Abstract

We construct a tractable endogenous growth model with endogenous investment specific technological change (ISTC) to explain why countries with different factor income tax combinations exhibit similar growth rates. Public and private capital stock externalities are assumed to augment ISTC. A specialized labor input exerts a positive externality in final good production. We show that the competitive equilibrium growth rate can be decomposed into a labor factor and a capital factor. Changes in factor income taxes, by affecting these factors, can have opposite effects on growth. Our model builds on the existing endogenous growth literature by providing an alternative, but compatible explanation for the offsetting growth effects of fiscal policy observed in the data. We also discuss how the presence of such externalities affect the magnitude of the factor income tax gap across countries.

Keywords: Endogenous Investment Specific Technological Change, Factor Income Taxation, Endogenous Growth Theory, Fiscal Policy.

JEL Codes: E2; E6; H2; O4

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1 Introduction

Why do countries with different factor income tax combinations exhibit similar growth rates? In this paper, we develop an endogenous growth model with endogenous investment specific technological change to understand this question.

Figure 1 plots the average aggregate annual real GDP growth rate from 1990 to 2007 against the factor income tax ratio for several advanced economies.\(^1\) Average growth for all countries (excluding Ireland) falls between 0.875\% and 2.462\%. The standard deviation of the average real GDP growth rates is low at 0.878 (excluding Ireland, the standard deviation is 0.4756). What is striking is that the range in the ratios of the average capital income tax rate to the average labor income tax rate in these economies is much more pronounced: 0.3951 to 1.725.\(^2\) Therefore, countries with almost similar growth rates are accompanied by totally different factor income tax combinations.

Figure 2 plots the range of individual factor income taxes for these countries where the tax on capital and labor income have been averaged over 1990–2007. Despite having similar growth rates, what is striking is that whereas the difference between factor income taxes is large in some countries, it is quite small in others.\(^3\)

\[\text{Insert Figure 1 and 2}\]

Figure 3 plots the levels of factor income tax rates across the G7 countries. The incidence of factor income taxation is quite disparate. In the US, UK, Canada, and Japan, the tax on capital income is greater than the tax on labor income. In contrast, for Germany, Italy, and France, the reverse is true.

\[\text{Insert Figure 3}\]

In other evidence, Jones (1995) also shows in a sample of 15 OECD countries from 1950 to 1987, that changes in investment rates do not have any significant long run growth effects.

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\(^1\)The growth rates are calculated from the OECD (2012) database: see Table (V.XV.OB). The countries are: Austria (AUS), Belgium (BEL), Canada (CAN), Denmark (DEN), Finland (FIN), France (FRA), Germany (GER), Greece (GRE), Ireland (IRE), Italy (ITA), Japan (JPN), Netherlands (NET), Portugal (PRT), Spain (SP), Sweden (SWE), United Kingdom (UK) and United States of America (USA). The base year is 2000.

\(^2\)Canada and Japan have data on capital and labor income tax estimates based on the approach used in Mendoza et al. (1994) and Trabandt and Uhlig (2009) from 1965 to 1996. For Germany, United Kingdom and United States of America, data is from 1965 to 2007. For France, the data is from 1970 to 2007. For Italy, the data is from 1980 to 2007. For Austria, Belgium, Denmark, Finland, Netherlands, Portugal and Sweden, the data is from 1995 to 2007. For Spain and Greece, the data is from 2000 to 2007. Finally, for Ireland, the data is from 2002 to 2007.

\(^3\)The data on factor income taxes are from Mendoza et al. (1994) and Trabandt and Uhlig (2009). The latter have used the approach in Mendoza et al. (1994) to estimate the tax rates for 17 OECD nations till 2007.
He shows that shocks to investments – both total and durables and in particular durable equipment – have only a short-run growth effect with no significant effect on long run growth.

Figures 1 - 3 and the evidence from Jones (1995) yield a puzzle since countries with different factor income tax combinations exhibiting similar growth rates is incompatible with a standard model of endogenous growth. The standard endogenous (AK) growth model predicts that fiscal policy has a large growth effect through its impact on the economy’s investment rate. Taken to the data, these models would predict a high correlation between the investment rate and the growth rate. The above evidence therefore suggests that changes in fiscal policy (or factor income taxes) must have offsetting changes in investments such that growth rates do not change.

The literature has tried to find extensions to the standard endogenous growth model that can explain the apparent absence of growth effects of fiscal policy. McGrattan (1998) develops a theoretical framework where government policy can be incorporated into a standard AK growth model by incorporating two types of capital: structures and equipment capital. She shows that the equilibrium growth rate depends on the investment rate and the capital-output ratio. The reason why fiscal policy has no growth effects is because its effect on the investment rate is offset by the effect of fiscal policy on the capital-output ratio. Because of these offsetting effects, total investment does not change that much. Jaimovich and Rebelo (2012) show that changes in tax rates can have non-linear effects on long-run output growth. To capture this non-linearity, they construct a model where low tax rates have negligible effects on growth but when disincentives to invest are large, larger tax rates have a strong negative effect on output growth. The mechanism in their model is based on a skewed distribution of agents between workers and innovators, which results in a small number of highly productive workers in equilibrium. In a related literature, Glomm and Ravikumar (1998) build a growth model where public education spending, financed by distortionary taxes affect human capital accumulation. Again, they find that despite being distortionary in nature, tax rates have negligible effects on growth rates. Finally, Stokey and Rebelo (1995) also show in a numerical exercise that big changes in tax on capital income (up to the order 30%) do not have large growth effects on the US economy.

1.1 Description of our model and main results

Our paper provides an alternative, but compatible, explanation based on the fact that different combinations of taxes can generate the same growth rate. We construct an endogenous growth model with endogenous investment specific technological change with three types of externalities: an externality from the stock of private and public capital in the process of
innovation; and an externality from labor allocated to research in final good production. In particular, the public capital stock – financed by distortionary taxes – and the private capital stock augment investment specific technological change (ISTC) as a positive externality.\textsuperscript{4} Our basic model follows Huffman (2008).\textsuperscript{5} Typically in the literature, the public input is seen as directly affecting final production directly either as a stock or a flow (e.g., see Futagami, Morita, and Shibata (1993), Chen (2006), Fischer and Turnovsky (1997, 1998), and Eicher and Turnovsky (2000)). We show that incorporating these externalities into a model of endogenous growth and ISTC leads to offsetting effects of factor income taxes on growth.\textsuperscript{6}

There are two sectors in the model: a final goods sector and a research sector. The final good sector produces a final good, using private capital, and labor. Labor supply is composite in the sense that one type of labor activity is devoted to final good production, and the other to research which directly reduces the real price of capital goods in the next period. The second sector (the research sector) captures the effect of public capital and private capital stock spillovers and research activity on reducing the real price of capital goods. The agent optimally chooses each labor activity. We assume two types of labor activities: one type is labor allocated for final goods production, or current production, and another type is labor allocated for enhancing investment specific technological change, or future capital accumulation, and therefore future production. Crucially, the agent might not be aware that his allocation of labor towards research also influences productivity of the current period’s final goods production. Therefore, although research labor allocation is done from the point of future capital accumulation and hence future output we will assume that the agent might be unaware of the spillover it has on current production. This implies that the process of augmenting knowledge - which is designed to influence the price of capital in the future - may affect present output too. Effectively, this means that the process of

\textsuperscript{4}Our setup also allows investment specific technological change to enhance the accumulation of public capital. For instance, providing better infrastructure today reduces the cost of providing public capital in the future.

\textsuperscript{5}A growing literature has attributed the importance of investment specific technological change to long run growth (see Greenwood et al. (1997, 2000); Whelan (2003)). Investment specific technological change refers to technological change which reduces the real price of capital goods. Greenwood et al. (1997, 2000) show that once the falling price of real capital goods is taken into account, this explains most of the observed growth in output in the US, with relatively little being left over to be explained by total factor productivity.

\textsuperscript{6}To the best of our knowledge, we are not aware of any paper in the literature in which public capital affects ISTC, either directly or as an externality. In a different context, Harrison and Weder (2000) build a two sector representative agent model with increasing returns to scale driven by externalities that come from sector specific as well as aggregate economic activity. Benhabib and Farmer (1996) show that small empirically plausible external effects lead to indeterminacy. Neither of these papers has a role for public capital. Lloyd-Braga, Modesto, and Seegmuller (2008) introduce positive government spending externalities in preferences. In our model, externalities from the public stock influence ISTC directly.
augmenting knowledge may make routine labor (in the final goods sector) more effective.

In the planner’s problem, we assume that public investment is financed by a fixed proportional income tax as in Barro (1990). We assume that the planner’s objective is to maximize the growth rate of the economy. In particular, fiscal policy is set with the objective of reaching the efficient growth rate. The planner maximizes the utility of the representative agent and internalizes the externalities in the research sector and final good sector. This yields the efficient allocation for a fixed tax rate. Corresponding to this allocation, we characterize the steady state balanced growth path and show that the growth rate depends on 1) a labor input devoted to research (the labor factor) and 2) the contribution to growth from public and private capital (the capital factor). We then derive the growth maximizing tax rate and show that it is determined by the relative importance of the public capital output ratio vis-à-vis the private capital output ratio in the investment specific technological change function. The implication of this is that if a planner was to choose the tax rate to maximize growth, the planner could maximize long run growth as long as the tax rate equals the relative contribution of public capital to investment specific technological change.

We then ask under what conditions can the planner’s allocations be implemented in the competitive decentralized equilibrium with identical and different factor income taxes. We assume that public investment is financed by distortionary factor income taxes on capital and labor income. We show that while the first best fiscal policy can be implemented as a competitive equilibrium, there is an indeterminate combination of capital tax rates and the labor tax rates that can replicate the planner’s allocations.

Our definition of indeterminacy is as follows: there is no unique combination of factor income taxes on capital and labor income that implements the planner’s allocations. In other words, we show that multiple factor income tax combinations - and therefore factor income tax gaps - can implement the efficient growth rate. This is consistent with the empirical evidence documented in Figures 1 -3. Indeterminacy obtains because the planner’s allocations yield a constant growth rate, and factor income taxes have offsetting effects on the capital factor and labor factor. In particular, an increase in the capital income tax reduces the capital factor, and reduces growth. However, an increase in the labor income tax

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7There is a large literature on political economy and institutional motives for designing fiscal policy. Our assumption that the policy setter maximizes the efficient growth rate is consistent with this approach to fiscal policy because several papers in this literature argues that even if politicians set fiscal policy to maintain constituent support, they achieve this by raising incomes through enacted policies (see Key (1966), Tufte (1978), Fiorina (1981), Kiewiet and Rivers (1985), Lewis-Beck (1990), Harrington (1993), Ghate (2003)). In an exhaustive study examining economic conditions and electoral outcomes in the U.S. and Western Europe, Kiewiet and Rivers (1985) find that a 1 percent decline in real income is associated with a reduction of the incumbent’s party vote share of between 0.5 percent and 1 percent. Lewis-Beck (1990) reports that voters consistently reveal that economic health is among the most important factors affecting their choices in elections.
exerts both offsetting income and substitution effects. We show that with ISTC, the income effect dominates the substitution effect, which increases labor supply, with the strength of the income effect made stronger the larger the extent of ISTC. The increase in labor supply increases the labor factor which augments capital accumulation and growth. We are able to analytically characterize the implementation of the planner’s growth rate.

How do the externalities affect the factor income tax gaps that implement the planner’s allocations? We first consider the case of a positive spillover from the specialized research labor activity on final good production. In this case, an increase in the spillover increases the planner’s allocation towards specialized labor. This increases the planner’s growth rate. To implement the higher growth rate, this requires an increase in the labor income tax, which raises the labor factor from the competitive growth rate, or a reduction in the capital income tax, which raises the capital factor. Implementing either leads to an increase in equilibrium factor income tax gap.

In contrast, when the weight on the positive spillover from the public and private capital stock falls, this leads to a higher contribution of the existing stock of ISTC on the future level of ISTC. That is, a lower weight on the stock externalities implies that the weight on the persistence of ISTC is higher. Therefore, the growth rate of the planner is higher. To raise the competitive equilibrium growth rate, a reduction in the tax on capital income raises the capital factor and an increase in the labor income tax raises the labor factor. An increase in both factors raise growth which requires an increase in the factor income tax gap to implement the planner’s growth rate.

Our general result is that to the extent that spillovers from a specialized labor input and the public and private capital stocks exist, an increase in these spillovers from the specialized labor, and a decrease in the spillover from public and private capital, increase the planner’s growth rate, and therefore increase the factor income tax gap required to implement the higher planner’s growth rate. Conversely, for a given level of externalities, maintaining the constancy of growth also requires different combinations of factor income taxes as in McGrattan (1998). We also show that when there are no externalities, equal factor income taxes always yield the optimal growth rate from the planner’s problem. Hence, the factor income tax gap is zero. Finally, we also conduct a simple numerical exercise to show that equilibrium factor income taxes generated by our model are in accordance with Figures 1 - 3.

1.1.1 Empirical Evidence on Externalities

With respect to the private capital stock, DeLong and Summers (1991) show that investment in machinery is associated with very strong positive externalities, and that increases in
investments in equipment implies higher growth. Hamilton and Monteagudo (1998) find that capital is associated with positive external effects in an estimated Solow growth model. Greenwood et al. (1997), show that the real price of capital equipment in the US – since 1950 – has fallen alongside a rise in the investment-GNP ratio. This suggests that the private capital stock exhibits a positive externality in investment specific technological change through the aggregate capital stock. Importantly, Greenwood et al. (1997, p. 342) say: "The negative co-movement between price and quantity.....can be interpreted as evidence that there has been significant technological change in the production of new equipment. Technological advances have made equipment less expensive, triggering increases in the accumulation of equipment both in the short and long run."

With respect to the nexus between public expenditures, R&D, and growth, Griliches (1979) examines how the indirect effects of research and development affect future output through induced changes in factor inputs. In his model, the accumulation of private capital is driven by the aggregate stock of knowledge and current and past stocks of research and development (R&D). Scott (1984) and Levin and Reiss (1984) estimate that the high spillovers from federal research and development spending dominates the crowding-out effect it has on private spending on R&D. The net effect is that public spending has a positive effect on productivity. Finally, David et al. (2000), show that public R&D spending is complementary to private R&D spending.

We also assume that the specialized labor input in the research sector exerts a positive externality in the production of the first sector, the final good. This assumption is motivated by both anecdotal evidence as well as the academic literature. For instance, Davidson (2012) documents evidence on the extent to which skill required for advanced manufacturing jobs. He argues that skilled factory workers these days are typically "hybrid-workers": they are both machinists (engaging in final good production) as well as computer programmers (engaging in research). For instance, in the US metal-fabricating sector, workers not only use cutting tools to shape a raw piece of metal, but they also write the computer code that instructs the machine to increase the speed of such operations. Globerman (1975) describes a class of machinists in the manufacturing sector called "tool and die makers", or also "mold makers" (see Bryce (1997)). The machinist therefore receives on-the-job training which enables him to work with machines and computers, which makes him multi-skilled. Even though on-the-job training is costly, Park (1996) shows, from an empirical study on manufacturing industries in Korea that employing "multi-skilled workers" makes a firm’s

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8 Primarily a skilled artisan, a tool and die maker works in an industrial environment where producing the final good requires two different skills – creative skills and machine knowledge. An example of such an activity, crucial to the manufacturing sector, is engineering drawing.
production more efficient in comparison to employing "single-skilled "or specialized workers to handle each individual activity.⁹ On the job training is undertaken for future benefits but it may also augment the efficiency of standard labor that has been assigned to produce output in the current period.

1.1.2 Related Literature

The setup of our model is technically similar to Huffman (2007, 2008) who explicitly models the mechanism by which the real price of capital falls when investment specific technological change occurs. Huffman (2008) builds a neoclassical growth model with investment specific technological change. Labor is used in research activities in order to increase investment specific technological change. In particular, the changing relative price of capital is driven by research activity, undertaken by labor effort. Higher research spending in one period lowers the cost of producing the capital good in the next period.¹⁰ Investment specific technological change is thus endogenous in the model, since employment can either be undertaken in a research sector or a production sector. His model includes capital taxes, labor taxes, and investment subsidies that are used to finance a lump-sum transfer. Huffman (2008) finds that a positive capital tax that is larger than a positive investment subsidy along with zero labor tax can replicate the first best allocation. Huffman’s models however do not incorporate public capital - a feature we show that is important in matching disparate factor income tax gaps observed in advanced economies.

Our paper is also related to the literature on fiscal policy and long run growth in the neoclassical framework. The literature started by Barro (1990) and Futagami, Morita, and Shibata (1993) – incorporate a public input – such as public infrastructure – that directly augments production. In Barro (1990), public services are a flow; while in Futagami, Morita, and Shibata (1993), public capital accumulates. However, in the large literature on public capital and its impact on growth spawned by these papers, the public input, whether it is modeled as a flow or a stock, doesn’t directly influence the real price of capital goods.¹¹ Since public capital affects the real price of capital explicitly in our model, this means that the public input affects future output through its effect on both future investment specific

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⁹Even though labor productivity in final good production is typically seen to be a function of the stock of knowledge (and therefore the externality comes from the level of ISTC), we assume that there is no difference in skills and ability in the labor force in the two productive activities, so that labor allocated to research is not an exact proxy for the stock of knowledge.

¹⁰Krusell (1998) also builds a model in which the decline in the relative price of equipment capital is a result of R&D decisions at the level of private firms.

technological change, as well as future private capital accumulation.

2 The Model

Consider an economy that is populated by identical representative agents with unit mass, who at each period $t$, derive utility from consumption of the final good $C_t$ and leisure $(1 - n_t)$. There is no population growth which implies that aggregate variables are also per-capita variables. The term $n_t$ represents the fraction of time spent at time $t$ in employment. The discounted life-time utility, $U$, of an infinitely lived representative agent is given by

$$U = \sum_{t=0}^{\infty} \beta^t [\log C_t + \log(1 - n_t)].$$

(1)

where $\beta \in (0, 1)$ denotes the period-wise discount factor. The total supply of labor for the agent at any time $t$ is given by $n_t$ such that

$$n_t \equiv n_{1t} + n_{2t},$$

(2)

where $n_{1t}$ is labor allocated for final goods production, or current production, and $n_{2t}$ is labor allocated for enhancing investment specific technological change, or future capital accumulation, and therefore future production. Crucially, the agent is not aware that his allocation of labor towards $n_{2t}$ also influences productivity of the current period’s final goods production.$^{12}$ Therefore, although $n_{2t}$ is employed from the point of future capital accumulation and hence future output he is unaware of the spillover it has on current production.

The final good is therefore produced by a neoclassical production function with capital $K_t$, $n_{1t}$, and $n_{2t}$. An important point is that the planner internalizes the effect of $n_{2t}$ on final goods production, while the agent will not. The production function is given by

$$Y_t = AK_t^\alpha n_{1t}^{1-\alpha} (n_{2t}^{1-\alpha})^\xi$$

(3)

where $A > 0$ is a scalar that denotes the exogenous level of productivity, $\alpha \in (0, 1)$ is the share of output paid to capital and $\xi > 0$ is the externality parameter capturing the effect that $n_2$ has on direct production. When $\xi > 0$, the planner internalizes the effect that $n_2$ has on direct production. When $\xi = 0$, there is no externality from $n_2$ on the production of the final good. Note, in this framework, as in Huffman (2008) the two labor activities $n_{1t}$ and $n_{2t}$ are assumed to be equally skilled, but are optimally allocated across different activities

$^{12}$This assumption is motivated by the empirical evidence on "multi-skilled" workers mentioned in the introduction.
by households.\textsuperscript{13}

Private capital accumulation grows according to the standard law of motion augmented by investment specific technological change,

\[ K_{t+1} = (1 - \delta)K_t + I_t Z_t, \tag{4} \]

where \( \delta \in [0, 1] \) denotes the rate of depreciation of capital and \( I_t \) represents the amount of total output allocated towards private investment at time period \( t \). We assume that, \( \delta = 1 \), to keep the model tractable. \( Z_t \) represents investment-specific technological change. The higher the value of \( Z_t \), the lower is the cost of accumulating capital in the future. Hence \( Z_t \) can also be viewed as the inverse of the price of per-unit private capital at time period \( t \). The term, \( I_t Z_t \), therefore represents the effective amount of investment driving capital accumulation in time period \( t+1 \).

In addition to labor time deployed by the representative firm towards R&D, the public capital stock, \( G \), plays a crucial role in lowering the price of capital accumulation. Typically the public input is seen as directly affecting final production – either as a stock or a flow (e.g., see Futagami, Morita, and Shibata (1993), Chen (2006), Fischer and Turnovsky (1997, 1998), and Eicher and Turnovsky (2000)). Instead, here we assume that the public input facilitates investment specific technological change. This means that the public input affects future output through future private capital accumulation directly. In the above literature, the public input affects current output directly. This is our point of departure. We therefore formalize the link between fiscal policy and growth through the effect that fiscal policy has on ISTC.

We assume that in every period, public investment is funded by total tax revenue. Public capital therefore evolves according to

\[ G_{t+1} = (1 - \delta)G_t + I_t^g Z_t, \tag{5} \]

where \( G_{t+1} \) denotes the public capital stock in \( t + 1 \), and \( I_t^g \) denotes the level of public investment made by the government in time period \( t \):

\[ I_t^g = \tau Y_t, \tag{6} \]

where \( \tau \in (0, 1) \) is the proportional tax rate.\textsuperscript{14} We assume that \( Z_t \) augments \( I_t^g \) in the same

\textsuperscript{13} Other papers in the literature - such as Reis (2011) - also assume two types of labor affecting production. In Reis (2011), one form of labor is the standard labor input, while the other labor input is entrepreneurial labor.

\textsuperscript{14} Since \( \delta = 1 \), equation (5) implies that \( G_{t+1} = I_t^g Z_t \), i.e., the ISTC adjusted public investment (flow) at
way as $I_t$ since it enables us to analyze the joint endogeneity of $Z$ and $G$. To derive the balanced growth path, we further assume that the period wise depreciation rate $\delta \in [0, 1]$ is same for both private capital and public capital.

### 2.1 Investment Specific Technological Change

To capture the effect of public capital on research and development, we assume that $Z$ grows according to the following law of motion,

$$Z_{t+1} = B n_{2t}^\theta Z_t^\gamma \left\{ \left( \frac{G_t}{Y_{t-1}} \right)^\mu \left( \frac{K_t}{Y_{t-1}} \right)^{1-\mu} \right\}^{1-\gamma}. \quad (7)$$

Here, $B > 0$ stands for an exogenously fixed scale productivity parameter and $\mu \in (0, 1)$ captures the impact of public investments on investment specific technological change. We assume that the parameters, $\theta \in (0, 1)$ and $\gamma \in (0, 1)$, where $\theta$ stands for the weight attached to research effort and $\gamma$ is the level of persistence the current year’s level of technology has on reducing the price of capital accumulation in the future.\(^{15}\) The term $\frac{G_t}{Y_{t-1}}$ represents the externality from public capital in enhancing investment specific technological change in time period $t + 1$. The aggregate capital-output ratio, $\frac{K_t}{Y_{t-1}}$, is also assumed to exert a positive externality effect on investment specific technological change. In particular, a higher aggregate stock of capital in $t$, $K_t$, relative to $Y_{t-1}$, raises $Z_{t+1}$. Like the externality from $n_2$, the planner internalizes the effect that stock of public capital and private capital has on investment specific technological change, while agents treat the effect of $\frac{G_t}{Y_{t-1}}$ and $\frac{K_t}{Y_{t-1}}$ on $Z_{t+1} –$ the bracketed term – as given.\(^{16}\) Note that when $\gamma = 1, \theta = 0$, ISTC is exogenous.

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\(^{15}\)This contrasts with Huffman (2008) where $\gamma = 1$ is required for growth rates of $Z$ and output to be along the balanced growth path. We require $\gamma \in (0, 1)$ for the equilibrium growth rate to adjust to the steady state balanced growth path.

\(^{16}\)We assume that $\delta = 1$ for analytical tractability. Our assumption of $\frac{G_t}{Y_{t-1}}$ augmenting $Z_{t+1}$ is for two reasons. First, if $G_t$ augmented output $Y_t$ instead, we can show that in equilibrium, the only possible balanced growth path is when the gross growth rate of all endogenous variables is 1 that is, all variables are at their steady state. This means, public capital will not affect the growth rate. Hence, allowing for ISTC to depend on the public input enables the balanced growth path to be affected by tax policy through ISTC. Second, if $Z_{t+1}$ was instead parametrized as

$$Z_{t+1} = B n_{2t}^\theta Z_t^\gamma \left\{ G_t^\mu K_t^{1-\mu} \right\}^{1-\gamma},$$

i.e., $G$ and $K$ are not normalized by $Y$, we can show that the growth rate if $Z$ will never converge to a balanced growth path.
2.2 The Planner’s Problem

We first solve the planner’s problem who internalizes all the externalities. This yields the socially efficient allocation for a fixed tax rate. The aggregate resource constraint the economy faces in each time period $t$ is given by

$$C_t + I_t \equiv Y_t(1 - \tau) = AK_t^\alpha n_{1t}^{1-\alpha} \left( n_{2t}^{1-\alpha} \right)^\xi (1 - \tau)$$  \hspace{1cm} (8)

where agents consume $C_t$ at time period $t$ and invest $I_t$ at time period $t$. Aggregate consumption and investment add up to after-tax levels of output, $Y_t(1 - \tau)$, where $\tau \in [0, 1]$ is the proportional tax rate that is assumed to be fixed in every time period.

Since the planner internalizes the size of public expenditure given by

$$\frac{G_{t+1}}{Y_t} = \tau Z_t,$$  \hspace{1cm} (9)

which follows from (5) and (6) after imposing $\delta = 1$, he takes the following law of motion of ISTC as a restriction:

$$Z_{t+1} = Bn_{2t}^\theta Z_t^\gamma Z_{t-1}^{(1-\gamma)\mu(1-\gamma)} \left( \frac{K_t}{Y_{t-1}} \right)^{(1-\mu)(1-\gamma)} ,$$  \hspace{1cm} (10)

which is obtained by substituting (9) in (7).

To obtain the efficient allocation, the planner maximizes the lifetime utility of the representative agent – given by (1) – subject to the economy wide resource constraint given by (8), the law of motion (4), the equation describing investment specific technological change (10) and the identity for total supply of labor given by (2).

2.2.1 First Order Conditions

The Lagrangian for the planner’s problem is given by,

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[ \log C_t + \log(1 - n_{1t} - n_{2t}) \right] + \sum_{t=0}^{\infty} \beta^t \lambda_{1t} \left[ AK_t^\alpha n_{1t}^{1-\alpha} \left( n_{2t}^{1-\alpha} \right)^\xi (1 - \tau) - C_t - \frac{K_{t+1}}{Z_t} \right]$$

$$+ \sum_{t=0}^{\infty} \beta^t \lambda_{2t} \left[ Bn_{2t}^\theta Z_t^\gamma Z_{t-1}^{(1-\gamma)\mu(1-\gamma)} \left( \frac{K_t}{Y_{t-1}} \right)^{(1-\mu)(1-\gamma)} - Z_{t+1} \right].$$

where $\lambda_{1t}$ and $\lambda_{2t}$ are the Lagrangian multipliers. Because our focus is on the balanced growth path corresponding to the efficient allocation, we assume that $\delta = 1$.

\footnote{Clearly, $C_t + I_t + I_t^\theta = Y_t$.}
The following first order conditions obtain with respect to \( C_t, K_{t+1}, n_{1t}, \) and \( n_{2t}, \) respectively\(^{18}\):

\[
\{ C_t \} : \frac{1}{C_t} = \lambda_t \tag{11}
\]

\[
\{ K_{t+1} \} : \frac{1}{C_t Z_t} = \frac{\alpha \beta Y_{t+1} (1 - \tau)}{C_{t+1} K_{t+1}} + \beta (1 - \gamma) (1 - \mu) \lambda_{2t+1} \frac{Z_{t+2}}{K_{t+1}} - \beta^2 \lambda_{2t+2} (1 - \gamma) \alpha \frac{Z_{t+3}}{K_{t+1}} \tag{12}
\]

\[
\{ Z_{t+1} \} : \lambda_{2t} = \beta \lambda_{2t+1} \gamma \frac{Z_{t+2}}{Z_{t+1}} + \frac{\beta}{Z_{t+1}} \left( \frac{I_{t+1}}{C_{t+1}} \right) + \beta^2 \lambda_{2t+2} \mu (1 - \gamma) \frac{Z_{t+3}}{Z_{t+1}} \tag{13}
\]

\[
\{ n_{1t} \} : \frac{1}{1 - n_t} = \frac{(1 - \alpha) Y_t (1 - \tau)}{C_t n_{1t}} - \beta \lambda_{2t+1} (1 - \gamma) (1 - \alpha) \frac{Z_{t+2}}{n_{1t}} \tag{14}
\]

and,

\[
\{ n_{2t} \} : \frac{1}{1 - n_t} = \frac{(1 - \alpha) \xi Y_t (1 - \tau)}{C_t n_{2t}} + \lambda_{2t} \theta \frac{Z_{t+1}}{n_{2t}} - \beta \lambda_{2t+1} (1 - \gamma) \xi (1 - \alpha) \frac{Z_{t+2}}{n_{2t}}. \tag{15}
\]

Equation (11) represents the standard first order condition for consumption, equating the marginal utility of consumption to the shadow price of wealth. Equation (12) is an augmented form of the standard Euler equation governing the consumption-savings decision of the household. Equation (13) is the Euler equation with respect to \( Z_{t+1} \). Equation (14) denotes the optimization condition with respect to labor supply \( (n_{1t}) \). Since \( 0 < \gamma < 1 \), the second term in the RHS is positive which constitutes a reduction in the marginal utility of leisure. This reduces \( n_1 \) relative to the standard case in which there is no investment specific technological change. Finally, equation (15) is the first order condition with respect to \( n_{2t} \).

### 2.2.2 Decision Rules

We now derive the closed form decision rules based on the above first order conditions using the method of undetermined coefficients, as shown in the following Lemma 1.

**Lemma 1** \( C_t, I_t, n_t, n_{1t}, n_{2t} \) are given by (16), (17), (18), where \( 0 < \Phi < 1 \) is given by (19), and \( 0 < x < 1 \) given by (20) is a constant.

\[
C_t = \Phi_P Y_t (1 - \tau), \quad I_t = (1 - \Phi_P) Y_t (1 - \tau) \tag{16}
\]

\(^{18}\)See Appendix A for derivations.
\[ n_t = n_P = \frac{(1 - \alpha)[(1 - \beta \gamma) - \beta^2 \mu(1 - \gamma) - \beta^2(1 - \gamma)(1 - \Phi_P)]}{(1 - \alpha)[(1 - \beta \gamma) - \beta^2 \mu(1 - \gamma) - \beta^2(1 - \gamma)(1 - \Phi_P)] + \Phi_Px_P \left[ 1 - \beta \gamma - \beta^2 \mu(1 - \gamma) \right]} \]  

(17)

\[ n_{1P} = x_P n_P, \quad n_{2P} = (1 - x_P)n_P, \]  

(18)

where \( \Phi_P \) is given by

\[ \Phi_P = 1 - \frac{\alpha \beta \left[ (1 - \beta \gamma) - \beta^2 \mu(1 - \gamma) \right]}{(1 - \beta \gamma) - \beta^2(1 - \gamma) + \alpha \beta^3 (1 - \gamma)}, \]  

(19)

and \( x_P \) is given by

\[ x_P = \frac{(1 - \alpha)\{(1 - \beta \gamma) - \beta^2 \mu(1 - \gamma) - \beta^2(1 - \gamma)(1 - \Phi_P)\}}{(1 + \xi)(1 - \alpha)\{(1 - \beta \gamma) - \beta^2 \mu(1 - \gamma) - \beta^2(1 - \gamma)(1 - \Phi_P)\} + \beta \theta(1 - \Phi_P).} \]  

(20)

**Proof.** See Appendix A for derivations. 

While decision rules for consumption and investment given by (16) suggest that levels of consumption and investment would fall if the proportional tax rate \( \tau \) increases, the share of after tax income spent on consumption given by \( \Phi_P \) increases when \( \mu \) rises, and thereby for investment it falls. Intuitively, when \( \mu \) rises the weight on the ratio of public capital to output, \( \frac{G_t}{Y_t} \), in augmenting investment specific technological change increases and so the weight on the ratio \( \frac{K_t}{Y_t} \) falls. Since the planner solves the optimization problem for the representative agent, the effect of increases in \( \mu \) on private investments is therefore expected.

2.2.3 The Balanced Growth Path

We can obtain the balanced growth path (BGP) corresponding to the efficient allocation by substituting (16), (17), (18), (19), and (20) into (7). Define \( \overline{M_P} \) a constant as

\[ \overline{M_P} = B((1 - x_P)n_P)^\theta(1 - \Phi_P)^{(1 - \mu)(1 - \gamma)}. \]  

(21)

Given the assumptions it is easy to show that we can obtain a constant growth rate for \( Z, K, G \) and \( Y \). This condition necessarily implies \( 0 < \Phi_P, x_P, n_P < 1 \) which always holds true. We therefore have the following Lemma 2.

**Lemma 2** On the steady state balanced growth path, the gross growth rate of \( Z, K, G \) and
There are several aspects of the equilibrium growth rate worth mentioning. First, the growth rate is independent of the technology parameter, $A$, but not $B$, as in Huffman (2008). Second, the growth rate of output, $g_y$, is less than $g_k$ along the balanced growth path because equation (7) is homogenous of degree $1 + \theta$. Lemma (2) therefore clearly establishes that the effect of the stock of public capital on $Z$ affects not just marginal productivity of factor inputs but also growth rate at the balanced growth path.

Finally, from (22), the tax rate exerts a positive effect on growth as well as a negative effect. This is similar to the equation characterizing the growth maximizing tax rate in models with public capital. The mechanism here is however different. For small values of the tax rate, a rise in $\tau$ leads to higher public capital relative to output, $Y_{t-1}$. This raises the future value of ISTC. An increase in ISTC reduces the real price of capital, stimulating investment and long run growth. However, for higher tax rates, further increases in the tax rate depresses after tax income, and investment. This reduces $G$ relative to $Y$, lowering $Z$, and depressing investment and long run growth. Hence, there is a unique growth maximizing tax rate.

Using the expression for $g_z$ in (22) we can characterize the growth maximizing tax rate as follows:

**Proposition 1** In the steady state, the planner maximizes growth by choosing the proportional tax rate given by $\tau = \mu$.

**Proof.** See Appendix A. $\blacksquare$

Proposition 1 sets a benchmark for the planner to set the optimal tax rate. If the planner wants to maximize balanced growth (corresponding to the efficient allocation), he should set the tax rate to $\mu$. The higher the weight attached to $\frac{G_t}{Y_{t-1}}$ in the investment specific technological change equation, the higher should be the optimal tax rate set by the planner. This result is intuitive since it suggests that the government would have to impose a higher tax rate on income if public capital were to play a greater role in driving investment specific technological change.

\[^{19}\text{See Bishnu, Ghate and Gopalakrishnan (2011).}\]
2.3 The Competitive Decentralized Equilibrium

We now solve the competitive decentralized equilibrium. Consider an economy that is populated by a set of homogenous and infinitely lived agents of unit mass with the aggregate population normalized to unity. There is no population growth and the representative firms are completely owned by agents. Firms pay taxes (or receive subsidies) on capital income \( \tau_k \in (-1, 1) \) while agents pay taxes (or receive subsidies) on labor income \( \tau_n \in (-1, 1) \). Agents derive utility from consumption of the final good and leisure given in equation (1). The wage payment \( w_t \) for both kinds of labor are the same since there is no skill difference assumed between both activities. Agents fund consumption and investment decisions from their after tax wages which they receive for supplying labor \( n_1 \) and \( n_2 \), and capital income earned from holding assets, which essentially equals the returns to capital lent out for production at each time period \( t \).

The government funds public investment, \( I^g_t \), at each time period \( t \) using a distortionary tax imposed on labor, \( \tau_n \in (-1, 1) \), and capital, \( \tau_k \in (-1, 1) \) respectively. The following is therefore the government budget constraint:

\[
I^g_t = w_t (n_{1t} + n_{2t}) \tau_n + \{ Y_t - w_t (n_{1t} + n_{2t}) \} \tau_k.
\]

2.3.1 The Firm’s Dynamic Profit Maximization Problem

The representative firm produces the final good based on (3). Hence, the production function is given by

\[
Y_t = AK^\alpha_t n_{1t}^{1-\alpha} \left( \frac{n_{1t}}{n_{2t}} \right)^\xi
\]

where the law of motion of private capital is given by (4). To determine the demand for factor inputs, competitive firms solve their dynamic profit maximization problems which, at time \( t \), have capital stock, \( K_t \), and the level of ISTC, \( Z_t \). The firm chooses \( K_{t+1}, n_{1t}, \) and \( n_{2t} \) optimally, taking all externalities and factor prices as given. As noted before, the firm might not be aware that \( n_{2t} \), employed from the point of lowering the price of future capital accumulation and hence future output, also has a spillover on current production. Let \( v(K_t, Z_t) \) denote the value function of the firm at time \( t \). The returns to investment in the credit markets are given by \( r_t \) and the wage is given by \( w_t \) at time period \( t \). The firm’s value function is given by:
\[ v(K_t, Z_t) = \max_{K_{t+1}, n_{1t}, n_{2t}} \left\{ [Y_t - w_t (n_{1t} + n_{2t})] (1 - \tau_k) - \frac{K_{t+1}}{Z_t} + \frac{1}{1 + r_{t+1}} v(K_{t+1}, Z_{t+1}) \right\}, \tag{24} \]

which it maximizes subject to (7).

The firm’s maximization exercise yields:\(^{20}\)

\[ \{K_{t+1}\} : \frac{1}{Z_t} = \left( \frac{1}{1 + r_{t+1}} \right) \frac{\alpha Y_{t+1} (1 - \tau_k)}{K_{t+1}} \]

\[ \{n_{1t}\} : w_t = \frac{(1 - \alpha) Y_t}{n_{1t}} \]

\[ \{n_{2t}\} : w_t (1 - \tau_k) = \left( \frac{\theta}{n_{2t}} \right) \sum_{j=0}^{\infty} \gamma_j \left[ \prod_{k=0}^{j} \frac{1}{1 + r_{t+k+1}} \right] I_{t+j+1}. \]

2.3.2 The Agents Problem

Since agents completely own the firms, they receive profits \( \pi_t \) as dividends \( \forall t \). Agents are also allowed to borrow and lend at the rate \( r_t \) by participating in the credit market. The agent maximizes (1) subject to the consumer budget constraint\(^ {21}\),

\[ a_{t+1} = \pi_t + (1 + r_t) a_t + w_t n_t (1 - \tau_n) - c_t, \tag{25} \]

and takes factor prices \( w_t \) and \( r_t \), profits \( \pi_t \), and all externalities as given.\(^ {22}\) Agents choose how much to consume, how much labor to supply, and their assets in period \( t + 1 \). Finally, the labor market clearing condition is given by

\[ n_t = n_{1t} + n_{2t}. \]

\(^{20}\)See Appendix B.

\(^{21}\)Because there is an unit mass of agents, any aggregate variable is equal to its per-capita magnitude.

\(^{22}\)Note that we are not taxing the dividends, \( \pi_t \), in the consumer budget constraint, but corporate capital income, \( [Y_t - w_t (n_{1t} + n_{2t})] \), as in Huffman (2008). Strictly speaking, \( \tau_k \) is therefore a corporate (profit) tax and not a tax on capital income. Taxing the firm’s corporate income at source, i.e., \( [Y_t - w_t (n_{1t} + n_{2t})] \), or at the level of the household, i.e., the dividend, \( \pi_t \), does not change the qualitative results of the model. These results are available from the authors on request.
2.3.3 First Order Conditions

The following is the Lagrangian for the agent,

\[ \mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[ \log c_t + \log(1 - n_t) + \lambda_t \left( \pi_t + (1 + r_t)a_t + w_t n_t(1 - \tau_n) - c_t - a_{t+1} \right) \right]. \]  

(26)

The optimization conditions with respect to \( c_t, a_{t+1}, \) and \( n_t \), are given by equations (27), (28), and (29) respectively:

\[ \{c_t\} : \frac{1}{c_t} = \lambda_t \]  

(27)

\[ \{a_{t+1}\} : \frac{\beta(1 + r_{t+1})}{c_{t+1}} = \frac{1}{c_t} \]  

(28)

\[ \{n_t\} : \frac{w_t(1 - \tau_n)}{c_t} = \frac{1}{1 - n_t} \]  

(29)

Once we substitute out for factor prices into the firm’s problem (equations (27), (28), and (29)), we obtain the following first order conditions for the competitive equilibrium:

\[ \{K_{t+1}\} : \frac{1}{c_tZ_t} = \frac{\alpha \beta Y_{t+1}(1 - \tau_k)}{c_{t+1} K_{t+1}} \]  

(30)

\[ \{n_{1t}\} : \frac{1}{1 - n_t} = \frac{(1 - \alpha)Y_t(1 - \tau_n)}{c_t n_{1t}} \]  

(31)

\[ \{n_{2t}\} : \frac{1}{1 - n_t} = \left( \frac{\beta \theta}{n_{2t}} \right) \left( \frac{1 - \tau_n}{1 - \tau_k} \right) \sum_{j=0}^{\infty} \beta^j \gamma^j \frac{I_{t+j+1}}{c_{t+j+1}}. \]  

(32)

Equation (30) is the standard Euler equation for the household. Compared to equation (12) in the planner’s problem, the effect of the stock-externalities because of \( K \) and \( G \) on the inter-temporal savings decision is absent. This is because agents do not internalize this externality. Equations (31) and (32) equate the after tax wage to the MRS between consumption and leisure. Compared to equations (14) and (15) respectively, the additional terms due to the externalities are also absent because the agents take the externality from \( n_2 \) as given.

2.3.4 Decision Rules

Based on the above first order conditions, Lemma 3 states the optimal decision rules for the agents.
Lemma 3 \( C_t, I_t, n_t, n_{1t}, n_{2t} \) are given by (33), (34), (35), where \( 0 < \Phi_{CE} < 1 \) is given by (36), and \( 0 < x_{CE} < 1 \) given by (37) is a constant.

\[
C_t = \Phi_{CE}AY_t, \quad I_t = (1 - \Phi_{CE})AY_t
\]  \hspace{1cm} (33)

where, \( A = \alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_n) - \frac{\alpha \beta^2 \theta (\tau_n - \tau_k)}{(1 - \beta \gamma)} \)

\[
n_t = n_{CE} = \frac{(1 - \alpha)(1 - \tau_n)}{(1 - \alpha)(1 - \tau_n) + x_{CE} \Phi_{CE} A}, \hspace{1cm} (34)
\]

\[
n_{1CE} = x_{CE} n_{CE}, \quad n_{2CE} = (1 - x_{CE}) n_{CE}, \hspace{1cm} (35)
\]

where \( \Phi_{CE} \) is given by

\[
\Phi_{CE} = 1 - \frac{\alpha \beta (1 - \tau_k)}{A}, \hspace{1cm} (36)
\]

and \( x_{CE} \) is given by

\[
x_{CE} = \frac{(1 - \alpha)(1 - \beta \gamma)}{\alpha \beta^2 \theta + (1 - \alpha)(1 - \beta \gamma)}. \hspace{1cm} (37)
\]

Proof. See Appendix B for details. ■

The above decision rules imply that depending upon the parameter values, there exists a feasible range of values that \( \tau_k \) and \( \tau_n \) can take such that

\[ 0 < A, \Phi_{CE}, n_{CE} < 1, \]

are true.\(^{23}\) The relationship between growth rates at the balanced growth path for private capital, public capital, output and investment specific technological change are identical to that for the planner’s version, as given in Lemma 2.

2.3.5 The Competitive Equilibrium Growth Rate

We would like to ascertain under what conditions the competitive equilibrium allocations implement the planner’s growth rate. From equations (33), (34), (35), (36), and (37), the growth rate under the competitive equilibrium is given by:

\[
g_{z_{CE}} = \left[ B \underbrace{n_{2CE}^\theta}_{\text{Labor factor}} \underbrace{(1 - A)^\mu (A)^{1 - \mu} (1 - \Phi_{CE})^{1 - \mu}}_{\text{Capital factor}} \right]^{\frac{1}{2 - \gamma}}. \hspace{1cm} (38)
\]

\(^{23}\)Restriction (53) in Appendix B is required on \( \tau_n \) and \( \tau_k \) for \( 0 < A, \Phi_{CE}, n_{CE} < 1. \)
The growth rate, $g_{z_{CE}}$, depends on two factors: a labor factor, $n_{2_{CE}}^\theta$, and a capital factor given by $\Upsilon = \left\{ (1 - A)^\mu (A)^{1-\mu} (1 - \Phi_{CE})^{1-\mu} \right\}^{1-\gamma}$, both of which depend on factor income taxes, $\tau_k$ and $\tau_n$.

The capital factor  In Appendix C we show that

$$\Upsilon = \left\{ \left[ (1 - \beta \gamma) \left[ (1 - \alpha) (\tau_n - \tau_k) + \tau_k \right] + \alpha \beta^2 \theta (\tau_n - \tau_k) \right]^{\mu} \alpha \beta (1 - \tau_k) \right\}^{1-\gamma}, \quad (39)$$

i.e., the capital factor, $\Upsilon$, unambiguously increases in $\tau_n$ and the tax gap $(\tau_n - \tau_k)$. We also show that $\Upsilon$ also decreases in $\tau_k$ as long as the following sufficient condition is satisfied:

$$1 - \beta \gamma < \beta^2 \theta. \quad (40)$$

Importantly, when $\tau_k = 1$, $\Upsilon = 0$, and there is no growth.\textsuperscript{24}

The labor factor  The research labor input $n_{2_{CE}}$ is given by

$$n_{2_{CE}} = (1 - x_{CE}) n_{CE}, \quad (41)$$

where

$$n_{CE} = \frac{\alpha^2 \beta \theta}{\alpha^2 \beta \theta + (1 - \alpha)(1 - \beta \gamma)}, \quad (1 - x_{CE}) = \frac{\alpha^2 \beta \theta + (1 - \alpha)(1 - \beta \gamma)}{\alpha^2 \beta \theta + (1 - \alpha)(1 - \beta \gamma)}.$$

Clearly, $(1 - x_{CE})$ is independent on factor income taxes. Hence, a change in taxes therefore affects $n_{2_{CE}}$ only through $n_{CE}$. In Appendix C, we show that

$$n_{CE} = \frac{(1 - \alpha) \left[ \alpha \beta^2 \theta + (1 - \alpha)(1 - \beta \gamma) \right]}{(1 - \alpha) \left[ \alpha \beta^2 \theta + (1 - \alpha)(1 - \beta \gamma) \right] + \Psi}, \quad (42)$$

where

$$\Psi = \frac{(1 - \alpha)}{(1 - \tau_n)} \left[ (1 - \beta \gamma) \left\{ \alpha (1 - \beta) + (1 - \alpha) + \alpha (1 - \beta) (\tau_n - \tau_k) - (1 - \alpha \beta) \tau_n \right\} - \alpha \beta^2 \theta (\tau_n - \tau_k) \right].$$

\textsuperscript{24}Equation (40) can be re-written as, $\beta \theta + \gamma > \frac{1}{\beta}$, which implies that if the returns from allocating resources to ISTC are greater than the returns from investing in an asset (which equals $\frac{1}{\beta}$ in the steady state), an increase in the tax on capital income will depress the capital factor.
As shown in Appendix C, if condition (40) holds, $\Psi$ decreases in the tax gap $(\tau_n - \tau_k)$ and $\tau_n$, and increases in $\tau_k$. As a result, $n_{CE}$ increases in $(\tau_n - \tau_k)$ and $\tau_n$, and decreases in $\tau_k$. The effect of a change in the factor income tax gap $(\tau_n - \tau_k)$ and $\tau_n$ on labor supply, and therefore the labor factor, can be summarized by Lemma 4.

**Lemma 4** Suppose

$$1 - \beta \gamma < \beta^2 \theta.$$  

Then, (i) An increase in $\tau_k$ lowers the capital factor, i.e., $\frac{\partial \tau}{\partial \tau_k} < 0$. (ii) A rise in the labor income tax rate, $\tau_n$, and the factor income tax gap, $(\tau_n - \tau_k)$, increases the labor factor, i.e., $\frac{\partial n_{CE}}{\partial (\tau_n - \tau_k)} > 0$, $\frac{\partial n_{CE}}{\partial \tau_n} > 0$, and $\frac{\partial n_{CE}}{\partial \tau_k} < 0 \implies \frac{\partial n_{CE}^\theta}{\partial (\tau_n - \tau_k)} > 0$ and $\frac{\partial n_{CE}^\theta}{\partial \tau_n} > 0$.

**Proof.** See Appendix C. ■

Lemma 4 implies that a smaller $\gamma$ makes $n_{CE}$ increase by more for an increase in $\tau_n$. Proposition 2 summarizes the effect of tax rates on the competitive equilibrium growth rate.

**Proposition 2** Since the labor factor and capital factor are increasing in $\tau_n$ and decreasing in $\tau_k$, the competitive equilibrium growth rate, $g_{CE}$, is increasing in the factor income tax gap, $(\tau_n - \tau_k)$. An increase in $g_{CE}$, is obtained by increasing $(\tau_n - \tau_k)$. The factor income tax gap must be increased by either raising $\tau_n$, or lowering $\tau_k$, or both.

**Proof.** Follows from $\frac{\partial \tau}{\partial \tau_n} > 0$, $\frac{\partial \tau}{\partial (\tau_n - \tau_k)} > 0$, and Lemma 4. ■

The intuition behind the above proposition is as follows. Assume that the sufficient condition, (40), holds, because of a high value of $\theta$.\(^{25}\) Since the competitive equilibrium growth rate $g_{CE}$ increases in the factor income tax gap $(\tau_n - \tau_k)$, an increase in $\tau_k$ requires a higher $\tau_n$ to decentralize the same growth rate $g_{CE}$. This suggests that fiscal policy has an offsetting effect on the agent’s growth rate. A higher $\tau_k$ lowers the capital factor $\Psi$. To mitigate the negative effect of $\tau_k$ on $\Psi$, we have to raise $\tau_n$ which not only has a positive effect on the labor factor $n_{2CE}^\theta$, but also on $\Psi$. This happens because although the substitution effect induces an increase in leisure, $1 - n_{CE}$, due to an increase in $\tau_n$ (the after tax wage has gone down), labor supply (and therefore the labor factor) increases because of the stronger income effect induced by ISTC. In particular, ISTC leads to an additional income effect, through consumption, compared to a case where ISTC is not endogenous. This can be seen from the below equation for, $\Phi_{CE}A$,

$$\Phi_{CE}A = \alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_n) - \frac{\alpha \beta^2 \theta (\tau_n - \tau_k)}{(1 - \beta \gamma)} - \alpha \beta (1 - \tau_k).$$

\(^{25}\)We can implement the planner’s allocations even if equation (40) is violated. However we assume this to be our main case because it is satisfied with reasonable parameter values. In the numerical section, we explore both possibilities.
When $\theta > 0$, an increase in $\tau_n$ lowers after-tax labor income and lowers consumption even more. Relative to the case where there is no endogenous ISTC, the after tax fraction of income allocated for private consumption, $\Phi_{CE}A$, is lowered by the term, $\frac{\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta_\gamma)}$. The drop in consumption causes leisure to fall more (relative to case when $\theta = 0$) and labor supply to increase by more (which follows from equation (29), where $c_t = w_t (1-\tau_n) (1-n_{CE})$).

An increase in $n_{CE}$ in turn implies a higher $n_{2CE}$, from equation (41) and noting that $1-x_{CE}$ is also increasing in $\theta$. Hence the labor factor rises. A rise in the labor factor increases $Z_{t+1}$ which increases capital accumulation and therefore future output and future consumption.

Without ISTC, it could be possible that labor supply falls if the substitution effect dominates the income effect. However with ISTC, the income effect dominates the substitution effect and labor supply, $n_{CE}$, rises.

Fiscal policy also offsets the effect of taxes because public capital crowds out private capital in our model. This is because, from (39) we know that $(1-A)$ increases in $\tau_k$ whereas, $A (1-\Phi_{CE})$ decreases. Proposition 2 therefore suggest that we can raise $g_z_{CE}$ to match the planner’s growth rate by increasing the factor income tax gap $(\tau_n-\tau_k)$ from an initial point where $g_z_{CE} < g_zp$. Further, since ISTC in our model is endogenous, a higher $\theta$ causes a bigger increase in $n_{CE}$ and therefore $n_{2CE}$. This translates into a bigger increase in $g_z_{CE}$ for a given increase in $\tau_n$. In terms of the capital factor, since the representative agent under-accumulates private capital because of taking the effect of $\Upsilon$ on $Z$ as given, $\tau_k$ must be lowered. As a result, an increase in the tax gap by raising $\tau_n$ and lowering $\tau_k$ increases $g_z_{CE}$.

In sum, as to which effect dominates depends on the sufficient condition, (40), identified in Proposition 2. For instance, the sufficient condition, (40) is also satisfied for higher values of $\gamma$, which in turn strengthens the income effect channel because of ISTC on labor supply, for an increase in $\tau_n$. A higher $\gamma$ also means that the weight on the capital stock externalities is weaker. As a result, the net effect is that a high $\gamma$ and a high $\theta$ makes the labor factor increase for an increase in $\tau_n$. Since condition (40), which is satisfied for a high $\gamma$ and $\theta$, causes the capital factor to fall when $\tau_k$ increases, the planner’s growth rate is decentralized using a combination of a high $\tau_n$ and a low $\tau_k$.

**The Effect of $\gamma$ and $\xi$**

Given the sufficient condition, (40), we graphically characterize the implementation of the planner’s growth rate (given in Proposition 1 and 2) to illustrate the effect of a change in the externality parameters on the factor income tax gap required to decentralize the planner’s equilibrium growth rate. First, as $\xi$ increases, the spillover from $n_2$ in final goods production increases. The planner therefore allocates more labor towards $n_2$. The increases the planner’s equilibrium growth rate $g_zp$. This is shown in Figure
where we assume $\tau_k = \overline{\tau}_k$, which yields a zero factor income tax gap. Starting with $\xi = 0$, the factor income tax gap required to decentralize the planner’s equilibrium growth rate corresponds to point ‘a’. Now suppose $\xi$ increases arbitrarily. Since the agent’s allocations do not depend, on $\xi$, the competitive equilibrium growth rate $g_{zCE}$ does not change. We know from Proposition 2 that in order to match a higher $g_{zP}$, the labor income tax must be increased for a given $\overline{\tau}_k$, which causes an increase in the factor income tax gap. The new factor income tax gap corresponds to point ‘b’.

[Insert Figure 4]

Now suppose $\gamma$ is arbitrarily increased from a low to a high value. The spillover from the capital factor for a higher $\gamma$ is low. This makes ISTC more persistent. This increases the growth rate of the planner. At the same time, the growth rate of the agent also increases because the weight on the externality from the capital factor is lower for a higher $\gamma$. This reduces the extent of under-accumulation of capital. As a result, the equilibrium factor income tax gap $(\tau_n - \tau_k)$ decreases. This is illustrated in Figure 5. Point ‘a’ corresponds to $\gamma = 0.5$ and point ‘b’ corresponds to $\gamma = 0.8$. The crucial difference is that both $\gamma$ and $\xi$ raise the planner’s growth rate, whereas only $\gamma$ raises the competitive equilibrium growth rate.

[Insert Figure 5]

3 Numerical Examples

In this section, we consider a few numerical examples to analyze how the magnitude of externalities $(\gamma, \xi)$ affect the factor income tax gap. In particular, we consider two examples: one where the sufficient condition given by equation (40) holds and another where the condition is violated. We first calibrate out factor income tax gaps that are broadly consistent with Figures 1 - 3. We start with two arbitrary values of $\gamma = \{0.1, 0.9\}$ corresponding to the case where the externality from the stock externalities are high and low, respectively. Then, starting with $\xi = 0$, we gradually raise $\xi$ to make it arbitrarily large, and calibrate out the factor income tax gap, $(\tau_n - \tau_k)$, for each change in $\xi$. In all the numerical experiments we fix $\alpha = 0.35$ and $\beta = 0.95$ as in Huffman (2008).

Suppose we set $\gamma = 0.9$. Other parameters are arbitrarily chosen as: $\mu = 0.5, \theta = 0.8$, and $B = 1.72$ which yields a growth rate of 2.5% as in Figure 1. This set of parameters

\footnote{We have chosen the parameters such that $n_2$ has a large weight on $Z$ and the externality from public and private capital on $Z$ has a small weightage. In addition, the effect of public capital to output ratio on $Z$ is moderate.}
satisfy condition (40). Table 1 summarizes the values of \( \tau_n \) for each value of \( \tau_k \) such that \( g_{zCE} = g_{zp} \) across different values of \( \xi = \{0, 0.1, 0.2\} \) and range \( \tau_k = \{-0.1, 0, 0.1, 0.2\} \).

Two observations emerge. First, as \( \xi \) increases, the equilibrium factor income tax gap needed to decentralize the planners growth increases. This is because, an increase in \( \xi \) increases the spillover from \( n_2 \) in … nal goods production. The planner therefore allocates more labor towards \( n_2 \): This increases \( g_{zP} \). To match a higher \( g_{zp} \), the labor income tax must be increased for a given \( \tau_k \), which causes an increase in the factor income tax gap. This requires \( \tau_n > \tau_k \) to decentralize the planners growth rate.

<table>
<thead>
<tr>
<th>( \tau_k )</th>
<th>( \tau_n - \tau_k (\xi = 0) )</th>
<th>( \tau_n - \tau_k (\xi = 0.1) )</th>
<th>( \tau_n - \tau_k (\xi = 0.2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1</td>
<td>0.445</td>
<td>0.45</td>
<td>0.46</td>
</tr>
<tr>
<td>0.0</td>
<td>0.415</td>
<td>0.42</td>
<td>0.42</td>
</tr>
<tr>
<td>0.1</td>
<td>0.37</td>
<td>0.38</td>
<td>0.385</td>
</tr>
<tr>
<td>0.2</td>
<td>0.335</td>
<td>0.345</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Table 1: Equilibrium factor income tax gaps under \( \gamma = 0.9 \)

When \( \gamma \) is high, spillover from the capital factor is low. This also makes ISTC more persistent. This increases the growth rate of the planner. To raise the competitive equilibrium growth rate, a reduction in the tax on capital income raises the capital factor and an increase in the labor income tax raises the labor factor. At the same time, since the effect of the externality from the capital factor is low, and the effect of public capital is low, \( (\tau_n - \tau_k) \) is narrower.\(^{27}\)

Suppose now \( \gamma = 0.1 \). Other parameters are arbitrarily chosen to be: \( \mu = 0.9, \theta = 0.1, \) and \( B = 1.74 \) which yields a growth rate of 2.5% which is roughly equal to the average growth rate for our sample of OECD countries in Figure 1.\(^{28}\) This set of parameters violates condition (40).

Table 2 summarizes the values of \( \tau_n \) for each value of \( \tau_k \) such that \( g_{zCE} = g_{zp} \) across different values of \( \xi = \{0, 0.1, 0.2\} \),and different values of \( \tau_k = \{0.3, 0.5, 0.7, 0.9\} \). Observe that not only are the individual factor income tax combinations higher than in Table 1, for lower \( \tau_n \), the tax gaps \( (\tau_n - \tau_k) \) are also higher. The tax gaps also become negative, i.e., \( \tau_k > \tau_n \), for higher values of \( \tau_n \).

\(^{27}\)We show in Appendix D that when there are no externalities, equal factor income taxes always yield the optimal growth rate from the planner’s problem. Hence, the factor income tax gap is zero.

\(^{28}\)Our choice of parameters are now such that \( n_2 \) has a small weightage on \( Z \) while the externality from public and private capital on \( Z \) has a high weightage. In addition, the effect of public capital to output ratio on \( Z \) is very high while that of private capital to output ratio is very small.
Table 2: Equilibrium factor income tax gaps under $\gamma = 0.1$

<table>
<thead>
<tr>
<th>$\tau_k$</th>
<th>$\tau_n - \tau_k (\xi = 0)$</th>
<th>$\tau_n - \tau_k (\xi = 0.1)$</th>
<th>$\tau_n - \tau_k (\xi = 0.2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.58</td>
<td>0.59</td>
<td>0.6</td>
</tr>
<tr>
<td>0.5</td>
<td>0.31</td>
<td>0.33</td>
<td>0.34</td>
</tr>
<tr>
<td>0.7</td>
<td>0.07</td>
<td>0.09</td>
<td>0.1</td>
</tr>
<tr>
<td>0.9</td>
<td>-0.09</td>
<td>-0.07</td>
<td>-0.05</td>
</tr>
</tbody>
</table>

First, we again observe that for an increase in $\xi$, there is a marginal increase in the tax gap $(\tau_n - \tau_k)$.

Second, as $\tau_k$ increases, the value of $\tau_n$ that decentralizes the planner’s growth rate for the given value of $\tau_k$ also increases. We also observe that as $\tau_k$ increases, the tax gap $(\tau_n - \tau_k)$ starts narrowing. For very high values of $\tau_k$ the corresponding value of $\tau_n$ could be smaller, such that the rankings get reversed and $\tau_n - \tau_k$ becomes negative. This is because, the condition given by equation (40) is now violated. The intuition is as follows. For a low value of $\gamma$, the income effect channel because of ISTC on labor supply is weakened, for an increase in $\tau_n$. Therefore, an increase in $\tau_n$ on the net, may not increase the labor factor. In addition, a low value of $\gamma$ also means that the weight on the capital stock externalities is stronger. Since the capital stock externalities consist of public and private capital, a higher $\tau_k$ may not have offsetting effects on the labor and capital factor, as in the previous case where the sufficient condition (40) is satisfied. As a result, a high $\tau_k$ and a low $\tau_n$ may implement the planner’s growth rate. This is consistent with Figure 2 where we generally observe high $\tau_k$ economies also have a lower $\tau_n$ (e.g., US, UK, Japan, and Denmark). Thus Table 1 is able to qualitatively match the factor income tax gaps in these economies even though the calibrated factor income tax gaps are smaller in magnitude in this experiment.

The numerical results above identify why the externalities are crucial for our results. While our model yields equilibrium factor income tax gaps that implement the planner’s growth rate, a change in the magnitude of the externalities widen/narrows the equilibrium factor income tax gaps required to implement the planner’s growth rate. Since Figures 1 - 3 show that both cases occur this provides a theoretical justification for our framework.

4 Conclusion

This paper constructs a simple and tractable endogenous growth model with endogenous investment specific technological change. Our theoretical model is motivated by the empirical observation that advanced economies – which are presumed to be on their balanced growth paths and therefore experience similar or identical growth rates – have widely varying factor income tax combinations. This observation is puzzling since it is incompatible with a
standard model of endogenous growth: in the standard model, fiscal policy can have large
growth effects through its impact on the economy’s investment rate. We see our contribu-
tion as providing an alterative, but compatible, explanation based on the fact that different
combinations of taxes can generate the same growth rate. Our innovation is to incorporate
aggregate public and private capital stock externalities in ISTC, as well as positive spillovers
driven by specialized labor in the research sector to explain this puzzle.

We characterize the balanced growth path of the economy corresponding to the socially
efficient allocation for a fixed tax rate and derive conditions under which the competitive
equilibrium can implement this growth rate. Our general result is that to the extent that
spillovers from a specialized labor input and the public and private capital stocks exist, an
increase in these spillover from specialized labor, and a decrease in the spillover from public
and private capital, increases the planner’s growth rate, and therefore increases the factor
income tax gap required to implement the higher planner’s growth rate. Conversely, for
a given level of externalities, maintaining the constancy of growth also requires different
combinations of factor income taxes. Finally, when there are no externalities, equal factor
income taxes always yield the optimal growth rate from the planner’s problem. Hence, the
factor income tax gap is zero. In the numerical section, we show that we can qualitatively
match the factor income tax gaps observed in the data.

In the future, we hope to extend our framework by comparing the growth and welfare
effects of optimal tax policy on research and development versus funding public investment.
In addition, our model characterizes the optimal tax rate along the balanced growth path.
Future work can model the transitional dynamics.
References


Technical Appendix (Not for Publication)

Appendix A: The Planner’s Version

The following first order conditions are therefore obtained with respect to \( C_t, K_{t+1}, Z_{t+1}, n_{1t}, \) and \( n_{2t} \):

\[
\{C_t\} : \frac{1}{C_t} = \lambda_{1t}
\]

\[
\{K_{t+1}\} : \frac{1}{C_t Z_t} = \frac{\alpha \beta Y_{t+1} (1 - \tau)}{C_{t+1} K_{t+1}} + \beta (1 - \gamma) (1 - \mu) \lambda_{2t+1} \frac{Z_{t+2}}{K_{t+1}} - \beta^2 \lambda_{2t+2} (1 - \gamma) \alpha \frac{Z_{t+3}}{K_{t+1}}
\]

\[
\{Z_{t+1}\} : \lambda_{2t} = \beta \lambda_{2t+1} \gamma \frac{Z_{t+2}}{Z_{t+1}} + \frac{\beta}{Z_{t+1}} \left( \frac{I_{t+1}}{C_{t+1}} \right) + \beta^2 \lambda_{2t+2} \mu (1 - \gamma) \frac{Z_{t+3}}{Z_{t+1}}
\]

\[
\{n_{1t}\} : \frac{1}{1 - n_t} = \frac{(1 - \alpha) Y_t (1 - \tau)}{C_t n_{1t}} - \beta \lambda_{2t+1} (1 - \gamma) (1 - \alpha) \frac{Z_{t+2}}{n_{1t}}
\]

and,

\[
\{n_{2t}\} : \frac{1}{1 - n_t} = \frac{(1 - \alpha) \xi Y_t (1 - \tau)}{C_t n_{2t}} + \lambda_{2t} \theta \frac{Z_{t+1}}{n_{2t}} - \beta \lambda_{2t+1} (1 - \gamma) \xi (1 - \alpha) \frac{Z_{t+2}}{n_{2t}}.
\]

We will use the method of undetermined coefficients. We have assumed,

\[
C_t = \Phi_P Y_t (1 - \tau), \quad I_t = (1 - \Phi_P) Y_t (1 - \tau), \quad I^g_t = \tau Y_t
\]

and

\[
n_1 = x n, \quad n_2 = (1 - x) n.
\]

From \( \{Z_{t+1}\} \),

\[
\{Z_{t+1}\} : Z_{t+1} \lambda_{2t} = \beta \lambda_{2t+1} \gamma Z_{t+2} + \beta^2 \lambda_{2t+2} \mu (1 - \gamma) \frac{Z_{t+3}}{Z_{t+1}} + \beta \left( \frac{1 - \Phi_P}{\Phi_P} \right).
\]

From \( \{n_{1t}\} \),

\[
\{n_{1t}\} : \frac{1}{1 - n_t} = \frac{(1 - \alpha) Y_t (1 - \tau)}{C_t n_{1t}} - \beta \lambda_{2t+1} (1 - \gamma) (1 - \alpha) \frac{Z_{t+2}}{n_{1t}},
\]

which implies

\[
\frac{xp n_P}{1 - n_P} = \frac{(1 - \alpha)}{\Phi_P} - \beta (1 - \gamma) (1 - \alpha) \lambda_{2t+1} Z_{t+2}.
\]
Therefore,

\[ \lambda_{2t+1}Z_{t+2} = \frac{(1-\alpha) - \frac{x_{pnp}}{1-n_p}}{\beta(1-\gamma)(1-\alpha)} . \]

This also implies for constant decision rules and a constant labor supply in every time period,

\[ \lambda_{2i-1}Z_i = \frac{(1-\alpha) - \frac{x_{pnp}}{1-n_p}}{\beta(1-\gamma)(1-\alpha)}, \quad \text{for all } i = t. \]

Substituting in \( \{Z_{t+1}\} \),

\[ \frac{(1-\alpha) - \frac{x_{pnp}}{1-n_p}}{\beta(1-\gamma)(1-\alpha)} \left[ 1 - \beta\gamma - \beta^2\mu(1-\gamma) \right] = \beta \left( \frac{1 - \Phi_P}{\Phi_P} \right). \]

This on rearranging gives

\[ \frac{n_p}{1-n_p} = \frac{(1-\alpha) \left[ 1 - \beta\gamma - \beta^2\mu(1-\gamma) - \beta^2(1-\gamma)(1-\Phi_P) \right]}{x_P\Phi_P \left[ 1 - \beta\gamma - \beta^2\mu(1-\gamma) \right]}. \]

Hence,

\[ n_p = \frac{(1-\alpha) \left[ 1 - \beta\gamma - \beta^2\mu(1-\gamma) - \beta^2(1-\gamma)(1-\Phi_P) \right]}{(1-\alpha) \left[ 1 - \beta\gamma - \beta^2\mu(1-\gamma) \right] + x_P\Phi_P \left[ 1 - \beta\gamma - \beta^2\mu(1-\gamma) \right]}. \]

Using

\[ \frac{n_p}{1-n_p} = \frac{(1-\alpha) \left[ 1 - \beta\gamma - \beta^2\mu(1-\gamma) - \beta^2(1-\gamma)(1-\Phi_P) \right]}{x_P\Phi_P \left[ 1 - \beta\gamma - \beta^2\mu(1-\gamma) \right]}, \]

we get

\[ \lambda_{2i-1}Z_i = \left( \frac{1 - \Phi_P}{\Phi_P} \right) \left( \frac{\beta}{1 - \beta\gamma - \beta^2\mu(1-\gamma)} \right). \]

From \( \{n_{2t}\} \)

\[ \{n_{2t}\} : \frac{(1-x_P) n_p}{1-n_p} = \frac{(1-\alpha)\xi}{\Phi_P} + \theta \lambda_{2t}Z_{t+1} - \beta(1-\gamma)\xi(1-\alpha) \lambda_{2t+1}Z_{t+2}. \]

This implies

\[ \frac{(1-x_P) n_p}{1-n_p} = \frac{(1-\alpha)\xi}{\Phi_P} + \left[ \theta - \beta(1-\gamma)\xi(1-\alpha) \right] \left( \frac{1 - \Phi_P}{\Phi_P} \right) \left( \frac{\beta}{1 - \beta\gamma - \beta^2\mu(1-\gamma)} \right). \]

Since

\[ \frac{n_p}{1-n_p} = \frac{(1-\alpha) \left[ 1 - \beta\gamma - \beta^2\mu(1-\gamma) - \beta^2(1-\gamma)(1-\Phi_P) \right]}{x_P\Phi_P \left[ 1 - \beta\gamma - \beta^2\mu(1-\gamma) \right]}, \]

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we get

\[
\frac{1 - x_P}{x_P} (1 - \alpha) \left[ 1 - \beta \gamma - \beta^2 \mu (1 - \gamma) - \beta^2 (1 - \gamma) (1 - \Phi_P) \right] \\
\frac{\Phi_P [1 - \beta \gamma - \beta^2 \mu (1 - \gamma)]}{(1 - \alpha) \left[ 1 - \beta \gamma - \beta^2 \mu (1 - \gamma) \right]}
\]

\[
= \frac{(1 - \alpha) \left[ (1 - \beta \gamma) - \beta^2 \mu (1 - \gamma) - \beta^2 (1 - \gamma) (1 - \Phi_P) \right] + \beta \theta (1 - \Phi_P)}{(1 - \alpha) \left[ 1 - \beta \gamma - \beta^2 \mu (1 - \gamma) \right]}
\]

Hence,

\[
x_P = \frac{(1 - \alpha) \left[ (1 - \beta \gamma) - \beta^2 \mu (1 - \gamma) - \beta^2 (1 - \gamma) (1 - \Phi_P) \right] + \beta \theta (1 - \Phi_P)}{(1 - \alpha) (1 + \xi) \left[ (1 - \beta \gamma) - \beta^2 \mu (1 - \gamma) - \beta^2 (1 - \gamma) (1 - \Phi_P) \right] + \beta \theta (1 - \Phi_P)}.
\]

Finally, from \( \{K_{t+1}\} \),

\[
\{K_{t+1}\} : \frac{1}{C_t Z_t} = \frac{\alpha \beta Y_{t+1} (1 - \tau)}{C_{t+1} K_{t+1}} + \beta (1 - \gamma) (1 - \mu) \lambda_{2t+1} \frac{Z_{t+2}}{K_{t+1}} - \beta^2 \lambda_{2t+2} (1 - \gamma) \alpha \frac{Z_{t+3}}{K_{t+1}}
\]

\[
\frac{1}{\Phi_P Y_t Z_t} = \frac{\alpha \beta}{\Phi_P (1 - \Phi_P) Y_t Z_t} + \frac{\beta (1 - \gamma) (1 - \mu)}{(1 - \Phi_P) Y_t Z_t} \left( \frac{1 - \Phi_P}{\Phi_P} \right)
\]

\[
+ \frac{\beta^2 (1 - \gamma) (1 - \mu) \gamma}{(1 - \Phi_P) Y_t Z_t} \lambda_{2t+2} Z_{t+3} - \frac{\beta^2 \alpha (1 - \gamma)}{(1 - \Phi_P) Y_t Z_t} \lambda_{2t+2} Z_{t+3}.
\]

Since

\[
\lambda_{2t-1} Z_t = \left( \frac{1 - \Phi_P}{\Phi_P} \right) \left( \frac{\beta}{1 - \beta \gamma - \beta^2 \mu (1 - \gamma)} \right),
\]

we get

\[
1 = \frac{\alpha \beta}{(1 - \Phi_P)} + \beta (1 - \gamma) (1 - \mu) + \frac{\beta^2 (1 - \gamma) (1 - \mu) [\gamma + \beta \mu (1 - \gamma)]}{[(1 - \beta \gamma) - \beta^2 \mu (1 - \gamma)]}
\]

\[
- \frac{\beta^3 (1 - \gamma) \alpha}{[(1 - \beta \gamma) - \beta^2 \mu (1 - \gamma)]}.
\]

On simplifying we get

\[
1 - \Phi_P = \frac{\alpha \beta [(1 - \beta \gamma) - \beta^2 \mu (1 - \gamma)]}{(1 - \beta \gamma) - \beta^2 (1 - \gamma) + \alpha \beta^3 (1 - \gamma)}.
\]
Conditions

As long as \((1 - \Phi_P) < 1\), we will get

\[
0 < x_P < 1.
\]

We know,

\[
(1 - \Phi_P) = \frac{\alpha \beta \left[ (1 - \beta \gamma) - \beta^2 \mu (1 - \gamma) \right]}{(1 - \beta \gamma) - \beta^2 (1 - \gamma) + \alpha \beta^3 (1 - \gamma)}
\]

Since,

\[
0 < (1 - \beta \gamma) - \beta^2 (1 - \gamma) = (1 - \beta) [1 + \beta (1 - \gamma)], \quad (1 - \Phi_P) > 0.
\]

To show

\[
(1 - \Phi_P) = \frac{\alpha \beta \left[ (1 - \beta \gamma) - \beta^2 \mu (1 - \gamma) \right]}{(1 - \beta \gamma) - \beta^2 (1 - \gamma) + \alpha \beta^3 (1 - \gamma)} < 1,
\]

we require,

\[
(1 - \beta \gamma) - \beta^2 (1 - \gamma) + \alpha \beta^3 (1 - \gamma)
\]

\[
> \alpha \beta \left[ (1 - \beta \gamma) - \beta^2 \mu (1 - \gamma) \right],
\]

or,

\[
(1 - \beta \gamma) (1 - \alpha \beta) - \beta^2 (1 - \gamma) + \alpha \beta^3 (1 - \gamma) + \alpha \beta^3 \mu (1 - \gamma) > 0.
\]

Rewriting the above LHS we get

\[
(1 - \beta \gamma) (1 - \alpha \beta) - \beta^2 (1 - \gamma) [1 - \alpha \beta (1 + \mu)].
\]

Since,

\[
(1 - \beta \gamma) > \beta^2 (1 - \gamma)
\]

and

\[
1 - \alpha \beta > 1 - \alpha \beta (1 + \mu),
\]

therefore

\[
(1 - \Phi_P) \in (0, 1).
\]

Since,

\[
x_P = \frac{(1 - \alpha) \left[ 1 - \beta \gamma - \beta^2 \mu (1 - \gamma) - \beta^2 (1 - \gamma) (1 - \Phi_P) \right]}{(1 - \alpha) (1 + \xi) \left[ (1 - \beta \gamma) - \beta^2 \mu (1 - \gamma) - \beta^2 (1 - \gamma) (1 - \Phi_P) \right] + \beta \theta (1 - \Phi_P)}
\]
Therefore

\[ 0 < x_P, \Phi_P < 1. \]

Finally, since

\[
n_P = \frac{(1 - \alpha) \left[ 1 - \beta \gamma - \beta^2 \mu (1 - \gamma) - \beta^2 (1 - \gamma) (1 - \Phi_P) \right]}{(1 - \alpha) \left[ 1 - \beta \gamma - \beta^2 \mu (1 - \gamma) - \beta^2 (1 - \gamma) (1 - \Phi_P) \right] + x_P \Phi_P \left[ 1 - \beta \gamma - \beta^2 \mu (1 - \gamma) \right]} + \frac{x_P \Phi_P}{1 - \alpha}
\]

and,

\[ 0 < x_P, \Phi_P < 1, \]

therefore,

\[ 0 < n_P < 1. \]

**Growth rate at the BGP**

\[ Y_t = A \left( n_{2t}^{1-\alpha} \right)^\xi K_t^{\alpha} n_{1t}^{1-\alpha} \]

On the balanced growth path (BGP),

\[ g_{yp} = g_{yp+1} = \frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}^{\alpha}}{K_t^{\alpha}} = g_{kP_{t+1}}^{\alpha} = g_{kP}^{\alpha}, \]

and \[ g_{kp} = \frac{K_{t+1}}{K_t} = \frac{I_t Z_t}{I_{t-1} Z_{t-1}} = g_{yp} \cdot g_{zp}. \]

Hence,

\[ g_{yp} = g_{zp}^{\alpha} \cdot g_{kp} = g_{gp} = g_{zp}^{1-\alpha}. \]

**Proposition 1**

\[ \hat{g}_{zp} = \left[ \hat{M}_P \{ (\tau)^\mu (1 - \tau)^{1-\mu} \} (1-\gamma) \right]^{\frac{1}{1-\gamma}}, \]

\[ \frac{\partial \hat{g}_{zp}}{\partial \tau} = 0, \]

\[ \Rightarrow \mu (\tau)^{\mu-1} (1-\tau)^{1-\mu} - (1-\mu) (\tau)^{\mu} (1-\tau)^{-\mu} = 0 \]

\[ \Rightarrow \tau = \mu. \]
Appendix B: Agent’s Version

We assume $\delta = 1$. The FOCs are:

$$\{K_{t+1}\} : \frac{-1}{Z_t} + \left(\frac{1}{1 + r_{t+1}}\right) \frac{\alpha Y_{t+1}(1 - \tau_k)}{K_{t+1}} = 0.$$  

$$\Rightarrow \{K_{t+1}\} : \frac{1}{Z_t} = \left(\frac{1}{1 + r_{t+1}}\right) \frac{\alpha Y_{t+1}(1 - \tau_k)}{K_{t+1}}. \quad (43)$$

$$\{n_{1t}\} : \frac{(1 - \alpha)Y_t(1 - \tau_k)}{n_{1t}} - w_t(1 - \tau_k) = 0$$  

$$\Rightarrow \{n_{1t}\} : w_t = \frac{(1 - \alpha)Y_t}{n_{1t}}. \quad (44)$$

Finally,

$$\{n_{2t}\} : w_t(1 - \tau_k) = \left(\frac{\theta}{n_{2t}}\right) \sum_{j=0}^{\infty} \gamma^j \left[\prod_{k=0}^{j} \frac{1}{1 + r_{t+k+1}}\right] I_{t+j+1}. \quad (45)$$

The Consumer’s Problem

$$\{c_t\} : \frac{1}{c_t} = \lambda_t,$$

$$\{a_{t+1}\} : \frac{\beta(1 + r_{t+1})}{c_{t+1}} = \frac{1}{c_t}$$

$$\{n_t\} : \frac{w_t(1 - \tau_n)}{c_t} = \frac{1}{1 - n_t}$$

From the firm’s FOC $\{K_{t+1}\}$:

$$\{K_{t+1}\} : \frac{1}{Z_t} = \left(\frac{1}{1 + r_{t+1}}\right) \frac{\alpha Y_{t+1}(1 - \tau_k)}{K_{t+1}}.$$  

Substituting for $(1 + r_{t+1})$ from $\{a_{t+1}\}$

$$\Rightarrow \frac{1}{Z_t} = \frac{\beta c_t}{c_{t+1}} \left[\frac{\alpha Y_{t+1}(1 - \tau_k)}{K_{t+1}}\right]$$

$$\Rightarrow \{K_{t+1}\} : \frac{1}{c_tZ_t} = \frac{\alpha \beta Y_{t+1}(1 - \tau_k)}{c_{t+1}K_{t+1}}$$

Similarly,

$$\{n_{1t}\} : \frac{1}{1 - n_t} = \frac{(1 - \alpha)Y_t(1 - \tau_n)}{c_t n_{1t}}$$
and,
\[
\{n_{2t}\} : \frac{1}{1 - n_t} = \left(\frac{\beta \theta}{n_{2t}}\right) \left(\frac{1 - \tau_n}{1 - \tau_k}\right) \sum_{j=0}^{\infty} \beta^j \gamma^j I_{t+j+1} / c_{t+j+1}.
\]

To summarize all FOCs,
\[
\{K_{t+1}\} : \frac{1}{c_t Z_t} = \frac{\alpha \beta Y_{t+1}(1 - \tau_k)}{c_{t+1} K_{t+1}}
\]
\[
\{n_{1t}\} : \frac{1}{1 - n_t} = \frac{(1 - \alpha) Y_t(1 - \tau)}{c_t n_{1t}}
\]
\[
\{n_{2t}\} : \frac{1}{1 - n_t} = \left(\frac{\beta \theta}{n_{2t}}\right) \left(\frac{1 - \tau_n}{1 - \tau_k}\right) \sum_{j=0}^{\infty} \beta^j \gamma^j I_{t+j+1} / c_{t+j+1}.
\]

When \(\tau_k = \tau_k = \tau\), we have
\[
\{K_{t+1}\} : \frac{1}{c_t Z_t} = \frac{\alpha \beta Y_{t+1}(1 - \tau)}{c_{t+1} K_{t+1}}
\]
\[
\{n_{1t}\} : \frac{1}{1 - n_t} = \frac{(1 - \alpha) Y_t(1 - \tau)}{n_{1t}}
\]
\[
\{n_{2t}\} : \frac{1}{1 - n_t} = \left(\frac{\beta \theta}{n_{2t}}\right) \sum_{j=0}^{\infty} \beta^j \gamma^j I_{t+j+1} / c_{t+j+1}.
\]

**The Decision Rules**

We use the method of undetermined coefficients to obtain the decision rules
\[
C_t = \Phi_{CE} AY_t,
\]
\[
I_t = (1 - \Phi_{CE}) AY_t
\]
\[
n_{1t} = x_{CE} n_{CE}
\]
\[
n_{2t} = (1 - x_{CE}) n_{CE}
\]
\[
n_t = n_{CE},
\]

where,
\[
\{Y_t - w_t(n_{1t} + n_{2t})\}(1 - \tau_k) + w_t(n_{1t} + n_{2t})(1 - \tau_n) = AY_t.
\]

\[\Rightarrow [\alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_n)]Y_t + w_t n_{2t}(\tau_k - \tau_n) = AY_t\]
\[ \Rightarrow [\alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_n)]Y_t + \left\{ \frac{\beta \theta A Y_t (1 - \Phi)}{(1 - \tau_k)(1 - \beta \gamma)} \right\} (\tau_k - \tau_n) = A Y_t \]

\[ \Rightarrow \alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_n) + \frac{\beta \theta A (1 - \Phi)}{(1 - \tau_k)(1 - \beta \gamma)} (\tau_k - \tau_n) = A \]

\[ \Rightarrow Y_t \left[ \alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_n) + \frac{\beta \theta A (1 - \Phi)}{(1 - \tau_k)(1 - \beta \gamma)} (\tau_k - \tau_n) \right] = A Y_t, \]

\[ \Rightarrow A = \left[ \alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_n) + \frac{\beta \theta (1 - \Phi) A}{(1 - \tau_k)(1 - \beta \gamma)} (\tau_k - \tau_n) \right]. \quad (46) \]

From the FOC of \( \{K_{t+1}\} \)

\[ \{K_{t+1}\} : \frac{1}{c_t Z_t} = \frac{\alpha \beta Y_{t+1}(1 - \tau_k)}{c_{t+1} K_{t+1}} \]

This implies,

\[ \frac{1}{\Phi_{CE} A Y_t Z_t} = \frac{\alpha \beta Y_{t+1}(1 - \tau_k)}{\Phi A Y_{t+1}(1 - \Phi_{CE}) A Y_t Z_t} \]

\[ \Rightarrow (1 - \Phi_{CE}) = \frac{\alpha \beta (1 - \tau_k)}{A}. \quad (47) \]

Substituting for \((1 - \Phi_{CE})A\) from 47 into 46,

\[ \Rightarrow A = \left[ \alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_n) + \frac{\beta \theta (1 - \Phi_{CE}) A}{(1 - \tau_k)(1 - \beta \gamma)} (\tau_k - \tau_n) \right] \]

\[ = \alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_n) - \frac{\alpha \beta^2 \theta (\tau_n - \tau_k)}{(1 - \beta \gamma)}. \quad (48) \]

When \(\tau_n = \tau_k = \tau\)

\[ A = [\alpha(1 - \tau) + (1 - \alpha)(1 - \tau)] \]

\[ = (1 - \tau). \]

From \(\{n_{1t}\}\) we get

\[ \{n_{1t}\} : \frac{x_{CE n_{CE}}}{1 - n_{CE}} = \frac{(1 - \alpha) Y_t (1 - \tau_n)}{\Phi_{CE} A Y_t} \]

\[ \Rightarrow \frac{x_{CE n_{CE}}}{1 - n_{CE}} = \frac{(1 - \alpha)(1 - \tau_n)}{\Phi_{CE} A} \]

\[ \Rightarrow \frac{n_{CE}}{1 - n_{CE}} = \frac{(1 - \alpha)(1 - \tau_n)}{x_{CE} \Phi_{CE} A} \]
\[ n_{CE} = (1 - \alpha)(1 - \tau_n) \]
\[ (1 - \alpha)(1 - \tau_n) + x_{CE} \Phi_{CE} A. \quad (49) \]

From \(\{n_{2l}\}\)
\[ \{n_{2l}\}: \frac{(1 - x)n_{CE}}{1 - n_{CE}} = \frac{\beta \theta}{(1 - \beta \gamma)} \left(1 - \frac{\tau_n}{1 - \tau_k}\right) \frac{(1 - \Phi_{CE})}{\Phi_{CE}} \]
\[ \Rightarrow \frac{(1 - \alpha)(1 - \tau_n)(1 - x_{CE})}{x_{CE}} = \frac{\beta \theta}{(1 - \beta \gamma)} \left(1 - \frac{\tau_n}{1 - \tau_k}\right) \frac{(1 - \Phi_{CE})}{\Phi_{CE}} \]
\[ \Rightarrow \frac{(1 - x_{CE})}{x_{CE}} = \frac{A \beta \theta (1 - \Phi_{CE})}{(1 - \alpha)(1 - \beta \gamma)(1 - \tau_k)}. \]
\[ \Rightarrow x_{CE} = \frac{(1 - \alpha)(1 - \beta \gamma)(1 - \tau_k)}{A \beta \theta (1 - \Phi_{CE}) + (1 - \alpha)(1 - \beta \gamma)(1 - \tau_k)} \quad (50) \]

Since,
\[ A(1 - \Phi_{CE}) = \alpha \beta (1 - \tau_k), \]
\[ \Rightarrow x_{CE} = \frac{(1 - \alpha)(1 - \beta \gamma)}{\alpha \beta \theta + (1 - \alpha)(1 - \beta \gamma)}. \]

From (36), we need
\[ 0 < 1 - \frac{\alpha \beta (1 - \tau_k)}{A} < 1, \]
which gives us
\[ 0 < \frac{\alpha \beta (1 - \tau_k)}{A} < 1, \]
or
\[ A > \alpha \beta (1 - \tau_k). \quad (51) \]

In addition, we also need
\[ 0 < A < 1 \quad (52) \]
to be satisfied. If equations (51) and (52) hold, we obtain
\[ 0 < A, \Phi_{CE}, n_{CE} < 1. \]

Equations (51) and (52) gives us a lower limit and an upper limit on \(\tau_n\), such that
\[ -\alpha \left[ 1 - \beta \theta - \beta^2 \theta \right] \frac{\tau_k}{(1 - \alpha)(1 - \beta \gamma) + \alpha \beta^2 \theta} < \tau_n < \frac{(1 - \beta \gamma)(1 - \alpha \beta)}{(1 - \alpha)(1 - \beta \gamma) + \alpha \beta^2 \theta} - \alpha \left[ (1 - \beta \gamma)(1 - \beta) - \beta^2 \theta \right] \frac{\tau_k}{(1 - \alpha)(1 - \beta \gamma) + \alpha \beta^2 \theta}. \quad (53) \]

In other words, for each \(\tau_k\) the lower and the upper bound on \(\tau_n\) must satisfy Restriction
(53).

**Appendix C**

\[
1 - A = 1 - \left[ \alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_n) - \frac{\alpha\beta^2 \theta(\tau_n - \tau_k)}{(1 - \beta \gamma)} \right]
\]
\[
= \frac{(1 - \beta \gamma) - \{\alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_n)\} (1 - \beta \gamma) + \alpha\beta^2 \theta(\tau_n - \tau_k)}{(1 - \beta \gamma)}
\]
\[
= \frac{(1 - \beta \gamma)[\tau_n - \alpha (\tau_n - \tau_k)] + \alpha\beta^2 \theta(\tau_n - \tau_k)}{(1 - \beta \gamma)}
\]
\[
= \frac{(1 - \beta \gamma)[\tau_k + (1 - \alpha)(\tau_n - \tau_k)] + \alpha\beta^2 \theta(\tau_n - \tau_k)}{(1 - \beta \gamma)}.
\]

Since

\[
A (1 - \Phi_{CE}) = \alpha\beta(1 - \tau_k)
\]
\[
1 - A = \frac{(1 - \beta \gamma)[(1 - \alpha)(\tau_n - \tau_k) + \tau_k] + \alpha\beta^2 \theta(\tau_n - \tau_k)}{1 - \beta \gamma}.
\]

This implies,

\[
\gamma = \left\{ \left[ \frac{(1 - \beta \gamma)[(1 - \alpha)(\tau_n - \tau_k) + \tau_k] + \alpha\beta^2 \theta(\tau_n - \tau_k)}{1 - \beta \gamma} \right]^\mu \right\}^{1 - \gamma} [\alpha\beta(1 - \tau_k)]^{1 - \mu}.
\]

In \( \gamma \), \( \alpha\beta(1 - \tau_k) \) decreases in \( \tau_k \). Further, suppose

\[
M_1 = \left[ \frac{(1 - \beta \gamma)[(1 - \alpha)(\tau_n - \tau_k) + \tau_k] + \alpha\beta^2 \theta(\tau_n - \tau_k)}{1 - \beta \gamma} \right]
\]
\[
M_2 = [\alpha\beta(1 - \tau_k)].
\]

Therefore,

\[
\frac{\partial \gamma}{\partial \tau_k} = (1 - \gamma) \gamma^{-\frac{\gamma}{1 - \gamma}} \left[ M_2 \mu \alpha \left\{ \frac{1 - \beta \gamma - \beta^2 \theta}{1 - \beta \gamma} \right\} - M_1 (1 - \mu) \alpha \beta \right] M_1^{\mu - 1} M_2^{-\mu}.
\]

Since, \( M_1 > 0 \) because \( 1 - A > 0 \) and \( M_2 > 0 \) by assumption,

\[
(1 - \beta \gamma) - \beta^2 \theta < 0,
\]

implies that \( \gamma \) will fall with an increase in \( \tau_k \).
From the labor supply term

\[ n_{CE} = \frac{(1 - \alpha)(1 - \tau_n)}{(1 - \alpha)(1 - \tau_n) + x_{CE} \Phi_{CE} A} \]

\[ = \frac{(1 - \alpha)}{(1 - \alpha) + \frac{x_{CE} \Phi_{CE} A}{(1 - \tau_n)}} \]

Note that

\[ x_{CE} \Phi_{CE} A = \frac{(1 - \alpha)(1 - \beta \gamma)}{\alpha \beta^2 \theta + (1 - \alpha)(1 - \beta \gamma)} [A - \alpha \beta (1 - \tau_k)] . \]

But

\[ A - \alpha \beta (1 - \tau_k) = \frac{(1 - \beta \gamma) [\alpha (1 - \beta) (1 - \tau_k) + (1 - \alpha)(1 - \tau_n)] - \alpha \beta^2 \theta (\tau_n - \tau_k)}{1 - \beta \gamma} . \]

Hence,

\[ x_{CE} \Phi_{CE} A = \frac{(1 - \alpha) [(1 - \beta \gamma) \{\alpha (1 - \beta) (1 - \tau_k) + (1 - \alpha)(1 - \tau_n)\} - \alpha \beta^2 \theta (\tau_n - \tau_k)]}{[\alpha \beta^2 \theta + (1 - \alpha)(1 - \beta \gamma)]} . \]

The term

\[ (1 - \beta \gamma) \{\alpha (1 - \beta) (1 - \tau_k) + (1 - \alpha)(1 - \tau_n)\} \]

can be re-written as

\[ (1 - \beta \gamma) \{\alpha (1 - \beta) + (1 - \alpha) - \alpha (1 - \beta) \tau_k - (1 - \alpha) \tau_n\} , \]

\[ = (1 - \beta \gamma) \{\alpha (1 - \beta) + (1 - \alpha) - \alpha \tau_k + \alpha \beta \tau_k - \tau_n + \alpha \tau_n\} \]

\[ = (1 - \beta \gamma) \{\alpha (1 - \beta) + (1 - \alpha) + \alpha (\tau_n - \tau_k) - \alpha \beta (\tau_n - \tau_k) - (1 - \alpha \beta) \tau_n\} \]

\[ = (1 - \beta \gamma) \{\alpha (1 - \beta) + (1 - \alpha) + \alpha (1 - \beta)(\tau_n - \tau_k) - (1 - \alpha \beta) \tau_n\} . \]

Hence,

\[ n_{CE} = \frac{(1 - \alpha) [\alpha \beta^2 \theta + (1 - \alpha)(1 - \beta \gamma)]}{(1 - \alpha) [\alpha \beta^2 \theta + (1 - \alpha)(1 - \beta \gamma)] + \Psi} , \]

where

\[ \Psi = \frac{(1 - \alpha)}{(1 - \tau_n)} [(1 - \beta \gamma) \{\alpha (1 - \beta) + (1 - \alpha) + \alpha (1 - \beta)(\tau_n - \tau_k) - (1 - \alpha \beta) \tau_n\} - \alpha \beta^2 \theta (\tau_n - \tau_k)] . \]

**Proof of Lemma 4**
Note that

\[
x_{CE} \Phi_{CE A} \frac{1}{1 - \tau_n} = x_{CE} \left[ \frac{\alpha (1 - \beta) (1 - \tau_k)}{(1 - \tau_n)} + (1 - \alpha) - \frac{\alpha \beta^2 \theta (\tau_n - \tau_k)}{(1 - \beta \gamma) (1 - \tau_n)} \right]
\]

\[
= x_{CE} \left[ \frac{\alpha (1 - \beta) (1 - \tau_k)}{(1 - \tau_n)} + (1 - \alpha) - \frac{\alpha \beta^2 \theta \tau_n}{(1 - \beta \gamma) (1 - \tau_n)} + \frac{\alpha \beta^2 \theta \tau_k}{(1 - \beta \gamma) (1 - \tau_n)} \right]
\]

Therefore,

\[
\frac{\partial^2 x_{CE} \Phi_{CE A}}{\partial \tau_n} = x_{CE} \left[ \frac{\alpha (1 - \beta) (1 - \tau_k)}{(1 - \tau_n)^2} - \frac{\alpha \beta^2 \theta (1 - \tau_k)}{(1 - \beta \gamma) (1 - \tau_n)^2} \right],
\]

which will be negative if

\[(1 - \beta \gamma) (1 - \beta) < \beta^2 \theta.\]

This condition will be satisfied if equation (40) holds. And this implies

\[
\frac{\partial n_{CE}}{\partial \tau_n} > 0.
\]

Further, since \(x_{CE}\) is independent of taxes,

\[
\frac{\partial n_{2CE}}{\partial \tau_n} > 0.
\]

Similarly, since

\[
\Psi = \frac{(1 - \alpha)}{(1 - \tau_n)} \left[ (1 - \beta \gamma) \{\alpha (1 - \beta) + (1 - \alpha) + \alpha (1 - \beta) (\tau_n - \tau_k) - (1 - \alpha \beta) \tau_n \} - \alpha \beta^2 \theta (\tau_n - \tau_k) \right],
\]

\[
\frac{\partial \Psi}{\partial (\tau_n - \tau_k)} = \frac{(1 - \alpha)}{(1 - \tau_n)} \left[ \alpha (1 - \beta) (1 - \beta \gamma) - \alpha \beta^2 \theta \right] (1 - \tau_k) < 0,
\]

if equation (40) holds, which further implies,

\[
\frac{\partial n_{CE}}{\partial (\tau_n - \tau_k)} > 0.
\]

Finally,

\[
\frac{\partial \Psi}{\partial \tau_k} = -\frac{(1 - \alpha)}{(1 - \tau_n)} \left[ (1 - \beta \gamma) \alpha (1 - \beta) - \alpha \beta^2 \theta \right] < 0,
\]

if equation (40) holds.

**Appendix D**

We know that,
\[
(1 - \Phi_P) = \frac{\alpha \beta \left[(1 - \beta \gamma) - \beta^2 \mu (1 - \gamma)\right]}{(1 - \beta \gamma) - \beta^2 (1 - \gamma) + \alpha \beta^3 (1 - \gamma)}
\]
\[
x_P = \frac{(1 - \alpha) \{(1 - \beta \gamma) - \beta^2 \mu (1 - \gamma) - \beta^2 (1 - \gamma)(1 - \Phi_P)\}}{(1 + \xi)/(1 - \alpha)\{(1 - \beta \gamma) - \beta^2 \mu (1 - \gamma) - \beta^2 (1 - \gamma)(1 - \Phi_P)\} + \beta \theta (1 - \Phi_P)}
\]
\[
n_P = \frac{(1 - \alpha) \{(1 - \beta \gamma) - \beta^2 \mu (1 - \gamma) - \beta^2 (1 - \gamma)(1 - \Phi)\}}{(1 - \alpha)\{(1 - \beta \gamma) - \beta^2 \mu (1 - \gamma) - \beta^2 (1 - \gamma)(1 - \Phi)\} + \Phi x \left[1 - \beta \gamma - \beta^2 \mu (1 - \gamma)\right]}.
\]

When \( \gamma = 1 \) and \( \xi = 0 \),
\[
1 - \Phi_P = \alpha \beta
\]
\[
x_P = \frac{(1 - \alpha)(1 - \beta)}{(1 - \alpha)(1 - \beta) + \alpha \beta^2 \theta}
\]
\[
n_P = \frac{(1 - \alpha)}{(1 - \alpha) + \Phi_P x_P}.
\]

In the competitive equilibrium under equal factor income taxes,
\[
A = 1 - \tau.
\]
\[
\Rightarrow (1 - \Phi_{CE}) = \alpha \beta
\]
\[
\Rightarrow n_{CE} = \frac{(1 - \alpha)}{(1 - \alpha) + x_{CE} \Phi_{CE}}
\]
\[
\Rightarrow x_{CE} = \frac{(1 - \alpha)(1 - \beta)}{\alpha \beta^2 \theta + (1 - \alpha)(1 - \beta)}.
\]

Clearly, when \( \gamma = 1 \) and \( \xi = 0 \), and \( \tau_n = \tau_k = \tau \),

As \( \gamma \to 1 \),
\[
1 - \Phi_P = 1 - \Phi_{CE}
\]
\[
x_P = x_{CE}
\]
\[
n_P = n_{CE}
\]
\[
\Rightarrow g_{z_{CE}} = g_{z_P}.
\]

Only equal factor income taxes under the no externality case, yields the planner’s growth
rate, except under a very restrictive parametric restriction,

\[ \left( \frac{1-\beta}{\beta} \right)^2 = \theta. \]

Under this equal factor income taxes are one among infinitely many factor income tax combinations that decentralize the planner’s growth rate. We can show this as follows.

For growth equalization, we need

\[ n_{CE} = \frac{(1-\alpha)(1-\tau_n)}{(1-\alpha)(1-\tau_n) + x_{CE}\Phi_{CE}A} = n_P, \]

\[ \Rightarrow \frac{x_{CE}\Phi_{CE}A}{(1-\tau_n)} = \Phi_Px_P \]
\[ \Rightarrow \frac{\Phi_{CE}A}{(1-\tau_n)} = \Phi_P \]
\[ \Rightarrow \frac{A - \alpha\beta(1-\tau_k)}{(1-\tau_n)} = 1 - \alpha\beta \]
\[ \Rightarrow A - \alpha\beta(1-\tau_k) = (1 - \alpha\beta)(1-\tau_n) \]

\[ \Rightarrow \alpha(1-\tau_k) + (1-\alpha)(1-\tau_n) - \frac{\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta)} - \alpha\beta(1-\tau_k) = (1 - \alpha\beta)(1-\tau_n). \]

Hence,

\[ (\alpha - \alpha\beta)(1-\tau_k) - (\alpha - \alpha\beta)(1-\tau_n) = \frac{\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta)} \]

which implies

\[ (1-\beta)(\tau_n-\tau_k) = \frac{\beta^2\theta(\tau_n-\tau_k)}{(1-\beta)}. \]

Clearly, as long as \( \frac{(1-\beta)}{\beta} \neq \sqrt{\theta} \), \( \tau_n = \tau_k \) always decentralizes planner’s growth rates. When \( \frac{(1-\beta)}{\beta} = \sqrt{\theta} \), any factor income tax combination decentralizes planner’s growth rate. As noted in the text, for \( \theta = 0.2 \), (or \( \theta = 0.5 \), as we have used in our numerical exercise) as in Huffman, the value of \( \beta = 0.69098 \) is very small and is not consistent with the literature. (When or \( \theta = 0.5 \), \( \beta = 0.58579 \) which is even smaller. ). We therefore rule out the possibility of equality.
5 Figures

Figure 1: Average growth rates for select OECD economies versus the ratio of tax on capital income to tax on labor income
Figure 2: Average factor income tax rates for select OECD economies
Figure 3: Time trend of factor income taxes for G7 economies
Figure 4: The effect of a change in $\xi$ on $(\tau_n - \tau_k)$
Figure 5: The effect of a change in $\gamma$ on $(\tau_n - \tau_k)$