Modelling Term Structures of Default Probabilities

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Abstract
The Merton model of measuring credit risk cannot generate default probabilities which replicate empirically observed default rates. This paper presents a simple model to address this problem. In the model, a firm defaults its obligations when its leverage ratio increases above a predefined level. We derive a closed-form solution of default probabilities based on the model as a function of the leverage ratio and short-term interest rate. The numerical results calculated from the solution show that the model is capable of producing term structures of default probabilities for non-investment ratings and BBB rating, that are consistent with some empirical findings. This model could provide new insight for future research on credit risk measurement and corporate bond analysis.
I. Introduction

Black and Scholes (1973) and Merton (1974) have been the pioneers in the modelling credit risk of corporates using a contingent-claim framework. They treat default risk equivalent to a European put option on a firm’s asset value, where the firm’s liability is the option strike. To extend the Merton model, structural models with more complex and dynamic liability structures have been considered by Black and Cox (1976), Longstaff and Schwartz (1995), Briys and de Varenne (1997), Collin-Dufresne and Goldstein (2001) and Hui et al. (2003). These models could explain empirical term structures of credit spreads of corporate bonds with different credit ratings to some extent. For example, Collin-Dufresne and Goldstein (2001) (hereafter referred to as CG) consider a mean-reverting liability that is tied to the interest rate dynamics. This assumption makes the liability-to-asset (i.e. leverage) ratio approach towards a constant target leverage ratio over time. The CG model helps reconcile some predictions of credit spreads with empirical observations. These include credit spreads that are larger for low-leverage firms and less sensitive to changes in firm value, and upward sloping term structures of credit spreads of speculative-grade bonds. However, the numerical results presented in the third section of this paper show that model default probabilities generated from both the CG model and the Merton model are substantially lower than the empirically observed cumulative default rates reported by Standard & Poor's (S&P's) (2002).

The objective of this paper is to develop a simple model that is capable of generating term structures of default probabilities which are consistent with some empirical findings. In the model, the underlying variable is the leverage ratio of a firm, which follows a standard Wiener process and is correlated with the risk-free interest rate. The proposed model does not specify the dynamics of the firm’s asset value and liability structure so that it is different from the structural models. It is however similar to the approach proposed by Cathcart and El-Jahel (1998), under which default occurs when a signaling process (instead of asset value) hits some constant default barrier. The Cathcart and El-Jahel model assumes the signaling process for each firm that determines the occurrence of default rather than the asset value of the firm. The signaling process can capture factors that can affect the default probability.
The value of the leverage ratio used in the proposed model is defined as the ratio of the total debt to market-value capitalisation. The stochasticity of the leverage ratio is therefore due to the combined effect of the dynamics of the market price of a firm’s debt and its equity value. Future changes in the liability structure of the firm give uncertainty to the total debt value. The use of the leverage ratio has the advantage of measuring a firm’s “going concern” equity value based on the information from the equity market. S&P's also uses leverage ratios as a key financial ratio to reflect the financial situations of companies with different credit ratings (see S&P's (2001)).

The use of leverage ratios for determining the occurrence of default is consistent with the studies by Anderson et al. (2002) who test the validity of using empirically calculated leverage ratios to assign bond ratings according to default risk. The empirical tests provide evidence supporting the use of such methodology. Longstaff and Schwartz (1995) also consider that the leverage ratio is a sufficient measure of default risk, without having to condition on the pattern of cash payments to be made prior to the maturity date of a bond in order to value the bond.

The correlation between the leverage ratio and interest rate can be observed from the historical behaviour of a firm in regard to its leverage ratio. With negative correlation between the leverage ratio and interest rate, the default probability could be a decreasing function of the interest rate in the model. This is consistent with the empirical findings of Longstaff and Schwartz (1995) and Duffee (1998 and 1999). This can be explained by a reason that firms reduce their demand for additional debt as the interest rate increases. We derive a closed-form solution for estimating model default probabilities as a function of the leverage ratio and interest rate explicitly.

The remainder of the paper is organised as follows. In the following section we discuss the proposed model and derive the closed-form solution for estimating model default probabilities. Numerical results of default probabilities calculated from the model based on market data are compared with the default rates reported by S&P's and model default probabilities generated from the CG and Merton models. The effects of the correlation between the leverage ratio and interest rate on default probabilities are also studied. The final section summarises the findings.

II. Analytical Model of Default Probabilities
A continuous-time framework is used to value the default probabilities with different time horizons of a firm. The firm’s leverage ratio and the short-term interest rate are stochastic variables in the model. The leverage ratio is assumed to follow a lognormal diffusion process. The dynamics of the interest rate is drawn from the term structure model of Vasicek (1977), i.e. the Ornstein-Uhlenbeck process. The risk-adjusted dynamic of the leverage ratio $L$ is modelled by the following stochastic differential equation:

$$dL = \alpha(t)Ldt + \sigma_L(t)Ldz_L,$$  

where $\alpha(t)$ and $\sigma_L(t)$ are the drift and the volatility of $L$ respectively and are time dependent. The drift $\alpha(t)$ could take into account the risk premiums of a non-traded firm’s asset and liability which form the leverage ratio.

The continuous stochastic movement of the interest rate $r$ follows:

$$dr = \kappa(t)[\theta(t) - r]dt + \sigma_r(t)dz_r,$$  

where $\sigma_r(t)$ is the instantaneous volatility. The short-term interest rate $r$ is mean-reverting to long-run mean $\theta(t)$ at speed $\kappa(t)$. The Wiener processes $dZ_L$ and $dZ_r$ are correlated with $dZ_LdZ_r = \rho dt$.

Applying the Ito’s lemma, the partial differential equation governing the price $P(L, r, t)$ of a corporate discount bond with time-maturity of $t$ based on the model is

$$\frac{\partial P}{\partial t} = \frac{1}{2} \sigma_L^2(t)L^2 \frac{\partial^2 P}{\partial L^2} + \frac{1}{2} \sigma_r^2(t)\frac{\partial^2 P}{\partial r^2} + \rho(t)\sigma_L(t)\sigma_r(t)L \frac{\partial^2 P}{\partial L \partial r} + \alpha(t)L \frac{\partial P}{\partial L} + \kappa(t)[\theta(t) - r] \frac{\partial P}{\partial r} - rP.$$  

The bond value is obtained by solving Equation (3) subject to the final payoff condition and the boundary condition. When the firm’s leverage ratio is above a predefined level $L_0$, bankruptcy occurs before bond maturity at $t = 0$. This is consistent with the event of bankruptcy being associated with high levels of debt relative to the market value of the firm’s assets. On the other hand, if the leverage ratio has never breached the predefined level $L_0$, the payoff to bondholders at bond maturity is the face value of the bond.
As shown in the Appendix, the corresponding default probability, \( P_{\text{def}}(L, t) \), of a corporate discount bond over a period of time \( t \) based on Equation (3) can be approximated by

\[
P_{\text{def}}(L, t) = 1 - N \left[ \ln \left( \frac{L}{L_0} \right) - b_2(t) \right] - \exp \left[ 4\beta \left( \ln \left( \frac{L}{L_0} \right) + b_2(t) \right) + 16\beta^2 b_1(t) \right] \times
\]

\[
N \left[ \ln \left( \frac{L}{L_0} \right) + b_2(t) + 8\beta b_1(t) \right] \right], \quad (4)
\]

where \( N \) is the cumulative normal distribution function, \( \beta \) is a real number parameter, and \( b_1(t) \) and \( b_2(t) \) are defined as follows:

\[
b_1(t) = \frac{1}{2} \int_0^t \sigma_1'(t')dt',
\]

\[
b_2(t) = \int_0^t \gamma(t')dt',
\]

\[
\gamma(t) = \alpha(t) + \rho(t)\sigma_L(t)\sigma_r(t)a_2(t)\exp[a_1(t)] - \frac{1}{2}\sigma_2^2(t),
\]

\[
a_1(t) = -\int_0^t \kappa(t')dt',
\]

\[
a_2(t) = -\int_0^t \exp[-a_1(t')]dt'.
\]

The parameter \( \beta \) is generally adjusted such that the approximate solution in Equation (4) provides the best approximation to the exact results by using a simple method developed by Lo et al. (2003) for solving barrier option values with time-dependent model parameters.

### III. Numerical Results of Model Default Probabilities

The computed default probabilities within a period of 15 years based on Equation (4) for firms with ratings of CCC, B, BB and BBB are presented in Exhibits 2, 3, 4 and 5 respectively and those with ratings of AAA, AA and A are presented in Exhibit 7. The model term structures of default probabilities are compared with the term structures of cumulative default rates of the corresponding ratings based on 9,769 companies' assigned long-term ratings from 1981 to 2001 reported by S&P's
The term structures of default probabilities generated from the CG model with a constant target leverage ratio of 0.315 (which is used by CG (2001) and corresponds to the leverage ratio of the BBB rating) and the Merton model are also shown in the Exhibits 2 to 5 and 7 for comparison purposes. The model parameters used for individual ratings are shown in Exhibit 1. The leverage ratios used for individual ratings are based on the industry median reported by S&P’s (2001). The values of $\sigma_L$ fall close to the asset volatilities of firms with individual ratings estimated by Delianedis and Geske (1999). The drift $\alpha$ is assumed to be 0 unless otherwise stated. Other common parameters used in calculations are $L_0 = 1.0$, $\kappa = 0.05$, $\theta = 1.0$, $\sigma_r = 0.03162$ and $\rho = 0$.

The term structures of default probabilities for the CCC rating based on the three models, i.e. the proposed model, CG model and Merton model, exhibit upward slopes at short tenors in Exhibit 2. At longer tenors, their shapes are flat. Intuitively, this is because the probability that the firm's leverage ratio reaches the default barrier increases over time. The flatten slopes at longer tenors however reflects that default probabilities of CCC-rated firms will not increase significantly over time as the firms survive at short terms. The shapes of the model term structures of default probabilities are consistent with S&P's default rates of CCC-rated firms. The default probabilities based on the proposed model are slightly higher than the default rates reported by S&P's, while the values of the term structures generated from the other two models are much lower than the empirical values. Another observation is that the default probabilities obtained from the proposed model and CG model are both higher than those obtained from the Merton model that does not capture early default risk.

The term structures of default probabilities for the B rating are illustrated in Exhibit 3 using the three models. The depicted term structures exhibit upward slopes at short tenors. At longer tenors, their shapes display different degrees of upward-sloping depending on the models. The shape and values of the term structure based on the proposed model match closely with the empirical default rates reported by S&P's at all tenors. The values of the term structures generated from the other two models are close to each other but much lower than the empirical values at all tenors.
Exhibit 4 shows the term structures of default probabilities for the BB rating. The depicted term structure based on the proposed model exhibits an upward slope. Its shape is consistent with the empirical findings of the term structure of the BB rating reported by S&P's, while the model term structure gives slightly higher values of default probabilities at tenors of longer than seven years. The differences in the values increase with the tenors. Conversely, the CG and Merton models present default probabilities much lower than the empirical default rates at all tenors. The default probabilities given by the CG model are lower than those given by the Merton model. It is because the target leverage ratio of 0.315 used in the CG model could reduce the default risk as the model implies that a BB-rated company intends to move its leverage ratio downward to the target leverage ratio. Exhibits 2 to 4 demonstrate that the proposed model could quite well explain the term structures of default rates of non-investment ratings while the other two models underestimate the corresponding default probabilities.

Exhibit 5 shows the term structures of default probabilities for the BBB rating. The depicted term structure based on the proposed model exhibits an upward slope which is steeper than the term structure of the BBB rating reported by S&P's. The model term structure gives higher values of default probabilities than the actual default rates at tenors of longer than six years. The differences in the values increase with the tenors. In Exhibit 6 below, the results however show that by using negative $\alpha$ the proposed model can generate model default probabilities which fit better with the term structure of default rates reported by S&P’s. Similar to the results in Exhibits 2 to 4, the term structures generated by the CG and Merton models in Exhibit 5 cannot match with the empirical default rates at all tenors.

In order to show the impact of changes in the drift $\alpha$ of the leverage ratio on model default probabilities, different values of $\alpha$ are used to generate the term structures of default probabilities for different ratings. In our search for optimal values of $\alpha$ such that model default probabilities fit the empirical values, we find that $\alpha = -0.015, -0.001, -0.005$ and -0.015 for ratings of CCC, B, BB and BBB respectively. In Exhibit 6, the model default probabilities generated from these values of $\alpha$ match more closely with the empirical values at all tenors, than the model default probabilities presented in Exhibits 2 to 5. The results show that the parameter $\alpha$ can
be used to calibrate the model default probabilities. For example, the average value of $\alpha$ for ratings of CCC, B and BB in Exhibit 5 can be used as a calibrated parameter of the drift of the leverage ratio for firms which have leverage ratios and associated volatilities fall into the category of non-investment ratings. The corresponding model default probabilities of the firms would be considered as calibrated estimates which are more accurate than those simply based on $\alpha = 0$. The results also indicate that firms generally intend to reduce their leverage ratios. This can be explained by an intuition that firms with relatively low ratings (i.e. high leverage ratios) tend to reduce their leverage ratios such that credit rating agencies may consider increasing the firms’ ratings because of their lowered leverage ratios.

In Exhibit 7, the default probabilities generated by the proposed model, CG model and Merton model for A, AA and AAA ratings are all lower than S&P’s default rates. In particular for AAA and AA ratings, most of the default probabilities predicted by all the models are zero. The reason is that the problem of downward-biased default risk of firms with good credit quality is common to models which assume continuous dynamics of the underlying variables. Therefore, criticism of the model based on low default probabilities is that of the underlying assumptions. This reflects that other default triggering mechanisms (i.e. other than basing on the dynamics of leverage ratios) should be considered for firms with credit quality similar to credit ratings of A or above.

We use the BB rating to illustrate the effect of the correlation $\rho$ between $L$ and $r$ on model default probabilities in Exhibit 8. The drift $\alpha = -0.005$ and $\rho = -0.75, -0.3, 0, 0.75$ are used to generate the term structures of default probabilities. It is observed that the effect of the correlation only becomes apparent at longer tenors and default probabilities increase with the decrease in $\rho$. The model default probabilities generated based on $\rho = -0.3$ are found to match more closely with the empirical default rates, than the model default probabilities generated based on the other values of $\rho$. The observation is consistent with the intuition that a low interest rate environment would encourage firms to increase their debts (i.e. leverage ratios) because of low interest rate costs involved and the firms' default probabilities would thus increase. The intuition also explains that the model default probabilities
generated by using negative correlation ($\rho = -0.3$) between $L$ and $r$ could fit the actual default rates well.

In summary, the numerical results show that the proposed model gives the basic shapes and values of the term structures of default probabilities for non-investment and BBB ratings, which broadly match with some empirical findings. This demonstrates that the leverage ratio is a determinant factor of default risk of firms with BBB ratings or below. As the value of the leverage ratio and its volatility measure a firm’s “going concern” equity value based on the information from the equity market, default risk of firms with credit quality of such ratings could be considered as being effectively reflected in the market.

IV. Conclusion

This paper develops a simple model with a closed-form solution to estimate default probabilities of firms based on their leverage ratios. The numerical results show that the model is capable of producing term structures of default probabilities which are consistent with some empirical findings reported by S&P's (2002). In particular, the model default probabilities fit actual default rates of firms with BBB and non-investment ratings quite well. However, other default triggering mechanisms should be considered for firms with credit ratings of A or above. The model could therefore provide new insight for future research on credit risk measurement of listed companies. More detailed empirical comparisons between the actual default rates and the model default probabilities are left to future research.

Endnotes

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1 The second approach is the reduced-form models in which time of default is assumed to follow a stochastic process governed by its own distribution that is characterised by an intensity or hazard rate process. This approach has been considered by Madan and Unal (1993), Jarrow and Turnbull (1995), Jarrow et al. (1997) and Duffie and Singleton (1999). Their models in general focus on more sophisticated characterisation of the hazard process. The derived pricing formulas
can be calibrated to market credit spreads. Some extensions explore assumptions surrounding recovery rate, risk-free interest rate processes, and contract boundary conditions.

Although this assumed process is consistent with many of the observed properties of interest rates, it can allow negative interest rates. However, this assumption may still be justifiable in the context of the valuation because given that the current value of interest rate and the mean-level are both positive, the dynamics always imply positive expected future interest rate.

References


Appendix:

Applying the Itô’s lemma, the price $P$ of a corporate bond with stochastic interest rate, which is a function of the leverage ratio $L$, the short-term interest rate $r$ and the time to maturity $t$ is governed by the partial differential equation

$$
\frac{\partial P(L, r, t)}{\partial t} = \frac{1}{2} \sigma_L^2(t) L^2 \frac{\partial^2 P}{\partial L^2} + \frac{1}{2} \sigma_L^2(t) \frac{\partial^2 P}{\partial r^2} + \rho(t) \sigma_L(t) \sigma_r(t) L \frac{\partial^2 P}{\partial L \partial r} + \alpha(t) L \frac{\partial P}{\partial L} + \kappa(t) \theta(t) - r \frac{\partial P}{\partial r} - r P .
$$

The solution of this partial differential equation has the form

$$
P(x, r, t) = B(r, t) \mathcal{P}(x, t) ,
$$

where $x = \ln(L/L_0)$, $L_0$ is a constant and $\mathcal{P}(x, t)$ satisfies the partial differential equation

$$
\frac{\partial \mathcal{P}(x, t)}{\partial t} = \frac{1}{2} \sigma_L^2(t) \frac{\partial^2 \mathcal{P}}{\partial x^2} + \gamma(t) \frac{\partial \mathcal{P}}{\partial x} ,
$$

and $B(r, t) = \exp \{ a_4(t) + a_3(t) a_2^2(t) \exp [2a_1(t)] + a_2(t) \exp [a_1(t)] r \}$ is simply the riskless bond function of the Vasicek model with explicitly time-dependent parameters.

Here the parameters $\gamma(t)$ and $a_n$’s are defined as follows:

$$
\begin{align*}
\gamma(t) &= \alpha(t) + \rho(t) \sigma_L(t) \sigma_r(t) a_2(t) \exp [a_1(t)] - \frac{1}{2} \sigma_L^2(t) , \\
a_1(t) &= - \int_0^t \kappa(t') dt' , \\
a_2(t) &= - \int_0^t \exp [-a_1(t')] dt' , \\
a_3(t) &= \frac{1}{2} \exp [-2a_1(t)] \int_0^t \sigma_r^2(t') \exp [2a_1(t')] dt' , \\
a_4(t) &= \int_0^t [\kappa(t') \theta(t') + 2a_3(t')] a_2(t') \exp [a_1(t')] dt' .
\end{align*}
$$

The solution of Eq. (3) is

$$
\mathcal{P}(x, t) = \int_{-\infty}^{\infty} dx' G(x, t; x', 0) \mathcal{P}(x, 0) ,
$$

and the kernel $G(x, t; x', 0)$ is given by

$$
G(x, t; x', 0) = \frac{1}{\sqrt{4\pi b_1(t)}} \exp \left\{ - \frac{[x' - x - b_2(t)]^2}{4b_1(t)} \right\} ,
$$

1
where

\[ b_1(t) = \frac{1}{2} \int_0^t \sigma_L^2(t') dt' , \]
\[ b_2(t) = \int_0^t \gamma(t') dt' . \]  

(7)

Furthermore, we can apply the method of images to incorporate an absorbing boundary, i.e. the leverage ratio upon default, along the x-axis with a drifted dynamics of the form \( x^*(t) = -b_2(t) - 4\beta b_1(t) \) into our model, where the parameter \( \beta \) is a real adjustable parameter controlling the movement of the leverage ratio upon default. The corresponding solution \( \mathcal{P}(x, t) \) is given by

\[ \mathcal{P}(x, t) = \int_{-\infty}^0 dx' K(x, t; x', 0) \mathcal{P}(x', 0) \]  

(8)

where

\[ K(x, t; x', 0) = G(x, t; x', 0) - G(x, t; -x', 0) \exp(-4\beta x') . \]  

(9)

Then, in terms of the kernel \( K(x, t; x', 0) \), we can also derive the probability of default over a period of time \( t \) as follows:

\[ P_{def}(x, t) = 1 - \int_{-\infty}^0 dx' K(x, t; x', 0) \mathcal{P}(x', 0) . \]  

(10)
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**EXHIBIT 1. Parameters used for individual ratings**
The leverage ratios $L$ are based on the industry medians reported by S&P’s (2001). The values of $\sigma_L$ fall close to the asset volatilities of firms with individual ratings estimated by Delianedis and Geske (1999).
EXHIBIT 2. Default probability term structures of CCC-rated firm.
The default probabilities of a CCC-rated firm with $L = 73.2\%$ and $\sigma_L = 0.315$ are plotted using $\alpha=0$. The other parameters used are $L_0 = 1.0$, $r = 5\%$, $\theta = 5\%$, $\kappa = 1.0$, $\sigma_r = 0.03162$ and $\rho = 0$. The term structures based on S&P’s data (2002), the CG model with a constant target leverage ratio equal to 0.315 and Merton model with the corresponding rating and parameters are illustrated for comparison.
EXHIBIT 3. Default probability term structures of B-rated firm.
The default probabilities of a B-rated firm with $L = 53.8\%$ and $\sigma_L = 0.288$ are plotted using $\alpha=0$. The other parameters used are $L_0 = 1.0$, $r = 5\%$, $\theta = 5\%$, $\kappa = 1.0$, $\sigma_r = 0.03162$ and $\rho = 0$. The term structures based on S&P’s data (2002), the CG model with a constant target leverage ratio equal to 0.315 and Merton model with the corresponding rating and parameters are illustrated for comparison.
The default probabilities of a BB−rated firm with $L = 49.5\%$ and $\sigma_L = 0.261$ are plotted using $\alpha = 0$. The other parameters used are $L_0 = 1.0$, $r = 5\%$, $\theta = 5\%$, $\kappa = 1.0$, $\sigma_r = 0.03162$ and $\rho = 0$. The term structures based on S&P’s data (2002), the CG model with a constant target leverage ratio equal to 0.315 and Merton model with the corresponding rating and parameters are illustrated for comparison.
EXHIBIT 5. Default probability term structures of BBB−rated firm.
The default probabilities of a BBB−rated firm with $L = 31.5\%$ and $\sigma_L = 0.235$ are plotted using $\alpha=0$.
The other parameters used are $L_0 = 1.0$, $r = 5\%$, $\theta = 5\%$, $\kappa = 1.0$, $\sigma_r = 0.03162$ and $\rho = 0$.
The term structures based on S&P’s data (2002), the CG model with a constant target leverage ratio equal to 0.315 and Merton model with the corresponding rating and parameters are illustrated for comparison.
EXHIBIT 6. Default probability term structures of CCC, B, BB and BBB with best−fitted $\alpha$.
The default probabilities of CCC, B, BB and BBB−rated firms are plotted using different values of $\alpha$, where $\alpha_{\text{CCC}} = -0.015$, $\alpha_{\text{B}} = -0.001$, $\alpha_{\text{BB}} = -0.005$ and $\alpha_{\text{BBB}} = -0.015$. The term structures based on S&P’s data (2002) are illustrated for comparison.
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EXHIBIT 7. Default probabilities (PDs in %) of AAA, AA and A-rated firms with $\alpha = 0$.

The default probabilities of AAA, AA and A-rated firms are computed using $\alpha = 0$. The parameters used are those in Exhibit 1, $L_0 = 1.0$, $r = 5\%$, $\theta = 5\%$, $\kappa = 1.0$, $\sigma_r = 0.03162$ and $\rho = 0$. The default probabilities based on S&P’s data (2002), CG model with a constant target leverage ratio equal to 0.315 and Merton model with the corresponding ratings and parameters are illustrated for comparison.
EXHIBIT 8. Default probability term structures of BB-rated firm with different $\rho$ and $\alpha = -0.005$.
The default probabilities of a BB-rated firm with $L = 49.5\%$ and $\sigma_L = 0.261$ are plotted using $\rho = -0.75, -0.3, 0$ and 0.75 and $\alpha = -0.005$. The other parameters used are $L_0 = 1.0$, $r = 5\%$, $\theta = 5\%$, $\kappa = 1.0$ and $\sigma_r = 0.03162$. The term structures based on S&P’s data (2002) are illustrated for comparison.