Abstract

Several European countries have recently envisaged to implement fiscal policies that constitute alternatives to monetary devaluation in the context of a monetary union. Social value-added tax is one of these alternatives: it consists to shift fiscal revenue from payroll tax to value-added tax, with the objective to address simultaneously competitiveness and employment problems. We analyze the consequence of such a policy in a model of international trade with heterogeneous firm à la Melitz. We depart from the CES case for taking account of the way changes in the tax rates may modify competition between producers, their margins, and the way these changes are reflected in prices. We first show that social VAT is neutral for zero trade balances. Then, in a two-country model, we show that, after the introduction of the social VAT, intensive and extensive margins increase in the net importing country regardless of the country that implements the policy. Both margins decrease in the net exporting country. Considering non-CES utility functions, the effects of social VAT are attenuated (amplified) if love for variety increases (decreases) with quantities.

Keywords: fiscal devaluation, social VAT, tax reform, international trade.

JEL Classifications: H20, F12.
1 Introduction

Several European countries, especially France, face with huge fiscal deficit prospects and high unemployment. In the long run, growing pension spendings and health expenditures due to ageing will make the government budget constraint more severe. In France, social spendings are mainly financed through social contributions that bear on wages, and can be harmful for employment and production. These factors have increased interest in revenue neutral tax shifts that consist to cut social contributions and increase fiscal revenue from other sources. It is in this context that the social value-added tax has been introduced and, after the presidential elections, removed by the French government in 2012. In the Euro zone, Germany in 2007 is a previous example. Social VAT is a revenue neutral or revenue enhancing shift from payroll tax to value-added tax, with the objective to address simultaneously competitiveness and employment problems. The present paper is an attempt to analyze the effect of social VAT on the structure of international trade and welfare.

Social VAT constitutes an alternative to monetary devaluation, especially in the context of a monetary union where changes in the nominal exchange rate cannot be an option. It has also similarities with another policy advocated by Keynes (1931) at the time of the gold standard, that consists in the introduction of a uniform ad valorem tariff on imports combined with a uniform subsidy on all exports. Both are generally called fiscal devaluations (Calmfors (1998)). Farhi et al. (2011) analyze the equivalence with monetary devaluation.\footnote{see also Correia (2011), de Mooij and Keen (2012) and Lipinska and von Thadden (2012).} All these instruments are presented as likely to promote competitiveness.

With social VAT, lowering payroll tax should lead to higher labor demand, net wages and aggregate supply and lower consumer prices. Higher VAT rate should increase consumer prices and reduce aggregate demand, labor demand and net wages. Neutrality of the social VAT means that the respective effects on each variable of changes in social contribution and value-added tax vanish. In a closed economy, such neutrality is often advocated in the long-run even if there may be some capital-labor substitution (see for instance Gauthier (2009), for France). In this paper, we leave aside this issue and focus on the consequences of social VAT that go through international trade.

In open economy, social VAT mimics monetary devaluation. The VAT base consists of total value of domestic consumption including imports, while payroll tax base is the wage bill. The difference between both bases results in a competitiveness effect (relative rise of imported products, and gain on export competitiveness). Central issues in the analysis are (i) how the fall in production costs is reflected in the producer prices, and (ii) how the increase in the VAT rate is reflected in consumer prices. Consider the case of a price elasticity that decreases with demand (usually considered as the most likely case). The producer prices do not fall as much as the cost, as well as firms do not fully pass on the VAT increases in the margin.\footnote{We can refer for instance to Peltzman (2000) for the US, Carbonnier (2007), Carbonnier (2008) for France, or Andrade et al. (2010) for cross-border pass-through.} Monopolistic competition here is an interesting framework in order to take account of the way foreign companies may react to social VAT by reducing their margins, and domestic firms
mays react by increasing their own. Margin rates depend on the price elasticity of demand. Nevertheless, in the CES case, changes in the payroll tax and value-added tax would be fully reflected in producer and consumer prices.

We depart from this case in order to analyze change in competition. We consider a model à la Melitz (2003) with firm heterogeneity. Following Zhelobodko et al. (2012), we assume non-CES utility function and introduce the concept of love for variety.\(^3\) In this framework, price elasticity of demand depends on the quantity consumed. We also follow Melitz (2003) by assuming that entry requires a sunk cost. Once the entry cost is paid, firms observe their marginal cost and then decide to produce or not.

We first consider the case of a zero trade balance and show that social VAT has no real effect. Bases of VAT and payroll tax are exactly the same, so that the social VAT does not create any competitiveness effect. We only observe changes in prices: labor cost in other countries adjust to labor cost in the country which has introduced social VAT. Consumer prices are not modified in the latter country, but decrease in other countries proportionally to the fall in labor cost. Social VAT thus results in deflation with no effects on quantities and welfare.

If trade balances are different from zero, base effect may appear. Let us consider the two-country case; one has a negative trade balance, while the other has a trade surplus. The net importing country consumes more than it produces, and then has a VAT base larger than the payroll tax base. This allows for better competitiveness in the net importing country. In the CES case, whatever the country that introduced the social tax, the net importing country sees its labor cost, which is also labor income, lowers in smaller proportions as the net exporting country. The relative increase in purchasing power in the net importing country allows an increase in domestic consumption goods and imported goods. In contrast, consumption declines in the net exporting country. From the point of view of the net importer, the volume of imports increases while the volume of exports declines. Simultaneously, we observe that export prices increase while import prices fall, reflecting an improvement in the terms of trade for the net importing country.

If we leave the CES case, changes in quantities alter the conditions of competition and how changes in taxes are reflected in the price. If love for variety increases with quantity,\(^4\) higher consumption by individuals in the net importing country leads to lower competition. Then, changes in taxes are less reflected in prices. The positive effect of fiscal devaluation is therefore attenuated for the net importing country. For the net exporter, decreases in the volume of consumption result in more competition, which reduces the negative effects of the social VAT. If love for variety decreases with quantity, these effects are reversed.

The Melitz’s framework with heterogeneous firms allows to analyze the effect of social VAT

\(^3\)See Zhelobodko et al. (2011) for a more extensive presentation of the case with heterogeneous firms. In their seminal paper, Dixit and Stiglitz (1977) have already allowed for variable elasticity of substitution and endogenous markup. More recently, Melitz and Ottaviano (2008) and Simonovska (2010) consider a linear demand system that also allows for endogenous markups. Melitz and Ottaviano (2008) use a model with horizontal product differentiation developed by Ottaviano et al. (2002).

\(^4\)This corresponds to decreasing price elasticity of demand. The absolute value of the price elasticity is proportional to the inverse of the love for variety.
on the structure of active firms. We show that extensive and intensive margins change in the same way. In the CES case, social VAT leads to entry of low productivity firms in the net importing country and an exit of less productive firms in the net exporting country. Beyond the CES case, these effects are mitigated if the elasticity of substitution is decreasing with quantities. Finally, we analyze the consequences of social VAT on welfare. To do so, we analyze the consequence on the number of firms that pay the fixed cost for learning their productivity. The more numerous firms are, the greater the number of varieties and the higher welfare. We analyze how variations in competition can modify the number of firms.

The rest of the paper is organized as follows. Section 2 is devoted to the presentation of the multi-countries model with firm heterogeneity. We analyze the consequences of social VAT in Section 3.

2 Firms heterogeneity and multi-countries model

There are $n$ countries that produce goods using only labor. In country $j$, $L_j$ workers supply inelastically $E_j$ efficiency units of labor.

2.1 Consumers

Consumers in country $j$ maximize utility derived from a continuum of horizontally differentiated goods provided by domestic and foreign monopolistically competitive firms. If a consumer in country $j$ consumes $x_{kj}(\omega)$ units of each variety $\omega$ produced in country $k$, for all varieties in the set $\Omega_{kj}$, she gets a utility $U_j$,

$$U_j = \sum_{k=1}^{n} \int_{\Omega_{kj}} u_j(x_{kj}(\omega))d\omega$$

where $u_j$ is a twice continuously differentiable function, strictly increasing and strictly concave.

Wage per efficient labor unit in country $j$ is denoted by $w_j$. Given a consumer price function $p_{kj}(\omega)$, the budget constraint of the consumer writes

$$\sum_{k=1}^{n} \int_{\Omega_{kj}} p_{kj}(\omega)x_{kj}(\omega)d\omega + S_j = w_jE_j + T_j$$  \hspace{1cm} (1)

where $T_j$ is a fiscal transfer to the consumer: positive is subsidy, negative if lump sum tax. $S_j$ represents the net savings of the agents: $S_j = A^t_j - (1 + \rho^{t-1})A^{t-1}_j$ where $A^t_j$ is the asset at the end of period $t$ and $\rho^t$ the interest rate on assets between period $t$ and $t+1$, defined in period $t$ and paid in period $t+1$. The sets of varieties $\Omega_{kj}$ are determined at equilibrium.

5Models à la Mélitz are not dynamic and neither is the present model, we just consider steady state equilibria when $A^t_j$ is constant. The motive of savings are supposed separables from the a parameters of the model (old age, bequest, prestige...) and the level of assets is given as a parameter and not determined by the model.
Let $\lambda_j$ be the Lagrange multiplier of the budget constraint. First-order conditions with respect to $x_{kj}(\omega)$ allow to define the inverse demand functions

$$p_{kj}(\omega) = \frac{u'_j(x_{kj}(\omega))}{\lambda_j}, \quad \omega \in \Omega_{kj}$$

(2)

### 2.2 Producers

The firms are distinguished by their marginal cost, denoted by $c$, drawn in a potentially country-dependent distribution function $\gamma_k(c)$ ($k = 1, \ldots, n$). In addition, if a firm from country $k$ exports to country $j$, it must pay a fixed cost $f_{kj} > 0$. For a firm with marginal cost $c$ in country $k$, the cost of producing $x$ units of a good and selling them in country $j$ is

$$C_{kj}(x; c) = (cx + f_{kj}) w_k$$

where $c$ and $f_{kj}$ are expressed in efficient labor unit of country $k$.

Firms are price setters; they maximize profits taking the inverse demand function into account. The functional form of the demand for variety $\omega$ from country $k$ is the same across consumers of country $j$. The market demand is then $L_j x_{kj}(\omega)$. Profit function of a firm with marginal cost $c$ in country $k$ is

$$\Pi_{kj}(x; \lambda_j, c) = (1 - t_j) \frac{u''_j(x)}{\lambda_j} L_j x - (1 - \tau_k) w_k (cL_j x + f_{kj})$$

where $t_j$ and $\tau_k$ are respectively the VAT rate in country $j$ and the labor subsidy rate in country $k$. One can see the wage rate $w_k$ as including social benefits perceived by a worker minus social contributions, so that an increase in $\tau_k$ is equivalent to a decrease in the social contribution rate.

The first order condition with respect to $x$ is equivalent to

$$u''_j(x) x + u'_j(x) = \lambda_j \frac{(1 - \tau_k) w_k}{1 - t_j} c$$

We assume sufficient conditions for existence and uniqueness of a positive solution in $x$, to hold:

(i) Existence : $\lim_{x \to 0} [u''_j(x) x + u'_j(x)] = \infty$, and $\lim_{x \to +\infty} [u''_j(x) x + u'_j(x)] \leq 0$.

(ii) Uniqueness : the profit function is strictly concave, i.e. $\frac{-xu''_j(x)}{u'_j(x)} < 2$.

Following Zhelobodko et al. (2012), we define the relative love for variety

$$r_u(x) = -\frac{xu''_j(x)}{u'_j(x)} > 0,$$
and rewrite the first order condition of the producer

\[ u'_j(x)(1 - r_{uj}(x)) = \tilde{\lambda}_{kj}c \]  

(3)

where \( \tilde{\lambda}_{kj} \equiv \frac{(1 - \tau_k)w_k}{1 - t_j} \lambda_j \).

Unlike the CES case where the elasticity of substitution is exogenous and constant, the value of \( r_{uj} \) varies with the consumption level, or equivalently, with the price level and the mass of varieties. This elasticity can be increasing or decreasing with consumption depending on the consumer preferences for more balanced bundles of varieties. For instance, Melitz and Ottaviano (2008) considers linear demands, which corresponds to decreasing elasticity of substitution.

Production per consumer is solution of FOC (3)

\[ \xi^*_j(\tilde{\lambda}_{kj}c) \equiv \arg\max_x \left[ u'_j(x) - \tilde{\lambda}_{kj}c \right] x \]  

(4)

From the strict concavity of the profit function \( \Pi_{kj}(x; \lambda_j, c) \) with respect to \( x \), we deduce that \( \xi^*_j \) is decreasing with respect to \( \tilde{\lambda}_{kj}c \). It is worthwhile to notice that, by contrast to the CES case, the elasticity of substitution is not the same across varieties. Consumers perceive varieties produced with the same unit cost \( c \) as equally substitutable. But, assuming for instance that love for variety increases with consumption, consumers view high-cost varieties as less differentiated than the low-cost varieties.

Finally, consumer price is

\[ p^*_kj(c) = \frac{(1 - \tau_k)w_k}{1 - t_j} \frac{c}{1 - r_{uj}(\xi^*_j(\tilde{\lambda}_{kj}c))} \]  

(5)

Unlike the CES case, the markup \( \left( 1 - r_{uj}(\xi^*_j(\tilde{\lambda}_{kj}c)) \right)^{-1} \) is endogenous and depends on the level of consumption in the same way as love for variety. Assuming increasing (decreasing) love for variety implies that the markup is lower (higher) for high-cost than low-cost varieties.

2.2.1 Cutoff condition

A firm in country \( k \) exports to country \( j \) if and only if profits are non-negative, that is

\[ \Pi_{kj}(\xi^*_j(\tilde{\lambda}_{kj}c); \lambda_j, c) \geq 0 \]

Let \( \Pi^*_j(c, \lambda) \) define the maximum profit per consumer before payment of the indirect tax, expressed in efficient labor unit of country \( k \) \( (\Pi_{kj}/(L_j(1 - \tau_k)w_k)) \),

\[ \Pi^*_j(c, \lambda) \equiv \left( \frac{u'_j(\xi^*_j(\lambda c))}{\lambda} - c \right) \xi^*_j(\lambda c) \]  

(6)

From the envelope theorem, we deduce that \( \Pi^*_j(c, \lambda) \) is decreasing in both arguments.
The non-negative profits condition defines a cutoff \( \tau_{kj} \) on the marginal cost above which firms do not produce. This cutoff satisfies\(^6\)

\[
\Pi_j^\star(\tau_{kj}, \tilde{\lambda}_{kj}) = \frac{f_{kj}}{L_j}
\]  

(7)

remembering \( \tilde{\lambda}_{kj} = \frac{(1-\tau_k)w_k}{1-t_j} \lambda_j \).

Since \( \Pi_j^\star(c, \lambda) \) is decreasing in both arguments, the cutoff condition defines a negative relationship between \( \tau_{kj} \) and \( \tilde{\lambda}_{kj} \):

\[
\tau_{kj} \equiv C_j\left(\tilde{\lambda}_{kj}, \frac{f_{kj}}{L_j}\right)
\]  

(8)

where \( C_j \) is decreasing in both arguments.

### 2.2.2 Free-entry condition

Following Zhelobodko et al. (2012) or Melitz and Ottaviano (2008), we assume that entry in country \( k \) requires a sunk cost \( F_k \), expressed in efficient labor unit of country \( k \). Once the entry cost is paid, firms observe their marginal cost and then decide to produce or not, recalling that a firm which chooses to produce and sells in country \( j \) incurs an additional fixed cost \( f_{kj} \).

Firms enter the market until expected profits net of entry cost become zero\(^7\)

\[
\sum_j \int_0^{\tau_{kj}} \left[ \left( (1-t_j)p^\star_{kj}(c) - (1-\tau_k)w_k c \right) L_j \xi_j(\tilde{\lambda}_{kj}c) - (1-\tau_k)w_k f_{kj} \right] \gamma_k(c)dc = (1-\tau_k)w_k F_k
\]  

(9)

Using (3), (5), (6) and (8), this yields

\[
\sum_j L_j \int_0^{C_j(\tilde{\lambda}_{kj}, f_{kj}/L_j)} \left( \Pi_j^\star(c, \tilde{\lambda}_{kj}) - \frac{f_{kj}}{L_j} \right) \gamma_k(c)dc - F_k = 0
\]  

(10)

Since \( C_{kj} \) and \( \Pi_j^\star \) are decreasing in \( \tilde{\lambda}_{kj} \), it is easy to show that the LHS is decreasing in \( \tilde{\lambda}_{kj} \), and then increasing in \( \tau_k \) and decreasing in \( t_j \) and \( \lambda_j \). Equation (10) implies that expected profits of a firm before entry, as well as aggregate profits, are zero. Consequently, dividends \( D_j \) in the budget constraints of the consumers (1) vanish.

\(^6\)The fixed cost \( f_{kj} \) is a labor cost. It is subsidized at rate \( \tau_k \) and the LHS corresponds to profits divided by \( (1-\tau_k)w_k \) (i.e. profits per efficient labor unit of country \( k \)).

\(^7\)The fixed cost \( F_k \) is a labor cost and is therefore subsidized at rate \( \tau_k \).
2.3 Equilibrium

Let $N_k$ denote the number of firms that enter the market in country $k$. At equilibrium, labor demand and supply must be equal in each country $k$:

$$N_k \left[ F_k + \sum_j \int_0^{\tau_{kj}} \left( cL_j \xi_j^* (\lambda_{kj} c) + f_{kj} \right) \gamma_k(c) dc \right] = L_k E_k$$

(11)

The government budget constraint in country $k$ is:

$$B_k + L_k T_k + \tau_k L_k w_k E_k = t_k L_k \sum_{j=1}^n N_j \int_0^{\tau_{kj}} p_{kj}^*(c) \xi_k^* (\lambda_{jk} c) \gamma_j(c) dc$$

(12)

where $B_k$ corresponds to the government budget balance. We do not consider the possibility for the government to purchase commodities.

Combining the consumer’s budget constraint (1), for a consumer of country $k$,

$$\sum_{j=1}^n N_j \int_0^{\tau_{kj}} p_{kj}^*(c) \xi_k^* (\lambda_{jk} c) \gamma_j(c) dc + S_k = w_k E_k + T_k$$

(13)

with the government budget constraint (12) yields

$$\frac{TB_k}{L_k} + (1 - t_k)(T_k - S_k) + (\tau_k - t_k) w_k E_k = 0$$

(14)

where $TB_k = B_k + L_k S_k$ is the trade balance of country $k$. Indeed, since government budget balance is not used for commodity purchases, the balance of trade for country $k$ is the difference between total firm revenue and total consumer spendings net of indirect taxes (commodity purchases of domestic products by consumers of country $k$ appear in both terms).

Given equations 9 and 11, the total firm revenue is:

$$N_k \sum_{j=1}^n \int_0^{\tau_{kj}} (1 - t_j) p_{kj}^* (c) L_j \xi_j^* (\lambda_{kj} c) \gamma_j(c) dc = (1 - \tau_k) w_k L_k E_k$$

Given equations 12 and 13, the total consumer spendings net of indirect taxes is:

$$L_k \sum_{j=1}^n N_j \int_0^{\tau_{kj}} (1 - t_k) p_{jk}^*(c) \xi_j^* (\lambda_{jk} c) \gamma_j(c) dc = (1 - \tau_k) L_k w_k E_k - L_k S_k - B_k$$

which directly gives the trade balance as the sum of the private net savings and the budget surplus. The sum over $k$ of the balances of trade is equal to zero:

$$\sum_k TB_k = \sum_k B_k + L_k S_k = 0$$

In order to define the international equilibrium, let us introduce some additional notations:
the expected revenue net of indirect tax earned by one firm of country \( k \) from its sales in country \( j \) and expressed in efficient labor unit of country \( k \)

\[
\Xi_{kj} = \frac{1 - t_j}{(1 - \tau_j) w_k} \int_0^{\tau_{kj}} p_{kj}^*(c) L_j \xi_j^*(\lambda_{kj} c) \gamma_k(c) dc
\]

can be written as a function of \( \tilde{\lambda}_{kj} \) and of the fixed cost \( f_{kj} \):

\[
\Xi_{kj}(\lambda, f) \equiv \int_0^{\tau_{kj}} \left( \frac{c L_j \xi_j^* (\lambda c)}{1 - r_{uj} (\xi_j^* (\lambda c))} \right) \gamma_k(c) dc
\] (15)

the expected labor cost of a firm of country \( k \) that sales in country \( j \), and expressed in efficient labor unit of country \( k \), can be written as a function of the same variables:

\[
\chi_{kj}(\lambda, f) \equiv \int_0^{\tau_{kj}} (c L_j \xi_j^* (\lambda c) + f) \gamma_k(c) dc
\] (16)

Notice that expected profit of a firm of country \( k \) that sales in country \( j \) can be rewritten as the difference between \( \Xi_{kj} \) and \( \chi_{kj} \):

\[
\int_0^{\tau_{kj}} \left( L_j \Pi_j^*(c, \lambda) - f \right) \gamma_k(c) dc = \Xi_{kj}(\lambda, f) - \chi_{kj}(\lambda, f)
\] (17)

Importantly, both variables \( \Xi_{kj} \) and \( \chi_{kj} \), as well as expected profit \( \Xi_{kj} - \chi_{kj} \) are decreasing with \( \tilde{\lambda}_{kj} \).

We normalize \( w_1 \) to unity. The \( n - 1 \) remaining wage rates \( (w_j)_{j=2,...,n} \), the \( n \) marginal utilities of income \( (\lambda_j)_{j=1,...,n} \), and the \( n \) numbers of firms \( (N_j)_{j=1,...,n} \) are solutions of a system of \( 3n - 1 \) equations, to be taken among the following \( 3n \) equations:

- Budget constraints of the consumer, for \( k = 1, ..., n \):

\[
\frac{1}{(1 - t_k) L_k} \sum_{j=1}^{n} N_j (1 - \tau_j) w_j \Xi_{jk} \left( \frac{\lambda_k (1 - \tau_j) w_j}{1 - t_k}, f_{jk} \right) = w_k E_k + T_k - S_k
\] (18)

- Labor market equilibria, for \( k = 1, ..., n \):

\[
\frac{L_k E_k}{N_k} = F_k + \sum_{j=1}^{n} \chi_{kj} \left( \frac{\lambda_j (1 - \tau_k) w_k}{1 - t_j}, f_{kj} \right)
\] (19)

\[8\] Since \( \overline{\tau}_j \left( \lambda, \xi_j \right) \) and \( \Pi_j^*(c, \lambda) \) are decreasing functions of their arguments, the result is immediate for expected profits \( \Xi_{kj} - \chi_{kj} \). Remembering that \( \xi_j^* (\lambda c) \) is also decreasing leads to the result for \( \chi_{kj} \). Finally, since

\[
\frac{c \xi_j^* (\lambda c)}{1 - r_{uj} (\xi_j^* (\lambda c))} = \frac{u_j' (\xi_j^* (\lambda c)) \xi_j^* (\lambda c)}{\lambda}
\]
is also decreasing in \( \lambda \) (from the concavity of the profit function in \( x \)), \( \Xi_{kj} \) is also decreasing in \( \lambda \).
• Free-entry conditions, for \( k = 1, \ldots, n \):

\[
\sum_{j=1}^{n} \left[ \Xi_{kj} \left( \frac{\lambda_j (1 - \tau_k) w_k}{1 - t_j}, f_{kj} \right) - \chi_{kj} \left( \frac{\lambda_j (1 - \tau_k) w_k}{1 - t_j}, f_{kj} \right) \right] = F_k
\]  

(20)

If the government commits itself to maintaining a target of budget balance \( B_k \), policy instruments \((\tau_k, t_k, T_k)_{k=1,\ldots,n} \) must satisfy the budget constraint of the government (14).

3 Social VAT

A fiscal devaluation in some country \( k \) consists in an increase in the labor subsidy rate \( \tau_k \), financed through an increase in the VAT rate \( t_k \) in the same country.

Using equation (14), the budget constraint of the consumer rewrites for any \( k = 1, \ldots, n \):

\[
\frac{1}{(1 - t_k)L_k} \sum_{j=1}^{n} N_j (1 - \tau_j) w_j \Xi_{jk} \left( \tilde{\lambda}_{jk}, f_{jk} \right) = \frac{(1 - \tau_k) w_k E_k}{1 - t_k} - \frac{TB_k}{(1 - t_k)L_k}
\]

Then, multiplying the latter equation by \( \lambda_k \) leads to the following 3\( n \) equations:

• Budget constraints of the consumers, for any \( k = 1, \ldots, n \):

\[
\sum_{j=1}^{n} N_j \tilde{\lambda}_{jk} \Xi_{jk} \left( \tilde{\lambda}_{jk}, f_{jk} \right) = \tilde{\lambda}_{kk} \left( L_k E_k - \frac{TB_k}{(1 - \tau_k) w_k} \right)
\]  

(21)

• Labor market equilibria, for any \( k = 1, \ldots, n \):

\[
\frac{L_k E_k}{N_k} = F_k + \sum_{j=1}^{n} \chi_{kj} \left( \tilde{\lambda}_{kj}, f_{kj} \right)
\]  

(22)

• Free-entry conditions, for any \( k = 1, \ldots, n \):

\[
\sum_{j=1}^{n} \left[ \Xi_{kj} \left( \tilde{\lambda}_{kj}, f_{kj} \right) - \chi_{kj} \left( \tilde{\lambda}_{kj}, f_{kj} \right) \right] = F_k
\]  

(23)

We first give conditions for social VAT to have no real effect, and then turn to the cases where it is not neutral.
3.1 Conditions for neutrality

Let us assume that policy instruments \((\tau_k, t_k, T_k)_{k=1,...,n}\) are set in order to balance imports and exports: \(TB_k = 0\) for any \(k\).

Then, unknowns of the preceding system of equations ((21), (22) and (23)) are the \(n^2 + n\) variables: \((\tilde{\lambda}_{kj})_{k,j=1,...,n}\), \((N_k)_{k=1,...,n}\). From the Walras law, one of the above \(3n\) equations is redundant. Moreover, from the definition of \(\tilde{\lambda}_{kj}\), one gets, for any \(k = 2,...,n\):

\[
\frac{\tilde{\lambda}_{k1}}{\lambda_{11}} = \frac{\tilde{\lambda}_{k2}}{\lambda_{12}} = \ldots = \frac{\tilde{\lambda}_{kn}}{\lambda_{1n}} = \frac{(1 - \tau_k)w_k}{(1 - \tau_j)w_j}
\]

that is \((n - 1)^2\) additional equations. Replacing for instance \(\tilde{\lambda}_{kj}\) for any \(k = 2,...,n\) and \(j = 2,...,n\) by \(\frac{\tilde{\lambda}_{k1} \lambda_{1j}}{\lambda_{11}}\), in the above \(3n\) equations, one can then take \(3n - 1\) of these equations and solve for \((\tilde{\lambda}_{k1}, \tilde{\lambda}_{11})_{k,j=1,...,n}\) and \((N_k)_{k=1,...,n}\).

**Consequence.** With zero trade balance in all countries \((TB_k = 0)\), the fiscal devaluation has no effect on quantities. Let \(\tilde{\lambda}_{kj}^*\) and \(N_k^*\) be the equilibrium values when all instruments are zero. After a fiscal devaluation in country 1, these equilibrium values remain unchanged. Then, quantities \(x_{kj}(c)\) and welfare are not modified. Only prices \(p_{kj}^*(c)\) and \(w_k\) will change. Changes in consumer prices and labor costs in countries \(k = 2,...,n\) are proportional to change in the labor cost in country 1. As soon as \(TB_k \neq 0\) in some country \(k\) (in fact, in at least two countries since \(\sum_k TB_k = 0\)), fiscal devaluation may lead to changes in quantities and welfare.

Nevertheless, it is possible to give a more general statement on neutrality. As it is now clear from equations (21), (22) and (23), real effects will come from changes in the ratios \(TB_k/(1 - \tau_k)w_k\), that is the real trade balance, or the sum of real government and consumers budget balances (expressed in unit of labor). A fiscal devaluation such as the social VAT would then have real effect only if it implies some change in the real trade balance \(B_k/(1 - \tau_k)w_k\) for some country \(k\).

3.2 Social VAT in the two-country case

We now turn to a case where real trade balance is not kept constant in the country that implements social VAT. For simplifying calculations, we consider only two countries (1 and 2). Let \(\hat{y}\) denote the proportional change in variable \(y\), that is \(\hat{y} = dy/y\). Differentiating the budget constraint of the government in country \(k\)

\[
TB_k + (1 - t_k)L_k(w_kE_k + T_k - S_k) - (1 - \tau_k)w_kL_kE_k = 0
\]

leads to

\[
d[(1 - t_k)L_k(w_kE_k + T_k - S_k)] = [(1 - \tau_k)w_k] \frac{(1 - \tau_k)w_kL_kE_k - dTB_k}{(1 - \tau_k)w_k}
\]

(24)
The left-hand side represents the variation of aggregate income of country-
consumers after indirect taxation. We differentiate other equilibrium equations:

- From the budget constraints of the consumer (18) (combined with (24)), one gets
  \[ \sum_{j=1}^{2} N_j (1 - \tau_j) w_j \Xi_{jk} \left\{ \hat{\Xi}_{jk} + [(1 - \tau_j) w_j] + \hat{N}_j \right\} = [(1 - \tau_k) w_k] (1 - \tau_k) w_k L_k E_k - dTB_k \] (25)

- From labor market equilibria (19), and the free-entry equations (20), we have
  \[ \sum_{j=1}^{2} \Xi_{kj} \hat{\Xi}_{kj} = \sum_{j=1}^{2} \chi_{kj} \hat{\chi}_{kj} = -\hat{N}_k \sum_{j=1}^{2} \Xi_{kj} \] (26)

For notational convenience, one gets from the differentiation of (15)
\[ \Xi_{kj} \hat{\Xi}_{kj} = G_{kj} \hat{\lambda}_{kj} \]

where
\[ G_{kj} \equiv \hat{\lambda}_{kj} \left[ \frac{\hat{c}_{kj} L_j \xi_j (\hat{\lambda}_{kj} \hat{c}_{kj}) \gamma_j (\hat{c}_{kj})}{1 - r_{kj}} \frac{\partial \hat{c}_{kj}}{\partial \hat{\lambda}_{kj}} + \int_{c_{kj}}^{c_{kj}} \frac{\partial}{\partial \lambda_{kj}} \left( \frac{\gamma_j (\hat{c}_{kj})}{1 - r_{kj}} \right) \right] \] (27)

Additionally, from (17), differentiation of \( \Xi_{kj} - \chi_{kj} \) leads to
\[ \Xi_{kj} \hat{\Xi}_{kj} - \chi_{kj} \hat{\chi}_{kj} = \left[ \int_{c_{kj}}^{c_{kj}} L_j \frac{\partial \Pi_j^\star (c, \hat{\lambda}_{kj})}{\partial \lambda_{kj}} \gamma_j (c) dc \right] \hat{\lambda}_{kj} = -\Xi_{kj} \hat{\lambda}_{kj} \] (28)

where the second equality results from the envelop theorem. Then, one obtains a system of linear equations with 8 unknowns (\( \hat{\lambda}_{kj}, \hat{w}_j \) and \( \hat{N}_j \)) that consists in the following 6 equations:

\[ \sum_{j=1}^{2} N_j (1 - \tau_j) w_j \left\{ G_{jk} \hat{\lambda}_{jk} + \Xi_{jk} [(1 - \tau_j) w_j] + \Xi_{jk} \hat{N}_j \right\} = [(1 - \tau_k) w_k] (1 - \tau_k) w_k L_k E_k - dTB_k, \] for \( k = 1, 2 \) (29)

\[ \sum_{l=1}^{2} G_{jl} \hat{\lambda}_{jl} = -\hat{N}_j \sum_{l=1}^{2} \Xi_{jl}, \] for \( j = 1, 2 \) (30)

\[ \sum_{j=1}^{2} \Xi_{kj} \hat{\lambda}_{kj} = 0, \] for \( k = 1, 2 \) (31)
and two additional equations that result from the definition of \( \tilde{\lambda}_{kj} \):

\[
\hat{\tilde{\lambda}}_{2j} - \hat{\tilde{\lambda}}_{1j} = [(1 - \tau_2)w_2] - [(1 - \tau_1)w_1], \text{ for } j = 1, 2
\]  

(32)

From the Walras law, one of these equations is redundant. We consider the price normalization \( w_1 = 1 \) before and after the reform, so that \( \hat{\tilde{w}}_1 = 0 \).

**Result 1.** For any country \( k \), the sign of \( \hat{\tilde{\lambda}}_{1k} \) and \( \hat{\tilde{\lambda}}_{2k} \) is the same and is opposite to the sign of \( \hat{\tilde{\lambda}}_{1j} \) and \( \hat{\tilde{\lambda}}_{2j} \), for \( j \neq k \).

**Proof.** From (31) and (32), one gets

\[
\Xi_{11}\hat{\tilde{\lambda}}_{11} + \Xi_{12}\hat{\tilde{\lambda}}_{12} = 0, \quad \Xi_{21}\hat{\tilde{\lambda}}_{21} + \Xi_{22}\hat{\tilde{\lambda}}_{22} = 0, \quad \text{and} \quad \hat{\tilde{\lambda}}_{21} - \hat{\tilde{\lambda}}_{11} = \hat{\tilde{\lambda}}_{22} - \hat{\tilde{\lambda}}_{12}
\]

and deduces

\[
\begin{align*}
\left(1 + \frac{\Xi_{21}}{\Xi_{22}}\right)\hat{\tilde{\lambda}}_{21} &= \left(1 + \frac{\Xi_{11}}{\Xi_{12}}\right)\hat{\tilde{\lambda}}_{11} \\
\left(1 + \frac{\Xi_{12}}{\Xi_{11}}\right)\hat{\tilde{\lambda}}_{12} &= \left(1 + \frac{\Xi_{22}}{\Xi_{21}}\right)\hat{\tilde{\lambda}}_{22}
\end{align*}
\]

which lead to the result.

**Result 1** means that any change in the fiscal policy increases intensive and extensive margins in one of the two country, while it decreases them in the other one. This is obtained from the free-entry condition for firms and the definition of \( \tilde{\lambda}_{kj} \).

**Result 2.** Assume a marginal change in the subsidy rate \( \tau_1 \), whereas the other country keeps \( \tau_2 \) constant. Then \( \hat{\tilde{\lambda}}_{11} \) has the same sign as \( dTB_1 - (1 - \tau_1)TB_1 \).

**Proof.** See Appendix.

The case \( dTB_1 - (1 - \tau_1)TB_1 = 0 \) means that the government of country 1 keeps the trade balance constant in real terms: \( d(TB_1/[(1 - \tau_1)w_1]) = 0 \), which is verified for exemple if both budget surplus and net savings keep constant in real terms. Assuming that the country with a negative trade balance introduces social VAT \( (TB_1 < 0) \), increase in labor subsidy is associated with reduction of the value of the government budget deficit and of the trade deficit. In this particular case, there is no real effect of the social VAT as with \( B_1 = 0 \).

Let us now assume \( dB_1 = dS_1 = 0 \) and consider a fiscal devaluation in country 1. In that case, the saving decision of consumers, linked to intertemporal distribution of the consumption, is not impacted by a permanent policy change. The government of country 1 increases simultaneously the labor subsidy rate \( \tau_1 \) and the indirect tax rate \( t_1 \), leaving \( B_1 \) and \( T_1 \) constant:

\[
(1 - t_1)\left(w_1E_1 + T_1\right)\frac{dt_1}{1 - t_1} = (1 - \tau_1)w_1E_1\frac{d\tau_1}{1 - \tau_1}
\]
In country 2, we assume that the labor subsidy rate $\tau_2$ and the indirect tax rate $t_2$ remain unchanged ($d\tau_2 = dt_2 = 0$). Since the sum of trade balance must remain equal to zero, country 2 trade balance $TB_2$ is kept constant. Any change in the wage rate $w_2$ leads to variation of the government transfer $T_2$:

$$(1 - t_2) L_2 dT_2 + (\tau_2 - t_2) L_2 E_2 dw_2 = 0 \quad (33)$$

From Result 2, assuming $dTB_1 = 0$ and $TB_1 < 0$ implies that $\hat{\lambda}_{11}$ and $\hat{\lambda}_{21}$ are negative. Moreover, both $\hat{\lambda}_{12}$ and $\hat{\lambda}_{22}$ are positive. We then deduce two immediate consequences in country 1: (i) demand addressed to each domestic and foreign firm rises, (ii) cutoffs increase, that is domestic and foreign firms with low productivity become active in country 1. The social VAT then leads to increase in intensive and extensive margins in the net importing country. The opposite result holds for the net exporting country.

We summarize these results in the following proposition.

**Proposition 1.** When country $k$ implements a social VAT policy without change in the trade balance $TB_k$, the net importing (exporting) country increases (decreases) its consumptions in domestic and foreign products, in the intensive and in the extensive margins.

In the non-CES case, variations in the intensive margin modify competition between firms. Let us assume that love for variety increases with quantity in both countries. Social VAT then raises love for variety in the net importing country (lower competition between firms and higher margins), and reduces love for variety in the net exporting country (higher competition and lower margins). Changes in tax rates should then be less reflected in consumer prices in the former country, and more reflected in the latter. Such changes in competition attenuate the consequences of social VAT when compared to the CES case.

In the opposite, if we assume that love for variety decreases with quantity in both countries. Social VAT then reduces love for variety in the net importing country (higher competition between firms and lower margins), and raises love for variety in the net exporting country (lower competition and higher margins). Changes in tax rates should then be more reflected in consumer prices in the former country, and less reflected in the latter. Such changes in competition exacerbate the consequences of social VAT when compared to the CES case.

Furthermore, one could think that the objective of the country setting social VAT policy is actually to reduced its trade balance deficit, that is to obtain $dTB_k > 0$. According to result 2 (and its proof in appendix), the gain in terms of trade balance crowd out the gain in terms of consumption in both intensive and extensive margin, making it zero when the trade balance increase exactly compensate the decrease of the real value of the trade balance due to the change of the term of trade. Actually, social VAT - whatever the country setting it - improves the term of trade in the country with negative trade balance and deteriorates it in the country with positive trade balance. This improvement of the term of trade in the net importing country may be use whether to increase its real consumption or to reduce its trade deficit.
Number of firms and welfare.

To derive consequence on welfare, one also needs to analyze the effect of social VAT on the numbers of firms \(N_1\) and \(N_2\). Indeed, utility of an individual in country \(j\) writes

\[
U_j = \sum_{k=1}^{n} N_k \int_{0}^{\bar{c}_{kj}} u_j \left( \xi_{kj}^* \left( \bar{\lambda}_{kj} c \right) \right) \gamma_k(c) dc
\]

But, as stated in the following result, the effect on the number of firms for each level of unit cost does not go necessarily in the same way than in intensive and extensive margins.

**Result 3.** Assuming a marginal change in the subsidy rate \(\tau_1\), whereas the other country keeps \(\tau_2\) constant, relative changes in the number of entries \(N_1\) and \(N_2\) satisfy:

\[
\hat{N}_1 = \left( \frac{G_{12}}{\Xi_{12}} - \frac{G_{11}}{\Xi_{11}} \right) \frac{\Xi_{11} \bar{\lambda}_{11}}{\Xi_{11} + \Xi_{12}}
\]

\[
\hat{N}_2 = \left( \frac{G_{22}}{\Xi_{22}} - \frac{G_{21}}{\Xi_{21}} \right) \frac{\Xi_{21} \bar{\lambda}_{21}}{\Xi_{21} + \Xi_{22}}
\]

**Proof.** See equations (38) and (39) in Appendix.

The ratio \(\frac{G_{kj}}{\Xi_{kj}}\) corresponds to the elasticity of \(\Xi_{kj}\) to \(\bar{\lambda}_{kj}\). In appendix, we show that this elasticity can be rewritten as

\[
\frac{G_{kj}}{\Xi_{kj}} = -1 - \int_{0}^{\bar{c}_{kj}} \frac{c L_j \xi_{kj}^*}{\Xi_{kj}} \left[ r_{uj} \left( \xi_{kj}^* \right) + \frac{\xi_{kj}^* \gamma_j}{1 - r_{uj} \left( \xi_{kj}^* \right)} \right]^{-1} \gamma_k(c) dc - \frac{\bar{c}_{kj} \xi_{kj}^*}{\left[ 1 - r_{uj} \left( \xi_{kj}^* \right) \right] \Xi_{kj}}
\]

with the following notations \(\xi_{kj}^* \equiv \xi_j^* \left( \bar{\lambda}_{kj} c \right)\) and \(\bar{\xi}_{kj}^* \equiv \xi_j^* \left( \bar{\lambda}_{kj} \bar{c}_{kj} \right)\).

To illustrate each of the effects, we consider the following two examples.

**Example 1.** First, let us consider CES utility functions with, for country \(j\), \(u_j(x) = \frac{x^\rho_j}{\rho_j}\. As a result love for variety, as well as firm markups, become constant: \(r_{uj}(x) = 1 - \rho_j\). Since profit maximization implies that \(r_{uj}\) lies between 0 and 1, the parameter \(\rho_j\) has to belong to the same interval and the elasticity of substitution \(\sigma_j = \frac{1}{1-\rho_j}\) is therefore larger that 1. From equation (4), demand writes

\[
\xi_{kj}^* = \left( \frac{\rho_j}{\bar{\lambda}_{kj} c} \right)^{\frac{1}{1-\rho_j}}
\]
We first leave aside the issue of heterogeneous firms by assuming that all firms have the same unit cost $c$:

$$\Xi_{kj} = L_j \xi_{kj} c \left(\xi_{kj}^*\right)^{1-r_{kj}}$$

and $\gamma_k(\tilde{c}_{kj}) = 0$

Then, it is straightforward to show that

$$\frac{G_{kj}}{\Xi_{kj}} = -\frac{1}{1-\rho_j}$$

From (34) and (35), we deduce that $\hat{N}_1$ and $\hat{N}_2$ have the same sign as $(\rho_1 - \rho_2) \hat{\lambda}_{11}$, or equivalently as $(r_{u_2} - r_{u_1}) \hat{\lambda}_{11}$.

If $TB_1$ is negative, social VAT allows all firms to sale more in country 1 (since $\hat{\lambda}_{11} < 0$ and $\hat{\lambda}_{21} < 0$). This results in higher number of firms in both countries only if love for variety is higher in country 1, that is, if there is less competition between firms in country 1. Welfare in country 1, which is the net importing country, then, increases unambiguously. In the net exporting country (country 2), the whole effect of social VAT is ambiguous. This effect of social VAT on the number of firms, that we call competition effect depends on the gap between the price elasticity of demands in countries 1 and 2.

**Example 2.** We now go further with CES utility functions but considering heterogeneous firms, or heterogeneous unit costs. Following Helpman et al. (2004) or Chaney (2008), we consider that productivity of a firm $1/c$ is drawn in a Pareto distribution. Otherwise stated, unit cost is drawn in a distribution over $[0, C]$ with the cumulative distribution function $\Gamma(c) = (c/C)^{m_k}$ and the probability distribution function $\gamma(c) = m_k c^{m_k-1}/C^{m_k}$. Hence functions $\Xi_{kj}$ and $\chi_{kj}$ are integrals from zero to $\tilde{c}_{kj}$ of functions proportional to $c^{m_k-1}/C^{m_k}$. These integrals exist if and only if $m_k > \frac{\rho_j}{1-\rho_j}$. The uniform function is obtained by setting $m_k = 1$, and requires $\rho_j < 1/2$. From (7), (15) and (37), one gets

$$\tilde{c}_{kj} = \rho_j (1-\rho_j) \frac{\rho_j^{1-\rho_j}}{\rho_j} \frac{1-\rho_j}{\rho_j} \left(\frac{L_j}{f_{kj}}\right)^{1-\rho_j}$$

$$\Xi_{kj} = \frac{m_k}{m_k (1-\rho_j) - \rho_j} \left(\frac{\tilde{c}_{kj}}{C}\right)^{m_k}$$

Thus

$$\frac{G_{kj}}{\Xi_{kj}} = -\frac{1}{1-\rho_j} + \frac{1}{1-\rho_j} - \frac{m_k}{\rho_j} = -\frac{m_k}{\rho_j}$$

Intensive margin effect  Extensive margin effect

From (34) and (35), we deduce that $\hat{N}_1$ and $\hat{N}_2$ have the same sign as $(\rho_2 - \rho_1) \hat{\lambda}_{11}$, or equivalently to $(r_{u_1} - r_{u_2}) \hat{\lambda}_{11}$. Surprisingly, the effect of competition is reversed when compared to Example 1. Here, numbers of firms increase when competition is higher in the net importing
country, that is, in the country where individuals consume more after the reform. The competition effect on the intensive margin is dominated by a cutoff effect on the extensive margin. The tax reform leads more firms with low productivity to become active in the net importing country, increasing labor demands in both countries. In the same time, low productivity firms become inactive in the net exporting country, with negative effect on labor demand. The result states that, if the net importing country involves more competition between firms, the net effect on labor demand in both countries is negative, and the two numbers of firms are rising.

We now go back to the general formula (36) of the ratio $G_{k,j}/\Xi_{k,j}$. In the general case with endogenous love for variety, the competition effect and the cutoff effect presented in the above two examples are affected by change in competition between firms. There is no reason to think that both number of firms should vary in the same way, so that the whole consequence of social VAT on welfare in both countries remains ambiguous.

## 4 Numerical analysis

Previous results may be highlighted through numerical analyses. First of all, a choice has to be made concerning the functional form of the utility function. This choice is particularly meaningful for the induced variation of love for variety with respect to the level of consumption. This drives the incidence of consumption taxes. Constant love for variety (the CES case) corresponds to full shifting of consumption taxes into prices, that is 100% of taxes paid by consumers. This is the usual case but empirical estimations find both overshifting and undershifting depending on markets. Decreasing love for variety corresponds to overshifting of taxes into prices, which means that the mark up increases when taxes increase: it is a relatively rare situation which may occur for markets with learning by consuming (such art) or with addiction (it has been proven for different alcoholic markets, see Kenkel (2005); Young and Bielinska-Kwapisz (2006); Carbonnier (2013)). The most current case according to empirical literature is undershifting of consumption taxes (Peltzman (2000); Carbonnier (2007, 2008); Andrade et al. (2010)), which correspond to increasing love for variety: the more somebody consume, the more he likes to diversify its bundle of consumption.

Figure 1 shows that the pricing in the CES case does not depend of the purchasing power of the consumers: all firms have the same mark up independ from the environement. In that usual case, the producing firms are very concentrated among the more productive and even mid-level productivity firm actually produce very low amount. The case of decreasing love for variety is very special and actually provides some counter-intuitive results, such as mark up increasing with respect to the marginal cost of production. The most likely case - increasing love for variety implying undershifting of taxes into prices - leads to far more intuitive results. For example, prices, mark ups and profits increase with respect to the purchasing power of consumers and mark ups and profits increase with respect to the productivity of the producing firm. The cutoff of actual production also increases with the purchasing power of consumers.

Consequently, we use the linear marginal utility case - with increasing love for variety - in
order to setup a simulation of the consequences of a tax devaluation. We simulate a tax devaluation using the two country model, with country 1 initially with trade balance deficit and country 2 with trade balance surplus. Country 1 implements a tax reform consisting in setting a 10% wage subvention (equivalent to a payroll tax reduction of 10 points of percentage) financed by a VAT increase. The impacts of such reform per level of firm productivity in each country are presented in Figure 2.
unambiguous as prices increase in country 1 whatever the producing country and decrease in country 2, for each level of firm productivity. Nevertheless, output is more ambiguous as it increases in country 1 in the intensive margins and in the extensive margins for own production, but it decreases in the extensive margins for exports. exactly the opposite occurs in country 2. The total output impact, as well as total consumption, import and export are reported in table 1.

<table>
<thead>
<tr>
<th>Table 1: Tax devaluation simulation: overall output and production variations</th>
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<td>Country 1</td>
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<td>Output</td>
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<td>Consumption</td>
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<td>Import</td>
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<td>Export</td>
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<tr>
<td>Country 2</td>
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<td>Consumption</td>
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<tr>
<td>Import</td>
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<tr>
<td>Export</td>
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</tbody>
</table>

Notes: Tax devaluation of 10% wage subvention financed by VAT set by country 1 in trade balance deficit.

It appears that output in value does not change in any country, which means that the tax devaluation has no impact on GDP in value, neither for country setting it nor for the other. However, it has impact on welfare, for several reasons. First, the absence of GDP change in value is due to a global price increase in the country consuming more; in volume, outputs decrease in both country. Given the unchanged volume of production factors, it means that tax devaluation leads to a productivity decrease. Indeed, the cutoff of productivity for a firm to actually sale toward the most consuming country (country with a trade deficit) increases: it means that less productive firms may enter that market, decreasing the mean productivity. However, the lower overall productivity does not lead to welfare decline in country 1 because of change in the terms of trade, leading to consumption reallocation from country 2 to country 1 (with greater consumption loss in country 2 than consumption win in country 1). International trade flows decrease from country 1 to country 2 but increase in the opposite direction.

5 Conclusion

In the present article, we model the impact of a policy of the kind of social VAT - that is subsidizing labor by taxing consumption - on international trade. We focus on a international
trade model à la Melitz in order to take into account the heterogeneity of firm productivities and the love for variety of consumers. We consider a general case (non-CES) for the monopolistical competition to take into account VAT incidence.

From the intensive and the extensive margins of consumptions, it appears that such a policy may not benefit to every countries. Actually, the benefiting country is not determined by the country who set the policy but by the balance of trade. The social VAT, whatever who set it, deteriorates the terms of trade of the country with trade surplus and case and ameliorate the terms of trade of the country with trade deficit.

This amelioration or deterioration is greater when VAT incidence is more heavily borne by consumers. Conversely, if VAT is shared between producers and consumers - that is if the love for variety of consumers increases with respect to the level of consumption - the impact of social VAT is weaker on every country.

References


A Proofs of Results 2 and 3

Using \([[(1 - \tau_1)w_1] = (1 - \tau_1)\) and \([[1 - \tau_2)w_2] = \hat{w}_2\), one gets from (29) and (30)

\[
N_1(1 - \tau_1)w_1 \left[ G_{11} \hat{\lambda}_{11} + \Xi_{11}(1 - \tau_1) + \Xi_{11} \hat{N}_1 \right] + N_2(1 - \tau_2)w_2 \left[ G_{21} \hat{\lambda}_{21} + \Xi_{21} \hat{w}_2 + \Xi_{21} \hat{N}_2 \right] = (1 - \tau_1) (1 - \tau_1) w_1 L_1 E_1 - dTB_1
\]

\[
G_{11} \hat{\lambda}_{11} + G_{12} \hat{\lambda}_{12} = -[\Xi_{11} + \Xi_{12}] \hat{N}_1
\]

\[
G_{21} \hat{\lambda}_{21} + G_{22} \hat{\lambda}_{22} = -[\Xi_{21} + \Xi_{22}] \hat{N}_2
\]

and, from (31) and (32),

\[
\hat{\lambda}_{12} = \frac{-\Xi_{11}}{\Xi_{12}} \hat{\lambda}_{11}, \quad \hat{\lambda}_{22} = \frac{-\Xi_{21}}{\Xi_{22}} \hat{\lambda}_{21},\text{ and } \hat{w}_2 = (1 - \tau_1) + \hat{\lambda}_{21} - \hat{\lambda}_{11}
\]

We deduce that \(\hat{\lambda}_{21}, \hat{\lambda}_{11}, \hat{N}_1\) and \(\hat{N}_2\) are solutions of the following system of four equations

\[
N_1(1 - \tau_1)w_1 \left[ G_{11} \hat{\lambda}_{11} + \Xi_{11} \hat{N}_1 \right] + N_2(1 - \tau_2)w_2 \left[ G_{21} \hat{\lambda}_{21} + \Xi_{21} \left( \hat{\lambda}_{21} - \hat{\lambda}_{11} \right) + \Xi_{21} \hat{N}_2 \right] = (1 - \tau_1)TB_1 - dTB_1
\]

\[
(1 + \frac{\Xi_{11}}{\Xi_{12}}) \hat{\lambda}_{11} = \left( 1 + \frac{\Xi_{21}}{\Xi_{22}} \right) \hat{\lambda}_{21}
\]

\[
\hat{N}_1 = \left( G_{12} \frac{\Xi_{11}}{\Xi_{12}} - G_{11} \right) \frac{\hat{\lambda}_{11}}{\Xi_{11} + \Xi_{12}} \quad (38)
\]

\[
\hat{N}_2 = \left( G_{22} \frac{\Xi_{21}}{\Xi_{22}} - G_{21} \right) \frac{\hat{\lambda}_{21}}{\Xi_{21} + \Xi_{22}} \quad (39)
\]

Then \(\hat{\lambda}_{11}\) is solution of

\[
D \hat{\lambda}_{11} = (1 - \tau_1)TB_1 - dTB_1
\]

where

\[
D \equiv N_1(1 - \tau_1)w_1 \left[ G_{11} + \frac{\Xi_{11}}{\Xi_{11} + \Xi_{12}} \left( G_{12} \Xi_{12} - G_{11} \right) \right] + \frac{N_2(1 - \tau_2)w_2}{1 + \frac{\Xi_{11}}{\Xi_{12}}} \left[ G_{21} \left( 1 + \frac{\Xi_{11}}{\Xi_{12}} \right) + \Xi_{21} \left( \frac{\Xi_{11}}{\Xi_{12}} - \Xi_{21} \right) + \frac{\Xi_{21}}{\Xi_{21} + \Xi_{22}} \left( G_{22} \frac{\Xi_{21}}{\Xi_{22}} - G_{21} \right) \left( 1 + \frac{\Xi_{11}}{\Xi_{12}} \right) \right]
\]

\[
= N_1(1 - \tau_1)w_1 \left[ \frac{\Xi_{12}}{\Xi_{11} + \Xi_{12}} \Xi_{11} + \frac{\Xi_{11}}{\Xi_{11} + \Xi_{12}} \frac{G_{12} \Xi_{12}}{\Xi_{12}} \right] - \Xi_{21} N_2(1 - \tau_2)w_2
\]

\[
+ N_2(1 - \tau_2)w_2 \frac{1 + \frac{\Xi_{11}}{\Xi_{12}}}{1 + \frac{\Xi_{21}}{\Xi_{22}}} \left[ \frac{\Xi_{22}}{\Xi_{21} + \Xi_{22}} \Xi_{21} + \frac{\Xi_{21}}{\Xi_{21} + \Xi_{22}} \frac{G_{22} \Xi_{22}}{\Xi_{22}} \right] + \Xi_{21} N_2(1 - \tau_2)w_2 \frac{1 + \frac{\Xi_{11}}{\Xi_{12}}}{1 + \frac{\Xi_{21}}{\Xi_{22}}} < 0
\]
Three of the four terms in the last expression of $D$ are negative. Factorizing the last two terms with $N_2(1 - \tau_2)w_2 \left(1 + \frac{\Xi_{11}}{\Xi_{12}}\right) / \left(1 + \frac{\Xi_{21}}{\Xi_{22}}\right)$, a sufficient condition for $D < 0$ is

$$A \equiv \frac{\Xi_{22}}{\Xi_{21} + \Xi_{22}} G_{21} + \frac{\Xi_{21}}{\Xi_{21} + \Xi_{22}} G_{22} \Xi_{21} + \Xi_{21} < 0$$

From (27) and (28), $G_{kj}$ satisfies $G_{kj} \hat{\lambda}_{kj} = -\Xi_{kj} \hat{\lambda}_{kj} + \chi_{kj} \hat{\chi}_{kj}$. Recalling that $\hat{\chi}_{kj}$ and $\hat{\lambda}_{kj}$ have opposite signs (see footnote 8), we obtain

$$G_{kj} < -\Xi_{kj} < 0.$$ 

and consequently

$$\frac{\Xi_{22}}{\Xi_{21} + \Xi_{22}} G_{21} < -\frac{\Xi_{22} \Xi_{21}}{\Xi_{21} + \Xi_{22}} \text{ and } \frac{\Xi_{21}}{\Xi_{21} + \Xi_{22}} G_{22} \Xi_{21} < -\frac{\Xi_{21} \Xi_{21}}{\Xi_{21} + \Xi_{22}}.$$ 

$A$, and therefore $D$, are negative, which leads to the result. ■

## B Decomposition of the ratio $G_{kj}/\Xi_{kj}$

From (27) and (28), $G_{kj}$ satisfies $G_{kj} \hat{\lambda}_{kj} = -\Xi_{kj} \hat{\lambda}_{kj} + \chi_{kj} \hat{\chi}_{kj}$, that is

$$\frac{G_{kj}}{\Xi_{kj}} = -1 + \frac{\chi_{kj}}{\Xi_{kj}} \hat{\chi}_{kj}$$

$$= -1 + \left(\hat{\epsilon}_{kj} L_j \hat{\xi}_{kj}^* + f_{kj}\right) \gamma_k \hat{c}_{kj} \frac{\partial \hat{c}_{kj}}{\partial \hat{\lambda}_{kj}} + \int^{\epsilon_{kj}}_{0} cL_j \frac{\partial \xi_{kj}^*}{\partial \lambda_{kj}} \gamma_k(c) dc$$

The second equality is obtained by differentiation of (16). We use $\xi_{kj}^*$ instead of $\xi_{j}^* \left(\hat{\lambda}_{kj} c\right)$ and $\hat{\xi}_{kj}^*$ instead of $\xi_{j}^* \left(\hat{\lambda}_{kj} \hat{c}_{kj}\right)$. From (3) and (7), one gets

$$\frac{\partial \xi_{kj}^*}{\partial \lambda_{kj}} = -\frac{\xi_{kj}^*}{\lambda_{kj}} \frac{r_{u_j} \left(\xi_{kj}^*\right) + \xi_{kj}^* r_{u_j}' \left(\xi_{kj}^*\right)}{1 - r_{u_j} \left(\xi_{kj}^*\right)}$$

$$\frac{\partial \hat{c}_{kj}}{\partial \hat{\lambda}_{kj}} = \frac{\hat{c}_{kj}}{\lambda_{kj}} = \frac{-\hat{c}_{kj}}{\lambda_{kj} \left[1 - r_{u_j} \left(\xi_{kj}^*\right)\right]}$$

Then

$$\frac{G_{kj}}{\Xi_{kj}} = -1 - \frac{\left(\hat{\epsilon}_{kj} L_j \hat{\xi}_{kj}^* + f_{kj}\right) \hat{\epsilon}_{kj} \gamma_k \hat{c}_{kj}}{\left[1 - r_{u_j} \left(\xi_{kj}^*\right)\right] \Xi_{kj}} - \int^{\epsilon_{kj}}_{0} \frac{cL_j \hat{\xi}_{kj}^*}{\Xi_{kj}} \left[r_{u_j} \left(\xi_{kj}^*\right) + \frac{\xi_{kj}^* r_{u_j}' \left(\xi_{kj}^*\right)}{1 - r_{u_j} \left(\xi_{kj}^*\right)}\right]^{-1} \gamma_k(c) dc$$

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