Banks and Liquidity Crises in Emerging Market Economies

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Abstract

This paper presents and analyzes a simple banking model in which banks have access to international capital markets and domestic asset markets. The model generates two types of equilibria: a no-default equilibrium and a mixed equilibrium. In the no-default equilibrium, all banks are symmetric and always solvent, while in the mixed equilibrium, some banks can be internationally illiquid and default simultaneously. The latter equilibrium captures the basic features of banking crises after financial liberalization in emerging market economies. In this case, a large capital inflow leads to high asset-price volatility and magnifies a banking crisis. The effects of various public policies are also examined.

1 Introduction

Countries have experienced banking crises at various times throughout history. In particular, since financial liberalization in many parts of the world in the 1980s, crises have become more frequent and more costly. Notable examples include Chile (1982), Mexico (1994), Argentina (1995),

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Brazil (1996), East Asia (1997), and Russia (1998). Some countries (e.g., in Latin America in the 1970s and 1980s) experienced crises because of inconsistent and unsustainable macroeconomic policies. In contrast, other countries (e.g., in East Asia in 1997) experienced crises despite having sound macroeconomic policies. However, in the latter case, the empirical evidence strongly indicates that after financial liberalization during the 1990s, their short-term external liabilities were growing faster than their international reserves. That is, the financial liberalization policies of these countries led to *internationally illiquid* financial systems, making them vulnerable to crises.

In addition, banking crises have often been accompanied by a sharp decline in asset prices. When under strain, some banks demand liquidity and sell their assets to the market. This causes asset prices to fall and, subsequently, puts other banks under strain, forcing them to sell as well. A collapse in asset prices can cause a widespread financial crisis. For example, Sarno and Taylor (1999) show that the East Asian crisis of 1997 was precipitated by such an event, which had been fuelled by strong capital inflows.

Here, I present a simple banking model that accounts for the observed effects of financial liberalization on banking crises. The model addresses the following basic characteristic of banking crises in emerging markets:

(i) Capital inflow increases the probability and size of a banking crisis.\(^1\)

(ii) Financial institutions take on significant amounts of short-term

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\(^1\)See Kaminsky and Reinhart (1999), Reinhart and Reinhart (2009), and Reinhart and Rogoff (2008).
(iii) A banking crisis is closely linked to an asset-price boom and burst.3

Most previous studies have considered a combination of either characteristics (i) and (ii), or (ii) and (iii). Examples of the former group include Chang and Velasco (2000a, b, 2001), while those in the latter group includes Allen and Gale (2004a, b). Here, I consider all three characteristic, which is crucial to the analysis and to the contribution of this study.

To analyze the effects of financial liberalization on banking crises, I extend the banking model developed by Chang and Velasco (2001) by incorporating interbank asset markets. Chang and Velasco (2001) develop an open-economy version of the Diamond and Dybvig (1983) banking model.4 They show that domestic bank runs, caused by panic among domestic depositors, may interact with panic by international creditors. That is, bank runs may occur when domestic banks are internationally illiquid. Despite the elegance and usefulness of the model, it appears to have two limitations. First, there is no aggregate uncertainty in the model: banking crises are “sunspot” phenomena. Important empirical evidence exists that refutes this approach.5 For example, Kaminsky and Reinhart (1999) study the relationship between banking crises and currency crises. Their results show that a banking crisis typically precedes debt before a crisis occurs.2

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3 See Kaminsky and Reinhart (1999) and Reinhart and Rogoff (2008).
4 See also Chang and Velasco (2000a, b).
5 There is a long-standing debate on whether banking crises are the result of self-fulfilling beliefs.
a currency crisis, which, in turn, exacerbates and deepens the banking crisis. Furthermore, they propose that banking crises are related to weak economic fundamentals.\textsuperscript{6} Second, their model does not include interbank asset markets. That is, they do not explicitly model a situation in which financial institutions trade assets, which plays an important role in a banking crisis.

My analysis is also based on the banking model proposed by Allen and Gale (1998, 2004a, b) and Allen, Carletti, and Gale (2009).\textsuperscript{7} Here, I extend their models to a small open economy. There are three periods in the usual way. Banks can borrow funds from domestic depositors and international creditors, and hold one-period liquid international assets or two-period long-term assets with a higher return. Banks face uncertain liquidity demands from their domestic depositors in the middle period. That is, there is the aggregate uncertainty that the overall level of the liquidity demands banks face is stochastic. Banks can meet their liquidity demands using liquid assets or by selling long-term assets in a competitive interbank asset market. In the market, prices are determined endogenously by the demand and supply of liquidity in each state of nature.

I show that two types of equilibria can emerge, depending on the liquidity risk: a \textit{no-default equilibrium} or a \textit{mixed equilibrium}. When the probability of high liquidity risk is high, a no-default equilibrium exists.

\textsuperscript{6}See also Gorton (1988) and Calomiris and Gorton (1991).

\textsuperscript{7}This paper also bears a theoretical similitude to the recent work of Carletti and Leonello (2013) studying a model with banking and asset markets. They focus on credit market competition and show that there is no trade-off between banking competition and financial stability.
In this case, all banks find it optimal to keep enough international reserves and avoid defaults. When the probability of high liquidity risk is low, holding liquid reserves becomes costly, and the mixed equilibrium can emerge. In the mixed equilibrium, ex-ante identical banks take different portfolios. Some banks, called risky banks, invest heavily in long-term assets and default in a bad state of nature when all consumers withdraw and a bank run occurs. As risky banks sell all their long-term assets in the bad state, asset prices drop significantly. In this case, creditors obtain the liquidation proceeds instead of the promised repayments. The remaining banks, called safe banks, hold enough liquidity to always meet their commitments, and buy long-term assets of the risky banks. This mixed equilibrium captures many features of a crisis in emerging market economies.

I then examine the effects of three popular public policies, a liquidity requirement, public deposit insurance and control on capital inflow. The liquidity requirement is a constraint imposed on all banks holding a certain proportion of liquidity reserves. Under the public deposit insurance policy, depositors receive some goods if their banks go bankrupt. A tax on short-or long-term capital inflow can change the foreign debt structure of banks. I show that crises can be eliminated at the expense of investment in higher-yielding assets when the liquidity requirement is sufficiently restrictive. In contrast, public deposit insurance and capital controls does not help to stabilize a financial system because it encourages banks to take a risky portfolio.

There is extensive literature on the implications of financial crises,
financial intermediaries, and financial liberalization in open economies. This research can be divided into three strands.

The first strand of literature focuses on the existence of multiple equilibria. In at least one equilibrium, there is a crisis, while in another, there is not. These studies include Calvo (1988), Obstfeld (1996), Cole and Kehoe (1996, 2000), and Chang and Velasco (2000a, b, 2001). However, the crises in these models are generated by sunspots, and domestic asset markets are not modeled explicitly.

The second strand is based on the business cycle view of crises. According to this view, a banking crisis can occur because of poor fundamentals arising from the business cycle. Allen and Gale (2000a) develop a model of a banking crisis triggered by poor fundamentals, and show that large movements in exchange rates are desirable to achieve optimal risk sharing. Allen and Gale (2000b) show that banking failures are contagious when aggregate shocks are present and the connections among banks are incomplete. Their model strategies are similar to mine. However, in their model, the liquidation value of the long-term asset is exogenous, the resulting equilibrium is typically symmetric, or both, are true. In my model, the value is determined endogenously in an asset market generating a mixed equilibrium.²

The third strand of literature focuses on the implications of international financial frictions, capital flows and crises. Caballero and Krishnamurthy (2001) emphasize the interaction between domestic and international collateral during financial crises. Aoki, Benigno, and Kiyotaki

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²Other important works that focus on asymmetric information and moral hazards. See Corsetti, Pesenti, and Roubini (1999) and Calvo and Mendoza (2000a,b).
(2009), Mendoza and Quadrini (2010), and Mendoza (2010) develop an open economy version of a real business cycle (RBC) model with collateral constraints. They use the model to analyze the effects of financial liberalization on asset prices and the vulnerability of financial systems. My approach differs to theirs on the volatility of asset prices after financial liberalization because, in my model, liquidity shortage can produce fire-sale pricing in the market.

The paper proceeds as follows. Section 2 describes the model. The constrained efficient allocation is derived in Section 3. Two equilibria, namely a no-default equilibrium and a mixed equilibrium, are analyzed in Section 4. Then, Section 5 examines the existence of equilibria, and Section 6 presents numerical examples. The role of policies is analyzed in Section 7 and, finally, Section 8 concludes the paper.

2 The Model

I consider a small open economy with three periods, indexed by $t = 0, 1, 2$. There is a single good in each period, which can be used for consumption, investment, or trade in an international market. For simplicity, the price of the good is fixed and normalized at one unit of international currency.\(^9\)

The economy is populated by a $[0, 1]$ continuum of ex ante identical domestic agents. Each agent has an endowment of one unit of the good

\(^9\)In developed countries, it is possible for domestic agents to borrow in the domestic currency and invest in foreign currency bonds. In contrast, in emerging countries foreign debt is usually denominated in foreign currency (dollars) rather than in domestic currency, because foreign creditors fear inflation tax. The dollarized economy is essentially a real economy, as described here.
only in period 0. The agents’ time preferences are subject to a random shock at the beginning of period 1. With probability $\lambda$, an agent is an early consumer who only values consumption in period 1, and with probability $1 - \lambda$, is a late consumer who only values consumption in period 2. Type realizations are i.i.d. across agents and are private information to the agents. The ex-ante uncertainty about consumers’ preferences generates a role for banks as liquidity providers, as in Diamond and Dybvig (1983). The mass of banks is normalized to one, implying that a deposit market is competitive.

Let $c_1$ and $c_2$ denote the consumption levels of early and late consumers, respectively, and let $u(c)$ be their utility function. It is assumed that $u$ is strictly increasing, strictly concave, and twice continuously differentiable.

There is aggregate uncertainty about the fraction of early consumers in the model. Aggregate uncertainty is represented by a state of nature $\theta \in \{L, H\}$, and the probability of being an early consumer is given by

$$\lambda_\theta = \begin{cases} 
\lambda_L & \text{with prob. } \pi, \\
\lambda_H & \text{with prob. } 1 - \pi,
\end{cases}$$

where $\lambda_L < \lambda_H$ and $0 < \pi < 1$. For simplicity, $\lambda$ takes two values.

The economy is open and small relative to the rest of the world in the sense that banks’ behavior has no impact on international prices. The rest of the world is risk neutral, and the gross return on a riskless asset is one. As in Chang and Velasco (2000, 2001), each bank can lend as much as it wants in the international market, while each bank can borrow at most an amount $f > 0$, where $f$ represents a country-level debt.
limit or credit ceiling. The existence of the limit to foreign borrowing will be taken as exogenously given, but this is not difficult to justify. It may capture the idea that government regulations or lack of investor protection and monitoring to alleviate asymmetric information in emerging economies limits credits from abroad. Then, \( f \) can be interpreted as a degree of “financial liberalization” in the economy.

There are two types of domestic assets, namely, a short asset and a long asset. Both are risk free. One unit of the good invested in the short asset in period \( t \) yields one unit in period \( t + 1 \) for \( t = 0,1 \). One unit of the good invested in the long asset in period 0 yields \( R > 1 \) units in period 2. On average, emerging economies grow quickly and offer a high-return investment opportunity. The setting captures this feature by assuming \( R > 1 \). There is a trade-off between liquidity and returns: long-term investments have higher returns, but take longer to mature.

There is a competitive domestic asset market in period 1.\(^{10}\) Let \( P \) denote the price of the long asset in terms of units of consumption in period 1. It is assumed that participation in this market is limited in the sense that only domestic banks can buy or sell the long asset. That is, the asset market is segmented, and foreign investors cannot participate in the market because of government regulations or lack of knowledge about local properties. This assumption captures the dimension of financial underdevelopment in emerging economies. Since the bank can do anything that an agent can do, there is no loss of generality in assuming that agents deposit their entire endowment in a bank in period 0.

\(^{10}\)There are no markets for Arrow securities contingent on the future state in period 0.
After the realization of preference shocks, banks with low liquidity reserves will be able to sell their long asset and buy the short asset from banks with high liquidity reserves.

The timing of events is as follows. In period 0, banks take deposits from agents and borrow funds from the international market, and then divide these resources between international reserves, short assets and long assets. In period 1, the preference shocks are realized and the domestic asset market opens. At the end of period 1, domestic depositors and foreign investors who invest short term in period 0 receive payment from their banks. In period 2, domestic depositors who do not withdraw in period 1 and foreign investors who invest long term in period 0 and short term in period 1 withdraw and consume their goods.

3 The Constrained Efficient Allocation

I begin with the constrained efficient allocation. The social planner treats agents symmetrically and makes all investment and consumption decisions to maximize the expected utility of a representative agent, subject to the constraint of using a fixed payment in period 1 and international borrowing constraints. The planner’s problem is described as follows:

$$\max \ E_\theta [\lambda u(c_1) + (1 - \lambda)u(c_2)],$$
subject to

\[ x + y \leq 1 + b_{01} + b_{02}, \]  
\[ \lambda_\theta c_1 + b_{01} \leq y + b_{1\theta}, \]  
\[ (1 - \lambda_\theta)c_{2\theta} + b_{1\theta} + b_{02} \leq Rx + y + b_{1\theta} - b_{01} - \lambda_\theta c_1, \]

\[ b_{01} + b_{02} \leq f, \]
\[ b_{1\theta} + b_{02} \leq f, \]
\[ c_1 \leq c_{2\theta}, \]

for any \( \theta = L, H \), where \( b_{01}, b_{02}, \) and \( b_{1\theta} \) denote short-term debt in period 0, long-term debt in period 0, and short-term debt in period 1 in state \( \theta \) from the international market, respectively. The first is a resource constraint in period 0, which states that the investment in the short and long assets must be less than or equal to the endowment plus short- and long-term international borrowing. Note that \( y \) comprises the short asset and international reserves because the gross return of both assets is the same. The second constraint is the budget constraint in period 1 in state \( \theta \). This constraint states that the consumption in period 1 and the repayment to the international market must be less than or equal to the amount of the short asset plus the short-term international borrowing. The third constraint is the budget constraint in period 2. Here, consumption in period 2 and the repayment to the international market must be less than or equal to the return from the long asset plus the amount of the short asset left over from period 1. The constraints (4) and (5) are the credit constraints in periods 0 and 1, respectively, which state that total international borrowing in any period in any state.
cannot exceed the credit limit \( f \). The final constraint is the incentive-compatibility or truth-telling constraint, which states that late consumers weakly prefer their own consumption to that of early consumers, for any state.

At the optimum, the borrowing constraints (4) and (5) in state \( H \) will be binding:

\[
\begin{align*}
    b_{01} + b_{02} &= f, \\
    b_{1H} + b_{02} &= f.
\end{align*}
\]

Otherwise, it would be possible to increase expected utility by borrowing more from abroad, since \( R > 1 \). From the two equations, \( b_{01} = b_{1H} \) is obtained in a straightforward manner. Similarly, at the optimum,

\[
\lambda_H c_1 = y + b_{1H} - b_{01} = y.
\]

If \( \lambda_H c_1 + b_{01} < y + b_{1H} \), it would be possible to increase expected utility by holding \( c_1 \) constant and reducing \( y \), since \( R > 1 \). Each bank holds just enough of the liquid asset to satisfy the highest liquidity demand \( \lambda_H c_1 \) by early consumers in state \( H \). This equation, in turn, leads to

\[
(1 - \lambda_H)c_{2H} + b_{1H} + b_{02} = Rx, \\
\text{or} \quad (1 - \lambda_H)c_{2H} = Rx - f.
\]

Thus, the planner’s problem is to choose \( y \) to maximize

\[
\pi \left[ \lambda_L u \left( \frac{y}{\lambda_H} \right) + (1 - \lambda_L) u \left( \frac{R(1 + f) - f - (R - 1 + \frac{\lambda_L}{\lambda_H})y}{1 - \lambda_L} \right) \right] \\
+ (1 - \pi) \left[ \lambda_H u \left( \frac{y}{\lambda_H} \right) + (1 - \lambda_H) u \left( \frac{R(1 + f - y) - f}{1 - \lambda_H} \right) \right].
\]
This gives the first-order condition that determines $y^*$:

$$
\frac{\pi \lambda_L + (1 - \pi) \lambda_H}{\lambda_H} u'(\frac{y}{\lambda_H}) - \pi \left( R - 1 + \frac{\lambda_L}{\lambda_H} \right) u' \left( \frac{R(1 + f) - f - (R - 1 + \frac{\lambda_L}{\lambda_H}) y}{1 - \lambda_L} \right) - R(1 - \pi) u' \left( \frac{R(1 + f - y) - f}{1 - \lambda_H} \right) = 0. \quad (7)
$$

Differentiating a second time with respect to $y$, we have

$$
\frac{\pi \lambda_L + (1 - \pi) \lambda_H}{(\lambda_H)^2} u''(\frac{y}{\lambda_H}) + \pi \left( \frac{R - 1 + \frac{\lambda_L}{\lambda_H}}{1 - \lambda_L} \right)^2 u'' \left( \frac{R(1 + f) - f - (R - 1 + \frac{\lambda_L}{\lambda_H}) y}{1 - \lambda_L} \right) + \frac{R^2(1 - \pi)}{1 - \lambda_H} u'' \left( \frac{R(1 + f - y) - f}{1 - \lambda_H} \right) < 0,
$$

since $u'' < 0$. Thus, the amount of the international liquidity reserves $y^*$ is determined uniquely. Then, it is straightforward to derive:

$$
c^*_1 = \frac{y^*}{\lambda_H},
$$

$$
c^*_2L = \frac{R(1 + f) - f - (R - 1 + \frac{\lambda_L}{\lambda_H}) y^*}{1 - \lambda_L},
$$

$$
c^*_2H = \frac{R(1 + f - y^*) - f}{1 - \lambda_H}.
$$

However, the optimal structure of the foreign debt $(b_{01}, \{b_{1\theta}\}_{\theta=L,H}, b_{02})$ is indeterminate. That is, any values of $b_{01} = b_{1H}$ and $b_{02}$ satisfying the binding (4) and (5) support a constrained efficient allocation.

### 4 Equilibrium

In what follows, I describe a decentralized economy in which banks offer demand deposit contracts to agents, as well as trade assets through an interbank asset market. The model generates two types of equilibria,
which will be discussed below. In one equilibrium, called a *no-default equilibrium*, bank runs do not occur, and all banks remain solvent. In the other equilibrium, called a *mixed equilibrium*, some banks experience a run and go bankrupt in state $H$, while others always remain solvent. A run occurs in the model only when the value of the bank’s portfolio in period 2 is not sufficient to repay at least $c_1$ to late consumers. That is, self-fulfilling runs do not occur.

### 4.1 The No-Default Equilibrium

First, consider first an equilibrium in which all banks initially offer identical *run-preventing contracts* and all depositors withdraw according to their time preferences. As a result of competition among banks, they create a portfolio of $y$ in the short asset and international reserves and $x$ in the long asset in period 0 to maximize the expected utility of a representative agent. As in Allen and Gale (1998, 2004), it is assumed that the deposit contract is incomplete in the sense that the repayments to both types of agents are not state contingent. Therefore, the problem faced by banks is as follows:

$$\max \ E_0[\lambda \ u(c_1) + (1 - \lambda) u(c_2)],$$

14
subject to

\[ x + y \leq 1 + b_{01} + b_{02}, \quad (11) \]
\[ \lambda_\theta c_1 + b_{01} \leq y + b_{1\theta} + P_\theta x, \quad (12) \]
\[ (1 - \lambda_\theta)c_{2\theta} + b_{1\theta} + b_{02} \leq R \left( x + \frac{y + b_{1\theta} - b_{01} - \lambda_\theta c_1}{P_\theta} \right), \quad (13) \]
\[ b_{01} + b_{02} \leq f, \quad (14) \]
\[ b_{1\theta} + b_{02} \leq f, \quad (15) \]
\[ c_1 \leq c_{2\theta}, \quad (16) \]

for any \( \theta = L, H \). Constraints (11)–(13) are the resource constraints in periods 0, 1, and 2, which have similar meanings to those in the planning problem, except for asset trading through the interbank market. In state \( \theta \), if \( y + b_{1\theta} - b_{01} - \lambda_\theta c_1 > 0 \), excess liquidity in period 1 can be used to buy \( (y + b_{1\theta} - b_{01} - \lambda_\theta c_1)/P_\theta \) units of the long asset from other banks. If \( y + b_{1\theta} - b_{01} - \lambda_\theta c_1 < 0 \), the long asset held by the bank must be sold in the market in period 1 to fund the liquidity shortfall. Constraints (14) and (15) denote the international credit constraints in periods 0 and 1, and the final constraint is the incentive constraint. Note that the international interest rate the banks face is zero, because they never default.

At the optimum, the borrowing constraint (14) will be binding:

\[ b_{01} + b_{02} = f. \]

Otherwise, it would be possible to increase expected utility by borrowing more from abroad, since \( R > 1 \). Similarly, when \( R/P_\theta \geq 1 \), it is optimal for the bank to borrow as much as possible from abroad and buy the long
asset in state \( \theta \) in period 1. Then,

\[ b_{1\theta} + b_{02} = f, \]

for any \( \theta = L, H \). From these three equations, we obtain \( b_{01} = b_{1L} = b_{1H} \).

The problem each bank solves in period 0 is to choose \( c_1 \) and \( y \) to maximize

\[
\pi \left[ \lambda_L u(c_1) + (1 - \lambda_L) u \left( \frac{R(1 + f - y + \frac{y - \lambda_L c_1}{P_L}) - f}{1 - \lambda_L} \right) \right] \\
+ (1 - \pi) \left[ \lambda_H u(c_1) + (1 - \lambda_H) u \left( \frac{R(1 + f - y + \frac{y - \lambda_H c_1}{P_H}) - f}{1 - \lambda_H} \right) \right],
\]

subject to \( c_1 > 0 \) and \( 0 \leq y \leq 1 + f \), and taking prices \( P_L \) and \( P_H \) as given.

The first-order conditions for this with respect to the choice of \( c_1 \) and \( y \) are

\[
[\pi \lambda_L + (1 - \pi) \lambda_H] u'(c_1) = \pi \lambda_L \frac{R}{P_L} u'(c_{2L}) + (1 - \pi) \lambda_H \frac{R}{P_H} u'(c_{2H}), \quad (17)
\]

\[
\pi \left( 1 - \frac{1}{P_L} \right) u'(c_{2L}) \leq (1 - \pi) \left( \frac{1}{P_H} - 1 \right) u'(c_{2H}), \quad (18)
\]

with equality if \( y < 1 + f \), or equivalently \( x > 0 \).

Since bankruptcy cannot occur in equilibrium, market clearing requires that the aggregate demand for liquidity does not exceed the aggregate liquidity supply \( y \):

\[
\lambda_L c_1 < \lambda_H c_1 \leq y.
\]

Since \( \lambda_L < \lambda_H \), there is excess liquidity in period 1 in state \( L \). In order for the interbank asset market to clear, it is necessary that:

\[
P_L = R. \quad (19)
\]
In this case, banks are willing to hold both the long asset and the excess liquidity between periods 1 and 2. If \( P_L < R \), they will hold only the long asset, while if \( P_L > R \) they will hold only the liquid asset. Hence, \( P_L = R \) must hold.

Suppose that \( \lambda_H c_1 < y \). Then, there is also excess liquidity in period 1 in state \( H \), which implies that \( P_H = R \). However, the long asset would dominate the short asset between periods 0 and 1 and every bank would only invest in the long asset (i.e., \( y = 0 \)), which is a contradiction. Hence, in equilibrium, the following equation must hold:

\[
\lambda_H c_1 = y. \tag{20}
\]

Now, substituting for \( P_L \) and \( c_1 \) from (19) and (20) into (17) and (18) and arranging these equations yields

\[
[\pi \lambda_L + (1 - \pi) \lambda_H] u' \left( \frac{y}{\lambda_H} \right) = \pi \left[ \lambda_H (R - 1) + \lambda_L \right] u' \left( \frac{R(1 + f) - f - (R - 1 + \frac{\lambda_H}{\lambda_H}) y}{1 - \lambda_L} \right) + \lambda_H R(1 - \pi) u' \left( \frac{R(1 + f - y) - f}{1 - \lambda_H} \right), \tag{21}
\]

which determines \( y \) uniquely.

It is straightforward to see that equation (21) is equivalent to (7), which implies that the value of \( y \) in the no-default equilibrium is the same as the value in the constrained efficient allocation. In addition, the values of \( c_1 \) and \( c_2 \) derived here are also equivalent to (8)–(10). The next proposition states this result.

**Proposition 1** The no-default equilibrium is unique and achieves the constrained efficient allocation.
The price $P_H$ must ensure that banks are willing to hold both the liquid asset and the long asset between periods 0 and 1. Then the price $P_H$ can be derived from (18), as follows:

$$P_H = \frac{(1 - \pi)Ru'(c_{2H})}{\pi(R - 1)u'(c_{2L}) + (1 - \pi)Ru'(c_{2H})} < 1. \quad (22)$$

Given $P_L = R > 1$, $P_H < 1$ must hold; otherwise the liquid asset is dominated by the long asset. The equilibrium prices defined by (19) and (22) fluctuate across states because of the inelasticity of liquidity supply in period 1. In addition, as in the planner’s problem, the structure of the foreign debt $(b_{01}, \{b_{1θ}\}_{θ=L,H}, b_{02})$ is indeterminate, because any values of $b_{01} = b_{1L} = b_{1H}$ and $b_{02}$ satisfying the binding international borrowing constraints can support the equilibrium.

Finally, for future reference, let $W^N$ denote the resulting expected utility in the no-default equilibrium.

### 4.2 The Mixed Equilibrium

Next, I characterize the mixed equilibrium, where the price drops enough to generate bankruptcy. As in the previous section, holding excess liquidity is costly, because doing so means forgoing the high return on the long asset.

First, in equilibrium, not all banks default simultaneously. Suppose that all banks make identical choices in period 0. If the fraction of early consumers is sufficiently high in state $H$, which violates the incentive constraint, all depositors try to withdraw their funds from their banks, and all banks try to sell the long asset for consumption goods in period
1. In this case, the price must be zero because no bank is willing to buy the long asset. However, this cannot be an equilibrium. Given a price of zero, a bank would be tempted to hold enough liquidity in period 0 and make a large capital gain by purchasing the long assets in period 1. Thus, an equilibrium in which banks can default must be mixed in the sense that at least two types of banks exists, which adopts different strategies.

Some banks remain solvent by holding a lot of the liquid asset in period 0 and offering deposit contracts that promise low payments in period 1. These banks are called safe banks. Other banks invest so heavily in the long asset and offer deposit contracts promising such high payments in period 1 that they may cause defaults. These banks are called risky banks. In state L, the safe banks have enough liquidity to meet depositors’ liquidity demands and to supply the remaining liquidity to the market. The risky banks can obtain the liquidity they need to honor their repayment by selling the long asset. In state H, the risky banks face high withdrawals and sell all their long assets to meet the liquidity needs of their own customers. Since the liquidity supply is inelastic in period 1, the market is less liquid, and the liquidity shortage leads to a drop in price (a so-called fire-sale price), which forces the risky banks to go bankrupt. The safe banks will earn large capital gains because they hold enough liquidity in excess of their depositors’ needs which enables them to buy the long asset at a fire-sale price.

Let us first consider the optimization problem of the safe banks. This is similar to the problem in the no-default equilibrium. Let $P_s$ denote the
price of the asset market in period 1 in state $\theta \in \{L, H\}$. Given the market price $P_0$, the safe banks offer the consumption profile $(c^s_1, \{c^s_{2\theta}\}_{\theta \in \{L, H\}})$ to depositors, the foreign debt profile $(b^s_{01}, b^s_{02}, \{b^s_{1\theta}\}_{\theta \in \{L, H\}})$, and the investment portfolio $(y^s, x^s)$ to maximize the expected utility of their customers. The problem faced by safe banks is as follows:

$$\max \ E_\theta[\lambda u(c^s_1) + (1 - \lambda)u(c^s_{2\theta})]$$ \quad (23)

subject to

$$x^s + y^s \leq 1 + b^s_{01} + b^s_{02}, \quad (24)$$

$$\lambda c^s_1 + b^s_{01} \leq y^s + b^s_{1\theta} + P_0 x^s, \quad (25)$$

$$(1 - \lambda) c^s_{2\theta} + b^s_{1\theta} + b^s_{02} \leq R \left( x^s + \frac{y^s + b^s_{1\theta} - b^s_{01} - \lambda c^s_1}{P_0} \right), \quad (26)$$

$$b^s_{01} + b^s_{02} \leq f, \quad (27)$$

$$b^s_{1\theta} + b^s_{02} \leq f, \quad (28)$$

$$c^s_1 \leq c^s_{2\theta}; \quad (29)$$

for any $\theta = L, H$. Constraints (24)–(25) are the resource constraints in periods 0, 1, and 2, which have similar meaning to those in no-default equilibrium. In state $\theta$, the safe bank has $y^s + b^s_{1\theta} - b^s_{01} - \lambda c^s_1$ units of excess liquidity and buys $(y^s + b^s_{1\theta} - b^s_{01} - \lambda c^s_1)/P_0$ units of the long asset from the risky banks. Constraints (27) and (28) are the credit constraints in periods 0 and 1, and the final constraint is the incentive constraint. Since the safe banks never default, the international interest rate they face is zero.

As in the no-default equilibrium, the international borrowing con-
straints (27) and (28) will be binding at the optimum:

\[ b_{01}^* + b_{02}^* = f, \quad (30) \]
\[ b_{1\theta}^* + b_{02}^* = f, \quad \theta = L, H. \quad (31) \]

If \( b_{01}^* + b_{02}^* < f \), it would be possible to increase expected utility by increasing \( b_{02}^* \), since \( R > 1 \). Similarly, if \( b_{1\theta}^* + b_{02}^* < f \), it would be possible to increase expected utility by increasing \( b_{1\theta}^* \) and buying the long-term asset in period 1, since \( R/P \theta > 1 \) for any \( \theta = L, H \). From the three constraints, we obtain \( b_{01}^* = b_{1L}^* = b_{1H}^* \), and we can rewrite the resource constraints (24) and (26) as:

\[ x^* + y^* = 1 + f, \quad (32) \]
\[ (1 - \lambda_\theta)c_{2\theta}^* + f = R \left( x^* + \frac{y^* - \lambda_\theta c_1^*}{P_\theta} \right), \quad (33) \]

for any \( \theta = L, H \).

The first-order conditions for the problem are:

\[ [\pi \lambda_L + (1 - \pi)\lambda_H]u'(c_1^*) = \pi \lambda_L \frac{R}{P_L} u'(c_{2L}^*) + (1 - \pi)\lambda_H \frac{R}{P_H} u'(c_{2H}^*), \quad (34) \]
\[ \pi \left(1 - \frac{1}{P_L}\right) u'(c_{2L}^*) \leq (1 - \pi) \left(1 - \frac{1}{P_H}\right) u'(c_{2H}^*), \quad (35) \]

with equality if \( x^* > 0 \). Given the asset price \( P_\theta \) in period 1, the vector \( \{c_1^*, \{c_{2\theta}^*\}_{\theta \in \{L,H\}}, (y^*, x^*)\} \) is determined by conditions (32)–(35). Note that the structure of the foreign debt \( \{b_{01}^*, b_{02}^*; \{b_{1\theta}^*\}_{\theta \in \{L,H\}}\} \) is indeterminate, because any values of \( b_{01}^*, \{b_{1\theta}^*\}_{\theta \in \{L,H\}}, \) and \( b_{02}^* \) satisfying (30) and (31) support an equilibrium.

Next, consider the optimization problem of the risky banks. In state \( L \), these banks can offer high repayments to depositors and can borrow
funds from the international market in period 1. In state $H$, they sell all their long assets to meet the liquidity needs of their depositors, and go bankrupt. In particular, international creditors reject new lending to the risky banks in period 1 (i.e., $b_{1H} = 0$) and try to withdraw their funds in the same way as domestic depositors do. Since the risky banks may fail to meet their obligation, the international interest rate they face will be greater than zero. Let $r_1$ and $r_2$ denote the short- and long-term interest rates they face in the international market, respectively. Given the market price $P$ and the interest rates $r_1$ and $r_2$, the risky banks offer consumption profile $(c^r_1, c^r_{2L})$ to depositors, the foreign debt profile $(b^r_{01}, b^r_{02}, b^r_{1L})$, and the investment portfolio $(y^r, x^r)$ to maximize the expected utility. The problem faced by the risky banks is as follows:

$$\max \pi [\lambda_L u(c^r_1) + (1 - \lambda_L)u(c^r_{2L})] + (1 - \pi)u \left( \frac{c^r_1}{c^r_1 + (1 + r_1)b^r_{01}}(y^r + P_H x^r) \right),$$

subject to

$$x^r + y^r \leq 1 + b^r_{01} + b^r_{02},$$

$$\lambda_L c^r_1 + (1 + r_1)b^r_{01} \leq y^r + b^r_{1L} + P_L x^r, \quad \lambda_L c^r_{2L} + b^r_{1L} + (1 + r_2)b^r_{02} \leq R \left( x^r - \frac{(1 + r_1)b^r_{01} + \lambda_L c^r_1 - y^r - b^r_{1L}}{P_L} \right),$$

$$b^r_{01} + b^r_{02} \leq f,$$

$$b^r_{1L} + b^r_{02} \leq f,$$

$$c^r_1 \leq c^r_{2L}.$$ 

Note that the risky banks liquidate all their assets and distribute the proceeds in proportion to the creditors’ claims in period 1 in state $H$. 

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Each domestic depositor receives a fraction $c^r_1/(c^r_1 + (1 + r_1)b^r_{01})$ of the asset value and international creditors receive the rest. Note too that the full amount of long-term debt is defaulted on in state $H$. Constraints (37)–(39) are the resource constraints in periods 0, 1, and 2, respectively. Risky banks demand $(1 + r_1)b^r_{01} + \lambda_L c^r_1 - y^r - b^r_{1L}$ units of the good and sell $((1 + r_1)b^r_{01} + \lambda_L c^r_1 - y^r - b^r_{1L})/P_L$ units of the long asset to the safe banks in state $L$, but sell all their long assets and demand $P_H x^r$ units of the good in state $H$. Constraints (40) and (41) denote the international credit constraint in periods 1 and 2, respectively, while constraint (42) is the incentive constraint in state $L$.

The resource constraint (37) and international borrowing constraints (40) and (41) are binding at the optimum:

$$y^r + x^r = 1 + f,$$

$$b^r_{01} + b^r_{02} = f,$$

$$b^r_{1L} + b^r_{02} = f.$$  

The first-order conditions for the problem give:

$$u'(c^r_1) + \frac{1 - \pi}{\pi \lambda_L} u'(\frac{c^r_1(y^r + P_H x^r)}{c^r_1 + (1 + r_1)b^r_{01}}) \frac{(y^r + P_H x^r)(1 + r_1)b^r_{01}}{(c^r_1 + (1 + r_1)b^r_{01})^2} = \frac{R}{P_L} u'(c^r_{2L}),$$

$$\pi R \left(1 - \frac{1}{P_L}\right) u'(c^r_{2L}) = (1 - \pi) u'(\frac{c^r_1(y^r + P_H x^r)}{c^r_1 + (1 + r_1)b^r_{01}}) \frac{c^r_1(1 - P_H)}{c^r_1 + (1 + r_1)b^r_{01}} + \mu_8,$$

$$\pi \left( r_2 - \frac{R}{P_L} r_1 \right) u'(c^r_{2L}) = (1 - \pi) u'(\frac{c^r_1(y^r + P_H x^r)}{c^r_1 + (1 + r_1)b^r_{01}}) \frac{(y^r + P_H x^r)(1 + r_1)c^r_1}{(c^r_1 + (1 + r_1)b^r_{01})^2} - \mu_5 - \mu_6 + \mu_7.$$
where $\mu_5, \mu_6, \mu_7, \mu_8$ are the Lagrange multipliers on the non-negativity constraints for $b_{1L}^r, b_{01}^r, b_{02}^r,$ and $y^r,$ respectively.

In equilibrium, domestic depositors must be indifferent between depositing their funds in a safe or risky bank; otherwise, one type of bank will attract no depositors. Let $W^s$ and $W^r$ denote the expected utility of safe and risky banks, respectively. The two expected utilities must be equalized, as follows:

$$W^s = W^r.$$  \hfill (49)

The asset market in period 1 must clear in both states. Let $\rho$ and $1 - \rho$ denote a proportion of safe banks and risky banks, respectively. In state $L,$ the risky banks demand liquidity $(1 - \rho)((1 + r_1)b_{01}^r + \lambda_L c_1^r - y^r - b_{1L}^r),$ and the safe banks supply their excess liquidity $\rho(y^s + b_{1L}^s - \lambda_L c_1^s - b_{01}^s)$ for the long asset. Market clearing requires that the demand for and the supply of liquidity be equal at price $P_L,$ as follows:

$$\rho(y^s + b_{1L}^s - \lambda_L c_1^s - b_{01}^s) = (1 - \rho)((1 + r_1)b_{01}^r + \lambda_L c_1^r - y^r - b_{1L}^r).$$ \hfill (50)

In state $H,$ the risky banks sell all their long assets $(1 - \rho)x^r$ at price $P_H,$ and the safe banks supply their excess liquidity $\rho(y^s + b_{1H}^s - \lambda_H c_1^s - b_{01}^s)$ for the long asset. Then, market clearing requires:

$$\rho(y^s + b_{1H}^s - \lambda_H c_1^s - b_{01}^s) = (1 - \rho)P_H x^r.$$ \hfill (51)

In state $H,$ the short-term debt is partially repudiated, while the long-term debt is not. The international creditors who offer short-term lending in period 0 receive $(1 + r_1)b_{01}^r$ in state $L$ and $(1 + r_1)b_{01}^r(y^r + P_H x^r)/(c_1^r + (1 + r_1)b_{01}^r)$ in state $H$ in period 1. Then, international creditors who
offer long-term lending receive \((1 + r_2)b^r_{01}\) in state L and nothing in state \(H\) in period 2. Thus, the no-arbitrage conditions are:

\[
1 = \pi(1 + r_1) + (1 - \pi)\left(\frac{(1 + r_1)(y^r + P_Hx^r)}{c^r_1 + (1 + r_1)b^r_{01}}\right),
\]

\[
1 = \pi(1 + r_2).
\]

As in Chang and Velasco (2000), a term structure of interest rates emerges endogenously. That is, the long-term debt is more expensive than the short-term debt \((r_1 < r_2)\), which is often relevant empirically.

The mixed equilibrium is characterized by the vector

\((c^1, \{c^2\}, \{b^*_0\}, \{b^*_1\}, y^r, c^r_1, \{b^r_0\}, \{b^r_1\}, y^r, \{P'_0\}, \{r_t\}, \rho),\)

satisfying (32)–(35), (39), and (43)–(53).

## 5 Existence of Equilibria

In the previous section, I characterized the two types of equilibria. In this section, I analyze the parameter space in which they exist. The key element for the existence of the equilibria is whether the strategies of the risky banks are optimal. The no-default equilibrium exists if no bank finds it optimal to default, given the prices \(P_L\) and \(P_H\) satisfying (19) and (22). On the other hand, the mixed equilibrium exists if some banks prefer portfolio allocations and deposit contracts that support default. That is, in order for the possibility of bank runs to arise in equilibrium, some banks must choose risky portfolios, while the remaining banks choose safe portfolios.

Let us consider the situation in which a bank chooses a risky portfolio when all other banks opt for a safe portfolio. The problem is similar to
that of the risky banks in the mixed equilibrium. The difference is that
the bank takes the market prices \( P_L = R \) and \( P_H \) defined by (22) as
given. The deviating bank chooses the consumption profile \((c^d_1, c^d_{2L})\), the
portfolio \((y^d, x^d)\), and the foreign debt profile \((b^d_{01}, b^d_{02}, b^d_{1L}, b^d_{1H})\) in order
to maximize the following expected utility:

\[
\max \pi \left[ \lambda_L \left( c^d_1 \right) + (1 - \lambda_L) \left( c^d_{2L} \right) \right] + (1 - \pi)u \left( \frac{c^d_1}{e^d_1 + (1 + r_1)b^d_{01}} \left( y^d + P_H x^d \right) \right)
\]

subject to

\[
x^d + y^d = 1 + b^d_{01} + b^d_{02}, \quad (55)
\]

\[
\lambda_L c^d_1 + (1 + r_1)b^d_{01} \leq y^d + b^d_{1L} + P_L x^d, \quad (56)
\]

\[
(1 - \lambda_L)c^d_{2L} + b^d_{1L} + (1 + r_2)b^d_{02} \leq R \left( x^d - \frac{(1 + r_1)b^d_{01} + \lambda_L c^d_1 - y^d - b^d_{1L}}{P_L} \right), \quad (57)
\]

\[
b^d_{01} + b^d_{02} \leq f, \quad (58)
\]

\[
b^d_{1L} + b^d_{02} \leq f, \quad (59)
\]

\[
c^d_1 \leq c^d_{2L}. \quad (60)
\]

Since the bank defaults in state \( H \), \( b^d_{1H} = 0 \) again. These constraints
have similar meanings to those of the maximization problem of the risky
banks in the mixed equilibrium. The solutions for the problem are given
by

\[
y^d + x^d = 1 + f, \quad (61)
\]

\[
b^d_{01} = b^d_{1L} = b^d_{1H} = 0, \quad b^d_{02} = f, \quad (62)
\]

\[
c^d_1 = c^d_{2L} = y^d + R x^d - \frac{f}{\pi}, \quad (63)
\]

\[
(1 - \pi)(1 - P_H)u' \left( y^d + P_H x^d \right) \leq \pi(R - 1)u'(c^d_{2L}), \quad (64)
\]

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with equality if $y^d > 0$, where $P_H$ is given by (22).

A deviating bank takes on as much long-term debt from the international market as possible, and invests a large proportion of its resources in the long asset. Then, it can provide good liquidity insurance and returns to depositors in state $L$, but defaults in state $H$.

Let $W^d$ denote the maximized expected utility corresponding to the solutions:

$$W^d = \pi u \left( y^d + Rx^d - \frac{f}{\pi} \right) + (1 - \pi)u \left( y^d + P_H x^d \right),$$

where $P_H$ is given by (22) and $(y^d, x^d)$ solves (61) and (64). Condition $W^d \geq W^N$ ensures that a bank has an incentive to choose a risky portfolio in a situation in which all other banks opt for a safe portfolio. That is, condition $W^d \geq W^N$ is necessary for the existence of the mixed equilibrium. The next proposition summarizes the existence of the equilibria.

**Proposition 2** If $W^N > W^d$, then there exists a no-default equilibrium.

It will be optimal for banks to avoid default in state $H$ when the probability of state $H$ occurring is high and the risk aversion of agents is high.

### 6 Examples

In this section, I illustrate the aforementioned equilibria using the numerical examples.
6.1 Basic Examples

First, I assume that depositors have the log utility function:

\[ u(c) = \log(c). \]

The liquidity shocks and the return on the long asset are assumed to be:

\[ \lambda_L = 0.8, \quad \lambda_H = 0.81, \quad \text{and} \quad R = 1.5. \]

In Example 1, I fix the probability of state \( L \), \( \pi = 0.6 \), and successively increase the credit ceiling, \( f \), as follows: 0.3 in Example 1A, 0.5 in Example 1B, and 0.7 in Example 1C. In Example 2, I fix the probability, \( \pi = 0.8 \), and successively increase \( f \), as follows: 0.3 in Example 2A, 0.5 in Example 2B, and 0.7 in Example 2C. These parameter values produce results that are typical of other simulations.

Table 1 shows the types of equilibria, the volatility of the asset prices \( P_L/P_H \), the proportion of safe banks \( \rho \), and the expected utility \( E[u] \), for various values of \( \pi \) and \( f \).

<table>
<thead>
<tr>
<th>Ex.</th>
<th>( \pi )</th>
<th>( f )</th>
<th>Types of eqm.</th>
<th>Price volatility ( (P_L/P_H) )</th>
<th>( \rho )</th>
<th>( E[u] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>0.6</td>
<td>0.3</td>
<td>No default</td>
<td>1.5000/0.6628=2.2631</td>
<td>1.0000</td>
<td>0.1728</td>
</tr>
<tr>
<td>1B</td>
<td>0.6</td>
<td>0.5</td>
<td>No default</td>
<td>1.5000/0.6628=2.2631</td>
<td>1.0000</td>
<td>0.2316</td>
</tr>
<tr>
<td>1C</td>
<td>0.6</td>
<td>0.7</td>
<td>No default</td>
<td>1.5000/0.6628=2.2631</td>
<td>1.0000</td>
<td>0.2872</td>
</tr>
<tr>
<td>2A</td>
<td>0.8</td>
<td>0.3</td>
<td>Mixed</td>
<td>1.2620/0.5189=2.4321</td>
<td>0.9723</td>
<td>0.1741</td>
</tr>
<tr>
<td>2B</td>
<td>0.8</td>
<td>0.5</td>
<td>Mixed</td>
<td>1.2947/0.4905=2.6396</td>
<td>0.9707</td>
<td>0.2328</td>
</tr>
<tr>
<td>2C</td>
<td>0.8</td>
<td>0.7</td>
<td>Mixed</td>
<td>1.3296/0.4646=2.8618</td>
<td>0.9700</td>
<td>0.2883</td>
</tr>
</tbody>
</table>

Table 1: Numerical examples

In Example 1, since no bank does has an incentive to choose a risky
portfolio, there exists a unique no-default equilibrium. In the no-default equilibrium, every bank has enough liquidity reserves to cover high liquidity demand. The equilibrium asset prices fluctuate across states, but relaxing the credit ceiling, which is interpreted as financial liberalization, has no impact on the prices and its volatility. Table 2 gives the allocations in the no-default equilibrium. As the credit ceiling, \( f \), increases, the amount of reserves, the long-term investment, and the payments to depositors increase, which improves the expected utility. Note that early consumers receive the same level of consumption, \( c_1 \), in each state. However, the late consumers have different levels of consumption in each state because they receive the residual value of the portfolio, and this depends on the asset price, \( P_0 \). Since there are no banking defaults in equilibrium, every bank faces low interest rates in the international capital market (i.e., \( r_1 = r_2 = 0 \)), which leads to the indeterminacy of the foreign debt structure. Note that these allocations are constrained efficient as stated in Proposition 1.

<table>
<thead>
<tr>
<th>Ex.</th>
<th>( \pi )</th>
<th>( f )</th>
<th>( (y, x) )</th>
<th>( (c_1, c_{2L}, c_{2H}) )</th>
<th>( (b_{01}, b_{1L}, b_{1H}, b_{02}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>0.6</td>
<td>0.3</td>
<td>(0.8888, 0.4112)</td>
<td>(1.0973, 1.6389, 1.6674)</td>
<td>Indeterminate</td>
</tr>
<tr>
<td>1B</td>
<td>0.6</td>
<td>0.5</td>
<td>(0.9427, 0.5573)</td>
<td>(1.1638, 1.7379, 1.7682)</td>
<td>Indeterminate</td>
</tr>
<tr>
<td>1C</td>
<td>0.6</td>
<td>0.7</td>
<td>(0.9966, 0.7034)</td>
<td>(1.2304, 1.8370, 1.8689)</td>
<td>Indeterminate</td>
</tr>
</tbody>
</table>

Table 2: Allocations in the no-default equilibrium

Under the parameters in Example 2, a bank has an incentive to choose a risky portfolio when all other banks opt for a safe portfolio because the

\[ \text{11The expected utility that a deviating bank can offer} \ W^d \text{ is 0.1634 in Example 1A, 0.2067 in Example 1B, and 0.2424 in Example 1C, which are less than the respective expected utilities in the no-default equilibrium.} \]
probability of state $H$, $1 - \pi$, is sufficiently low. That is, the no-default equilibrium no longer exists, and there exists a mixed equilibrium. Table 1 shows that, in contrast to the no-default equilibrium, capital inflow has a significant impact on the prices and their volatility. Capital inflow increases the asset price in state $L$, $P_L$, and decreases the price in state $H$, $P_H$, resulting in an increase in the price volatility. In addition, large capital inflow increases the proportion of risky banks, $1 - \rho$, which means that capital inflow magnifies the size of a banking crisis. However, as capital inflow increases, the expected utility increases because the level of consumption improves.

Table 3 gives the allocations in the mixed equilibrium. The safe banks hold large amounts of liquid international reserves and offer deposit contracts promising low payments in period 1. The risky banks invest only in the long-term asset and offer deposit contracts promising high payments in period 1. The risky banks borrow long term from the international creditors up to their credit limit, while the safe banks are indifferent between the structure of the foreign debt in every example. When liquidity demands are low ($\theta = L$), the safe banks have excess liquidity, which they supply to the market by buying the long asset. The risky banks obtain the liquidity they need to honor their deposit contracts by selling

\[^{12}\text{The expected utility that a deviating bank can offer } W^d\text{ is 0.2444 in Example 2A, 0.2980 in Example 2B, and 0.3473 in Example 2C, which are larger than the respective expected utilities in the no-default equilibrium.}\]

\[^{13}\text{The intuition is simple. Since the return on the long asset is larger than the international interest rate, the risky banks can take advantage of this opportunity by increasing the long-term investments when the credit ceiling } f\text{ increases. As a result, there are more risky banks, and the amount of the long asset sold in state } H\text{ becomes quite large, which cause a sharp fall in the price.}\]
the long asset. When liquidity demands are high ($\theta = H$), the market for the long asset is less liquid because the safe banks must devote more of their liquidity to satisfying the needs of their own customers. This liquidity shortage leads to a drop in the price of the long asset, which forces the risky banks to go bankrupt. Subsequently, they liquidate all their long assets. The increase in the supply of the long asset can lead to a sharp drop in prices. In this case, there is “cash-in-the-market” pricing. The safe banks hold just enough liquidity in excess of their customers’ needs to enable them to buy up the long asset at a fire-sale price. The low price compensates them for the cost of holding the extra liquidity when liquidity demands are low and prices are high.

In the mixed equilibrium, the ratio of short-term debt to international liquidity reserves can be increasing in capital inflow. Let us define the ratio as

$$\eta = \frac{\rho(\bar{\lambda}c_1^s + b_{01}^s) + (1 - \rho)(\bar{\lambda}c_1^r + (1 + r_1)b_{01}^s)}{\rho y^s + (1 - \rho)y^r},$$

where $\bar{\lambda} = \pi\lambda_L + (1 - \pi)\lambda_H$ is the average fraction of early consumers. The term $\rho(\bar{\lambda}c_1^s + b_{01}^s) + (1 - \rho)(\bar{\lambda}c_1^r + (1 + r_1)b_{01}^s)$ represents the average total short liabilities of the banking system in period 1, while the term

<table>
<thead>
<tr>
<th>Ex.</th>
<th>$\pi$</th>
<th>$f$</th>
<th>$g^s$, $x^s$</th>
<th>$(y^r, x^r)$</th>
<th>$(c_1^s, c_2^s, \gamma_{\lambda}^s)$</th>
<th>$(b_{01}^r, b_{1L}^r, b_{1H}^r, b_{02}^r)$</th>
<th>$(b_{01}^s, b_{1L}^s, b_{1H}^s, b_{02}^s)$</th>
<th>$(b_{01}^r, b_{1L}^r, b_{1H}^r, b_{02}^r)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2A</td>
<td>0.8</td>
<td>0.3</td>
<td>(0.9085, 0.3915)</td>
<td>(0.0000, 1.3000)</td>
<td>(1.0979, 1.6155, 1.8040)</td>
<td>Indeterminate</td>
<td>(0.0000, 0.0000, 0.0000, 0.3000)</td>
<td></td>
</tr>
<tr>
<td>2B</td>
<td>0.8</td>
<td>0.5</td>
<td>(0.9652, 0.5348)</td>
<td>(0.0000, 1.5000)</td>
<td>(1.3251, 1.5750, 0.6746)</td>
<td>Indeterminate</td>
<td>(0.0000, 0.0000, 0.0000, 0.5000)</td>
<td></td>
</tr>
<tr>
<td>2C</td>
<td>0.8</td>
<td>0.7</td>
<td>(1.0212, 0.6788)</td>
<td>(0.0000, 1.7000)</td>
<td>(1.4848, 1.6750, 0.7898)</td>
<td>Indeterminate</td>
<td>(0.0000, 0.0000, 0.0000, 0.7000)</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Allocations in the mixed equilibrium
\( \rho y^a + (1 - \rho) y^r \) is the total international liquidity reserves of the financial system in the same period. A higher ratio implies that international reserves are not sufficient to repay maturing debt, capturing a difficult liquidity situation of an economy. In all the examples, \( b_{01}^a = 0 \) holds, while \( b_{01}^r \) is indeterminate and takes a value in \([0, f]\). Suppose that \( b_{01}^r = \nu f \) (or equivalently, \( b_{02}^r = (1 - \nu) f \)), where \( \nu = 0.2 \). In the case of Example 2, the ratio \( \eta \) is increasing in \( f \) and takes the following values: 1.0686 when \( f = 0.3 \); 1.1062 when \( f = 0.5 \); and 1.1396 when \( f = 0.7 \). These are consistent with empirical evidence provided by Chang and Velasco (1998) and Radelet and Sachs (1998).

### 6.2 Risk Aversion

So far, I have assumed that depositors have a log utility function. This implies a constant relative risk aversion equal to one. I now assume that depositors have a constant relative risk aversion utility function (CRRA), given by

\[
u(c) = \frac{c^{1-\sigma}}{1-\sigma},
\]

where \( \sigma \geq 1 \) represents the degree of risk aversion. Table 4 illustrates the equilibrium values corresponding to \( \sigma = 1, 2, \) and \( 3 \). Other parameters take the same values (\( \lambda_L = 0.8, \lambda_H = 0.81, \) and \( R = 1.5 \)) as those in Example 2.

Note that as the relative risk aversion increases, depositors require banks to hold more liquidity reserves and to provide better insurance against liquidity shocks. Then, a higher relative risk aversion reduces the profit of a deviating bank from the no-default equilibrium, which
Table 4: Numerical examples for $u(c) = c^{1-\sigma}/(1 - \sigma)$ where $\sigma > 1$.

<table>
<thead>
<tr>
<th>Ex.</th>
<th>$\sigma$</th>
<th>$\pi$</th>
<th>$f$</th>
<th>Types of eqn.</th>
<th>Price volatility ($P_L/P_H$)</th>
<th>$\rho$</th>
<th>$E[u]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2A</td>
<td>1</td>
<td>0.8</td>
<td>0.3</td>
<td>Mixed</td>
<td>1.2620/0.5189=2.4321</td>
<td>0.9723</td>
<td>0.1741</td>
</tr>
<tr>
<td>2B</td>
<td>1</td>
<td>0.8</td>
<td>0.5</td>
<td>Mixed</td>
<td>1.2947/0.4905=2.6396</td>
<td>0.9707</td>
<td>0.2328</td>
</tr>
<tr>
<td>2C</td>
<td>1</td>
<td>0.8</td>
<td>0.7</td>
<td>Mixed</td>
<td>1.3296/0.4646=2.8618</td>
<td>0.9700</td>
<td>0.2883</td>
</tr>
<tr>
<td>3A</td>
<td>2</td>
<td>0.8</td>
<td>0.3</td>
<td>Mixed</td>
<td>1.5000/0.3952=3.7955</td>
<td>0.9888</td>
<td>-0.8465</td>
</tr>
<tr>
<td>3B</td>
<td>2</td>
<td>0.8</td>
<td>0.5</td>
<td>Mixed</td>
<td>1.5000/0.4012=3.7388</td>
<td>0.9918</td>
<td>-0.7981</td>
</tr>
<tr>
<td>3C</td>
<td>2</td>
<td>0.8</td>
<td>0.7</td>
<td>Mixed</td>
<td>1.5000/0.4049=3.7046</td>
<td>0.9936</td>
<td>-0.7550</td>
</tr>
<tr>
<td>4A</td>
<td>3</td>
<td>0.8</td>
<td>0.3</td>
<td>No default</td>
<td>1.5000/0.4237=3.5402</td>
<td>1.0000</td>
<td>-0.3598</td>
</tr>
<tr>
<td>4B</td>
<td>3</td>
<td>0.8</td>
<td>0.5</td>
<td>No default</td>
<td>1.5000/0.4237=3.5402</td>
<td>1.0000</td>
<td>-0.3198</td>
</tr>
<tr>
<td>4C</td>
<td>3</td>
<td>0.8</td>
<td>0.7</td>
<td>No default</td>
<td>1.5000/0.4237=3.5402</td>
<td>1.0000</td>
<td>-0.2862</td>
</tr>
</tbody>
</table>

means that it enlarges the range of parameters for which the no-default equilibrium exists. When a relative risk aversion is sufficiently close to one ($\sigma = 2$), the mixed equilibrium still exists, but there are fewer risky banks. In this equilibrium, the risky banks also hold some liquidity reserves between periods 0 and 1, and reduce the amount of the long asset they sell in period 1. Then, the asset market becomes more liquid, and the price of the long asset in state $L$ increases, resulting in significant asset-price volatility. When the relative risk aversion is sufficiently high ($\sigma = 3$), the equilibrium is default-free, because no bank has an incentive to adopt a risky portfolio.$^{14}$

### 6.3 Short-Term Debt

I have considered a situation in which the domestic banks can borrow both short- and long-term from the international capital market. Next, suppose that all foreign loans contracted in period 0 mature in period 1.

$^{14}$The basic equilibrium properties are the same as those in Example 1.
That is, no long-term borrowing is allowed in period 0 (i.e., $b_{02} = 0$, or equivalently, $r_2 = \infty$). Transaction and information costs may prevent banks from borrowing long term. This modification is irrelevant for banks adopting a safe portfolio, because they never default and their short-term debt is automatically renewed in period 1. In contrast, banks taking a risky portfolio are restricted because they prefer long-term debt, as shown in Example 2. Tables 5 and 6 illustrate the mixed equilibrium with no long-term international borrowing in period 0. Since this constraint reduces an incentive to adopt a risky portfolio, the number of safe banks increases and the asset market becomes more liquid, resulting in high asset-price volatility.

<table>
<thead>
<tr>
<th>Ex.</th>
<th>$\pi$</th>
<th>$f$</th>
<th>Types of eqm.</th>
<th>Price volatility ($P_L/P_H$)</th>
<th>$\rho$</th>
<th>$E[u]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5A</td>
<td>0.8</td>
<td>0.3</td>
<td>Mixed</td>
<td>1.3743/0.4579=3.0013</td>
<td>0.9831</td>
<td>0.1734</td>
</tr>
<tr>
<td>5B</td>
<td>0.8</td>
<td>0.5</td>
<td>Mixed</td>
<td>1.4433/0.4271=3.3793</td>
<td>0.9846</td>
<td>0.2320</td>
</tr>
<tr>
<td>5C</td>
<td>0.8</td>
<td>0.7</td>
<td>Mixed</td>
<td>1.5000/0.4063=3.6919</td>
<td>0.9856</td>
<td>0.2874</td>
</tr>
</tbody>
</table>

Table 5: Numerical examples for foreign short-term debt

<table>
<thead>
<tr>
<th>Ex.</th>
<th>$\pi$</th>
<th>$f$</th>
<th>($y^s, x^s$)</th>
<th>($y^r, x^r$)</th>
<th>($c^r_1, c^r_{2L}, c^s_{2H}$)</th>
<th>($c^r_1, c^r_{2L}, \varphi(y^r + P_H x^r)$)</th>
<th>($b^s_{01}, b^s_{1L}, b^s_{1H}, b^s_{02}$)</th>
<th>($b^r_{01}, b^r_{1L}, b^r_{1H}, b^r_{02}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5A</td>
<td>0.8</td>
<td>0.3</td>
<td>(0.8989, 0.4011)</td>
<td>(0.0000, 1.3000)</td>
<td>(1.0971, 1.6237, 1.7641)</td>
<td>(1.4816, 1.5266, 0.4824)</td>
<td>Indeterminate</td>
<td>(0.3000, 0.3000, 0.0000, 0.0000)</td>
</tr>
<tr>
<td>5B</td>
<td>0.8</td>
<td>0.5</td>
<td>(0.9523, 0.5477)</td>
<td>(0.0000, 1.5000)</td>
<td>(1.1632, 1.7206, 1.8782)</td>
<td>(1.6259, 1.5610, 0.4716)</td>
<td>Indeterminate</td>
<td>(0.5000, 0.5000, 0.0000, 0.0000)</td>
</tr>
<tr>
<td>5C</td>
<td>0.8</td>
<td>0.7</td>
<td>(1.0059, 0.6941)</td>
<td>(0.0000, 1.7000)</td>
<td>(1.2294, 1.8176, 1.9919)</td>
<td>(1.7617, 1.6026, 0.4713)</td>
<td>Indeterminate</td>
<td>(0.7000, 0.7000, 0.0000, 0.0000)</td>
</tr>
</tbody>
</table>

Table 6: Allocations in the mixed equilibrium with short-term foreign debt, where $\varphi = c^r_1/(c^r_1 + (1 + r_1)f)$.

In this situation, the short-term obligations of risky banks are too high
relative to their liquid reserves. In addition, domestic agents who have
deposited funds in these banks will compete for the liquidated value of
the asset with international creditors in state $H$. Then, domestic agents
receive only $\varphi(y^r + P_H x^r)$, while international creditors obtain the rest
$(1 - \varphi)(y^r + P_H x^r)$, where $\varphi = c_1^r/(c_1^r + (1 + r_1)f)$.

Panic by domestic depositors and international creditors is often ob-
served in crises of emerging economies, and produces a reversal in cap-
ital flows. In the mixed equilibrium, the net capital outflow in period
0 is $-f < 0$. In period 1, in state $H$, there is a capital outflow of
$\rho b_{01}^r + (1 - \rho)(1 - \varphi)(y^r + P_H x^r)$ and an inflow of $\rho b_{11}^r H + (1 - \rho)b_{1H}^r$. Thus
the net outflow is

$$(1 - \rho)(1 - \varphi)(y^r + P_H x^r) > 0.$$ 

Again, note that international creditors refuse to roll over the short-term
debt of the risky banks (i.e., $b_{1H}^r = 0$). As a result, there is a sudden
reversal in capital flows (a so-called “sudden stop”) in the event of a

6.4 Discussion

The model presented here captures the basic features of financial crises
in emerging market economies. Then, it is natural to ask whether the ex-
pected utility in the mixed equilibrium is larger than that in constrained
efficient allocation. The examples provide a somewhat surprising result.
The expected utilities provided to agents by the planner are 0.1729 in
Example 2A, 0.2318 in Example 2B, and 0.2873 in Example 2C, which
are less than the values in the mixed equilibrium. The intuition for this result is related to the contingencies of the banking contracts. Under the assumption that banks are restricted to using non-contingent banking contracts, the choices of the planner and banks are distorted. As we have seen, some banks cannot meet their commitments and go bankrupt in one state under incomplete contracts. In this case, depositors receive only the liquidated value of their banks’ portfolio, rather than the promised payment. However, this means that default relaxes the constraint of incomplete contracts, and allows the banks to offer the deposit contract more contingent on the state of nature, resulting in more efficient risk sharing. This result implies that there is no justification for government interventions to prevent a financial crisis, which contrasts sharply with the results of models with multiple equilibria.\textsuperscript{15}

7 Policy Implications

In this section, I examine various policy implications. As stated earlier, banking defaults restore the contingency of deposit contracts, and the public policy cannot improve welfare. However, a government would place more value on financial stability than ex ante efficiency, because a crisis may have significant negative impacts on the real sector (e.g., increasing unemployment, decreasing output, etc.), which are not modeled here. Can the government eliminate a crisis at the expense of welfare? To answer this question, I consider three popular policies that are often implemented in practice: a liquidity requirement, government deposit

\textsuperscript{15}Allen and Gale (2004) provide a more detailed discussion.
insurance, and control on capital inflow. To gain some insight into the complex effects of the policy interventions, I employ Example 2 presented in the previous section.

7.1 Liquidity Requirement

The liquidity requirement is a constraint imposed on all banks holding a certain proportion of liquidity reserves in the bank’s portfolio.\textsuperscript{16} Specifically, the government forces banks to invest at least the fraction $\xi$ of their available resources $(1 + f)$ in liquidity reserves in period 0 as:

$$y \geq \xi(1 + f),$$

where $0 \leq \xi \leq 1$. Since this constraint reduces the profit of a bank deviating from the no-default equilibrium, it enlarges the range of parameters for which the no-default equilibrium exists. This policy may be irrelevant to the safe banks, because they already hold enough international liquid reserves. Table 5 compares the mixed equilibrium with liquidity requirements, $\xi = 0$, 0.2, and 0.5. As shown, the policy eliminates the possibility of a crisis at the expense of the expected utility, as long as it is sufficiently severe ($\xi \geq 0.5$). However, if the liquidity constraint is not restrictive ($\xi = 0.2$), the mixed equilibrium still exists under the constraint. In the equilibrium, the size of banking defaults, the volatility of the asset market, and the welfare loss increase. These examples imply that a tepid liquidity requirement could harm rather than stabilize the banking system.

\textsuperscript{16}The liquidity coverage ratio is envisioned by Basel III.
7.2 Government Deposit Insurance

I next consider the public deposit insurance provided by the government and financed by taxes.\textsuperscript{17} The importance of insuring depositors in the event of a run is generally acknowledged, because it reduces their incentive to withdraw their funds early.\textsuperscript{18} Under the deposit insurance policy, depositors receive some funds from the government when their banks go bankrupt. Specifically, I assume that in state $H$ in period 1 the government imposes a lump-sum tax on the safe banks, $\tau$, and transfers $\phi$ to the depositors of the risky banks. Then the budget constraint of the safe banks in period 1 in state $H$ is modified as:

$$\lambda^s_H c^s_1 + b^s_{01} + \tau \leq y^s + b^s_{1H}.$$
while in state $H$, all domestic depositors of the risky banks receive:

$$\frac{c_1'(y^r + P_H x^r)}{c_1' + (1 + r_1)b_{01}} + \phi.$$

Therefore, the government resource constraint is given by:

$$(1 - \rho)\phi = \rho \tau.$$

Note that since the deposit insurance policy benefits a deviating bank, it enlarges the range of parameters for which the mixed equilibrium exists.

<table>
<thead>
<tr>
<th>Ex.</th>
<th>$\pi$</th>
<th>$\delta f$</th>
<th>DI ($\phi, \tau$)</th>
<th>Types of eqm.</th>
<th>Price volatility ($P_L/P_H$)</th>
<th>$\rho$</th>
<th>$E[u]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2A</td>
<td>0.8</td>
<td>0.3</td>
<td>(0.0000, 0.0000)</td>
<td>Mixed</td>
<td>1.2620/0.5189=2.4321</td>
<td>0.9723</td>
<td>0.1741</td>
</tr>
<tr>
<td>2B</td>
<td>0.8</td>
<td>0.5</td>
<td>(0.0000, 0.0000)</td>
<td>Mixed</td>
<td>1.2947/0.4905=2.6396</td>
<td>0.9707</td>
<td>0.2328</td>
</tr>
<tr>
<td>2C</td>
<td>0.8</td>
<td>0.7</td>
<td>(0.0000, 0.0000)</td>
<td>Mixed</td>
<td>1.3296/0.4646=2.8618</td>
<td>0.9700</td>
<td>0.2883</td>
</tr>
<tr>
<td>7A</td>
<td>0.8</td>
<td>0.3</td>
<td>(0.1000, 0.0084)</td>
<td>Mixed</td>
<td>1.1704/0.5794=2.0200</td>
<td>0.9222</td>
<td>0.1729</td>
</tr>
<tr>
<td>7B</td>
<td>0.8</td>
<td>0.5</td>
<td>(0.1000, 0.0069)</td>
<td>Mixed</td>
<td>1.2190/0.5251=2.3215</td>
<td>0.9356</td>
<td>0.2318</td>
</tr>
<tr>
<td>7C</td>
<td>0.8</td>
<td>0.7</td>
<td>(0.1000, 0.0060)</td>
<td>Mixed</td>
<td>1.2638/0.4854=2.6036</td>
<td>0.9431</td>
<td>0.2875</td>
</tr>
<tr>
<td>8A</td>
<td>0.8</td>
<td>0.3</td>
<td>(0.2000, 0.0553)</td>
<td>Mixed</td>
<td>1.1350/0.5493=2.0663</td>
<td>0.7834</td>
<td>0.1669</td>
</tr>
<tr>
<td>8B</td>
<td>0.8</td>
<td>0.5</td>
<td>(0.2000, 0.0339)</td>
<td>Mixed</td>
<td>1.1836/0.5050=2.3438</td>
<td>0.8550</td>
<td>0.2281</td>
</tr>
<tr>
<td>8C</td>
<td>0.8</td>
<td>0.7</td>
<td>(0.2000, 0.0249)</td>
<td>Mixed</td>
<td>1.2268/0.4701=2.6135</td>
<td>0.8894</td>
<td>0.2847</td>
</tr>
</tbody>
</table>

Table 8: The effects of government deposit insurance: $\phi = \rho \tau/(1 - \rho)$.

Table 6 shows the equilibrium values with and without the public deposit insurance. In these examples, the government sets the transfer level ($\phi = 0, 0.1, \text{ and } 0.2$) in period 0, which means that the tax level $\tau$ is determined endogenously by the government budget constraint. Table 6 shows that the public deposit insurance stabilizes the asset price volatility, but increases the size of defaults and decreases welfare, because more banks choose a risky portfolio. Interestingly, the effect of the increased capital inflow on the number of risky banks is very different. In the
model, the public deposit insurance increases the fragility of the banking system, which contrasts with the Diamond and Dybvig (1983) model, but is consistent with the empirical evidence provided by Demirguc-Kunt and Detragiache (2002) and Ioannidou and Penas (2010).

7.3 Capital Control

The conventional view is that too much short-term borrowing would put an economy into a situation of internationally illiquidity. Some countries experiencing a crisis then imposed on short-term capital inflows to remedy excessive short-term indebtedness. Notable examples include Chile and Colombia in the 1990s. Valdés-Prieto and Soto (1996) and Cardenas and Barrera (1997) estimate that a shift in composition toward longer debt maturities is precisely what the taxes seem to have accomplished in both countries. In the mode of Chang and Velasco (2001), shorter foreign debt maturity, more fragile of the financial system. In contrast, my examples in subsection 6.3 show that short-term foreign debt maturity leads to less risky banks because short maturity reduces a profit of a deviating bank. The result implies that a tax on short-term capital inflow may have a destabilizing effect on the banking system.

8 Conclusions

I developed a simple banking model in a small open economy in which the financial system is opened to include an international capital market. The model generates two types of equilibria. In the no-default equilib-
rium, all banks are symmetric and solvent for all states, while in the mixed equilibrium, some banks are internationally illiquid and default simultaneously in the event of a large liquidity shock. The mixed equilibrium captures the basic observed features of banking crises in emerging market economies after financial liberalization: 1) capital inflow increases the size of a financial crisis; 2) domestic banks are internationally illiquid in the event of a crisis; and 3) banks’ assets are traded at fire-sale prices.

The simple examples presented in Section 6 emphasize the general equilibrium effects of the liquidity regulation, the deposit insurance policy, and controls on capital inflow, which most partial-equilibrium models exclude. These policies impact banks’ portfolio choices, which, in turn, affect the number of risky banks and asset prices. The model shows that a liquidity requirement can stabilize the financial system, while deposit insurance and tax on short-term capital inflow cannot do so.

Since my objective here is to construct a simple banking model, I make the rather sharp simplifying assumptions: the exogenous international credit limit, ignoring foreign direct investment (FDI) and real deposit contract. Relaxing these assumptions is a promising direction for future research.
References


