Intra- and Intergroup Conflicts:
Theory and Experiment in Continuous Time

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September 2015

Abstract

Standard one- and two-population models for evolutionary games are the
two limiting cases of a uniparametric family combining intra- and intergroup
interactions. Our setup interpolates between both extremes with a coupling
parameter $\kappa$. For the example of a hawk-dove game, we perform a replicator
dynamics analysis of this coupled model. We confirm the existence of a bifurcation
in the dynamics of the system and identify three regions for equilibrium selection,
one of which does not appear in standard one- and two-population models. We then
design an experiment in continuous time exploring the dynamics and equilibrium
selection. The data largely confirm the theory but a behavioral analysis with logit-
response dynamics outperforms the explanatory power of replicator dynamics.

Keywords: evolutionary game theory, experiment in continuous time, hawk-dove game,
logit-response dynamics, replicator dynamics.

JEL Classification: C62, C73, C91, C92.

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1 Introduction

Evolutionary game theory makes an important distinction whether players interact within a single population or between two (or more) disjunct populations (Friedman, 1991; Weibull, 1995). When matched with opponents in a single population, players earn the expected payoff as if playing against the aggregate strategy of their own population, so only symmetric strategies can survive. With a two-population matching, each member of the group of, say, row players is matched against a rival from the group of column players. Here, polarization in behaviors can occur and the populations may specialize in different pure strategies. The same evolutionary forces may accordingly imply qualitatively different results, so the distinction of single- and two-population settings is crucial.

The compartmentalization of one- and two-population models may, however, not always be appropriate. A two-population analysis requires that players exclusively receive their payoffs from interactions with the external population. Likewise, in a one-population setting, players never interact with opponents from other populations. But these assumptions may not be warranted: why should players in a two-population game not at least occasionally interact within their own population? Why should agents in a single population setting not sometimes be exposed also to interactions with agents from other populations? It seems plausible that the interaction will often be mixed.

An example with human players can be found in Mailath (1998). He illustrates the issue of single- vs. two-population with a game where traders either bargain among traders within their own village or they may encounter visitors from a different population. The game is hawk-dove in both cases but the analysis is one-population in the first case and two-population in the second and the evolutionary selection mechanism differ starkly. These are, however, only the two polar cases and traders may also interact both with players from their own village and with visitors at the same time. Another example would be workers who need to decide whether to cooperate within their team (intragroup) but they also decide on cooperation, say, as sales representatives, outside the firm (intergroup).

For non-humans, examples where the one- and the two-population cases overlap are abundant in resource conflicts. “Competition occurs when a number of animals (of the same or of different species) utilize common resources, the supply of which is short (Birch, 1957, p.6).” Animals of the same species will predominantly compete for resources (intra-species competition) but there will also be inter-species competition. Think of different predatory mammals fighting for preys and water, or of various sessile organisms competing for light interception and soil. Inevitably intra-specific and inter-specific competition overlap.¹

¹Connell and Sousa (1983) and Schoener (1983) provide surveys of works on inter-specific competition overlap.
The notion that intragroup and intergroup interactions overlap makes sense when agents do not condition their strategy on the population from which an opponent stems. This will be the case when players cannot identify where a rival is from. Even if they can identify the group to which other players belong, they may still not be able or willing to condition their strategy on this identification. On the one hand, boundedly rational players may choose the same action for intragroup and intergroup interactions due to limited learning in complex environments. On the other hand, rational agents may do the same in order to establish a global reputation or as a result of costly cognitive resources they employ to organize their reasoning. Either way, one- vs. two-population cases may not be as cleanly separated as often assumed.

Another case where players may not perfectly condition their strategies are decision errors. If one allows for errors in the conditioning of strategies, in the sense that players may mistakenly take a visitor for a home player or vice versa, the interaction between the populations would also be coupled and not perfectly separate.

In this paper, we consider the interaction of one- and two-population dynamics. We analyze a uniparametric family combining the two models by interpolating between both extremes with a coupling parameter \( \kappa \) measuring the weight of the intergroup interaction. One- and two-population matching are obtained for \( \kappa = 0 \) and \( \kappa = 1 \), respectively. For replicator dynamics, we analyze the dynamics of this system theoretically. We also analyze the overlap of one- and two-population cases in an experiment in continuous time generalizing the study by Oprea, Henwood, and Friedman (2011).

Our application is a hawk-dove game. The hawk-dove game is the paradigm for the analysis of conflicts over scarce resources (Maynard Smith, 1982). Originally introduced by Maynard Smith and Price (1973) in the context of animal conflict, it became highly influential also for human interactions due to its fairly simple definition which nevertheless generates very rich dynamics as a population game. Oprea, Henwood, and Friedman (2011) analyze the hawk-dove game for sign-preserving dynamics for the one- and the two-population case. Their experiment confirms that the symmetric mixed equilibrium is more likely to be selected in the one-population treatment whereas separation is stronger in the two-population treatment.

Our theoretical analysis confirms the existence of a bifurcation in the dynamics of the system, and the replicator dynamics predicts three regions with different stable equilibria. First, for \( \kappa < \kappa^* \), subjects randomize, as in the one-population (\( \kappa = 0 \))

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2 See some theoretical and experimental works on this idea in Section 2.

3 Selten (1980) takes an approach opposite to ours by assuming that players can condition their strategy based on the information available to them.

4 Gómez-Gardeñes et al. (2012) use a similar model to analyze simulations of games on overlapping graphs.
setting. For $\kappa > \kappa_p^*$ pure play emerges, as in the two-populations ($\kappa = 1$) analysis. Interestingly, for the intermediate values $\kappa \in (\kappa_m^*, \kappa_p^*)$, a qualitatively novel prediction emerges where one population plays pure hawk and the second population randomizes. This hybrid case neither occurs with a single population nor with two populations.

One way of interpreting these theoretical results is that the existing analyses of undiluted one- and two-population cases are robust with respect to perturbations. We find $\kappa_m^* = 1/2$ and we show that $\kappa_p^*$ will vary between one half and one. In other words, the prediction of the one-population case extends for much overlap with a second population while only a more moderate statement can be made regarding the realm of the two-population case. Hence, the scope for pure equilibria will be smaller. In that sense, the two-population analysis seems somewhat less robust. Nevertheless, we conclude that, theoretically, both the one- and two-population cases are not strongly affected by perturbations.

Our experimental results qualitatively confirm the predictions but there are also departures from the theory. We find that mixed behavior is observed throughout where predicted, including the pure one-population treatment and the coupled variants with $\kappa < 1/2$. We also see sound separation of hawk and dove play in our pure two-population treatment. These findings confirm and extend the experiments of Oprea, Henwood, and Friedman (2011). Among the discrepancies between the replicator dynamics prediction and the experimental results is a general bias in the mixed strategies: in the treatments where the mixed equilibrium was expected, the frequency of hawk play was lower than predicted. Concerning the coupling regime in which the pure equilibrium is predicted, we also observe that, with $\kappa \in (\kappa_p^*, 1)$, the separating effect is less pronounced. The data further indicate that the splitting point (understood as the realization of $\kappa$ for which the populations split into two groups of ‘mostly hawks’ and ‘mostly doves’) experiences notable variations across sessions.

We discuss a behavioral model correcting the evolutionary predictions. Logit-response dynamics (Alós-Ferrer and Netzer, 2010) introduces a probabilistic element in the behavior and can be seen as a dynamic interpretation of the concept of quantal response equilibrium (McKelvey and Palfrey, 1995). The aforementioned discrepancies between theory and data can largely be reconciled in this model and variations between sessions are well captured with different values of the rationality parameter.

In the next section, we review relevant literature. Section 3 introduces the model for coupled populations and a proposition summarizes the predictions. Section 4 describes the experiment. Section 5 reports on the experimental results, and Section 6 provides a further behavioral analysis. Section 7 concludes.

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5This is in line with Cason, Friedman, and Hopkins (2014). They compare best-response dynamics and its perturbed counterpart focusing on the limit cycles in the Rock-Paper-Scissors game.

6Capra et al. (2002) apply this model to learning experiments in market environments.
2 Some relevant literature

On evolutionary games. The implementation of interactions as population games recovers the spirit of the “mass action” interpretation of the mixed Nash equilibrium (Björnerstedt and Weibull, 1996; Young, 2011). Each player in a population can simply play a pure strategy, but the frequencies of the strategies in the population may correspond to a mixed Nash equilibrium. This relaxes the reasoning skills required for mixed play.

When the (static) game exhibits multiplicity of equilibria, the evolutionary approach is a powerful criterion, among many other examples, for equilibrium selection and learning (Friedman, 1996). In a seminal contribution to evolutionary game theory in economics, Friedman (1991) compares the theoretical conditions for static stability in evolutionary games involving one and two (or more) groups of individuals, with applications to, e.g., male-female mating problems. Weibull (1995) attributes the first multi-population replicator dynamics analyses to Taylor and Jonker (1978) and Maynard Smith (1982). Weibull (1995) also analyzes a different version of replicator dynamics for the $N$-population case. Following Nowak and May (1992), many papers have also analyzed how the structure of a population may affect its behavior (Lieberman, Hauert, and Nowak, 2005; Taylor et al., 2004).

On the coupling of interactions. As mentioned in the Introduction, our starting point is that players choose the same action when playing the same game in encounters with players from different populations. The exact mechanism why players might choose the same action in similar interactions is beyond the scope of this paper, and here we refer to various approaches suggesting that players may do so.

For example, Samuelson (2001) argues that some agents facing a problem of multiple strategic interactions may balance the gains from better decision-making against the cost of using scarce cognitive resources. This can result in the application of the same choice in several of the interactions. Jehiel (2005) formalizes the notion of “analogy-based expectation equilibrium” where players best respond to beliefs that are correct on average over various analogous situations. Huck, Jehiel, and Rutter (2011) provide experimental evidence.

Grimm and Mengel (2014) analyze learning across games in experiments with the concepts of “belief bundling” and “action bundling”. Both can imply that players simplify their decision environment by choosing the same action in different games, as a form of “best response bundling.” See also Mengel (2012) and the discussion contained therein.

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7Huck, Normann, and Oechssler (1999) also test learning models in an experiment.
Figure 1. Population structures. Network representation of 1P and 2P protocols with 24 players.

3 Theory

We choose the hawk-dove game for our analysis of intra- and intergroup conflict. The action set is $S = \{s_1, s_2\}$, where $s_1$ stands for hawk and $s_2$ stands for dove. The hawk-dove game can be parametrized as

$$
\hat{\Pi} = \begin{pmatrix}
 a + \frac{1}{2}(v - c) & a + v \\
 a & a + \frac{1}{2}v
\end{pmatrix},
$$

(1)

where the matrix element $\pi_{ij}$ defines the payoff from choosing action $s_i$ when playing against pure strategy $s_j$. The parameters $v < c$ represent the valuation of the good and the cost of the conflict, respectively. And $a > 0$ is an endowment of the players.

We analyze the replicator dynamics with an even number of players divided into two populations of equal size, labeled as $X$ and $Y$.\footnote{Page and Nowak (2002) explain different approaches to deterministic dynamics in population games. See Szabó and Fáth (2007) or Sandholm (2010) for comprehensive surveys.} Define the simplex $S^X = \{s^X = (s^X_1, s^X_2) : \sum_{a=1,2} s^X_a = 1\}$ such that any point in it represents the fraction of population $X$ employing each available strategy.\footnote{Each individual of a population could be also using a mixed strategy. This does not alter the analysis.} $S^Y$ analogous for population $Y$. The product $\Omega = S^X \times S^Y$ is the set of strategy profiles and also the state space of the dynamical system. To shorten the notation, let $x$ and $y$ be the share of the strategy $s_1$ in populations $X$ and $Y$, respectively. The state vector of population $X$ is $\omega_x = (x, 1 - x)^T$, and $\omega_y = (y, 1 - y)^T$ for population $Y$. Then, the dimensionality of the problem reduces to two, $x$ and $y$, because the state of the dynamical system is a composite in the form $\omega = (x, 1 - x; y, 1 - y)^T \in \Omega$. Let $\mathcal{L}_{\hat{\Pi}} : [0, 1] \times [0, 1] \to \mathbb{R}$ be a linear operator for a given $2 \times 2$ matrix $\hat{\Pi}$. Its application over a bi-dimensional vector $v$ is defined as $\mathcal{L}_{\hat{\Pi}}[v] = \langle e, \hat{\Pi}v \rangle$, where we define $e = (1, -1)^T$ and $\langle \cdot, \cdot \rangle$ is the inner product in the vector space. The matrix $\hat{\Pi}$ is given in (1).

Consider two standard matching protocols: a one-population protocol (1P) and a two-population protocol (2P). In 1P, players interact randomly with other players from
Table 1. Fixed points. Replicator dynamics with $v = 12$ and $c = 18$.

<table>
<thead>
<tr>
<th>Pure-Pure: $p^*_1 = (0, 0)$</th>
<th>Pure-Mixed/Mixed-Pure $p^*_5 = (0, 2/3(1- \kappa))$</th>
<th>Mixed-Mixed: $p^*_9 = (2/3, 2/3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^*_2 = (1, 0)$</td>
<td>$p^*_6 = (1, (2 - 3\kappa)/[3(1 - \kappa)])$</td>
<td></td>
</tr>
<tr>
<td>$p^*_7 = (0, 1)$</td>
<td>$p^*_8 = (2/3(1 - \kappa), 0)$</td>
<td></td>
</tr>
<tr>
<td>$p^*_4 = (1, 1)$</td>
<td>$p^*_3 = (0, 1)$</td>
<td></td>
</tr>
</tbody>
</table>

Their population and so, technically, every player in a population earns the payoff of her choice against the aggregate strategy of her own population. In 2P, the row players (population $X$) play against the column players (population $Y$). See Figure 1 for a graphical representation of these population structures. In the 1P case and focusing on population $X$, we can write the rate of growth of the strategy $s_1$ in the population as $\dot{x} = x(1 - x)\mathcal{L}_X[\omega_x]$. Focusing on the same population for the 2P case, we write $\dot{x} = x(1 - x)\mathcal{L}_X[\omega_y]$.

As our key novelty, we introduce the coupling parameter $\kappa \in [0, 1]$ that integrates the two protocols in a linear fashion. This coefficient is restricted to the unit interval such that its extremes $\kappa = 0$ and $\kappa = 1$ correspond to the one-population and the two-population settings, respectively. We then define the linear combination $\dot{x} = x(1 - x)[(1 - \kappa)\mathcal{L}_X[\omega_x] + \kappa\mathcal{L}_Y[\omega_y]]$ generalizing the study to situations of simultaneous existence of strategic interactions at both intra- and intergroup levels. The (instantaneous) payoff function for a player belonging to the population $X$ and choosing strategy $s_i \in S$ is given as

$$\pi_X(s_i; x, y)(t) = (1 - \kappa)[\pi_{i1}x(t) + \pi_{i2}(1 - x(t))] + \kappa[\pi_{i1}y(t) + \pi_{i2}(1 - y(t))].$$

(2)

Finally, we obtain the dynamics of the model as a system of coupled ODEs:\textsuperscript{10}

$$\begin{align*}
\dot{x} &= x(1 - x)\frac{1}{2}[v - c(x + \kappa(y - x))] \\
\dot{y} &= y(1 - y)\frac{1}{2}[v - c(y + \kappa(x - y))].
\end{align*}$$

(3)

The rate of growth of each strategy in the population is determined solely as a function of: (i) the current state of the system $(x, y)$, (ii) the value of the good $v$ and the cost of the conflict $c$, and (iii) the coupling parameter $\kappa$.

This parameter $\kappa$ represents the strength of the coupling between the two populations of players, while $(1 - \kappa)$ is the weight of the interaction within each group. Depending on the context of application of the model, this can mirror different effects. For example, if we consider the viewpoint of a trader in the example by Mailath (1998), then $\kappa$ measures the fraction of participants at the trading fair coming from a neighboring city. And if

\textsuperscript{10}Formulation of $\dot{y}$ and $\pi_Y$ comes from $\dot{x}$ and $\pi_X$ by symmetry, exchanging population labels $(x \leftrightarrow y)$. 

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we focus on the idea of best response bundling in the experiment by Grimm and Mengel (2014) our model can be seen as the mean field abstraction of a treatment with two hawk-dove games, where $\kappa$ tunes the frequency in which each matching protocol appears.

The following proposition formalizes the analysis of the dynamical system (3) and relates it to our notion of intra- and intergroup conflict. See proof in the Appendix.

**Proposition 1.** Given the replicator dynamics defined in (3):

(a) if $\kappa < \kappa^*_m$, the mixed Nash equilibrium is selected,

(b) if $\kappa > \kappa^*_p$, the pure Nash equilibria are selected,

(c) in the intermediate range $\kappa^*_m \leq \kappa \leq \kappa^*_p$, a hybrid equilibrium is selected where one population plays hawk and the other one randomizes.

The cutoff points satisfy $\kappa^*_m = 1/2 \leq \kappa^*_p$, and $\kappa^*_p = v/c$ if $c < 2v$ or $\kappa^*_p = 1 - v/c$ if $c > 2v$. If $c = 2v$, then $\kappa^*_p = \kappa^*_m = 1/2$ and case (c) is void.

Proposition 1 contains the 1P and 2P cases from previous research (Oprea, Henwood, and Friedman, 2011) as limit cases. The prediction for region (a) is as in the 1P matching ($\kappa = 0$) and the one for region (b) is as in the 2P matching ($\kappa = 1$). The hybrid equilibrium in region (c) is novel and exists neither as a Nash equilibrium nor as an attractor of the 1P or 2P settings. Table 1 gives the coordinates of the fixed points with parameters corresponding to the games played in the experiment (Section 4).

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11 The definition can include asymmetric coupling with groups of different size or in situations where the populations weight the two conflicts in a different way, suitable in the meaning of animal competition.
We illustrate in Figure 2 the predictions for equilibrium selection. For values of \( \kappa < \kappa_m^* = 1/2 \) the only attractor in the phase space corresponds to both populations mixing with coefficients \( (2/3, 2/3) \). For \( \kappa > \kappa_p^* = 2/3 \) full separation emerges: one group plays hawk and the other one plays dove. For the intermediate range of coupling \( \kappa \in [1/2, 2/3] \), replicator dynamics predicts stability in a pure-mixed configuration such that the more hawkish population plays hawk with full share of choices \( (x = 1) \), while the more dovish group plays a mixed strategy with a share \( y = 1/6 \) of the action hawk.

Proposition 1 generates immediate testable hypotheses. On top of relying on the point predictions (mixed equilibrium for \( \kappa < 1/2 \), pure equilibria for \( \kappa > 2/3 \)), we make use of a "separation index", \( \Delta s(\kappa) \in [0, 1] \). This index is defined as \( \Delta s(\kappa) = \bar{s}_1(\kappa, X) - \bar{s}_1(\kappa, Y) \). That is, \( \Delta s(\kappa) \) is the share of the hawk strategy in the more hawkish population minus the share of the hawk strategy in the more dovish population,\(^{12}\) for a given value of the treatment variable \( \kappa \). Interpreting Proposition 1 in a qualitative fashion and using \( \Delta s(\kappa) \), we obtain as our main hypotheses

\[
\Delta s(0) = \Delta s(0.2) = \Delta s(0.4) < \Delta s(0.6) < \Delta s(0.8) = \Delta s(1). 
\]

4 Experiment

For the experiment, we choose the payoff parameters \( a = 3 \), \( v = 12 \), and \( c = 18 \). This results in the following hawk-dove game

\[
\tilde{\Pi} = \begin{pmatrix} 0 & 15 \\ 3 & 9 \end{pmatrix},
\]

with three Nash equilibria (as standard two-player game) of the form \( (\sigma_X, \sigma_Y) \in \{ (1, 0), (0, 1), (2/3, 2/3) \} \), where \( \sigma_l \) denotes the probability that strategy hawk is chosen by player \( l \in \{ X, Y \} \).

Our treatment variable is the value of the coupling parameter \( \kappa \). We consider \( \kappa \in \{ 0, 0.2, 0.4, 0.6, 0.8, 1 \} \). The cases \( \kappa \in \{ 0, 0.2, 0.4 \} \) correspond to Proposition 1 (a), the cases \( \kappa \in \{ 0.8, 1 \} \) to Part (b), and \( \kappa = 0.6 \) corresponds to Proposition 1 (c).

We use a within-subjects design. All subjects play all six treatments consecutively. To avoid order effects or hysteresis, we randomize the order of the treatments at the session level (see Table 2). To prevent reputation effects and to maintain the one-shot character of the experiment, we employ random matching such that the composition of the groups changes at the beginning of each treatment. Players are independently and randomly assigned their initial actions in each treatment. Further, subjects are paid

\(^{12}\)The label “population X” is arbitrarily assigned to the more hawkish group in the steady state for the analysis of the experimental data in the rest of the paper.
Table 2. Sequence of treatments in each session.

<table>
<thead>
<tr>
<th>Period</th>
<th>Session 1</th>
<th>Session 2</th>
<th>Session 3</th>
<th>Session 4</th>
<th>Session 5</th>
<th>Session 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\kappa = 0.8$</td>
<td>$\kappa = 0.2$</td>
<td>$\kappa = 1$</td>
<td>$\kappa = 0.8$</td>
<td>$\kappa = 0.4$</td>
<td>$\kappa = 1$</td>
</tr>
<tr>
<td>2</td>
<td>$\kappa = 0.2$</td>
<td>$\kappa = 1$</td>
<td>$\kappa = 0.4$</td>
<td>$\kappa = 0$</td>
<td>$\kappa = 0.8$</td>
<td>$\kappa = 0.6$</td>
</tr>
<tr>
<td>3</td>
<td>$\kappa = 0$</td>
<td>$\kappa = 0.6$</td>
<td>$\kappa = 0.6$</td>
<td>$\kappa = 0.4$</td>
<td>$\kappa = 0.2$</td>
<td>$\kappa = 0.2$</td>
</tr>
<tr>
<td>4</td>
<td>$\kappa = 0.6$</td>
<td>$\kappa = 0$</td>
<td>$\kappa = 0$</td>
<td>$\kappa = 0.6$</td>
<td>$\kappa = 1$</td>
<td>$\kappa = 0.4$</td>
</tr>
<tr>
<td>5</td>
<td>$\kappa = 0.4$</td>
<td>$\kappa = 0.8$</td>
<td>$\kappa = 0.2$</td>
<td>$\kappa = 1$</td>
<td>$\kappa = 0$</td>
<td>$\kappa = 0.8$</td>
</tr>
<tr>
<td>6</td>
<td>$\kappa = 1$</td>
<td>$\kappa = 0.4$</td>
<td>$\kappa = 0.8$</td>
<td>$\kappa = 0.2$</td>
<td>$\kappa = 0.6$</td>
<td>$\kappa = 0$</td>
</tr>
</tbody>
</table>

only for one treatment selected randomly in order to avoid wealth effects or hedging behavior across treatments (Blanco et al., 2010). This randomization is implemented with a public dice roll at the end of each session.

Other experimental procedures were as follows. All participants received hardcopies of the instructions at the beginning of the session, and afterwards these were summarized orally. Each session began with a trial part consisting of three periods of 90 seconds each in which the players had the opportunity to familiarize themselves with the software. These three periods were not paid, the subjects were aware of this, and we chose payoff matrices that differed from the hawk-dove games that would be used in the actual treatments. The six periods in which we ran the treatments had a time length of 210 seconds. Subjects reported a good understanding of the task and software in an anonymous questionnaire they filled in at the end of the sessions.

The experiment is conducted in continuous time using the software called ConG, a package made recently available in open-access by Pettit et al. (2014). We have generated a laboratory environment in which the subjects play in (virtually) continuous time, and they can make their choices in a fully asynchronous manner, experiencing the updating of the system in real time.

Figure 3 shows two examples of the graphical display faced by subjects in our experiment. On the left side of the screen, players see the payoff matrices and the selection tool. Every agent gets framed as a row player who needs to choose either $A$ (hawk) or $B$ (dove). The selection can be made with the radio buttons and/or with the up and down arrow keys of the keyboard. Subjects can change their action at any point in time and their choices have an immediate impact on both games. The current choice is highlighted with a blue shadow in the row that is selected.

Players see two payoff matrices, the left one refers to the interaction with the “own group” and the second one refers to the interaction with the “other group”. The entries of these two matrices displayed are determined as $(1 - \kappa)\hat{I}$ and $\kappa\hat{I}$, respectively, with

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13 English translation of the instructions available as supplementary material.
14 We have extended the basic package of ConG with a new payoff function, a customized matching scheme, and a new graphical interface adapted to the information set that needs to be displayed according to our design.
Figure 3. Experimental display. Screenshot (translated from German) of two terminals at the end of two treatments. Left: hawkish player in $\kappa = 1$. Right: mixing behavior in $\kappa = 0.4$.

$\hat{\Pi}$ defined in (5) and $\kappa$ being the treatment variable (not known by players). Subjects were explained that the level of their payoff flow was determined as the sum of what they were earning simultaneously in the interactions with the own and with the other group. The upper left corner indicates the accumulated payoff during the period and the remaining time.

The right half of the screen provides the players with all relevant information on the state of the play. The top chart plots the average strategy (i.e., the share of subjects choosing hawk) of each population. The middle chart documents the player’s own choice, which can only alternate between A (hawk) and B (dove). The bottom chart plots a dark red solid line representing the payoff flow that the player is earning at a given point in time. The red shadow helps to give the player the notion that the actual payoff they earn is the accumulated over time. The three charts share the same horizontal axis, time. Note that all changes of any factor are shown in the corresponding charts without any noticeable delay.

We ran six experimental sessions at the DICELab for experimental economics in Düsseldorf in April and May 2015. Each session comprised 24 subjects (two populations of 12 subjects each): 144 subjects in total. Generally, participants were recruited from the local subject pool which contains students of various fields at the Heinrich Heine University of Düsseldorf, using ORSEE (Greiner, 2015).

5 Results

Figure 4 presents the data from all sessions and from all treatments. Vertical axis represents the share of players who choose action hawk in the two populations of each session, at each instant. Horizontal axis represents time in seconds. This figure shows the evolution of the average strategy of the populations over time.

Observed group behavior is qualitatively in line with the predictions. In the
treatments with $\kappa \in \{0, 0.2, 0.4\}$, aggregate choices in both populations are expected to converge to the mixed equilibrium where $2/3$ of the agents in each group should choose hawk. We can see this mixing behavior in the first three rows of the plot, and we find that the average strategies oscillate around that value in all 18 charts. When we consider the last 60 seconds of play as the region for computing the steady state of the system, we find that the separation index $\Delta s(\kappa)$ is between 0 and 0.13 in all six sessions of these three treatments.

For $\kappa = 0.6$, theory suggests the hybrid case where one population should choose the pure strategy hawk while an average of $1/6$ of hawkish play should emerge in the other population. Figure 4 shows this kind of outcome for Sessions 3, 5 and 6 where we observe $\Delta s = 0.37$, 0.31, and 0.65, respectively. In the other three sessions, the steady state is more in line with the mixed equilibrium and we observe separation indices in the vicinity of 0.15, resembling the observations in the treatments with lower $\kappa$. Therefore, the level of coupling for which the system evolves towards a polarized configuration varies from session to session.

With $\kappa \in \{0.8, 1\}$, the average strategies of the two populations are predicted to separate in the steady state. In Figure 4, the last two rows of the graph show much separation at the end of the treatments. Average separation in the two-population treatment ($\kappa = 1$) is 0.93. For $\kappa = 0.8$ we obtain $\Delta s(0.8)$ between 0.70 and 0.95 except in Session 4. In this session we observe a period of experimentation with mixing behavior, with departure towards the pure equilibrium delayed and slow, such that $\Delta s(0.8) = 0.45$.

Figure 5 summarizes this information. Panel (a) compares the aggregate results
Figure 5. Aggregate results. (a): share of hawk choices in steady state by $\kappa$ in all sessions, compared to bifurcation diagram in Figure 2. (b): $\Delta s(\kappa)$ for each session.

of all sessions to the bifurcation predicted by replicator dynamics. For each session, population label $X$ has been arbitrarily assigned to the more hawkish population in the steady state (last 60 seconds). We observe three main effects. First, for levels of $\kappa$ in which the intragroup interaction is more intense, mixing behavior occurs, but with a general downward bias in the share of hawk choices in all populations. Second, the scatter plot for $\kappa = 0.6$ and $0.8$ shows considerable dispersion, and even though the existence of the bifurcation is intuitive, the intensity of the polarizing effect is clearly weaker than predicted by replicator dynamics. Third, most of the deviation from perfect separation in the last two treatments ($\kappa \in \{0.8, 0.1\}$) is driven by the dovish populations. For $\kappa = 1$, the share of strategy hawk in the steady state of the hawkish populations is always higher than $x = 0.99$ in five cases, with $x = 0.96$ in Session 1. Nevertheless, the share of strategy hawk in the dovish populations is always non-zero and reaches values of almost 20% in several sessions. For $\kappa = 0.8$ both the hawkish and the dovish populations deviate from perfect polarization but still a similar argument applies.

We consider that this kind of behavior conforms a pattern of loss aversion. The configurations with high values of $\kappa$ polarize the conflict and promote high inequality in the payoffs. As a consequence of this, some individuals within the dovish group that are able to identify how their group is doomed to earn the lowest payoffs deviate systematically from their best response, in what we consider is an attempt to break the coordination in the less profitable outcome. We observe an interesting case of the tensions to avoid being a dove in the treatment $\kappa = 0.8$ in Session 1 (see Figure 4). Given this level of coupling between the two populations, the dynamic analysis predicts separation in the choice of strategies. There has to exist a group being mostly hawkish

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15See also the discussion in Oprea, Henwood, and Friedman (2011, Section 4).
and one being mostly dovish but it is not relevant (macroscopically) which group plays each role because the system of differential equations is symmetric and both polarized solutions are fully equivalent. In this observation we capture a constant oscillation in which the populations visit both polarized configurations alternating in time.

Panel (b) of Figure 5 plots the separation indices for each treatment, classified by session. In spite of the subtleties already described above, the statistical analysis confirms qualitatively the hypotheses (4) generated by replicator dynamics. Two-sided Wilcoxon signed-rank tests yield the following results,

\[ \Delta s(0) \overset{p > 0.999}{=} \Delta s(0.2) \overset{p > 0.999}{=} \Delta s(0.4) \overset{p = 0.031}{<} \Delta s(0.6) \overset{p = 0.031}{<} \Delta s(0.8) \overset{p = 0.062}{\leq} \Delta s(1), \]  

There are no significant differences between the consecutive separation indices with \( \kappa \in \{0, 0.2, 0.4\} \). When \( \kappa = 0.6 \), the separation index becomes significantly larger than before, and for \( \kappa = 0.8 \) it is also significantly larger than for \( \kappa = 0.6 \). Last, the separation index for \( \kappa = 1 \) is weakly significantly larger than for \( \kappa = 0.8 \), a slight deviation from the predicted equality. The following Section 6 extends the model in order to include the behavioral aspects observed in this Section.

## 6 Behavioral model

We analyze the logit-response dynamics (Alós-Ferrer and Netzer, 2010), a model in which players update their actions in a probabilistic manner.\(^\text{16}\) Better alternatives are chosen more frequently than others, but the system can still show configurations not predicted in the standard model. This can be seen as the dynamic version of the logit QRE (Goeree, Holt, and Palfrey, 2005; McKelvey and Palfrey, 1995).

Applied to our hawk-dove game, a player of population \( X \) observing a choice profile in the populations \((x, y)\) and given the chance to revise her strategy chooses action \( s_1 \) (hawk) with probability

\[ p_X(s_1; x, y) = \frac{1}{1 + e^{-\lambda \Delta u}}, \]  

where \( \Delta u = \frac{1}{2} [v - c(x + \kappa(y - x))] \) is the payoff advantage (in population \( X \)) of the strategy hawk with respect to the strategy dove. One obtains \( p_Y \) from \( p_X \) under reversion of labels \( x \) and \( y \). The comparison of strategies affects the dynamics of the system weighted by \( \lambda \in [0, \infty) \), a ‘rationality’ parameter. This parameter captures deviations from the best response: when \( \lambda = 0 \) the revision mechanism is independent from the payoff structure of the game and the system evolves towards an equal share of strategies.

\(^{16}\)The initial model (3) assumes (macroscopically) that the share of each strategy in the populations evolves according to the strategy fitness in a proportional fashion. This is obtained with players that follow imitation rules to revise their choices. For example, Sandholm, Dokumaci, and Lahkar (2008) discuss several revision protocols and their relation to replicator dynamics.
Figure 6. Logit-response dynamics. (a) Stable manifold as a function of $\lambda$ and $\kappa$. (b) Bifurcation diagrams for several values of $\lambda$. (c) Location of the critical value $\kappa_{\text{crit}}$ as a function of $\lambda$. (d) Share of strategy $s_1$ in the mixed equilibrium as a function of $\lambda$.

in the populations, and when $\lambda \to \infty$ it tends to the replicator dynamics.

Hence, the deterministic dynamics is defined by the following system of coupled differential equations (with $p_X$ and $p_Y$ defined in $7$),

$$\begin{align*}
\dot{x} &= p_X(s_1;x,y) - x \\
\dot{y} &= p_Y(s_1;x,y) - y.
\end{align*}$$

(8)

A rest point of (8) corresponds to the logit quantal response equilibrium for the given value of the parameter $\lambda$.\textsuperscript{17} We illustrate the predictions of this model in Figure 6. In panels (a-b) we can see how the stable manifold of the logit-response dynamics in the hawk-dove game also contains a bifurcation in the system as a function of $\kappa$.

For every value of the rationality parameter $\lambda$ sufficiently high, there exists a critical value of $\kappa$ such that equilibrium stability shifts from a mixing configuration towards a more polarized one. As it is shown in panel (b), the prediction for $\lambda = 0$ is independent of the coupling condition and corresponds to uniform randomization. When $\lambda$ increases, the system shows a bifurcation diagram that becomes closer to the prediction with replicator dynamics (see Figure 2-c) the higher the value of $\lambda$.

Overall, the new parameter $\lambda$ can jointly explain both biases that we observe in the experimental results: when the populations play mixed strategies, the share of the action hawk is lower than predicted in the mixed NE, and the polarization for $\kappa = 0.6$ and $\kappa = 0.8$ is clearly less pronounced than predicted by replicator dynamics. Figure 7 shows the fit of the model to the experimental data. We compute one different value of $\lambda$ for each experimental session.\textsuperscript{18}

\textsuperscript{17}See Kreindler and Young (2013, Section 4) and Zhuang, Di, and Wu (2014).

\textsuperscript{18}We perform a grid search on the values for $\lambda$ with resolution $\Delta \lambda = 10^{-2}$, and considering the distance $\sum_k |x^*(\kappa) - \bar{s}_1(\kappa,X)|^2 + |y^*(\kappa) - \bar{s}_1(\kappa,Y)|^2$ as the objective function to be minimized, where
7 Conclusion

Coordination and learning in group behavior is a fundamental motivation for the experimental study of evolutionary dynamics. We have analyzed how human behavior exhibits the dynamical bifurcation from mixing to polarizing equilibria in the hawk-dove game. This depends on the tradeoff between the one-population and the two-population structures and the transition occurs at an intermediate range of values of the coupling parameter $\kappa$. We have naturally extended the study by Oprea, Henwood, and Friedman (2011) finding evidence of how the evolutionary predictions may hold true when humans play a hawk-dove game in the lab.

Our results confirm the hypothesis at a qualitative level, and we can conclude that the predictions for both one-population and two-population settings are robust with respect to the presence of simultaneous inter- and intragroup interactions, respectively. Nevertheless, the transition regime experiences notable variation across sessions, and the bifurcation in human behavior is better captured by the logit-response dynamics.

This paper provides strong support for the use of continuous time experiments in the analysis of intra- and intergroup conflicts. Our observation of actual bifurcations,

$$(x^*, y^*)$$ is the logit-response prediction (as a function of $\kappa$) and $s_1$ is the share of strategy hawk in the steady state in the corresponding experimental treatment.
together with other recent developments in the field, such as the analysis of limit-cycles in Rock-Paper-Scissors games (Cason, Friedman, and Hopkins, 2014) represents a remarkable improvement in the level of resolution to which behavioral studies can aspire with these novel techniques.

References


A Proof of Proposition 1

Location of fixed points. We obtain the zero-growth curves directly from (3). Because of the linearity of the system, these nullclines are simple lines in the plane,

\[
\begin{align*}
\dot{x} &= 0 \rightarrow x = 0 & \dot{y} &= y = 0 \\
\dot{x} &= 0 \rightarrow x = 1 & \dot{y} &= y = 1 \\
\dot{x} &= 0 \rightarrow x = \frac{v - \kappa cy}{(1 - \kappa)c} & \dot{y} &= y = \frac{v - \kappa cx}{(1 - \kappa)c} 
\end{align*}
\]  

(9)

Fixed points are located at the different intersections between a horizontal and a vertical nullcline. Obviously, the corners of the state space are fixed points and represent possible equilibria in which both populations play pure strategies,

\[
\begin{align*}
p_1^* &= (0, 0), & p_2^* &= (1, 0), & p_3^* &= (0, 1), & p_4^* &= (1, 1). 
\end{align*}
\]  

(10)

Four other possible points are such that one population plays a pure strategy and the other one is mixing,

\[
\begin{align*}
p_5^* &= (0, \frac{v}{(1 - \kappa)c}), & p_6^* &= (1, [v - \kappa c]/[(1 - \kappa)c]), \\
p_7^* &= \left(\frac{v}{(1 - \kappa)c}, 0\right), & p_8^* &= \left([v - \kappa c]/[(1 - \kappa)c], 1\right). 
\end{align*}
\]  

(11)
Finally we also obtain a possible configuration in which both populations mix strategies in a symmetric manner,

\[ p_9^* = (v/c, v/c). \]  

(12)

Point \( p_9^* \) lies always inside the unit square because of \( 0 < v < c \) in the definition of the hawk-dove game. Nevertheless, the fixed points in (11) only exist for the range of coupling \( \kappa \in [0, v/c] \).

**Linear stability analysis.** Jacobian \( J \) with matrix elements \( J_{mn} = \partial \dot{n}_m / \partial n \) defined by

\[
2 \times J_{xx}(x, y) = v - 2vx + c[3x^2(1 - \kappa) - \kappa y - 2x(1 - \kappa - \kappa y)] \\
2 \times J_{xy}(x, y) = \kappa cx(1 - x),
\]

with \( J_{yx} \) and \( J_{yy} \) given by symmetry \((x \leftrightarrow y)\). Stable points are those fixed points for which both eigenvalues of the Jacobian matrix (evaluated at the point’s coordinates) are negative (Hofbauer and Sigmund, 2003). The eigenvalues are the two roots of the characteristic polynomial \( \det [\nu I_2 - J] \).

For \( p_1^* \) and \( p_4^* \), we obtain \( \nu_1 = \nu_2 = v/2 > 0 \), and \( \nu_1 = \nu_2 = (c-v)/2 > 0 \), respectively. Therefore, these two symmetric equilibria in pure strategies are never attractors of the system. For the antisymmetric equilibria in pure strategies \((p_2^*, p_3^*)\), the eigenvalues are \( \nu_1 = (c-v-\kappa c)/2 \), and \( \nu_2 = (v-\kappa c)/2 \). If \( c < 2v \), then \( \nu_2 > \nu_1 \) and the polarized configurations are stable for \( \kappa \in [v/c, 1] \). If \( c > 2v \), then \( \nu_1 > \nu_2 \) and the polarized configurations are stable for \( \kappa \in [1/v,c, 1] \).

Considering the points \( p_5^* \) and \( p_8^* \), the eigenvalues are \( \nu_1 = v + v/(2\kappa - 1) \), and \( \nu_2 = [(1 - \kappa)cv - v^2]/[2(\kappa - 1) - c] \). \( \nu_1 \) is negative for \( \kappa > 1/2 \), and \( \nu_2 \) is so for \( \kappa < 1 - v/c \). Then, these two points are stable for \( \kappa \in [1/2, 1 - v/c] \), but only if \( c > 2v \). For the points \( p_6^* \) and \( p_7^* \), the eigenvalues are \( \nu_1 = (c-v)(2\kappa - 1)/[2(\kappa - 1)] \), and \( \nu_2 = (c-v)(\kappa c-v)/[2(\kappa - 1)c] \). \( \nu_1 \) is negative for \( \kappa \in (1/2, 1) \) and \( \nu_2 \) is so for \( \kappa < v/c \). Thus, these points are stable for the range \( \kappa \in [1/2, v/c] \), provided \( c < 2v \).

Finally, the symmetric equilibrium in mixed strategies \( p_9^* \) yields eigenvalues \( \nu_1 = v(v-c)/2c \), and \( \nu_2 = v(c-v)(2\kappa - 1)/2c \). The first one is constant and always negative given \( c > v \) by definition of the hawk-dove game. The second one is negative for \( \kappa < 1/2 \). Then, \( p_9^* \) is stable in the interval \( \kappa \in [0, 1/2] \).