The wrong man for the job: biased beliefs and job mismatching

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Abstract

Gender segregation in the job market has been pervasive over time, with women being less likely to achieve high-skilled positions. In this paper we derive a model to explain the emergence of gender segregation in the labour markets as a result of females’ biased beliefs regarding their ranking position with respect to other candidates, when abilities are equally distributed among them. Women due to their underconfidence, are less prone to apply to skilled labor jobs than men. and even in case firms can perfectly screen candidates for a job, inefficient matching in the job market may persist. Therefore we provide a theoretical foundation to sustain the importance of implementing a calibrated affirmative action in order to restore the efficiency of the job matching between high skilled firms and candidates, by increasing the diversity of qualified applicants.

What a man thinks of himself, that is which determines, or rather indicates, his fate.

- Henry David Thoreau -

1 Introduction

There is strong evidence that women, while globally facing higher unemployment rates than men, seem also to be segregated in some segments of the labor market: they are

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underrepresented in managerial and legislative occupations and over-represented in mid-skill occupations\(^1\) (Bourmpoula et al., 2012). In this paper, we provide a theoretical foundation to explain the emergence of gender gap and segregation in the job market, as a consequence of different levels of self-confidence of men and women, when abilities are equally distributed among them.

Different reasons have been proposed to explain the existence of the gender gap in the workplace. First, women may have innate lower (higher) abilities than men in some sectors and are thus less (more) likely to be selected when applying. However, even if discussion about this topic is still open, recent research suggests that men and women do not differ much in their cognitive abilities and it is rather social and cultural factors that influence perceived or actual performance differences (Hyde, 2005, Spelke, 2005). Second, women and men face a different trade-off when formulating their career and family plans, and a link between relative wages and fertility has been showed (Erosa et al., 2002, Galor and Weil, 1996). In particular, Dessy and Djebbary (2010) show that the shorter reproductive capability of women with respect to men causes them to be more constrained in their career-family choices, so that failure in coordination of women’s marriage-timing decision lead to persisting gender differences in career choices.

Third, some studies have suggested the existence of a glass ceiling (Cotter et al., 2001)\(^2\), which leads organization to discriminate women’s promotion and thus prevent them to achieve the highest rank in the firm, when having equal abilities than men (Bas-sanini and Saint-Martin, 2008). However, a recent research by the Institute of Leadership & Management (2011), claims that women managers are rather impeded in their careers by lower ambitions and expectations, which lead them to a cautious approach to career opportunities, than by a glass ceiling.

A fourth possible explanation of gender segregation has thus been developed, which relies on different preferences of men and women regarding the job environments where they would like to work, ultimately affecting their job entry decisions. Laboratory (Gneezy et al., 2003, Masclet et al., 2015) and natural field experiments (Flory et al., 2015) provide evidence of women being less likely to apply to competitive work-settings (for a review of these studies, see Niederle and Vesterlund (2011)). In particular, this phenomenon is associated with women i) having different distributional preferences (Bala-foutas et al., 2012), ii) being more risk averse (Charness and Gneezy, 2012) and iii) less

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\(^1\)The global female labour force was estimated to be 1.3 billion in 2012, – about 39.9 per cent of the total labour force of 3.3 billion.

\(^2\)Carol Hymowitz and Timothy D. Schellhardt were the first to use the term “glass ceiling” in their March 24, 1986 article in the Wall Street Journal, “The Glass Ceiling: Why Women Can’t Seem to Break the Invisible Barrier That Blocks Them from the Top Job.”
Self-confidence in the ability to successfully win a contest and the gender of the competitors seems to play an important role in these studies. Wieland and Sarin (2012) and Kamas and Preston (2012a) indeed show that there is no difference in the choice of the payment scheme (i.e. the decision to compete in tournament) with respect to gender when considering gender neutral tasks. In particular, Kamas and Preston (2012b), in a recent experiment investigate the extent to which differences in a taste to compete or differences in ability, confidence, risk aversion, or personality characteristics explain gender differences in willingness to compete and conclude that gender differences in confidence, and to a lesser extent risk attitudes, explain this pattern. Moreover, Günther et al. (2010) observe that "women tend not to compete with men in areas where they (rightly or wrongly) think that they will lose anyway". Whether women have lower self confidence that men is a long lasting question (Lenney, 1977), which seems to be sustained by studies in social psychology (Furnham, 2001, Haynes and Heilman, 2013) showing females as less likely to perceive themselves as qualified to run for political office (Lawless and Fox, 2005), or expressing lower career-entry and career-peak pay expectations (Bylsma and Major, 1992, Schweitzer et al., 2014). In particular, when considering the job market, Barbulescu and Bidwell. (2012) showed that, among MBA students, women’s lower expectations of job demand’s success is one of the causes of their lower number of applications to finance and consulting jobs with respect to men. Even if such expectations had not an empirical foundation (i.e. the authors found no evidence that women were less likely to receive job offers in any of these fields), they lead women to accept lower salary offers than the ones accepted by their male counterparts (Bowles et al., 2005). In line with these results, in a recent experiment Mobius et al. (2014) found that women are significantly more conservative than men in updating their beliefs about their own ability. One implication is that high-ability women who receive the same mix of signals as high-ability men will tend to end up less confident4.

In the present study we provide a first theoretical foundation to explain the crucial role of self-confidence in explaining the observed gender gap in the workplace, as put into evidence by previous experimental results (Kamas and Preston, 2012b, Buser et al., 2014). In our study, differences in how women and men perceive themselves as having

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3For a more general discussion of the contributions of laboratory and field experiments in explaining gender differences on labor market outcomes see Azmat and Petrongolo (2014).

the abilities to fulfill the job responsibilities with respect to other candidates, affect the workers’ application decision, with women being less likely to apply to the skilled segment of the market. We find that biased beliefs of women about their relative abilities with respect to men lead to inefficient job matching equilibria: low skilled but self-confident males are recruited in the skilled segment of the job market while high-skilled, but under-confident, women are relegated to the unskilled one. Our results are consistent with the very recent study by Flory et al. (2015). They show, in a natural field experiment where almost 9000 job-seekers are randomized into different compensation regimes, that women are less likely than men to apply to competitive work settings as much as they shy away from jobs characterized by uncertainty over the payment such that the worker is allowed to believe that she has the ability to influence the outcome through her own labour. Thus, even if the worker is not performing against anyone else, so that competition does not play any role, females are less confident than men about their ability to successfully fulfill the job request.

The role of self-confidence is of primarily importance in our life, having a realistic (and positive) view of ourselves and of our own abilities may increase our motivation, letting us to engage in what we really want and can do. On the other hand, a too much high level of confidence may push people to accept challenges or tasks that are not fitted for their true abilities, increasing their probability to fail when trying to reach their objectives. An under-confident person, conversely, may pass up opportunities or decide not to try (hard enough) to reach his personal aims. A number of studies in psychology and economics have analyzed the role of self confidence both from a practical and theoretical point of view. In particular, when considering the role played by self confidence in the labor market, most of the literature has been primarily focused on the agency model. In Sautmann (2011) and Santos Pinto (2008), Santos Pinto (2010) studies, the principal, who is aware of the agent’s overconfidence, takes advantage of it by paying the worker a lower wage. Bénabou and Tyrole (2002) analyze the role of self-confidence in influencing how people process information and make decisions in order to explain some “irrational” behaviors such as self-handicapping or self-deception. In the studies by Falk et al. (2006a), Falk et al. (2006b) and Andolfatto et al. (2009) self confidence is analyzed in a job searching framework. While Andolfatto et al. (2009) apply the ideas of Bénabou and Tyrole (2002) in a model of labor market search, Falk et al. (2006a) Falk et al. (2006b) show that wrong beliefs about relative ability affects unemployment duration, in turn determining worker’s potential starting wages.

Santos Pinto (2008) refers to the term positive (negative) self-image as the agent’s over(under) estimation of the productivity of effort.
In our model we reasonably assume that abilities are equally distributed among men and women while under confidence is not. As a consequence, the observed gender gap appears to be not (only) a problem of fairness, but a problem of efficiency. Since high-skilled women are less likely to apply for top positions, firms are thus selecting their workers not necessarily in a group containing the best fitted candidates. Women self-select into job positions according to their (mis)perceptions about their opportunity to be successfully recruited. Moreover, once segregated in the unskilled segment of the labour market, women’s wrong beliefs would not be disconfirmed by evidence, making the gender gap to persist over time. We sustain the hypothesis that segregation in the job application and in the hiring process is (also) caused by biased workers’ self-selection. The unequal distribution of males and females between the skilled and unskilled sector is thus endogenous on workers’ application choices, when sorting decision depends on different gender’s misperception of relative abilities to fit the job.

When considering sorting decisions in the labor market and in educational attainments, in a recent study Filippin and Paccagnella (2012) provide evidence that the level of confidence of young agents may consistently affect their future lives: they show that even small differences in initial confidence of people about their ability may lead to diverging patterns of human capital accumulation between otherwise identical individuals. Larkin and Leider (2012) and Dohmen and Falk (2011) experimentally demonstrate that different incentive schemes invite different employees to join the organization, depending on their behavioral biases. In a study closely relate to our topic, Santos Pinto (2012) analyze the emergence of the gender pay gap as a result of males and females different levels of self-confidence in the classic labor market signaling model by Spence (1973): overconfident men are more likely to invest in education than underconfident women, which in turn lead to a higher productivity of men with respect to women, thus generating a gender pay gap.

Differently than in our study, Santos Pinto (2012) aims at explaining the gender gap in wages while we focus our attention at the previous level: we want to investigate the gender gap in the job application process which results in gender segregation, due to high qualified but underconfident women renouncing to apply to the high skilled job market. Moreover, the author explains the existance of a gender pay gap according to different (observable) educational investements made by biased males and females, while in our model the role of the bias does affect their career choice, when holding the same level of ability and education. This job mismatching causes an efficiency loss in the job market, which can be canceled out by calibrating a suitable affirmative action that guarantees the participation of high skilled women in the job market.
Previous researchers have analyzed models of directed search where workers do not randomly search among all possible jobs, but apply for jobs that are more likely to be appropriate for their skills and interests. In particular, Galenianos and Kircher (2009) develop a model where both the success probability of unemployed workers and the wages posted by firms are equilibrium outcomes. Wage dispersion is the result of allowing every worker to apply for multiple jobs. Differently that in their paper, in our model we allow for heterogeneity of both workers and firms, rather than assuming that all workers and all firms are identical. Moreover, as in Chade et al. (2004) and Nagypal (2004) the wages are exogenously given but, a fundamental modification of our paper is played by the role of biased beliefs of (female) workers regarding their relative abilities.

During the past years, several policies have been proposed to establish gender (and minorities) equality in the job market (see Anderson (2004), for an historical view of the affirmative action agenda). Affirmative action, first instituted in US in the 1960s and 1970s by employers and educational institutions, is a policy designed to increase the employment and educational opportunities available for disadvantage groups. Often, it is addressed to qualified women and other minorities and gives them preference in hiring, promotion, and admission. Coate and Loury (1993) provide mixed results regarding whether positive discrimination policies eliminate negative stereotypes in the employer’s beliefs. Affirmative action such as exogenous imposed quotas on the labour force composition have been often criticized for being unfair and inefficient. Opponents to such a policy, claim that it is unfair to hire an individual for a job on anything other than his qualifications and skills. However, the problem relieved in this paper is that females do not even apply to high qualified jobs, because mistakenly perceiving themselves as having relative lower ability with respect to other candidates. Since applications to a job offer is a time consuming process, they prefer to apply to lower-skilled jobs, thus avoiding the risk to be disregarded and to be unemployed when competing for a high skilled position. In a recent laboratory experiment, Balafoutas and Sutter (2012) provide evidence that affirmative actions encourage women to enter competition more often, without negatively affecting efficiency. Niederle et al. (2013) obtained a similar result when experimentally testing the effect of quota in favor of women in competitive tournaments. Moreover, in contrast with a common critique to the implementation of quotas, they did not find a decrease in the minimum performance threshold when achieving a more diverse set of winners. Finally, in a recent field experiment investigating affirmative actions in Colombia (Ibañez et al., 2015), the authors find that the gains of attracting female applicants far outweigh the losses in male applicants. In the present study we thus provide a theoretical foundation of the positive effect of
programmes designed to positively encourage the correct development of self-image in women and to promote new role models.

The reminder of this paper is organized as follows. Section 2 presents the model. In the following section, we compute the equilibrium of the model when workers have unbiased beliefs about their abilities and then the equilibrium when female workers exhibit underconfidence. In Section 3 we analyze the role of affirmative actions. Finally, Section 4 discusses the results and concludes. Proofs of all results are in the Appendix.

2 The model

Consider a job market with the following matching process. There is a continuum of workers of measure one, where half of them are male and half are female, and two firms \( G \) and \( B \). Firm \( B \) has a number of vacancies of measure one and therefore all workers can be employed in it, while firm \( G \) has vacancies of measure \( z < 1 \) and therefore it cannot be that all workers are employed in \( G \). Wages are fixed and equal to \( w_G \) and \( w_B \) for firm \( G \) and \( B \) respectively, with \( w_G > w_B > 0 \).

Each worker is of type (ability) \( k \in \{H, L\} \) with equal probability\(^7\). Ability and gender are not correlated, therefore the population is partitioned in four groups of equal size: high ability \( (H) \) male workers, \( H \) female workers, low ability \( (L) \) male workers and \( L \) female workers.

Each worker \( i \) receives an informative signal \( s_i \in \{H, L\} \) about her/his type. The signal is informative because for both \( k \in \{H, L\} \) \( \Pr(i = k|s_i = k) = \sigma > \frac{1}{2} \). Therefore, \( \frac{1}{2} \) of the workers receive a signal \( s_i = H : \frac{\sigma}{2} \) are truly \( H \) type workers, while \( \frac{1-\sigma}{2} \) are \( L \) workers who have received a wrong signal.

The quality of the signals and the correct distribution of abilities in the pool of applicants are public information and, in particular, each worker knows that there are \( \frac{1}{2} \) workers of type \( H \), equally divided among man and woman.

After having observed the signal, each worker decides whether to apply to firm \( G \) or \( B \) of the labor market\(^8\). Each worker can apply at most to one firm. The utility of an employed worker is equal to her wage while the payoffs of unmatched workers are normalized to zero. Since there is no unemployment benefit, participation constraints are satisfied.

\(^7\)We could more generally assume that each worker, irrespective of her gender, is \( H \) type with probability \( \tau \). Results would not change.

\(^8\)Alternatively, we could assume that the worker has to pay a specific cost \( C_k \) with \( C_G > 0 \) and \( C_B \) normalized to zero (investment in education, for instance) to apply to sector \( k \). Results are similar in a model with sunk costs of application.
Firm $G$ gives priority to $H$ workers when hiring. With probability $p$ firm $G$ observes applicants’ type, while with complementary probability firm $G$ is not able to observe their type. If more than $z$ workers apply to $G$ then a rationing occurs. If firm $G$ observes applicants’ type, it hires $H$ workers first and, if some jobs are still vacant, workers of type $L$ are enrolled too.\footnote{In our model, we assume that employers do not have prior beliefs regarding gender ability, i.e. negative stereotypes. In particular, firms select workers only considering the applicants’ abilities.} Differently, when applicants’ type is not observed by firm $G$, it randomly selects $z$ workers among their applicants.

Along the rest of the paper we assume the following:

**Assumptions A1** $\frac{1}{2} > z > w_B \frac{w_G}{w} \text{ and } w_G > 2w_B$

The assumption that $\frac{1}{2} > z$ joint with the fact that half of the workers are of type $H$ implies that, if $p = 1$, $L$ type workers are are not hired in firm $G$ if all workers who receive a signal $s_i = H$ applies to firm $G$. The assumptions that $w_G > 2w_B$ and $z > w_B \frac{w_G}{w}$ imply that if $p = 0$ and firm $G$ hires at random, then all workers prefer to apply to firm $G$ than working in firm $B$.

Firm $B$ offers an unlimited number of job offers while firm $G$ has a limited number of vacancies. Workers therefore compete for a job position in $G$ which provides high payoff conditional on success while firm $B$ is characterized by probability equals to 1 of getting a job offer but for a low pay.

In our model search is directed, firms publicly post vacancies and commit to a wage and each worker chooses the job to which to apply. We assume that agents can only apply to one firm, not to both. This is a reasonable assumption since job applications is a time consuming process: workers need to adjust their resume in order to present themselves as the ones perfect fitting for the job, according to the firm’s specific requirements. As a consequence, candidates do not apply to all the job offers in the market, but just select the ones that they think to be the more achievable from their own point of view. Finally, to define whether a position is achievable or not a candidate has thus to take into consideration many factors: the expected salary, the working environment, but also his (perceived) relative ability with respect to others competitors in the application process. Our model resembles to the academic market: universities are vertically differentiated (in our model the market is segmented in two levels), entry salaries are fixed at large extent and there are a limited number of jobs available, at least in top universities; as in our model, candidates may apply to a limited number of jobs (due to time and effort constraints)\footnote{In the american job market each candidate can send a limited number of f expresses of interests by candidates.to the departments they applied, (currently only five) which are considered as credible signal of sincere interest in the position.}.
2.1 The job matching equilibrium

We first derive the job matching equilibria in the benchmark case in which workers do not have biased beliefs regarding their ability. In both subsections 2.1.1 and 2.1.2, we examine the job matching equilibria when considering different levels of transparency in the job market, so that firm G is able to infer with probability $p$ its applicants’ types.

2.1.1 The benchmark case

In this section we examine the matching equilibria in the job market that will be used as a benchmark when we introduce the assumption that female workers have biased beliefs about their own ability. By assumption each worker can either apply for a job in $G$ or in $B$.

**Proposition 1** If $p \geq \bar{p}$, with $\bar{p} \equiv \frac{2w_G - w_H}{2w_G - 2\sigma}$, then in equilibrium workers who receive a signal $s_i = H$ apply to firm $G$ and those who receive a signal $s_i = L$ apply to firm $B$ (separating equilibrium). If $p \leq \frac{z w_G - w_B}{z(2\sigma - 1)w_G} \equiv \hat{p}$, both types of workers apply to firm $G$ (pooling equilibrium). If $p \in \left[\bar{p}, \hat{p}\right]$, there is a semiseparating equilibrium such that workers who receive a signal $s_i = H$ apply to firm $G$ and those who receive a signal $s_i = L$ are indifferent: a fraction $\xi(p) \in (0, \frac{1}{2})$ of them apply to firm $B$ and a fraction $(1 - \xi(p))$ apply to firm $G$, with $\xi(p)$ satisfying the following equality:

$$p(1 - \sigma) \frac{z}{\xi(1 - \sigma) + \sigma \frac{z}{2}} + (1 - p) \frac{z}{\xi + \frac{z}{2}} w_G = w_B$$

Proof of proposition 1 is in the Appendix. The previous proposition states that when firm $G$ ability to screen candidates is high enough ($p \geq \hat{p}$) then only workers receiving a signal $s_i = H$ apply to G and, among them, $\frac{\sigma}{2}$ are truly $H$ types and $\frac{1 - \sigma}{2}$ are $L$ types who received the wrong signal. Since we assume that $z < \frac{1}{2}\sigma$, the probability to hire a high ability worker is equal to $p + (1 - p)\sigma$. When the probability $p$ to screen candidates decreases, the efficiency of the matching decreases as well. Namely, if $p < \bar{p}$, the probability of hiring a $H$ worker is $p + \frac{(1 - p)\sigma}{2}$. Thus, in absence on any workers’ biased beliefs about own relative ability, a more transparent job market where hiring is based on merit induces a more efficient matching in the job market.

2.1.2 Underconfidence and job matching

In this section we introduce the following modification with respect to the benchmark case: female workers have now biased beliefs regarding their ability in the job market; namely, any female worker at time $t = 0$ underestimates the probability of being of type
H. In particular when a female receives a signal \( s_i = H \), she underestimates the quality of the signal and believes that she is of type \( L \) with a larger probability: parameter \( \rho \) measures the woman’s bias, that we define as underconfidence, with \( 0 \leq \rho \leq +\infty \). Men, differently, are defined as honest overconfident (see footnote at page ??).

Formally, a female worker \( i \) who observes \( s_i = H \) at time \( t = 0 \) assigns probability \( \Pr_f(i = H \mid s_i = H) = \frac{1}{1+\rho} \sigma \) to be a \( H \) type worker (the subscript \( f \) stays for "female"). Therefore, \( \Pr_f(i = L \mid s_i = H) = \frac{1+\rho - \sigma}{1+\rho} \) is the probability that a female worker \( i \) assigns to be a \( L \) type worker when she receives a signal \( s_i = H \). Similarly, \( \Pr_f(i = L \mid s_i = L) = \min(1, \frac{1+\rho}{1+\rho} \sigma) \). Think, for example, of a student who receives a good mark on an exam (i.e. a signal about her ability): she may think the mark was not caused by her ability but by a mistake of the professor, or that it depends on luck; she does not correctly infer the information of the signal, because of her prior beliefs about her own ability. In particular, underconfidence is different than holding a negative stereotype against women: in our model a woman, when receiving a signal \( s_i = H \) assigns lower probability of being better than other candidates, independently on their gender.

Now workers differ not only with respect to their "intrinsic ability", but also with respect to their level of selfconfidence (which is correlated with their gender). We have thus to analyze the decision of four different types of workers: high ability male workers, high ability female workers, low ability male workers and low ability female workers. The following proposition describes which equilibria emerge depending on the parameters. In particular, sorting is defined as inefficient when (some) workers who have received signal \( s_i = H \) apply to firm \( B \) and (some) workers who received a signal \( s = L \) apply to firm \( G \).

First we assume that \( z \leq \frac{\sigma}{4} \) so that workers are facing a very competitive job market when applying for high skilled positions. In particular, the vacancies available in \( G \) are limited so that if only \( H \) male workers apply to firm \( G \) some of them remain unemployed.

**Proposition 2** If \( \rho > \frac{2\sigma - 1}{1-\sigma} \) and \( p \in \left[ \frac{1}{w_G}, \frac{1}{2z-4z(1-\sigma)} \right] \) in equilibrium male workers apply to firm \( G \) and female workers apply to firm \( B \) (gender segregation and inefficient sorting). If \( \rho \geq \frac{4z^2 - z w_B - w_B}{w_B - 4z w_G (1-p)} \) and \( p \geq \frac{1}{w_G} \) male workers who observe \( s_i = H \) apply to firm \( G \) and all other workers apply to firm \( B \) (gender segregation with efficient sorting). All men, independently on their signals and female workers who observe \( s_i = H \) apply to firm \( G \) while all other workers apply to firm \( B \) when \( 0 < \rho < \frac{\rho \sigma w_G}{4(w_B - 3w_G(1+p))(1+\sigma)} - 1 \) and \( p \leq \frac{4z w_B - 3w_B}{4z(4\sigma - 2)} \) (partial gender segregation, semipooling). If \( p \in \left[ \frac{1}{w_G}, \frac{2z^2 - 2zw_B}{w_G(1-\sigma)} \right] \) and \( \rho < \frac{2\sigma - 1}{1-\sigma} \) in equilibrium workers who observe \( s_i = H \) apply to firm \( G \) and the other workers apply to firm \( B \) (no gender
segregation and efficient sorting). If \( \rho \leq \frac{1-\sigma}{\sigma} \) and \( p \leq \frac{1}{w_G} - \frac{z w_G - w_B}{2z(1-\sigma(\rho+1))} \) or \( \rho > \frac{1-\sigma}{\sigma} \) and \( p \leq \frac{z w_G - w_B}{z w_G} \) then all workers apply to firm \( G \) (no gender segregation, pooling).

According to the previous proposition,\(^{11}\) when females have biased beliefs regarding their relative ability and the number of vacant jobs in the skilled segment of the job market is sufficiently low, we observe the emergence of equilibria characterized by an inefficient matching between firms and workers. In particular, when all males apply to \( G \) and all females apply to \( B \), we have that firms are not selecting their workers within the best pool of applicants. An equal representation of females and males in the skilled segment of the job market thus is not only a matter of fairness but, likewise important, also a matter of efficiency.

**Corollary 1** Suppose \( \rho \) is large, in particular \( \rho \geq \frac{2\sigma-1}{1-\sigma} \). If \( p \) is small every worker applies to firm \( G \). When \( p \) increases then only male workers apply to firm \( G \). An increase in the precision \( p \) does not increase the average quality of the applicants, but it induces gender segregation, because no women apply for the job.

Interestingly enough, the above effect underlined in the corollary induces a (wrong) statistical inference that reinforces women’s underconfidence. When the precision in the hiring process increases and, ex-post, better workers are hired on average, it also happens that more men than women are hired. However, this is not due to the fact that women are on average less talented than men (i.e. positive correlation between (male) gender and ability), but to the fact that they now apply less frequently than men because they are underconfident.

It is worthy noticing that when there is efficient sorting and gender segregation, i.e. only male workers who observe \( s_i = H \) apply to firm \( G \), a wage increase would first attract male workers who observes \( s_i = L \) rather than female workers and therefore firm \( G \) does not have any incentive to solve the gender segregation problem.

We observe different results if we assume that \( \frac{z}{4} < z \leq \frac{1}{4} \), as shown in the following proposition. In particular, we explicit our results when considering the case highlighted in our Corollary 1. When \( z \) is sufficiently small, so that \( z \leq \frac{z}{4} \), we observe that an increase in the precision \( p \) leads to an efficient matching. Only men who received a signal \( s_i = H \) (both high ability male workers who received the correct signal, that is \( \frac{z}{4} \) of the applicants, and low ability male workers who received the wrong signal, that

\(^{11}\) Proof of Proposition 2 is provided in the Appendix.

\(^{12}\) The analysis of the equilibria when \( \frac{1}{4} \leq z \leq \frac{5}{2}, \frac{5}{2} \leq z \leq \frac{1+\sigma}{4} \) and \( \frac{1+\sigma}{4} \leq z \leq \frac{1}{2} \), together with proof of Proposition 3, is relegated to the Appendix.

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is $\frac{1-z}{4}$ of the applicants) apply to firm $G$: since $z \leq \frac{2}{3}$ all high ability men are hired while low ability (male) workers are disregarded. Differently, as shown in the following proposition, when $z$ increases it is no more possible for firm $G$ to maximize efficiency when hiring workers and females candidates are underconfident about their ability.

**Proposition 3** The equilibrium where only male workers who observe $s_i = H$ apply to firm $G$ while all other workers apply to firm $B$ exists if and only if

$$\tilde{z} = \frac{(w_G - w_B)\sigma + w_B}{4w_G}$$

and $\tilde{z} < \frac{1}{3}$, and when $p \geq \frac{(4zw_G - w_B)(1 + \rho)(1 - \sigma)}{\sigma(1 - 4z)w_G}$. Differently, when $\hat{z} \leq z$, it is no more possible to observe the equilibrium where only male workers who observe $s_i = H$ apply to firm $G$ while all other workers apply to firm $B$. The conditions for all other equilibria to exist are identical as in proposition 2.

Previous proposition states that when the positions available in the more competitive and remunerative firm increases and is included in the interval

$$\left[\frac{(w_G - w_B)\sigma + w_B}{4w_G}, \frac{1}{2}\right],$$

we observe that when firm $G$ has high ability to screen candidates and females are sufficiently underconfident, it is no more possible for firm $G$ to maximize efficiency when hiring workers. As a consequence, restoring fairness in the job market is, once again, important in term of efficiency.

In the next section we thus present an affirmative action as a possible solution to close the gender gap in the high skilled segment of the job market. The intuition is simple. Suppose that a quota of female workers must be hired by firm $G$. The ex-ante probability of being hired now will increase for a woman, irrespective of her ability. Therefore, also a biased woman who is underconfident may prefer to apply now. It is important to note that a quota has to be calibrated so that it will be high enough to restore efficiency but low enough not to prevent high ability men to apply for jobs in firm $G$. A gender quota does not introduce any gender bias ex-ante but restores an efficient sorting in the job application phase. Moreover, predictions of the effects of the gender quota are different depending on whether females’ underemployment in top positions is the result of a correlation between ability and gender or if it depends on women’s underconfidence about their relative ability to get the job. While in the fist case we should expect firms to hire no more women than what is made compulsory by the quota in the latter case the introduction of the optimal quota (even if lower than $\frac{1}{2}$), will induce an equal proportion of men and women in the high skilled segment.\textsuperscript{13}

\textsuperscript{13}Our predictions about the effects of the gender quota are consistent with a job environment characterized by no negative stereotype against women.
3 Affirmative Action

In our model firms’ ability to screen workers is not sufficient to restore an efficient matching, because underconfidence prevents women to apply for the skilled segment of the market and therefore there is an inefficient self selection of candidates. The ability to screen workers is useless when there are no workers to screen. Actually, as the above corollary points out, a job market, where firms are able to identify who is the best one among their job candidates, may be more inefficient and induce more segregation than a less meritocratic one, when some workers are underconfident.

In this section we want to study which policies can be undertaken to restore an efficient matching with equality of opportunity.

An affirmative action is usually designed to improve the employment or educational opportunities of individuals in disadvantaged group. These policies sought to eliminate the injustices so frequently associated with discrimination, but there is disagreement about how to design them, and the introduction of exogenous quota are particularly ostracized. Nevertheless, in our study we are presenting a theoretical explanation in favor of the introduction of quota as a tool to close the gender gap in a job environment where discrimination is not at stake.

First, in our model, gender quotas are not introduced to fight females’ discrimination in the job market, since we assume that there is no negative discrimination with respect to the gender of candidates\textsuperscript{14}. In particular, we are focusing on the supply-side of the labour market, claiming that an affirmative action would increase the pool of qualified applicants by offsetting those self-defeating beliefs that prevent women from achieving the career goals they deserve.

Second, in this study we are focusing on gender quota, a very controversial type of affirmative action. Gender quota has been criticized as a form of reverse discrimination (?), which favors candidates by considering observable characteristics (i.e gender), different than merit. However, this is true only if we assume that abilities are not distributed equally among men and women. If you think that the gender gap observed in reality actually reflects the gender gap in abilities, then introducing a quota to increase the number of females in the job market will effectively lead to a reverse discrimination and to an efficiency loss. However, if males and females have the same abilities but not the same self-confidence when evaluating them (?), then introducing an exogenously

\textsuperscript{14} Previous research provides evidence of the presence of employer’s discrimination against women. In our model we prove that quota are important in re-establishing gender equality in the job market even when it is distorted “only” by women’s bias. Everything the same, adding discrimination will make our results stronger.
imposed quota would lead qualified but underconfident women to apply, thus increasing the diversity and efficiency of the labor force, without discriminating men. Recent studies have indeed supported the introduction of affirmative quota to increase women’s willingness to compete (??). Laboratory (??) and field (??) experiments demonstrated that gender quotas do not result in less able women overtaking most able men. In the present study, besides providing a theoretical evidence which explains the emergence of gender segregation in the job market as a result of women’s underconfidence, we now demonstrate the efficacy of calibrated quota in closing the gender gap and restoring efficiency.

Suppose \( z \leq \frac{\sigma}{\tau}, \rho > \frac{2\tau - 1}{1-\sigma}, \) and \( p \in \left[ \frac{1}{w_G} \frac{1}{w_G} \frac{2z w_G - w_B}{2z (1 - \tau)}, \frac{1}{w_G} \frac{1}{w_G} \frac{2z w_G - w_B}{2z (1 - \tau)} \right] \). In equilibrium, see Proposition 2 above, without any affirmative action, male workers apply to firm \( G \) and female workers apply to firm \( B \) (gender segregation and inefficient sorting). Firms will thus select their workers on a pool of candidates only composed by men, half of them \( H \) type, and half of them \( L \) type. We thus propose the following affirmative action: each woman who apply to firm \( G \) has an exogenous probability \( \phi \) to be hired independently on any evaluation of her skill. Clearly, if \( \phi = 1 \), the affirmative action turns out to be discriminatory because it prevents men to be hired. Such an extreme policy induces the same level of inefficiency observed in the job market when there is not any affirmative action, such that \( \phi = 0 \), and women are underconfident about their relative abilities.

Let \( \phi z \) de the quota of jobs reserved to female workers. In particular once firm \( G \) has received all the applications, \( \phi z \) jobs are randomly allocated to female applicants. Then the remaining jobs are allocated among the remaining applicants (that is, all men and those women not already hired) in the standard way.

The following four conditions must be satisfied to restore the efficient separating equilibrium such that workers apply to \( G \) if and only if they observe \( \hat{s}_i = H \).

\( H \) type female workers apply if:

\[
4\phi zw_G + p \left( \frac{1}{1 + \rho} \frac{(1 - \phi)z}{\sigma (1 - \phi z)} w_G + (1 - p) \frac{(1 - \phi)z}{2 - \phi z} w_G \right) \geq w_B
\]

\( H \) type male workers apply if:

\[
p2(1 - \phi)\frac{z}{\sigma (1 - \phi z)} w_G + (1 - p) \frac{(1 - \phi)z}{2 - \phi z} w_G \geq w_B
\]
L type female workers do not apply (assuming \( \frac{1 + \rho}{1 - \sigma} > 1 \)) if:

\[
4\phi z w_G + (1 - p) \left( \frac{1 - \phi}{2 - \phi z} w_G \right) \leq w_B
\]

L male workers do not apply if:

\[
p(1 - \sigma) \frac{2(1 - \phi) z}{\sigma} w_G + (1 - p) \left( \frac{1 - \phi}{2 - \phi z} w_G \right) \leq w_B
\]

Efficient sorting might be impossible to restore, while a non discriminatory equilibrium is easier to obtain. Suppose for instance that \( \rho \to \infty \) and therefore all women believe to be of type L.

We thus get:

\[
\phi w_G + (1 - \phi)(p\frac{1}{1 + \rho} \sigma) \frac{2z}{\sigma} + (1 - p)2z w_G = w_B
\]

which solution is \( \phi \geq \frac{w_B - w_G \left( 2z(1-p)+2p \frac{1}{\rho+1} \right)}{w_G - w_G \left( 2z(1-p)+2p \frac{1}{\rho+1} \right)} \)

\[
\begin{align*}
1 & \text{ if } \frac{1}{\rho+1} (w_G - 2zw_G + \rho w_G - 2z) = 0 \\
C & \text{ if } \frac{1}{\rho+1} (w_G - 2zw_G + \rho w_G - 2z) = 0 \\
0 & \text{ if } \frac{1}{\rho+1} (w_G - 2zw_G + \rho w_G - 2z) = 0
\end{align*}
\]

**Proposition 4** Suppose \( p \geq \bar{p} \), and \( \rho \geq \bar{\rho} \). If the affirmative action \( \phi \in \left[ \frac{(1+p)[4\theta_B-(1-p)(\theta^H_G-\theta^H_L)]-4p\theta^H_G}{(1+p)(1+p)(\theta^H_G-\theta^H_L)-4p\theta^H_G}, \frac{4\theta_B-(1-p)(\theta^H_G-\theta^H_L)}{(1+p)(1+p)(\theta^H_G-\theta^H_L)-4p\theta^H_G} \right] \), is settled, then the efficient separating equilibrium is restored.

Proposition (4) implies that a social planner, in order to maximize the probability to get an efficient matching between high-skilled firms and workers in the job market, should not just limit his intervention in improving the ability of firms to discriminate among agents’ abilities. Indeed, when applying such a policy in an environment where a part of the workers (i.e. women) are underconfident when evaluating their relative ability, then the market ends up in an inefficient separating equilibrium, incurring in a high welfare loss. To restore efficiency, the social planner should thus impose a gender quota so that the participation of high ability women in the skilled segment of the market is assured, without impeding men’s candidature, when being of type H.
4 Discussion

In this paper we have presented a stylized model which identifies in the bias of women regarding their relative abilities an explanation of the emergence of gender segregation in the job market. Our results are consistent with recent experimental evidence showing that females are underconfident with respect to their abilities (???).

First, we have shown that in a job market where abilities are distributed equally among women and men we may end up in an inefficient matching between workers and firms because of females’ underconfidence: high abilities female workers, when being underconfident regarding their relative abilities, do not apply to the skilled segment of the market, and this result is exacerbated when the ability of the market to rank and select workers according to their abilities is high. Self-confident males are then enrolled in the high skilled segment both when having high and low abilities, facing a softer competition. As a consequence, when women are biased, the probability to end up in an efficient matching between high skilled firms and high abilities workers is reduced.

Second, in this paper we provide a theoretical base to explain the importance of implementing affirmative actions to restore efficiency in the job market. Imposing a suitable quota to the participation of women in the high skilled segment incentives them to apply when having high ability (even if being underconfident) but not when having low ability. As a consequence, the quota does not prevent qualified men to participate in the high skilled job market, thus maximizing the efficient job matching. We have shown that incentivizing women to participate in the high skilled segment of the job market is not (only) a question of fairness.

Alternative explanations of the low proportion of women in many high-profile jobs include discrimination, different family-career plans, or different abilities. More recently, an increasing number of studies (???) have advanced the hypothesis that women are less prone to enter into competitive environments. Additionally, a bunch of the literature has observed that women exhibit a lack of self-confidence in their own abilities compared to men (???). In this paper we thus provide a theoretical base to explain the emergence of gender segregation as a result of self selection of (under-confident) women in mediocre careers where abilities are not an issue. In order to counterbalance the negative effect of the bias of women in reaching an efficient matching between firms and workers we thus sustain the importance of implementing calibrated affirmative action, to induce them to participate in the skilled segment of the job market, even when mistakenly perceiving themselves as not having the competence to well perform in such a job. Affirmative action, even if receiving divergent attention, has been showed to incentivize women’s
participation without affecting efficiency of the market (??). However, more effective solutions may be implemented to recover the efficient matching between firms and workers in the long term. In particular, since the gender gap in self-confidence seems to develop early in life (??) and depending on factors such as socioeconomic environments and parental attitudes (??), our results suggest that a policy which intervene to equally encourage the development of self-image in young women and men would be beneficial in improving the gender equality and the efficiency of the job market.

A Appendix A

Proof of proposition 1

Separating equilibrium. First of all we prove the existence of the equilibrium such that only workers who receive a signal $s_i = H$ apply to firms in firm $G$ if $p \geq \bar{p}$. A worker who receive a signal $s_i = H$ has not incentive to deviate if and only if:

$$\left\{ p \left[ \sigma \left( \frac{2z}{\sigma} \right) + (1 - \sigma) \left( \max \left( 0, \frac{z - \sigma \frac{1}{2}}{(1 - \sigma) \frac{1}{2}} \right) \right) \right] + (1 - p) 2z \right\} w_G \geq w_B \quad (A.1)$$

with probability $p$ firm $G$ is able to observe the applicants’ type; Since with probability $\Pr(i = k | s_i = k) = \sigma > \frac{1}{2}$ the signal is correct, the worker will be enrolled in $G$ depending on the size of the segment $z$ and on the proportion of true $H$ types applicants. In particular, the probability of a (true) $H$ type worker to get a job in $G$ is equal to $\frac{z}{z}$. With probability $\Pr(i \neq k | s_i = k) = 1 - \sigma < \frac{1}{2}$ the worker receiving a signal $s_i = H$ is a $L$ type applicant and thus, if the size $z$ of positions available in firm $G$ is small enough, he will be not enrolled and will get zero utility. However, if there are still vacant jobs in $G$ after that all true $H$ types applicants are hired, that is $z - \sigma \frac{1}{2}$, he will get a job while competing with other $L$ type applicants $(1 - \sigma) \frac{1}{2}$. Finally, with probability $1 - p$, firm $G$ is not able to observe the applicants’ type and thus will select workers at random. In such a situation, both truly $H$ type workers and those $L$ workers who have received a wrong signal compete for a position in $G$ such that they will be hired with probability $\frac{z}{z} + \frac{z}{2} = 2z$. By assumption $z < \sigma \frac{1}{2}$ so that segment $G$ is smaller than the proportion of true $H$ type workers receiving a signal $s_i = H$. Therefore the above inequality can be written as:

$$2z w_G \geq w_B \quad (A.2)$$

which holds by Assumption A1.
Workers who receive a signal $s_i = L$ have not incentive to deviate from the above equilibrium if and only if:

$$\left\{ p \left[ \sigma \left( \max \left( 0, \frac{z - \sigma \frac{1}{2}}{(1 - \sigma) \frac{1}{2}} \right) \right) + (1 - \sigma) \left( \frac{2z}{\sigma} \right) \right] + (1 - p)2z \right\} w_G \leq w_B \quad (A.3)$$

that is with probability $\sigma$ the signal is correct and the worker is a true $L$ type so when deviating and applying to a position in firm $G$, with probability $p$ they will be able to infer his type and he will be enrolled only if there is a sufficient number of vacancies $\frac{z - \sigma \frac{1}{2}}{(1 - \sigma) \frac{1}{2}}$. With probability $1 - \sigma$ the signal $s_i = L$ is not correct and thus the worker will compete with with others $H$ type workers for a job in $G$ and will be enrolled with probability $\frac{z}{2}$. Finally, with complementary probability $1 - p$, firm $G$ is not able to observe the applicants’ type and thus will select workers at random.

By assumption $z < \sigma \frac{1}{2}$ and thus we have that deviating from equilibrium is not profitable when:

$$p \geq \frac{2zw_G - w_B \sigma}{2zw_G} \frac{2\sigma - 1}{\sigma^2 - 1} = \bar{p} \quad (A.4)$$

*Pooling equilibrium.* Consider now a strategy profile such that all workers apply to firm $G$.

Workers who observed $s_i = H$ apply to $G$ if and only if:

$$\{ p \left[ \sigma \left( \min \left( 2z; 1 \right) \right) + (1 - \sigma)0 \right] + (1 - p)z \} w_G \geq w_B \quad (A.5)$$

that is, with probability $p$ the $H$ worker type is recognized by forms in $G$ and thus he’s getting a job for sure if $z$ is sufficiently big, otherwise the probability to be hired depends on the size of the segment and on the proportion of $H$ workers in the job market, that is $\frac{z}{2} + \frac{z^2}{2}$. Since $z < \sigma \frac{1}{2}$, when being a true $L$ type, the workers will not be hired in $G$.

By rearranging equation (A.5) we have that:

$$p \geq \frac{w_B - zw_G}{z (\sigma^2 - 1) w_G} \quad (A.6)$$

which always holds since the numerator of equation (A.6) is negative by assumption A1 while its denominator is positive.

Workers who observed $s_i = L$ apply to $G$ if and only if:

$$\left[ p(1 - \sigma) \left( \min \left( 2z; 1 \right) \right) + (1 - p)z \right] w_G \geq w_B \quad (A.7)$$
that is, if the signal is correct in signalling his type, he will not be enrolled in $G$ when firms are able to screen candidates with probability $p$: High ability types will be hired and no more vacant jobs will be available since by assumption $z < \frac{1}{2}$. With probability $(1 - \sigma)$ the worker is a truly $H$ type, thus he will get a position in $G$ competing with other $H$ type candidates. By rearranging equation A.7 we get:

$$p \leq \frac{zw_G - w_B}{z (2\sigma - 1) w_G} = \tilde{p}$$

(A.8)

Semiseparating equilibrium. Finally, Let $\xi \in (0, \frac{1}{2})$ be the fraction of workers who observe $s_i = L$ and apply to firm $B$. These workers are indifferent whether to apply to firm $G$ or $B$ if and only if:

$$(p(1 - \sigma) + (1 - p)\frac{z}{\xi + \frac{1}{2}})w_G = w_B,$$

(A.9)

equality A.9 can be written as $p = \frac{1}{w_G} - \frac{w_B - zw_G}{\xi + \frac{1}{2} + \frac{1}{2} + (1 - p)\frac{z}{\xi + (\xi + 1)}}$ and this expression is decreasing in $\xi$; therefore a semiseparating equilibrium exists only if $p \in [\frac{1}{w_G} - \frac{zw_G - w_B}{\Delta w_G}, \frac{2zw_G - w_B}{2zw_G - 2\sigma - 1}]$.

A strategy profile such that no worker applies to firm $G$ cannot exist, because any worker can deviate and successfully apply to firm $G$.

Proof of Proposition 2

(i) Gender Segregation and Inefficient Sorting. Consider first an equilibrium where all male workers apply to firm $G$ and all female workers apply to firm $B$.

Male workers who receive a signal $s_i = H$ will apply to $G$ if the following condition is satisfied:

$$\{p [\sigma \min (1, 4z)] + (1 - \sigma) \max (0, 4z - 1)] + (1 - p)2z \} w_G \geq w_B$$

(A.10)

with probability $\sigma$ the signal is correct and the truly $H$ type male worker will be enrolled for sure if there is enough vacancies in the firms, otherwise he will compete with other male workers for a position, so that $\frac{z}{1 + \frac{1}{2} + 4z} = 4z$. Since, by assumption, $z < \frac{4}{3}$, previous equation is equal to:

$$p \geq \frac{w_B - 2zw_G}{2z (2\sigma - 1) w_G}$$

(A.11)
Such condition is always satisfied since, by assumption A.1, the numerator is negative and the denominator is positive.

Female workers who observe $s_i = H$ prefer not to apply to $G$ when all male workers apply if:

$$\left[p \left( \frac{1}{1 + \sigma} \right) 4z + (1 - p) 2z \right] w_G \leq w_B$$

(A.12)

while L male workers apply to firm $G$ only if

$$(p(1 - \sigma)4z + (1 - p)2z) w_G \geq w_B$$

(A.13)

and therefore the necessary conditions are $\rho > \frac{2\sigma - 1}{1 - \sigma}$, $p \in \left[ \frac{1}{w_G} \frac{2w_G - w_B}{2z - 4zw_G}, \frac{1}{w_G} \frac{2w_G - w_B}{2z - 4zw_G(1 - \sigma)} \right]$.

The larger is $\rho$ the larger is the set of parameter values for which this equilibrium exists.

(ii) Gender Segregation Efficient Sorting. Consider now an equilibrium where only male workers who received a signal $s_i = H$ apply to firm $G$, while all female workers and male workers who observe $s_i = L$ apply to firm $B$.

Female workers who received a signal $s_i = H$ prefer to not apply to firm $G$ if:

$$\left\{ p \left( \frac{1}{1 + \sigma} \right) \left( \min \left( 1, \frac{4z}{\sigma} \right) \right) + (1 - p)4z \right\} w_G \leq w_B$$

(A.14)

with probability $\frac{1}{1 + \sigma}$ they are truly $H$ type workers thus they are enrolled for sure if there are enough vacancies, otherwise they will compete with High ability male candidates for a job. Since we assume that $z < \frac{\sigma}{4}$, the latter case applies.

Similarly, male workers who observe $s_i = L$ do not apply to firm $G$ if:

$$\left[p(1 - \sigma) \left( \min \left( 1, \frac{4z}{\sigma} \right) \right) + (1 - p)4z \right] w_G \leq w_B$$

(A.15)

Rearranging equations A.14 and A.15 we have that such an equilibrium exists if $\rho \geq \frac{4zw_G - w_B}{w_H - 4zw_G(1 - p)}$ and $p \geq \frac{1}{w_G} \frac{4zw_G - w_B}{4z(2 - \frac{\sigma}{4})}$.

(iii) Partial Gender segregation and semipooling: All men, irrespective of their signals, and female workers who observe $s_i = H$ apply to $G$ if the following conditions hold.

Female workers who received a signal $s_i = H$ apply to $G$ if:

$$\left\{ p \left( \frac{1}{1 + \sigma} \right) \frac{z}{\sigma^1 + (1 - \sigma)^{\frac{1}{2}}} + (1 - p)\frac{4z}{3^2} \right\} w_G \geq w_B$$

(A.16)
that is, if \( \rho \leq \frac{pzw_G}{w_B - \frac{1}{2}zw_G(1-p)(\sigma+1)} - 1 \). Men who observe \( s = L \) apply if:

\[
\left\{ p(1 - \sigma) \frac{z}{\sigma \frac{1}{2} + (1 - \sigma) \frac{1}{4}} + (1 - p) \frac{4}{3} z \right\} w_G \geq w_B \quad (A.17)
\]

that is if \( \rho \leq \frac{1}{w_G} \frac{zw_G}{2(z + 2(z + 1))} \).

Finally, female workers who observe \( s_i = L \) do not apply if:

\[
\left\{ p \left( 1 - \frac{1 + \rho}{1} \sigma \right) \frac{2z}{\sigma \frac{1}{2} + (1 - \sigma) \frac{1}{4}} + (1 - p) \frac{4}{3} z \right\} w_G \leq w_B \quad (A.18)
\]

This condition is always satisfies when \( \rho > 0 \) and equation A.17 is satisfied.

(iv) No Gender Segregation, Efficient Sorting. Consider the equilibrium where workers who observe \( s_i = H \) apply to \( G \) and while other workers apply to \( B \).

Female workers who observe a signal \( s_i = H \) prefer to apply when all workers who have received \( s_i = H \) apply if:

\[
\left[ p \left( \frac{1 - \sigma}{1} \frac{2z}{\sigma \frac{1}{2} + (1 - \sigma) \frac{1}{4}} \right) + (1 - p)2z \right] w_G \geq w_B \quad (A.19)
\]

that is if \( \rho \leq \frac{1}{w_G} \frac{2zw_G - w_B}{2z - 2(z - 1)} \).

Male workers who observe \( s_i = L \) do not apply to firm \( G \) only if:

\[
(p(1 - (1 + \rho)\sigma)2z + (1 - p)z)w_G \leq w_B \quad (A.20)
\]

that is if \( \rho \geq \frac{1}{w_G} \frac{2zw_G - w_B}{2z - 2(z - 1)} \). Notice that \( \frac{1}{w_G} \frac{2zw_G - w_B}{2z - 2(z - 1)} > \frac{1}{w_G} \frac{2zw_G - w_B}{2z - 2(z - 1)(1 - \sigma)} \) implies \( \frac{2\sigma - 1}{(1 - \sigma)} > \rho \).

Therefore this equilibrium exists if \( p \in \left[ \frac{1}{w_G} \frac{2zw_G - w_B}{2z - 2(z - 1)}, \frac{1}{w_G} \frac{2zw_G - w_B}{2z - 2(z - 1)} \right] \) and \( \rho < \frac{2\sigma - 1}{(1 - \sigma)} \).

(v) No Gender Segregation and Pooling. All workers apply to firm \( G \) if female workers who observe \( s_i = L \) apply to \( G \), that is:

\[
(p(1 - \min(1,(1 + \rho)\sigma))2z + (1 - p)z)w_G \geq w_B. \quad (A.21)
\]

The above condition is satisfied if \( \rho \leq \frac{1 - \sigma}{\sigma} \) and \( p \leq \frac{zw_G - w_B}{zw_G - 2z(1 - \sigma(\rho + 1))} \) or if \( \rho > \frac{1 - \sigma}{\sigma} \) and \( p \leq \frac{zw_G - w_B}{zw_G} \).

Proof of Proposition 3
(ib) Gender Segregation and Inefficient Sorting. When $\frac{q}{4} \leq z \leq \frac{1}{4}$, conditions A.10
A.12 and A.13 are still satisfied: all male workers apply to firm $G$ and all female workers
apply to firm $B$ when $\rho > \frac{2\sigma - 1}{4} \frac{w_G - w_B}{w_G}$, $p \in \left[ \frac{1}{w_G} \frac{2zw_G - w_B}{2z - 4z(1 - \sigma)}, \frac{1}{w_G} \frac{2zw_G - w_B}{2z - 4z(1 - \sigma)} \right]$.

Parameters are different when $\frac{1}{4} \leq z \leq \frac{1}{2}$. In particular, when $\sigma \geq \frac{w_G + w_B}{2w_G}$ and
$z \leq \frac{w_G + w_B}{4w_G} < \frac{q}{2}$ or when $\sigma \leq \frac{w_G + w_B}{2w_G}$ and $z \leq \frac{w_G + w_B}{4w_G} < \frac{1}{2}$, female workers who observe
$s_i = H$ prefer to apply to $B$ when all male workers apply if:

$$p = \left\{ \left[ \frac{1}{1 + \rho} \right] + \left( \frac{1 + \rho - \sigma}{1 + \rho} \right) (4z - 1) \right\} + (1 - p) 2z \right\} w_G \leq w_B$$  \hfill (A.22)

condition A.22 implies that:

$$p \geq \frac{(2zw_G - w_B)(1 + \rho)}{(1 - 2\sigma + \rho)(1 - 2z) w_G}$$  \hfill (A.23)

The denominator is positive only if $\rho \geq \frac{(2z - 1)(1 - \sigma)}{1 - 2z}$ while the numerator is always
positive by Assumption A1 $w_G \geq 2w_B$, and since $z \geq \frac{1}{4}$.

$L$ male workers apply to firm $G$ only if

$$\{p[\sigma (4z - 1) + (1 - \sigma)] + (1 - p)2z \} w_G \geq w_B$$  \hfill (A.24)

which implies that $p \leq \frac{1}{w_G} \frac{2zw_G - w_B}{(2z - 1)(1 - \sigma)}$.

(ii) Gender Segregation Efficient Sorting. When $\frac{q}{4} \leq z \leq \frac{w_B + (w_G - w_B)\sigma}{4w_G} < \frac{1}{4}$, conditions
A.14 and A.15 are still satisfied: only male workers who receive signal $s_i = H$ apply
to firm $G$ while all other workers apply to firm $B$ when $\rho \geq \frac{4zw_G - w_B}{w_B - 4zw_G(1 - p)}$ and
$p \geq \frac{1}{w_G} \frac{4zw_G - w_B}{4z(2 - \frac{1}{2})}$. Differently when $z \geq \frac{w_B + (w_G - w_B)\sigma}{4w_G}$ there is no equilibrium where all
females are segregated in the low skilled firm and only males who received a signal $s_i = H$
apply to $G$.

In order for this equilibrium to exist it must be that female workers who received a
signal $s_i = H$ apply to firm $B$, as shown in the following condition:

$$\left\{ p \left[ \frac{1}{1 + \rho} \sigma \right] + \left( \frac{1 + \rho - \sigma}{1 + \rho} \right) (4z - \sigma) \right\} + (1 - p)4z \right\} w_G \leq w_B$$  \hfill (A.25)

Since we are assuming that $\frac{q}{2} > z > \frac{q}{4}$, when deviating from the equilibrium and candid-
ates abilities are observable with probability $p$, female workers are getting a position in
firm $G$ with probability $\frac{1}{1 + \rho} \sigma$ when being a true $H$ type. Differently, with probability $\frac{1 + \rho - \sigma}{1 + \rho}$ the signal $s_i = H$ is wrong and thus they have to compete with male candidates who also receive the wrong signal $s_i = H$ for the positions in firm $G$ not already filled by $H$ male candidates.

Rearranging equation A.25 we have that previous condition is satisfied when:

$$p \geq \frac{(4zw_G - w_B)(1 + \rho)(1 - \sigma)}{\sigma p(1 - 4z)w_G} \quad \text{(A.26)}$$

A.26 must be lower or equal than one, meaning that $(4zw_G - w_B)(1 + \rho)(1 - \sigma) \leq \sigma p(1 - 4z)w_G$. Rearranging previous equation it must be that:

$$\rho \geq \frac{4zw_G - w_B(1 - \sigma)}{w_B - 4zw_G(1 - p)} \quad \text{(A.27)}$$

which is true only if $z \leq \frac{w_B + (w_G - w_B)\sigma}{4w_G}$.

(iiib) Partial Gender segregation and semipooling: When $\frac{2}{3} \leq z \leq \frac{1 + \sigma}{4}$, conditions A.16 A.17 and A.18 are still satisfied: all men, irrespective of their signals, and female workers who observe $s_i = H$ apply to $G$ while all other workers apply to firm $B$ when $\rho \leq \frac{\rho 2zw_G}{2(w_B - \frac{3}{2}zw_G(1 - p))(\sigma + 1)} - 1$ and $p \leq \frac{1}{w_G} \left[ \frac{w_B - \frac{3}{2}zw_G}{z_w - z_{4w_G}} \right]$.

Differently, when $\frac{1 + \sigma}{4} \leq z \leq \frac{3}{4}$ female workers who observe $s_i = H$ apply to $G$ when all male workers apply if:

$$\left\{ p \left[ \left( \frac{1}{1 + \rho} \right) + \left( \frac{1 + \rho - \sigma}{1 + \rho} \right) \frac{4z - 1 - \sigma}{2 - \sigma} \right] + (1 - p) \frac{4z}{3} \right\} w_G \geq w_B \quad \text{(A.28)}$$

which is satisfied for every $p$ when $\rho \leq \frac{2\sigma - 1}{\sigma + 1}$.

Respectively, female and male workers who observe $s_i = L$ apply to firm $B$ only if :

$$\left\{ p \left[ (1 + \rho) \sigma \frac{(4z - 1 - \sigma)}{2 - \sigma} + 1 - (1 + \rho) \sigma \right] + (1 - p) \frac{4z}{3} \right\} w_G \leq w_B \quad \text{(A.29)}$$

$$\left\{ p \left[ \sigma \frac{(4z - 1 - \sigma)}{2 - \sigma} + (1 - \sigma) \right] + (1 - p) \frac{4z}{3} \right\} w_G \geq w_B \quad \text{(A.30)}$$
Conditions A.29 and A.30 are satisfied when 

\[ p \in \left[ \frac{4zw_G - 3w_B}{(4zw_G - 3w_B)(2 - 4\sigma - 3\sigma\rho)}, \frac{4zw_G - 3w_B(2 - \sigma)}{(4zw_G - 3w_B)(2 - 4\sigma)} \right] \]

(ivb) No Gender Segregation, Efficient Sorting. When \( \frac{\sigma}{2} \leq z \leq \frac{\sigma}{2} \) previous conditions A.19 and A.20 are still satisfied when 

\[ p \geq \frac{w_G}{2z - 2z^2(1 - \sigma)} \] and \( p < \frac{2\sigma - 1}{1 - \sigma} \).

The equilibrium where workers who observe \( s_i = H \) apply to G and while other workers apply to B exists when \( \frac{\sigma}{2} \leq z \leq \frac{1}{2} \) if female workers who observe a signal \( s_i = H \) prefer to apply when all workers who have received \( s_i = H \):

\[ \left\{ p \left[ \frac{1}{1 + \rho} \right] + \left( \frac{1 + \rho - \sigma}{1 + \rho} \left( 2z - \sigma \right) \right) + (1 - p)2z \right\} w_G \geq w_B \quad \text{(A.31)} \]

that is if 

\[ p \leq \frac{w_G}{2z - 2z^2(1 - \sigma)} \frac{(4zw_G - w_B)(1 + \rho)(1 - \sigma)}{\sigma(1 + \rho)} \] .

Male workers who observe \( s_i = L \) do not apply to firm G only if:

\[ \left\{ (p \sigma \left( \frac{2z - \sigma}{1 - \sigma} \right) + (1 - \sigma)) + (1 - p)2z \right\} w_G \leq w_B \quad \text{(A.32)} \]

that is if

\[ p \geq \frac{w_G}{2z - 2z^2(1 - \sigma)} \frac{(2zw_G - w_B)(1 - \sigma)}{\sigma(1 - \rho + \sigma)} \]

Noticing that \( \frac{w_G}{2z - 2z^2(1 - \sigma)} \frac{(4zw_G - w_B)(1 + \rho)(1 - \sigma)}{\sigma(1 - \rho + \sigma)} > \frac{w_G}{2z - 2z^2(1 - \sigma)} \frac{(2zw_G - w_B)(1 - \sigma)}{\sigma(1 - \rho + \sigma)} \)

implied \( \frac{2\sigma - 1}{(1 - 2\sigma) + \rho} > \rho \). Therefore this equilibrium exists if 

\[ p \in \left[ \frac{w_G}{2z - 2z^2(1 - \sigma)} \frac{(2zw_G - w_B)(1 - \sigma)}{\sigma(1 - \rho + \sigma)} , \frac{w_G}{2z - 2z^2(1 - \sigma)} \frac{(4zw_G - w_B)(1 + \rho)(1 - \sigma)}{\sigma(1 - \rho + \sigma)} \right] \]

and \( \rho < \frac{2\sigma - 1}{(1 - 2\sigma) + \sigma} \)

(vb) No Gender Segregation and Pooling. Finally, when \( \frac{\sigma}{4} \leq z \leq \frac{1}{2} \) condition A.21 is still satisfied so that all workers apply to firm G if 

\[ p \leq \frac{1 - \sigma}{\sigma} \quad \text{and} \quad p \leq \frac{zw_G - w_B}{zw_G - 2z(1 - \sigma(\rho + 1))} \] or if 

\[ p > \frac{1 - \sigma}{\sigma} \quad \text{and} \quad p \leq \frac{zw_G - w_B}{zw_G} \]

B Appendix B: Affirmative actions