i$ and $D_i$ can choose between doing so and realizing the payoffs specified under the contract or...
separating. If either \( U_i \) and \( D_i \) chooses to separate, then both firms enter the rematching market where they can potentially form new partners, as in the earlier analysis. We assume that forming a structured partnership is less costly than integration, where contracting costs \( C > 0 \) are small and are split equally between the two parties.

Our approach to contracting reflects the idea that after firms jointly take a product to the market, a court should be able to verify how they divide sales revenue between them. We do not consider contracts that effectively punish a firm for separating from its initial partner, for example through termination fees, because such contracts will be difficult to enforce. If an initial pair splits up, then a court may have difficulty verifying the extent to which collaboration actually took place and which firm was responsible for separation. Moreover, courts are often unwilling to enforce breach damages that they deem to be excessive (Chung, 1998).

As a result, contracting cannot force a pair that collaborates to go to the product market, but it can fix the way they divide the value generated if they choose to do so. It follows that a structured partnership can be summarized by parameter \( \beta \in [0, 1] \), where \( U_i \) earns \( \beta \Delta \) and \( D_i \) earns \( (1 - \beta)\Delta \) from going to the product market after collaborating. We will say that structured partnership \( \beta \) is implementable if both \( U_i \) and \( D_i \) both have a strict incentive to realize the payoffs specified in the contract rather than take their outside options: \( \beta \Delta > \pi_{U_i} \) and \( (1 - \beta)\Delta > \pi_{D_i} \). Otherwise we say that \( \beta \) is not implementable. A non-implementable structured partnership effectively has no bite, because at least one firm can credibly threaten to take its outside option. Firms will then have an incentive to renegotiate the contract rather than separate and lose value. We assume that in this case, the firms will renegotiate to the same agreement as in an unstructured partnership, effectively bargaining with Nash weights equal to one half.

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22 As described below, we also allow the firms to renegotiate if either \( U_i \) and \( D_i \) can credible argue they would otherwise opt for separation.

23 We assume that only initial pairs can form structured partnerships. Firms that meet in the rematching market would never do so, since neither firm has an incentive to give up bargaining power after investment decisions have been made.

24 We comment more below on precisely what is meant by ‘small’, and how our results would change if the cost of forming a structured partnership was larger.

25 We examine and discuss more extensively termination-fee contracts in Section 5.3.
A strategy in this setting is just as in Section 4, but now firms can now choose between three organizational forms in period 1, where opting for a structured partnership also involves a choice of $\beta$. We generalize our earlier approach by making the following additional requirements on equilibrium choices of organizational form. First, no pair in a structured partnership can strictly increase their joint payoff by integrating. Second, no integrated pair can strictly increase their joint payoff by forming a structured partnership. Third, no pair in an unstructured partnership can strictly increase both firms’ payoffs by forming a structured partnership. Fourth, no firm in a structured partnership can strictly increase its payoff if the two firms instead formed an unstructured partnership.\(^{26}\)

In order to describe how the possibility of structured partnerships can affect equilibrium outcomes, consider condition

$$\Delta - \left(\frac{I + C}{2}\right) \geq \left[1 - \left(\frac{1}{2}\right)^{m-1}\right] \alpha \Delta, \quad (5)$$

and the following definition.

**Definition 2.** Define the critical market size for structured partnerships, $N^{**}$, as follows. If

$$\alpha > \frac{1}{\Delta} \left(\Delta - \frac{I + C}{2}\right), \quad (6)$$

then $N^{**}$ is the value of $m \in \mathbb{R}^+$ for which (5) holds with equality. If (6) is violated, then $N^{**} = +\infty$.

The critical market size for structured partnerships can be given the following interpretation. If the market is below this size, $N < N^{**}$, then there exists some structured partnership that a downstream firm would be willing to accept in period 1, that would lead to collaboration in period 2, and that both firms would agree to implement in period 3.\(^{27}\) If instead $N \geq N^{**}$, then no

\(^{26}\)These requirements reflect how firms will integrate if doing so is in their joint interest, as in Section 4, and also that forming a structured partnership requires mutual consent.

\(^{27}\)To see why this is the case, notice that the left-hand-side of (5) is the maximum value that an upstream firm can ever receive in a structured partnership, given that the downstream firm must earn back its cost from forming the structured partnership and investing. The right-hand-side of (5) is an investing upstream firm’s outside option when $m - 1$ pairs do not invest, as derived in the proof of Lemma 2.
structured partnership that is implementable would give downstream firms sufficient incentives to pay the contracting and investment costs.

Comparing Definitions 1 and 2 shows that the critical market size for structured partnerships is larger than the critical market size from Section 3, by our assumption that contracting costs are small. That is, \( N^{**} \geq N^* \) holds for \( C \) sufficiently small, where the inequality is strict whenever the \( N^* \) is finite.\(^{28}\) This suggests that structured partnerships may play a role in promoting investment in relatively large markets, as reflected in the following result.

**Proposition 3.** Suppose that firms can either form unstructured partnerships, structured partnerships, or integrate in period 1, and that the cost \( C > 0 \) of forming a structured partnership is sufficiently small. Then in equilibrium, all firms will invest. Moreover:

(i) If \( N < N^* \), then all firms form unstructured partnerships.

(ii) If \( N^* \leq N < N^{**} \), then \( M \) pairs form structured partnerships and \( N - M \) pairs form unstructured partnerships, where \( N - N^* < M \leq N - N^* + 1 \).

(iii) If \( N^{**} \leq N \), then \( K \) pairs integrate, \( M \) pairs form structured partnerships, and \( N - K - M \) pairs form unstructured partnerships, where \( N - N^{**} < K \leq N - N^{**} + 1 \), and where \( (N - K) - N^* < M \leq (N - K) - N^* + 1 \) if \( \lceil N^* \rceil < \lceil N^{**} \rceil \) and \( M = 0 \) if \( \lceil N^* \rceil = \lceil N^{**} \rceil \).

Structured partnerships can help in precisely the same circumstances where Section 4 suggested some firms would opt for integration. Specifically, they help whenever investment incentives would break down if all firms formed unstructured partnerships. Structured partnerships support investment by granting more surplus to downstream firms than they would obtain under Nash bargaining, which plays a key role in situations off the equilibrium path where some pairs do not invest. One asymmetry (the downstream firm receives higher surplus) then helps counteract another (the upstream firm has a better outside option), limiting shifts in bargaining positions towards the upstream firm and making the hold-up problem less severe.\(^{29}\)

\(^{28}\)A necessary and sufficient condition for \( N^{**} \geq N^* \) to hold is \( C < \bar{\Delta} - \Delta \).

\(^{29}\)In this sense, an equivalent way of understanding such a structured partnership is that it effectively specifies a bargaining weight for the downstream firm that exceeds one half.
In moderately large markets, $N^* \leq N < N^{**}$, structured partnerships will crowd out all integration, by providing investment incentives while allowing firms to save on integration costs. But Proposition 3 shows that integration still has a role to play when markets are sufficiently large, $N \geq N^{**}$. The issue with such markets is that structured partnerships are difficult to implement. Large markets provide little surplus for firms to bargain over, at least if they expect all other pairs to separate, since the value of firms' outside options is increasing in rematching market size. This means that even the most favorable structured partnership for a downstream firm, one that grants it the entire surplus following collaboration, cannot allow it to recover its costs. Put another way, no structured partnership that would allow the downstream firm to recover its costs can actually be implemented, because the upstream firm could credibly threaten to separate after collaboration has taken place.

Proposition 3 shows that large markets with sufficiently large asymmetries are characterized by the coexistence of three organizational firms: unstructured partnerships, structured partnerships, and integration. From a joint payoff perspective, unstructured partnerships are more profitable than structured partnerships, and both are more profitable than integration. The reason that firms opt for different organizational forms is again because they serve as strategic substitutes. Forming a structured partnerships helps promote collaboration in unstructured partnerships, and integrating does the same in both unstructured partnerships and structured partnerships. Each pair knows that deviating to a less costly organizational form would stop other pairs from collaborating, which would increase rematching market size and destroy their own investment incentives. It follows that even though both structured partnerships and integration may appear inefficient, because they are costly, these organizational forms generate externalities that help sustain an efficient aggregate outcome.

Figure 3 illustrates the distribution of organizational forms as a function of parameter values. Just as in Figure 2, the vertical axis shows the total market size, $N$, the horizontal axis shows the asymmetry in the firms' components, $\alpha$, and the dashed vertical line to the left denotes the value of $\alpha$ for which (4) holds with equality. The additional dashed vertical line to the right denotes the value of $\alpha$ for which (6) holds with equality. Looking at both figures, part of the area that previously
showed integration has now been replaced by a new area showing structured partnerships.

Examining how the distribution of organizational forms in large markets varies with $\alpha$ again highlights the key role of asymmetries. For low values of $\alpha$ (small asymmetries), to the left of both dashed lines, all firms form unstructured partnerships. For intermediate values of $\alpha$ (moderate asymmetries), between the two dashed lines, some firms form unstructured partnerships and others form structured partnerships. Finally, for high values of $\alpha$ (large asymmetries), to the right of both dashed lines, firms choose all three organizational forms. This comparison suggests that the net impact of increased asymmetries should be to push firms away from unstructured partnerships and towards integration, but should have an ambiguous impact on structured partnerships.

6 Extensions and Robustness

We now examine several extensions to the basic model.
6.1 Asymmetries in the number of firms

A main feature of our analysis has been the link between investment incentives and asymmetries, specifically the asymmetry in the relative importance of firms’ component. We now consider an additional type of asymmetry: in the number of upstream and downstream firms. In what follows, firms form unstructured partnerships in period 1, as in the baseline analysis of Section 3, the difference being that the number $N_U$ upstream firms need not equal the number $N_D$ of downstream firms. The number of initial pairs is therefore given by $\min(N_U, N_D)$. In period 2, firms in each pair choose whether to invest, whereas unmatched firms have no action to take. Period 3 proceeds just as in the baseline, except now the rematching market includes all firms that separate and all firms that were not initially matched.\(^{30}\)

We now demonstrate how asymmetries in firm numbers can affect investment incentives.

**Proposition 4.** Suppose that all firms form unstructured partnerships, and fix $\bar{\Delta}$, $I$, and $N$ at values such that $\left(\frac{1}{2}\right)^{N-1} \bar{\Delta} < I$. Then there exist $\alpha > 1/2$ and $\bar{\Delta} < \bar{\Delta}$ such that in equilibrium (i) no firm invests if $N_d = N_u = N$, and (ii) all initially-matched firms invest if $N_d = N$ and $N_u \geq N + 1$. Moreover, for any parameter values such that $N^*$ is finite, no firm will invest in equilibrium if both $N_d$ and $N_u$ are sufficiently large.

Proposition 4 shows that if upstream firms outnumber downstream firms, then the hold-up problem in partnerships may be less severe than suggested in the baseline. Specifically, there are parameter values for which a low-trust equilibrium with no investment exists according to Proposition 1, but where introducing additional upstream firms into the market causes this equilibrium to break down, thereby guaranteeing full investment.\(^{31}\) It follows that investment incentives in partnerships then remain intact even though the market exceeds the critical size $N^*$.

The intuition is that the presence of extra upstream firms in the rematching market increases

\(^{30}\)We continue to assume that separations are for good, and that firms in pairs that separate are randomly re-matched. However, unlike in the baseline analysis, some firms will remain unmatched in each stage of period 3, as long as $N_U \neq N_D$.

\(^{31}\)If $\left(\frac{1}{2}\right)^{N-1} \bar{\Delta} < I$ was violated, then $N^*$ would be infinite for all values of $\alpha$ and $\bar{\Delta}$, and a low-trust equilibrium would never exist in the baseline.
the value of a downstream firm’s outside option. As with structured partnerships, one type of asymmetry (here rematching market size) helps counteract another (in importance of components), which decreases the severity of the hold-up problem. However, there are two important caveats. First, if downstream firms outnumber upstream firms, then the effect would be reversed. One asymmetry would then reinforce the other and make the hold-up problem even more severe. Second, the effect of asymmetric firm numbers will be negligible in very large markets. In the limit as both \( N_u \) and \( N_d \) become large, investment incentives reduce to those in the baseline evaluated at \( N = \min(N_U, N_D) \). Aggregate investment will then break down in partnerships, regardless of whether the asymmetry \( N_u - N_d \) is large or small.\(^{32}\) Firms looking to restore investment incentives will then have to consider alternative organizational forms, as analyzed in Sections 4 and 5.

### 6.2 Large firms with upstream and downstream capabilities

In this extension, we allow for the possibility that some (or all) firms may be large. We define a large firm as one that can unilaterally go to the product market after collaborating, without having to find a partner. The payoff for a large upstream firm that does so is \( \alpha \Delta \), and for a large downstream firm is \( (1 - \alpha) \Delta \). These payoffs correspond to those from our earlier analysis when firms that initially collaborated separated and eventually went to the product market together with firms that did not. The interpretation is that the product may then have one high- and one lower-quality component. A large upstream firm may specialize in producing the car body, but it can also produce the chassis in house, if necessary, at a lower quality level. Thus, a large upstream firm also has some downstream capabilities.\(^{33}\)

Our next result demonstrates that the presence of large firms in the market will tend to reduce investment incentives.

**Proposition 5.** Suppose that either all firms are large, or all upstream firms are large and all

\(^{32}\)Formally, the reason for this result is that the marginal effect of increased firm numbers tends to zero as the market becomes large, since the value of outside options follows a geometric series.

\(^{33}\)We maintain all other assumptions from our baseline analysis, including that firms that do not collaborate in period 1 cannot create value in the product market.
downstream firms are not. Then if $N^*$ is finite, no firm will invest in equilibrium, regardless of market size, $N$.

The hold-up problem becomes more severe if all firms are large, as the analysis effectively reduces to that in the baseline with infinite market size. The reason is that all firms that collaborate and then separate can go alone to the product market. The firms then both capture the entire subsequent value, $\alpha \Delta$ and $(1 - \alpha) \Delta$, since there is no need for bargaining with additional partners. These outside options correspond to those from the baseline with an arbitrarily large rematching market, where firms that collaborate and separate also have all the bargaining power vis-a-vis their new partners. This result highlights how bargaining affects investment in different ways. The need for bargaining between initial partners following collaboration is what generates the hold-up problem. But the severity of this problem is limited by the need to bargain after rematching, forcing firms to share value with their new partners, and making separation less attractive.

If upstream firms are large but downstream firms are not, then investment incentives will fall further still. In this case, an upstream firm can immediately go to the product market after separation and capture the full value associated with its product. The downstream firm is unable to do so, and instead must enter the rematching market and bargain with a new partner. This additional asymmetry in favor of upstream firms further increases their bargaining power, reducing downstream-firm incentives for investment.\footnote{The situation is more complex if downstream firms are large but upstream firms are not. The hold-up problem can then be either more or less severe than in the baseline. The intuition is that increased downstream-firm bargaining power can help sustain investment incentives in relatively large markets, by reducing the asymmetry in favor of upstream firms. But it has the opposite effect in small markets, by allowing downstream firms to hold up their upstream partners.}

6.3 Robustness to contractual solutions

In Section 5, we examined the robustness of our results to one particular type of contract, which we termed \textit{structured partnerships}. Here, we briefly discuss why other commonly-observed contractual solutions may be of limited use in our setting.
Payments to collaborate. Payments provided by the “strong” partner to the “weak” partner to encourage the latter to invest will not solve the problems associated with shifting bargaining positions. The reason is that investment is not verifiable by a court. Thus, the weak partner would sign the contract, accept the payment, and then refuse to invest if doing so was in its own interest.

Termination-fee contracts. Termination-fee contracts may appear to be a potential solution to the problem of shifting bargaining positions. However, they will only work under very stringent assumptions. To see this, consider a simple model where the rematching market is modeled in a reduced form. Let \( U \) and \( D \)'s payoffs in the absence of investment be normalized to zero. As before, let \( I \) denote each firm's investment cost, and let \( \Delta \) denote a collaborating pairs' joint payoff from going to the product market. We now assume exogenous disagreement payoffs of \( \alpha \Delta \) and \( (1 - \alpha) \Delta \) for \( U \) and \( D \), respectively, with \( \alpha > \frac{1}{2} \) and \( \Delta > \Delta \). These same disagreement payoffs would emerge endogenously in our earlier analysis given an arbitrarily large rematching market.

Collaboration yields net payoffs of \( \alpha \Delta + \frac{1}{2}(\Delta - \Delta) \) and \( (1 - \alpha) \Delta + \frac{1}{2}(\Delta - \Delta) \) to \( U \) and \( D \), respectively. Suppose that \( (1 - \alpha) \Delta + \frac{1}{2}(\Delta - \Delta) < \frac{1}{2} I \), so that \( D \) would refuse to invest. \( U \) could then offer a termination-fee contract that pays \( F \) to \( D \) if \( U \) terminates the contract. In this case, once investment has occurred, \( U \)'s outside option is \( \alpha \Delta - F \), and \( D \)'s outside option is \( (1 - \alpha) \Delta + F \). The new gross payoffs that accrue to \( U \) and \( D \) after bargaining are therefore \( \alpha \Delta + \frac{1}{2}(\Delta - \Delta) - F \) and \( (1 - \alpha) \Delta + \frac{1}{2}(\Delta - \Delta) + F \), respectively. It follows that by setting \( F^* = \frac{1}{2} I - (1 - \alpha) \Delta - \frac{1}{2}(\Delta - \Delta) \), \( U \) can induce \( D \) to invest. \( U \) then enjoys the entire net value created by collaboration, \( \Delta - I \), whereas \( D \) earns a net payoff of zero.

That being said, the ability of termination fees to induce investment is limited. One important reason is that \( D \) may be tempted to renege from the very beginning. Given termination-fee contract \( F^* \), \( D \) may not invest, which may force \( U \) to terminate their relationship. In general, \( D \)'s temptation to renege will be very strong: its payoff from reneging, \( F^* \), exceeds its payoff from collaborating, which is zero.\(^{35}\) Thus, termination fees will not help unless the courts can indemnify \( U \) against reneging by \( D \).

\(^{35}\)More generally, given termination-fee contract \( F \), \( D \) will prefer to renege if \( F > (1 - \alpha) \Delta + \frac{1}{2}(\Delta - \Delta) + F - \frac{1}{2} I \), which holds whenever investment incentives break down, \((1 - \alpha) \Delta + \frac{1}{2}(\Delta - \Delta) - \frac{1}{2} I < 0.\)
Another issue with termination-fee contracts relates to their enforceability. In practice, U.S. courts have refused to enforce breach damages that are deemed to be excessive (Chung, 1998). This suggests that, if $F^*$ is ‘too high’ to be enforceable by courts, then $U_i$ will not be able to induce $D_i$ to invest. Moreover, $F^*$ will be high precisely in situations with particularly large shifts in bargaining positions that threaten investment incentives.

6.4 Other robustness issues

To further address issues of robustness, we also briefly discuss the extent to which our results depend on (i) our choice of tie-breaking rule, (ii) the details of the bargaining procedure, and (iii) the fact that structured partnership costs are small.

(i). Our analysis assumes that if firms are indifferent in period 3 between going to the product market or separating, then they will choose to separate. This tie-breaking rule is relevant off the equilibrium path, where non-collaborating pairs expect all pairs that collaborate to immediately go to the product market, leaving themselves indifferent about separating. Assuming the opposite tie-breaking rule would effectively shut down the rematching market and eliminate the hold-up problem. But such a rule would be unreasonable, since each firm in a non-collaborating pair views separation as a weakly dominant strategy. If there was any positive probability that a collaborating pair would separate, no matter how small, then all non-collaborating pairs would have a strict incentive to separate as well, as rematching could then lead to strictly positive value.

(ii). Our bargaining setup assumes that some asymmetry favors upstream firms over their downstream partners, and that outside options become more valuable with an increase in rematching market size. Other bargaining procedures that share these features would lead to similar qualitative results. The important thing for market size to matter is that some bargaining friction should prevent firms in a strong position from extracting all surplus from potential partners. The friction in our setup is the assumption of sequential rematching, which prevents a firm that collaborates and then separates from pitting other firms against one another à la Bertrand, for example through bidding to be the firm’s partner.\(^{36}\)

\(^{36}\)In the absence of such a friction, the only relevant question about size would be whether the rematching market
Our assumption of small costs $C$ for structured partnerships ensures that such partnerships can actually occur in equilibrium. If $C > L$, then no pair would opt for a structured partnership, because it would be cheaper to choose integration. If $C > \Delta - \Delta$, then the critical market size from Section 3 would exceed the critical size for structured partnerships, $N^* > N^{**}$, so structured partnership would never restore investment incentives. A sufficiently small value of $C$ is also necessary for a pure strategy equilibrium to exist when $N \geq N^{**}$, as in the case described in Proposition 3 (iii).\(^{37}\)

### 6.5 Alternative interpretation of the model

We conclude by offering an alternative interpretation of our model. Rather than producing product components, such as a chassis and car body, firms could be engaging in collaborative R&D. For instance, the upstream firm is a biotech company and the downstream firm is a pharmaceutical company, where collaboration results in a new idea to treat a specific disease. The partners can either implement the idea and generate joint value $\Delta$ or separate. In the latter case, with probability $\alpha$ the idea is appropriated by the upstream firm, and with probability $1 - \alpha$ by the downstream firm. Rematching then occurs, and a firm that appropriated an idea can implement it with its new partner. Doing so yields lower joint value $\Delta < \Delta$ because the idea was to some extent tailored to the capabilities of the firms that collaborated.

Although this set-up may seem slightly different than the one we have analyzed, they will both give identical results. With the R&D interpretation, firms would be uncertain about their payoffs from entering the rematching market, since payoffs would depend on which firm captured the idea. Firms would therefore bargain based on the expected value of their outside options, taking into account the relevant probabilities, $\alpha$ and $1 - \alpha$. Our analysis here has shown that a firm’s outside

\(^{37}\)The key point for existence of a pure strategy equilibrium is that $N^{**}$ should be sufficiently close to $N^{**}|C=0$, defined as the value that $N^{**}$ would take if $C = 0$. If not, then a pair that integrates in Proposition 3 (iii) may profitably deviate to a structured partnership, if the resulting number of non-integrated pairs satisfied $N^{**} \leq N - K + 1 < N^{**}|C=0$. All firms in structured partnerships will still invest following the deviation, even though $N^{**} \leq N - K + 1$, since the cost of forming a structured partnership is sunk.
option is proportional to the relative importance of its component, also captured by \( \alpha \) and \( 1 - \alpha \), which is why both set-ups yield the same investment incentives and bargaining outcomes.

For this argument to hold, there is no need for ideas to be imperfect substitutes. If two firms both appropriate an idea and rematch together, they might generate value \( 2\Delta \), which could exceed the value \( \Delta \) created in either of their initial pairs. However, from an ex ante perspective, separations would still be inefficient, as the firms that separate cannot earn more in expectation than they would by bargaining in their initial pairs, as shown in the proof of Lemma 1. It follows that on the equilibrium path, all firms that collaborate will immediately agree and leave the market, just as in our analysis. A collaborating firm that deviates by separating, and then appropriates an idea, realizes that it will never rematch with another firm that did the same, and so will bargain over at most \( \Delta \) in later stages.

7 Conclusion

Collaboration, information sharing and co-design within supply chains are widely acknowledged phenomena. This paper examines the stability of upstream-downstream partnerships in a setting where joint investment creates power imbalances. Endogenous shifts in bargaining positions may arise because of knowledge misappropriation, asymmetric learning, or simply because different product components have different values outside the relationship. These shifts create a hold-up problem between partners that can destroy investment incentives.

We identify three necessary conditions for inefficiencies in unstructured partnerships to occur: (i) initial asymmetries in appropriability between upstream and downstream firms, \( \alpha > 1/2 \), (ii) the non-specificity of investment, \( \Delta \), and (iii) a market size, \( N \), sufficiently large that initial asymmetries can effectively be exploited.

A contribution of our analysis is to distinguish asymmetric-numbers bargaining problems (i.e., the outcome of our asymmetric fundamental transformation) from small- and large-numbers bargaining problems (McLaren, 2003). In many real-world situations (although not all), the presence of a bilateral monopoly does not appear to impede cooperation. Faced with ‘mutual destruction',
most decision-makers seem to be able to arrive to satisfactory agreements. Asymmetries in power can be a more significant problem. To deal with these asymmetries, firms rely on a number of ‘dependence balancing mechanisms’. We focused on two equity-based mechanisms (integration and structured partnerships); however, many others (e.g., bilateral investment, reputations) are possible.

Our model can generate multiple equilibria, co-existence of organizational forms and performance heterogeneity, even though all pairs in the economy are ex ante identical. Unstructured partnerships are more profitable than more ‘hierarchical’ governance forms because they save on integration costs. Yet, it is precisely the positive externality generated by more hierarchical governance forms that allows others to be sustained. The business ecosystem rests on a delicate balance. To an external observer, ‘hierarchy’ may appear inefficient, but the ‘market’ cannot work properly without some degree of integration in the economy.

References


Appendix: Proofs

Proof of Lemma 1. Suppose that \( n \) out of \( m \) collaborating pairs separate, and index these pairs by \( i: (U_i, D_i) \), with \( i = 1, \ldots, n \). For pair \( i \) to separate, we must have \( V_{ii} = \Delta \leq \pi_{U_i} + \pi_{D_i} \), so their joint payoff in the product market must not exceed the joint value of their outside options. Summing over \( i \) gives

\[
\sum_{i=1}^{n} \Delta \leq \sum_{i=1}^{n} (\pi_{U_i} + \pi_{D_i}).
\]

The total expected payoff, gross of investment costs, for all firms that separate at \( s = 1 \) cannot exceed the total value that firms from the \( n \) collaborating pairs can generate as of \( s = 2 \). This implies

\[
\sum_{i=1}^{n} (\pi_{U_i} + \pi_{D_i}) \leq n \Delta.
\]

Combining with \( \sum_{i=1}^{n} (\pi_{U_i} + \pi_{D_i}) \geq \Delta \) rules out \( n \geq 1 \). If instead \( n = 0 \), then the same reasoning shows that each collaborating pair \( j \) will go to the product market, because \( \Delta > \pi_{U_j} + \pi_{D_j} \). Hence, in equilibrium, no collaborating pair will separate.

Now suppose firms \( (U_j, D_k) \) are matched at some stage \( s \geq 1 \) but that neither firm was part of a collaborating pair. In this case, \( V_{jk} = 0 \). All firms can guarantee themselves zero payoff by separating, so \( \pi_{U_j} + \pi_{D_i} \geq 0 \). It follows that \( V_{jk} \leq \pi_{U_j} + \pi_{D_i} \), so the firms will separate, given our tie-breaking rule that firms separate when indifferent. \( \square \)

Proof of Lemma 2. Lemma 1 shows that on the equilibrium path, the rematching market consists only of firms that did not collaborate. Consider a one-stage deviation where collaborating pair \( i \) in fact separates at \( s = 1 \). Then \( U_i \) and \( D_i \) rematch into new pairs \( (U_i, D_j) \) and \( (U_j, D_i) \), that can earn \( \alpha \Delta \) and \( (1 - \alpha) \Delta \) respectively in the product market. If instead \( (U_i, D_j) \) or \( (U_j, D_i) \) separate, then Lemma 1 shows that neither \( D_j \) nor \( U_j \) would ever go to the product market, implying outside options \( \pi_{D_j} = \pi_{U_j} = 0 \) for all \( s \geq 2 \).

Consider the outside option of \( U_i \), when \( m - 1 \) downstream firms remain in the game with which it has not yet matched. We show by induction that this outside option is equal to

\[
\pi_{U_i} (m - 1) = \left[ 1 - \left( \frac{1}{2} \right)^{m-1} \right] \alpha \Delta. \tag{7}
\]

First suppose that \( U_i \) and \( D_j \) are matched when \( m - 1 = 0 \). If \( (U_i, D_j) \) separates, then no further rematching is possible, so \( U_i \) faces outside option \( \pi_{U_i} (m - 1 = 0) = 0 \). This outside option corresponds to (7) evaluated at \( m - 1 = 0 \). Hence, \( U_i \) and \( D_j \) will go to the product market when \( m - 1 = 0 \), by \( V_{ij} = \alpha \Delta > 0 = \pi_{U_i} (0) + \pi_{D_j} \).

Suppose instead that \( U_i \) and \( D_j \) are matched when \( m - 1 \geq 1 \). The induction hypothesis is as follows: conditional on reaching stage any later stage, where \( m' - 1 \) downstream firms remain in
the game with which $U_i$ has not yet matched (with $m' - 1 \leq m - 2$), $U_i$ will immediately go to the product market with its partner, and will face outside option (7) evaluated at $m = m'$.

If $(U_i, D_j)$ separates, then $U_i$ will rematch with some $D_k$ in the next stage, where $m - 2$ downstream firms remain with which $U_i$ has not yet matched. By the induction hypothesis, $(U_i, D_k)$ will then go to the product market, where (1) implies $\Pi_{U_i} = (\alpha \Delta + \pi_{U_i}(m-2) - \pi_{D_k})/2$. Substituting for $\pi_{U_i}(m-2)$ from (7) along with $\pi_{D_k} = 0$ yields $\Pi_{U_i} = \alpha \Delta (1 - \left(\frac{1}{2}\right)^{m-1})$. This payoff represents $U_i$’s outside option when matched with $D_j$, so that $\pi_{U_i}(m-1) = \alpha \Delta (1 - \left(\frac{1}{2}\right)^{m-1})$, as required by (7). If $s \geq 2$, then this means that $(U_i, D_j)$ will indeed go to the product market: $V_{ij} = \alpha \Delta > \alpha \Delta (1 - \left(\frac{1}{2}\right)^{m-1}) = \pi_{U_i}(m - 1) + \pi_{D_j}$.

The proof for $D_i$ is entirely analogous, but with $\alpha$ replaced by $1 - \alpha$. Thus, the outside option of $D_i$, when $m - 1$ upstream firms remain in the game with which it has not yet matched, is equal to

$$\pi_{D_i}(m - 1) = \left[1 - \left(\frac{1}{2}\right)^{m-1}\right](1 - \alpha) \Delta. \quad (8)$$

To complete the inductive argument, it remains to confirm that collaborating pair $(U_i, D_i)$ will go to the product market at $s = 1$. This is indeed the case: $V_{ii} = \Delta > \Delta (1 - \left(\frac{1}{2}\right)^{m-1}) = \pi_{U_i}(m-1) + \pi_{D_i}(m-1)$, as required, using (7) and (8). From (1) and (2), the gross payoffs to $U_i$ and $D_i$ from going to the product market are $\Pi_{U_i} = (\Delta + \pi_{U_i}(m-1) - \pi_{D_i}(m-1))/2$ and $\Pi_{D_i} = (\Delta - \pi_{U_i}(m-1) + \pi_{D_i}(m-1))/2$, where again using (7) and (8) gives the payoffs specified in the lemma.

Proof of Proposition 1. We first rule out any candidate equilibrium where $m - 1$ pairs collaborate, with $1 \leq m - 1 \leq N - 1$. Lemma 1 shows that on the equilibrium path, each collaborating pair $i$ immediately goes to the product market at $s = 1$; all other firms separate and remain in the rematching market for all later stages. Firms $U_i$ and $D_i$ in collaborating pair $i$ earn $\Pi_{U_i} - I/2$ and $\Pi_{D_i} - I/2$ respectively, with $\Pi_{U_i}$ and $\Pi_{D_i}$ given by Lemma 2. Firms $\Pi_{U_j}$ and $\Pi_{D_j}$ in a non-collaborating pair $j$ both earn zero.

Suppose that $D_i$ deviates by not investing, so that no collaboration takes place between pair $i$. Then by Lemma 1, $D_i$ will enter the rematching market and remain in the game for all later stages, earning a payoff of zero. To rule out this deviation, we require $\Pi_{D_i} - I/2 \geq 0$, where Lemma 2 implies

$$\frac{\Delta}{2} - \left(\alpha - \frac{1}{2}\right) \left[1 - \left(\frac{1}{2}\right)^{m-1}\right] \Delta - \frac{I}{2} \geq 0 \quad (9)$$
which is equivalent to (3). Now suppose instead that $U_j$ and $D_j$ jointly deviate by investing, so the number of non-collaborating pairs is $m - 2$. $U_j$ and $D_j$ then earn $\Pi_{U_j} - I/2$ and $\Pi_{D_j} - I/2$, with $\Pi_{U_j}$ and $\Pi_{D_j}$ given by Lemma 2, but with $m - 1$ replaced by $m - 2$. To rule out this deviation, we must show that it is not profitable for at least one firm. Lemma 2 showed that $\Pi_{U_j} \geq \Pi_{D_j}$, so the condition for ruling out the deviation is $\Pi_{D_j} - I/2 \leq 0$, or equivalently

$$\frac{\Delta}{2} - \left(\alpha - \frac{1}{2}\right) \left[1 - \left(\frac{1}{2}\right)^{m-2}\right] \Delta - \frac{I}{2} \leq 0.$$  \hspace{1cm} (10)

It follows immediately from $\alpha > 1/2$ and $\Delta > 0$ that (9) and (10) cannot both hold, so there is no equilibrium with $1 \leq m - 1 \leq N - 1$.

Now consider a candidate equilibrium where all firms invest. Then Lemma 2 evaluated at $m - 1 = 0$ implies $\Pi_{U_i} = \Pi_{D_i} = (\Delta - I)/2$. As argued above, if either $U_i$ or $D_i$ deviate by not investing, then they will remain in the game for all later stages and earn a payoff of zero. By $\Delta - I > 0$, this deviation is not profitable, so an equilibrium exists where all firms invest.

Finally consider a candidate equilibrium where no firm invests, so where all firms earn zero. Suppose that $U_j$ and $D_j$ jointly deviate by investing. By the same argument as above, the condition to rule out this deviation is (10) evaluated at $m - 2 = N - 1$. Given (3) and Definition 1, this condition is equivalent to $N \geq N^*$. Hence, an equilibrium exists where no firm invests if and only if $N \geq N^*$.

Proof of Proposition 2. We first adapt the argument of Lemma 1 to show that all collaborating pairs immediately go to the product market at $s = 1$, after any history. Suppose instead that $m$ pairs in partnerships and $n$ integrated pairs collaborate and then separate. If pair $i$ in a partnership instead went to the product market, then (1) and (2) imply that $U_i$ would earn $(\bar{\Delta} + \pi_{U_i} - \pi_{D_i})/2 - I/2$ and $D_i$ would earn $(\bar{\Delta} + \pi_{D_i} - \pi_{U_i})/2 - I/2$. The fact that they separate implies either $(\bar{\Delta} + \pi_{U_i} - \pi_{D_i})/2 - I/2 \leq \pi_{U_i} - I/2$ or $(\bar{\Delta} + \pi_{D_i} - \pi_{U_i})/2 - I/2 \leq \pi_{D_i} - I/2$, where both inequalities reduce to $\bar{\Delta} \leq \pi_{U_i} + \pi_{D_i}$.

If integrated pair $j$ that separates instead went to the product market, they would earn a joint payoff of $\bar{\Delta} - I - L$. The fact that they separate implies $\bar{\Delta} - I - L \leq \pi_{U_j} + \pi_{D_j} - I - L$, or equivalently $\bar{\Delta} \leq \pi_{U_j} + \pi_{D_j}$. Summing over all $i$ and $j$ therefore gives $(m + n)\bar{\Delta} \leq \sum_{i=1}^{m} (\pi_{U_i} + \pi_{D_i}) + \sum_{j=1}^{n} (\pi_{U_j} + \pi_{D_j})$.

The total expected payoff, gross of costs, for all firms that separate at $s = 1$ cannot exceed the total value that firms from the $m + n$ collaborating pairs that separate can generate as of $s = 2$.  

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This implies $\sum_{i=1}^m (\pi_{U_i} + \pi_{D_i}) + \sum_{j=1}^n (\pi_{U_j} + \pi_{D_j}) \leq (m + n)\Delta$. It then follows from $\bar{\Delta} > \Delta$ that $m + n = 0$. Indeed, when $m = n = 0$, firms in each pair will prefer to immediately go to the product market and earn $(\bar{\Delta} - I)/2 > 0$, rather than separate and earn zero.

We now show that an integrated pair $j$ will always collaborate, regardless of how many pairs form partnerships. Collaborating gives $U_j$ and $D_j$ a joint payoff of $\bar{\Delta} - I - L > 0$. If the firms did not invest and then went to the product market, they would earn a joint payoff of $-L < 0$, equal to the cost of integration. If $U_j$ and $D_j$ did not invest and then separated, then each firm would remain in the game for all later stages and earn $-L/2 < 0$. Either way, $U_j$ and $D_j$ maximize their joint payoff by collaborating.

This in turn implies that on the equilibrium path, each pair $i$ that forms a partnership must also collaborate. If pair $i$ did not collaborate, then $U_i$ and $D_i$ would separate, remain in the game for all later stages, and earn zero. The firms could therefore increase their joint payoff by integrating and both investing, to earn $\bar{\Delta} - I - L > 0$. Thus, for pair $i$ in a partnership, $U_i$ and $D_i$ both earn an equilibrium payoff of $(\bar{\Delta} - I)/2$, given Lemma 2 evaluated at $m - 1 = 0$. For integrated pair $j$, the joint payoff to $U_j$ and $D_j$ is $\bar{\Delta} - I - L$.

Now consider a candidate equilibrium where $K$ pairs integrate and $N - K \geq N^*$ pairs form partnerships. Since all integrated pairs immediately go to the product market, the incentives for collaborating pairs are identical to those considered in Proposition 1, but with $N$ replaced by $N - K$. Recall that in any subgame, firms play the equilibrium with the lowest level of aggregate investment. Thus, Proposition 1 combined with $N - K \geq N^*$ implies that no firm in a partnership will invest. This contradicts the fact that all pairs collaborate on the equilibrium path, and implies that $N - K < N^*$ must hold in equilibrium.

Consider a candidate equilibrium where $N - K \leq N < N^*$. Suppose $K \geq 1$, and that integrated pair $j$ deviates to a partnership. The number of partnerships then becomes $N - K + 1$, and Proposition 1 implies these pairs will all collaborate, by $N - K + 1 \leq N < N^*$. It follows that $U_j$ and $D_j$ earn a joint payoff of $\bar{\Delta} - I$ from the deviation, which exceeds their joint payoff $\bar{\Delta} - I - L$ in the candidate equilibrium. Thus, any candidate equilibrium with $N - K \leq N < N^*$ must have $K = 0$. All pairs then form partnerships, and all collaborate by $N < N^*$. The joint payoff to each pair is therefore $\bar{\Delta} - I$, which exceeds their joint payoff from deviating to integration, $\bar{\Delta} - I - L$. Hence, when $N < N^*$, there is a unique equilibrium where all pairs form partnerships and collaborate.

Finally, consider a candidate equilibrium where $N - K < N^* \leq N$. If integrated pair $j$ deviates
to a partnership, then the number of partnerships becomes \( N - K + 1 \). Proposition 1 implies that all of these pairs will collaborate if \( N - K + 1 < N^* \), in which case the deviation increases pair \( j \)'s joint payoff from \( \Delta - I - L \) to \( \Delta - I \). Thus, any candidate equilibrium with \( N \geq N^* \) must have \( N - K + 1 \geq N^* \), where combining with \( N - K < N^* \) pins down the number of integrated pairs: \( N - N^* < K \leq N - N^* + 1 \). We now confirm that there is no incentive to deviate from this candidate equilibrium. If integrated pair \( j \) deviates to a partnership, then Proposition 1 implies they will not collaborate, by \( N - K + 1 \geq N^* \), so the deviation reduces pair \( j \)'s joint payoff from \( \Delta - I - L \) to zero. For pair \( i \) in a partnership, deviating to integration will reduce its joint payoff from \( \Delta - I \) to \( \Delta - I - L \). If either \( U_i \) and \( D_i \) instead deviate by not investing, then their payoff would drop from \((\Delta - I)/2\) to zero. Thus, when \( N \geq N^* \), there is a unique equilibrium where \( K \) pairs integrate, with \( N - N^* < K \leq N - N^* + 1 \), and where all pairs collaborate.

**Proof of Proposition 3.** As in the proof of Proposition 2, we show that all collaborating pairs immediately go to the product market at \( s = 1 \), after any history. Suppose that \( m \) pairs in unstructured partnerships, \( n \) integrated pairs, and \( l \) pairs in structured partnerships collaborate and then separate. The total expected payoff, gross of costs, for firms that separate at \( s = 1 \) cannot exceed the total value that firms from these \( m + n + l \) collaborating pairs can generate as of \( s = 2 \). This implies \( \sum_{i=1}^{m}(\pi_{U_i} + \pi_{D_i}) + \sum_{j=1}^{n}(\pi_{U_j} + \pi_{D_j}) + \sum_{k=1}^{l}(\pi_{U_k} + \pi_{D_k}) \leq (m + n + l)\Delta \). Rematching is independent of initial organizational form, hence \( \pi_{U_i} = \pi_{U_j} = \pi_{U_k} \equiv \pi_U \) and \( \pi_{D_i} = \pi_{D_j} = \pi_{D_k} \equiv \pi_D \), which implies \( (m + n + l)(\pi_U + \pi_D) \leq (m + n + l)\Delta \).

For any one of the \( l \) pairs, indexed by \( k \), the structured partnership cannot be implementable, since otherwise neither \( U_k \) nor \( D_k \) would choose to separate. It follows that \( U_k \) and \( D_k \) bargain over \( \Delta \) with weights equal to one half, just like pairs in unstructured partnerships. Thus, (1) and (2) imply that \( U_k \) and \( D_k \) would earn \((\Delta + \pi_U - \pi_D)/2 - I/2 - C/2 \) and \((\Delta + \pi_D - \pi_U)/2 - I/2 - C/2 \) respectively from going to the product market. The fact that they separate implies either \((\Delta + \pi_U - \pi_D)/2 - I/2 - C/2 \leq \pi_U - I/2 - C/2 \) or \((\Delta + \pi_D - \pi_U)/2 - I/2 - C/2 \leq \pi_D - I/2 - C/2 \), which is equivalent to \( \Delta \leq \pi_U + \pi_D \). Combined with \((m + n + l)(\pi_U + \pi_D) \leq (m + n + l)\Delta \), it follows from \( \Delta > \Delta \) that all collaborating pairs in structured partnerships go immediately to the product market, \( l = 0 \). An identical argument to that in the proof of Proposition 2 then shows that \( m = n = 0 \), so all other collaborating pairs do the same.

As a result, the proof of Proposition 2 immediately shows that an integrated pair will always collaborate, and that all pairs in unstructured partnerships must collaborate on the equilibrium path. For pair \( k \) in a structured partnership, not investing will give both \( U_k \) and \( D_k \) a payoff of
-C/2, since they can never rematch with a firm that collaborated. If \( U_k \) and \( D_k \) instead integrated and both invested, then their joint payoff would increase to \( \overline{\Delta} - I - L > 0 \). It follows that on the equilibrium path, all firms will invest.

Consider pair \( k \), with structured partnership \( \beta^* \equiv 1 - (I + C)/2\overline{\Delta} \), that collaborates. If \( \beta^* \) is implementable, then pair \( k \) immediately goes to the product market, where \( U_k \) earns \( \beta^*\overline{\Delta} - I/2 - C/2 = \overline{\Delta} - I - C > 0 \) and \( D_k \) earns \( (1 - \beta^*)\overline{\Delta} - I/2 - C/2 = 0 \). Deviating to no investment yields \(-C < 0\), which is not profitable. The same argument shows that if neither \( U_k \) nor \( D_k \) invest, then both firms can strictly increase their payoffs by jointly deviating to investment. It follows that pair \( k \) will collaborate if \( \beta^* \) is implementable.

We now show that \( \beta^* \) is always implementable when \( N < N^{**} \). Suppose that \( m - 1 \) pairs do not collaborate. Then if pair \( k \) collaborates and then separates, \( U_k \) will earn \( \pi_{U_k}(m - 1) \) given by (7), and \( D_k \) will earn \( \pi_{D_k}(m - 1) \) given by (8). Hence, \( \beta^* \) is always implementable if \( \beta^*\overline{\Delta} = \overline{\Delta} - I/2 - C/2 > \pi_{U_k}(m - 1) \) and \( (1 - \beta^*)\overline{\Delta} = I/2 + C/2 > \pi_{D_k}(m - 1) \) for all \( m - 1 \leq N - 1 \). By Definition 2, (5) holds with equality when evaluated at \( m - 1 = N^{**} - 1 \), where (7) then implies \( \pi_{U_k}(N^{**} - 1) = \overline{\Delta} - I/2 - C/2 \). Combining with \( \pi_{U_k}(m - 1) + \pi_{D_k}(m - 1) \leq \overline{\Delta} \) then yields \( \pi_{D_k}(N^{**} - 1) \leq \overline{\Delta} - \overline{\Delta} + I/2 + C/2 < I/2 + C/2 \). Both \( \pi_{U_k}(m - 1) \) and \( \pi_{D_k}(m - 1) \) are increasing in \( m - 1 \), so \( \pi_{U_k}(m - 1) < \overline{\Delta} - I/2 - C/2 \) and \( \pi_{D_k}(m - 1) < I/2 + C/2 \) hold for all \( m - 1 \leq N - 1 < N^{**} - 1 \), as required for implementation.

Suppose that \( N < N^{**} \) and consider a candidate equilibrium where some pair integrates. If the pair instead formed joint partnership \( \beta^* \), they would collaborate and go to the product market, increasing their joint payoff to \( \overline{\Delta} - I - C > \overline{\Delta} - I - L \). It follows that no pair will integrate in equilibrium if \( N < N^{**} \).

Suppose furthermore that \( N < N^* \). Proposition 2 showed that there was a unique equilibrium where all firms form unstructured partnerships, invest, and earn \( (\overline{\Delta} - I)/2 > 0 \). To show that this still constitutes an equilibrium, we rule out any deviation from pair \( i \) to a structured partnership. If pair \( i \) does not collaborate following the deviation, then both \( U_i \) and \( D_i \) earn \(-C/2 < 0\), which is not profitable. If pair \( i \) collaborate, then they will immediately go to the product market and earn joint payoff \( \overline{\Delta} - I - C \). There exists no value of \( \beta \) satisfying both \( \beta\overline{\Delta} - I/2 - C/2 \geq (\overline{\Delta} - I)/2 \) and \( (1 - \beta)\overline{\Delta} - I/2 - C/2 \geq (\overline{\Delta} - I)/2 \), so this deviation cannot be profitable to both \( U_i \) and \( D_i \).

We now rule out any candidate equilibrium with some pair \( k \) in a structured partnership. By deviating to an unstructured partnership and collaborating, \( U_k \) and \( D_k \) would earn \( \Pi_{U_k} - I/2 \) and \( \Pi_{D_k} - I/2 \) respectively, given Lemma 2. The number of non-collaborating pairs satisfies
$m - 1 \leq N - 1 < N^{*} - 1$. Definition 1 then implies $\Pi_{D_k} - I/2 > 0$, which combined with $\Pi_{U_k} > \Pi_{D_k}$ implies $\Pi_{U_k} - I/2 > 0$. Thus, pair $k$ will indeed collaborate following the deviation, earning joint payoff $\Delta - I$, which strictly exceeds their joint payoff in the candidate equilibrium, $\Delta - I - C$. There is no value of $\beta$ satisfying both $\beta \Delta - I/2 - C/2 \geq \Pi_{U_k} - I/2$ and $(1 - \beta) \Delta - I/2 - C \geq \Pi_{D_k} - I/2$, so either $U_k$ or $D_k$ strictly benefits from the deviation.

Now suppose instead that $N^{*} \leq N < N^{**}$, and let $M$ denote the number of pairs in structured partnerships. We show there is no equilibrium with $M \leq N - N^{*}$. Recall that all firms must invest on the equilibrium path, and that in any subgame, firms play the equilibrium with the lowest aggregate investment. Thus, for $M \leq N - N^{*}$ to constitute an equilibrium, it must be that $m$ pairs not collaborating is inconsistent with equilibrium pay in the ensuing subgame, for all $1 \leq m \leq N$.

First consider $m = N$, so no pair collaborates. Then $D_i$ in an unstructured partnership earns zero, and would instead earn $\Pi_{D_i} - I/2$ if pair $i$ jointly deviated to investment, given Lemma 2 evaluated at $m - 1 = N - 1$. From Definition 1, (9) does not hold strictly at $m - 1 = N^{*} - 1$, so it cannot hold strictly at $m - 1 = N - 1 \geq N^{*} - 1$. It follows that $D_i$ does not profit from this deviation: $\Pi_{D_i} - I/2 \leq 0$. The same applies to any $D_j$ in a non-implementable structured partnership, since pair $j$ would bargain as under an unstructured partnership. Hence, for $m = N$ to be inconsistent with equilibrium play, there must be some pair with structured partnership $\beta_N$ that (i) is implementable: $\beta_N \Delta > \pi_{U_N} (N - 1) \text{ and } (1 - \beta_N) \Delta > \pi_{D_N} (N - 1)$; and (ii) ensures $U_N$ and $D_N$ both profit from jointly deviating to investment: $\beta_N \Delta > I/2 \text{ and } (1 - \beta_N) \Delta > I/2$. In particular, this argument implies $M \geq 1$. Moreover (7) and (8) show that both $\pi_{U_i} (m - 1)$ and $\pi_{D_i} (m - 1)$ are increasing in $m$, which means that (i) and (ii) also hold for all $m \leq N - 1$. Hence, pair $N$ will also collaborate whenever $m \leq N - 1$.

Now consider $m = N - 1$, so only pair $N$ collaborates. If $M = 1$, then none of the remaining pairs $i$ collaborating is consistent with equilibrium play in this subgame. To see why, $D_i$ in an unstructured partnership earns zero, and jointly deviating with $U_i$ to investment yields $\Pi_{D_i} - I/2$, using Lemma 2 evaluated at $m - 1 = N - 2$. This deviation payoff satisfies $\Pi_{D_i} - I/2 \leq 0$, since (9) cannot hold strictly at $m - 1 = N - 2$, because $1 = M \leq N - N^{*}$ implies $N - 2 \geq N^{*} - 1$. Thus, for $m = N$ to be inconsistent with equilibrium play, we require $M \geq 2$. Specifically, there must be a pair with structured partnership $\beta_{N-1}$ that (i) is implementable: $\beta_{N-1} \Delta > \pi_{U_{N-1}} (N - 2)$ and $(1 - \beta_{N-1}) \Delta > \pi_{D_{N-1}} (N - 2)$; and (ii) ensures that $U_{N-1}$ and $D_{N-1}$ both profit from jointly deviating to investment: $\beta_{N-1} \Delta > I/2 \text{ and } (1 - \beta_{N-1}) \Delta > I/2$. By the same logic as above, pair $N - 1$ will also collaborate whenever $m \leq N - 2$. 

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Continuing in this way until \( m = N - M + 1 \) shows that there must be at least \( M \) pairs with structured partnerships \( \beta_N, \beta_{N-1}, \ldots, \beta_{N-M+1} \), that are implementable for all \( m' \leq N - M +1 \), and that give all these firms an incentive to invest. This means in particular that if \( m = N - M \), and all \( M \) pairs in structured partnerships collaborate and all \( N - M > 0 \) pairs in unstructured partnerships do not, then no firm in a structured partnership will deviate to no investment. Applying the same argument as above, \( D_i \) in an unstructured partnership does not profit from a joint deviation with \( U_i \) to investment, since \( N - M \geq N^* \) implies deviation payoff \( \Pi_{D_i} - I/2 \leq 0 \). It follows that \( N - M > 0 \) pairs not collaborating is consistent with equilibrium play in this subgame. Thus, there is no equilibrium with \( M \leq N - N^* \).

We now show that there is no equilibrium with \( M > N - N^* + 1 \). For such an equilibrium did exist, there must be at least \( [N^*] \) pairs in structured partnerships that are implementable for all \( m'' \), and that give all these firms an incentive to invest. Since \( M > N - N^* +1 \geq N - [N^*] +1 \), there exists some pair \( k \) with structured partnership \( \beta \) such that (i) and (ii) continue to hold even if pair \( k \) deviated to an unstructured partnership. This is because at least \( N - [N^*] +1 \) pairs remaining in structured partnerships will all collaborate, regardless of how the other \( [N^*] -1 \) pairs behave. The number of non-collaborating pairs after the deviation is therefore \( m \leq [N^*] -1 < N^* \). Condition (9) holds strictly when evaluated at any \( m-1 < N^* -1 \), so both \( U_k \) and \( D_k \) strictly profit from investing following their deviation. They earn respectively \( \Pi_{U_k} - I/2 > 0 \) and \( \Pi_{U_k} - I/2 > 0 \), with \( \Pi_{U_k} + \Pi_{U_k} = \Delta \), whereas they earned \( \beta \Delta - I/2 - C/2 \) and \( (1-\beta)\Delta - I/2 - C/2 \) respectively in the candidate equilibrium. There is no value of \( \beta \) satisfying both \( \Pi_{U_k} - I/2 \leq \beta \Delta - I/2 - C/2 \) and \( \Pi_{D_k} - I/2 \leq (1-\beta)\Delta - I/2 - C/2 \), so either \( U_k \) or \( D_k \) strictly profits from the deviation.

The next step is to show that an equilibrium exists with \( N - N^* < M \leq N - N^* +1 \), where all \( M \) pairs in structured partnerships choose \( \beta^* \equiv 1 - (I + C)/2\Delta \), and where all firms invest. By \( N < N^{**} \), \( \beta^* \) is always implementable and provides strict incentive for investment. Each firm in an unstructured partnership earns \( (\Delta - I)/2 > 0 \), where deviating to no investment yields zero. Deviating to integration would reduce any pair’s joint payoff, since \( \Delta - I - L < \Delta - I - C < \Delta - I \). If pair \( i \) in an unstructured partnership deviates to a structured partnership, then they would earn joint payoff \( \Delta - I - C < \Delta - I \). There is no value of \( \beta \) satisfying both \( \beta \Delta - C/2 - I/2 \geq (\Delta - I)/2 \) and \( (1-\beta)\Delta - I/2 - C/2 \geq (\Delta - I)/2 \), so either \( U_i \) or \( D_i \) would find this deviation unprofitable. If pair \( k \) in a structured partnership deviates to an unstructured partnership, then the number of unstructured partnerships becomes \( N - M +1 \geq N^* \). Then only firms in structured partnerships will invest. In particular, if pair \( k \) collaborated following the deviation, then \( D_k \) would earn \( \Pi_{D_k} - I/2 \leq 0 \), since
(9) does not hold strictly at \( m - 1 = N - M \geq N^* - 1 \). Hence, the deviation yields zero to \( U_k \) and \( D_k \), and is therefore unprofitable. We conclude that \( N - N^* < M \leq N - N^* + 1 \) constitutes an equilibrium.

Now suppose that \( N \geq N^{**} \). We first rule out an equilibrium where \( K \leq N - N^{**} \) pairs integrate. For \( K \leq N - N^{**} \) to constitute an equilibrium, having only these \( K \) pairs collaborate must be inconsistent with equilibrium play in the ensuing subgame. Condition (9) is violated for \( m - 1 \geq N - K - 1 \), by \( N - K \geq N^{**} - N^* \). Thus, for any pair \( i \) in an unstructured partnership, or with a non-implementable structured partnership, jointly deviating to investment will reduce \( D_i \)'s payoff: \( \Pi_{D_i} - I/2 < 0 \). Now suppose pair \( k \) with implementable structured partnership \( \beta \) jointly deviates to investment. Definition 2 and (7) imply \( \pi_{U_k}(m - 1) \geq \Delta - I/2 - C/2 \) for all \( m - 1 \geq N^{**} - 1 \). Similarly, \( \pi_{U_k}(m - 1) \geq \Delta - I/2 \) for all \( m - 1 \geq N^{**}|C=0 - 1 \), where \( N^{**}|C=0 \) is defined as the value of \( m \) for which (5) would hold with equality if \( C \equiv 0 \). Let \( C > 0 \) be sufficiently small such that \( N - K \geq N^{**} \) implies \( N - K \geq N^{**}|C=0 \). Hence, since \( \beta \) is implementable, we have \( \beta \Delta > \pi_{U_k}(N - K - 1) \geq \Delta - I/2 \), and so \( (1 - \beta)\Delta < I/2 \). Thus, jointly deviating to investment would reduce \( D_k \)'s payoff: \( (1 - \beta)\Delta - I/2 - C/2 < -C/2 \). It follows that \( K \leq N - N^{**} \) cannot hold in equilibrium.

We now rule out an equilibrium with \( K > N - N^{**} + 1 \). Suppose integrated pair \( j \) deviates to structured partnership \( \beta^* \). Then all remaining integrated pairs will invest, and the number of non-integrated pairs becomes \( N - K + 1 \). By \( N - K + 1 < N^{**} \), \( \beta^* \) is implementable for pair \( j \) regardless of how other non-integrated pairs behave. It follows that pair \( j \) will invest following the deviation and earn \( \Delta - I - C \), which exceeds their joint payoff \( \Delta - I - L \) in the candidate equilibrium. Thus, \( K > N - N^{**} + 1 \) cannot hold in equilibrium.

To complete the proof, we show that an equilibrium exists with \( N - N^{**} < K \leq N - N^{**} + 1 \), where \( (N - K) - N^* < M \leq (N - K) - N^* + 1 \) if \( [N^*] < [N^{**}] \) and \( M = 0 \) if \( [N^*] = [N^{**}] \). Suppose all firms invest, and each of the \( M \) pairs in structured partnerships chooses \( \beta^* \). Conditional on all integrated pairs collaborating, incentives for the \( N - K < N^{**} \) non-integrated pairs are identical to the case \( N < N^{**} \) analyzed above, but with \( N \) replaced by \( N - K \). The following conditions are therefore necessary and sufficient to rule out deviations from firms in non-integrated pairs. If \( [N^*] = [N^{**}] \), so that \( N - K < N^* < N^{**} \), then all non-integrated pairs must form unstructured partnerships. If instead \( [N^*] < [N^{**}] \), so that \( N^* \leq N - K < N^{**} \), then the number of pairs \( M \) in structured partnerships must satisfy \( (N - K) - N^* < M \leq (N - K) - N^* + 1 \), where notice that \( \beta^* \) is implementable by \( N - K < N^{**} \). Thus, to conclude, we must show that no
integrated pair \( j \) will deviate to a different organizational form. The number of non-integrated pairs following such a deviation is \( N - K + 1 \geq N^{**} > N^* \). Let \( C > 0 \) be sufficiently small such that \( N - K + 1 \geq N^{**} \) implies \( N - K + 1 \geq N^{**}|_{C=0} \). Firms will play the equilibrium in this subgame with the least aggregate investment, which we now argue has no investment from non-integrated pairs. For pair \( j \) in an unstructured partnership or a non-implementable structured partnership, \( D_j \) would earn \( \Pi_{D_j} - I/2 < 0 \) from collaboration, since \( N - K + 1 > N^* \). For pair \( j \) in an implementable structured partnership, \( D_j \) would earn \( (1 - \beta)\Delta - I/2 - C/2 < -C/2 \) from collaboration, by \( N - K + 1 \geq N^{**}|_{C=0} \), following the same argument as above. Thus, the deviation yields pair \( j \) a joint payoff of at most zero, which is less than their joint payoff of \( \Delta - I - L \) from integration in the candidate equilibrium. It follows that no integrated pair will deviate to another organizational form.

**Proof of Proposition 4.** Consider inequality (3) evaluated at \( m - 1 = N - 1 \). First, (3) holds strictly when \( \Delta = 0 \), by \( \Delta > I \). Second, (3) is violated when \( \Delta = \Delta \) and \( \alpha = 1 \), by \( (\frac{1}{2})^{N-1}\Delta < I \). Third, the left-hand-side of (3) is decreasing in \( \Delta \), for any \( \alpha > 1/2 \). Fourth, (3) holds strictly at \( \alpha = 1/2 \), by \( \Delta > I \). Fifth, the left-hand-side of (3) is decreasing in \( \alpha \), for any \( \Delta > 0 \). It follows that when \( \alpha = 1 \), there exists a unique \( \Delta_0 \in (0, \Delta) \) such that (3) holds with equality at \( \Delta = \Delta_0 \), is violated if and only if \( \Delta \in (\Delta_0, \Delta] \). Moreover, for any \( \Delta \in (\Delta_0, \Delta] \), we can define \( \alpha(\Delta) \) as the unique \( \alpha \) for which (3) holds with equality. The function \( \alpha(\Delta) \) is continuous and strictly decreasing in \( \Delta \), with \( \alpha(\Delta_0) = 1 \) and \( \alpha(\Delta) > 1/2 \).

Fix \( \Delta = \Delta_0 + \epsilon \), where \( \epsilon > 0 \) is small, and let \( \alpha = \alpha(\Delta) \). Suppose first that \( N_u = N_d = N \). By the definition of \( \alpha(\Delta) \), (3) holds with equality when evaluated at \( m - 1 = N - 1 \). Definition 1 then implies \( N^* = N \), so that no pair will collaborate in equilibrium by Proposition 1.

Now suppose \( N_u \geq N + 1 \) and \( N_d = N \). By the same argument as in Lemma 1, all collaborating pairs will immediately go to the product market at \( s = 1 \), and all non-collaborating pairs will separate. The latter firms will enter the rematching market and remain in the game for all subsequent stages, as will the \( N_u - N \) upstream firms that were not initially matched.

Say \( m - 1 \) pairs do not collaborate, and consider the payoffs for \( U_i \) and \( D_i \) that collaborate. If pair \( i \) separated, then there would be \( m - 1 \) downstream firms with which \( U_i \) could potentially match, so \( \pi_{U_i} \) is given by (7). There would be \( m + (N_u - N) - 1 \) upstream firms with which \( D_i \) could potentially match, so \( \pi_{D_i} \) is given by (8) with \( m - 1 \) replaced by \( m + (N_u - N) - 1 \). Substituting into (1) and (2) shows that \( U_i \) and \( D_i \) will earn the following by going to the product market:
\[ \Pi_{U_i} = \frac{\Delta}{2} + \frac{\alpha}{2} \left[ 1 - \left( \frac{1}{2} \right)^{m-1} \right] \Delta - \left( \frac{1}{2} - \alpha \right) \left[ 1 - \left( \frac{1}{2} \right)^{m+(N_u-N)-1} \right] \Delta, \]  
\[ \Pi_{D_i} = \frac{\Delta}{2} - \frac{\alpha}{2} \left[ 1 - \left( \frac{1}{2} \right)^{m-1} \right] \Delta + \left( \frac{1}{2} - \alpha \right) \left[ 1 - \left( \frac{1}{2} \right)^{m+(N_u-N)-1} \right] \Delta. \]  

Consider a candidate equilibrium with no collaboration, so where all firms earn zero. If pair \( i \) jointly deviates to investment, then \( U_i \) will earn \( \Pi_{U_i} - I/2 \) and \( D_i \) will earn \( \Pi_{D_i} - I/2 \), with both (11) and (12) evaluated at \( m - 1 = N - 1 \). Since \( N = N^* \), (3) holds with equality when evaluated at \( m - 1 = N - 1 \), or equivalently (10) holds with equality when evaluated at \( m - 2 = N - 1 \). Comparing (3) and (10) with (12) shows that \( \Pi_{D_i} - I/2 > 0 \) following the deviation, by \( N_u - N \geq 1 \).

Recall that \( \Delta \) was fixed at \( \Delta_0 + \epsilon \), and that \( \alpha = \alpha(\Delta) \), where \( \alpha(\Delta) \) is continuous and strictly decreasing in \( \Delta \) over \([\Delta_0, \Delta]\), with \( \alpha(\Delta_0) = 1 \). Thus, by setting \( \epsilon > 0 \) sufficiently small, \( \alpha(\Delta) \) can be made arbitrarily close to 1. Comparing (11) and (12) shows that \( \Pi_{U_i} > \Pi_{D_i} \) when evaluated at \( \Delta > 0 \) and \( \alpha = 1 \), where continuity implies that \( \Pi_{U_i} > \Pi_{D_i} \) also holds whenever \( \alpha \) is sufficiently close to 1. Hence, for \( \epsilon \) sufficiently small, \( \Pi_{D_i} - I/2 > 0 \) implies \( \Pi_{U_i} - I/2 > 0 \). It follows that both \( U_i \) and \( D_i \) find the deviation profitable, so no equilibrium exists in which all pairs do not collaborate.

Now consider a candidate equilibrium where \( m - 1 \) pairs do not collaborate, with \( 1 \leq m - 1 \leq N - 1 \). When \( N_u = N_d = N \), the payoffs to \( U_i \) and \( D_i \) in collaborating pair \( i \) were given by Lemma 2. The proof of Proposition 1 showed that in this case, conditions (9) and (10) were both necessary to rule out profitable deviations, but could not simultaneously hold. From (11) and (12), the payoffs to \( U_i \) and \( D_i \) when \( N_u \geq N + 1 \) and \( N_d = N \) are identical to those in Lemma 2 when evaluated at \( \alpha = 1 \), where all expressions in square brackets take on values between zero and one. Thus, by continuity, (11) and (12) can be made arbitrarily close to the payoffs in Lemma 2 for all \( N_u \geq N + 1 \), by fixing \( \epsilon \) sufficiently small, so that \( \alpha = \alpha(\Delta) \) is sufficiently close to 1. It follows that at least one of the deviations corresponding to (9) and (10) will be profitable, so that \( 1 \leq m - 1 \leq N - 1 \) cannot constitute an equilibrium.

Finally, consider a candidate equilibrium where all pairs collaborate. \( U_i \) then earns \( \Pi_{U_i} - I/2 \) and \( D_i \) earns \( \Pi_{D_i} - I/2 \), with (11) and (12) both evaluated at \( m - 1 = 0 \). A firm that deviates by not investing remains in the game for all later stages and earns zero. From (11) and (12), the equilibrium payoffs to \( U_i \) and \( D_i \) are both equal to \((\Delta - I)/2 > 0\) when \( \alpha = 1 \). Thus, by continuity, both these equilibrium payoffs will be strictly positive when \( \epsilon \) is sufficiently small, so that \( \alpha = \alpha(\Delta) \) is sufficiently close to 1. It follows that for \( \Delta = \Delta_0 + \epsilon \) and \( \alpha = \alpha(\Delta) \) with \( \epsilon > 0 \) fixed sufficiently
small, there is a unique equilibrium where all pairs collaborate.

To complete the proof, we show that whenever $N^*$ is finite, no pair will collaborate if both $N_d$ and $N_u$ are sufficiently large. Consider a candidate equilibrium with no investment, so where all firms earn zero. If pair $i$ jointly deviates to investment, then $D_i$ will earn $\Pi_D - I/2$, with (12) evaluated at $m = N = N_d$. The first and second exponents in (12) are then $N_d - 1$ and $N_u - 1$ respectively. Thus, as $N_u$ and $N_d$ become large, $\Pi_D$ tends to $\Delta/2 - (\alpha - 1/2)\Delta$. By Definition 1, $\Delta/2 - (\alpha - 1/2)\Delta < I/2$ holds whenever $N^*$ is finite. It follows that $\Pi_D - I/2 < 0$ for $N_d$ and $N_u$ sufficiently large, so that this deviation reduces $D_i$’s payoff. Thus, no pair will collaborate in equilibrium.

Proof of Proposition 5. Suppose that all upstream firms are large, and consider a candidate equilibrium where no firm invests. Each firm then earns an equilibrium payoff of zero. If firms $U_i$ and $D_i$ jointly deviate to investment, then $\pi_{U_i}$ is bounded below by $\alpha \Delta$, which is what $U_i$ could earn by separating and unilaterally choosing to go to the product market. The outside option $\pi_{D_i}$ is bounded above by $(1 - \alpha)\Delta$, which is what $D_i$ could generate in the product market after separating from $U_i$. It therefore follows from (2) that the gross deviation payoff of $D_i$ is bounded above by $(\Delta - \alpha \Delta + (1 - \alpha)\Delta)/2$. By Definition 1, finite $N^*$ implies $\Delta/2 - (\alpha - 1/2)\Delta < I/2$, which is equivalent to $(\Delta - \alpha \Delta + (1 - \alpha)\Delta)/2 - I/2 < 0$. Hence, $D_i$ earns a strictly negative net payoff from the deviation. It follows that no investment will occur in equilibrium if all upstream firms are large, regardless of whether downstream firms are large or not. □
Supplementary appendix: Risk and equilibrium selection

The idea of coordination failure in investment decisions plays an important role in our analysis. Specifically, we have assumed that in any subgame, firms play the equilibrium with the lowest level of aggregate investment. Applied to Section 3, this criterion selects the inefficient equilibrium with no investment, rather than the equilibrium with full investment, whenever the market size exceeds the critical value $N^*$. We now show that the qualitative results from our baseline analysis continue to hold under a weaker assumption about coordination failure: that firms only play the inefficient equilibrium if it is less risky than the efficient one.

To capture the notion of risk in our setting, we apply the concept of $p$-dominance (Morris et al., 1995). An equilibrium $s^* = (s^*_i, s^*_{i-1})$ is $p$-dominant, for given $p \in [0, 1]$, if the strategy $s_i = s^*_i$ of each player $i$ is a best response to any conjecture that places probability at least $p$ on the equilibrium strategy profile $s_{-i} = s^*_{-i}$ being played by the other players. In particular, player $i$ must find $s^*_i$ optimal even if he conjectures that with probability $1 - p$, others will play some $s'_{i-1} \neq s^*_{i-1}$ which would give him a very low payoff. We now show that in our setting, for any given parameter values (other than a knife-edge case), there is always a unique equilibrium that is $p$-dominant for at least some $p < 1/2$. This is the equilibrium that can be viewed as less risky, and that we assume players will play.\(^{38}\)

The analysis of Section 3 showed that an equilibrium always exists where all firms invest. For this equilibrium be $p$-dominant, for a particular value of $p$, no firm can have an incentive to stop investing, for any conjecture that places probability at least $p$ on all other pairs investing.\(^{39}\) Moreover, the relevant incentive constraint is again that of the downstream firm, since Lemma 2 shows that it earns weakly less than its upstream partner, regardless of the investment decisions of other pairs. Given the payoff listed in this Lemma, the condition for $p$-dominance is therefore

$$\Delta - (1 - p) \left( \alpha - \frac{1}{2} \right) \left[ 1 - \left( \frac{1}{2} \right)^{N-1} \right] \Delta \geq I - \frac{I}{2}.$$  \hspace{1cm} (13)

That is, the downstream firm’s expected payoff from investing must be exceed its cost, given the

\(^{38}\)It follows that every Nash equilibrium is $p$-dominant for $p = 1$, but not necessarily for smaller values of $p$. In $2 \times 2$ coordination games, an equilibrium is risk dominant if and only if it is $p$-dominant for at least some $p < 1/2$. Thus, $p$-dominance for $p < 1/2$ generalizes the notion of risk dominance to a broader class of games. $p$-dominant equilibria for $p < 1/2$ also have attractive robustness properties in terms of incomplete information and of evolutionary stability.

\(^{39}\)We restrict attention to conjectures that place probability 1 on the firm’s initial partner investing. This is consistent with the stability requirement used throughout our analysis, that firms within a pair can coordinate their investment decisions.
conjecture that places probability $1-p$ on the worst possible scenario, given its equilibrium strategy: where no other pair invests, which minimizes its bargaining power.

We now turn to the equilibrium with no investment, which exists when $N \geq N^*$. By a similar reasoning, given the payoffs in Lemma 2, the criterion for $p$-dominance is

$$\frac{\Delta}{2} - p \left( \alpha - \frac{1}{2} \right) \left[ 1 - \left( \frac{1}{2} \right)^{N-1} \right] \Delta \leq \frac{I}{2}. \quad (14)$$

That is, the downstream firm’s expected payoff from jointly deviating to investment with its partner cannot exceed its cost, given the conjecture that places probability $1-p$ on the best possible scenario, given its deviation: where all other pairs invest, which maximizes its bargaining power.

Comparing (13) and (14) shows that for any given parameter values, other than the knife-edge case where both constraints hold with equality at $p = 1/2$, it is always true that one constraint is violated for all $p < 1/2$, and the other is satisfied for at least some $p < 1/2$. Thus, our selection criterion always picks out a single equilibrium, the one that is “less risky”. This will be the inefficient equilibrium if and only if (14) holds strictly when evaluated at $p = 1/2$.

To say more, consider how (14) compares to inequality (3), which was used to define the critical market size $N^*$. The left-hand-sides of these constraints coincide, when the latter is evaluated at $m - 1 = N - 1$, except that the term $(\alpha - 1/2)$ in (3) is multiplied by $p$ in (14). Thus, following Definition 1, we can define $N^*_p$ as the unique value of $N \in \mathbb{R}$ for which (14) holds with equality, where $N^*_p \equiv +\infty$ if no such $N$ exists.\(^{40}\)

Thus, using our criterion of $p$-dominance for $p < 1/2$, firms will play the inefficient equilibrium whenever $N \geq N^*_p=1/2$. This qualitative result echoes that in the baseline, where the condition for inefficiency was $N \geq N^*$. It also follows immediately from (3), (14), and Definition 1, that all comparative static from Corollary 1 apply to $N^*_p=1/2$ just as with $N^*$. Inefficiently low investment is more likely in large markets (large $N$) with easily redeployable components ($\Delta$ close to $\overline{\Delta}$) and large asymmetries ($\alpha$ close to 1).\(^{41}\)

\(^{40}\)From (3), (14), and Definition 1, it follows immediately that (i) $N^*_p = N^*$ for $p = 1$; (ii) $N^*_p \geq N^*$ for all $p < 1$, with a strict inequality whenever $N^*$ is finite; (iii) $N^*_p$ is increasing in $p$, and strictly increasing whenever $N^*_p$ is finite.

\(^{41}\)It is possible to further weaken our selection criterion as follows. For any given $p < 1/2$, assume that firms only play the inefficient equilibrium if it is $p$-dominant for all $p \in [p, 1]$. The relevant incentive constraint is then (14) but with $p$ replaced by $p$. All the above conclusions then go through, but with a critical value $N_{p=1/2}^* > N_{p=1/2}^*$. Thus, the size of $(1/2 - p) > 0$ captures “how much less risky” the inefficient equilibrium must be, compared to the efficient equilibrium, for firms to refrain from investing, according to this selection criterion.