Currency Unions and International Assistance

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Revised July 2013

Abstract

This paper discusses the relationship between the formation of a currency union and the emergence of fiscal assistance between the countries of the union. Countries trade off the benefits of flexible exchange rates and the risk shared through mutual, voluntary fiscal transfers. It is shown that countries have more incentives to smooth their consumptions by implementing a system of voluntary fiscal transfers. Fiscal transfers therefore become more important and easier to sustain in a currency area. For some set of economic parameters, a sustainable risk sharing scheme offers a higher benefit than the exchange rate flexibility so that a currency area is optimal.

1 Introduction

The recent controversy about the refinancing of high deficit countries within the Euro-zone has highlighted the difficulty in adopting a common fiscal policy amongst sovereign states. The inclination or reluctance of the core Euro-zone countries to provide fiscal assistance to peripheral countries does not only renew the debate about whether the Euro-zone is an optimal currency area but also casts doubt about whether it might become so with the currently low levels of fiscal assistance between its member states. In this paper we discuss the theory of optimal currency areas taking into account the costs and benefits of international assistance between countries. Our focus is on the emergence of international assistance schemes that may insure countries against consumption shocks.\(^1\) Amongst others\(^2\), Forni and Reichlin (1999) have shown that there exists a large potential insurable income risk in the E.U. (about 45%) and that risk diversification is highly incomplete. This suggests that considerable welfare gains could be obtained if member states adopted an effective fiscal assistance mechanism.

To examine those issues, we consider a model of two trade partner countries that choose to adopt a fixed or flexible exchange rate regime and that are able to make transfers to each other on a

\(^{1}\)Arguably the current crisis for the euro-zone originates in financial rather than real shocks or the moral hazard of bad governance created by the expectation of future bailouts. We don’t address either of these issues, but believe that our analysis of fiscal transfers and optimal currency area will be of relevance in studying these issues too.

\(^{2}\)See also French and Poterba, 1999; Baxter and Jermann, 1997; Lewis, 1998.
voluntary basis. In other words, they implement voluntary fiscal transfers in the current time period without making any contractually enforceable promises for the long term. Risk-sharing is limited by self-enforcing constraints of the type introduced by Thomas and Worrall (1988). A country will only provide insurance if it perceives that the long-term benefit of risk sharing offsets the cost of the current fiscal transfers. This is a natural assumption for a currency union between sovereign states where no central fiscal systems implements risk sharing and where no supra-national legal authority enforces transfers.

Our baseline model embeds three features that are usually considered inimical to an optimum currency area. Namely, labor markets display wage stickiness, business cycles are not positively correlated and currency exchanges take place without any transactions costs. As a result, unsynchronized business cycles exacerbate the negative impact of wage stickiness on consumption and currency unions do not benefit from lower exchange rate transaction costs (e.g. Mundell, 1961 and 1973, Bayoumi, 1994; Alesina and Barro, 2002). However, the scope for voluntary insurance schemes is more important in a currency area when business cycles are not perfectly correlated precisely because wage stickiness exacerbates the amplitude of consumption fluctuations. It is thus possible that consumption sharing incentives increase the benefits of a currency area and reverse the Mundell’s result. The present paper shows the conditions under which this conjecture is fulfilled.

The paper presents a model where households consume differentiated products and supply their labor force to the firms that sell those products. Households have preferences for money holdings and set their wages one period in advance. The exchange rate is fixed in the currency area and set by the market forces under flexible exchange rates. There is no international financial assets. Our analysis firstly reveals that a currency union can be optimal if it is associated with more redistribution of consumption than would occur under flexible exchange rates. A currency area is preferred if it involves consumption sharing and flexible exchange rate regimes do not support any form of redistribution. The currency area is even better if households are more risk averse and have more elastic labor supply and if firms sell more differentiated products. Although those results are consistent with Ching and Devereux (2003) and Devereux (2004), we go beyond their analysis. Indeed, we consider that transfers can emerge under both flexible and fixed exchange rates and that they are endogenously sustained by the countries considering their short- and long-term costs and benefits. Consumption sharing mechanisms are then shown to be easier to sustain in currency areas and for larger shocks. For some set of economic parameters, consumption sharing is sustained in the currency area whereas no transfers can be sustained under flexible exchange rates. Finally, we show that, in the latter parameter constellation, currency areas can also be preferred because the benefit of the associated redistributive mechanism balances the cost of wage stickiness. Our analysis therefore shows that the choice for a currency area cannot be disentangled from the choice for risk sharing or redistributive systems. Indeed, in our model, currency areas make redistribution more likely. Empirically, this is no real surprise as long-lived currency areas have usually adopted redistributive fiscal systems (e.g. U.S.A., U.K., etc.) and as the issue of international assistance becomes increasingly critical among the members of Euro area.
Literature review The paper relates to various strands of the literature on optimum currency areas and risk sharing initiated by Mundell (1973). A first strand includes static analysis with uncertainty. Drèze (2000) argues that transfers between regions can be used as a means of insurance against regional income shocks. Alesina and Perotti (2004) studies the political economic balance between a better insurance and a wider population heterogeneity. Persson and Tabellini (1996) discuss risk sharing and moral hazard. Ching and Devereux (2003) and Devereux (2004) discuss a static model of international trade with preference shocks and with fiscal redistribution within currency areas. By borrowing their production and labor supply set-up, the present model allows us to present a relevant set of parameters for which currency areas are optimal and voluntary transfers are sustained.

A second strand of this literature is the new dynamic public economics. Kocherlakota (2010) stresses the political constraints of countries and emphasizes the commitment and dynamic optimization problem that countries face in choosing a currency union. This extends to macroeconomic questions Thomas and Worrall’s (1988) analysis about risk-sharing mechanisms under self-enforcing constraints. As in our approach, Castro and Koumintingé (2010) emphasize risk sharing and limited commitment. They however assume that the formation of a union enables full risk sharing, that trade with countries outside of the union is restricted by limited enforcement and that the technology of enforcement changes after the formation of union.

A last strand relates to the so-called New Open Economy Macroeconomics that traditionally focuses on monetary policies under the assumption of complete financial markets (Obstfeld and Rogoff 1995, Corsetti et al. 2010, etc.). Under this assumption, financial markets offer an important risk sharing mechanism so that fiscal assistance policies between countries is not likely to relevant. However, financial markets are far from being complete in practice. For this reason Corsetti, Dedola and Leduc (2005) study the monetary policies when those markets are partially complete. Our analysis goes further and discusses the situation where financial and monetary markets are absent. Our main hypothesis is that the money and asset holdings of the (median) households are not sufficient large to permit the perfect consumption smoothing. We further add the discussion of incentives and sustainability of risk sharing mechanisms, which, to our knowledge, has not been covered. From the empirical viewpoint, this view is supported by the literature on the international portfolio puzzle. From a factual viewpoint, the recent social turmoil’s in Southern E.U. countries most probably highlight that poor and middle class households did not hold those perfectly diversified international asset portfolios and are claiming for the design of income smoothing mechanisms, which are not currently offered by markets and transnational institutions.

[Comment comparing with Arellano and Heathcote: Those authors show a calibrated quantitative model, while this paper show the point theoretically. Yet the debt contracts in Arellano and Heathcote are state uncontingent whereas the transfers are here state contingent.]

The paper is organized as it follows. Section 2 presents the baseline model and Section 3 studies the equilibria under fixed and flexible and with fiscal transfers between countries. In Section 4 discusses when the currency area performs better than flexible exchange rates. Section 5 discusses to transaction costs. Section 6 concludes. The Appendix contains the proofs.
2 Model

The model includes a home and foreign country, \( i = H, F \). Each country hosts a unit mass of domestic households and a unit mass of firms producing differentiated tradeable varieties.

**Demands** Each household \( h \in [0, 1] \) residing in country \( i \) is endowed with the utility function

\[
U_i(h) = V \left[ C_i^\mu(h) \left( \frac{M_i(h)}{P_i} \right)^{1-\mu} \right] - \frac{\ell_i(h)^{1+\psi}}{1+\psi}
\]

where

\[
C_i(h) = \left[ \int_0^1 (d_{iH}(\omega, h))^\rho \, d\omega + \int_0^1 (d_{iF}(\upsilon, h))^\rho \, d\upsilon \right]^{1/\rho}
\]

is the aggregate composite of the demands for local varieties \( \omega \in [0, 1] \) and foreign varieties \( \upsilon \in [0, 1] \), \( M_i(h)/P_i \) is the real money balance or cash holdings and \( \ell_i(h) \) is the household \( h \)'s labor supply (worked hours). In this specification, \( \psi \) measures the inverse of the Frisch elasticity of labor supply while \( \mu \) denotes the preference and share of expenditure for cash holdings. The parameter \( \rho \) is the intensity of the preference for product variety, which increases with the elasticity of substitution between product varieties, \( \sigma \), as \( \rho \equiv 1 - 1/\sigma \). We assume that the function \( V \) displays constant relative risk aversion as \( V(x) \equiv x^{1-\gamma}/(1-\gamma) \) with \( \gamma > 1 \). Finally, the parameter expresses the intensity of preference for leisure. In this model, this parameter plays no role and is normalized to one w.l.o.g.

The household \( h \) maximizes its utility subject to its budget constraint

\[
\int_0^1 d_{iH}(\omega, h) p_{iH}(\omega) \, d\omega + \int_0^1 d_{iF}(\upsilon, h) p_{iF}(\upsilon) \, d\upsilon + M_i(h) = Y_i(h) + T_i + M_i^0
\]

where \( Y_i(h) = w_i(h) \ell_i(h) + \Pi_i \) is the household \( h \)'s income (wage bill and equal share of local profits), \( T_i \) is a transfer received by country \( i \) and \( M_i^0 \) is the endowment of local money. Money is the unit of numéraire and is exogenously supplied by the country’s central bank. For simplicity, money is fully depreciated in each time period. In the above expression, all values are denominated in country \( i \)'s currency.

Under this specification, it is well known that the composite consumption and the demand for money are given by

\[
\frac{P_i C_i(h)}{\mu} = \frac{M_i(h)}{1-\mu} = Y_i(h) + T_i + M_i^0
\]

where

\[
P_i = \left[ \int_0^1 (p_{iH}(\omega))^{1-\sigma} \, d\omega + \int_0^1 (p_{iF}(\upsilon))^{1-\sigma} \, d\upsilon \right]^{1/\sigma}
\]

is a local price index. The demand for variety \( \omega \) produced in country \( j \) from consumers in country \( i \) is equal to

\[
d_{ij}(\omega, h) = \mu \left[ \frac{p_{ij}(\omega)}{P_i^{1-\sigma}} \right]^{1-\sigma} [Y_i(h) + T_i + M_i^0]
\]
where $\sigma = 1/(1 - \rho)$ is the elasticity of substitution between varieties. The indirect utility from consumption is therefore given by $V [\xi (Y_i + T_i + M^0_i) / P_i]$ where $\xi \equiv \mu^\rho (1 - \mu)^{1-\mu}$.

We define the nominal exchange rate $\varepsilon$ as the foreign currency per unit of home currency. [inverse definition of Corsetti; is it acceptable in the lit?] So, denoting the prices of local products to local consumers as $p_H(\omega) \equiv p_{HH}(\omega)$ and $p_F(v) \equiv p_{FF}(v)$, the nominal exchange rate imposes that $p_{HF}(v) = \varepsilon p_F(v)$ and $p_{FH}(\omega) = (1/\varepsilon) p_H(\omega)$. Naturally, $\varepsilon = 1$ in the case of a currency union. It is easy to check from equation (1) that $P_H = \varepsilon P_F$. Also, since there are no transfers from outside the two countries, we have $T_H = -\varepsilon T_F$. Denoting the aggregate of the local income as $Y_i \equiv \int_0^1 Y_i(h)dh$, we can measure the total world income as $Y = Y_H + \varepsilon Y_F$, which is dominated in the domestic currency. Similarly, the world money supply in the Home currency is given by $M^0 = M^0_H + \varepsilon M^0_F$.

The aggregate demand for local and foreign varieties $\omega$ and $\xi$, are then given by

$$d_H(\omega) = \mu \frac{(p_H(\omega))^{-\sigma}}{P_H^{1-\sigma}} (Y + M^0) \quad \text{and} \quad d_F(v) = \mu \frac{p_F(v)^{-\sigma}}{P_H^{1-\sigma}} \frac{(Y + M^0)}{\varepsilon}.$$ 

We can now present the production side of the economy.

**Production** Local firms hire imperfectly substitutable labor services from households and set their product prices. Let us consider a firm producing the variety $\omega$ in country $i$. On the one hand, when that firm hires $\ell_i(\omega, h)$ units of differentiated labor service from each local household $h$, it is able to produce an amount of

$$F_i[\ell_i(\omega, h)] \equiv \frac{1}{a_i} \left[ \int_0^1 \ell_i(\omega, h) \frac{\sigma + 1}{\sigma} dh \right]^{\theta - 1}$$

units of its own variety $\omega$, where $\theta > 1$ measures the elasticity of substitution between labor services. In this expression, $1/a_i$ is the country productivity shock. In this model, it is the source of ex-post heterogeneity between countries. We further assume that those shocks have the same probability distribution across countries so that countries are homogenous from an ex-ante view point.

The firm choose the labor input mix $\ell_i(\omega, \cdot)$ that minimizes its cost per output $\int_0^1 w_i(h)\ell_i(\omega, h)dh$ subject to $F_i[\ell_i(\omega, h)] = 1$. Its demand of labor service per unit of output can therefore be computed as

$$\ell_i(\omega, h) = a_i \left( \frac{w_i(h)}{w_i} \right)^{-\theta}$$

where

$$w_i \equiv \left[ \int_0^1 (w_i(h))^{1-\theta} dh \right]^{1/(1-\theta)}$$

is a local wage index. As a consequence, the cost per output is equal to $a_i w_i$ and the firm’s profit can be written as

$$\pi_i(\omega) = (p_i(\omega) - a_i w_i) d_i(\omega)$$

On the other hand, the firm $\omega$ chooses the price $p_i(\omega)$ that maximizes this profit function taking exchange rates and prices indices as givens. Because its demand is iso-elastic, it sets the price $p_i(\omega) = p_i \equiv a_i w_i / \rho$. The firms producing in the same country therefore have same profits $\pi_i(\omega) \equiv \pi_i$ and product supply $d_i(\omega) \equiv d_i$. Because each country host a unit mass of firm, $\pi_i$ is equal to the aggregate profit $\Pi_i$ while $d_i$ also denotes the aggregate product supply in that country, $d_i = \int_0^1 d_i(\omega)d\omega$. 


Labor market and wage stickiness  Households set their wages one period ahead and adapt their labor supply as to balance the demand of local firms. Price rigidities therefore stem from the labor market. Let us consider the household $h$ in country $i$. On the one hand, at the pre-set wage, this household matches its labor service supply $\ell_i(\omega, h)$ with each firm’s demand, which is equal to the labor input per output (2) times the firm’s output $d_i(\omega)$. The amount of labor service supplied to all firms, $\ell_i(h) \equiv \int_0^1 \ell_i(\omega, h) d\omega$, is then equal to

$$\ell_i(h) = a_i \left( \frac{w_i(h)}{w_i} \right)^{-\theta} d_i$$

which is iso-elastic in the household’s wage, $w_i(h)$. On the other hand, the household sets its wage $w_i(h)$ one period in advance and maximizes her expected utility

$$EU(h) = EV \left[ \xi w_i(h) \ell_i(h) + \Pi_i + T_i + M^0_i \right] - \frac{E \ell_i(h)^{1+\psi}}{1+\psi}$$

taking aggregate profits, transfers and money supply as givens and where $E$ is the expectation operator (over states of nature that will be define later). This gives the wage

$$w_i(h) = \frac{\theta}{\theta - 1} \frac{E \ell_i(h)^{\psi+1}}{\xi V'[\cdot] \frac{\ell_i(h)}{P_i}}$$

As $\theta$ falls, households offer more differentiated labor services, have more power on the labor market and set their wages higher above their marginal utility of labor (see the right hand side). As all households face the same labor demand, they set the same wage so that $w_i(h) \equiv w_i$ and $\ell_i(h) \equiv \ell_i(h, \omega) \equiv \ell_i$. By the same token, consumption and earnings are the same for all households in the same country: $C_i(h) \equiv C_i$, $M_i(h) \equiv M_i$ and $Y_i(h) \equiv Y_i$. Similarly, all firms in the same country set the same prices, hire the same amount of labor, and produce the same quantity of each variety. So, the indices $h$ and $\omega$ referring to households and firms can be dropped without loss of generality. We summarize the above results as it follows.

Equilibrium  In equilibrium we get the following relationships. Product prices and wages are set by firms and households such that

$$p_i = a_i w_i / \rho \quad \text{and} \quad w_i = \frac{\theta}{\theta - 1} \frac{E \ell_i^{\psi+1}}{\xi V'[\cdot] \frac{\ell_i}{P_i}}$$

(3)

Labor supply clears with its demand so that

$$\ell_i = a_i d_i$$

(4)

National income equalizes the value of production

$$Y_i = w_i \ell_i + \Pi_i = p_i d_i = w_i \ell_i / \rho$$

(5)

The demand for goods and money holding divide as constant share of disposable income

$$\frac{P_i C_i}{\mu} = \frac{M_i}{1 - \mu} = Y_i + T_i + M^0_i$$

(6)
where the local price indices are given by

\[ P_H = \varepsilon P_F = (1/\rho) \left[ (w_H a_H)^{1-\sigma} + (\varepsilon w_F a_F)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \]  

(7)

The demand for each variety are equal to

\[ d_H = \mu \frac{p_H^{1-\sigma}}{P_H^{1-\sigma}} (Y + M^0) \quad \text{and} \quad d_F = \mu \frac{p_F^{1-\sigma}}{P_F^{1-\sigma}} \frac{Y + M^0}{\varepsilon} \]  

(8)

where \( Y = w_H \ell_H / \rho + \varepsilon w_F \ell_F / \rho \) is the world income evaluated at home prices and \( M^0 \) is the aggregate money supply also evaluated at home prices. The above relationship allows us to express the exchange rate as

\[ \varepsilon = \left( \frac{a_H}{a_F} \right)^{\rho} \left( \frac{\ell_H}{\ell_F} \right)^{\frac{1}{\sigma}} \]  

(9)

So, productivity shocks can be absorbed through changes in either the exchange rate or the labor supplies (and thus production). Finally, the utility can be written as

\[ U_i = V \left[ \xi w_i \ell_i / \rho + T_i + M_i^0 \right] - \frac{\ell_i^{1+\psi}}{1 + \psi} \]

In this model with homothetic preferences, the money supply policy matters but the total amount of money supply is irrelevant. Any long-run proportional rise in money supply will indeed automatically matched by the same proportional rise in wages and prices. For this reason and for the clarity of exposition, we normalize money supplies to \( M_i^0 = m_0 \equiv (1-\mu) / \mu \) w.l.o.g.. In this case, money supplies follow the households’ preferences for cash holdings so that exogenous changes in the parameter \( \mu \) do not alter nominal prices and wages.

Finally it is convenient to define the ‘relative’ productivity shock

\[ b_i \equiv \frac{1}{2} \left( a_i^{1-\sigma} + a_j^{1-\sigma} \right) \]

and the ‘global’ index of inverse productivity

\[ A \equiv \left( a_H^{1-\sigma} + a_F^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \]

3 Exchange rate regimes and international transfers

In this section we discuss the two exchange rates system and the effect of transfers international redistribution through fiscal transfers.

3.1 Currency union

We now consider the equilibrium in which countries \( H \) and \( F \) form a currency union so that \( \varepsilon = 1 \). The total money supply be \( M^0 = 2m_0 \) is equal to money demand \( M_H + M_F \). Since countries are ex-ante symmetric and use the same currency, households set the same ex-ante wages: \( w_i \equiv w \).
Naturally, price indices become equal across countries: \( P^c \equiv P_i \), where the superscript \( c \) denotes the variable under a currency area. Aggregating the demands (6) across each country, we get the world income \( Y = 2 \). Plugging labor demands (8) in the labor market conditions (4), we get each country \( i \)'s employment level

\[
\ell^c_i (w^c) = \rho \frac{b_i}{w^c}
\]

The higher productivity country (higher \( b_i \)) has a higher employment level. Employment is independent of transfers. Then, using the wage set in (4) and the labor supply (10) one can verify that the wage \( w^c \) satisfies

\[
(w^c)^{\gamma + \psi} = \kappa_0 \frac{\mathbb{E} \left[ b_i^{1+\psi} \right]}{\mathbb{E} \left[ b_i (b_i + T_i + m_0)^{-\gamma} \right] A^{\gamma - 1}}
\]

and \( \kappa_0 \equiv \rho^{\gamma + \psi - 1} \xi^{\gamma - 1} (\theta/(\theta - 1)) \). Because it is set ex-ante, the value of \( w^c \) depends on the probability distribution of productivity shocks \( b_i \) and transfers \( T_H \) but does not depend any specific shock realization. We denote by \( w^c_0 \) the wage set in the absence of transfers.

National incomes follows the employment levels so that \( Y_i = w \ell^c_i / \rho \). The utility in country \( i \) can be written as

\[
U^c_i (T_i, w) = V \left[ \xi \rho \frac{w \ell^c_i / \rho + T_i + m_0}{w^c A} \right] - \frac{(\ell^c_i)^{1+\psi}}{1 + \psi}
\]

which is an increasing and concave function of the received transfer.

We now consider equilibrium output and consumption under a flexible exchange rate regime.

### 3.2 Flexible exchange rate

The introduction of an exchange rate provides an additional instrument to allow relative prices to adjust and smoothen the impact of shocks on production and employment. Under a flexible exchange rate regime, money demand equates its supply within each country so that \( M_i = m_0 \). Again, since countries are ex-ante symmetric, households set the same ex-ante wages \( w_i = w \). From (13) and (6) we get \( P_t C_t = 1 \) and

\[
\frac{m_0}{1 - \mu} = \frac{w \ell_i + T_i + m_0}{\rho \ell_i}
\]

The latter identity solves for the employment level as

\[
\ell^f_i (T_i, w) = \frac{\rho}{w} (1 - T_i)
\]

where the superscript \( f \) denotes the variable under flexible exchange rates. Transfers and labor supply are perfect substitutes. Households would like to increase both their consumption and money holdings. They are however refrained to consume more because of the money supply does not change. As a result, they prefer to decrease their work participation after receiving a transfer.\(^3\)

\(^3\)Corsetti, Martin and Pesenti (2008) show the same property. Those authors nevertheless approximate the economy around zero transfer so that this property disappears in their analysis.
Using (9), the exchange rate writes as
\[ \varepsilon (T_H) = \left( \frac{b_F}{b_H} \frac{1 - T_H}{1 + T_H} \right)^{\frac{1}{\sigma}} \]  
and the price indices become
\[ P_H = \varepsilon P_F = \left( \frac{w}{\rho} \right) \left[ a_H^{1-\sigma} + (\varepsilon a_F)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \]
which are functions of transfers and wages. The exchange rate and price indices depend on transfers \( T_H \). When the domestic country \( H \) gives a transfer \( T_H = -\varepsilon T_F < 0 \) to the foreign country, the latter increases its product demand so that the exchange rate \( \varepsilon \) appreciates. This leads to a rise in the domestic price index and a fall in the domestic purchasing power.

Using the wage set in (4) and the labor supply (10) we compute the wage as
\[ (w^f)^{\gamma+\psi} = \kappa_0 \frac{\mathbb{E} \left[ (1 - T_i)^{1+\psi} \right]}{\mathbb{E} \left[ \mu^\gamma (1 - T_i) B_i^{-1} A^{-1} \right]} \]  
where \( B_i \) is defined as
\[ B_H = \left[ \frac{b_H}{2} + \varepsilon^{1-\sigma} b_F / 2 \right]^{\frac{1}{1-\sigma}} \]
and \( B_F = \varepsilon B_F \) where \( \varepsilon \) is the above function of \( T_H \). The wage \( w^f \) depends on the probability distribution of shocks and transfers \( T_H \). We denote by \( w^f_0 \) the wage set in the absence of transfers. In the absence of shocks, wages are equal under fixed and flexible exchange rates (i.e. \( w^f_0 = w^c_0 \) if \( a_i = a_j = \text{cst} \)). Finally, the utility becomes
\[ U^f_i (T_i, w^f) = V \left[ \frac{\xi \rho}{\mu} \frac{1}{w^f B_i A} \right] - \left( \frac{\ell^f_i}{1 + \psi} \right) \]  

### 3.3 Fiscal transfers, redistribution and consumption sharing

Consumption and utility levels vary with the realization of the productivity shocks. Nevertheless, to smooth those fluctuations, countries have the possibility to assist each other and redistribute consumption by making international transfers to each other. In a full fledged federation this redistribution mechanism is embedded in the fiscal system. In an association of countries, those transfers must be made in an explicit way. For a reason that is explained below, we here focus on the easiest transfer mechanism where transfers equalize consumption.

In a currency area, each country’s consumption is given by \( \left( w^c \ell^c_i / \rho + T_i + m_0 \right) / P^c, \ i = H, F. \) Equalizing the consumption levels across countries gives the transfer
\[ T^*_H = \frac{w^c}{2\rho} (\ell^c_F - \ell^c_H) = \frac{1}{2} (b_F - b_H) \]
where the asterisk * denotes the variables under consumption sharing. The transfers compensate the country with bad productivity shock and employment levels. Knowing those transfers, households set their wage \( w^* \) so that
\[ (w^*)^{\gamma+\psi} = \kappa_0 \frac{\mathbb{E} \left[ b_i^{1+\psi} \right]}{\mathbb{E} \left[ \mu^\gamma b_i A^{-1} \right]} \]
and their utility becomes
\[ U_c^i(T^*_i, w^*) = V \left[ \xi \frac{\rho}{\mu} \frac{1}{w^*} + A \right] - \left( \frac{\ell^c_i}{1 + \psi} \right) \]

where \( \ell^c_i = \rho b_i / w^* \).

Under flexible exchange rates, the aggregate consumption of country \( i \) is given by \( \xi m_0 / [(1 - \mu) P_i] \). Equalizing consumption implies that price indices are the same, \( P_H = P_F \), which implies that the exchange rate across the countries is sterilized so as \( \varepsilon = 1 \). By (14) and (15) we get
\[ T^*_H = \frac{1}{2} (b_F - b_H) \quad \text{and} \quad \ell^f_i = \frac{b_i}{w_f} \]

Those are the same transfers as under common currency. Since labor supplies are also the same function of the wage, the wage is set to the same level \( w^* \). The labor supply turns out to be the same: \( \ell^f_i = \ell^c_i = \frac{\rho b_i}{w^*} \). To sum up, a fiscal policy of equalizing consumption yields the same welfare as under flexible exchange rate and currency area.

Note that, in the currency area, the transfers equalizing consumption are also those chosen by a utilitarian planner who maximizes the welfare objective \( W^c \equiv U_H^c(T_H, w^c) + U_F^c(T_F, w^c) \). Indeed, since \( \ell^c_i \) is independent of transfers, the maximum of this objective implies
\[ \frac{dW^c}{dT_H} = V' \left[ \frac{\xi w \ell_H / \rho + T_H + m_0}{P^c} \right] \xi P^c - V' \left[ \frac{\xi w \ell_F / \rho - T_H + m_0}{P^c} \right] \xi P^c = 0 \]

which implies consumption equalization and therefore yields the transfers given in (18). By contrast, under a flexible exchange rate regime, the consumption sharing transfers are not the one chosen by a utilitarian planner. Transfers indeed affect labor supplies in this regime. The marginal utility from an additional transfer is no longer a function of consumption only. Equalizing marginal utility is no longer equivalent to equalizing consumption. As a result, the good productivity country has an incentive to reduce its transfer below the consumption sharing level. In turn this entices the low productivity country to raise its labor participation. A full fledge analysis of utilitarian redistribution under this regime is however more complicated. It is also less relevant as transfers will have to take into account the disutility of labor in each country and state of nature, a task that government would achieve with much (political) difficulty in reality. As a case in point, the actual debate about international redistribution (e.g. in the E.U.) is concerned with how transfers received or granted compare to national incomes. They do not take into account the benefits in terms of leisure.

### 4 Sustainable transfer systems

In this section, we discuss the conditions under which transfer systems are not only beneficial but also sustainable. Our first question is whether the risk sharing motivation is sufficient to overcome the inefficiencies in the currency area. We therefore investigate when a currency area becomes more attractive when it is associated with consumption sharing transfers. Our second question is whether such transfers can be implement on a voluntary basis. In other words, we investigate whether one country has an incentive to voluntarily assist the other country by giving a transfer when the latter has a bad economic outcome. The issue is of particular importance in the context of associations of
countries, like the E.U., where there exist no significant and binding assistance mechanisms and no fiscal systems across the borders of their member countries.

For the sake of simplicity, we shall assume that the productivity shocks in countries $H$ and $F$ is governed by a two-state symmetric stochastic process $s = \{1, 2\}$ where the two countries alternate between a good and bad inverse productivity, $a_H = z \in (0, 1)$ and $a_B = 1$. More formally, the country inverse productivities $(a_H^s, a_F^s)$ are given by $(a_G, a_B) = (z, 1)$ in state $s = 1$ and $(a_B, a_G) = (1, z)$ in state $s = 2$. The domestic country has a good productivity shock is state 1 while the foreign one has the same good productivity shock in state 2. To preserve symmetry we assume that each state occurs with equal probability. In this two-state two-country set-up, that there exists a one-to-one mapping between the state and the country $i$. In particular, we can denote the utility levels $(U_H^s(T_H^s, w), U_F^s(T_F^s, w))$ as $(U_G(T_G, w), U_B(T_B, w))$ in state $s = 1$ and $(U_B(T_B, \cdot), U_G(T_G, \cdot))$ in state $s = 2$. Of course, transfers remains the same; only their direction changes. This allows us to dispense the use to the superscript $s$ in many mathematical expressions.

This framework turns out to interesting for three reasons. The first reason is that the context of a negative shock correlation is precisely the one in which the Mundell’ argument is formulated: currency area cannot not be optimal if business cycles are not positively correlated. However, such a setting is also the one in which the incentives for risk sharing are the strongest. One may conjecture that weaker shock correlations reduce insurance incentives and therefore should make currency areas less attractive. The second reason is that the choice of a two state model obviously makes the formal analysis less cumbersome. The third reason is that, in this framework, the transfers implemented by a utilitarian planner corresponds to those that implemented for the purpose of risk sharing. Indeed, the utilitarian planer’s objective $W^s = U_H^s(T_H^s, w) + U_F^s(T_F^s, w)$ and the expected utility $\sum_{s=1}^{2}[\frac{1}{2}U_H^s(T_H^s, w) + \frac{1}{2}U_F^s(T_F^s, w)]$ simplifies to the same expression $U_G(T_G, w) + U_B(T_B, w)$. As a result, the fiscal transfers (18) implementing consumption sharing in a currency area corresponds to those that maximize risk sharing.

In the subsequent analysis, symmetry will allow us to refer to a country only by its productivity state $s \in \{G, B\}$. We will therefore dispense reference to each country’s label $H$ or $F$ and use the simpler notation, $U_s(T, w), s \in \{G, B\}$ and $E_s U_s(T, w)$. Our first question is whether efficient risk sharing gives a sufficient motivation to overcome the inefficiencies in the currency area.

4.1 Currency union with fiscal transfers

We now compare the regime of flexible exchange rates in the absence of transfers with a currency union that implements consumption sharing transfers. We compute

$$\text{E}_s U_s^T(T_s^*, w) = -\frac{1 + \kappa_1}{1 + \psi} \left(\frac{\rho}{w^*}\right)^{1+\psi} \text{E}_s \theta^{1+\psi}$$  \hspace{1cm} (19)

$$\text{E}_s U_s^f(0, w_0^f) = -\frac{1 + \kappa_1}{1 + \psi} \left(\frac{\rho}{w_0^f}\right)^{1+\psi}$$  \hspace{1cm} (20)

where $\kappa_1 \equiv \left(\mu \rho^{-1} + \frac{\theta - 1}{\theta}\right)^{-1}$ is a constant. In this model, only four parameters determine the dominance of a common currency area: the productivity shocks $z$, the elasticity of substitution $\sigma$, the risk
aversion $\gamma$ and the inverse elasticity of labor supply $\psi$. The present comparison indeed does not depend on $\theta$ and $\mu$ because the ratio $\hat{w}^*/\hat{w}^f$ and the term $E_s b_s^{1+\psi}$ are independent of those parameters. Comparing the above expressions, we can make the following proposition.

**Proposition 1** For large enough shocks ($z \to 0$), the currency union with consumption sharing transfers dominates the flexible exchange rate regime with no transfers. For small enough shocks ($z \to 1$), the currency union dominates if and only if $\gamma > \sigma (\psi \sigma - 1)$. That is, for sufficiently high risk aversion, large labor supply elasticity and weak product substitutability.

**Proof.** See Appendix A. ■

The above proposition presents an important result in its own right. It shows that despite the model being designed to give an advantage to the flexible exchange rate regime, a currency union can be optimal if it is associated with more redistribution of resources than would occur under a flexible exchange rate regime. High risk aversion entices households to prefer consumption sharing within the currency area even if price stickiness makes this area costly. Currency areas are also less harmful when labor supply is more elastic because households are more flexible to balance their consumption reduction with additional leisure in bad times. Finally, currency areas are less harmful when products are weak substitutes. Because product demands for local goods are then less elastic, local firms have less incentives to cut local production and employment after a bad productivity shock. So, the power in the product market and the intensity of competition with foreign firms are a important factors for the benefit of a currency area.

Figure 1 extends the above results to intermediate shock levels, $z \in [0.8, 1]$. It depicts the loci where currency areas with fiscal transfers yields the same expected welfare as a flexible exchange rate system without those transfers. The areas below (resp. above) those loci represents the parameters where a currency area dominates (resp. is dominated by) a flexible exchange rate system. Currency areas are preferred for stronger productivity shocks $1/z$, stronger risk aversion $\gamma$, more elastic labor supply $1/\psi$ and lower elasticity of substitution $\sigma$. The literature assesses the value of relative risk aversion in the range $\gamma \in [1.4, 7.1]$. Backus, Kehoe and Kydland (1992) and Corsetti, Dedola and Leduc (2004) use a low value $\gamma = 2$. Basu and Fernald (1997) estimate of elasticity of substitution in a range $\sigma \in [4, 6]$. Mendoza [1991] using a sample of industrialized countries sets that elasticity equal to $\sigma = 3.8 (\rho = 0.74)$. Stockman and Tesar [1995] estimate a lower elasticity $\sigma = 1.8 (\rho = 0.44)$. It is generally agreed that $\sigma$ is low for traded products. Estimates of the Frisch elasticity range from 0.1 to more than 1.0, but cluster around 0.4. Devereux (2004) explores the labor supply elasticity in the range $\psi^{-1} \in [0.5, 2]$. Reichling and Whalen (2012) recently suggest the range $[0.27, 0.53]$.

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4The impact of labor supply elasticity on the preference for a currency area is discussed in Devereux (2004).

5Todter (2008) assesses the coefficients of relative risk aversion in the range $\gamma \in [1.4, 7.1]$. Backus, Kehoe and Kydland (1992) and Corsetti, Dedola and Leduc (2004) use a low value $\gamma = 2$. Basu and Fernald (1997) estimate of elasticity of substitution in a range $\sigma \in [4, 6]$. Mendoza [1991] using a sample of industrialized countries sets that elasticity equal to $\sigma = 3.8 (\rho = 0.74)$. Stockman and Tesar [1995] estimate a lower elasticity $\sigma = 1.8 (\rho = 0.44)$. It is generally agreed that $\sigma$ is low for traded products. Estimates of the Frisch elasticity range from 0.1 to more than 1.0, but cluster around 0.4. Devereux (2004) explores the labor supply elasticity in the range $\psi^{-1} \in [0.5, 2]$. Reichling and Whalen (2012) recently suggest the range $[0.27, 0.53]$. 

INSERT FIGURE 1 HERE
The above argument leaves unexplained the reason why a currency union is associated with stronger risk-sharing mechanism. In contrast to Alesina et al. (1995) and Personn and Tabellini (1996), this argument hinges on the existence of risk sharing motives rather than public goods. It is certainly conceivable that institutional factors play a role. For instance, a constitution usually provides a legal framework for redistributive policies. However, it is less clear how the redistributive policies may be implemented in an association of sovereign states that does not foresee such redistribution plans. In the next sub-sections we examine how the adoption of common currency may enhance risk sharing mechanisms.

4.2 Voluntary transfer systems

We now consider countries’ choice to implement those risk sharing mechanisms and make the assumption that sovereign countries offer risk sharing transfers on a voluntarily basis. Redistribution for any risk-sharing purpose is therefore limited by participation or self-enforcing constraints, which bind when a country perceives that risk sharing is not in its own interest. In contrast to the previous subsection, there is here no institutional factors that enforce risk sharing. We show that a currency union may become beneficial because it relaxes the self-enforcing constraints and permits better risk sharing.

To discuss the possibility of fiscal transfers when countries have limited commitment, we assume that countries interact repeatedly over an infinite horizon. In this setting, there exists no asset that can be transferred between time periods. As claimed in the introduction, we take the view that money and asset holdings by households are not sufficient large to permit any consumption smoothing. Since transfers cannot be legally enforced, countries can renege on any agreement if they find it in their interest not to make a transfer. Hence any assistance programme has to be designed to be self-enforcing. We apply an approach similar to Thomas and Worrall (1988) who examine self-enforcing wage contracts between employers and employees. Thus, we presume that countries make a tacit agreement on a programme of mutual assistance and specify a state contingent transfer to be made from the good to the bad productivity country. We assume that any breach of this tacit agreement results in a breakdown in which no transfers are made.

Let household and countries have the same discount factor be \( \delta \in (0, 1) \). We now compare the sustainability conditions for each exchange rate regime.

4.2.1 Sustaining consumption sharing

In this subsection we study the discount factors for which countries sustain the assistance with consumption sharing transfers. We here look at the benefit from sustaining the set of transfers under full consumption sharing.

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6This contrasts with a large body of international macroeconomic literature but may be comforted by some empirical evidence. In particular, money holdings are supposed to be ineffective in transmitting wealth from one period to the other. We make this assumption not only for the sake of analytical tractability but also because money holdings are usually too small to smooth incomes during long recession times and because our focus is on the incentives for risk sharing through voluntary transfers mechanisms.
When the consumption sharing transfers are sustained, households set their wages to \( w^* \) and obtain the expected utility \( E_s U_s(T^*_s, w^*) = \frac{1}{2} U_G(T^*_G, w^*) + \frac{1}{2} U_B(T^*_B, w^*) \). A country may choose to breach the agreement when it has a good productivity shock and is called upon to make a transfer. In this case, it gives no transfer and its households obtain a utility level \( U_G(0, w^*) \). After the breach, there is no assistance scheme and countries get \( E_s U_s(0, w_0) = \frac{1}{2} U_G(0, w_0) + \frac{1}{2} U_B(0, w_0) \). This system of transfers is sustainable if and only if

\[
U_G(T^*_G, w^*) - U_G(0, w^*) + \frac{\delta}{1 - \delta} [E_s U_s(T^*_s, w^*) - E_s U_s(0, w_0)] \geq 0 \quad (21)
\]

In this expression, the first (negative) difference represents the short run loss in sustaining the current assistance while the term with the second (positive) difference denotes the net present value of the expected gains in sustaining future assistance compared to returning to a system with no transfers. Larger discount factors relaxes this inequality. So, there exists a critical discount factor \( \delta_c \) above which this system of transfers is sustainable. The analytical value of this discount factor for each exchange rate regime is provided in Appendix B. For small shocks and money holdings (\( z \) and \( \mu \) close enough to one and \( m_0 \) close to zero), the critical discount factor \( \delta_c \) is given by

\[
\delta_c \simeq 1 - \frac{11 + \kappa_1}{4 \kappa_1} \frac{1 + \psi}{\gamma + \psi} \sigma (1 - z) \gamma \gamma (1 - z)
\]

\[
\delta_f \simeq 1 - \frac{11 + \kappa_1}{4 \kappa_1} \frac{1 + \psi}{\gamma + \psi} \sigma (1 - z) \gamma \theta \gamma - \sigma (\sigma \psi - 1) (\theta - 1) \sigma^2 + \sigma (1 - z)
\]

The above result implies three important points. First, the critical discount factor \( \delta_c \) under currency area rises with the amplitude of productivity shocks, \( 1 - z \), and reached one when those shocks vanish (\( z = 1 \)). This means that smaller shocks reduce the incentives to sustain the transfers under currency areas. Second, \( \delta_f \) falls with larger shock amplitudes, \( 1 - z \), if \( \gamma > \sigma (\sigma \psi - 1) \). In this case, larger shocks enhance the incentives to sustain the transfers under flexible exchange rates. However, when the last condition is not met, flexible exchange rates never support those transfers because \( \delta_f > 1 \). Indeed, whereas the transfers equalize consumption, they do not maximize expected utility and may be rejected by infinitely patient countries. Finally, one can check that that \( \delta_c < \delta_f \), for any \( \delta_f < 1 \). Currency areas therefore sustain the consumption sharing transfers for a larger set of economic parameters.

**Proposition 2** Suppose small enough money holdings and productivity shocks (\( \mu \to 1 \), \( m_0 \to 0 \) and \( z \to 1 \)). Then, consumption sharing transfers are more likely to be sustained for larger shock amplitudes \( (1 - z) \) in a currency union. This is true in the flexible exchange rate system only if \( \gamma > \sigma (\sigma \psi - 1) \). They are more likely to be sustained under currency areas than under flexible exchange rate systems \( (\delta_c < \delta_f) \).

**Proof.** See Appendix B for the detailed derivation of the above formulas. ■

We can discuss additional comparative statics properties. Expanding the constant \( \kappa_1 \), one can show that \( \delta_c \) falls with larger \( \gamma \), \( \theta \), \( \psi \) and \( \sigma \). As a result, larger risk aversion \( \gamma \), smaller labor supply elasticities \( \psi^{-1} \) and stronger product substitutability \( \sigma \) increase the incentives to sustain the transfers in the currency area. Those properties are intuitive. Larger productivity shocks raise the insurance
incentives and therefore the incentive to share consumption. Those incentives are naturally stronger when households are more risk averse and they are not eager to substitute their consumption with their labor supply. Finally, those incentives are also stronger when products are better substitutes. Higher product substitutability indeed intensifies firms’ competition and entices firms to rebalance production away from the low productivity country. As this increases the variance in income and consumption, households prefer to smooth their consumption with transfers. The same comparative statics for $f^3$ are however not as trivial.

Figure 2 generalizes the above results for larger shocks and money holdings. The solid and dashed red inked curves respectively depict the values of $c^f$ and $f^f$ in the fixed and flexible exchange rate regimes. This numerical example confirms that the consumption sharing transfers are sustained for lower discount factors under currency area ($c^f < f^f$). Those properties can be reproduced for other relevant parameters as those in the ranges given in the sub-section 4.1. Moreover, the critical discount factor $f^f$ can be shown to be larger than one (for instance for the same parameters and $\sigma = 4$, not shown in the figure). Then, consumption sharing can never be sustained under flexible exchange rates systems.

\section*{4.2.2 Sustaining no transfer}

We now consider the case where the countries can sustain no transfers. To find the circumstances where no transfers can be sustained, we need only consider some small transfers $T_s \in \{T_G, T_B\}$. As argued above, a country will breach when it gets a good productivity shock and is called upon to make a transfer. No small transfer can be sustained under the following condition:

$$\lim_{T_s \to 0} U_G(T_G, w) - U_G(0, w) + \frac{\delta}{1 - \delta} [E_s U(T_s, w) - E_s U(0, w_0)] \leq 0$$

where $w$ is the wage set by the household when they expect the small transfers $T_s$. As before, the first (negative) difference represents the short run loss from sustaining the current small fiscal assistance and the last term denotes the net present value of the expected benefit of sustaining the future small fiscal assistance. Because smaller discount factors relaxes this inequality, there exists a critical discount factor $\delta^c$ below which small transfers are never sustainable. Small transfers create several effects. They negatively impact the purchasing power of the giving country and entices its households to augment their labor supply. The opposite effect takes place in the recipient country. In addition, small transfers also alter exchange rates and wage setting. The overall impact to those effects on the benefit of sustaining or stopping small transfers are determined in Appendix C. For small shocks and money holdings ($z$ and $\mu$ close enough to one), the critical discount factor $\delta^c$ is approximated by

$$\delta^c \simeq 1 - \frac{1}{2} \left( \frac{\psi}{\gamma + \psi} + \frac{\gamma}{\gamma + \psi} \frac{\theta - 1}{\theta} \right) \gamma (\sigma - 1) (1 - z)$$

which rises to one as the shock amplitude $1 - z$ falls to zero. On the other hand, we can approximate

$$\delta^f \simeq 1 - \frac{1}{2} \left[ \left( \frac{\psi}{\gamma + \psi} + \frac{\gamma}{\gamma + \psi} \frac{\theta - 1}{\theta} \right) \frac{\theta}{\sigma ((\theta - 1) \sigma + 1)} - \frac{(\sigma - 1) - \sigma}{\gamma \sigma^2 (\gamma + \psi)} \right] \gamma (\sigma - 1) (1 - z)$$
This falls with the shock amplitude if and only if
\[ \psi \geq \psi_0 \equiv -\theta + (\sigma - 1)(\gamma - 2\theta + 1) + (\gamma - 1)(\sigma - 1)(\theta - 1)(\sigma - 1 - \gamma) \theta \sigma \gamma \]

When this condition is satisfied, \( \delta_f \) rises to one as the shock amplitude falls to zero. Otherwise, \( \delta_f \) lies above one and small transfers are never sustained. [More about \( \delta_c < \delta_f \)]

**Proposition 3** For small enough productivity shocks and money holdings, transfers are more likely to sustained for larger shocks \((1 - z)\) in a currency union. This is also true in the flexible exchange rate system only if \( \psi \geq \psi_0 \).

**Proof.** See details in Appendix C. ■

Figure 2 depicts in blue ink the values of this critical discount factor for various shocks in the case of currency area and flexible exchange rates (\( \delta_c \) and \( \delta_f \)). In this numerical example, the consumption sharing transfers are sustained for lower discount factors under currency area because \( \delta_c \) is smaller than \( \delta_f \). Those properties can be reproduced for other relevant parameters.

According to Figure 2, we can make the following argument about the sustainability of exchange rate regimes. Fix the shock structure and increases the households’ impatience (lower \( \delta \)). Then, countries first become unable to sustain consumption sharing under a flexible exchange rate regime. They then become unable to sustain any transfers under the latter regime. For even higher impatience, they stop supporting consumption sharing and finally any transfer under currency areas. Figure 2 nevertheless shows that transfers are more likely to be sustained under currency areas. This is because wage stickiness leads to stronger consumption variations and incentive for risk sharing.

### 4.3 Sustainable and optimal currency areas

Because they permit consumption smoothing, transfer mechanisms are substitutes for exchange rate instruments. When currency areas sustain transfer mechanisms, they may be able to recover the cost of a fixed exchange rate. This will be the case under three conditions: currency areas with consumption sharing are sustainable; flexible exchange rate regimes support no transfers; and the former regime performs better than the latter. This implies that the following three conditions: \( \delta \in [\delta_c, \delta_f] \), \( \delta_f > \delta_c \) and \( E_s U_s^c(T_s^*, w) \geq E_s U_s^f(0, w_f^0) \). Such a case is illustrated in Figures 1 and 2 with the parameters \( z = 0.95 \) and \( \delta = 0.97 \). For such parameters, Figure 1 show that a currency area with such transfers is better than the flexible exchange rate without transfers while Figure 2 shows that no transfers are sustained under flexible exchange rates whereas the consumption sharing transfers are sustained in a currency area. For small shocks, the above conditions form a non-empty set of parameters requires that

\[ \gamma > \sigma (\psi \sigma - 1) \]
\[ 1 + \psi + \rho (\gamma - 1) \frac{\theta - 1}{\theta} > 2 \left( \psi + \gamma \rho \frac{\theta - 1}{\theta} \right) \frac{\theta}{\sigma ((\theta - 1)\sigma + 1)} - 2 \frac{(\sigma - 1) \gamma - \sigma}{\gamma \sigma^2} \]
\[ \rho > \rho_0 \equiv \frac{1}{\theta} - 2 \frac{(\sigma - 1)}{\gamma \sigma^2} \]

Many parameters fulfill this condition provided that the elasticity of product substitution \( \sigma \) and inverse elasticity of labor supply \( 1/\psi \) are not too high. For instance, the parameters of Figures 1 and
2 (σ = 2, γ = 4, ψ = 1, θ = 5) imply that the above conditions are satisfied and that δ should be lie between δ^f = 1 − 0.23(1 − z) and δ^c = 1 − 0.64 (1 − z). This leads to our main result.

**Proposition 4** For small productivity shocks, there exist discount factors such that currency areas are preferred to flexible exchange rate systems under condition (23) and (24). In this case, consumption sharing transfers are sustained in the currency area whereas no transfers are sustained under flexible exchange rates.

The proposition presents the conditions for the optimality of currency areas when they implement transfer mechanisms that maximize optimal consumption sharing without affecting the participation of the country with the good productivity shock. By continuity, for slightly lower discount factors, there exists a set of parameters such that currency areas are also optimal but implement sustainable mechanisms where consumption sharing is constrained by the participation of that country. This naturally extends the scope of the proposition.

This result qualifies Mundell’s argument about currency unions. Under his argument, a currency cannot be optimal if shocks are not positively correlated. In this paper, we have shown that negatively correlated shocks indeed harm households when there are no transfers between countries. However, wage stickiness increases the consumption variance and enhances risk sharing incentives in currency unions. For a set of economic parameters, the consumption sharing transfers become sustainable and currency unions may outperform flexible exchange rate regimes. In other words, transfers are offered on a voluntary basis and there is no need to any form of contract between countries such as fiscal treaties, common constitution, etc. The main issue however is about the empirical relevance of the parameters for which currency areas are preferred.

## 5 Transaction costs

A major advantage of currency areas lies the elimination of transaction costs in exchanging currencies. In this section, we extend the previous analysis in the presence of transaction costs. As in Bayoumi (1995 I.M.F.), we choose a simple transaction cost in the form of iceberg costs, \( \tau \geq 1 \), according to which a share \( \tau − 1 \) of the value of exports is lost during the currency exchange transactions. Since such costs do not exist in the currency area, we focus on the flexible exchange rate regime.

In the presence of transaction costs, the wages \( w^f_0 \) and expected utility \( E_sU^f_s(0, w^f_0) \) are still given by the expressions (16) and (20) where the indices must be changed to

\[
B^{1-\sigma}_H = \frac{1}{2} \left( b_H + b_F \tau^{1-\sigma} \epsilon^{1-\sigma} \right) \quad \text{and} \quad B^{1-\sigma}_F = \frac{1}{2} \left( b_F + b_H \tau^{1-\sigma} \epsilon^{\sigma-1} \right)
\]

and where \( \epsilon \) is a function of shocks, transfers and transaction cost \( \tau \) (see details in Appendix D). So, larger transaction costs \( \tau \) reduce the expected utility (to more negative values) if wages \( w^f_0 \) also fall to lower (positive) values. This happens if \( E \left[ (B_iA)^{\gamma-1} \right] \) increases with larger \( \tau \). This also true for small enough shocks and transaction costs because transaction costs have a direct positive effect on \( B_H \) and \( B_F \) but have no first order effects on the exchange rate (that is, \( \partial B_i / \partial \tau > 0 \) and

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7This case is studied by Picard and Worrall (2010).
\[
\lim_{\tau \to 1, b \mu / b \tau \to 1} \frac{d\varepsilon}{d\tau} = 0.
\]
So, for small enough shocks and transaction costs, the wage and expected utility fall with larger transaction costs. Figure 4 depicts the sets of parameters for which currency areas with consumption sharing transfers dominate flexible exchange rate regimes with no transfers at all. It clearly shows that currency areas become beneficial for a larger set of economic parameters and in particular for small enough shocks. Indeed, for small productivity shocks, both regimes yield similar expected utilities while transaction costs lead to first order reduction of expected utility in the flexible exchange rate regime. To sum up, currency areas are more likely to be preferred in the presence of transaction costs.

A second concern is about the sustainability of transfer mechanisms. One must discuss two views on transaction costs. In the first view, transaction costs are associated with the resources used by currency traders, which include the office and personnel costs as well as the opportunity cost of holding currencies. In this view, transaction costs take place in all situations, including the one where countries implement the consumption sharing transfers and - in this model - the exchange rate tends to constant value. Transaction costs reduce households' utility in all states of nature and at all levels of transfers. The instantaneous and expected utilities \( U^f_G(T^*_G, w^*) \) and \( E_s U^f_s(T^*_s, w^*) \) under consumption sharing transfers fall by a similar amount while the instantaneous utility under deviation \( U^f_G(0, w^*) \) and the expected utility under no transfer \( E_s U^f_s(0, w^f_0) \) also decrease. The overall effect is difficult to disentangle analytically. However, calibrated examples show that transaction costs have only small effects on the critical discount factors \( \delta^f \) and \( \delta^f_s \). Figure 4 plots those discount factors for the same transaction costs as in Figures 2 and 3. The discount factor \( \delta^f \) (blue inked dashed curve) is almost unaltered by transaction costs while the discount factor \( \delta^f_s \) moves up (red inked dashed curve).\(^8\) This means that transaction costs make consumption sharing transfers less sustainable. In that view, transactions not only make fiscal assistance less likely under flexible exchange rates but they improve the welfare in the currency union as we have discussed in the previous paragraph. As a result, the presence of transaction costs gives an advantage to currency unions.

In the second view, transaction costs are associated to the exchange rate fluctuation or volatility. Typically, currency traders, firms and households need to hedge their currency positions at a cost. Although the present model is not tailored to discuss hedging issues, it can shed some (admittedly imperfect) light on the impact of this form of transaction costs. Indeed, when countries grant the consumption sharing transfers, the transfers neutralize the exchange rate to a constant, which eliminates exchange rate volatility and therefore transaction costs. As a result, the instantaneous and expected utilities \( U^f_G(T^*_G, w^*) \) and \( E_s U^f_s(T^*_s, w^*) \) under consumption sharing transfers remains the same as before. By contrast, when a country deviates or when it makes no transfer, the exchange rate fluctuates in response to the productivity shocks so that transaction costs are present and create inefficiencies. Then, both the instantaneous utility under deviation \( U^f_G(0, w^*) \) and the expected utility under no transfer \( E_s U^f_s(0, w^f_0) \) fall with larger transaction costs. Those effects relax the self-enforcing

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\(^8\)In the simulation of , it is necessary to compute the consumption sharing transfers and the exchange rate in the presence of transaction costs.
constraint (21). As a result, transaction costs make full consumption sharing more sustainable under flexible exchange. It can further be shown that they diminish the threshold \( \delta^f \) to lower values. This is because it increases the benefit sustaining transfers \( E_sU_s^f(T_s^*, w^*) - E_sU_s^f(0, w_0^f) \) (to more positive values) and decreases the benefit of a deviation \( U_G^f(0, w^*, \tau) - U_G^f(T_G^*, w^*) \) to lower (positive) values.\(^9\) This threshold is shown in Figure 4 by the red inked dotted curves. Higher transaction costs move those curves down. In this view, transactions costs act as an additional cost when a country refuses to sustain fiscal assistance. They therefore improve the incentives to maintain fiscal transfers under flexible exchange rates and make this regimes more advantageous. To sum up, the optimality of currency areas and the sustainability of fiscal assistance depend on the form of transaction costs in the currency markets.

6 Welfare maximizing risk sharing

[Tim’s simulation]

7 Conclusion

This paper discusses the relationship between the formation of a currency union and the emergence of fiscal assistance between the countries of the union. Countries trade off the benefits of flexible exchange rates and the risk shared through mutual, voluntary fiscal transfers. It is shown that countries have more incentives to smooth their consumptions by implementing a system of voluntary fiscal transfers. Such fiscal assistance is more important and easier to sustain in a currency area. For a large set of economic parameters, a sustainable risk sharing scheme offers a higher benefit than the exchange rate flexibility so that a currency area is optimal. The desirability and sustainability of currency areas is mainly enhanced by product differentiation, labor supply flexibility and risk aversion.

The present paper has abstracted from important economic features and sets the path in several research directions. First, since international assets can be used for consumption smoothing, it will be important to introduce some incomplete asset markets in the model. Second, Given the current E.U. banking crisis, it may be interesting to embed banking sector liquidity issues. Third, it will be interesting to expand the number of state of nature. [more]

\(^9\) Indeed, \( \delta^f \) increases with \( E_sU_s^f(T_s^*, w^*) - E_sU_s^f(0, w_0^f) \) and decreases with \( U_G^f(0, w^*, \tau) - U_G^f(T_G^*, w^*) \); see appendix B.
8 References


Appendix A

In the case of two negatively correlated shocks we successively have

\[
\frac{E_s U^c_s(T^*_s)}{E_s U^f_s(0)} = \mathbb{E}[b_1^{1+\psi}] \left(\frac{w^f(0)}{w^*}\right)^{1+\psi} = \left(\frac{E_s b_1^{1+\psi}}{E_s b_G} \frac{E_s b_G}{E_s B 1+\psi}\right)^{1+\psi}
\]

where \(\varepsilon_G = \varepsilon_B^{-1} = \left(\frac{b_G}{b_B}\right)^{\frac{1}{\sigma}}\) while \(B_G = (b_G/2 + \varepsilon_G^{1-\sigma} b_B/2)^{\frac{1}{1-\sigma}}\) and \(B_B = (b_B/2 + \varepsilon_B^{1-\sigma} b_G/2)^{\frac{1}{1-\sigma}}\). Also, \(b_G = z^{1-\sigma} / \left[\frac{1}{2} (z^{1-\sigma} + 1)\right]\), \(b_B = 1/\left[\frac{1}{2} (z^{1-\sigma} + 1)\right]\) so that \(E_s b_s = 1\). Note firstly that those expressions do not depend on the parameters \(\theta\) and \(\mu\). Secondly, when \(z \to 0\), we have \(b_G = 2\), \(b_B = 0\) and \(\varepsilon_G = \varepsilon_B^{-1} = 0\) so that \(\lim_{z \to 0} \varepsilon_B^{-1} b_G = \infty\) and \(E_s U^c_s(T^*_s)/E_s U^f_s(0) = 2 (2^{1+\psi} + 1)\), which is larger than one because \(\gamma > 1\). Finally, for \(z\) close to 1 we can approximate by keeping the first order terms in \((z-1)\) so that

\[
\frac{E_s U^c_s(T^*_s)}{E_s U^f_s(0)} \approx 1 - \frac{1}{8} \frac{(\psi + 1) (\gamma - 1)}{(\gamma + \psi)} [\gamma - \sigma (\sigma \psi - 1)] (z-1)^2
\]

So, the currency area yields a higher welfare (less negative expected utility) if and only if \(E_s U^c_s(T^*_s)/E_s U^f_s(0) < 1 \iff \gamma > \sigma (\sigma \psi - 1)\).
Appendix B

Rewriting the equality (21) shows that full risk-sharing is sustainable for discount factors above the critical level \( \bar{\delta} = 1 / (1 - \lambda) \) where

\[
\bar{\delta} = \frac{E_s U_s(T_s^*, w*) - E_s U_s(0, w_0)}{U_G(T_G^*, w*) - U_G(0, w*)}
\]

(25)

When \( \bar{\lambda} \) is close to zero we will be allowed to approximate \( \bar{\delta} \simeq 1 + \bar{\lambda} \).

Common currency

Under a common currency, this yields

\[
\bar{\lambda}_c = \frac{E_s U_c(T_c^*, w*) - E_s U_c(0, w_0)}{U_G(T_G^*, w*) - U_G(0, w*)}
\]

(26)

where \( E_s U_c(T_c^*, w*) \) is given by (19) and \( E_s U_c(0, w_0) \) can be show to be equal to (19) where the wage is given by (11) where \( T_i = 0 \). For \( \mu \to 1 \), we successively compute the wages \((w^*)^{\gamma + \psi} = \kappa_0 (E_s b_s^{1+\psi}) / (E_s b_s A^{\gamma-1}) \) and \((w_0)^{\gamma + \psi} = \kappa_0 (E_s b_s^{1+\psi}) / (E_s b_s A^{1-\gamma}) \). The expected utilities are equal to

\[
E_s U_c(T_c^*, w*) = -\left( \frac{\rho}{w^*} \right)^{1+\psi} \left( \frac{1}{1 + \psi} \right) E[b_s^{1+\psi}]
\]

\[
E_s U_c(0, w_0) = -\left( \frac{\rho}{w_0} \right)^{1+\psi} \left( \frac{1}{1 + \psi} \right) E[b_s^{1+\psi}]
\]

and the contemporaneous utilities under deviation and cooperation in the good productivity shock are given by

\[
U_c(0, w^*) = V \left[ \xi \rho \frac{b_G}{w^* A} \right] - \left( \frac{\ell_c^{1+\psi}}{1 + \psi} \right) \frac{1}{\theta - 1} \frac{1}{1 + \psi}
\]

and

\[
U_c(T_c^*, w^*) = V \left[ \xi \rho \frac{1}{w^* A} \right] - \left( \frac{\ell_c^{1+\psi}}{1 + \psi} \right)
\]

After simplifications, this gives the analytical value

\[
\bar{\lambda}_c = \left( 1 + \rho \frac{\gamma - 1}{1 + \psi} \right) \frac{1 - (E b_s^{1+\gamma})^{1+\psi}}{1 - b_s^{1+\gamma}}
\]

For small shocks, we compute

\[
\bar{\lambda}_c \simeq \left( 1 + \rho \frac{\gamma - 1}{1 + \psi} \right) \frac{1}{4 \gamma + \psi} (\sigma - 1) (z - 1)
\]

It can readily be shown that the threshold increases in \( z \) and decrease in \( \gamma, \theta, \psi \) and \( \sigma \) for \( z < 1 \).
Flexible exchange rates

Under flexible exchange rates, the threshold (25) is given by

$$
\bar{\lambda}' = \frac{E_t U_t^f (T^*, w^*) - E_t U_t^f (0, w_0^f)}{U_t^G (T^*, w^*) - U_t^G (0, w^*)}
$$

where $EU^f (T^*, w^*)$ is equal to $EU^c (T^*, w^*)$ in (19) and $EU^f (0, w_0^f)$ is given by (20). For $\mu \to 1$, we get the wages $(w^*)^{\gamma+\psi} = \kappa_0 (E_s b_s^{1+\psi}) / A^{\gamma-1}$ and $(w_0^f)^{\gamma+\psi} = \kappa_0 / [A^{\gamma-1} E_s (B_s^0)^{\gamma-1}]$ where $B_s^0 = \left( b_s / 2 + (\varepsilon_s^0)^{1-\sigma} b_r / 2 \right)^{\frac{1}{1-\sigma}}$ and $\varepsilon_s^0 = (b_r / b_s)^{\frac{1}{2}}, s \neq r$. The expected utilities with and without transfers are equal to

$$
E_s U_s^f (T_s^*, w^*) = - \left( \frac{\rho}{w^*} \right)^{1+\psi} \left( \frac{1}{\gamma - 1} \frac{\theta}{1 - 1}\frac{1}{\rho} + \frac{1}{1 + \psi} \right) E [b_s^{1+\psi}]
$$

$$
E_s U_s^f (0, w_0^f) = - \left( \frac{\rho}{w_0^f} \right)^{1+\psi} \left( \frac{1}{\gamma - 1} \frac{\theta}{1 - 1}\frac{1}{\rho} + \frac{1}{1 + \psi} \right)
$$

The instantaneous utility under deviation is given by

$$
U_t^f (0, w^*) = V \left[ \xi \rho - \frac{1}{w^*B_G^0}\right] - \left( \frac{\xi^f G}{1 + \psi} \right)
$$

where we here use $\xi^f G = \rho / \bar{\omega}^*$, $B_G^0 = \left( b_G / 2 + (\varepsilon_G^0)^{1-\sigma} b_B / 2 \right)^{\frac{1}{1-\sigma}}$. The instantaneous utility under cooperation is given by

$$
U_t^f (T_G^*, w^*) = V \left[ \xi \rho - \frac{1}{w^*B_G^0}\right] - \left( \frac{\xi^f G}{1 + \psi} \right)
$$

where we must here use $\xi^f G = \rho b_G / w^*$ and $B_G^1 = 1$ because we here use the exchange rate under consumption sharing, $\varepsilon_G = 1$. So, we compute the following analytical value:

$$
\bar{\lambda}' = \left( 1 + \rho \frac{\gamma - 1}{1 + \psi} \frac{\theta - 1}{\theta} \right) \frac{1 - \left( E_s (B_s^0)^{\gamma-1}\right) \frac{1+\psi}{1+\psi} \left( E_s b_s^{1+\psi}\right) \frac{1+\psi}{1+\psi}}{1 - (B_G^0)^{\gamma-1} - \frac{\gamma-1}{1+\psi} \rho \frac{\theta - 1}{\theta} - (1 - b_G^{1+\psi}) (E_s b_s^{1+\psi})^{-1}}
$$

For $z$ close to 1 we keep the first order terms to approximate this function as

$$
\bar{\lambda}' \simeq \frac{1}{4} \left( 1 + \rho \frac{\gamma - 1}{1 + \psi} \frac{\theta - 1}{\theta} \right) \rho \frac{1 + \psi}{\gamma + \psi} \frac{\theta}{(\theta - 1)\sigma + 1} \left[ \gamma - \sigma (\sigma \psi - 1) \right] (z - 1)
$$

which increases in $z$ if and only if $\gamma > \sigma (\sigma \psi - 1)$. If this condition is not satisfied, $\bar{\lambda}'$ falls with larger $z$ and $\bar{\delta}'$ falls from above 1 to 1. In this case, $\bar{\delta}' > 1$. Flexible exchange rates never sustain transfers equalizing consumption. By contrast, if $\gamma > \sigma (\sigma \psi - 1)$, $\bar{\delta}'$ increases in $z$ up to 1 when $z$ is close to 1. Then, flexible exchange rates sustain transfers equalizing consumption. Moreover, flexible exchange rates are less likely to sustain transfers equalizing consumption than currency areas. Indeed, since
$\lambda^c$ and $\lambda^f$ are negative for $z$ close to 1, we successively get $\delta^c < \delta^f \iff \lambda^c < \lambda^f \iff \lambda^c/\lambda^f > 1$ or equivalently

$$\gamma \sigma \frac{(\theta - 1) \sigma + 1}{\theta [\gamma - \sigma (\sigma \psi - 1)]} > 1$$

This condition is satisfied because the LHS has a positive denominator and it is a decreasing function of $\gamma$ that reaches a value larger than one at $\gamma \to \infty$.

**Appendix C**

Rewriting the inequality (22) shows that transfers are unsustainable if the discount factors below a critical level $\delta$ where $\delta = 1/(1 - \lambda)$ where

$$\Delta = \lim_{T_0 \to 0} \frac{E_s U_s(T_s, w) - E_s U_s(0, w_0)}{U_G(T_G, w) - U_G(0, w)}$$

(27)

where $w$ is the wage set under the expectation of the small transfers $T_s \in \{T_G, T_B\}$. For $T_s \to 0$, both numerator and denominator are nil so that we apply the L’Hopital’s rule.

**Common currency**

Under a common currency, we must compute the value of

$$\Delta^c = \lim_{T_s \to 0} \frac{E_s U_s(T_s, w^c) - E_s U_s(0, w_0^c)}{U_G(T_G, w^c) - U_G(0, w^c)}$$

(28)

where $w^c$ is the wage evaluate with the transfers $T_s$ and $w_0^c$ is here the wage $w^c$ evaluated at zero transfers. We again focus on $\mu \to 1$. We write

$$U_s^c(T_s, w^c, \ell_s) = V \left[ \xi \rho \frac{w^c \ell_s / \rho + T_s}{w^c A} \right] - \frac{(\ell_s)^{1+\psi}}{1+\psi}$$

where we make explicit the labor supply $\ell_s$ for the sake of clarity. Labor supply is so as $\ell_s(w) = \rho b_s / w$ . The term in the expectation of the numerator in (28) becomes

$$\Phi_s \equiv \lim_{T_s \to 0} \frac{\partial U_s^c}{\partial T_s} + w_0^c \lim_{T_s \to 0} \frac{\partial U_s^c}{\partial w} \frac{d \ln w^c}{d T_s} + \lim_{T_s \to 0} \frac{\partial U_s^c}{\partial \ell_s} \frac{\partial \ln \ell_s}{d \ln w} \frac{d \ln w^c}{d T_s}$$

which must be evaluated at $(T_s, w, \ell_s) = (0, w_0^c, \ell_s(w_0^c))$. We evaluate

$$\lim_{T_s \to 0} \frac{\partial U_s^c}{\partial T_s} = \frac{\rho}{w_0^c \ell_s} \left( \xi \frac{\ell_s}{A} \right)^{1-\gamma}$$

$$\lim_{T_s \to 0} \frac{\partial U_s^c}{\partial w} = -\xi \rho \frac{T_s}{(w_0^c)^2 A} V' \left[ \xi \frac{\ell_s}{A} \right]$$

and

$$\lim_{T_s \to 0} \frac{d \ln \ell_s}{d \ln w} = -1$$

while

$$\lim_{T_s \to 0} \frac{d \ln w^c}{d T_s} = \frac{-\gamma}{\gamma + \psi} \frac{1}{b_G (b_G / b_B)^{\gamma - 1} + 1}$$

and

$$\lim_{T_s \to 0} \frac{\partial U_s^c}{\partial \ell_s} = \frac{1}{\ell_s} \left( \xi \frac{\ell_s}{A} \right)^{1-\gamma} \left[ 1 - \frac{\theta - 1}{\theta} b_{\psi+\gamma} b_s E_r b_{r-\gamma}^\psi \right]$$
Similarly, the denominator in (28) becomes $\Psi_G \equiv \lim_{T \to 0} \frac{d}{dT} \left[ U_G^f (T_G, w^c, \ell_G^c (w^c)) - U_G^c (0, w^c, \ell_G^c (w^c)) \right] = \frac{\partial U_G^c}{\partial T}$, which must be evaluated at $(T_G, w, \ell_G) \to (0, w^c_0, \ell_G^c (w^c_0))$. After simplifications, we successively get

$$\lambda^c = \frac{1}{2} \Phi_G - \frac{1}{2} \Phi_B \equiv \frac{1}{2} \left( 1 - \left( \frac{b_T}{b_B} \right)^{\gamma} \right) \left( \frac{\psi}{\gamma + \psi} + \frac{\gamma}{\gamma + \psi} \rho - 1 \right)$$

**Flexible exchange rates**

Under flexible exchange rates, we must evaluate

$$\lambda^f = \lim_{T \to 0} \frac{\frac{d}{dT} \left[ U^f_s (T_s, -U^f_s (0, w^f_0)) \right]}{\frac{d}{dT} \left[ U_G^f (T_G, w^f) - U_G^f (0, w^f) \right]}$$

where $w^f$ is the wage with small transfers $T_s$ and $w^f_0$ is the wage $w^f$ evaluated at $T_s = 0$. We again focus on $\mu \to 1$. In the two state model, this gives

$$U_s (T_s, w^f, P_s, \ell_s) = V \left[ \frac{w^f \ell_f / \rho + T_s}{P_s} \right] - \left( \frac{\ell_f}{2} \right)^{1+\gamma}$$

where $P_s$ is a function of wage and exchange rate, $P_s (w^f, \varepsilon_s (T_s)) = (w^f / \rho) \left[ (a_s)^{1-\sigma} + (\varepsilon_s a_s)^{1-\sigma} \right]^{1-\sigma}$, while the exchange rate $\varepsilon_s$ is a function of the transfers

$$\varepsilon_s (T_s) = \left( \frac{b_r 1 - T_s}{b_s + 1 + T_s} \right)^{\frac{1}{\sigma}}, \quad r \neq s \in \{ G, B \}$$

We define $\Phi^0_s = P_s \left( w^f_0, \varepsilon_s (0) \right)$. Note that the labor supply $\ell_f$ is a function of transfer and wage: $\ell_f^s (T_s, w)$. The term in the expectation of the numerator in (29) is explicitly written as

$$\Phi_s \equiv \lim_{T_s \to 0} \frac{d}{dT_s} \left[ U_s (T_s, w^f, P_s (w^f, \varepsilon_s (T_s)), \ell_s^f (T_s, w^f)) - U_s \left( 0, w^f_0, P_s (w^f_0, \varepsilon_s (0)), \ell_s^f \left( 0, w^f_0 \right) \right) \right]$$

which must be evaluated at $(T_s, w^f, P_s, \ell_f^s) = (0, w^f_0, P^0_s, \ell_f^s (0, w^f_0))$. This gives

$$\Phi_s = \lim_{T_s \to 0} \frac{\partial U_s}{\partial T_s} + \lim_{T_s \to 0} \frac{\partial U_s}{\partial w} \frac{d \ln w}{dT_s} + \lim_{T_s \to 0} \frac{\partial U_s}{\partial P_s} \frac{d P^0_s}{dT_s} \left( \frac{\partial \ln P_s}{\partial \ln w} \frac{d \ln w}{dT_s} + \frac{\partial \ln P_s}{\partial \ln w} \frac{d \ln w}{dT_s} \right)$$

where

$$\lim_{T_s \to 0} \frac{\partial U_s}{\partial T_s} = \left( \frac{1}{P_s} \right)^{1-\gamma}, \quad \lim_{T_s \to 0} \frac{\partial U_s}{\partial w} = \left( \frac{1}{w^f_0} \right) \left( \frac{1}{P_s^0} \right)^{1-\gamma}, \quad \text{and} \quad \lim_{T_s \to 0} \frac{\partial U_s}{\partial P_s} = \left( -\frac{1}{P_s} \right) \left( \frac{1}{P_s^0} \right)^{1-\gamma}$$

and

$$\lim_{T_s \to 0} \frac{\partial \ln \varepsilon_s}{\partial T_s} = -\frac{2}{\sigma}, \quad \lim_{T_s \to 0} \frac{\partial \ln P_s}{\partial \ln w} = 1, \quad \lim_{T_s \to 0} \frac{\partial \ln P_s}{\partial \ln \varepsilon_s} = \frac{b_r^2}{b_r^2 + b_s^2}, \quad \lim_{T_s \to 0} \frac{d \ln \ell_s^f}{dT_s} = \lim_{T_s \to 0} \frac{d \ln \ell_s^f}{dT_s} = -1$$

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while
\[
\lim_{T_s \to 0} \frac{\partial U_s}{\partial \ell^f_s} = \frac{1}{\ell_s} \left[ 1 - \frac{\theta - 1}{\theta} \rho \frac{E_r [(B_r(0))^{\gamma - 1}]}{(B_s(0))^{\gamma - 1}} \right] \left( \frac{1}{P^f_s} \right)^{1-\gamma}
\]

The denominator in (29) is explicitly written as
\[
\Psi_G \equiv \lim_{T_s \to 0} \frac{d}{dT_G} \left[ U_G \left( T_G, w^f, P_G \left( w^f, \varepsilon_G (T_G) \right), \ell^f_G (T_G, w^f) \right) - U_s \left( 0, w^f, P_G \left( w^f, \varepsilon_G (0) \right), \ell^f_s \left( 0, w^f \right) \right) \right]
\]
which must be evaluated at \((T_s, w^f, P_s, \ell^f_s) = (0, w^f_0, P_s^0, \ell^f_s(0, w^f_0))\). This gives
\[
\Psi_G = \lim_{T_s \to 0} \frac{\partial U_G}{\partial T_G} + \lim_{T_s \to 0} \frac{\partial U_G}{\partial \ln P_G} \left( \frac{\partial \ln P_G}{\partial \ln \varepsilon_G} + \frac{\partial \ln \varepsilon_G}{\partial T_G} \right) + \lim_{T_s \to 0} \frac{\partial U_s}{\partial \ln \ell_s} \frac{d \ln \ell_s}{d T_s} \tag{30}
\]
So, using the above values we get
\[
\Lambda^f = \left[ -\left( \frac{1}{P^G_s} \right)^{1-\gamma} \left\{ 1 + \frac{2}{\sigma} \frac{b^2_s}{b^2_s + b^2_B} \right\} - \left[ 1 - \frac{\theta - 1}{\theta} \rho \frac{E_r [(B_r(0))^{\gamma - 1}]}{(B_s(0))^{\gamma - 1}} \right] \left( \frac{d \ln w^f}{d T_G} + 1 \right) \right] \\
\left[ 1 + \frac{2}{\sigma} \frac{b^2_s}{b^2_s + b^2_B} \right] - \left[ 1 - \frac{\theta - 1}{\theta} \rho \frac{E_r [(B_r(0))^{\gamma - 1}]}{(B_B(0))^{\gamma - 1}} \right] \left( \frac{d \ln w^f}{d T_G} + 1 \right) \]
\[
- \left( \frac{1}{P^G_s} \right)^{1-\gamma} \left[ \frac{2}{\sigma} \frac{b^2_s}{b^2_s + b^2_B} + \frac{\theta - 1}{\theta} \rho \frac{E_r [(B_r(0))^{\gamma - 1}]}{(B_B(0))^{\gamma - 1}} \right]
\]

The effect of transfers on wages can be computed as
\[
(\gamma + \psi) \frac{d \ln w^f}{d T_G} = 1 + \frac{2(\gamma - 1)}{\sigma} \left( 1 - \left( \frac{b_G}{b_B} \right)^{z - 1} \right) \frac{1}{\left( 1 + \left( \frac{b_G}{b_B} \right)^{z - 1} \right) \left( 1 + \left( \frac{b_G}{b_B} \right)^{\frac{\gamma - 1}{\sigma}} \right)}
\]
This is nil for \(z = 1 \iff b_G = b_B\).

Finally, for small shocks we get
\[
\Lambda^f \approx \frac{\gamma \theta \rho}{(\theta - 1) \sigma + 1} (z - 1) - 2 \frac{\sigma + \theta - 1}{1 + \sigma (\theta - 1)} \frac{d \ln w^f}{d T_G}
\]
where
\[
\frac{d \ln w^f}{d T_G} \approx \frac{(\gamma - 1)(\gamma + \sigma)(\sigma - 1)}{2(\gamma + \psi) \sigma^2} (z - 1)
\]
So, we get
\[
\Lambda^f \approx (z - 1)(\sigma - 1) \frac{\gamma^2(\theta - 1)(\sigma - 1) + \gamma \theta \sigma \psi + (\theta + \sigma - 1)(\sigma + \gamma - \sigma \gamma)}{2\sigma^2(\gamma + \psi)((\theta - 1)\sigma + 1)}
\]
Appendix D

In the presence of transaction costs, prices of exported goods are multiplied by $\tau$ so that $p_{HF} = \varepsilon \tau p_F$ and $p_{FH} = (1/\varepsilon) \tau p_H$ where the exchange rates between $H$ and $F$ is still denoted by $\varepsilon$. The price indices depend on transaction costs as $P_{H}^{1-\sigma} = p_{H}^{1-\sigma} + (\varepsilon \tau p_{F})^{1-\sigma}$ and $P_{F}^{1-\sigma} = p_{F}^{1-\sigma} + (\tau p_{H}/\varepsilon)^{1-\sigma}$. Prices are again markups on costs: $p_i = a_i w_i / \rho$. In the flexible exchange rate regime, the money supply equalizes the money demand so that $m_0 = (1 - \mu)(Y_i + T_i + M_i^0)$. The labor supply is equal the labor demand by domestic firms: $\ell_i = a_i (d_{ii} + \tau d_{ji})$. The exchange rate can then be computed as the solution of the equation

$$
\frac{1 - T_H}{1 - T_F} = \frac{b_H}{b_F} \frac{1 + \tau^{1-\sigma} Z(\varepsilon, \tau)}{\tau^{1-\sigma} + \varepsilon Z(\varepsilon, \tau)}
$$

where

$$Z(\varepsilon, \tau) \equiv \left( \frac{P_H}{\varepsilon P_F} \right)^{1-\sigma} = \frac{b_H + b_F \tau^{1-\sigma} \varepsilon^{1-\sigma}}{b_F \varepsilon^{1-\sigma} + b_H \tau^{1-\sigma}}$$

For $\tau = 1$, this yields $Z(\varepsilon, 1) = 1$ and the exchange rate resumes to (15). It can be shown that $\lim_{\tau \to 1, b_H/b_F \to 1} d\varepsilon/d\tau = 0$. As in Corsetti [put reference??], transaction costs create a home bias so that the terms of trade, $P_H/P_F = \varepsilon (Z(\varepsilon, \tau))^{1-\sigma}$, differs from the nominal exchange rates $\varepsilon$. 
Supplementary Document
This supplementary document offers an extensive mathematical presentation of the Appendices.

Appendix A
In the case of two negatively correlated shocks we successively have

\[
\frac{E_s U^c_s(T^*_s)}{E_s U^f_s(0)} = E \left[ b_s^{1+\psi} \right] \frac{w^f(0)}{w^*}^{1+\psi} = \left( E_s b_s^{1+\psi} \right)^{\frac{\gamma-1}{\gamma+\psi}} \left( \frac{E_s b_s}{E_s B_s^{\gamma-1}} \right)^{\frac{\gamma+\psi}{\gamma+\psi}}
\]

where

\[ b_G = \frac{z^{1-\sigma}}{2(z^{1-\sigma}+1)}, \quad b_B = \frac{1}{2(z^{1-\sigma}+1)} \quad \text{and} \quad \varepsilon_G = \varepsilon_B^{\gamma-1} = \left( \frac{b_B}{b_G} \right)^{\frac{1}{\gamma}} = z^{\frac{\sigma-1}{\gamma}} \]

and

\[ B_G = \left( b_G/2 + \varepsilon_G^{-1} b_B/2 \right)^{\frac{1}{\gamma}} \quad \text{and} \quad B_B = \left( b_B/2 + \varepsilon_B^{-1} b_G/2 \right)^{\frac{1}{\gamma}} \]

Note that \( E_s b_s = 1 \). Those expressions do not depend on the parameters \( \theta \) and \( \mu \). When \( z \to 0 \), we have \( b_G = 2, b_B = 0 \) and \( \varepsilon_G = \varepsilon_B = 1 \), so that \( \lim_{z \to 0} \varepsilon_B^{-1} B_G = \infty \) and \( E_s U^c_s(T^*_s)/EU^f_s(0) = 2^{(\gamma+1)\frac{\gamma-1}{\gamma+\psi}} \), which is larger than one because \( \gamma > 1 \).

When \( z \to 1 \), we have \( b_G = b_B = \varepsilon_G = \varepsilon_B^{-1} = 1 \), so that \( EU^c_s(T^*_s)/EU^f_s(0) = 1 \). For \( z \) close to 1, we can approximate

\[
\left( \frac{E_s b_s}{E_s B_s^{\gamma-1}} \right)^{\frac{\gamma+\psi}{\gamma+\psi}} \simeq 1 - \frac{1}{8} \frac{(\psi+1)}{\gamma+\psi} \rho^2 (\gamma-1) (\sigma+\gamma) (z-1)^2 \tag{31}
\]

Also, using

\[
(E_s b_s^m)^n = \left( \frac{1}{2} \frac{(z^{(1-\sigma)m} + 1)^n}{(z^{1-\sigma}+1)^m} \simeq 1 + \frac{1}{8} mn (\sigma-1)^2 (m-1) (z-1)^2 \tag{32}
\]

for \( z \) close to 1, we get

\[
(E_s b_s^{1+\psi})^{\frac{\gamma-1}{\gamma+\psi}} \simeq 1 + \frac{1}{8} (1+\psi) \frac{\gamma-1}{\gamma+\psi} (\sigma-1)^2 \psi (z-1)^2 \tag{33}
\]

Multiplying (31) and (33) and keeping highest order terms, we get

\[
\frac{E_s U^c_s(T^*_s)}{E_s U^f_s(0)} \simeq 1 - \frac{1}{8} \rho^2 (\psi+1) (\gamma-1) (\gamma-\sigma (\sigma \psi - 1)) (z-1)^2
\]

So, the currency area yields a higher welfare (less negative expected utility) if and only if

\[
\frac{E_s U^c_s(T^*_s)}{E_s U^f_s(0)} < 1 \iff \gamma > \sigma (\sigma \psi - 1)
\]

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Appendix B

Rewriting the equality (21) shows that full risk-sharing is sustainable for discount factors above the critical level $\delta = 1 / (1 - \lambda)$ where

$$\chi = \frac{E_s U_s(T_s^*, w^*) - E_s U_s(0, w_0)}{U_G(T_G^*, w^*) - U_G(0, w^*)}$$

(34)

Common currency

Under a common currency, this yields

$$\chi^c = \frac{E_s U_s^c(T^*, w^*) - E U_s^c(0, w^c_0)}{U_G^c(T_G^*, w^*) - U_G^c(0, w^c)}$$

(35)

where $EU_s^c(T^*, w^*)$ is given by (19) and $EU_s^c(0, w^c_0)$ can be show to be equal to (19) where the wage is replaced by

$$(w^c_0)^\gamma + \psi = \kappa_0 \frac{E_s b_s^{1+\psi}}{E_s [b_s (b_s + 1 - \mu)]^{-\gamma} A^{\gamma - 1}}$$

So, for $\mu \to 1$, we successively compute the wages

$$(w^*)^{\gamma + \psi} = \kappa_0 \frac{E_s b_s^{1+\psi}}{E_s [b_s A^{\gamma - 1}]}$$

and

$$(w^c_0)^{\gamma + \psi} = \kappa_0 \frac{E_s b_s^{1+\psi}}{E_s [b_s^{1-\gamma} A^{\gamma - 1}]}$$

and

$$(w^* / w^c_0)^{\gamma + \psi} = E_s b_s^{1-\gamma}$$

the expected utilities

$$EU_s^c(T_s^*, w^*) = - \left( \frac{\rho}{w^*} \right)^{1+\psi} \left( \frac{1}{\gamma - 1} \left( \frac{\theta}{1 - \rho} + \frac{1}{1 + \psi} \right) E [b_s^{1+\psi}] \right)$$

$$EU_s^c(0, w^c_0) = - \left( \frac{\rho}{w^c_0} \right)^{1+\psi} \left( \frac{1}{\gamma - 1} \left( \frac{\theta}{1 - \rho} + \frac{1}{1 + \psi} \right) E [b_s^{1+\psi}] \right)$$

and the contemporaneous utilities under deviation and cooperation

$$U_G^c(0, w^*) = V \left[ \xi \rho \frac{b_G}{w^* A} \right] - \left( \ell_G^{c} \right)^{1+\psi}$$

$$U_G^c(T_G^*, w^*) = V \left[ \xi \rho \frac{1}{w^* A} \right] - \left( \ell_G^{c} \right)^{1+\psi}$$

We get the denominator of (35) equal to

$$U_G^c(T_G^*, w^*) - U_G^c(0, w^*) = \frac{1}{1 - \delta} \left[ \xi \rho \frac{1}{w^* A} \right]^{1-\gamma} - \frac{1}{1 - \gamma} \left[ \xi \rho \frac{b_G}{w^* A} \right]^{1-\gamma}$$

$$= \frac{1}{1 - \gamma} \left( \xi \rho \frac{1}{w^* A} \right)^{1-\gamma} \left( 1 - b_G^{1-\gamma} \right)$$
Under flexible exchange rates, the threshold \((34)\) is given by

\[
EU^c(T^*, w^*) - EU^c(0, w_0^c) = -\left(\frac{1}{\gamma - 1} \frac{\theta}{1 - \rho} + \frac{1}{1 + \psi}\right) E_s [b_s^{1+\psi}] \left[\left(\frac{\rho}{w^*}\right)^{1+\psi} - \left(\frac{\rho}{w_0^c}\right)^{1+\psi}\right]
\]

\[
= -\left(\frac{1}{\gamma - 1} \frac{\theta}{1 - \rho} + \frac{1}{1 + \psi}\right) E_s [b_s^{1+\psi}] \left(\frac{\rho}{w^*}\right)^{1+\psi} \left[1 - \left(\frac{w^*}{w_0^c}\right)^{1+\psi}\right]
\]

\[
= -\left(\frac{1}{\gamma - 1} \frac{\theta}{1 - \rho} + \frac{1}{1 + \psi}\right) E_s [b_s^{1+\psi}] \left(\frac{\rho}{w^*}\right)^{1+\psi} \left[1 - \left(E [b_s^{1-\gamma}]\right)^{1+\psi}\right]
\]

So, we compute

\[
\bar{\lambda}^c = -\left(\frac{1}{\gamma - 1} \frac{\theta}{1 - \rho} + \frac{1}{1 + \psi}\right) E \left[b_s^{1+\psi}\right] \left(\frac{\rho}{w^*}\right)^{1+\psi} \left[1 - \left(E [b_s^{1-\gamma}]\right)^{1+\psi}\right]
\]

\[
= -\left(\frac{1}{\gamma - 1} \frac{\theta}{1 - \rho} + \frac{1}{1 + \psi}\right) E \left[b_s^{1+\psi}\right] \left(\frac{\rho}{w^*}\right)^{1+\psi} \left[1 - \left(E [b_s^{1-\gamma}]\right)^{1+\psi}\right]
\]

\[
= -\left(\frac{1}{\gamma - 1} \frac{\theta}{1 - \rho} + \frac{1}{1 + \psi}\right) \rho \left[1 - \left(E b_s^{1-\gamma}\right)^{1+\psi}\right]
\]

\[
= -\frac{1}{1 - \gamma} \left(\theta/(\theta - 1)\right) \left[1 - \left(E b_s^{1-\gamma}\right)^{1+\psi}\right]
\]

This gives a threshold equal to

\[
\bar{\lambda}^c = \left(1 + \rho \frac{\gamma - 1}{1 + \psi} \frac{1}{\theta}\right) \frac{1 - \left(E b_s^{1-\gamma}\right)^{1+\psi}}{1 - b_s^{1-\gamma}}
\]

For small shocks, we compute

\[
\bar{\lambda}^c \simeq \left(1 + \rho \frac{\gamma - 1}{1 + \psi} \frac{1}{\theta}\right) \frac{1 + \psi}{4} \frac{1}{\gamma + \psi} (\sigma - 1) (z - 1)
\]

It can readily be shown that the threshold increases in \(z\) and decrease in \(\gamma, \theta, \psi\) and \(\sigma\) for \(z < 1\).

**Flexible exchange rates**

Under flexible exchange rates, the threshold \((34)\) is given by

\[
\bar{\lambda}' = \frac{E_s U_s^f(T^*, w^*) - E_s U_s^f(0, w_0^f)}{U_s^f(T_G^*, w^*) - U_s^f(0, w^*)}
\]

where \(EU^f(T^*, w^*)\) is given by \(EU^c(T^*, w^*)\) in \((19)\) and \(EU^f(0, w_0^f)\) is given by \((20)\). For \(\mu \to 1\), we get the wages

\[
(w^*)^{\gamma+\psi} = \kappa_0 \frac{E_s b_s^{1+\psi}}{E_s b_s A^{\gamma-1}} = \kappa_0 \frac{E_s b_s^{1+\psi}}{A^{\gamma-1}}
\]

\[
(w_0^f)^{\gamma+\psi} = \kappa_0 \frac{1}{E_s (B_0^s)^{\gamma-1} A^{\gamma-1}} = \kappa_0 \frac{1}{A^{\gamma-1} E_s (B_0^s)^{\gamma-1}}
\]

\[
(w^*)^{\gamma+\psi} = \frac{E_s b_s^{1+\psi} E_s (B_0^s)^{\gamma-1} A^{\gamma-1}}{E_s b_s A^{\gamma-1}} = E_s b_s^{1+\psi} E_s (B_0^s)^{\gamma-1}
\]

\[
(w_0^f)^{\gamma+\psi} = \frac{1}{E_s (B_0^s)^{\gamma-1} A^{\gamma-1}} = \frac{1}{A^{\gamma-1} E_s (B_0^s)^{\gamma-1}}
\]
where $B^0_y = (b_y/2 + \varepsilon_s^{-1-\sigma} b_y/2)^{1-\sigma}$ and $\varepsilon_s = \left( \frac{b_y}{b_r} \right)^{\frac{1}{\sigma}}$, $s \neq r$.

The expected utilities with and without transfers are equal to

\[
E_s U_s^f(T_s^*, w^*) = -\left( \frac{\rho}{w_s^*} \right)^{1+\psi} \left( \frac{1}{\gamma - 1} \frac{\theta}{1 + \rho} \frac{\gamma}{1 + \psi} \right) E \left[ b_s^{1+\psi} \right]
\]

\[
E_s U_s^f(0, w_0^*) = -\left( \frac{\rho}{w_0^*} \right)^{1+\psi} \left( \frac{1}{\gamma - 1} \frac{\theta}{1 + \rho} \frac{1}{1 + \psi} \right)
\]

while the instantaneous utility under deviation is given by

\[
U_G^f(0, w^*) = V \left[ \xi \rho \frac{1}{w^* B_G A} \right] - \left( \frac{\ell_G^0}{1+\psi} \right)
\]

where we here use $\ell_G^0 = \rho/\tilde{w}^*$, $B_G^0 = (b_G/2 + \varepsilon_G^{-1-\sigma} b_G/2)^{1-\sigma}$ and $\varepsilon_G = \left( \frac{b_G}{b_r} \right)^{\frac{1}{\sigma}}$. The instantaneous utility under cooperation is given by

\[
U_G^f(T_s^*, w^*) = V \left[ \xi \rho \frac{1}{w^* B_G^* A} \right] - \left( \frac{\ell_G^*}{1+\psi} \right)
\]

where we must here use $\ell_G^* = \rho b_G/ w^*$ and $B_G^* = (b_G/2 + \varepsilon_G^{-1-\sigma} b_G/2)^{1-\sigma}$ where $\varepsilon_G = 1$ is the exchange rate under consumption sharing. This implies that $B_G^* = (b_G/2 + b_G/2)^{1-\sigma} = 1$. So, at $\mu \to 1$, we compute

\[
\bar{\lambda}^f = \left( 1 + \rho \frac{\gamma - 1}{1 + \psi - 1} \right) \frac{1 - \left( E_s (B_s^0)^{\gamma-1} \right)^{\frac{1+\psi}{1+\psi}} \left( E_s b_s^{1+\psi} \right)^{\frac{1}{1+\psi}}}{\left( 1 - (B_G^0)^{\gamma-1} \right)^{-\frac{\gamma-1}{1+\psi}} \rho^\frac{\gamma-1}{1+\psi} \left( 1 - B_G^{1+\psi} \right) \left( E_s b_s^{1+\psi} \right)}
\]

For $z = 1$, this tends to zero. For $z$ close to 1 we can keep the first order terms to approximate this function as

\[
\bar{\lambda}^f \approx \frac{1}{4} \left( 1 + \rho \frac{\gamma - 1}{1 + \psi - 1} \right) \frac{1 + \psi}{\gamma + \psi (\theta - 1) \sigma + 1} \left[ \gamma - \sigma (\sigma \psi - 1) \right] (z - 1)
\]

which increases in $z$ if and only if $\gamma > \sigma (\sigma \psi - 1)$. If this condition is not satisfied, $\bar{\lambda}^f$ falls with larger $z$ and $\bar{\delta}^f$ falls from above 1 to 1. In this case, $\bar{\delta}^f > 1$. Flexible exchange rates never sustain transfers equalizing consumption. By contrast, if $\gamma > \sigma (\sigma \psi - 1)$, $\bar{\delta}^f$ increases in $z$ up to 1 when $z$ is close to 1. Then, flexible exchange rates sustain transfers equalizing consumption. Moreover, flexible exchange rates are less likely to sustain transfers equalizing consumption than currency areas. Indeed, since $\bar{\lambda}^e$ and $\bar{\lambda}^f$ are negative for $z$ close to 1, we successively get $\bar{\delta}^e < \bar{\delta}^f \iff \bar{\lambda}^e < \bar{\lambda}^f \iff \bar{\lambda}^e / \bar{\lambda}^f > 1$ or equivalently

$$
\frac{\gamma \sigma - 1}{\theta - 1} \sigma + 1 \left[ \gamma - \sigma (\sigma \psi - 1) \right] > 1
$$

This condition is satisfied because the LHS has a positive denominator and it is a decreasing function of $\gamma$ that reaches a value larger than one at $\gamma \to \infty$. 

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Appendix C

Rewriting the inequality (22) shows that transfers are unsustainable if the discount factors below a critical level $\delta$ where $\delta = 1/(1 - \lambda)$ where

$$\lambda = \lim_{T_s \to 0} \frac{E_s U_s(T_s, w) - E_s U_s(0, w_0)}{U_G(T_G, w) - U_G(0, w)}$$

(36)

where $w$ is the wage set under the expectation of the small transfers $T_s \in \{T_G, T_B\}$.

**Common currency**

Under a common currency, we must compute the value of

$$\lambda^c = \lim_{T_s \to 0} \frac{E_s U_s^c(T_s, w^c) - E_s U_s^c(0, w_0^c)}{U_G^c(T_G, w^c) - U_G^c(0, w^c)}$$

where $w^c$ is the wage evaluate with the transfers $T_s$ and $w_0^c$ is here the wage $w^c$ evaluated at zero transfers. At $T_s \to 0$, both numerator and denominator are nil so that we apply the L’Hopital’s rule such that

$$\lambda^c = \lim_{T_s \to 0} \frac{E_s \frac{d}{dT_s} [U_s^c(T_s, w^c) - U_s^c(0, w_0^c)]}{\lim_{T_s \to 0} \frac{d}{dT_G} [U_G^c(T_G, w^c) - U_G^c(0, w^c)]}$$

(37)

We again focus on $\mu \to 1$. We write

$$U_s^c(T_s, w^c, \ell_s) = V \left[ \xi \rho \frac{w^c \ell_s / \rho + T_s}{w^c A} \right] - \frac{(\ell_s)^{1+\psi}}{1 + \psi}$$

where we make explicit the labor supply $\ell_s$ for the sake of clarity. Labor supply is so as $\ell^c_s(w) = \rho b_s / w$. The term in the expectation of the numerator in (37) becomes

$$\Phi_s \equiv \lim_{T_s \to 0} \frac{d}{dT_s} [U_s^c(T_s, w^c, \ell^c_s(w^c)) - U_s^c(0, w_0^c, \ell^c_s(w_0^c))]$$

$$= \lim_{T_s \to 0} \left[ \frac{\partial U_s^c}{\partial T_s} + \frac{\partial U_s^c}{\partial w} \frac{dw^c}{dT_s} + \frac{\partial U_s^c}{\partial \ell_s} \frac{d\ell_s}{dT_s} \frac{d\ell^c_s}{dw^c} \right]$$

$$= \lim_{T_s \to 0} \frac{\partial U_s^c}{\partial T_s} + w_0^c \lim_{T_s \to 0} \frac{\partial U_s^c}{\partial w} \frac{d\ln w^c}{dT_s} + \lim_{T_s \to 0} \frac{\partial U_s^c}{\partial \ell_s} \frac{d\ell^c_s}{dw^c} \frac{d\ln w^c}{dT_s}$$

(38)

which must be evaluated at $(T_s, w, \ell^c_s) \to (0, w_0^c, \ell^c_s(w_0^c))$. Since

$$\lim_{T_s \to 0} \frac{\partial U_s^c}{\partial T_s} = \xi \rho \frac{1}{w_0^c A} V' \left[ \frac{\ell^c_s}{A} \right] = \frac{\rho}{w_0^c \ell^c_s} \left( \frac{\ell^c_s}{A} \right)^{1-\gamma}$$

(39)

$$\lim_{T_s \to 0} \frac{\partial U_s^c}{\partial w} = -\xi \rho \frac{T_s}{(w_0^c)^2} \frac{1}{A} V' \left[ \frac{\ell^c_s}{A} \right]$$

(40)

the second term in the bracket of (38) is nil at $T_s \to 0$. Also, we compute

$$\lim_{T_s \to 0} \frac{d\ln \ell^c_s}{d\ln w} = -1$$

and

$$\lim_{T_G \to 0} \frac{d\ln w^c}{dT_G} = -\frac{\gamma}{\gamma + \psi} b_G (b_G/b_B)^{\gamma - 1} + 1$$

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\[
\lim_{T_s \to 0} \frac{\partial U_c^s}{\partial \ell^s_c} = \xi \frac{1}{A} V' \left[ \frac{\ell^s_c}{A} \right] - \ell^s_c
\]

\[
= \frac{1}{\ell^s_c} \left( \xi \frac{\ell^s_c}{A} \right)^{1-\gamma} - \ell^s_c + \psi
\]

\[
= \frac{1}{\ell^s_c} \left( \xi \frac{\ell^s_c}{A} \right)^{1-\gamma} \left[ 1 - \ell^s_c \left( \frac{\xi}{A} \right)^{\gamma-1} \right]
\]

\[
= \frac{1}{\ell^s_c} \left( \xi \frac{\ell^s_c}{A} \right)^{1-\gamma} \left[ 1 - \left( \frac{\rho b^c_w}{w^0_c} \right)^{\psi+\gamma} \left( \frac{\xi}{A} \right)^{\gamma-1} \right]
\]

\[
= \frac{1}{\ell^s_c} \left( \xi \frac{\ell^s_c}{A} \right)^{1-\gamma} \left[ 1 - \frac{\rho - 1}{\theta} \frac{b^c_w b^{1-\gamma}}{E_r b^{1+\psi}} \right]
\]

where we successively used the definitions and values of \( V, \ell^c \) and \( w^c_0 \). So, we get

\[
\Phi_s = \left( \frac{\xi \rho b^s_w}{A w^0_c} \right)^{1-\gamma} \left\{ \frac{1}{b^s_w} + \left[ 1 - \frac{\theta - 1}{\theta} b^{\psi+\gamma} \frac{E_r b^{1-\gamma}}{E_r b^{1+\psi}} \right] \left( \frac{b_G/b_B}{b_G/b_B} \right)^{\gamma-1} - 1 \right\} d_T G
\]

where \( d_T G / d_T s = 1 \) if \( s = G \) and \(-1\) otherwise.

Because \( d_T G = -d_T B < 0 \), the numerator in (37) is equal to

\[
-\frac{1}{2} \Phi_G + \frac{1}{2} \Phi_B
\]

Similarly, the denominator in (37) becomes

\[
\Psi_G \equiv \lim_{T_g \to 0} \frac{d}{d T_G} [U^{c}_G(T_G, w^c, \ell^c_G(w^c)) - U^{c}_G(0, w^c, \ell^c_G(w^c))] = \frac{\partial U^{c}_G}{\partial T_G}
\]

which must be evaluated at \( (T_G, w, \ell^c_G) \to (0, w^c, \ell^c_G(w^c)) \), which itself tends to \((0, w^c_0, \ell^c_G(w^c_0))\). Using (39)

\[
\Psi_G = \frac{1}{b_G} \left( \frac{\xi \rho b_G}{A w^c_0} \right)^{1-\gamma}
\]
Therefore, we successively get

\[ \Lambda^c = \frac{1}{2} \Phi_G - \frac{1}{2} \Phi_B \]

\[ \frac{\Phi_G}{2} - \frac{\Phi_B}{2} \]

\[ \frac{1}{2} b_G \left\{ \frac{1}{b_G} + \left[ 1 - \frac{\theta - 1}{\theta} b_{G}^{\psi + \gamma} E_r b_r^{1-\gamma} \right] \frac{\gamma}{\gamma + \psi} \frac{1}{b_G} \left( b_G/b_B \right)^{\gamma - 1} - 1 \right\} \]

\[ - \frac{1}{2} b_B \left( \frac{b_B}{b_G} \right)^{1-\gamma} \left\{ \frac{1}{b_B} - \left[ 1 - \frac{\theta - 1}{\theta} b_{G}^{\psi + \gamma} E_r b_r^{1-\gamma} \right] \frac{\gamma}{\gamma + \psi} \frac{1}{b_G} \left( b_G/b_B \right)^{\gamma - 1} - 1 \right\} \]

\[ \frac{1}{2} \left( \left( 1 - \frac{b_B}{b_G} \right)^{\gamma} + 2 \left( 1 - \frac{\theta - 1}{\theta} \right) \frac{\gamma}{\gamma + \psi} \left( b_G/b_B \right)^{\gamma - 1} + 1 \right) \]

\[ \frac{1}{2} \left( \frac{b_G}{b_B} \right) \left( 1 - \frac{\theta - 1}{\theta} \right) \frac{\gamma}{\gamma + \psi} \]

\[ \frac{1}{2} \left( 1 - \frac{b_B}{b_G} \right) \left( \frac{\psi}{\gamma + \psi} + \frac{\gamma}{\gamma + \psi} \frac{\theta - 1}{\theta} \right) \]

**Flexible exchange rates**

Under flexible exchange rates, a similar argument yields

\[ \Lambda^f = \frac{\lim_{T_s \to 0} E_s \frac{d}{dT_s} \left[ U_s(T_s, -U_s^f(0,w^f_0)) \right]}{\lim_{T_G \to 0} \frac{d}{dT_G} \left[ U_G(T_G,w^f) - U_G^f(0,w^f) \right]} \]  \( \text{(41)} \)

where \( w^f \) is the wage with small transfers \( T_s \) and \( w^f_0 \) is here the wage \( w^f \) evaluated at \( T_s = 0 \).

We need to use the results of section 3 again. We use the utility function

\[ U_i = V \left[ \frac{w^f \ell_i / \rho + T_i + \frac{1}{\mu} \mu^0}{P_i} \right] - \frac{\ell_i^{1+\psi}}{1+\psi} \]

We again focus on \( \mu \to 1 \). In the two state model, this gives

\[ U_s(T_s, w^f, P_s, \ell_s) = V \left[ \frac{w^f \ell_s / \rho + T_s}{P_s} \right] - \frac{\ell_s^{1+\psi}}{1+\psi} \]

where \( P_s \) is a function of wage and exchange rate, \( P_s(w^f, \varepsilon_s(T_s)) = (w^f / \rho) \left[ (a_s)^{1-\sigma} + (\varepsilon_s a_r)^{1-\sigma} \right]^{1-\sigma} \), while the exchange rate \( \varepsilon_s \) is a function of the transfers

\[ \varepsilon_s(T_s) = \left( b_r \frac{1 - T_s}{b_s 1 + T_s} \right)^{\frac{1}{2}}, \quad r \neq s \in \{G, B\} \]
Note that $\ell_s^f$ is the labor supply $\ell_s^f(T_s, w)$. We define $P_s^0 = P_s\left(w_0^f, \varepsilon_s(0)\right)$.

The term in the expectation of the numerator in (41) is explicitly written as

$$
\Phi_s \equiv \lim_{T_s \to 0} \frac{d}{dT_s} \left[ U_s(T_s, w^f, P_s(\varepsilon_s(T_s)), \ell_s^f(T_s, w^f)) - U_s(0, w_0^f, P_s(w_0^f, \varepsilon_s(0)), \ell_s^f(0, w_0^f)) \right]
$$

and gives

$$
\Phi_s = \lim_{T_s \to 0} \left[ \frac{\partial U_s}{\partial T_s} + \frac{\partial U_s}{\partial w} \frac{d w^f}{dT_s} + \frac{\partial U_s}{\partial P_s} \left( \frac{\partial P^T_s}{\partial w} \frac{d w^f}{dT_s} + \frac{\partial P_s}{\partial \varepsilon_s} \frac{d \varepsilon_s}{dT_s} \right) + \frac{\partial U_s}{\partial \ell_s} \left( \frac{d \ell_s^f}{dT_s} + \frac{d \ell_s^f}{dw} \frac{d w^f}{dT_s} \right) \right]
$$

which must be evaluated at $(T_s, w^f, P_s, \ell_s^f) \to (0, w_0^f, P_s^0, \ell_s^f(0, w_0^f))$. We can write

$$
\Phi_s = \lim_{T_s \to 0} \left[ \frac{\partial U_s}{\partial T_s} + \frac{\partial U_s}{\partial w} \frac{d w^f}{dT_s} + \frac{\partial U_s}{\partial P_s} \left( \frac{\partial P^T_s}{\partial w} \frac{d w^f}{dT_s} + \frac{\partial P_s}{\partial \varepsilon_s} \frac{d \varepsilon_s}{dT_s} \right) + \frac{\partial U_s}{\partial \ell_s} \left( \frac{d \ell_s^f}{dT_s} + \frac{d \ell_s^f}{dw} \frac{d w^f}{dT_s} \right) \right]
$$

where

$$
\lim_{T_s \to 0} \frac{\partial U_s}{\partial T_s} = \frac{1}{P_s^0} V' \left[ \frac{w_0^f \ell_s^f / \rho}{P_s^0} \right] = \left( \frac{1}{w_0^f \ell_s^f / \rho} \right) \left( \frac{w_0^f \ell_s^f / \rho}{P_s^0} \right)^{1-\gamma} = \left( \frac{1}{P_s^0} \right)^{1-\gamma}
$$

$$
\lim_{T_s \to 0} \frac{\partial U_s}{\partial w} = \frac{\ell_s^f / \rho}{P_s^0} V' \left[ \frac{w_0^f \ell_s^f / \rho}{P_s^0} \right] = \left( \frac{1}{w_0^f} \right) \left( \frac{1}{P_s^0} \right)^{1-\gamma}
$$

$$
\lim_{T_s \to 0} \frac{\partial U_s}{\partial P_s} = -\frac{w_0^f \ell_s / \rho}{P_s^0} V' \left[ \frac{w_0^f \ell_s / \rho}{P_s^0} \right] = \left( -\frac{1}{P_s^0} \right) \left( \frac{1}{P_s^0} \right)^{1-\gamma}
$$

and

$$
\lim_{T_s \to 0} \frac{\partial \ln \varepsilon_s}{\partial T_s} = -\frac{2}{\sigma}
$$

$$
\lim_{T_s \to 0} \frac{\partial \ln P_s}{\partial \ln w} = 1
$$

$$
\lim_{T_s \to 0} \frac{\partial \ln P_s}{\partial \ln \varepsilon_s} = \frac{\varepsilon_s a_r}{a_1^{1-\sigma} + (\varepsilon_s a_r)^{1-\sigma}} = \frac{(\varepsilon_s)^{1-\sigma} b_r}{b_s + (\varepsilon_s)^{1-\sigma} b_r} = \frac{b_r^1}{b_r^1 + b_s^1}
$$

$$
\lim_{T_s \to 0} \frac{d \ln \ell_s}{d \ln w} = -1
$$

$$
\lim_{T_s \to 0} \frac{d \ln \ell_s}{d T_s} = -1
$$

Also using the definition of $V$, gathering terms, replacing $\ell_s$ by $\rho/w_0^f$, replacing $P_s^0$ by $w_0^f/\rho$
\[\left[(a_s)^{-\sigma} + (\varepsilon_s a_r)^{-\sigma}\right]^{-1}} = (w_0^f/\rho)AB_s(0)\] and substituting for \(w_0^f\), we successively get

\[
\lim_{T_s \to 0} \frac{\partial U_s}{\partial \ell_s^c} = \frac{w_0^f/\rho}{P_0^s} V' \left[ \frac{w_0^f \ell_s/\rho}{P_0^s} \right] - \ell_s^0
\]

\[
= \frac{1}{\ell_s} \left[ \left( \frac{w_0^f \ell_s/\rho}{P_0^s} \right)^{1-\gamma} - \ell_s^{\psi+1} \right]
\]

\[
= \frac{1}{\ell_s} \left[ \left( \frac{w_0^f \ell_s/\rho}{P_0^s} \right)^{1-\gamma} \right] \left[ 1 - \ell_s^{\psi+1} \left( \frac{w_0^f \ell_s/\rho}{P_0^s} \right)^{\gamma-1} \right]
\]

\[
= \frac{1}{\ell_s} \left[ 1 - \ell_s^{\psi+1} \left( \frac{1}{P_0^s} \right)^{\gamma-1} \right] \left( \frac{1}{P_0^s} \right)^{1-\gamma}
\]

\[
= \frac{1}{\ell_s} \left[ 1 - \left( \frac{\rho}{w_0^f} \right)^{\psi+1} \left( \frac{1}{P_0^s} \right)^{\gamma-1} \right] \left( \frac{1}{P_0^s} \right)^{1-\gamma}
\]

\[
= \frac{1}{\ell_s} \left[ 1 - \left( \frac{w_0^f}{\rho} \right)^{-\psi-\gamma} \rho^{\psi+1} \left( \frac{\rho}{AB_s(0)} \right)^{\gamma-1} \right] \left( \frac{1}{P_0^s} \right)^{1-\gamma}
\]

\[
= \frac{1}{\ell_s} \left[ 1 - \frac{\theta - 1}{\theta} E_r [(B_s(0))^{\gamma-1}] \right] \left( \frac{1}{P_0^s} \right)^{1-\gamma}
\]

(52)

Note that the effect through wages has no first order effect in (43) because the terms in \(d \ln w^f/dT_s\) cancel out. One can indeed check that

\[
\frac{\partial U_s}{\partial w} w_0^f + \frac{\partial U_s}{\partial \ell_s} P_0^s \frac{\partial \ln P_s}{\partial \ln w} = 0
\]

and

\[
\lim_{T_s \to 0} \frac{d \ln \ell_s^f}{d \ln w} \frac{d \ln w^f}{dT_s} + \frac{d \ln \ell_s^f}{dT_s} = - \left( \frac{d \ln w^f}{dT_G} + 1 \right)
\]

Expression (43) becomes

\[
\Phi_s = \lim_{T_s \to 0} \frac{\partial U_s}{\partial T_s} + \lim_{T_s \to 0} \frac{\partial U_s}{\partial P_s} P_0^s \frac{\partial \ln P_s}{\partial \ln \varepsilon_s} + \lim_{T_s \to 0} \frac{\partial U_s}{\partial \ell_s} \ell_s^f \left( \frac{d \ln \ell_s^f}{d \ln w} \frac{d \ln w^f}{dT_s} + \frac{d \ln \ell_s^f}{dT_s} \right)
\]

\[
= \left( \frac{1}{P_0^s} \right)^{1-\gamma} \left\{ 1 + \frac{2}{\sigma} \frac{b_r^2}{b_r^2 + b_s^2} \left[ 1 - \frac{\theta - 1}{\theta} E_r [(B_s(0))^{\gamma-1}] \right] \left( \frac{d \ln w^f}{dT_G} + 1 \right) \right\}
\]

(53)

In this expression \(dT_G/dT_s = 1\) for \(s = G\) and \(-1\) for \(s = B\). So for small \(T_G < 0 < T_B\), the numerator in (41) is equal to

\[-\frac{1}{2} \Phi_G + \frac{1}{2} \Phi_B\]

The denominator in (41) is explicitly written as

\[
\Psi_G = \lim_{T_G \to 0} \frac{1}{dT_G} \left[ U_G \left( T_G, w_f, P_G \left( w_f^f, \varepsilon_G (T_G) \right), \ell_s^f (T_G, w_f) \right) - U_s \left( 0, w_f, P_G \left( w_f^f, \varepsilon_G (0) \right), \ell_s^f (0, w_f) \right) \right]
\]
and gives
\[ \Psi_G = \lim_{T_G \to 0} \left[ \frac{\partial U_G}{\partial T_G} + \frac{\partial U_G}{\partial T_s} \left( \frac{\partial \ln P_G}{\partial \ln \varepsilon_G} + \frac{\partial U_s}{\partial \ln \ell_s} \right) \right] \]
which must be evaluated at \((T_s, w^f, P_s, \ell^f_s) \to (0, w^f_0, P^0_s, \ell^f_s(0, w^f_0))\). So, using (44), (46), (47) and (49) this gives
\[ \Psi_G = \left( \frac{1}{P^0_G} \right)^{1-\gamma} \left[ \frac{2 \gamma \frac{b_B^1}{b_B^2 + b_G^1}}{\theta - 1} \rho \left( \frac{E_r[(B_r(0))^{\gamma-1}]}{(B_G(0))^{\gamma-1}} \right) \frac{\ln w^f}{dT_G} + 1 \right] \]
For small \(T_G < 0 < T_B\), the denominator in (41) is equal to
\[-\Psi_G\]
For small \(T_G < 0 < T_B\), the ratio in (41) therefore is equal to
\[ \lambda^f = \left\{ \left( \frac{1}{P^0_G} \right)^{1-\gamma} \left[ 1 + \frac{2 \gamma \frac{b_B^1}{b_B^2 + b_G^1}}{\theta - 1} \rho \left( \frac{E_r[(B_r(0))^{\gamma-1}]}{(B_G(0))^{\gamma-1}} \right) \frac{\ln w^f}{dT_G} + 1 \right] \right\} \]
For small shocks we get
\[ \lambda^f \simeq \frac{\gamma \theta \rho}{(\theta - 1) \sigma + 1} (z - 1) - 2 \frac{\sigma + \theta - 1}{1 + \sigma (\theta - 1)} \frac{\ln w^f}{dT_G} \]
We still need to compute the effect of transfers on wages: \(d \ln w^f/dT_G\). Towards this aim, we use
\[ (w^f)^{\gamma + \psi} = \kappa_0 \frac{\frac{1}{2} (1 - T_G)^{1+\psi} + \frac{1}{2} (1 + T_G)^{1+\psi}}{\frac{1}{2} (1 - T_G) B_G^{\gamma-1} A^{\gamma-1} + \frac{1}{2} (1 + T_G) B_B^{\gamma-1} A^{\gamma-1}} \]
where \(B_G\) and \(B_B\) are the functions
\[ B_G(T_G) = \left( b_G/2 + \frac{b_B (1 - T_G)}{b_G + T_G} \right)^{\frac{1-\sigma}{\sigma}} b_B/2 \]
\[ B_B(-T_G) = \left( b_B/2 + \frac{b_B (1 + T_G)}{b_B - T_G} \right)^{\frac{1-\sigma}{\sigma}} b_G/2 \]
Note that
\[ \lim_{T_G \to 0} \frac{B_G}{B_B} = \left( \frac{b_B}{b_G} \right)^{\frac{1}{\sigma}} \]
Also we successively compute

\[ \lim_{T_G \to 0} \frac{d \ln B_G}{dT_G} = \frac{1}{(1 - \sigma)} \frac{1}{b_G/2 \left( \frac{b_B}{b_G} \right)^{\frac{1-\sigma}{\sigma}}} \frac{d}{dT_G} \left( \frac{b_G/2 + \left( \frac{b_B}{b_G} \frac{1 - T_G}{1 + T_G} \right)^{\frac{1-\sigma}{\sigma}} b_B}{2} \right) \]

\[ = \frac{1}{(1 - \sigma)} \frac{1}{1 + \left( \frac{b_G}{b_B} \right)^{\frac{1}{\sigma}}} \lim_{T_G \to 0} \frac{d}{dT_G} \left( \frac{1 - T_G}{1 + T_G} \right)^{\frac{1-\sigma}{\sigma}} \]

\[ = -\frac{2}{\sigma} \frac{1}{1 + \left( \frac{b_G}{b_B} \right)^{\frac{1}{\sigma}}} \]

and, by a similar argument,

\[ \lim_{T_G \to 0} \frac{d \ln B_B}{dT_G} = \frac{2}{\sigma} \frac{1}{1 + \left( \frac{b_G}{b_B} \right)^{\frac{1}{\sigma}}} \]

We successively have that

\[ (\gamma + \psi) \frac{d \ln w^f}{dT_G} = -\frac{d}{dT_G} \left( (1 - T_G) B_G^{\gamma-1} + (1 + T_G) B_B^{\gamma-1} \right) \]

\[ = -\frac{-B_G^{\gamma-1} + B_B^{\gamma-1}}{B_G^{\gamma-1} + B_B^{\gamma-1}} - (\gamma - 1) \frac{B_G^{\gamma-1} \frac{d \ln B_G}{dT_G} + B_B^{\gamma-1} \frac{d \ln B_B}{dT_G}}{B_G^{\gamma-1} + B_B^{\gamma-1}} \]

\[ = \frac{1}{1 + \left( \frac{b_G}{b_B} \right)^{\frac{2-\sigma}{\sigma}}} \left[ 1 - \left( \frac{b_G}{b_B} \right)^{\frac{2-\sigma}{\sigma}} + (\gamma - 1) \frac{2}{\sigma} \left( 1 - \frac{b_G}{b_B} \right)^{\frac{2-\sigma}{\sigma}} \right] \]

This is nil for \( z = 1 \iff b_G = b_B \). For \( z \) close to 1, this yields

\[ \frac{d \ln w^f}{dT_G} \approx \frac{(\gamma - 1) (\gamma + \sigma) (\sigma - 1)}{2 (\gamma + \psi) \sigma^2} (z - 1) \]

So we get

\[ \lambda^f \approx (z - 1)(\sigma - 1) \frac{\gamma^2(\theta - 1)(\sigma - 1) + \gamma \theta \sigma \psi + (\theta + \sigma - 1)(\sigma + \gamma - \sigma \gamma)}{2 \sigma^2 (\gamma + \psi)((\theta - 1)\sigma + 1)} \]