Strategic Substitutes, Complements and Ambiguity: An Experimental Study *

Sara le Roux†
Department of Economics,
Oxford Brookes University, England

David Kelsey
Department of Economics,
University of Exeter, England

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Abstract

We report the results from a set of experiments conducted to test the effect of ambiguity on individual behaviour in games of strategic complements and strategic substitutes. We test whether subjects’ perception of ambiguity differs when faced by a local opponent as opposed to a foreign one. Interestingly, though subjects often choose an ambiguity safe strategy (not part of a Nash equilibrium), we do not find much difference in the ambiguity levels when faced by foreign subjects.

Keywords: Ambiguity; Choquet expected utility; strategic complements; strategic substitutes; Ellsberg urn.

JEL Classification: C72, C91, D03, D81

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†Address for Correspondence: Sara le Roux, Department of Economics, Oxford Brookes University, Wheatley Campus, Oxford OX33 1HX. E-mail: sle-roux@brookes.ac.uk Tel: +44 (0) 1865 485918
1 Introduction

In this paper we report the results of an experimental study on the effects of ambiguity on individual decision-making. Eichberger and Kelsey (2002) make the theoretical prediction that ambiguity has opposite effects in games of strategic complements and substitutes. We test this prediction in our experiments. In the case of strategic substitutes, increasing the level of ambiguity would cause a shift in equilibrium strategies in a Pareto improving direction, whereas for strategic complements, an increase in ambiguity would cause a shift in equilibrium, away from the ex-post Pareto optimal level. Thus it was hypothesised that ambiguity had an adverse effect in the case of games with strategic complements, but was helpful in attaining a Pareto efficient outcome in the case of games with strategic substitutes. In order to implement this, we need to find a way of introducing ambiguity into a game setting, without lying to the subjects. We achieve this by adapting the experimental design of Kilka and Weber (2001) for games.

Kilka and Weber (2001) find that subjects are more ambiguity-averse when the returns of an investment are dependent on foreign securities than when they are linked to domestic securities. In our games subjects were either matched with a local opponent or with a foreign one. The foreign opponent was intended to be the analogy of the foreign securities used in the Kilka and Weber (2001) paper. We hypothesised that subjects will be more ambiguity-averse when their opponents are individuals of a foreign country than when they are matched with local individuals. In order to test this hypothesis, we recruited subjects both locally at the University of Exeter as well as overseas in St. Stephens College, India.

In addition we also alternated the main games with Ellsberg Urn type decision problems to evaluate whether individuals display ambiguity-averse, ambiguity-neutral or ambiguity-seeking behaviour. This was done in order to test whether there was any difference in ambiguity attitude between the games and the single person decision problems. Moreover, it allowed us to elicit an independent measure of subjects’ ambiguity-attitudes.
We find that subject behaviour broadly correlates with our hypothesis. However, subjects do not display an increase in ambiguity when faced by foreign opponents. This is in line with findings reported in Kelsey and le Roux (2013). Another interesting observation from the data is that even though subjects display ambiguity-averse behaviour when faced by other opponents (whether local or foreign), they often display ambiguity seeking behaviour when faced by nature/in single-person decision situations. This is consistent with an earlier study (Kelsey and le Roux (2015)), where subjects showed differences in ambiguity attitudes based on the scenario they were facing.

Organisation of the Chapter In Section 2, we describe the theory being tested in the experiments. Section 3 and 4 describe the experimental model and design employed, Section 5 consists of data analysis and results, Section 6 reviews related literature and Section 7 provides a summary of results together with future avenues of research.

2 Preferences and Equilibrium under Ambiguity

2.1 Modelling Ambiguity

The Ellsberg paradox is a well-documented violation of the Subjective Expected Utility (SEU) Savage (1954). We use the three-ball version of the Ellsberg urn in our experiments. Consider an urn which contains 90 balls which are labelled X, Y, or Z. There are 30 balls labelled X, but subjects are not told how many are labelled Y and Z. Subjects are asked to choose between acts \( f, g, f', g' \) as shown in the table below (Pay-offs below are shown in terms of Experimental Currency Units - ECU):
Table 1: Acts available to Subjects

<table>
<thead>
<tr>
<th>Act</th>
<th>30 balls</th>
<th>60 balls</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td>(f)</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>(g)</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>(f')</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>(g')</td>
<td>0</td>
<td>100</td>
</tr>
</tbody>
</table>

When asked to choose between acts \(f\) and \(g\), subjects generally prefer the unambiguous act \(f\). However, when asked to choose between \(f'\) and \(g'\), subjects prefer \(g'\) to \(f'\), again avoiding the ambiguous act. This inconsistency could be solved by representing beliefs by a non-additive set function \(\nu\). Non-additive set functions allow that \(\nu(X \cup Z) \neq \nu(X) + \nu(Z)\), which would be compatible with the choices in the Ellsberg paradox.

2.2 Equilibrium under Ambiguity (EUA)

We present an equilibrium concept for strategic games with ambiguity. In a Nash equilibrium, players are believed to behave consistently with the actual behaviour of their opponents, and can attach a set of additive probabilities to the opponent’s actions. In the presence of ambiguity, beliefs are represented by capacities. The support of a capacity is a player’s belief of how the opponent will act. Formally, the support of a neo-additive capacity, \(\nu(A) = \delta \alpha + (1 - \delta) \pi(A)\), is defined by \(\text{supp}(\nu) = \text{supp}(\pi)\). Thus the support of a neo-additive belief is equal to the support of its additive component.\(^1\)

**Definition 1** A pair of neo-additive capacities \((\nu_1^*, \nu_2^*)\) is an Equilibrium Under Ambiguity (EUA) if for \(i = 1, 2\), \(\text{supp}(\nu_i^*) \subseteq R_{-i}(\nu_i^*)\).

Here \(R_i\) denotes the best-response correspondence of player \(i\) given that his beliefs are represented by \(\nu_i\), and is defined by \(R_i(\nu_i) = R_i(\pi_i, \alpha_i, \delta_i) := \arg\max_{s_i \in S_i} V_i(s_i; \pi_i, \alpha_i, \delta_i)\).

This definition of equilibrium is taken from Eichberger and Kelsey (2014), who adapt an earlier definition in Dow and Werlang (1994). These papers show that an EUA will exist for

\(^1\)This definition is justified in Eichberger and Kelsey (2011).
any given ambiguity-attitude of the players. In games, one can determine $\pi_i$ endogenously as the prediction of the players from the knowledge of the game structure and the preferences of others. In contrast, we treat the degrees of optimism, $\alpha_i$ and ambiguity, $\delta_i$, as exogenous. In equilibrium, each player assigns a strictly positive decision weight to his/her opponent’s best responses given the opponent’s belief. However, each player lacks confidence in his/her likelihood assessment and responds in an optimistic way by over-weighting the best outcome, or in a pessimistic way by over-weighting the worst outcome.

3 Experimental Model

In this section, we look at the games used in our experiments, followed by a brief look at the Ellsberg decision problems being studied by us. Henceforth we will use male pronouns he, his etc. to denote the Row Player, while female pronouns she, hers etc. will be used to denote the Column Player.\footnote{This convention is for the sake of convenience only and does not bear any relation to the actual gender of the subjects in our experiments.}

3.1 The Games

The games used in our experimental sessions can be seen in Figure 1. Games (SC1) and (SC2) (as labelled in Figure 1) are games with strategic complements games and positive externalities; while Games (SS1) and (SS2), were games with strategic substitutes and negative externalities.

**Proposition 3.1** If both players are ambiguity averse, i.e. $\alpha = 1$, and have preferences that can be represented by neo-additive capacities, then:

1) In the case of games with strategic complements and positive (resp. negative) externalities, the equilibrium strategy under ambiguity of an agent with neo-additive beliefs, is decreasing (resp. increasing) in ambiguity.

2) In the case of games with strategic substitutes and negative (resp. positive) externalities, the
equilibrium strategy under ambiguity of an agent with neo-additive beliefs, is decreasing (resp. increasing) in ambiguity.

Games (SC1) and (SC2) are games with strategic complements games and positive externalities. This can be verified if we fix the order $T < C < B$ and $L < M < R$. Both games have one pure Nash equilibrium: $(C, M)$. The equilibrium under ambiguity for these games is $(T, M)$.

Game (SS1) is a strategic substitutes game with negative externalities and multiple Nash equilibria, if we fix $T > C > B$ and $L > M > R$. The game has three pure Nash equilibria: $(T, R)$, $(C, M)$ and $(B, L)$, none of which are focal. The equilibrium under ambiguity for this game is $(B, R)$. Game (SS2) is a strategic substitutes game with negative externalities if we fix $T > C > B$ and $L > M > R$. The game has a unique Nash equilibrium: $(C, M)$. The equilibrium under ambiguity for this game is $(B, R)$. For both strategic substitutes games the Nash equilibrium Pareto dominates the equilibrium with ambiguity.

### 3.2 Ellsberg Urn Experiments

The game rounds were alternated with single person decision problems similar to the Ellsberg Urn experiment. Subjects were presented with an urn containing 90 balls, of which 30 were
labelled $X$, and the remainder were an unknown proportion of $Y$ or $Z$ balls. The decisions put to the subjects took the following form:

“An urn contains 90 balls, of which 30 are labelled $X$. The remainder are either $Y$ or $Z$.

Which of the following options do you prefer?

a) Payoff of $\lambda$ if an $X$ ball is drawn.

b) Payoff of 100 if a $Y$ ball is drawn.

c) Payoff of 100 if a $Z$ ball is drawn.”

Payoff “$\lambda$” attached to the option $X$ was changed from round to round, with $\lambda = 95, 90, 80, 100, 105$ (in that order), to measure the ambiguity threshold of subjects.

In our Ellsberg urn experiments, we use balls labelled $X, Y$ and $Z$, rather than following the traditional practice of using Red, Blue and Yellow coloured balls.\(^3\) This is because in a previous set of experiments conducted by us (Kelsey and le Roux (2015)), we used the traditional Ellsberg Urn setup and found that subjects often chose Blue (the ambiguous option), simply because they had a fondness for the colour blue. Similarly, we found a large number of Chinese subjects chose Red, because it was considered "auspicious" in Chinese culture. In this study we use balls labelled $X, Y$ and $Z$, in order to avoid biases of this sort.

4 Experimental Design

The games described above were used in paper-based experiments, conducted at St. Stephen’s College in New Delhi, India, and at the Finance and Economics Experimental Laboratory in Exeter (FEELE), UK. The experiments were conducted with three different treatments - under the first treatment, subjects were only matched with locally recruited subjects; under the second treatment, Exeter subjects were only matched with subjects recruited in India; and under the

\(^3\)In the traditional Ellsberg urn setup, the urn would contain Red, Blue and Yellow coloured balls. The number of Red balls in the urn would be known, while the remaining Blue and Yellow coloured balls would be ambiguous in number.
final treatment, subjects were matched against both internationally as well as locally recruited subjects, for the purpose of payment.

Treatments 1 and 3 consisted of 60 subjects each and Treatment 2 had 61 subjects. In total there were 181 subjects who took part in the experiment, 81 of whom were males and the remaining 100 were females. Each session lasted a maximum of 45 minutes including payment.

Subjects first read through a short, comprehensive set of instructions at their own pace, following this the instructions were also read out to all the participants in general. The subjects were asked to fill out practice questions to check that they understood the games correctly. Once the practice questions had been answered and discussed, the actual set of experimental questions were handed out to the subjects. Subjects were randomly assigned the role of either a Row Player or a Column Player at the beginning of the experiment, for the purpose of matching in the games, and remained in the same role for the rest of the experiment.

Each subject had to select one option per round: Top/Centre/Bottom if they were a Row Player or Left/Middle/Right if they were a Column Player, and in case of the Ellsberg urn rounds X, Y or Z. In the Ellsberg urn rounds, the pay-offs attached to drawing a Y or Z ball were held constant at 100, while those attached to drawing an X ball varied as 95, 90, 80, 100, 105.

Once subjects had made all decisions, a throw of dice determined one game round and one Ellsberg urn round for which subjects would be paid. In order to prevent individuals from self-insuring against payoff risks across rounds, we picked one round at random for payment (See Charness and Genicot (2009)). Players’ decisions were matched according to a predetermined matching, and pay-offs were announced.

Instead of using a real urn we used a computer to simulate the drawing of a ball from the

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4The experimental protocols can be found at the following link: http://saraleroux.weebly.com/experimental-protocols.html
5If all rounds count equally towards the final payoff, subjects are likely to try and accumulate a high payoff in the first few rounds and then care less about how they decide in the following rounds. In contrast, if subjects know that they will be paid for a random round, they treat each decision with care.
The computer randomly assigned the number of Y and Z balls in the urn so that they summed to 60, while keeping the number of X balls fixed at 30 and the total number of balls in the urn at 90. The computer then simulated an independent ball draw for each subject. If the label of the ball drawn by the computer matched that chosen by the subject, it entitled him to the payoff specified in the round chosen for payment.

5 Data Analysis and Results

5.1 Row Player Behaviour

In Treatment I, subjects were matched against other local subjects (this treatment analyses data from Delhi vs. Delhi and Exeter vs. Exeter sessions)\(^7\). See Figure 2, for a summary of row player behaviour. We find that a large majority of our subjects, 63\% in SC1 and 73\% in SC2, chose the ambiguity-safe option \(T\) in our experiments. In comparison, only 13\% (SC1) and 17\% (SC2) of subjects chose \(C\), the choice under Nash. Binomial Test A (See Table 2) finds that subjects chose the ambiguity-safe option significantly more often than either of the other options available to her. Similarly, in SS1 and SS2 we find that 50\% and 67\% of subjects chose the ambiguity-safe option \(B\). It is interesting to note that when multiple Nash equilibria are present (in SS1), 40\% of the subjects select the Nash \((C, M)\) – which gives an equal payoff to both players. This might indicate that fairness constraints affect these subjects more than ambiguity. As such, Binomial Test B finds that subjects choose the ambiguity-safe option \(B\), significantly more often in SS2, but fails to reject this hypothesis for SS1 (See Table 2, Row 5).

\(^6\)The computer simulated urn can be found at the following link: http://saraleroux.weebly.com/experimental-protocols.html

\(^7\)A probit regression showed that the dummy variable for location (Delhi/Exeter) does not have a significant impact on choosing the ambiguity safe option. Thus for the purpose of analysing subject behaviour in Treatment I, we have combined the data from sessions where Delhi subjects played against other Delhi subjects (local vs. local) with data from the Exeter vs. Exeter session, without the loss of efficiency.
Treatment II consisted of matching Exeter subjects against an Indian opponent. Subjects were told that the same experiments had been run in India and that they would be matched up against an Indian subject whose responses had been already collected. Cultural studies conducted in the past have shown that western societies are individual-oriented, while Asian cultures tend to be collectivist. Members of Asian cultures have larger social networks that they can fall back upon in the event of an emergency/loss. This makes them more risk/ambiguity-seeking than their western counterparts (Weber and Hsee (1998)). As such, we expected that subjects would be more ambiguous when matched against opponents who are from a different socio-cultural background than themselves.

We find that 85% and 87% of subjects chose the ambiguity-safe strategy $T$ in $SC1$ and $SC2$, respectively, compared to 7% (2) and 13% of subjects who chose $C$ (the choice under Nash equilibrium). When compared to the base treatment, it is clear that subjects perceived greater ambiguity in this situation (when faced by the foreign subject). As can be seen in Table 2 (Row 6), subjects chose the ambiguity safe option significantly more often than the other two options available to them. In the strategic substitutes game $SS1$ and $SS2$, 50% and 67% of subjects chose the $B$, the choice under EUA. Even though we perceive heightened ambiguity on
the part of the subjects, about half of them (43%), opt for the Nash outcome which would result in equitable payoffs for both players. Binomial Test B cannot be rejected for SS1, however, we do reject the null at a 5% level of significance for SS2.

<table>
<thead>
<tr>
<th>Test:</th>
<th>Binomial Test A</th>
<th>Binomial Test B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Null Hypothesis ( (H_0) ): ( \text{prob}(T) = \text{prob}(C + B) = 0.5 )</td>
<td>( \text{prob}(B) = \text{prob}(T + C) = 0.5 )</td>
<td></td>
</tr>
<tr>
<td>Alt. Hypothesis ( (H_1) ): ( \text{prob}(T) &gt; \text{prob}(C + B) )</td>
<td>( \text{prob}(B) &gt; \text{prob}(T + C) )</td>
<td></td>
</tr>
<tr>
<td>Game: ( SC_1 )</td>
<td>( SC_2 )</td>
<td>( SS_1 )</td>
</tr>
<tr>
<td>Treatment I</td>
<td>Reject @ 5%</td>
<td>Reject @ 1%</td>
</tr>
<tr>
<td>Treatment II</td>
<td>Reject @ 1%</td>
<td>Reject @ 1%</td>
</tr>
<tr>
<td>Treatment III vs. LS</td>
<td>Reject @ 5%</td>
<td>Reject @ 5%</td>
</tr>
<tr>
<td>Treatment III vs. FS</td>
<td>Fail to Reject</td>
<td>Fail to Reject</td>
</tr>
</tbody>
</table>

In Treatment III, Exeter subjects were asked to make decisions versus both the local (Exeter) as well as the foreign (Indian) opponent. They were allowed to choose different actions against the foreign opponent and the domestic one. See the bottom half of Figure 2, for a summary of Row Player behaviour. We find that fewer subjects chose the ambiguity-safe option \( T \), against the foreign opponent than against the local opponent, in both \( SC_1 \) and \( SC_2 \). It is interesting to note that subjects chose the ambiguity-safe option significantly more often against the local opponent, but not against the foreign subject (See Table 2, Rows 7 and 8). We find similar behaviour in game \( SS_1 \), where fewer subjects took the ambiguity-safe option versus the foreign subject than against the local subject. Play in \( SS_2 \), was closer to our expectations, and subjects chose the ambiguity-safe option more against the foreign subject than the local one.

5.2 Column Player Behaviour

See Figure 3, for a summary of Column Player behaviour in Treatments I, II and III. In Treatment I, we find that 70% and 87% of subjects chose the Nash strategy \( M \) in \( SC_1 \) and \( SC_2 \), respectively. Binomial Test C finds that subjects choose the Nash/EUA option significantly more often than either of the other two choices (See Table 3, Row 5). In the strategic substitutes

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8Note in the case of \( SC_1 \) and \( SC_2 \), the equilibrium action under ambiguity coincides with the Nash strategy, for the Column player.
In Treatment II, 87% and 100% of subjects choose the Nash/EUA strategy $M$, in $SC1$ and $SC2$ respectively. Unsurprisingly, Binomial Test $C$ can be rejected at a 1% level for both $SC1$ and $SC2$ (See Table 3, Row 6). In games $SS1$ and $SS2$, we find 71% and 65% of subjects chose the ambiguity-safe strategy $R$. Binomial Test $D$ is rejected at 1% for $SS1$ and at 5% for $SS2$ (Table 3, Row 5). As such, we do see evidence that subjects seek to take the ambiguity-safe option against the foreign subject. Moreover, we encouraged subjects to write a short account at the end of the experiment, about their reactions and what they were thinking about when they made their choices during the experiment. A number of subjects concluded that they preferred to stick with a safe (but definite) payoff rather than take a chance and lose out, since they were not sure what prompted the foreign opponent’s decision choices. It was clear that the situation was perceived by them as being ambiguous, and they were willing to forego the possibility of getting a higher payoff, in order to get a certain payoff.
### Table 3: Binomial Tests C and D - Results

<table>
<thead>
<tr>
<th>Test:</th>
<th>Binomial Test C</th>
<th>Binomial Test D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Null Hypothesis $H_0$:</td>
<td>$\text{prob}(M) = \text{prob}(L + R) = 0.5$</td>
<td>$\text{prob}(R) = \text{prob}(L + M) = 0.5$</td>
</tr>
<tr>
<td>Alt. Hypothesis $H_1$:</td>
<td>$\text{prob}(M) &gt; \text{prob}(L + R)$</td>
<td>$\text{prob}(R) &gt; \text{prob}(L + M)$</td>
</tr>
<tr>
<td>Game:</td>
<td>$SC_1$</td>
<td>$SC_2$</td>
</tr>
<tr>
<td>Treatment I</td>
<td>Reject @ 5%</td>
<td>Reject @ 1%</td>
</tr>
<tr>
<td>Treatment II</td>
<td>Reject @ 1%</td>
<td>Reject @ 1%</td>
</tr>
<tr>
<td>Treatment III vs. LS</td>
<td>Reject @ 1%</td>
<td>Reject @ 1%</td>
</tr>
<tr>
<td>Treatment III vs. FS</td>
<td>Reject @ 1%</td>
<td>Reject @ 1%</td>
</tr>
</tbody>
</table>

In Treatment III, subjects were matched against both local as well as foreign opponents and as such, we expect the ambiguity perceived by the subjects to be higher in the case of a foreign opponent. See Figure 3 (lower half of figure), for a summary of Column Player behaviour in Treatment III. It is clear that a large majority of the subjects are choosing the Nash in $SC_1$ and $SC_2$; however, fewer subjects chose it against the foreign subject. *Binomial Test C* shows that the Nash/EUA option was the significant choice of subjects, in both $SC_1$ and $SC_2$, and against both local and foreign opponents. In game $SS_1$, we find that 70% and 77% of subjects chose the ambiguity-safe strategy $R$, against the local and foreign subject respectively. In $SS_2$, half the subjects chose the ambiguity-safe strategy while the other half chose the Nash against the local opponent. When faced with the foreign opponent, 60% chose the ambiguity-safe option while 40% chose the choice under Nash. It can be noted that in both the strategic substitutes rounds, the ambiguity-safe option was chosen more often against the foreign subject. As before, we conduct *Binomial Test D* and reject the null at 5% against the local opponent and 1% against foreign opponents for $SS_1$. We fail to reject the null for $SS_2$, since the decisions were very close to the 50–50 mark. However, it is clear in both $SS_1$ as well as $SS_2$, that the ambiguity-safe option was chosen more often against the foreign subject.

### 5.3 Behaviour in Ellsberg Urn Rounds

The strategic complement and substitute games were alternated with Ellsberg Urn decisions, in order to elicit an ambiguity threshold of the subjects. Moreover, it enabled us to evaluate
whether the ambiguity of subjects remained consistent between single person decisions, and situations where they were faced by ambiguity created by interacting with other players. The payoff on drawing $X$ (the unambiguous event) was varied as $\lambda = 95, 90, 85, 100$ or $105$ ECU, depending on the round being played.

As can be seen in Figure 4, for $\lambda = 100$ (the standard Ellsberg urn decision problem), 73% (133) of subjects chose $X$, while 27% (48) chose to bet on $Y$ and $Z$.\footnote{We consider the sum of the people who chose $Y$ and $Z$, rather than the number of people who chose $Y$ or $Z$ balls individually, in order to negate any effect of people choosing $Y$ just because it appeared before $Z$ on the choice set.} This result is consistent with previous Ellsberg urn studies, with a majority of subjects displaying ambiguity-averse behaviour.

When there is a premium attached to $X$, i.e., when $\lambda = 105$, a majority of subjects (73%) opt for $X$. However, what is more interesting to note is that 27% of subjects opt for $Y + Z$. This is very interesting because these subjects are willing to take a lower payoff, in order to choose $Y$ or $Z$ - the balls whose proportion is unknown! We believe this captures ambiguity-seeking behaviour on the part of the subjects.

Even a small penalty on $X$ from $\lambda = 100$ to $\lambda = 95$, leads to a big rise in the number of subjects choosing $Y + Z$. When $\lambda = 95$, 74% (134) of subjects choose $Y + Z$. This goes up substantially to 81% (146) of subjects choosing $Y + Z$, when $\lambda = 85$. Most subjects are
not ambiguity-averse enough to bear a small penalty, in order to continue choosing $X$ (the unambiguous event). It is interesting to note here that 19% (35) of subjects chose $X$, even when $X = 85$, thus displaying strong ambiguity-averse behaviour.

At the individual level, of the 133 subjects that chose $X$ when $\lambda = 100$, 68 switched to $Y + Z$ at $\lambda = 95$, 7 switched to $Y + Z$ at $\lambda = 90$, 5 switched to $Y + Z$ at $\lambda = 85$, while 21 subjects chose $X$ for all values of $\lambda$. Looking more closely at the choices of the subjects who always chose $X$, we find that 9 of these subjects always chose the ambiguity-safe options in the game rounds.\(^1\) Thus, a very small subset of our subject pool (5%), showed strong ambiguity-averse behaviour.

Looking at the 49 subjects who chose $Y + Z$ when $\lambda = 105$, we find that 11 of these subjects never chose $X$ in the Urn rounds. However, 12 of these 49 subjects always chose the ambiguity-safe options in the game rounds – these subjects seem to have a context-dependant ambiguity-attitude: ambiguity-loving in single person decisions and ambiguity-averse in the game environment.

Table 4: Binomial Tests E and F - Results

<table>
<thead>
<tr>
<th>Test:</th>
<th>Binomial Test E</th>
<th>Binomial Test F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Null Hypothesis $H_0$: $\text{prob}(X) = \text{prob}(Y + Z) = 0.5$</td>
<td>$\text{prob}(X) = \text{prob}(Y + Z) = 0.5$</td>
<td></td>
</tr>
<tr>
<td>Alt. Hypothesis $H_1$: $\text{prob}(X) &gt; \text{prob}(Y + Z)$</td>
<td>$\text{prob}(Y + Z) &gt; \text{prob}(X)$</td>
<td></td>
</tr>
<tr>
<td>$\lambda = 105$</td>
<td>Reject @1%</td>
<td></td>
</tr>
<tr>
<td>$\lambda = 100$</td>
<td>Reject @1%</td>
<td></td>
</tr>
<tr>
<td>$\lambda = 95$</td>
<td></td>
<td>Reject @1%</td>
</tr>
<tr>
<td>$\lambda = 90$</td>
<td></td>
<td>Reject @1%</td>
</tr>
<tr>
<td>$\lambda = 85$</td>
<td></td>
<td>Reject @1%</td>
</tr>
</tbody>
</table>

We conducted Binomial Tests E and F as described in Table 4. We find that subjects choose the unambiguous ball $X$ significantly more often for $\lambda = 105$ and 100, but prefer the ambiguous balls $Y + Z$ for lower values of $\lambda$. On the whole, subjects seem to prefer “betting” on $Y$ and $Z$. Responses gathered from the subjects showed that subjects viewed the urn rounds as “gambles”. The justification given for this was that the computer could have picked any

\(^{10}\) of these were Column Players and the remaining 2 were Row Players.
of the three options - thus $Y$ or $Z$ balls could have been more in number than $X$ balls, that were capped at 30 balls. The subjects thus displayed an optimistic attitude towards ambiguity. Moreover, some subjects treated these rounds as based on luck rather than reasoning.\textsuperscript{11}

\section{Related Literature}

\subsection{Papers on Games}

Our study builds upon the theoretical paper by Eichberger and Kelsey (2002). They find that in a game with positive (resp. negative) externalities, ambiguity prompts a player to put an increased (resp. decreased) weight on the lowest of his opponent’s actions. The marginal benefit that the player gets as a result of his own action then, gets decreased (resp. increased) in the case of a game with strategic complements (resp. substitutes). In the presence of positive externalities, players often have the incentive to use a strategy below the Pareto optimal level, and so, the resultant Nash equilibrium is inefficient. In the case of strategic substitutes, increasing the level of ambiguity would cause a shift in equilibrium strategies towards a Pareto efficient outcome, whereas for strategic complements, an increase in ambiguity would cause a shift in equilibrium, away from the ex-post Pareto efficient outcome. Thus it was hypothesised that ambiguity had an adverse effect in case of games with strategic complements, but was helpful in attaining a Pareto efficient outcome in the case of games with strategic substitutes. Ambiguity thus causes a decrease in equilibrium actions in a game of strategic complements and positive externalities or one that consists of the reverse case, i.e., strategic substitutes and negative externalities.

Di Mauro and Castro (2008) conduct a set of experiments designed to test the Eichberger and Kelsey (2002) hypothesis that it is ambiguity that causes an increase in contribution towards

\footnote{One subject in particular noted that—“The urn question is pure luck, because majority of the unmarked balls are either $Y$ or $Z$, and choosing either is a gamble.”}
the public good, and not altruism. In order to negate the chance that altruism, or a feeling of reciprocation prompted the subjects’ actions, the subjects were informed that their opponent would be a virtual agent and the opponent’s play was simulated by a computer. Subjects played in two scenarios, one with risk, the other with ambiguity. It was noted that contributions were significantly higher when the situation was one of ambiguity. The results showed that there was indeed evidence that ambiguity was the cause of increased contribution, as hypothesised by Eichberger and Kelsey (2002), and not altruism. This is akin to the results found in our paper that ambiguity significantly affects the decisions made by individuals, in a manner that depends directly on the strategic nature of the game in consideration.

Another paper that studies strategic ambiguity in games experimentally, is Eichberger, Kelsey, and Schipper (2008). In common with the present paper, it used the identity of the opponent to introduce ambiguity in the experiment. While in our paper we look at subject’s behaviour when faced with local and foreign opponents, they studied games in which subjects faced either a granny (who was described as being ignorant of economic strategy), a game theorist (who was described as a successful professor of economics), or another student as an opponent. The key hypothesis being tested was that ambiguity has the opposite effect in games of strategic complements and substitutes. Ambiguity averse actions were chosen significantly more often against the granny than against the game theorist, irrespective of whether the game was one of strategic complements, strategic substitutes or one with multiple equilibria. When the level of ambiguity the subjects faced while playing the granny was compared to that of the subjects faced playing against each other, it was found that the players still found the granny a more ambiguous opponent.

The paper also tested whether ambiguity had the opposite effect in games of strategic complements and substitutes. Similar to our study where we found that ambiguity had opposite effects depending on the strategic nature of the game, Eichberger, Kelsey, and Schipper (2008)
too conclude that comparative statics broadly supported the theoretical prediction. Subjects were also found to react to variations in the level of ambiguity, which was tested by altering the cardinal payoff in the game while keeping the ordinal payoff structure unchanged. It can thus be seen that subjects react not only to ambiguity on the part of the opponent, but also to subtle changes in the payoff structures of the experiment.

Kelsey and Laban (2014) study experiments with a stag hunt and bargaining coordination game. They use a between-subjects design to vary the identity of the opponent to see whether cultural norms or identity play a part in coordination decisions. They find that players do appear to use cultural stereotypes to predict behaviour, especially in the bargaining game. In particular, British subjects act in manner that indicates they believe Asian subjects will behave more cautiously. In our experiments we failed to see subjects react more ambiguously towards foreign opponents and as such, our subjects did not vary their actions on cultural stereotypes.

Ivanov (2011), discusses the findings of a series of experiments on one-shot normal form games run to distinguish between eighteen types of players. A person was classified on the basis of his attitude to ambiguity - as being either ambiguity averse, ambiguity neutral, or ambiguity loving; on the basis of his attitude to risk - as being risk averse, risk neutral or risk loving; and whether he played strategically or naively. A person who played in a naive manner was modelled as having a uniform belief in every game he played, whereas if he played strategically, his beliefs were different for every game and were thus unrestricted. The study finds that about 32% of the subjects taking part in the experiment were ambiguity loving, as opposed to 22% who were ambiguity averse. The majority of subjects (46%) were found to be ambiguity neutral. While being tested on the basis of their attitude to risk, 62% of the subjects were found to be risk averse, 36% to be risk neutral, and a mere 2% were risk loving. 90% of the subjects played in a strategic manner, while 10% played naively. These results are opposite to ours, since we find more subjects who are ambiguity averse than those who are ambiguity
seeking, in the game rounds.

The study by Ivanov (2011) questions the fact that there are more subjects who are ambiguity loving/neutral, than those who are ambiguity averse, given that on average a majority of them play strategically. This is attributed to players’ altruistic behaviour, i.e., they played in a manner that would maximise the sum of both players’ payoffs. This may be because a player is willing to compromise with his opponent, in order to do well himself.

While our study concentrated on investigating individual behaviour in the presence of ambiguity, Keller, Sarin, and Sounderpandian (2007) investigate whether individuals deciding together as pairs (termed dyads in the paper) display ambiguity averse behaviour. Participants were initially asked to state how much they were willing to pay for six monetary gambles. Five of the six gambles put before the subjects involved ambiguity, while the sixth involved no ambiguity. Once the participants had all disclosed their individual willingness to pay, they were randomly paired with another subject and each pair had to re-specify how much they were willing to pay for the six gambles. It was found that the pairs displayed risk averse as well as ambiguity averse preferences. It was observed that the willingness-to-pay among pairs of individuals deciding together, was lower than the average of their individual willingness-to-pay for gambles. They thus conclude that ambiguity averse behaviour is prevalent in group settings.

In our experiments, we did not allow subjects to interact with each other. We believed that this would reduce the level of ambiguity they would perceive, when asked to make decisions against each other. In contrast, Keck, Diecidue, and Budescu (2012), conduct an experiment in which subjects made decisions individually, as a group, and individually after interacting and exchanging information with others. Subjects were asked to make binary choices between sure sums of money and ambiguous and risky bets. They found that individuals are more likely to make ambiguity neutral decisions after interacting with other subjects. Moreover, they find that ambiguity seeking and ambiguity averse preferences among individuals are eliminated by
communication and interaction between individuals; and as such, groups are more likely to make ambiguity neutral decisions.

### 6.2 Papers on Ellsberg Urns

The Ellsberg urn experiments conducted by us investigated whether there was any correlation between ambiguity-averse behaviour in the game rounds and ambiguity attitude in single person decision problems. Moreover, we wanted to evaluate whether there was any threshold at which individuals switched from being ambiguity averse to being ambiguity neutral (or seeking) in their preferences.

Eliaz and Ortoleva (2011), study a three-colour Ellsberg urn with increased ambiguity, in that the amount of money that subjects can earn also depends on the number of balls of the chosen colour in the ambiguous urn. The subjects thus face ambiguity on two accounts: the unknown proportion of balls in the urn as well as the size of the prize money. In their experiment, both winning and the amount that the subject could possibly win were both perfectly correlated - either positively or negatively, depending on which of the two treatments was run by them. In the experiment, most subjects preferred betting in the positively correlated treatment rather than the negative one. Moreover, subjects also showed a preference for a gamble when there was positively correlated ambiguity, as opposed to a gamble without any ambiguity. This behaviour of the subjects, is compatible with our findings that subjects preferred betting on \( Y/Z \) where there was ambiguity, rather than on \( X \), the known choice.

Binmore, Stewart, and Voorhoeve (2011), attempt to test whether subjects are indeed ambiguity averse. They investigate whether the apparent ambiguity averse behaviour, predominantly reported by a number of papers, can be captured by the Hurwicz criterion. They report that subject behaviour in experiments conducted by them is inconsistent with the Hurwicz criterion. Instead, they find that the principle of insufficient reason has greater predictive power with re-
spect to their data, than ambiguity averse behaviour. This may also explain why behaviour in
games appears different to behaviour in Ellsberg urn type experiments. It is harder to apply
the principle of insufficient reason to games. Our results are consistent with these findings,
since we find that subjects are not willing to pay even a moderate penalty to avoid ambiguity
in the Ellsberg urn rounds where the payoff attached to $X$ were $95/90/85\text{ECU}$. This might
be because in the absence of information, subjects use the principle of insufficient reason and
attach a $50 : 50$ probability to the remaining $60\ Y$ and $Z$ balls left in the urn. The principle
of insufficient reason would imply that the probability distribution attached to the $X$, $Y$ and
$Z$ balls in the urn is $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. It would thus be rational to choose $Y$ or $Z$ and get a payoff of
$100\text{ECU}$, than to choose $X$ and suffer a penalty, i.e., get payoffs $95/90/85\text{ECU}$.

In our experiments we did not allow the subjects to communicate or interact with each other.
Charness, Karni, and Levin (2012), test whether individuals display a non-neutral attitude
towards ambiguity, and given a chance to interact, can subjects persuade others to change their
ambiguity attitude. They find that though a number of their subjects displayed an incoherent
attitude towards ambiguity, a majority of subjects displayed ambiguity neutral preferences. A
small minority of subjects (smaller than the number of subjects who were ambiguity-incoherent)
displayed ambiguity averse and ambiguity seeking behaviour. More interestingly, they find that
if subjects are allowed to interact with each other, given the right incentives, ambiguity neutral
subjects often manage to convince ambiguity seeking and ambiguity incoherent subjects to
change their mind and follow ambiguity neutral behaviour.

Halevy (2005), extends the standard Ellsberg type experiment to demonstrate that ambi-
guity preferences are associated with compound objective lotteries. The study finds that the
subject pool can be divided into those who are ambiguity neutral and reduce compound ob-
jective lotteries, i.e., they have behaviour which is consistent with SEU; and those who fail
to reduce compound lotteries. The latter category of individuals display different preferences
over ambiguity and compound lotteries, and are consistent with models that capture ambiguity seeking/averse preferences. It is concluded (in the study), that there is no unique theory that can capture all the different preference patterns observed in a given sample. As such, these experimental findings are consistent with Epstein (1999), where ambiguity aversion is defined as a behaviour that is not probabilistically sophisticated and thus cannot be aligned with a specific functional form or model.

7 Conclusions

Subject behaviour was found to be consistent with our hypothesis. In the games we find that subjects do indeed choose the equilibrium action under ambiguity more often than either of the other actions. We expected subjects to display a greater level of ambiguity-averse behaviour when faced by a foreign opponent. However, though we observe ambiguity averse behaviour on the whole in the games, we fail to see an escalation in the level of ambiguity when subjects are faced with foreign opponents. These results are consistent with an earlier study, where subjects’ perception of ambiguity in a public goods setting was analysed (Kelsey and le Roux (2013)).

One would expect that the ambiguity-safe option would be chosen more often against the foreign subject and not otherwise. Moreover, it is interesting that our findings are opposite to those of Kilka and Weber (2001), who found that subjects are more ambiguity-averse when the returns of an investment are dependent on foreign securities than when they are linked to domestic securities. One can note that decisions regarding financial markets are much more complex than the act of dealing with other people. It is easier for subjects to conceptualise another person whom they may be faced against, rather than investments in known/unknown financial markets. Follow-up experiments may be run, where subjects are given a choice of whether they would like to face a foreign opponent in a game, or invest in a foreign security.
It can be noted that the behaviour in game $SS2$ supports our hypothesis that when faced by both the foreign subject and the local subject simultaneously, the safe act would be taken more often against foreign subject. One of the reasons for not picking the ambiguity-safe option more often against the foreign subject, may be that subjects were trying to be dynamically consistent when making their choices. Another reason for this behaviour could be that subjects could see the other local subjects sitting in the experimental laboratory, whereas the foreign subject seemed very far away. They thus chose to play cautiously against the local subject, while taking their chances against the foreign subject.

Another reason for subjects choosing the same action against both foreign and local opponents, may be that some students were afraid that if they chose a different option against the foreign subject, they might appear racist.\textsuperscript{12} In an attempt to appear fair, subjects may have chosen the same option against both opponents. We could avoid this complication in future experiments, by comparing different groups of a similar race, such as African-Americans and Africans. In future experiments, we could have treatments where subjects are allowed to choose which opponent they would like to face, local or foreign. Furthermore, we could check if they are willing to pay a penalty in order to avoid facing the foreign opponent. It would be interesting if subjects were willing to pay a penalty, to avoid an ambiguous foreign opponent, since we find little evidence of willingness to pay a penalty, to avoid the ambiguous balls in the Ellsberg urn experiments conducted by us.

In the Ellsberg Urn rounds we find that for $\lambda = 105$ and $\lambda = 100$ subjects prefer to opt for $X$ rather than $Y$ or $Z$, but even the smallest reduction on $\lambda$ leads to subjects choosing $Y$ or $Z$ (which is the ambiguous choice). When the payoff attached to $X$ was 95, 90, or 85, $Y + Z$ was chosen significantly more often than $X$. We notice that the subjects are unwilling to bear even a small penalty in order to stick with $X$ balls (the unambiguous choice). At the individual level,

\textsuperscript{12}This was part of an overheard conversation between subjects, who were talking to each other at the end of the experiment.
we found steep drops in the number of subjects choosing $X$, for every reduction in the value of $\lambda$. A very small subset of our subject pool (5%), showed strong ambiguity-averse behaviour always choosing the ambiguity-safe option in the game rounds and $X$ in the Urn rounds, while 27% of our subjects showed mildly ambiguity-seeking behaviour by opting for $Y + Z$ when $\lambda = 105$.

In addition, we note that subjects seem to have a context-dependant ambiguity-attitude: ambiguity-loving in single person decisions and ambiguity-averse in the game environment. This is consistent with our earlier study (Kelsey and le Roux (2015)), where we found that the ambiguity-attitude of subjects was dependent on the scenario they were facing. It might be interesting to elicit subjects' preferences on whether they would like to face an opponent or an Ellsberg urn.

It is our belief that subjects find it more ambiguous to make decisions against other people than against the random move of nature, over which everyone is equally powerless. This might even explain why people are more concerned with scenarios involving political turmoil or war - situations dependent on other people, but appear to discount the seriousness of possible natural disasters - which are beyond anyone’s control.

A Appendix

Proposition 3.1 1) In the case of games with strategic complements and positive (resp. negative) externalities, the equilibrium strategy under ambiguity of an agent with neo-additive beliefs, is decreasing (resp. increasing) in ambiguity.

Proof.

In order to illustrate the theorem, we use Game $(SC2)$ in Table 5 below.
The game one pure Nash equilibrium: $(C, M)$. Moreover, if we order the strategy spaces as follows: $T < C < B$ and $L < M < R$, the game is one of strategic complements and positive externalities.

Let the Row Player have the following beliefs: $v^{RP}(L) = 1 - \delta^{RP}$ and $v^{RP}(M, R) = 0$. Then the Choquet expected payoff for the Row Player would be:

\[
\begin{align*}
V^{RP}(T) &= 115, \\
V^{RP}(C) &= 135\delta^{RP}, \\
V^{RP}(B) &= 5 + 100\delta^{RP}.
\end{align*}
\]

Thus, $T$ is the best response for the Row Player if $\delta^{RP} \geq \frac{115}{135}$ or $\delta^{RP} \geq 0.85$. Intuitively, this means that if the Row Player is sufficiently ambiguous about the opponent’s behaviour, he would opt for $T$, which is the ambiguity safe option.

Similarly, if the Column Player has the following beliefs: $v^{CP}(B) = 1 - \delta^{CP}$ and $v^{CP}(T, C) = 0$. Then the Choquet expected payoff for the Column Player would be:

\[
\begin{align*}
V^{CP}(L) &= 100 + (95 - 105)\delta^{CP}, \\
V^{CP}(M) &= 100, \\
V^{CP}(R) &= 135(1 - \delta^{CP}).
\end{align*}
\]

Thus, $M$ is the best response for the Column Player if $\delta^{CP} \geq \frac{135 - 100}{135}$ or $\delta^{CP} \geq 0.26$. Intuitively, this means that if the Column Player is sufficiently ambiguous about the opponent’s behaviour,
he would opt for $M$, which is the ambiguity safe option.

Hence, the best response for both players in a game with strategic complements and positive externalities, given sufficient ambiguity is one that decreases ambiguity. $\square$

**Proposition 3.1 2)** In the case of games with strategic substitutes and negative (resp. positive) externalities, the equilibrium strategy under ambiguity of an agent with neo-additive beliefs, is decreasing (resp. increasing) in ambiguity.

**Proof.**

In order to illustrate the theorem, we use Game $(SS2)$ in Table 6 below.

<table>
<thead>
<tr>
<th></th>
<th>$L$</th>
<th>$M$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>95, 5</td>
<td>5, 0</td>
<td>5,90</td>
</tr>
<tr>
<td>$C$</td>
<td>0, 5</td>
<td>100, 100</td>
<td>100, 90</td>
</tr>
<tr>
<td>$B$</td>
<td>90, 95</td>
<td>90, 100</td>
<td>90, 90</td>
</tr>
</tbody>
</table>

The game has one pure Nash equilibrium: $(C, M)$. Moreover, if we order the strategy spaces as follows: $T > C > B$ and $L > M > R$, the game is one of strategic substitutes and negative externalities.

Let the Row Player have the following beliefs : $v^{RP}(L) = 1 - \delta^{RP}$ and $v^{RP}(M, R) = 0$.

Then the Choquet expected payoff for the Row Player would be:

$$V^{RP}(T) = 95 - 90\delta^{RP},$$

$$V^{RP}(C) = 100\delta^{RP},$$

$$V^{RP}(B) = 90.$$

Thus, $B$ is the best response for the Row Player if $\delta^{RP} \geq \frac{90}{100}$ or $\delta^{RP} \geq 0.9$. Intuitively, this means that if the Row Player is sufficiently ambiguous about the opponent’s behaviour, he would opt for $B$, which is the ambiguity safe option.
Let the Column Player have the following beliefs: \( v^{CP}(T) = 1 - \delta^{CP} \) and \( v^{CP}(C,B) = 0 \).

Then the Choquet expected payoff for the Column Player would be:

\[
V^{CP}(M) = 100\delta^{CP},
\]
\[
V^{CP}(R) = 90.
\]

The Column Player would thus prefer \( R \) if, \( \delta^{CP} \geq \frac{90}{100} \) or \( \delta^{CP} \geq 0.9 \). Intuitively, this means that if the Column Player is sufficiently ambiguous about the opponent’s behaviour, he would opt for \( R \), which is the ambiguity safe option.

Hence, the best response for both players in a game with strategic substitutes and negative externalities, given sufficient ambiguity is one that decreases ambiguity. \( \square \)

References


