Common Labor Market, Attachment and Spillovers in a Large Federation

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Abstract: In this paper, we extend the home-attachment model to a setting with multiple (more than two) regions and consider non-cooperative policy making for provision of different types of federal public goods in the presence of a common labor market. Migration and working place choices are independent. We show that the optimal redistributive policy implemented by the central government always yields equalization of private consumption levels across regions. This result holds whether or not policy makers are able to anticipate migration responses to their policy choices. In the decentralized leadership games, regional governments make contributions at levels that fully internalize externalities.

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1. Introduction

In Europe and United States of America (USA), one can easily find evidence of large common labor markets in operation. Following Wildasin (1991, p. 758), we characterize a common labor market as “…one in which at least some portion of the work force is able to switch from jobs in one jurisdiction to jobs in another jurisdiction (within the time frame of the analysis, and possibly at a nonzero cost).” In this paper, we consider a large federal economy, with many regions (states, provinces or member nations), in which individuals are attached to home (as in, e.g., Mansoorian and Myers (1993), Wellisch (1994) and Boadway et al (2013)) and who nonetheless may work in regions other than the ones where they reside. The combination of a common labor market and home attachment in a large economy is one of the key contributions that we make to the literature.

Naturally, common labor markets involve substantive flows of commuters and should yield a high degree of wage convergence across regions, particularly for occupations that face fierce competition between residents and commuters. Commuting is common in Europe. Over one million individuals accepted employment in a European country other than their countries of residence during 2010-2011 (European Commission, 2012).

![Fig. 1: Commuting flows in Europe (European Commission, 2012).](image)
Figure 1 shows examples of commuting flows among some European countries. It is visually clear that all nations displayed in the figure are linked directly or indirectly with all other nations, in a large labor-market network. Switzerland, Luxembourg, Austria, and Netherlands are net recipients, while in France, United Kingdom, Sweden, Germany and Italy the commuting outflows are larger than the inflows.

Large commuting flows are also observable in the USA. According to Agrawal and Hoyt (2014), 75 million people reside and work in multiple-state MSAs in the USA. Figure 2 displays the two largest commuting networks in this nation in 2011. Note that New York, Connecticut, New Jersey and Pennsylvania are all connected directly or indirectly with each other – New York and New Jersey are “commuting hubs” and Connecticut and Pennsylvania are “commuting spokes.” District of Columbia, Maryland and Virginia are all directly connected with each other.

Starting with Wildasin (1991), the fiscal federalism literature (see, e.g., Wildasin (1991), Wellisch and Wildasin (1996), Rothstein and Hoover (2006) and Wilson (2007)) has examined fiscal and redistributive implications of government policies in the presence of a common labor market. Among other important results, Wildasin (1991) demonstrates that a central authority may provide grants to lower level governments that induce the latter to behave efficiently in the implementation of redistributive schemes. Wellisch and Wildasin (1996) study the fiscal impacts of immigration in a federation characterized by decentralized tax and redistributive policy making. They show that immigration creates fiscal externalities and that these externalities can be fully internalized if the central government implements an interregional system of fiscal grants.
Wilson (2007) also examines fiscal and welfare effects of immigration control policies. Among other important results, he shows that a host country may benefit from attracting a large flow of non-impoverished workers from another country in the presence of a common labor market. Unlike this literature, we do not consider either decentralized income redistribution or immigration. In our model, interregional income redistribution is implemented by a central authority. The central authority makes redistributive income transfers after it observes the policies chosen by the regional governments (see, e.g., Caplan et al (2000)). Regional governments’ policies govern the supply of multiple types of federal public goods: (i) they share contributions to a federal public good whose aggregation technology is not restricted to summation (see, e.g., Cornes (1993) and Silva (2014)); and (ii) they also provide at least one public good that generates spillover consumption benefits to all other regions (see, e.g., Wellisch (1994) and Aoyama and Silva (2010)).

In line with most of the fiscal federalism literature, we first consider a federation with two regions. This allows us to clearly demonstrate that our model is a hybrid of models used in the “common-labor-market” literature and in the “home-attachment” literature. In particular, our model builds on Wildasin (1991) and Wellisch (1994). The motivation for setting a model that is similar in many respects (especially with regard to the interregional consumption spillovers) to the one utilized in Wellisch (1994) is the fact that he demonstrates that non-cooperative regional policy making is inefficient.\footnote{Mansoorian and Myers (1993) and Wellisch (1994) demonstrate that decentralized policy making is efficient in the presence of home attachment if regional governments provide public goods whose consumption benefits are enjoyed by residents only. Aoyama and Silva (2010) build a model after Wellisch (1994) and demonstrate that the efficiency of a subgame-perfect equilibrium for a decentralized leadership game in which regional governments provide public goods that produce interregional spillovers prior to the implementation of interregional transfers by a benevolent central government, where public policies are made in anticipation of migration responses of workers who are attached to home, under two circumstances: (i) when regional governments have quasi-linear payoff functions; and (ii) when regional governments have Rawlsian payoff functions. Aoyama and Silva (2013) also build a model after Wellisch (1994) in which regional governments possess Rawlsian payoff functions and demonstrate that the decentralized non-cooperative equilibrium is efficient in the presence of home attachment and interregional spillovers.} Wellisch (1994), however, assumes that labor markets are regional, ignoring the possibility that workers supply labor in the region in which they do not reside. Our first important result shows that non-cooperative regional policy making is efficient in the presence of a common labor market and of a benevolent central authority that cares about redistribution
when regional and central policies fully account for migration responses. Our second
important result demonstrates that non-cooperative regional policy making is also
efficient if regional and central authorities are unable to commit to public policies vis-à-
vis the imperfectly mobile population of consumers. In other words, the subgame-perfect
equilibrium for the decentralized leadership game played by regional and central
authorities is efficient whether or not these players take migration choices as given.
Boadway et al (2013) shows that regional governments make choices that internalize
interregional externalities when migration occurs prior to policy decisions. Boadway et al
(2013) also shows that the labor market allocations will typically be inefficient, since the
regionally independent labor markets will generally be characterized by different
marginal labor productivities. In our model, the regional marginal productivities of labor
are necessarily equal, since this is a natural implication of a common labor market – firms
in both regions choose labor quantities in order to equate the marginal productivity of
labor to the common wage rate.

We later extend the model to allow for many regions. As far as we know, we are the
first ones to build a home-attachment model with spillovers and a common labor market
for a federation containing more than two regions. We postulate that “nature” determines
each individual’s attachment benefits prior to the commencement of all decision making.
Each individual receives a “message” from nature that tells him/her that he/she enjoys
attachment benefits for a predetermined pair of regions only. The individual does not
enjoy any attachment benefit with respect to regions other than the two regions that
nature selects for him/her. The assumption that each individual derives attachment
benefits for two regions only does not seem to be too demanding, as it is unlikely that in
practice individuals enjoy attachment benefits for several regions. Home attachment is
typically determined by culture, family or social customs.

Nature’s selection procedure implies that the total population is arbitrarily partitioned
into home-attachment groups. These groups are not necessarily of the same size. This
asymmetric aspect of the model enables us to obtain a non-cooperative equilibrium in
which regional populations are of different sizes. This certainly has a very strong
empirical appeal. We show that the main results for the small economy (with two
regions) also hold in the large economy.
We organize the remainder of the paper as follows. Section 2 examines allocations (social optimum and subgame-perfect equilibria for decentralized leadership games) for a small federal economy containing two regions. Section 3 extends the basic model to one in which the federal economy contains many regions. Section 4 provides concluding remarks.

2. Model with two regions

Consider a small economy with two regions, indexed by \( j, j = 1,2 \). In this economy, individuals supply labor to competitive firms that produce a numeraire good. We assume that the other inputs used by the firms (e.g., land and capital) are held fixed. There are \( M_j \geq 2 \) firms in region \( j \). All firms use the same technology. This technology is represented by a production function, \( F(.) \), which is concave (characterized by constant returns to scale), increasing in all arguments, with diminishing marginal products, and which satisfies the standard Inada conditions (i.e., all inputs are essential). Let \( f^j(l_{mj}) \equiv F(l_{mj};z_{mj}) \) denote the reduced form for the production function used by a representative firm, \( m, m = 1,...,M_j \), in region \( j \), where \( l_{mj} \) is the quantity of labor hired and \( z_{mj} \) is the vector of inputs that are held fixed. For simplicity and to keep things as symmetrical as possible, we assume that \( z_{mj} = z_j \) for all \( m = 1,...,M_j \). Since we do not require that \( z_1 = z_2 \), the representative firm in region 1 may produce more or less than the representative firm in region 2 even if both firms hire the same quantity of labor.

Firms hire labor from a common competitive labor market. Individuals who work in one region may reside in any of the two regions of the economy. Following the literature on home-attachment, we ignore mobility costs (including commuting costs) other than the psychic costs associated with attachment benefits foregone. Including commuting costs in the model is an interesting extension that should be addressed in future work. To properly model commuting costs, one would have to build a “spatial model” (see, e.g., Brueckner and Kim (2003), Brueckner and Helsley (2011) and Agrawal and Hoyt (2014)) where the distance between residence and working places would be taken into account. The model would also need to account for the location of one’s house and differing property and income tax payments. In equilibrium, assuming standard housing size and quality and no amenity differences across locations, all individuals would have to be indifferent between commuting costs and property and income tax payments (which would be an increasing function of the

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2 This assumption guarantees that, in any equilibrium, all regions are populated.

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market wage. Assume throughout that the price of the numeraire good is equal to 1. The representative firm in region \( j \) chooses a non-negative amount of labor input, \( l_{mj} \), to maximize \( f^j(l_{mj}) - w l_{mj} \). Assuming an interior solution, the first order condition informs us that the firm hires labor up to the point where the marginal product of labor is just equal to the marginal cost of labor, \( f^{j}_{l_m} = w \), \( m = 1, \ldots, M_j \), \( j = 1,2 \). Hence, we obtain \( l_{mj}(w) = l_j(w) \), \( m = 1, \ldots, M_j \) and \( j = 1,2 \). In words, all firms within a region demand the same quantity of labor.

The labor market clears when the total demand for labor equals the total supply:

\[
M_1l_1(w) + M_2l_2(w) = N\bar{T},
\]

where \( N > 0 \) and \( \bar{T} > 0 \) are respectively the total population of workers and the total amount of labor units that each worker can supply in the market. We assume that each worker supplies all of his available labor units in the market inelastically provided that \( w > 0 \). The market-clearing condition (1a) allows us to define the market wage as an implicit function of the labor market characteristics, \( w = w(\bar{T},M_1,M_2,N) \).

All consumers in the economy have identical preferences with respect to the consumption goods. There are \( n_j \) consumers in region \( j \), with \( n_1 + n_2 = N \). The representative consumer in region \( j \) derives utility from consumption of \( x_j \) units of numeraire good, \( G_1 \) units of a public good provided in region 1, \( G_2 \) units of a public good provided in region 2 and \( Q \) units of a public good that is produced from contributions from both regions. We assume that \( Q = \Phi(q_1,q_2) \), where \( \Phi(.) \) is a concave, continuous and differentiable transformation function, increasing in both arguments, and \( q_j \) is the contribution made by region \( j \).^4 The representative consumer in region \( j \) obtains utility from consumption of private and public goods according to the following concave utility function, \( u^j = u(x_j,G_1,G_2,Q) \), where, in most of what follows, we assume

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^4 Examples include \( \Phi(q_1,q_2) = q_1^\theta q_2^{1-\theta} \) and \( \Phi(q_1,q_2) = \theta q_1 + (1-\theta)q_2 \), \( 0 < \theta < 1 \).
that the marginal utility from private consumption is diminishing and \( \lim_{x_j \to 0} u'_x = \infty. \)

Note that \( u'_x \equiv \partial u / \partial x_j \), \( j = 1,2 \).

Consumers are free to choose their region of residence. However, we consider situations where consumers are attached to regions. Attachment is captured through an idiosyncratic attachment benefit function. Let \( n \in [0, N] \) denote a consumer in the economy. This individual gets an attachment benefit equal to \( a(N - n) \) if she resides in region 1 and an attachment benefit equal to \( an \) if she resides in region 2, where \( a > 0 \) is the attachment intensity. Hence, the total utility individual \( n \) derives from residing in region 1 is \( u(x_1, G_1, G_2, Q) + a(N - n) \), while the total utility this individual derives from residing in region 2 is \( u(x_2, G_1, G_2, Q) + an \). In the migration equilibrium, there is an individual, \( n_1 \), who is indifferent between residing in region 1 and residing in region 2:

\[
\begin{align*}
&u(x_1, G_1, G_2, Q) + a(N - n_1) = u(x_2, G_1, G_2, Q) + an_1. \\
&\text{(1b)}
\end{align*}
\]

Condition (1b) allows us to write the following migration response function:

\[
\begin{align*}
n^i(x_1, x_2, G_1, G_2, Q) &= [u(x_1, G_1, G_2, Q) - u(x_2, G_1, G_2, Q) + an] / 2a. \\
&\text{(1c)}
\end{align*}
\]

Since \( n_2 = N - n_1 \), we also have

\[
\begin{align*}
n^i(x_1, x_2, G_1, G_2, Q) &= [u(x_2, G_1, G_2, Q) - u(x_1, G_1, G_2, Q) + an] / 2a. \\
&\text{(1d)}
\end{align*}
\]

The representative consumer in region \( j \) has \( i_j + \tau_j \) units of income and spends \( x_j + t_j \) units of this income to finance her private and public consumption levels. The representative consumer in region \( j \) earns market income from supplying labor in the market and from being a shareholder in all firms located in the region:

\[
i_j = w + M_j \left[ f^i \left( I_j \right) - w I_j \left( \right) \right] / n_j. \]
\[
\text{(1e)}
\]

In addition to earned market income, the representative consumer in region \( j \) also receives or pays a transfer equal to \( \tau_j \) units of income. This income transfer amount is

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5 This assumption guarantees that \( x_j > 0, n_j > 0, \forall j \), in the social optimum and equilibria examined in this paper.
controlled by the central government. We assume that \( n_1 \tau_1 + n_2 \tau_2 = 0 \); that is, the income transfers are purely redistributive. As for her expenditures, the representative consumer in region \( j \) pays for consumption of the numeraire good and pays a tax equal to \( t_j \) units of income to her regional government for the provision of public goods. The budget constraint faced by regional government \( j \) is

\[
n_j t_j = G_j + q_j, \tag{1f}
\]

where, for simplicity, we assume that it takes one unit of numeraire good to produce one unit of each type of public good.

### 2.1. Social optimum

We first derive the socially optimal allocation. This is a useful benchmark for future comparisons. We assume that the social planner is utilitarian. Let \( W = W^1 + W^2 \) denote the social welfare level, where \( W^j \) is region \( j \)’s welfare level, \( j = 1, 2 \). Note that

\[
W^1 = \int_0^n \left[ u(x_1, G_1, G_2, Q) + a(N - n) \right] dn \quad \text{and} \quad W^2 = \int_n^N \left[ u(x_2, G_1, G_2, Q) + an \right] dn .
\]

Hence,

\[
W = n_1 \left[ u(x_1, G_1, G_2, Q) + a \left( N - \frac{n_1}{2} \right) \right] + n_2 \left[ u(x_2, G_1, G_2, Q) + a \left( N - \frac{n_2}{2} \right) \right]. \tag{2a}
\]

We initially assume that the social planner takes the migration response functions into account when it chooses private and public expenditure levels, \( \{x_j, G_j, q_j\}_{j=1,2} \), to maximize social welfare. The social planner chooses non-negative \( \{x_j, G_j, q_j\}_{j=1,2} \) to maximize (2a) subject to (1c), (1d) and the economy-wide resource constraint:

\[
n_1 x_1 + n_2 x_2 + G_1 + G_2 + q_1 + q_2 = I(\bar{I}, M_1, M_2, N), \tag{2b}
\]

where \( I(\bar{I}, M_1, M_2, N) > 0 \) is the total income produced in the economy. We obtain equation (2b) from aggregating the individual budget constraints in each region and combining the implied results with conditions (1e), (1f) and the redistributive constraint, \( n_1 \tau_1 + n_2 \tau_2 = 0 \). It is important to note that the total income produced in the economy does not depend on the distribution of consumers between regions because (some) consumers/workers can reside in one region and work in the other:
\[ I(\overline{t}, M_1, M_2, N) = \sum_{j=1}^{2} M_j f^i \left( I_j \left( w(\overline{t}, M_1, M_2, N) \right) \right). \]  \hspace{1cm} (2c)

Letting \( \lambda \geq 0 \) be the Lagrangian multiplier associated with constraint (2b), the first order conditions for an interior solution to the social planner’s problem are constraint (2b) and the following conditions \( (j = 1, 2) \):

\[ n_1 u_x^1 + \sum_{j=1}^{2} n_x^j \left( u^j + a(N - n_j) \right) - \lambda \left( n_1 + \sum_{j=1}^{2} n_x^j x_j \right) = 0, \]  \hspace{1cm} (2d)

\[ n_2 u_x^2 + \sum_{j=1}^{2} n_x^j \left( u^j + a(N - n_j) \right) - \lambda \left( n_2 + \sum_{j=1}^{2} n_x^j x_j \right) = 0, \]  \hspace{1cm} (2e)

\[ \sum_{j=1}^{2} n_j u_{G_k}^j + \sum_{j=1}^{2} n_{G_k}^j \left( u^j + a(N - n_j) \right) - \lambda \left( \sum_{j=1}^{2} n_{G_k}^j x_j - 1 \right) = 0, \]  \hspace{1cm} (2f)

\[ \sum_{j=1}^{2} n_j u_{G_k}^j + \sum_{j=1}^{2} n_{G_k}^j \left( u^j + a(N - n_j) \right) - \lambda \left( \sum_{j=1}^{2} n_{G_k}^j x_j - 1 \right) = 0, \]  \hspace{1cm} (2g)

\[ \left( \frac{\partial \Phi}{\partial q_1} \right) \left[ \sum_{j=1}^{2} n_j u_{Q}^j + \sum_{j=1}^{2} n_{Q}^j \left( u^j + a(N - n_j) \right) \right] - \lambda \left( \frac{\partial \Phi}{\partial q_1} \sum_{j=1}^{2} n_{Q}^j x_j - 1 \right) = 0, \]  \hspace{1cm} (2h)

\[ \left( \frac{\partial \Phi}{\partial q_2} \right) \left[ \sum_{j=1}^{2} n_j u_{Q}^j + \sum_{j=1}^{2} n_{Q}^j \left( u^j + a(N - n_j) \right) \right] - \lambda \left( \frac{\partial \Phi}{\partial q_2} \sum_{j=1}^{2} n_{Q}^j x_j - 1 \right) = 0, \]  \hspace{1cm} (2i)

where \( u^j_{G_k} = \frac{\partial u^j}{\partial G_k} \), \( u^j_{Q} = \frac{\partial u^j}{\partial Q} \), \( n_{G_k}^j = \frac{\partial n^j}{\partial x_k} \), \( n_{Q}^j = \frac{\partial n^j}{\partial G_k} \), \( n_{Q}^j = \frac{\partial n^j}{\partial Q} \), \( k = 1, 2 \).

Consider equations (2d) and (2e). From equations (1c) and (1d), we know that \( n_{x_k}^2 = -n_{x_k}^1 \), \( k = 1, 2 \). This fact and equation (1b) imply that equations (2d) and (2e) simplify as follows:

\[ 2an_j u_{x}^j = \lambda \left[ u_{x}^j (x_j - x_k) + 2an_j \right], \hspace{1cm} j, k = 1, 2, \hspace{1cm} j \neq k, \]  \hspace{1cm} (2j)

where, in writing equations (2j), we make use of the fact that \( n_{x_j}^j = u_{x}^j / 2a, \) \( j = 1, 2 \).

Combining the two equations (2j) in order to eliminate \( \lambda \), we obtain

\[ 2an_1 n_2 (u_{x}^1 - u_{x}^2) = Nu_{x}^1 u_{x}^2 (x_1 - x_2). \]  \hspace{1cm} (2k)
Equation (2k) is the condition that governs the socially optimal policy with respect to income redistribution across regions.

We claim that equation (2k) implies that $x_1 = x_2$ in the social optimum. Suppose not. Let $x_i > x_j$. Then, $u_x^1 < u_x^2$ because $u_{xx} < 0$. But, this violates equation (2k) because the left hand side is negative and the right hand side is positive. A similar argument applies if $x_j > x_i$. Hence, we must have $x_1 = x_2$ and subsequently $u_x^1 = u_x^2$.

Now, note that if $x_1 = x_2$, we obtain $n_1 = n_2 = N/2$ from (1b) and $n_1 + n_2 = N$. The following proposition summarizes the important results we have obtained so far:

**Proposition 1.** If the social planner’s problem fully accounts for migration responses to the choices of private and public expenditure levels, the socially optimal allocation involves $x_1 = x_2$, $u_x^1 = u_x^2$ and $n_1 = n_2 = N/2$.

Let $x = x_1 = x_2$. The resource constraint (2b) becomes

$$N x + \sum_{j=1}^{2} (G_j + q_j) = I\left(\bar{T}, M_1, M_2, N\right),$$

where $I\left(\bar{T}, M_1, M_2, N\right)$ is given by equation (2c). Since $n_{G_j}^2 = -n_{G_j}^1$, $n_{Q_j}^2 = -n_{Q_j}^1$, $x_1 = x_2$ and $\lambda = u_x^1 = u_x^2$, conditions (2f) – (2i) become

$$N \begin{pmatrix} u_{G_j} \\ u_x \end{pmatrix} = \frac{\partial \Phi}{\partial q_j} = 1, \quad j = 1, 2, \quad (2m)$$

$$N \begin{pmatrix} u_{Q_j} \\ u_x \end{pmatrix} = \frac{\partial \Phi}{\partial q_j} = 1, \quad j = 1, 2. \quad (2n)$$

Equations (2m) and (2n) are the Samuelson conditions for optimal provision of the federal public goods.

The socially optimal allocation is characterized by conditions (1b), (2c), $u_x^1 = u_x^2$, $x_1 = x_2$, $n_1 = n_2 = N/2$, (2l), (2m) and (2n). As we mentioned above, this allocation is obtained under the assumption the social planner is able to fully anticipate the migration responses to its policy choices. However, we now demonstrate that the social optimum remains unchanged if the social planner is unable to account for the migration responses
when it chooses the private and public expenditure levels. To see this formally, suppose that the social planner chooses \( \{ x_j, G_j, q_j \}_{j=1,2} \) to maximize (2a) subject to (2b), taking the population distribution, \( \{ n_1, n_2 \} \), as given. Letting \( \mu \geq 0 \) be the Lagrangian multiplier associated with constraint (2b), the first order conditions for an interior solution are, for \( j = 1, 2 \):

\[
\begin{align*}
  n_j u_x^j - \mu n_j &= 0 , \\
  \sum_{j=1}^2 n_j u_x^j - \mu &= 0 , \quad k = 1, 2 , \\
  \left( \frac{\partial \Phi}{\partial q_k} \right) \sum_{j=1}^2 n_j u_x^j - \mu &= 0 , \quad k = 1, 2 .
\end{align*}
\]  

Combining (3a) – (3c), we obtain

\[
\begin{align*}
  u_x^1 &= \mu = u_x^2 , \\
  \sum_{j=1}^2 n_j \left( \frac{u_x^j}{u_x^j} \right) &= 1 , \quad k = 1, 2 , \\
  \left( \frac{\partial \Phi}{\partial q_k} \right) \sum_{j=1}^2 n_j \left( \frac{u_x^j}{u_x^j} \right) &= 1 , \quad k = 1, 2 .
\end{align*}
\]  

Now, observe that \( u_x^1 = u_x^2 \) implies \( x_1 = x_2 \). Hence, we obtain \( n_1 = n_2 = N/2 \) from (1b) and \( n_1 + n_2 = N \). Given these results, the resource constraint becomes equation (2l) and the Samuelson conditions (3e) and (3f) become equations (2m) and (2n), respectively. In sum, the social optimum is again the symmetric Pareto optimal allocation.

**Proposition 2.** In the presence of a common labor market, the social optimum corresponds to the symmetric Pareto optimal allocation whether or not the social planner is able to anticipate the migration responses to its policy choices.

Proposition 2 is important because it informs us that under a common labor market the socially optimal allocation is unique and symmetric. This follows because the distribution of population across regions does not affect the economy-wide resource constraint. As we discussed above, the total income is unaffected by the population distribution – see equation (2c).
2.2. Decentralized leadership

We now consider a setting in which the federation is characterized by decentralized leadership. The regional governments choose their public good contributions in full anticipation of the central government’s interregional redistribution mechanism. The decentralized leadership game has two stages; namely, simultaneous choices of public good contributions in the first stage followed by the central government’s choices of private consumption levels in the second stage. In addition, we initially assume that the central government chooses its redistribution policy accounting for the migration responses to its choices. The payoffs for the regional governments are the regional welfare functions. The payoff for the central government is the social welfare function. The equilibrium concept is subgame perfection.

Consider the second stage of the game. This is equivalent to the problem that the social planner solved when it accounted for the migration responses in the previous section, except that here the central government does not control the public good expenditure levels. Thus, the conditions that characterize the solution in the second stage are (2b) and (2k). The latter implies that $x_1 = x_2 = x$, $u_x^1 = u_x^2$ and $n_1 = n_2 = N/2$. It follows that condition (2l) holds and can be written in terms of the central government’s best response function:

$$x(G_1, G_2, q_1, q_2) = \left( 1 - \sum_{k=1}^{2} (G_k + q_k) \right) / N .$$  \hspace{1cm} (4a)

Plugging $n_j = N/2$ and (4a) into the payoff for regional government $j$, we obtain

$$\left( \frac{N}{2} \right) \left[ u(x(G_1, G_2, q_1, q_2), G_1, G_2, Q) + \frac{3aN}{4} \right] .$$  \hspace{1cm} (4b)

Each regional government wishes to maximize the payoff function (4b) — hence, each regional government wishes to maximize $u(x(G_1, G_2, q_1, q_2), G_1, G_2, Q)$. In the first stage, regional government $j$ chooses non-negative $\{G_j, q_j\}$ to maximize (4b) subject to $Q = \Phi(q_1, q_2)$, taking the choices of the other government as given. It is straightforward to show that the first order conditions for the problems faced by the regional governments yield equations (2m) and (2n).
Proposition 3. The subgame perfect equilibrium for the decentralized leadership game where the center redistributes income between regions taking into account the migration responses to its policy choices corresponds to the symmetric Pareto optimal allocation.

Proposition 3 is remarkable in light of the negative results in Wellisch (1994). Our model has two key departures from Wellisch’s model: (i) the presence of a common labor market; and (ii) ex-post income redistribution undertaken by the central government. As discussed above, a common labor market implies that the economy’s income is independent from the population distribution across regions. This, in turn, implies that the economy-wide’s resource constraint is also independent from the population distribution across regions, a fact that yields the result that the center’s optimal income redistribution policy equates marginal utilities of income across regions. As the regional governments make their choices in full anticipation of the center’s redistributive policy and also knowing that the population will be divided equally across regions, they find it optimal to make choices that fully internalize all interregional externalities. They make contributions at levels that satisfy the Samuelson conditions for optimal provision of public goods.

A common labor market is necessary for the efficiency of the subgame-perfect equilibrium for the decentralized leadership game. Intuitively, the reader can confirm this by comparing our efficiency result with results that are available in the literature for models in which labor markets are assumed to be regional and independent from each other. Caplan et al (2000) shows that the subgame-perfect equilibrium for the decentralized leadership game they consider is socially optimal. In their setting, the center promotes interregional income transfers ex-post, but in full anticipation of how its choices affect the migration equilibrium. Their efficiency result, however, depends crucially on the assumption that the federal public good is pure, where the total amount is given by the sum of the regional contributions. This is made clear by subsequent work (see, e.g., Silva and Yamaguchi (2010) and Caplan and Silva (2011)), in which the assumption that the federal public good is pure is relaxed. These papers demonstrate that, all else equal, the subgame-perfect equilibrium for the decentralized leadership game is not socially optimal.
Aoyama and Silva (2010) builds a model after Wellisch (1994) and shows that the subgame-perfect equilibrium for a decentralized leadership game with interregional spillovers is socially efficient under two circumstances: (i) if regional and central governments’ payoff functions are Rawlsian; and (ii) if the regional governments’ payoff functions are quasi-linear and the central government’s payoff function is a Bergson-Samuelson transformation of regional welfare levels. In both cases, the center’s income redistribution policy yields perfect incentive equivalence, which motivates the regional governments to make choices that internalize interregional spillovers. More recently, Boadway et al (2013) also shows that the subgame-perfect equilibrium for a decentralized leadership game, with ex-post interregional income transfers implemented by a utilitarian central government and in which there are interregional externalities other than those implied by a pure federal public good, is socially optimal. However, this result depends crucially on their modeling assumption that the regional welfare functions are concave transformations of quasi-linear utilities (i.e., one of the two cases examined in Aoyama and Silva (2010)), which in the face of interregional income redistribution imply that the center’s optimal redistributive policy equates welfare levels across regions; hence, there is perfect incentive equivalence and the efficiency result is obtained. As discussed in Aoyama and Silva (2010), it is straightforward to show that if the modeling assumption that regional utilities represent quasi-linear preferences is relaxed and the welfare function is characterized by diminishing marginal utility of income, the efficiency results of Boadway et al (2013) do not hold in general.

It is now important to note that the subgame perfect equilibrium for the decentralized leadership game remains the same even if the governments are unable to anticipate the migration responses to their choices. To see this, assume that the regional governments and the center take the population distribution \( \{n_1, n_2\} \) as given when they make their choices. In the second stage, the center chooses private consumption levels in order to maximize welfare subject to (2b). Its optimal choices yield \( u^*_1 = u^*_2 \) as in the problem solved by the social planner under similar circumstances in the previous section. Hence, we have \( x_1 = x_2 = x \) and \( n_1 = n_2 = N/2 \). We also get the center’s best response function (4a) and the regional payoff functions (4b). Therefore, the first order conditions in the
first stage yield equations (2m) and (2n). The following propositions summarizes this important result:

**Proposition 4.** *In the presence of a common labor market, the center’s optimal redistributive policy equates marginal utilities of income across region whether or not the center is able to anticipate the migration responses to its redistributive policy. As an immediate consequence, the regions find it optimal to make choices that internalize all externalities in the first stage. The subgame perfect equilibrium for the decentralized leadership game corresponds to the symmetric Pareto optimal allocation whether or not the governments are able to anticipate the migration responses to their policy choices.*

Proposition 4 is important because it reveals that the efficiency of the subgame-perfect equilibrium for the decentralized leadership game does not depend on the timing of the migration equilibrium. Again, this fact follows from the common labor market structure and the ex-post income redistribution policy implemented by the center. It is also important to note that the allocation of labor is efficient since labor units are allocated in order to equate marginal products of labor within and across regions. Boadway et al (2013) shows that if in a sequential game the migration choices occur prior to the policy choices made by regional and central governments, the subgame-perfect equilibrium for this game is socially efficient and the population is equally split between regions; however, the allocation of labor units is not efficient in general because marginal products of labor are not necessarily equal across regions. Unlike our efficiency result for a sequential game in which regional and central governments take the population distribution across regions as given, the efficiency result in Boadway et al (2013) under similar circumstances depends on their modeling assumption that regional welfare functions represent quasi-linear preferences.

3. **Large economy**

Consider now a large economy with \( J > 2 \) regions. For the most part, the model is the same as before except that \( j = 1, \ldots, J \). In particular, we assume that: (i) each region has a regional government; (ii) each regional government is utilitarian and cares about the welfare of its residents only; and (iii) there is a central government in the federation, which is utilitarian and cares about the welfare of all residents in the economy.
Firms compete for labor in a common labor market. The individuals who work in a particular region may reside in any region of the federation. Let \( w \) denote the wage in the labor market. The representative firm in region \( j \) chooses a non-negative amount of labor input, \( l_{mj} \), to maximize \( f^j(l_{mj}) - w l_{mj} \). The first order condition is \( f^j_w = w \), \( m = 1, \ldots, M_j \), \( j = 1, \ldots, J \). Thus, \( l_{mj}(w) = l_j(w) \), \( m = 1, \ldots, M_j \) and \( j = 1, \ldots, J \). All firms within a region demand the same quantity of labor.

The clearing condition for the labor market is

\[
\sum_{j=1}^J M_j l_j(w) = N \bar{T},
\]

where \( N = \sum_{j=1}^J n_j \). Equation (5) enables us to define the market-clearing wage as a function of the vector of the numbers of regional firms, total labor force and the maximum of labor units each worker is able to offer in the market, \( w = w(\bar{T}, M, N) \), for \( M = (M_1, \ldots, M_J) \).

The representative consumer in region \( j \) derives utility from consumption of \( x_j \) units of numeraire good, \( G_k \) units of a public good provided in region \( k \), \( k = 1, \ldots, J \), and \( Q \) units of a public good that is produced from contributions from all regions, \( Q = \Phi(q) \), where \( q = (q_1, \ldots, q_J) \) and \( \Phi(\cdot) \) is a continuous, differentiable concave transformation of regional contributions, \( q_k \), \( k = 1, \ldots, J \), and increasing in all arguments. This consumer’s utility function is \( u(x_j, G_k, Q) \), \( j = 1, \ldots, J \), where \( G = (G_1, \ldots, G_J) \). The properties of this utility function are the same as before.

We now extend the attachment model to \( J > 2 \) regions. In doing this, we assume that any individual is free to establish his/her residence in any region, but derives attachment benefits from establishing residence in two regions only. These attachment benefits are exogenously assigned to each individual by nature (prior to the beginning of the game examined here). Although any individual selects his/her region of residence from a menu containing \( J \) regions (namely, all regions in the federation), in equilibrium, he/she will prefer to reside in one of the regions in which he/she derives attachment benefits because
the total utility enjoyed by any individual in at least one of these regions will be higher than the total utility that he/she enjoys in a region in which he/she does not derive any attachment benefit.

Assume that nature partitions the entire population, $N$, into population groups, $N_{r,s} > 0$, where $r = 1, ..., J-1$ and $s = r+1, ..., J$ denote regions in the federation and $\sum_{r=1}^{J-1} N_{r,r+1} = N$. Individuals assigned to a particular population group derive attachment benefits from residing in either of the two regions that characterize the group. Consider the individuals who are assigned to population group $N_{1,h}$, $h = 2, ..., J$. An individual $n^{(1,h)} \in [0, N_{1,h}]$ gets utility $u(x_{i}, G_j, Q) + a(N_{1,h} - n^{(1,h)})$ from residing in region 1 and utility $u(x_{h}, G_j, Q) + an^{(1,h)}$ from residing in region $h$. In equilibrium, we have

$$u(x_{i}, G_j, Q) + a(N_{1,h} - n_{1,h}) = u(x_{h}, G_j, Q) + an_{1,h}, \quad h = 2, ..., J,$$

where $n_{1,h}$ is the individual who is indifferent between residing in region 1 and residing in region $h$, $h = 2, ..., J$. Letting $n_{k,1} \equiv N_{1,h} - n^{(1,h)}$, $h = 2, ..., J$, equations (6) can be rewritten as follows:

$$u(x_{i}, G_j, Q) + an_{h,1} = u(x_{h}, G_j, Q) + an_{1,h}, \quad h = 2, ..., J.$$  

(7a)

Now, consider the individuals who are assigned by nature to population group $N_{r,h}$ for a pair of regions $r$ and $\hat{h}$, where $r \in \{2, ..., J-1\}$ and $\hat{h} \in \{\hat{r}+1, ..., J\}$. Let $n^{(r,\hat{h})} \in [0, N_{r,\hat{h}}]$ denote an individual who is attached to either region $r$ or $\hat{h}$, and who gets utility $u(x_{r}, G_j, Q) + a(N_{r,\hat{h}} - n^{(r,\hat{h})})$ from residing in region $r$ and $u(x_{\hat{h}}, G_j, Q) + an^{(r,\hat{h})}$ from residing in region $\hat{h}$. Letting $n_{r,\hat{h}}$ denote the individual who is indifferent between residing in region $r$ or $\hat{h}$ and letting $n_{h,r} \equiv N_{r,\hat{h}} - n_{r,\hat{h}}$, we can write the following migration equilibrium equations:

$$u(x_{r}, G_j, Q) + an_{h,r} = u(x_{\hat{h}}, G_j, Q) + an_{r,\hat{h}}, \quad r = 2, ..., J-1, \quad \hat{h} = \hat{r} + 1, ..., J.$$  

(7b)

\footnote{By symmetry, $N_{h,1} = N_{1,h}$, $h = 2, ..., J$.}
The migration equilibrium is characterized by equations (7a) and (7b).\(^7\) Letting \(n_j\) denote the population in region \(j\), \(j=1,\ldots,J\), we have
\[
n_1 = \frac{1}{j} \sum_{h=2}^{J} n_{1,h} \quad \text{and} \quad n_r = n_{r,1} + \sum_{h=r+1}^{J} n_{r,h},
\]
\(
r = 2,\ldots,\hat{J}-1, \hat{r} = r+1,\ldots,J, \text{ and } N = \sum_{j=1}^{j} n_j.
\)

3.1. Decentralized leadership

Consider a setting where the large federation features decentralized leadership. The payoff for the government in region 1 is
\[
W^1 = \sum_{h=2}^{J} \left\{ f_0^{n_h} \left[ u(x_t, G, Q) + a \left( N_{1,h} - n_1^{1,h} \right) \right] dn^{1,h} \right\}.
\]
This can be simplified to yield
\[W^1 = n_1 u(x_t, G, Q) + a \left[ \sum_{h=2}^{J} n_{1,h} \left( N_{1,h} - \frac{n_{1,h}}{2} \right) \right]. \tag{8a}\]
The payoff for the government in region \(r\), \(r = 2,\ldots,J-1\), is
\[W^r = n_r u(x_t, G, Q) + a \left[ \sum_{h=r}^{J} n_{r,h} \left( N_{r,h} - \frac{n_{r,h}}{2} \right) \right]. \tag{8b}\]
The payoff for the central government is
\[W = \sum_{j=1}^{J} \left\{ n_j u(x_j, G, Q) + a \left[ \sum_{h=j}^{J} n_{j,h} \left( N_{j,h} - \frac{n_{j,h}}{2} \right) \right] \right\}. \tag{8c}\]

We can now derive the economy-wide resource constraint for the large federation. We obtain this constraint by aggregating individual and governmental budget constraints while also aggregating individual market incomes. The budget constraint facing an individual who resides in region \(j\) is again
\[x_j + t_j = i_j + \tau_j, \quad j=1,\ldots,J. \]
Equations (1e) and (1f) that describe the market income for an individual who resides in region \(j\) and the budget constraint faced by the government in region \(j\), respectively, hold in the large

\(^7\) As discussed in the text, every individual also considers the benefits of establishing residence in any of the \(J-2\) regions where he or she does not enjoy an attachment benefit. Hence, there is a larger number (indeed, a continuum) of potential conditions that may characterize the migration equilibrium. However, as we will show below, the full set of conditions that describe every individual’s residential choice between a region where he or she enjoys an attachment benefit and a region where he or she does not derive an attachment benefit can be ignored. In equilibrium, the utility from residing in any region in which an individual does not enjoy attachment benefit is strictly lower than the utility that this individual obtains from residing in the preferred choice between the two regions in which he or she obtains attachment benefits.
federation for \( j = 1, \ldots, J \). The interregional income redistribution constraint faced by the central government is now

\[
\sum_{j=1}^{J} n_j \tau_j = 0. \tag{8d}
\]

Hence, aggregating individual and government budget constraints and accounting for equation (8d), we obtain

\[
\sum_{j=1}^{J} (n_j x_j + G_j + q_j) = I(\overline{I}, \overline{M}, N), \tag{8e}
\]

where \( I(\overline{I}, \overline{M}, N) \) is the total income produced in the economy. It is again important to note that the total income produced in the economy does not depend on the population distribution across regions:

\[
I(\overline{I}, \overline{M}, N) = \sum_{j=1}^{J} M_j f^j (I_j(w(\overline{I}, \overline{M}, N))). \tag{8f}
\]

Equation (8f) is an immediate implication of the common labor market, since all workers earn the same wage in spite of the region in which they work.

In what follows, we consider a decentralized leadership game in which the center and the regions take migration decisions as given when they make their choices. Proposition 4 informs us that the center’s optimal redistributive policy is the same whether it accounts for migration responses or not if the federation contains two regions. It is straightforward, albeit algebraically cumbersome, to demonstrate that, in the large federation, one of the center’s optimal redistributive policies in a decentralized leadership game in which the center accounts for migration responses to its choices is exactly the optimal redistributive policy that the center chooses in the decentralized leadership game examined below.\(^8\)

Taking migration decisions as given, regional and central governments play a two-stage game. Regional government \( j \), \( j = 1, \ldots, J \), chooses non-negative \( \{G_j, q_j\} \) to maximize its payoff function, taking the choices of all other regional governments as given in the first stage. Having observed \( \{G, q\} \), where \( q = (q_1, \ldots, q_J) \), the center chooses \( \{x_1, \ldots, x_J\} \) in the second stage.

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\(^8\) A proof of this claim is available from the authors upon request.
Consider the second stage. The center’s first order conditions yield
\[ u_x^1 = u_x^h, \quad h = 2, \ldots, J, \] (9a)
in addition to the economy-wide resource constraint, (8e). Conditions (9a) yield \( x_i = x_h \) for \( h = 2, \ldots, J \). Letting \( x = x_i = \ldots = x_j \), the economy-wide resource constraint yields
\[ x(G, Q) = \left( I \tilde{(I, M, N)} - \sum_{j=1}^J G_j - Q \right) / N. \] (9b)
Consider now the first stage. Accounting for (9b), regional government 1 wishes to maximize
\[ n_u(x(G, Q), G, Q) + a \left[ \sum_{h=2}^{J} n_{1,h} \left( N_{1,h} - \frac{n_{1,h}}{2} \right) \right]. \] (9c)
Also accounting for (9b), regional government \( r, \ r = 2, \ldots, J \) wishes to maximize
\[ n_u(x(G, Q), G, Q) + a \left[ \sum_{h \neq r} n_{r,h} \left( N_{r,h} - \frac{n_{r,h}}{2} \right) \right]. \] (9d)
Taking migration decisions as given, the regional governments must also take the regional population distribution as given. This implies that each regional government in the first stage makes choices to maximize \( u(x(G, Q), G, Q) \), a common objective function. Hence, they make choices that in the aggregate yield the Samuelson conditions for efficient contributions to public goods. This fact yields the following important result:

**Proposition 5.** In the presence of a common labor market and attachment for a large federation, all regional government find it desirable to provide contributions to federal public goods (with shared provision or not) that satisfy the Pareto-efficient conditions for provision of such goods. The center finds it optimal to equalize private consumption levels across regions. In sum, the subgame-perfect equilibrium for the decentralized leadership game is Pareto efficient.

Proposition 5 has important policy implications. The working hypothesis that there is a common labor market in the federation is easy to accept if workers who choose to work in regions other than their regions of residence do not have to travel long distances. Thus, for a fixed geographic area that constitutes the federation, the larger the number of regions, the larger will be the commuting flows across regions. This phenomenon is quite
apparent in the European Union. In a context in which the federation is large and the existence of a common labor market is very plausible, Proposition 5 informs us that attachment to regions may not be an impediment for efficiency or equity. Federal public goods may be efficiently provided and each individual in the federation may have access to the same basket of private and public goods. Furthermore, since the exogenously determined population groups, \( N_{r,s} \), are not necessarily symmetric, the efficient and equitable allocation implied by Proposition 5 is quite consistent with an uneven population distribution in equilibrium.

The fact that the subgame-perfect equilibrium for the decentralized leadership game does not imply that the population distribution is symmetric can be demonstrated with a simple example. Suppose that \( J = 3 \), \( N_{1,2} = 40 \), \( N_{1,3} = 40 \), \( N_{2,3} = 20 \). In equilibrium, \( n_1 = (N_{1,2} + N_{1,3})/2 = 40 \), \( n_2 = (N_{1,2} + N_{2,3})/2 = 30 \), \( n_3 = (N_{1,3} + N_{2,3})/2 = 30 \).

Note that no individual finds it optimal to move to a region in which he or she does not enjoy an attachment benefit. Since utilities of consumption are equalized in the subgame-perfect equilibrium, all individuals enjoy a net strictly positive utility premium if they reside in their most preferred region relative to all other regions in which he or she does not derive attachment benefits.

4. Conclusion

In this paper, we extend the home-attachment model to a setting with more than two regions and consider non-cooperative policy making for provision of different types of federal public goods in the presence of a common labor market. Our model is a hybrid of standard home-attachment and common-labor-market models in the fiscal federalism literature. We show that in the presence of a common labor market, the optimal redistributive policy implemented by the central government always yields equalization of private consumption levels across regions. This result holds despite the center’s ability of anticipating migration responses to its income-transfer choices. Anticipating equalization of private consumption levels across regions, all regional governments have incentives to make contributions at levels that fully internalize externalities.

Our results have interesting policy implications. Unlike Wellisch (1994), we show that home attachment is not necessarily an impediment for efficient behavior at the
regional government level. This finding is largely due to the fact that there is a common labor market. Essentially, the presence of a common labor market separates the decision of where one supplies labor from the decision of where one establishes his/her residence. An individual can reside in a region in which he/she derives attachment benefits and work in some other region in which he/she does not derive any attachment benefit. This type of behavior seems to provide an empirically supportable behavioral hypothesis for observable residential and job choices in both Europe and the USA, where there are significant commuting flows.

References


