Growth Effects of Environmental Policy, when Production and Pollution Abatement Technologies are Embodied in Separate Intermediates

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Abstract

A Schumpeterian growth model is developed to investigate how environmental policy affects economic growth. In contrast to previous models, production and pollution abatement technologies are embodied in separate intermediate good types. In this setting, I show analytically that a tightening of the environmental policy unambiguously dampens economic growth. Simulation results indicate that the growth rate of pollution emission is more sensitive to changes in the environmental policy than the growth rate of output. In addition, old and new stylized facts concerning economic growth and pollution emission are presented, and the model is shown to match these stylized facts.
1 Introduction

Extensive empirical work provides evidence suggesting that a tighter environmental policy stimulates environmental innovation (e.g., Brunnermeier and Cohen 2003; Johnstone and Labonne, 2006; Popp, 2006; Arimura et al., 2007; Ambec et al., 2011; and Haščič et al., 2012). The question is then, whether the stimulation comes at the expense of other types of research. If so, environmental policy might have a negative effect on economic growth.

The goal of this paper is to investigate how environmental policy affects economic growth, when environmental policy also affects the direction of research efforts. The studies closest to the one conducted in this paper are Hart (2004, 2007) and Ricci (2007). These analyzes are based on Schumpeterian growth models, where intermediate goods can be improved along two dimensions: productivity and environmental friendliness. As more environmentally friendly intermediates are less productive, R&D firms face a design trade-off, when attempting to develop higher intermediate good qualities. As emphasized by Ricci (2007), a tightening of the environmental policy has two opposing effects on economic growth in such models. Firstly, a tighter environmental policy will reward innovators to the extent that new intermediate good vintages are relatively more environmentally friendly. This will increase research efforts. Secondly, when the environmental policy is tightened, R&D firms have an incentive to focus more on the environmentally friendliness of their new intermediate good designs. This will lower the productivity gain from R&D. Economic growth is increased if the first effect dominates the second and vice versa.

In Hart (2004, 2007) R&D firms have two design options: clean and dirty. This discrete set of choices enable an analytical solution, and it is shown that a tighter environmental policy might enhance growth and reduce environmental damage under certain conditions. In contrast, Ricci (2007) assumes that R&D firms have a continuum of design options. The model cannot be solved analytically, but simulations indicate that for plausible parameter values, a tightening of the environmental policy dampens economic growth.

In this paper, I develop a Schumpeterian growth model where production and pollution abatement technologies are embodied in separate intermediate good types. As a consequence, both the intermediate goods sector and the R&D sector are bifurcated into two subsectors: one for each intermediate good type. How should this bifurcation really be understood in the context of the previous literature? Imagine a firm obtaining a patent on a new quality of a certain engine type. If production and pollution abatement technolo-
gies are embodied in the same intermediate goods (as in Hart 2004, 2007 and Ricci 2007), then the new engine quality is both more powerful and more environmentally friendly than the previous qualities. If the two types of technology are embodied in separate intermediates, one firm would obtain a patent on a more powerful engine, while another (or the same) firm would obtain a patent on a catalyst, that could be implemented into the engine to reduce pollution emission. Hence, innovation arrivals of pollution abatement technologies are detached from innovation arrivals of production technologies. If a firm develops both production and pollution abatement technologies, these will be developed in separate R&D units. One unit can be successful during a certain time interval, while the other fails. If technologies are embodied in the same intermediate, the firm will have only one R&D unit. Either the firm develops a more productive and more environmentally friendly intermediate good quality, or no innovation occurs.

I argue, that separating the two technology types, results in a more realistic representation of the innovation process, for at least two reasons. Firstly, it seems natural to assume that the innovation arrivals of production and pollution abatement technologies are independent. Certainly, it is possible to invent a better catalyst without also inventing a more powerful engine. Secondly, to a large extent, private firms are only willing to conduct research, when the developed ideas can be protected. As patents are granted very specific components rather than entire systems, it seems appropriate to assume that production and pollution abatement technologies are developed separately.

This paper relates to a growing body of literature investigating how environmental policy affects the direction of technological change in endogenous growth models. Smulders and de Nooij (2003), Grimaud and Rouge (2008), and Acemoglu et al. (2012) assume that output is produced using a constant elasticity of substitution production function with two input types: clean and dirty. Production of the dirty input requires a natural resource or energy input, while the clean input does not. Environmental policy can then skew incentives such that it becomes relatively more attractive to conduct research in clean input technologies. Smulders and de Nooij (2003) find that policies reducing the level of the energy input without changing the growth rate, has no impact on long run economic growth, whereas policies reducing the growth rate of the energy input, reduces long run economic growth. Acemoglu et al. (2012) show that if the environmental policy is tightened sufficiently to avoid an environmental disaster, growth is initially dampened as the clean technologies catch up. But, long run growth is unaffected by the policy.
In contrast, the present study finds that a tightening of the environmental policy unambiguously dampens long-run economic growth. The reason is that pollution emission becomes relatively more expensive, which decreases demand for production technologies. It then becomes less profitable to conduct research in production technologies, and less resources are devoted to research in these technologies. On the other hand, it becomes relatively more attractive to conduct research in pollution abatement technologies, as the demand for pollution abatement technologies increases. As a result, the growth rates of output and pollution emission unambiguously fall. This also holds when resources are reallocated from manufacturing to R&D, as a consequence of the policy change.

The paper will proceed as follows. In the next section, stylized facts related to economic growth, pollution emission, environmental taxation, and pollution abatement expenditures are presented. The aim is to construct a model that is able to match these stylized facts. Section 3 presents the basic model, and it is shown that the model matches the stylized facts. The policy implications of the model are derived analytically in Section 4. The model is slightly generalized in Section 5, and the model is simulated in Section 6. Section 7 concludes.

2 Some Stylized Facts

In this section, I present some stylized facts related to economic growth, pollution emission, environmental policy, and pollution abatement expenditures.1 Some of these facts serve as motivation for certain modeling assumptions made in the subsequent section, and the model is shown also to match the remaining facts.

2.1 Pollution intensities

The first set of stylized facts are related to pollution intensities, defined as pollution emissions over GDP. A low pollution intensity means that little pollution is emitted compared to the economic value created. Following Brock and Taylor (2005, 2010), I focus on air pollution. In particular, I focus on the air pollutants CO, NO\(_x\), SO\(_2\) (or SO\(_x\)), VOC (volatile organic compound), and CO\(_2\). Brock and Taylor (2005) show that these pollution intensities decreased almost monotonically for the US over the period 1940-1998.

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1 Some of the stylized facts presented below are to my knowledge new to the literature. The rest have been presented in either Brock and Taylor (2005, 2010) or Botta and Kozluk (2014).
As shown in the left panel of Figure 1, the pattern has continued in more recent years. From the right panel of Figure 1, it is clear that aggregate US emissions have decreased substantially for most pollutants (CO, NO\textsubscript{x}, SO\textsubscript{2}, and VOC) over the period 1970-2012. Hence, the decrease in these pollution intensities are not only caused by an increase in GDP, but also by a decrease in emissions. In contrast, aggregate CO\textsubscript{2} emission increased over the period, and the decrease in CO\textsubscript{2} pollution intensity is therefore a consequence of relatively faster growth in GDP.

**FIGURE 1:** US pollution emissions and intensities, 1970-2012.

*Data sources:* EPA, BEA, and Boden, Marland, and Andres (2013).

*Notes:* Pollution intensity defined as emission divided by GDP. The CO\textsubscript{2} emission is confined to emission from fossil fuel use and cement production.

Generally, OECD countries experienced a decline in these five pollution intensities during the period 1990-2012.\(^2\) In fact, it seems like the pollution intensities are reversely related to income and environmental policy stringency. Let GDP per capita measure income, and let environmental policy stringency be measured by the economy wide environmental policy stringency (EPS) index described in Botta and Kozluk (2014). The EPS index is defined from zero to six, where zero is when environmental policy is none existing, and six is a very stringent environmental policy.

Table 1 present OLS regression results for log transformed pollution intensities regressed on income and the EPS index. The regression results indicate that both income and environmental policy stringency are negatively correlated with pollution intensities. As income is strongly correlated with the EPS index, it seems appropriate to have both variables in the regression. It turns out that the coefficients for the EPS index are negative and significant at the one pct. level in all cases, whereas the income coefficients are negative and significant at the one pct. level in most cases.

\(^2\)Disregarding Mexico due to insufficient data, the exceptions the clear decreasing trend in pollution intensities are: CO\textsubscript{2}: Portugal, Spain, and Turkey. SO\textsubscript{2}: Iceland. VOC: Chile.
Table 2 provides OLS regression results when including country fixed effects. Overall, the results are very similar, but now all regression coefficients are negative and statistically significant at the one pct. level. In addition, the $R^2$ values are notably higher.

**TABLE 1:** Pollution intensity regressed on GDP per capita and/or the EPS index for OECD countries over the period 1990-2012 using OLS.

<table>
<thead>
<tr>
<th>Income</th>
<th>$\ln(CO_2)$</th>
<th>$\ln(CO)$</th>
<th>$\ln(NO_x)$</th>
<th>$\ln(SO_x)$</th>
<th>$\ln(VOC)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.23*</td>
<td>-0.08*</td>
<td>-0.26*</td>
<td>-0.08*</td>
<td>-0.96*</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.04)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>EPS</td>
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<td>-0.10*</td>
<td>-0.38*</td>
<td>-0.28*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.02)</td>
<td></td>
</tr>
<tr>
<td>Country FE</td>
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<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>Observations</td>
<td>594</td>
<td>510</td>
<td>510</td>
<td>510</td>
</tr>
<tr>
<td></td>
<td>$R^2$</td>
<td>0.27</td>
<td>0.13</td>
<td>0.16</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>Adjusted $R^2$</td>
<td>0.16</td>
<td>0.27</td>
<td>0.28</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Data source: OECD.

Note: Standard errors in brackets. (*) indicates significant at a 1 pct. level. Environmental policy stringency (EPS) is measured using the EPS index presented in Botta and Kozluk (2014). The index is defined from 0 (not existing) to 6 (most stringent). Income is GDP per capita (10,000 US 2005-dollars per capita). Pollution intensity defined as emission divided by GDP (unit: tons of emission per thousand US 2005-dollars).

It seems fair to conclude that for developed countries, pollution intensities fall with income and environmental policy stringency. Aggregate pollution emission might, however, still increase with income. For instance, US pollution emission increased in the period 1948-1970 for $SO_2$, $CO$, and $VOC$ (see Brock and Taylor 2010), while US income increased. Hence, a theoretical model should predict that income grows faster than pollution emission, while allowing both de- and increasing pollution emission.

**TABLE 2:** Pollution intensities regressed on GDP per capita and the EPS index for OECD countries using OLS, 1990-2012.

<table>
<thead>
<tr>
<th>Income</th>
<th>$\ln(CO_2)$</th>
<th>$\ln(CO)$</th>
<th>$\ln(NO_x)$</th>
<th>$\ln(SO_x)$</th>
<th>$\ln(VOC)$</th>
</tr>
</thead>
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<tr>
<td></td>
<td>-0.37*</td>
<td>-0.21*</td>
<td>-1.10*</td>
<td>-0.80*</td>
<td>-0.74*</td>
</tr>
<tr>
<td></td>
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<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.04)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>EPS</td>
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<td>-0.43*</td>
<td>-0.31*</td>
<td></td>
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<td>(0.00)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
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</tr>
<tr>
<td>Country FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>Observations</td>
<td>594</td>
<td>510</td>
<td>510</td>
<td>510</td>
</tr>
<tr>
<td></td>
<td>$R^2$</td>
<td>0.66</td>
<td>0.63</td>
<td>0.73</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>Adjusted $R^2$</td>
<td>0.71</td>
<td>0.84</td>
<td>0.79</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Data source: OECD.

Note: Standard errors in brackets. (*) indicates significant at a 1 pct. level. Environmental policy stringency (EPS) is measured using the EPS index presented in Botta and Kozluk (2014). The index is defined from 0 (not existing) to 6 (most stringent). Income is GDP per capita (10,000 US 2005-dollars per capita). Pollution intensity defined as emission divided by GDP (unit: tons of emission per thousand US 2005-dollars).

### 2.2 Environmental policy

Figure 2 depicts the evolution of the EPS index over time. From the left panel, it is clear that the EPS values for all individual OECD countries for which data is available, has
increased from 1995 to 2012. The right panel shows that the average EPS value in the OECD increased systematically through the period 1990-2012.

**FIGURE 2:** Evolution of the environmental policy stringency (EPS) index over time.
*Data source:* OECD.
*Notes:* Environmental policy stringency (EPS) is measured using the EPS index presented in Botta and Kozluk (2014). The index is defined from 0 (not existing) to 6 (most stringent). Due to missing data, the 1990 value is used for IRL.

Figure 3 depicts tax revenues from environmentally related taxes as share of GDP for some of the largest OECD economies in the period 1994-2012. In contrast to the EPS index, the tax revenues from environmental taxes as share of GDP remains remarkably constant over the period despite many economic events, e.g. business cycles, policy changes, and the Financial Crisis of 2008.

The theoretical model will focus on environmental taxes. As the environmental tax revenue from environmental taxes seems to be a constant share of GDP, I will assume that the government adjusts environmental tax rates to ensure this relation. Using this policy rule, environmental tax rates must increase over time, as pollution intensities fall. Higher environmental tax rates then translate into a higher EPS index value.

**FIGURE 3:** Revenues from environmentally related taxes, 1994-2012.
*Data source:* OECD.stat.
2.3 Pollution abatement costs

An important component in most growth models designed to answer environmental related questions are the pollution abatement expenditures. Data on the subject is relatively scarce, and this paper will therefore focus on the US case for which the longest time series are available. The left panel of Figure 4 shows that aggregate US pollution abatement expenditures was approximately a constant share of GDP over the period 1975-1994. The notable increase in pollution abatement expenditures as share of GDP from 1972 to 1975 can probably be attributed to the Clean Air Act of 1970, which changed US air pollution policy substantially.\textsuperscript{3}

The left panel of Figure 4 also show that pollution abatement expenditures were roughly a constant share of GDP for the business sector during the period. The right panel of Figure 4 depicts pollution abatement expenditure shares by sector. The business sector had a share of over 60 pct. for the whole period, whereas each of the other two sectors never had a share above 30 pct.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{US pollution abatement expenditures, 1972-1994. \textit{Data sources:} Vogan (1996) and BEA.}
\end{figure}

The empirical evidence presented in Figure 4 have two important implications. Firstly, aggregate and business sector pollution abatement expenditures as share of GDP can be constant, while pollution emissions and intensities fall (see Figure 1 and 4). A theoretical model should therefore allow for falling pollution emissions and intensities, when aggregate and business sector pollution abatement expenditures are some constant share of GDP. Secondly, as the business sector accounts for most of the aggregate pollution abatement expenditures, it seems natural to focus on this sector in theoretical work.

\textsuperscript{3}The Air Act Amendments of 1977 and 1990 added major amendments to the the Clean Air Act of 1970, but do not seem to have affected the pollution abatement expenditures much. For more information about the Clean Air Act, see Davidson and Norbeck (2012) and Brownell et al. (2015)
3 The Model

The model is an extension of the Schumpeterian model presented in Aghion and Howitt (1998), p. 85-92. Time is continuous. There are two types of intermediate goods: one used for production and one for pollution abatement. These two intermediate good types will be referred to as production and abatement intermediates, respectively. In the presented model, GDP ends up proportional to output (of final goods), and the difference between the two will therefore be ignored, when relating the model to the stylized facts presented above. This is without loss of generality.

3.1 The final goods sector

There is a continuum of firms in the final goods sector indexed by \( j \in [0, N_t] \). Firms in the final goods sector operate under perfect competition. Aggregate output (of final goods), \( Y_t \), is given by

\[
Y_t = \int_0^{N_t} Y_{jt} \, dj, \quad j \in [0, N_t]
\]

(3.1)

where \( Y_{jt} \) is final goods production by firm \( j \) at time \( t \), and \( N_t \) is the amount of firms with a positive production in the final goods sector.

All final goods are consumed, and aggregate consumption, \( C_t \), is therefore given by

\[
C_t = Y_t
\]

(3.2)

Final goods are produced using production intermediates as input. In addition, there is a quasi-fixed cost associated with production.\(^4\) The production technology in the final goods sector is given by

\[
Y_{jt} = \max \left( 0; \int_0^1 x_{ijt}^\alpha A_{it} \, di - F A_t^{\text{MAX}} \right), \quad i \in [0, 1], \quad \alpha \in (0, 1), \quad F > 0
\]

(3.3)

where \( x_{ijt} \) is production intermediate \( i \) used by firm \( j \) at time \( t \), \( A_{it} \) is a parameter capturing the productivity of intermediate good \( i \), and \( A_t^{\text{MAX}} \equiv \max \left( (A_{it})_{i \in [0,1]} \right) \) is the leading edge production technology. Note that the quasi-fixed cost, \( F A_t^{\text{MAX}} \), is proportional to the leading edge production technology. The idea is that more advanced production

\(^4\)A quasi-fixed cost is a cost independent of the output level, but as opposed to a fixed cost, it is only present when production activity occurs (e.g., lights in a factory).
technologies also require more advanced facilities.\textsuperscript{5}

Pollution is an unavoidable by-product of production. Inspired by the pollution function in Gradus and Smulders (1993), pollution emission of firm $j$, $P_{jt}$, is given by

$$P_{jt} = \frac{Y_{jt}^\beta}{Z_{jt}^\chi}, \quad \beta \in (0, 1), \quad \chi \in (0, 1)$$

(3.4)

where $Z_{jt}$ is abatement activity of firm $j$. Aggregate pollution emission, $P_t$, is given by

$$P_t = \int_0^{N_t} P_{jt} \, dj$$

(3.5)

The government imposes a tax on pollution emission. In order to reduce the environmental tax bill, firms reduce pollution emission by allocating resources to pollution abatement. Abatement activity is conducted using only abatement intermediates as input. Total abatement activity of firm $j$ is given by

$$Z_{jt} = \int_0^1 z_{hjt}^\mu B_{ht} \, dh, \quad h \in [0, 1], \quad \mu \in (0, 1)$$

(3.6)

where $z_{hjt}$ is abatement intermediate $h$ used by firm $j$ at time $t$, and $B_{ht}$ is a parameter capturing the efficiency of abatement intermediate $h$.

The profit of firm $j$, $\pi^f_{jt}$, is given by

$$\pi^f_{jt} = \int_0^1 x_{ijt}^a A_{it} \, di - FA_{t}^{MAX} - \int_0^1 x_{ijt} P_{it} \, di$$

$$- \int_0^1 z_{hjt} q_{ht} \, dh - \frac{\left(\int_0^1 x_{ijt}^a A_{it} \, di - FA_{t}^{MAX}\right)^\beta}{\left(\int_0^1 z_{hjt}^\mu B_{ht} \, dh\right)^\chi} \tau_t$$

(3.7)

where $p_{it}$ is the price of production intermediate $i$, $q_{ht}$ is the price of abatement intermediate $h$, and $\tau_t$ is the pollution tax rate. All prices are in terms of final goods. Each firm $j$ maximizes $\pi^f_{jt}$ with respect to $(x_{ijt})_{i \in [0, 1]}$ and $(z_{hjt})_{h \in [0, 1]}$ taking $q_{ht}$, $p_{it}$, and $\tau_t$ as given.

\textsuperscript{5}There are two things worth noticing. Firstly, from a pure modeling perspective, the proportionality between $A_{t}^{MAX}$ and the quasi-fixed cost ensures that $N_t$ is finite and constant over time in equilibrium. This makes the model more tractable. Note also that substituting $A_{t}^{MAX}$ with a measure of the average technological level $A_t = \int_0^1 A_{it} \, di$ does not change the qualitative results, as the two technology measures grow at the same rate in equilibrium. Secondly, it can be shown that the model features constant returns to scale on the aggregate level in equilibrium, with respect to labor input in manufacturing.
The first-order conditions are given by

\[ p_{it} = \alpha x_{it}^{\alpha - 1} A_{it} \left( 1 - \beta \frac{P_{jt}}{Y_{jt}} \right), \quad \forall i \in [0, 1] \]  
(3.8)

\[ q_{ht} = \chi \mu z_{ht}^{\mu - 1} B_{ht} \frac{P_{jt}}{Z_{jt}} \tau_t, \quad \forall h \in [0, 1] \]  
(3.9)

Firms will have an incentive to enter the final goods sector until profits are driven to zero. As all final good firms are identical, \( N_t \) adjusts such that the equilibrium condition

\[ \pi^f_{jt} = 0, \quad \forall j \in [0, N_t] \]  
(3.10)

is satisfied.

Motivated by the data presented in Figure 3, I assume that the government adjusts the pollution tax rate, such that the tax revenue from the pollution tax is a constant share of output:

\[ P_t \tau_t = \phi Y_t, \quad \phi \in (0, 1) \]  
(3.11)

In equilibrium \( P_{jt} = \hat{P}_t = \frac{1}{N_t} P_t, Y_{jt} = \hat{Y}_t = \frac{1}{N_t} Y_t, x_{ijt} = \hat{x}_{it}, z_{hti} = \hat{z}_{ht}, \) and \( Z_{jt} = \hat{Z}_t \) since all the firms in the final goods sector are identical, the production technology exhibits decreasing marginal product, and a quasi-fixed cost is present. As final good firms take the tax rate, \( \tau_t \), as given, equilibrium expressions for the intermediate good prices are obtained by substituting (3.11) into (3.8) and (3.9):

\[ p_{it} = \alpha \hat{x}_{it}^{\alpha - 1} A_{it} (1 - \phi \beta), \quad \forall i \in [0, 1] \]  
(3.12)

\[ q_{ht} = \chi \mu \phi \hat{Y}_t \hat{z}_{ht}^{\mu - 1} B_{ht}, \quad \forall h \in [0, 1] \]  
(3.13)

Given \( \hat{Y}_t, \hat{Z}_t, \) and prices, an increase in \( \phi \) decreases \( \hat{x}_{it} \) and increases \( \hat{z}_{ht} \). When \( \phi \) is increased, pollution emission becomes more expensive, which decreases demand for (polluting) production intermediates and increases demand for abatement intermediates.

3.2 The technological process and the R&D sector

The R&D sector is divided into two subsectors: one developing higher production intermediate good qualities, and one higher abatement intermediate good qualities. Each of
these subsectors are again divided into a continuum of subsectors as there are labs developing higher qualities of each $i$ and $h$ intermediate. In both subsectors, entrepreneurs are conducting research using only labor as input. If an entrepreneur invents a higher quality of an intermediate good, he/she receives a patent (of infinite duration) on that intermediate good quality. As the previous intermediate good qualities cannot compete, the entrepreneur becomes a monopolist of that intermediate good. The value of a patent on the second highest quality is thereby destroyed. Hence, innovation is associated with creative destruction.

Innovations arrive randomly following a Poisson process. Denote labor input in sub-subsector $i$ by $n^A_{it}$, and labor input in sub-subsector $h$ by $n^B_{ht}$. The Poisson arrival rates in the sub-subsectors $i$ and $h$, are given by $\lambda n^A_{it}$ and $\eta n^B_{ht}$, respectively. When a new intermediate good quality is developed, the leading edge technology for that intermediate good type is increased due to positive spillover effects. Specifically, the leading edge technologies evolve according to

$$A^\text{MAX}_t = A^\text{MAX}_0 \lambda \ln(\gamma) n^A_t, \quad A^\text{MAX}_0 > 0 \quad \text{given,} \quad \lambda > 0, \quad \gamma > 1$$

$$B^\text{MAX}_t = B^\text{MAX}_0 \eta \ln(\xi) n^B_t, \quad B^\text{MAX}_0 > 0 \quad \text{given,} \quad \eta > 0, \quad \xi > 1$$

where $B^\text{MAX}_t \equiv \max \left( (B_{ht})_{h \in [0,1]} \right)$ is the leading edge abatement technology, $n^A_t \equiv \int_0^1 n^A_{it} \, di$, $n^B_t \equiv \int_0^1 n^B_{ht} \, dh$, the dots denote derivatives with respect to time, and $\ln(\gamma)$ and $\ln(\xi)$ capture the sizes of the spillover effects.

A new quality of an intermediate good developed at time $t'$ will have the same productivity as the leading edge technology of that intermediate good type. Hence, a production and an abatement intermediate developed at time $t'$ have the productivities $A^\text{MAX}_{t'}$ and $B^\text{MAX}_{t'}$, respectively. It turns out that in equilibrium research is conducted equally on each intermediate good, given the type. As there exists a continuum of intermediates, the expected number equals the actual number of innovation arrivals due to the law of large numbers. Hence, the leading edge technologies evolve according to non-stochastic processes while the technological level for each intermediate is stochastic.

As shown in Aghion and Howitt (1998) the distribution of the relative technology pa-
rameters $a_{it} \equiv A_{it}/A_{t}^{\text{MAX}}$ and $b_{ht} \equiv B_{ht}/B_{t}^{\text{MAX}}$ will converge to the distributions

\[ G_A(a) \equiv a^{\frac{1}{\ln(\gamma)}} , \quad \forall a \in [0, 1] \]  \hspace{1cm} (3.16)
\[ G_B(b) \equiv b^{\frac{1}{\ln(\xi)}} , \quad \forall b \in [0, 1] \]  \hspace{1cm} (3.17)

in the long-run. For simplicity, let $(a_{it})_{i \in [0,1]}$ and $(b_{ht})_{h \in [0,1]}$ be distributed according to $G_A(a)$ and $G_B(b)$ from $t = 0$.

### 3.2.1 The two R&D subsectors

There is free entry into both R&D subsectors, and all R&D firms take the wage rate as given. As the probability of inventing a new intermediate good quality is proportional to the labor input, firms in the R&D sector will demand an infinite amount of labor if the expected value of research is greater than the wage rate. In the opposite case, where the expected value is smaller than the wage rate, the demand for labor is zero. Hence, the research arbitrage conditions are given by

\[ w_t = \frac{\lambda V^{T}_{it}}{1 + \lambda} \quad \text{if} \quad n_{it}^A > 0 \]  \hspace{1cm} (3.18)
\[ w_t = \frac{\eta V^{T}_{ht}}{1 + \eta} \quad \text{if} \quad n_{ht}^B > 0 \]  \hspace{1cm} (3.19)

where $T_{it}$ and $T_{ht}$ denote the number of qualities of intermediate $i$ and $h$, respectively, developed at time $t$; $V^{T}_{it}$ and $V^{T}_{ht}$ denote the value of inventing a higher quality of intermediate good $i$ and $h$, respectively; and $w_t$ is the wage rate. The research arbitrage equations ensure that in equilibrium, the cost of having a researcher working for one unit of time equals the expected value of his/her work.

The value of a new innovation is the net present value of the expected future profit stream associated with the monopoly obtained on that intermediate good quality

\[ V^{T}_{it} = \int_t^{\infty} \tilde{\pi}_{it}^A e^{-\int_t^{s} (r_u + n_{iu}^A \lambda) du} ds \]  \hspace{1cm} (3.20)
\[ V^{T}_{ht} = \int_t^{\infty} \tilde{\pi}_{ht}^B e^{-\int_t^{s} (r_u + n_{hu}^B \eta) du} ds \]  \hspace{1cm} (3.21)

where $\tilde{\pi}_{it}^A$ and $\tilde{\pi}_{ht}^B$ are the optimized profit streams associated with a monopoly of intermediate $i$ and $h$, respectively, and $r_t$ is the (risk free) real interest rate. The instantaneous profit stream is discounted by a modified interest rate, which reflects the opportunity cost.
of investing in R&D firms (the real interest rate), and the risk of losing the monopoly (the flow probability that a higher quality of the intermediate good is developed).

Since a new quality of any intermediate good of a certain type invented at time $t'$ is associated with the leading edge technology of that intermediate good type, the value of inventing a higher quality of any intermediate good is the same, given the type. It then follows from (3.18) and (3.19) that the same amount of research will be conducted on each intermediate good, given the type. Hence, $n_t^A \equiv \int_0^1 n^A_i d_i = n^A_t$ and $n_t^B \equiv \int_0^1 n^B_{ht} dh = n^B_{ht}$ in equilibrium.

### 3.3 The intermediate goods sector

In the intermediate goods sector, previously successful entrepreneurs from the R&D sector produces intermediate goods. They operate under monopolistic competition, as each monopolist has the exclusive rights to produce the highest existing quality of a certain intermediate good. Innovations are assumed to be drastic such that a monopolist can charge any price for his/her intermediate goods without fearing entry from previous monopolists.

The sector is divided into two subsectors: one for each intermediate good type. Given the technological design, intermediates are produced using only labor as input. One unit of labor can produce one unit of any intermediate.

#### 3.3.1 The production intermediate subsector

In the production intermediate subsector, a continuum of firms indexed by $i \in [0, 1]$, produce production intermediates. Each firm $i$ has a monopoly on the intermediate good $x_{it}$ and earns the profit, $\pi^A_{it}$, given by

$$\pi^A_{it} = \int_0^N (p_{it} - w_t)x_{ijt} \, dj$$

The monopolists face the demand function given by (3.12) when maximizing profits. The first-order condition for firm $i$ can be rewritten as

$$\hat{x}_{it} = \left( \frac{\alpha^2(1 - \phi\beta)}{w_t} A_{it} \right)^{\frac{1}{1+\alpha}}$$
where it is used that \( x_{ijt} = \hat{x}_{it} \) in equilibrium. The price of production intermediate \( i \) is derived by substituting (3.23) into (3.12):

\[
p_{it} = \frac{1}{\alpha} w_t
\]

By substituting (3.23) and (3.24) into (3.22), the equilibrium profit stream for monopolist \( i, \tilde{\pi}^A_{it} \), is obtained:

\[
\tilde{\pi}^A_{it} = (1 - \alpha)\alpha (1 - \phi \beta) N_t A_{it} \hat{x}^\alpha_{it}
\]

The total profits of all firms in the production intermediate subsector, \( \pi^A_t \), are given by

\[
\pi^A_t = \int_0^1 \tilde{\pi}^A_{it} \, di = (1 - \alpha)\alpha (1 - \phi \beta) \left( Y_t + N_t F A^\text{MAX}_t \right)
\]

Using that \( a_{it} \) is distributed according to (3.16), it follows from (3.1), (3.3), and (3.23) that \( Y_t \) can be expressed as

\[
Y_t = \left( \frac{\alpha^2 (1 - \phi \beta) A^\text{MAX}_t w_t}{\Lambda_A - F} \right)^{\frac{1}{1 - \mu}} N_t A^\text{MAX}_t, \quad \Lambda_A \equiv \frac{1 - \alpha}{1 - \alpha + \ln(\gamma)}
\]

3.3.2 The abatement intermediate subsector

In the abatement intermediate subsector, a continuum of firms indexed by \( h \in [0, 1] \), produce abatement intermediates. Each firm \( h \) has a monopoly on the intermediate good \( z_{ht} \) and earns the profit, \( \pi^B_{ht} \), given by

\[
\pi^B_{ht} = \int_0^{N_t} (q_{ht} - w_t) z_{htj} \, dj
\]

The monopolists face the demand function given by (3.13) when maximizing profits. The first-order condition for firm \( h \) can be rewritten as

\[
\hat{z}_{ht} = \left( \frac{\chi \mu \phi \hat{Y}_t}{\hat{Z}_{ht} w_t} B_{ht} \right)^{\frac{1}{1 - \mu}}
\]
where it is used that \( z_{ht} = \hat{z}_{ht} \) in equilibrium. Abatement activity per firm is obtained by substituting (3.29) into (3.6):

\[
\hat{Z}_t = \left( \frac{\chi \mu^2 \phi \hat{Y}_t}{w_t} \right)^\mu A_B^{1-\mu} B_t^{\mathrm{MAX}}, \quad A_B = \frac{1 - \mu}{1 - \mu + \ln(\xi)}
\] (3.30)

Substituting (3.30) into (3.29) and rearranging terms:

\[
\hat{z}_{ht} = \frac{\chi \mu^2 \phi}{w_t} \Lambda_B^{-1} B_t^{1-\mu} \hat{Y}_t
\] (3.31)

The price of abatement intermediate \( h \) is derived by substituting (3.31) into (3.13):

\[
q_{ht} = \frac{1}{\mu} w_t
\] (3.32)

By substituting (3.31) and (3.32) into (3.28), the equilibrium profit stream for monopolist \( h \), \( \tilde{\pi}_B^{ht} \), is derived as

\[
\tilde{\pi}_B^{ht} = (1 - \mu) \chi \mu \phi \Lambda_B^{-1} B_t^{1-\mu} Y_t
\] (3.33)

The total profits of all firms in the abatement intermediate subsector, \( \pi^B_t \), are given by

\[
\pi^B_t = \int_0^1 \tilde{\pi}_B^{ht} \, dh = (1 - \mu) \chi \mu \phi Y_t
\] (3.34)

### 3.4 Consumers, the government, and the labor market

The consumer side of the economy is modeled as a single representative household solving the problem:

\[
\max_{(c_t)_{t=0}^\infty} \int_0^\infty \ln(c_t) \, e^{-\rho t} \, dt, \quad \rho > 0
\]

\[
\text{st.} \quad \dot{\bar{W}}_t = w_t \bar{L} + r_t \bar{W}_t + I_t - c_t \bar{L}, \quad \bar{W}_t > 0 \quad \text{given}, \quad \lim_{t \to \infty} \bar{W}_t \, e^{-\int_0^\infty r_t \, dt} \geq 0, \quad c_t \geq 0
\]

where \( \bar{L} > 0 \) is aggregate labor supply, \( c_t \equiv \frac{C_t}{\bar{L}} \), \( \bar{W}_t \) is financial wealth, and \( I_t \) is a lump-sum government transfer.\(^6\) Using optimal control theory, the following two conditions for

---

\(^6\)The more general case of CRRA preferences will be studied in Section 5.
optimal behavior are derived:

\[
\begin{align*}
\frac{\dot{c}_t}{c_t} = \frac{\dot{C}_t}{C_t} = r_t - \rho \\
\lim_{t \to \infty} \tilde{W}_t e^{-\int_0^\infty r_t \, dt} &= 0
\end{align*}
\] (3.35)

The representative household owns all firms in the economy. However, only the firms in the intermediate goods sector earn positive profits. Hence, the financial wealth of the representative household is the net present value of all existing intermediate good firms, which is given by

\[
\tilde{W}_t = V_t + \tilde{V}_t
\] (3.37)

where \( V_t \equiv \int_0^1 V_{it}^T \, di \) and \( \tilde{V}_t = \int_0^1 \tilde{V}_{ht}^T \, dh \). The saving of the representative household finances R&D activity. These R&D activities increase the value of active intermediate good firms and thereby the representative household’s financial wealth.

The government keeps a balanced budget at all times. That is, the entire tax revenue is transferred to the representative household at all moments in time:

\[
I_t = \phi Y_t
\] (3.38)

The labor market operates under perfect competition, and labor can be devoted to either manufacturing or research. The labor market clearing condition is given by

\[
\tilde{L} = L_t + n_t^A + n_t^B
\] (3.39)

where \( L_t \) is labor devoted to manufacturing. In equilibrium, labor input devoted to manufacturing must equal the total supply of intermediate goods. Hence, the manufacturing clearing condition is given by

\[
L_t = x_t + z_t
\] (3.40)

where \( x_t \equiv \int_0^{N_i} \left( \int_0^1 x_{ijt} \, di \right) \, dj \) and \( z_t \equiv \int_0^{N_i} \left( \int_0^1 z_{ht} \, dh \right) \, dj \).
3.5 Market equilibrium

To close the model, the allocation of labor has to be determined. This will be done in several steps. As a starting point, \( N_t \) is determined using the equilibrium condition (3.10). All firms in the final goods sector obtain the same profit denoted \( \tilde{\pi}_f^t \). Substituting (3.12) and (3.13) into (3.7)

\[
\tilde{\pi}_f^t = \frac{1}{N_t} Y_t (1 - \alpha(1 - \phi \beta) - \chi \mu \phi - \phi) - FA_t^{\text{MAX}} \alpha(1 - \phi \beta)
\]

The variable \( N_t \) can be isolated using equilibrium condition (3.10):

\[
N_t = \frac{1 - \alpha(1 - \phi \beta) - \chi \mu \phi - \phi}{\alpha(1 - \phi \beta) F} \frac{Y_t}{A_t^{\text{MAX}}} \quad (3.41)
\]

To ensure that \( N_t > 0 \), the following parameter restriction is imposed:

**Parameter Restriction 1.** \( 1 - \alpha(1 - \phi \beta) - \chi \mu \phi - \phi > 0 \).

The wage rate is determined using (3.23), (3.27), (3.31), (3.40), and (3.41):

\[
w_t = \Omega_1 \frac{Y_t}{L_t}, \quad \Omega_1 \equiv (1 - \chi \mu \phi - \phi) \alpha + \chi \mu^2 \phi \quad (3.42)
\]

To determine aggregate output, substitute (3.41) and (3.42) into (3.27):

\[
Y_t = \frac{\Lambda_A \Psi}{(1 - \chi \mu \phi - \phi)} N_t^{1-\alpha} L_t^\beta A_t^{\text{MAX}}, \quad \Psi \equiv (1 - \phi \beta) \alpha \left(\frac{(1 - \chi \mu \phi - \phi) \alpha}{\Omega_1 \Lambda_A}\right)^{\alpha} \quad (3.43)
\]

Substituting (3.43) into (3.41):

\[
N_t = \left(\frac{1 - \alpha(1 - \phi \beta) - \chi \mu \phi - \phi}{(1 - \chi \mu \phi - \phi) F}\right)^{\frac{1}{\alpha}} \left(\frac{\alpha(1 - \chi \mu \phi - \phi)}{\Omega_1}\right)^{\frac{1 - \alpha}{\alpha}} L_t \quad (3.44)
\]

Using (3.23), (3.43), and (3.42), \( \hat{x}_{it} \) can be derived as

\[
\hat{x}_{it} = \frac{\alpha(1 - \chi \mu \phi - \phi) a_{it}^{\frac{1}{\alpha}}}{\Omega_1 \Lambda_A} \frac{L_t}{N_t} \quad (3.45)
\]
Expressions for $\tilde{\pi}^A_{it}$ and $\tilde{\pi}^B_{ht}$ can be derived using (3.25), (3.33), (3.43), and (3.45):

$$
\tilde{\pi}^A_{it} = (1 - \alpha)\Psi a_{it}^{\frac{\alpha}{1 - \alpha}} N_t^{1 - \alpha} L_t^\alpha A_{it}
$$

(3.46)

$$
\tilde{\pi}^B_{ht} = (1 - \mu)\chi\mu\Lambda A \Psi \frac{1}{(1 - \chi\mu - \phi)} N_t^{1 - \alpha} L_t^\alpha A_{MAX}^t
$$

(3.47)

The final step is to use the research arbitrage equations (3.18) and (3.19) to determine the labor allocation.

It is shown in Appendix A.1 that in equilibrium, the two following conditions must hold:

$$
1 = \lambda \Omega_2 L_t = \frac{\lambda \Omega_2 L_t}{r_t + n^A_t \lambda (1 + \frac{\alpha}{1 - \alpha} \ln(\gamma))}, \quad \Omega_2 = \frac{1 - \alpha}{\Lambda A} \frac{1 - \chi\mu\phi - \phi}{(1 - \chi\mu\phi - \phi)\alpha + \chi \mu^2 \phi}
$$

(3.48)

$$
1 = \eta \Omega_3 L_t = \frac{\eta \Omega_3 L_t}{r_t + n^B_t \eta \Lambda_B^{-1} - n^A_t \alpha \ln(\gamma)}, \quad \Omega_3 = \frac{(1 - \mu)\chi\mu}{\Lambda B} \frac{\phi}{(1 - \chi\mu\phi - \phi)\alpha + \chi \mu^2 \phi}
$$

(3.49)

From (3.2), (3.35), (3.43), and (3.44):

$$
g_{Y,t} = g_{C,t} = g_{L,t} + g_{A,t} = r_t - \rho
$$

(3.50)

where $g_{Y,t} \equiv \dot{Y}_t / Y_t$, $g_{C,t} \equiv \dot{C}_t / C_t$, $g_{L,t} \equiv \dot{L}_t / L_t$, and $g_{A,t} \equiv A_{MAX}^t / A_{MAX}^t$.

Consider the following proposition:

**Proposition 1.** If the parameters in the model are such that $n^A_t > 0$, $n^B_t > 0$, and $L_t > 0$ for all $t \geq 0$, and that (3.39) is satisfied, then $g_{L,t} = 0$ for all $t \geq 0$.

The proposition is proved in Appendix A.2. It follows directly from (3.48), (3.49), and Proposition 1 that the labor allocation is constant in equilibrium. To ensure that $n^A > 0$ and $n^B > 0$ in equilibrium, the following two parameter restrictions are imposed:

**Parameter Restriction 2.** $\bar{L} + \frac{\rho}{\eta} \Lambda_B > \frac{1 - \phi(1 - \mu)\chi\mu\phi - \phi}{(1 - \alpha)(1 - \chi\mu - \phi)} \frac{\rho}{\Lambda A}$

**Parameter Restriction 3.** $\bar{L} + \frac{\rho}{\Lambda A} > \frac{1 - \phi(1 - \mu)\chi\mu\phi - \phi}{(1 - \alpha)(1 - \chi\mu - \phi)} \frac{\rho}{\eta} \Lambda_B$

Using Proposition 1, the labor allocation in equilibrium is determined by (3.39), (3.48),
(3.49), and (3.50):

\[
L = \frac{(1 - \chi \mu \phi - \phi) \alpha + \chi \mu^2 \phi}{1 - \phi} \Gamma, \quad \Gamma \equiv \bar{L} + \rho \left( \frac{\Lambda_A}{\lambda} + \frac{\Lambda_B}{\eta} \right)
\]

(3.51)

\[
n^A = \frac{(1 - \alpha)(1 - \chi \mu \phi - \phi)}{1 - \phi} \Gamma - \frac{\Lambda_A}{\lambda} \rho
\]

(3.52)

\[
n^B = \frac{(1 - \mu) \chi \mu \phi}{1 - \phi} \Gamma - \frac{\Lambda_B}{\eta} \rho
\]

(3.53)

Define a balanced growth path as a path where the variables \(C_t, Y_t, Y_{jt}, A_t^{MAX}, w_t, p_t, q_{ht}, B_t^{MAX}, P_t,\) and \(P_{jt}\) all grow at constant rates while \(N_t\) is constant. Consider the following Proposition:

**Proposition 2.** The variables \(C_t, Y_t, Y_{jt}, A_t^{MAX}, w_t, p_t, q_{ht}, B_t^{MAX}, P_t,\) and \(P_{jt}\) all grow at the constant rate \(g_A = n^A \lambda \ln(\gamma),\) \(N_t\) is constant over time, and \(B_t^{MAX}\) grows at the constant rate \(g_B = n^B \eta \ln(\xi).\) In addition, \(P_t\) and \(P_{jt}\) grow at the constant rate \(g_P = \beta n^A \lambda \ln(\gamma) - \chi n^B \eta \ln(\xi).\)

The proposition is proved in Appendix A.3. According to Proposition 2, the economy is always on a balanced growth path.

### 3.6 Stylized facts revisited

The model matches some of the stylized facts presented in Section 2 by assumption, and in this section, I show that it also matches the remaining facts.

The model should predict that pollution intensity falls with income. Define pollution intensity as: \(\mathcal{P}_t \equiv \frac{P_t}{Y_t}.\) The growth rate in pollution emission is given by

\[
g_P = \beta g_A - \chi g_B
\]

(3.54)

The sign of \(g_P\) is ambiguous. It then follows that the growth rate of pollution intensity is given by: \(g_P = (\beta - 1)g_A - \chi g_B < 0.\) Thus, the model allows pollution emission to decrease or increase with income, but pollution intensity unambiguously falls with income.

The environmental policy stringency measured by the EPS index correspondence to \(\tau_t.\)

Consider the government’s tax rule expressed in terms of pollution intensity: \(\phi = \mathcal{P}_t \tau_t.\) As the pollution intensity falls, the pollution tax rate must increase. Thus, the model predicts a negative correlation between the EPS index and pollution intensity.

The business sector of the economy is the final goods sector and the intermediate goods sector combined, as both sectors are part of the production process. Hence, the business
sector’s pollution abatement expenditures are simply the cost of producing abatement intermediates, which is given by: $z_t w_t = \chi \mu^2 \phi Y_t$. As the expression is proportional to $Y_t$, the business sector’s pollution abatement expenditures are a constant share of output. Aggregate pollution abatement expenditures is the sum of the business sector’s pollution abatement expenditures and pollution abatement R&D expenditures. The pollution abatement R&D expenditures are given by: $n^B w_t = \Omega t^{\mu^2} Y_t$. As the labor allocation is constant, aggregate pollution abatement expenditures are a constant share of output.

4 Policy Implications

To study how a tightening of the environmental policy affects economic growth, an unexpected increase in the environmental tax revenue parameter, $\phi$, is analyzed. This policy change will be referred to as a tightening of the environmental policy. In reality, the government possess many other environmental policy instruments beside environmental taxes. For the mechanisms described in this model, what matters is that pollution emission becomes more expensive, when the environmental policy is tightened. The parameter $\phi$ can therefore be interpreted rather broadly as the environmental policy stringency. Note, however, that the measure is different from the EPS index measure described in Botta and Kozluk (2014), which correspondence to the pollution tax rate, $\tau_t$, in the model. A constant $\phi$ implies an increase in the pollution tax rate over time. Essentially, $\phi$ measures the technology adjusted environmental policy stringency.

4.1 Labor allocation and growth effects

The following proposition states the labor allocation effects of a tightening of the environmental policy:

**Proposition 3.** Assuming that Parameter Restriction 1, 2 and 3 hold, then:

1. $\frac{d \tau}{d \phi} < 0$ and $\frac{d z}{d \phi} > 0$
2. $\frac{d n^A}{d \phi} < 0$ and $\frac{d n^B}{d \phi} > 0$
3. $\frac{d L}{d \phi} > 0$ for $\alpha \geq \mu$

The proposition is proved in Appendix B.1. The proposition highlights three labor allocation effects of a tightening of the environmental policy. The first effect is a *production direction* effect. A tightening of the environmental policy increases the price of pollution
emission. This causes the demand for abatement intermediates to increase while the
demand for production intermediates decreases. Holding labor input in manufacturing
constant, labor must be reallocated from the production to the abatement intermediate
subsector to ensure that supply equals demand.

The second effect is a research direction effect. As demand for abatement intermediates
increase, so does the value of a patent in the abatement intermediate subsector relative
to the cost of conducting research (the wage rate). The opposite holds for the production
intermediate subsector. Holding the labor supply in the R&D sector constant, labor is
reallocated towards the abatement R&D subsector, to ensure that the research arbitrage
equations are fulfilled. This reallocation of labor will affect the value of patents in the
two subsectors through the expected duration of a patent.

The final effect is a labor force allocation effect. Labor might be reallocated from
manufacturing to R&D or vise versa, depending on the relative sizes of $\alpha$ and $\mu$. If $\alpha < \mu$
the total profits in the intermediate goods sector falls, given the labor allocation between
manufacturing and R&D. The reason is that intermediate goods with the price $(1/\alpha)w$
are substituted for intermediates with the relatively lower price $(1/\mu)w$. As total profits in
the intermediate goods sector is reduced, the incentive to conduct research is diminished.
Labor then flows from R&D to manufacturing until the research arbitrage equations are
fulfilled.

The following proposition stating the growth effects of a tightening of the environmental
policy follows almost immediately from Proposition 3:

**Proposition 4.** Assuming that Parameter Restriction 1, 2 and 3 hold, then $\frac{d\gamma_A}{d\phi} < 0$ and
$\frac{d\gamma_P}{d\phi} < 0$.

According to Proposition 4, a tightening of the environmental policy decreases the
growth rates of aggregate output and pollution emission. The question is then: why
can the potential inflow of labor from manufacturing to R&D never be strong enough to
increase the growth rate of aggregate output? When a tightening of the environmental
policy causes labor to flow from manufacturing to R&D, the reason is that profits in the
intermediate goods sector has increased. But, this overall increase is entirely driven by
the abatement intermediate subsector, as profits in the production intermediate subsector
decreases unambiguously, due to the production direction effect. Hence, the labor flow
from manufacturing is directed entirely to the abatement R&D subsector, and the labor
allocated to the production R&D subsector will unambiguously fall.
4.2 Environmental sustainability

In line with Groth and Ricci (2011), the environmental quality, $E_t$, evolves according to the following law of motion:

$$
\dot{E}_t = \zeta(\bar{E} - E_t) - \delta P_t, \quad E_0 \in (0, \bar{E}] \quad \text{given,} \quad \bar{E} > 0, \quad \zeta > 0, \quad \delta > 0
$$

(4.1)

The environmental quality is negatively affected by pollution emission, but regenerates when it is below some critical level, $\bar{E}$. The speed of regeneration depends positively on the gap between the critical level and the environmental quality.

I define an environmentally sustainable equilibrium as an equilibrium where $E_t > 0$ for all $t \geq 0$. The law of motion given by (4.1) can then be used to assess whether a given environmental policy results in an environmentally sustainable equilibrium.

It is shown in Appendix B.2 that aggregate pollution emission is given by:

$$
P_t = \Xi N\left(\frac{A_{\text{MAX}}}{B_{\text{MAX}}}\right)^\beta
$$

(4.2)

where $\Xi$ is a constant defined in Appendix B.2. In equilibrium, the growth rate of aggregate pollution emission is given by

$$
g_P = \beta g_A - \chi g_B
$$

(4.3)

The following proposition state the necessary and sufficient conditions for an environmentally sustainable equilibrium:

**Proposition 5.** $g_P \leq 0$ is a necessary and sufficient condition for an environmentally sustainable equilibrium.

The proposition is proved in Appendix B.3. Note that $\Xi$ is a function of $\phi$, and thus, environmental policy might affect whether the economy is in an environmentally sustainable equilibrium. Unfortunately, the minimum $\phi$ value ensuring a sustainable environmental policy cannot be derived analytically.

The conditions stated in Proposition 5 are intuitive. As the growth rate of pollution emission is constant in steady state, this growth rate cannot be positive, as this would result in a permanently increasing pollution emission. As the environment’s regenerating capacity cannot cope with a pollution emission level above some threshold, this would
result in an environmentally unsustainable equilibrium. Assuming that the growth rate of pollution emission is at most zero, the initial pollution emission must be low enough for the environment’s regenerating capacity to cope with it.

5 Extended Model

The assumption of logarithmic preferences in the instantaneous utility function might be considered problematic, as the degree of relative risk aversion seems to be above one empirically (see Attanasio and Weber 1993; and Okubo 2011). To relax this assumption, consider the model from Section 3 with the following instantaneous utility function:

\[
\begin{aligned}
&\begin{cases}
\frac{c_t^\theta - 1}{1 - \theta}, & \theta > 0, \quad \theta \neq 1 \\
\ln(c_t), & \theta = 1
\end{cases}
\end{aligned}
\]

(5.1)

where \( \theta \) is the degree of relative risk aversion. Most of the derivations from Section 3 are still valid. However, the real interest rate is now given by: \( r = g_A \theta + \rho \). This means that the modified discount rates in the two R&D subsectors are given by

\[
\begin{align*}
r^A &= \rho + g_A \left( \theta + \frac{1}{\ln(\gamma)} + \frac{\alpha}{1 - \alpha} \right) \\
r^B &= \rho + g_A (\theta - 1) + g_B \frac{1}{\Lambda_B \ln(\xi)}
\end{align*}
\]

where \( r^A \) and \( r^B \) are the modified discount rates in the production and abatement R&D subsector, respectively. Now, the modified discount rate in the abatement R&D subsector, \( r^B \), is affected positively (negatively) by \( g_A \) for \( \theta > 1 \) (\( \theta < 1 \)). This reflects two opposing effects. Firstly, the representative household’s desire to consumption smooth increases with the degree of relative risk aversion. When the representative household has a higher tendency to consumption smooth, the equilibrium real interest rate increases. As a consequence, the two modified discount rates \( r^A \) and \( r^B \) are increasing in the degree of relative risk aversion, \( \theta \). Specifically, the effect is captured by the term \( \theta g_A \) in the two expressions. Secondly, when production grows, so does the pollution emission and the demand for abatement intermediates. A higher demand for abatement intermediates increases the profits obtained in the abatement intermediate subsector. This effect is reflected by the term \(-g_A\) in the expression for \( r^B \).
The two effects balance out in the expression for \( r^B \) when \( \theta = 1 \), as in the model presented in Section 3. As \( \theta \) seems to be well above one empirically, the following parameter restriction is imposed:

**Parameter Restriction 4.** \( \theta > 1 \)

The labor allocation is then given by

\[
\begin{align*}
n^A &= \frac{\Omega_2 L}{\Sigma_1} - \frac{\rho}{\lambda \Sigma_1}, & \Sigma_1 &\equiv \ln(\gamma) \left( \theta + \frac{1}{\ln(\gamma)} + \frac{\alpha}{1 - \alpha} \right) \\
n^B &= \Lambda_B \Omega_3 L - \frac{\rho}{\eta} \Lambda_B - \Sigma_2 n^A, & \Sigma_2 &\equiv (\theta - 1) \frac{\Lambda}{\eta} \ln(\gamma) \Lambda_B \\
L &= \frac{\bar{L} + \rho \left( \frac{\Sigma_3}{\lambda} + \frac{\Lambda_B}{\eta} \right)}{1 + \Lambda_B \Omega_3 + \Sigma_3}, & \Sigma_3 &\equiv \frac{1 - \Sigma_2}{\Sigma_1} 
\end{align*}
\]

(5.2) (5.3) (5.4)

Two parameter restrictions are imposed to ensure that \( n^A, n^B, \) and \( L \) are positive.

**Parameter Restriction 5.** \( \frac{\bar{L} + \rho \left( \frac{\Sigma_3}{\lambda} + \frac{\Lambda_B}{\eta} \right)}{1 + \Lambda_B \Omega_3 + \Sigma_3} > \frac{\rho}{\Lambda_B} \)

**Parameter Restriction 6.** \( \frac{\bar{L} + \rho \left( \frac{\Sigma_3}{\lambda} + \frac{\Lambda_B}{\eta} \right)}{1 + \Lambda_B \Omega_3 + \Sigma_3} > \frac{\rho}{\Lambda_B} \frac{1}{\Omega_3 - (\theta - 1) \frac{\Lambda}{\eta} \ln(\gamma) \frac{\Sigma_2}{\Sigma_1}} \)

It turns out that the qualitative policy implications of the extended model are very similar to those of the model presented in Section 3. Consider the following proposition:

**Proposition 6.** Assuming that Parameter Restriction 1, 4, 5 and 6 hold, then

(i) \( \frac{dL}{d\phi} \geq 0 \) for \( \frac{\alpha}{1 - \alpha} \geq 1 - \frac{\rho}{\Lambda_B} \left( \frac{1 - (\theta - 1) \frac{\Lambda}{\eta} \ln(\gamma) \Lambda_B}{\ln(\gamma) \left( \theta + \frac{1}{\ln(\gamma)} + \frac{\alpha}{1 - \alpha} \right)} \right) \Lambda_B^{-1} \)

(ii) \( \frac{dn^A}{d\phi} < 0 \) and \( \frac{dn^B}{d\phi} > 0 \).

(iii) \( \frac{dg^n_A}{d\phi} < 0 \) and \( \frac{dg^n_B}{d\phi} < 0 \)

The proposition is proved in Appendix C. The qualitative growth effects of a tighter environmental policy are unaffected by the generalization. A natural question to ask is whether the results hold if \( \theta < 1 \). Unfortunately, this cannot be determined analytically, but simulations indicate that the results do hold for \( \theta < 1 \) for plausible parameter values.

### 6 Simulations

In this section, the model from Section 5 is simulated in order to investigate its quantitative policy implications. To obtain somewhat plausible parameter values, the model is calibrated to the US economy. The point of departure is the year 1994, as this is the
only year for which both environmental tax revenue and pollution abatement cost data is available. The US tax revenue from environmentally related taxes was 1.09\% of GDP in 1994. As $\phi$ should be interpreted as a measure of the overall environmental policy stringency and not just the tax revenue from environmental taxes as share of output, the $\phi$ value for the US, $\phi^{US}$, must have been at least 1.09\%. In fact, $\phi^{US}$ is probably considerably higher, as US environmental policy relies little on environmental taxes.\footnote{ENTER SOURCE}

The simulations will therefore be conducted for $\phi^{US} = 2.18\%$ and $\phi^{US} = 3.27\%$, i.e. two and three times the minimum $\phi^{US}$ value. Using a $\phi^{US}$ value considerably higher, e.g. $\phi^{US} = 10\%$, does not change the main conclusions of the simulation exercise.

The parameter $\chi$ is computed by matching the business sector’s pollution abatement expenditures, with that of the US in 1994, which was about 0.73\% of GDP. Hence, $\chi \mu^2 \phi = 0.73\%$. Given some $\mu$-value, $\chi$ can be computed. Note that $\mu$ must be at least 0.59 and 0.48 to ensure that $\chi < 1$ for $\phi^{US} = 2.18\%$ and $\phi^{US} = 3.27\%$, respectively. I set $\mu$ equal to 0.6, but other allowable $\mu$ values provide similar results.

The parameter $F$ only has a strong impact on the variable $N$, and it is set somewhat arbitrarily to 0.1. Total labor supply is normalized to one, and $\beta$ is estimated using the procedure described in Appendix D. Parameters present in other growth models are assigned values from previous papers: $\alpha = 0.4$ and $\lambda = 0.5$ as in Ricci (2007), and $\theta = 2.5$ and $\rho = 0.2$ as in Groth and Ricci (2007). Assuming that it is equally difficult to invent new production and abatement intermediate good qualities, $\eta = \lambda = 0.5$.

The last step is to calibrated the model such that it matches the US growth rates of real GDP and pollution emission. Unfortunately, it is difficult to operationalize the variable $P_t$, as it is unclear how to construct a single pollution emission measure. As in Section 2, I focus on air pollution and assume that $g_P$ is the unweighted average of the emission growth rates for the five pollutants studied in Section 2 (CO$_2$, CO, NO$_x$, SO$_2$, and VOC) for the period 1970-2012. Using only one of the five pollutants to compute $g_P$ does not change the main conclusions of the exercise. The values of $\ln(\gamma)$ and $\ln(\xi)$ are adjusted such that the model matches the growth rates of US real GDP (2.85\%) and pollution emission (-1.85\%) for the period 1970-2012. The baseline parameter assumptions are reported in Table 3, and the remaining parameter values ($\chi$, $\ln(\gamma)$, and $\ln(\xi)$) are computed using the procedure described above.

Figure 5 depicts the growth rates of output and pollution emission for the two $\phi^{US}$
values. In both simulations, the growth rate of output is weakly affected by a tightening of the environmental policy compared to the growth rate of pollution emission. Increasing $\phi$ from 2 to 3 pct. decreases the growth rate of output by 0.019 and 0.013 pp for $\phi^{US} = 2.18\%$ and $\phi^{US} = 3.27\%$, respectively. The same change in $\phi$ reduces the growth rate of pollution emission by $2.92$ and $1.69$ pp for $\phi^{US} = 2.18\%$ and $\phi^{US} = 3.27\%$, respectively. In addition, it seems like the slope of the $g_A$ curve is only weakly affected by the $\phi^{US}$ value, whereas the slope of the $g_P$ curve seems to be strongly affected. Intuitively, the results should

be understood the following way. As the model must match the pollution abatement expenditures of the US business sector, which is a small number, $\chi$ times $\phi^{US}$ must be small as well. When $\phi^{US}$ is increased, $\chi$ decreases proportionally, such that the product of the two parameters is unchanged. Obviously, $\phi$ affects labor input in the abatement R&D subsector, as the demand for abatement intermediates is based solely on the environmental tax. The parameter $\chi$ also affects the labor input in the abatement R&D subsector. A higher $\chi$ means a higher efficiency of abatement technologies, and thereby a higher demand for these technologies. As both $\chi$ and $\phi^{US}$ are small, so is the labor input in the abatement R&D subsector. Since US pollution intensities have decreased notably during the last plus 40 years, pollution abatement research must have had a strong impact on

### TABLE 3: Baseline parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.40</td>
<td>$\eta$</td>
<td>0.50</td>
<td>$\rho$</td>
<td>0.02</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.70</td>
<td>$\mu$</td>
<td>0.60</td>
<td>$F$</td>
<td>0.10</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.50</td>
<td>$\theta$</td>
<td>2.50</td>
<td>$\bar{L}$</td>
<td>1.00</td>
</tr>
</tbody>
</table>

**FIGURE 5:** Growth rates of output and pollution emission as function of $\phi$, using the baseline parameter values.
pollution emission despite the relatively small input. Consequently, \( \ln(\xi) \) becomes large when calibrating the model. This explains the relatively large impact of environmental policy on pollution emission. The relatively small economic growth effect is simply a consequence of the low \( \chi \) and \( \phi \) values. Only a small fraction of overall R&D input is allocated to the abatement R&D subsector. If environmental policy doubles or triples the abatement R&D subsector input, this requires little reallocation of the overall labor supply. As a consequence, the input in the production R&D subsector is almost unaffected by significant environmental policy changes, which leads to a weak effect on the economic growth rate.

Another interesting question is how environmental policy affects the level of certain central variables, given the technological level. Figure 6 plots central economic variables as function of \( \phi \) using the baseline parameter values and the technological level \((A_t^{\text{MAX}}, B_t^{\text{MAX}}) = (1, 1)\). As expected, a tighter environmental policy reduces profits in the production intermediate subsector and increases the profits in the abatement intermediate subsector. Note that the profits in the production intermediate subsector is relatively large compared to the profits in the abatement intermediate subsector. Furthermore, a tighter environmental policy reduce output and the wage rate. Finally, pollution emission and pollution intensity are reduced when the environmental policy is tightened.

Note that all the level effects are amplified when \( \phi^{\text{US}} \) is reduced. The reason is that a low \( \phi^{\text{US}} \) implies a large \( \chi \) value, as the model must match the US business sector pollution abatement expenditures. A larger \( \chi \) value implies more efficient pollution abatement. Thus, final goods sector firms will react with relatively more pollution abatement when the environmental policy is tightened for higher \( \chi \) values. As the response to the environmental policy change is stronger, so are the level effects.

Overall, the simulation exercise suggests that environmental policy has a small effect on economic growth compared to the effect on pollution emission growth. One interpretation of this would be that the simulation exercise suggests that it is cheap to reach environmental goals. I make no such claim, and remind the reader that even small changes in a growth rate has large level effects in the long-run. In addition, the simulation exercise showed how environmental policy might affect the level of several central variables. These level effects might potentially be important from a political perspective, as current generations might be strongly affected by environmental policy, due to these level effects.
Profits in the production intermediate subsector

Tax revenue from the pollution tax as share of output

φ
US
= 2.18%
φ
US
= 3.27%

Output

Tax revenue from the pollution tax as share of output

φ
US
= 2.18%
φ
US
= 3.27%

Pollution emission

Tax revenue from the pollution tax as share of output

φ
US
= 2.18%
φ
US
= 3.27%

Profits in the abatement intermediate subsector

Wage rate

Pollution intensity

FIGURE 6: Central variables as function of φ, given the technological level \((A_t^{\text{MAX}}, B_t^{\text{MAX}}) = (1, 1)\), and using the baseline parameter values.

7 Concluding Remarks

The policy implications presented in this paper concerning the growth rates of output and pollution emission are consistent with those presented in Ricci (2007). Specifically, both analyzes suggest that there is a trade-off between environmental quality and economic growth. Also, both analyzes suggest that the growth rate of pollution emission is more sensitive to environmental policy compared to the economic growth rate. This consistency is somewhat surprising and reassuring as the two modeling approaches are based on diametrically opposing views on the embodiment of production and pollution abatement technologies in intermediate goods used in the production process.

The analysis conducted in this paper had at least three advantages compared to much of the existing literature. Firstly, environmental policy implications concerning economic growth were unambiguous and derived analytically. Secondly, the model was shown to match certain stylized facts. In contrast, many previous studies do not relate model predictions to real world data. Thirdly, the separation of production and pollution abatement technologies in different intermediate good types resulted in a more realistic modeling approach, as innovation arrivals of pollution abatement technologies were detached from those of production technologies.
References


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A Calculations and Proofs for Section 3

A.1 Deriving (3.48) and (3.49)

Using (3.46) and (3.20), the value of a new patent in the production intermediate subsector is derived as

\[ V_{it}^{T+1} = \int_t^\infty \pi_i A_t e^{-\int_t^s r_u + n_i^4 \lambda du} ds \]

\[ = \int_t^\infty (1 - \alpha) \Psi \left( \frac{N_t}{L_t} \right)^{1-\alpha} \left( \frac{A_t^{\text{MAX}}}{A_t^{\text{MAX}}} \right)^{1-\alpha} A_t^{\text{MAX}} L_s e^{-\int_t^s r_u + n_i^4 \lambda du} ds \]

\[ = (1 - \alpha) \Psi \left( \frac{N_t}{L_t} \right)^{1-\alpha} \left( A_t^{\text{MAX}} \right)^{1-\alpha} \int_t^\infty L_s \left( A_t^{\text{MAX}} \right)^{1-\alpha} e^{-\int_t^s r_u + n_i^4 \lambda du} ds \]

where it is used that \( N_t/L_t \) is constant over time in equilibrium. This follows directly from (3.44).

It can easily be verified using that \( n_i^d = n_i^d \) in equilibrium and (3.14), that

\[ A_t^{\text{MAX}} = A_t^{\text{MAX}} e^{\int_0^t \lambda \ln(\gamma) n_i^4 ds} \]

for some \( t' \in [0, t] \). Substituting (A.1) into (A.4) yields

\[ V_{it}^{T+1} = (1 - \alpha) \Psi \left( \frac{N_t}{L_t} \right)^{1-\alpha} A_t^{\text{MAX}} \int_t^\infty L_s e^{-\int_t^s r_u + (1 + \frac{\alpha}{1-\alpha} \ln(\gamma)) n_i^4 du} ds \]

From (3.42) and (3.43), it can be shown that the wage rate, \( w_t \), can be expressed as

\[ w_t = \frac{\Omega_1 A_A \Psi}{(1 - \chi \mu \phi - \phi)} \left( \frac{N_t}{L_t} \right)^{1-\alpha} A_t^{\text{MAX}} \]

Dividing both sides of (A.5) by \( w_t \) yields

\[ \frac{V_{it}^{T+1}}{w_t} = (1 - \alpha) (1 - \chi \mu \phi - \phi) \frac{\Omega_1 A_A}{\Omega_2} \int_t^\infty L_s e^{-\int_t^s r_u + (1 + \frac{\alpha}{1-\alpha} \ln(\gamma)) n_i^4 du} ds \]

The expression can be differentiated with respect to time using Leibniz rule.

Define the variable \( f_s \equiv -\int_t^s r_u + \left( 1 + \frac{\alpha}{1-\alpha} \ln(\gamma) \right) \lambda n_i^4 du \). Using Leibniz rule, it can
easily be verified that
\[
\frac{df_s}{ds} = r_t + \left(1 + \frac{\alpha}{1 - \alpha} \ln(\gamma)\right) \lambda n_t^A
\]

From the research arbitrage equation (3.18), it follows that \( v_{it}^{T_{t+1}} = 1/\lambda \). Thus, the derivative of \( v_{it}^{T_{t+1}} \) with respect to time equals zero. Using Leibniz rule, it follows that differentiating (A.6) with respect to time yields
\[
0 = \Omega_2 \left(r_t + \left(1 + \frac{\alpha}{1 - \alpha} \ln(\gamma)\right) \lambda n_t^A\right) v_{it}^{T_{t+1}} - L_t \Omega_2 \quad \Leftrightarrow \quad 1 = \frac{\lambda \Omega_2 L_t}{r_t \left(1 + \frac{\alpha}{1 - \alpha} \ln(\gamma)\right) \lambda n_t^A}
\]

Now consider the value of a patent in the abatement intermediate subsector. From (3.21) and (3.47) it follows that
\[
\bar{V}_{ht}^{T_{t+1}} = \int_t^\infty \bar{n}_h^B e^{-\int_t^s r_u + n_h^B \eta du} ds
\]

(A.7)
\[
= \frac{(1 - \mu) \chi \mu \phi \Lambda_A}{(1 - \chi \mu \phi - \phi) \Lambda_B} \Psi \left(\frac{N_t}{L_t}\right)^{1-\alpha} \int_t^\infty L_s e^{-\int_t^s r_u + \left(1 + \frac{1}{1 - \mu} \ln(\xi)\right) n_h^B \eta - \ln(\gamma) \lambda n_t^A du} ds
\]

(A.8)

where it is used that \( n_h^B = n_t^B \) in equilibrium, and that
\[
B_t^{MAX} = B_t^{MAX} e^{\int_t^\infty \ln(\xi) n_h^B ds}
\]

Dividing both sides of (A.7) with \( w_t \):
\[
\bar{v}_{ht}^{T_{t+1}} = \frac{(1 - \mu) \chi \mu \phi}{\Omega_1 \Lambda_B} \int_t^\infty L_s e^{-\int_t^s r_u + \left(1 + \frac{1}{1 - \mu} \ln(\xi)\right) n_h^B \eta - \ln(\gamma) \lambda n_t^A du} ds
\]

(A.9)

It follows from (3.19), that \( \bar{v}_{ht}^{T_{t+1}} = 1/\eta \). Differentiating (A.9) with respect to time yields
\[
0 = \left(r_t + \left(1 + \frac{1}{1 - \mu} \ln(\xi)\right) n_t^B \eta - n_t^A \Lambda \ln(\gamma)\right) \bar{v}_{ht}^{T_{t+1}} - L_t \Omega_3 \quad \Leftrightarrow \quad 1 = \frac{\eta \Omega_3 L_t}{r_t \left(1 + \frac{1}{1 - \mu} \ln(\xi)\right) n_t^B \eta - n_t^A \Lambda \ln(\gamma)}
\]
A.2 Proof of Proposition 1

Proof. Substituting (3.50) into (3.48) and (3.49)

\[ 1 = \frac{\lambda \Omega_2 L_t}{\rho + n_t^A \lambda A^{-1} + g_{L,t}} = \frac{\eta \Omega_3 L_t}{\rho + n_t^B \eta B^{-1} + g_{L,t}} \]  

(A.10)

From (A.10)

\[ n_t^A = \Omega_2 \Lambda_A L_t - \frac{\Lambda_A}{\lambda} \rho - \frac{\Lambda_A}{\lambda} g_{L,t} \]  

(A.11)

\[ n_t^B = \Omega_3 \Lambda_B L_t - \frac{\Lambda_B}{\eta} \rho - \frac{\Lambda_B}{\eta} g_{L,t} \]  

(A.12)

Substituting (A.11) and (A.12) into (3.39)

\[ \bar{L} - L_t = n_t^A + n_t^B = (\Omega_2 \Lambda_A + \Omega_3 \Lambda_B) L_t - (\rho + g_{L,t}) \left( \frac{\Lambda_A}{\lambda} + \frac{\Lambda_B}{\eta} \right) \]  

⇔

\[ g_{L,t} = \frac{(1 + \Omega_2 \Lambda_A + \Omega_3 \Lambda_B) L_t - \rho \left( \frac{\Lambda_A}{\lambda} + \frac{\Lambda_B}{\eta} \right) - \bar{L}}{\left( \frac{\Lambda_A}{\lambda} + \frac{\Lambda_B}{\eta} \right)} \]  

⇔

\[ \dot{L}_t = \left( \frac{L_t}{\frac{\Lambda_A}{\lambda} + \frac{\Lambda_B}{\eta}} \right) D_t \]  

(A.13)

where

\[ D_t \equiv \left( (1 + \Omega_2 \Lambda_A + \Omega_3 \Lambda_B) L_t - \left( \bar{L} + \rho \left( \frac{\Lambda_A}{\lambda} + \frac{\Lambda_B}{\eta} \right) \right) \right) \]

It is clear from (A.13) that if \( L_0 = 0 \) then \( L_t = 0 \) for all \( t \geq 0 \). This will imply that \( \pi^A_{it} = \pi^B_{ht} = 0 \), cf. (3.46) and (3.47). This in turns imply that \( n_t^A = n_t^B = 0 \), and the labor market clearing condition (3.39) is violated. Hence, \( L_0 \) must be strictly positive.

If \( D_0 > 0 \), then \( \dot{L}_0 > 0 \) and \( D_t, \dot{L}_t > 0 \) for all \( t \geq 0 \). In fact, it follows from (A.13) that \( \frac{\partial g_{L,t}}{\partial t} > 0 \). Hence, \( L_t \) will continue to grow with an increasing growth rate. For some \( t > 0, L_t > \bar{L} \) which violates (3.39). Thus, \( D_0 > 0 \) leads to a contradiction.

If \( D_0 < 0 \), then \( \dot{L}_0 < 0 \) and \( D_t, \dot{L}_t < 0 \) for all \( t \geq 0 \). It follows from (A.13) that \( \frac{\partial g_{L,t}}{\partial t} < 0 \). Hence, \( L_t \) will decrease over time at an ever decreasing growth rate. For some \( t > 0, L_t < 0 \). Thus, \( D_0 < 0 \) leads to a contradiction.

If \( D_t = 0 \), then \( \dot{L}_0 = 0 \) and \( D_t, \dot{L}_t = 0 \) for all \( t \geq 0 \). This does not lead to a
contradiction. Hence, $D_t$ must be zero. This implies that

$$L_t = \frac{\bar{L} + \rho \left( \frac{\Lambda_A}{\theta} + \frac{\Delta_B}{\eta} \right)}{1 + \Omega_2 \Lambda_A + \Omega_3 \Lambda_B}$$

(A.14)

It follows from (A.14) that $L_t = L$, i.e. labor input in manufacturing is constant over time. Hence, $g_{L,t} = g_L = 0$ in equilibrium.

\begin{proof}

A.3 Proof of Proposition 2

Proof. As $L_t = L$ according to Proposition 1, $N_t = N$. This follows directly from (3.44). It then follows from (3.2) and (3.43) that $C_t$ and $Y_t$ grow at the same rate as $A^\text{MAX}_t$. As $Y_{jt} = \hat{Y}_t = \frac{1}{N} Y_t$ in equilibrium, $\hat{Y}_t$ also grows at the same rate as $A^\text{MAX}_t$.

From (3.42) it follows that $w_t$ grows at the same rate as $A^\text{MAX}_t$. It then follows immediately from (3.24) and (3.32) that $p_{it}$ and $q_{ht}$ grows at the same rate as $A^\text{MAX}_t$.

Since $n_t^A = n^A$, it follows from (3.14) that $g_A = \lambda \ln(\gamma) n^A$. Likewise, it follows from (3.15) that $g_B = \eta \ln(\xi) n^B$.

As $Y_t$ and $w_t$ grow at the same rate, it follows from (3.30) that $\hat{Z}_t$ grows at the rate $g_B$. It then follows from (3.4) and (3.5) that $P_{jt}$ and $P_t$ grow at the rate $g_P = \beta g_A - \chi g_B$.

In equilibrium $P_{jt} = \hat{P}_t = \frac{1}{N} P_t$. Hence,

$$P_t = N \frac{\hat{Y}_t^\beta}{\hat{Z}_t^\chi} \Rightarrow \ln(P_t) = \ln(N) + \beta \ln(\hat{Y}_t) - \chi \ln(\hat{Z}_t) \Rightarrow g_P = \beta g_A - \chi g_B$$

\end{proof}

B Proofs and Calculations for Section 4

B.1 Proof of Proposition 3

Proof. Rewrite (3.51) as

$$L = \left( \alpha + \frac{(\mu - \alpha)\chi \mu \phi}{1 - \phi} \right) \Gamma$$

Differentiating the expression with respect to $\phi$:

$$\frac{dL}{d\phi} = \frac{(\mu - \alpha)\chi \mu}{(1 - \phi)^2} \Gamma \gtrless 0 \quad \text{for} \quad \alpha \lesssim \mu$$

(B.1)
Labor input in the production intermediate subsector is given by

\[ x = \int_0^N \left( \int_0^1 \hat{x}_{it} \, di \right) \, dj = \frac{\alpha(1 - \chi \mu \phi - \phi)}{\Omega_1} L = \Theta_1 L, \quad \Theta_1 \equiv \frac{\alpha(1 - \chi \mu \phi - \phi)}{\Omega_1} \quad \text{(B.2)} \]

Labor input in the abatement intermediate subsector is given by

\[ z = \int_0^N \left( \int_0^1 \hat{z}_{ht} \, di \right) \, dj = \frac{\chi \mu^2 \phi}{\Omega_1} L = \Theta_2 L, \quad \Theta_2 \equiv \frac{\chi \mu^2 \phi}{\Omega_1} \quad \text{(B.3)} \]

From (B.2)

\[ \frac{dx}{d\phi} = \frac{d\Theta_1}{d\phi} L + \Theta_1 \frac{dL}{d\phi} = -\frac{\alpha \chi \mu}{(1 - \phi)^2} \Gamma < 0 \]

From (B.3)

\[ \frac{dz}{d\phi} = \frac{d\Theta_2}{d\phi} L + \Theta_2 \frac{dL}{d\phi} = \frac{\chi \mu^2}{(1 - \phi)^2} \Gamma > 0 \]

Differentiating (3.52) with respect to \( \phi \)

\[ \frac{dn^A}{d\phi} = -\frac{(1 - \alpha) \chi \mu}{(1 - \phi)^2} \Gamma < 0 \]

Differentiating (3.53) with respect to \( \phi \)

\[ \frac{dn^B}{d\phi} = \frac{(1 - \mu) \chi \mu}{(1 - \phi)^2} \Gamma > 0 \]

B.2 Deriving Expression for Pollution Emission

To obtain a equilibrium expression for \( P_t \), the equilibrium values for \( \hat{Y}_t \) and \( \hat{Z}_t \) are needed.

Using (3.43), (3.44), (3.30) and (3.42):

\[ \hat{Y}_t = \Xi_1 A_{t}^{\text{MAX}} \quad \text{(B.4)} \]

\[ \hat{Z}_t = \Xi_2 B_{t}^{\text{MAX}} \quad \text{(B.5)} \]
where

\[ \Xi_1 \equiv \frac{\alpha(1 - \phi \beta) F}{1 - \alpha(1 - \phi \beta) - \chi \mu \phi - \phi} \]
\[ \Xi_2 \equiv \left( \frac{\chi \mu \phi^2}{\alpha(1 - \chi \mu \phi) - \phi} \right) \left( \frac{1 - \chi \mu \phi - \phi}{1 - \alpha(1 - \phi \beta) - \chi \mu \phi - \phi} \right)^{\frac{1}{\alpha}} \Lambda_{A^\mu} \Lambda_{B^1} \]

Pollution emission can then be expressed as

\[ P_t = \Xi \left( \frac{A_{t} \text{MAX}}{B_{t} \text{MAX}} \right)^\beta, \quad \Xi \equiv \frac{\Xi_1}{\Xi_2} \]

**B.3 Proof of Proposition 5**

*Proof.* From (3.54) it follows that \( g_P > 0 \) if \( \beta g_A > \chi g_B \). Hence, \( P_t \to \infty \) for \( t \to \infty \).

As \( P_t \) increases monotonically over time, there will be some \( t' \in t \) for which \( P_{t'} = \frac{\zeta}{\delta} \bar{E} \).

Then \( P_t > \frac{\zeta}{\delta} \bar{E} \) for all \( t > t' \). Hence, \( E_t \) will decrease over time. In addition, it will do so with a faster and faster past, as \( P_t \) increases over time. Hence, there will exist a \( t'' \in t \) for which \( E_{t''} = 0 \). Hence, \( g_P > 0 \) is not compatible with an environmentally sustainable equilibrium.

Assume that \( g_P = 0 \). Then \( P_t = P_0 > 0 \) for all \( t \geq 0 \). It then follows that

\[ \bar{E}_t = \zeta(\bar{E} - E_t) - \delta P_0 \implies E_t = \left( E_0 - \frac{\bar{P}}{\zeta} \right) e^{-\zeta t} + \frac{\bar{P}}{\zeta}, \quad \bar{P} = \zeta \bar{E} - \delta P_0 \]

It follows directly from the above expression that \( E_t \to \frac{\bar{E}}{\zeta} \) for \( t \to \infty \). In addition, \( E_t \) is monotonically in- or decreasing over time. Hence, \( E_t \) will move monotonically from \( E_0 \) to \( \frac{\bar{P}}{\zeta} \). To ensure an environmentally sustainable equilibrium

\[ \frac{\bar{P}}{\zeta} = \frac{\zeta \bar{E} - \delta P_0}{\zeta} > 0 \iff P_0 < \frac{\zeta}{\delta} \bar{E} \]

Hence,

\[ P_0 = N \Xi \left( \frac{A_{0} \text{MAX}}{B_{0} \text{MAX}} \right)^\beta < \frac{\zeta}{\delta} \bar{E} \]

\( \square \)
C Proof of Proposition 6

Proof. Differentiating (5.4) with respect to $\phi$ yields

$$\frac{dL}{d\phi} = -L (\Lambda_B \Omega_3 + \Sigma_3 \Omega_2)^{-1} \left( \Lambda_B \frac{d\Omega_3}{d\phi} + \Sigma_3 \frac{d\Omega_2}{d\phi} \right)$$

The sign of the expression is determined by the last term in the expression. Deriving $\Lambda_B \frac{d\Omega_3}{d\phi}$ and $\Sigma_3 \frac{d\Omega_2}{d\phi}$:

$$\Lambda_B \frac{d\Omega_3}{d\phi} = \frac{(1 - \mu) \chi \alpha}{(\alpha(1 - \chi \mu \phi - \phi) + \chi \mu^2 \phi)^2}$$

$$\Sigma_3 \frac{d\Omega_2}{d\phi} = \frac{-\chi \mu^2}{\Lambda_A (\alpha(1 - \chi \mu \phi - \phi) + \chi \mu^2 \phi)^2}$$

Hence,

$$\frac{dL}{d\phi} \lesssim 0 \quad \text{for} \quad (1 - \mu) \alpha \lesssim (1 - \alpha) \mu \frac{\Sigma_3}{\Lambda_A} \iff \frac{\alpha}{1 - \alpha} \lesssim \frac{\mu}{1 - \mu} \frac{\Sigma_3}{\Lambda_A}$$

Differentiating (5.2) with respect to $\phi$ yields

$$\frac{dn^A}{d\phi} = \Omega_2 \left( \frac{-\chi \mu^2}{(1 - \chi \mu \phi - \phi) \Omega_1} - \frac{\Lambda_B \frac{d\Omega_3}{d\phi} + \Sigma_3 \frac{d\Omega_2}{d\phi}}{\Lambda_B \Omega_3 + \Sigma_3 \Omega_2} \right) \frac{L}{\Sigma_1}$$

where it is used that $\frac{d\Omega_2}{d\phi} = \Omega_2 \frac{-\chi \mu^2}{(1 - \chi \mu \phi - \phi) \Omega_1}$. The sign is determined by the expression in the parenthesis in the last expression. Evaluating the sign:

$$\frac{-\chi \mu^2}{(1 - \chi \mu \phi - \phi) \Omega_1} \left( \Lambda_B \Omega_3 + \Sigma_3 \Omega_2 \right) < \frac{\alpha \Lambda_B \Omega_3}{\phi \Omega_1} + \Sigma_3 \frac{-\chi \mu^2}{(1 - \chi \mu \phi - \phi) \Omega_1} \quad \iff \quad 0 < \Lambda_B \Omega_3 \frac{\alpha(1 - \chi \mu \phi - \phi) + \chi \mu^2 \phi}{\phi (1 - \chi \mu \phi - \phi)}$$

where it is used that $\frac{d\Omega_3}{d\phi} = \frac{\Omega_3}{\phi \Omega_1}$. Since the expression is true, $\frac{dn^A}{d\phi} < 0$.

Differentiating (5.3) with respect to $\phi$ yields

$$\frac{dn^B}{d\phi} = \left( \frac{\alpha}{\Omega_1 \phi} - \frac{\Lambda_B \frac{d\Omega_1}{d\phi} + \Sigma_3 \frac{d\Omega_2}{d\phi}}{\Lambda_B \Omega_3 + \Sigma_3 \Omega_2} \right) \Omega_3 \Lambda_B L - \Sigma_2 \frac{dn^A}{d\phi}$$
As $\theta$ is assumed to be greater than one, $\Sigma_2 > 0$. Hence, $\frac{dn_B}{d\phi} > 0$ if

$$\frac{\alpha}{\Omega_1 \phi} - \frac{\Lambda_B d\Omega_3 + \Sigma_3 d\Omega_2}{\Lambda_B \Omega_3 + \Sigma_3 \Omega_2} > 0 \iff \alpha(1 - \chi \mu \phi - \phi) + \chi \mu^2 \phi > 0$$

The expression is true by assumption. Hence, $\frac{dn_B}{d\phi} > 0$.

\[\square\]

### D Calibrating the Model

The growth rate of pollution emission is given by

$$\frac{\hat{P}_t}{P_t} = \beta \hat{Y}_t - \chi \hat{Z}_t \tag{D.1}$$

Assuming that the growth rate in $\hat{Z}_t$ is constant, the discrete time version of (D.1) is approximated as

$$\tilde{P}_t = \kappa + \beta \hat{Y}_t + \epsilon_t, \quad \kappa \equiv \chi g^B$$

where $\hat{X}_t \equiv \ln \left( \frac{X_{t+1}}{X_t} \right)$, and $\epsilon_t$ is the error term. According to the model, $\kappa < 0$ and $\beta \in (0, 1)$.

Output, $Y_t$, is approximated by real GDP, and $\hat{P}_t$ is approximated by an unweighted average of the growth rates in CO$_2$, CO, NO$_x$, SO$_2$, and VOC emission. Using data for the period 1970-2012 an OLS regression suggests that $\beta$ is around 0.7 and statistically significant on a one pct. level. The constant, $\kappa$, is around -0.04 and statistically significant on a one pct. level. Note that $g_P = 0.0185$ and $g_A = 0.0285$, which fits quite well with the $\beta$ and $\kappa$ values, as $\kappa = g_P - \beta g_A \approx 0.04$.  

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