Abstract

This paper analyzes the interaction between the choice of public or private education by comparing their respective utilities and nationwide human capital accumulation by extending Cardak’s (2004) overlapping-generations model. Cardak (2004) assumed that human capital is accumulated solely through parental level of human capital and educational expenditure. We agree that human capital accrues through parental level of human capital and educational expenditure, but also includes time span allocated to human capital accumulation, depending on Glomm and Ravikumar (1992). In addition, this paper considers two cases that education choice is influenced by leisure in public and private education or not, which embodies the difference in parental sense of values. Moreover, we examine the effects on nationwide human capital accumulation of the two public education policies that
increase investment in public education and the time span allocated to human capital accumulation when public education is provided. We focus the model building of Cardak (2004) that the choice of public or private education is determined not by respective human capital levels but utilities. We find that the efficiency of public education policies on nationwide human capital accumulation depends on parental sense of values and private education.

**Key words:** Human capital, Overlapping-generations, Education choice
1 Introduction

Studies of public and private education typically compare disparate models (e.g., Glomm and Ravikumar, 1992; Gradstein and Justman, 1997; Saint and Verdier, 1993). In addition, studies often treat public and private education as complements in developing human capital (Benabou, 1996; Eckstein and Zilcha, 1994; Kaganovich and Zilcha, 1999). Cardak(2004a, b) departs from the literature by analyzing public and private education as mutually exclusive, rather than separately comparable alternatives, and by assuming that individuals choose between them by comparing their respective utilities. However, his production function for human capital becomes linear when private education is provided—a result influenced by assuming that human capital accumulation is solely determined by parental level of human capital and educational expenditure.

This study analyzes the interaction between the choice of public or private education and nationwide human capital accumulation by extending Cardak’s(2004a) overlapping-generations model. We agree that human capital accumulates through parental level of human capital and educational expenditure, but add the element of individuals’ time span allocated to human capital accumulation, depending on Glomm and Ravikumar(1992). We find that the human capital production function becomes concave when private education is provided, similar to that shown in Cardak(2004b). In addition, this paper considers two cases that education choice is influenced by leisure in public and private education or not. We define the influence of the leisure on the choice of public or private education as the difference in parental sense of values. Moreover, we examine the effects on nationwide human capital accumulation
of public education policies that increase investment in public education and the time span allocated to human capital accumulation when public education is provided.

We find that when the choice between public or private education is determined not by comparing their respective human capital levels but utilities, there is no assurance that these policies stimulate nationwide human capital accumulation and the efficiency of public education policies on nationwide human capital accumulation depends on the difference in parental sense of values and private education.

Section 2 presents the basic model. Section 3 analyzes the relationship between the difference in parental sense of values and the choice of public or private education. Section 4 considers the effects of public education policies that increase investment in public education and the time span allocated to human capital accumulation when public education is provided.

2 Model

Consider an overlapping-generations economy in which economic activity extends over an infinite discrete period. Lives of individuals in generation $t$ extend over two periods: $t$ and $t+1$. Individuals bear children in the second period. This paper assumes that population size in a period is constant and makes it a standard at 1.
2.1 Human Capital Accumulation

Individual \(i\) of generation \(t\) is born to a parent endowed with \(h_{i,t}\) units of human capital. His parent invests him \(q_{i,t}\) units of educational expenditure at time \(t\). Individual \(i\) allocates \(n\) units of his endowment to leisure and the remaining \(1 - n\) units to human capital accumulation. He acquires \(h_{i,t+1}\) units of human capital at time \(t + 1\). Then

\[
h_{i,t+1} = (1 - n)\beta (q_{i,t})^\gamma (h_{i,t})^\delta; \quad \beta, \gamma, \delta \in (0, 1), \quad \beta + \gamma + \delta = 1
\]

\(n\) and \(q_{i,t}\) is determined by the choice in equation (2) and (3).

\[
n = \begin{cases} 
  n^u \cdots \text{public education chosen} & ; 0 < n < 1 \\
  n^r \cdots \text{private education chosen} 
\end{cases}
\]

\[
q_{i,t} = \begin{cases} 
  E_t & \text{if } e_{i,t} = 0 \cdots \text{public education chosen} \\
  e_{i,t} & \text{if } e_{i,t} > 0 \cdots \text{private education chosen} 
\end{cases}
\]

where \(n^u\) and \(n^r\) are leisure in public and private education, and \(E_t\) and \(e_{i,t}\) are expenditures on public and private education by individual \(i\) of generation \(t - 1\) at time \(t\). This paper assumes that \(n^u\) and \(n^r\) are determined by government and private school, respectively. \(E_t\) is defined as

\[
E_t = \frac{\tau H_t}{P_t} = \frac{\tau \int_0^\infty h_{i,t} \cdot f_{i}(h_{i,t}) \, dh_{i,t}}{P_t}; \quad 0 < \tau < 1
\]

In equation (4), \(\tau\), \(H_t\), and \(P_t\), respectively, denote the income tax rate,
average human capital endowment, and the proportion of the population attending public education at time $t$. We show the population distribution at time $t$ using the density function of human capital, $f_t(h_{i,t})$.

### 2.2 Individuals

Labor income $y_{i,t+1}$ earned by individual $i$ of generation $t$ at time $t + 1$ is equal to acquired human capital $h_{i,t+1}$ at time $t + 1$:

$$y_{i,t+1} = h_{i,t+1}$$  \hspace{1cm} (5)

There are no inheritances. At time $t + 1$, consumption $c_{i,t+1}$ by individual $i$ is determined as

$$c_{i,t+1} = \begin{cases} 
(1 - \tau) y_{i,t+1} & \text{public education chosen} \\
(1 - \tau) y_{i,t+1} - e_{i,t+1} & \text{private education chosen}
\end{cases}$$  \hspace{1cm} (6)

In this paper, we consider two cases that education choice is influenced by leisure of public and private education, $n^u$ and $n^r$, or not. Education choice of individuals are determined by their parents, and we define the difference whether education is influenced by the leisure or not as the one by parental sense of values. In this model, we define the case that education choice is not influenced by $n^u$ and $n^r$, as "the case that parents devote to education of their children" and the one that education choice is influenced by the leisure as "the case that parents do not devote to education of their children".
2.2.1 The Case that Parents Devote to Education of Their Children

In this case, there are two cases of provision of education to next generation.

The Case that Individuals Provide Public Education to Next Generation

The utility of individual $i$ of generation $t$ during two periods $V^u$ is determined as

$$V^u = (1 - \alpha_1 - \alpha_2) \log n + \alpha_1 \log c_{i,t+1} + \alpha_2 \log E_{t+1} ; \quad \alpha_1, \alpha_2, 1 - \alpha_1 - \alpha_2 \in (0, 1)$$

In this case, individuals can not determine both $n$ and $E_{t+1}$. Moreover, there is no inheritance. Therefore, the utility-maximization problem does not occur. The consumption at time $t + 1$, $c^u_{t+1}$, is derived as

$$c^u_{t+1} = \begin{cases} 
(1 - \tau) (1 - n^u)^\beta (E_t)^\gamma (h_{i,t})^\delta \cdot \text{public education chosen} \\
(1 - \tau) (1 - n^r)^\beta (e_{i,t})^\gamma (h_{i,t})^\delta \cdot \text{private education chosen} 
\end{cases}$$

The Case that Individuals Provide Private Education to Next Generation

Individual $i$ of generation $t$ chooses $c_{i,t+1}$ and $e_{i,t+1}$ so as to maximize his utility during two periods, $V^r$.

$$\begin{align*}
\text{Maximize} & \quad V^r = (1 - \alpha_1 - \alpha_2) \log n + \alpha_1 \log c_{i,t+1} + \alpha_2 \log e_{i,t+1} \\
\text{subject to} & \quad c_{i,t+1} = (1 - \tau) y_{i,t+1} - e_{i,t+1}, \quad y_{i,t+1} = h_{i,t+1}
\end{align*}$$
Under private education, individuals can determine educational expenditure of next generation. Therefore, the utility-maximization problem occurs in this case. The optimal consumption $c_{t+1}^r$ and educational expenditure $e_{t+1}^r$ at time $t+1$ are derived as (7) and (8).

$$c_{t+1}^r = \frac{\alpha_1 (1 - \tau) y_{t+1}}{\alpha_1 + \alpha_2} = \frac{\alpha_1 (1 - \tau) h_{t+1}}{\alpha_1 + \alpha_2} \quad (7)$$

$$e_{t+1}^r = \frac{\alpha_2 (1 - \tau) y_{t+1}}{\alpha_1 + \alpha_2} = \frac{\alpha_2 (1 - \tau) h_{t+1}}{\alpha_1 + \alpha_2} \quad (8)$$

### 2.2.2 The Case that Parents Do Not Devote to Education of Their Children

In this case, there are two cases of provision of education to next generation, which is the same as section 2.2.1.

The Case that Individuals Provide Public Education to Next Generation

The utility of individual $i$ of generation $t$ during two periods $V_i^\bar{u}$ is determined as

$$V_i^\bar{u} = (1 - \alpha_1 - \alpha_2) \log n_i^u + \alpha_1 \log c_{i,t+1} + \alpha_2 \log E_{t+1}, \; \alpha_1, \alpha_2 \in (0, 1)$$

In this case, individuals can not determine both $n_i^u$ and $E_{t+1}$, and there is no inheritance, which is the same as section 2.2.1. Therefore, the utility-maximization problem does not occur. The consumption at time $t+1$, $c_{t+1}^\bar{u}$, is derived as
\[ c_{t+1} = (1 - \tau) (1 - n^n) \beta (e_{i,t})^\gamma (h_{i,t})^\delta \]

The Case that Individuals Provide Private Education to Next Generation

Individual \( i \) of generation \( t \) chooses \( c_{i,t+1} \) and \( e_{i,t+1} \) so as to maximize utility during two periods \( V^r \).

Maximize
\[ V^r = (1 - \alpha_1 - \alpha_2) \log n^r + \alpha_1 \log c_{i,t+1} + \alpha_2 \log e_{i,t+1} \]
subject to
\[ c_{i,t+1} = (1 - \tau) y_{i,t+1} - e_{i,t+1}, \quad y_{i,t+1} = h_{i,t+1} \]

The optimal consumption \( c_{t+1} \) and educational expenditure \( e_{t+1} \) at time \( t + 1 \) are derived as (9) and (10).

\[
\begin{align*}
\tilde{c}_{t+1} &= \frac{\alpha_1 (1 - \tau) y_{i,t+1}}{\alpha_1 + \alpha_2} = \frac{\alpha_1 (1 - \tau) h_{i,t+1}}{\alpha_1 + \alpha_2} \quad \text{(9)} \\
\tilde{e}_{t+1} &= \frac{\alpha_2 (1 - \tau) y_{i,t+1}}{\alpha_1 + \alpha_2} = \frac{\alpha_2 (1 - \tau) h_{i,t+1}}{\alpha_1 + \alpha_2} \quad \text{(10)}
\end{align*}
\]

3 Parental Sense of Values and Education Choice

The choice of public or private education is determined by their respective utilities. Education choice of an individual is determined by his parent.

3.1 The Case that Parents Devote to Education of Their Children

In this case, although individuals prefer leisure, it does not influence on education choice of next generation. Therefore, this case can be assumed as the one that parents devote to education of their children. In this case, the
choice of public or private education is determined by comparing $V^u$ and $V^r$. The level of human capital that satisfies $V^u = V^r$ at time $t + 1$, $h^*_{t+1}$, is derived by

$$h^*_{t+1} = \left( \frac{\alpha_1 + \alpha_2}{\alpha_1} \right)^{\frac{\alpha_1}{\alpha_2}} \frac{E^*_t \left( \alpha_1 + \alpha_2 \right)}{\alpha_2 (1 - \tau)}$$  \hspace{1cm} (11)$$

In equation (11), $E^*_t$ is the educational expenditure which satisfies $V^u = V^r$ at time $t + 1$ when public education is provided. Parents with human capital endowments $h_{i,t+1} \leq h^*_{t+1}$ prefer public education for their children at time $t + 1$, and parents with $h^*_{t+1} < h_{i,t+1}$ prefer private education. From (11), the human capital endowments $h^*_t$ that satisfies $V^u = V^r$ at time $t$ is derived as

$$h^*_t = \left( \frac{\alpha_1 + \alpha_2}{\alpha_1} \right)^{\frac{\alpha_1}{\alpha_2}} \frac{E^*_t \left( \alpha_1 + \alpha_2 \right)}{\alpha_2 (1 - \tau)}$$  \hspace{1cm} (12)$$

In equation (12), $E^*_t$ is the educational expenditure which satisfies $V^u = V^r$ at time $t$ when public education is provided. Parents with human capital endowments $h_{i,t} \leq h^*_t$ prefer public education for their children at time $t$, and parents with $h^*_t < h_{i,t}$ prefer private education. In this case, $P_t$ is defined as

$$P_t = \int_0^{h^*_t} f_t (h_{i,t}) \, dh_{i,t}$$  \hspace{1cm} (13)$$
3.2 The Case that Parents Do Not Devote to Education of Their Children

In this case, the leisure in public and private education, \( n^u \) and \( n^r \), influence on education choice of next generation. That is, this case can be assumed as the one that parents do not devote to education of their children. In this case, the choice of public or private education is determined by comparing \( V^\tilde{u} \) and \( V^\tilde{r} \). The level of human capital that satisfies \( V^\tilde{u} = V^\tilde{r} \) at time \( t + 1 \), \( h_{t+1}^* \), is derived by

\[
h_{t+1}^* = \left( \frac{n^u}{n^r} \right)^{\frac{1-\alpha_1}{\alpha_2}} \left( \frac{\alpha_1 + \alpha_2}{\alpha_1} \right)^{\frac{\alpha_1}{\alpha_2}} E_{t+1}^\tilde{u} \left( \alpha_1 + \alpha_2 \right) \frac{\alpha_1}{\alpha_2 (1 - \tau)} \tag{14}
\]

In equation (14), \( E_{t+1}^\tilde{u} \) is the educational expenditure which satisfies \( V^\tilde{u} = V^\tilde{r} \) at time \( t + 1 \) when public education is provided. Parents with human capital endowments \( h_{i,t+1} \leq h_{t+1}^* \) prefer public education for their children at time \( t + 1 \), and parents with \( h_{t+1}^* < h_{i,t+1} \) prefer private education. From equation (14), the human capital endowments \( h_t^\ast \) that satisfies \( V^\tilde{u} = V^\tilde{r} \) at time \( t \) is derived as

\[
h_t^\ast = \left( \frac{n^u}{n^r} \right)^{\frac{1-\alpha_1}{\alpha_2}} \left( \frac{\alpha_1 + \alpha_2}{\alpha_1} \right)^{\frac{\alpha_1}{\alpha_2}} E_t^\tilde{u} \left( \alpha_1 + \alpha_2 \right) \frac{\alpha_1}{\alpha_2 (1 - \tau)} \tag{15}
\]

In equation (15), \( E_t^\tilde{u} \) is the educational expenditure which satisfies \( V^\tilde{u} = V^\tilde{r} \) at time \( t \) when public education is provided. Parents with human capital endowments \( h_{i,t} \leq h_t^\ast \) prefer public education for their children at time \( t \), and parents with \( h_t^\ast < h_{i,t} \) prefer private education. In this case, \( P_t \) is defined as
\[ P_t = \int_0^{h_t} f_t(h_{i,t}) \, dh_{i,t} \quad (16) \]

This paper assumes that \( n^u > n^r \). Then we find that \( n^u/n^r > 1 \) and \( h^*_t > h^*_t \).

### 3.3 Human Capital Production Functions

Incorporating (2), (3) and (4) into (1), the production function for human capital when public education is provided is derived as

\[ h_{i,t+1} = h(n^u, h_{i,t}, E_t) = (1 - n^u)^\beta \left( \frac{\tau H_t}{P_t} \right)^\gamma (h_{i,t})^\delta \quad (17) \]

From equation (8) or (10), \( e_{i,t} \) is derives as

\[ e_{i,t} = \frac{\alpha_2 (1 - \tau) y_{i,t}}{\alpha_1 + \alpha_2} = \frac{\alpha_2 (1 - \tau) h_{i,t}}{\alpha_1 + \alpha_2} \quad (18) \]

Incorporating (2), (3) and (18) into (1), the production function for human capital when private education is provided is derived as

\[ h_{i,t+1} = h(n^r, h_{i,t}, e_{i,t}) = (1 - n^r)^\beta \left\{ \frac{\alpha_2 (1 - \tau)}{\alpha_1 + \alpha_2} \right\}^\gamma (h_{i,t})^{\gamma + \delta} \quad (19) \]

We assume that all individuals’ human capital endowments converge to steady state \( h^u_t \) when public education is provided, and they converge to steady state \( h^r_s \) when private education is provided. From (17) and (19), \( h^u_t \) and \( h^r_s \) are derived as follows:
We assume that $h_t^u < h_t^r$. From (20) and (21), $E_t = \frac{\tau H_t}{P_t}$ is determined to satisfy the following condition:

$$E_t = \frac{\tau H_t}{P_t} < \frac{(1 - n^r)^{\beta(1-\delta)}}{(1 - n^u)^{\gamma}} \left\{ \frac{\alpha_2 (1 - \tau)}{\alpha_1 + \alpha_2} \right\}^{\frac{1-\delta}{\gamma-\delta}}$$

(22)

Cardak’s (2004a) production function for human capital becomes linear when private education is provided, but here it becomes concave, similar to that shown in Cardak (2004b). Moreover, production functions for human capital intersect. From (20) and (21), human capital level which satisfies $h(n^u, h_{i,t}, E_t) = h(n^r, h_{i,t}, e_{i,t})$ at time $t$, $h_t^{**}$, is derived as

$$h_t^{**} = \left( \frac{1 - n^u}{1 - n^r} \right)^{\frac{\beta}{\gamma}} \left\{ \frac{\alpha_2 (1 - \tau)}{\alpha_1 + \alpha_2} \right\} \left( \frac{\tau H_t}{P_t} \right)$$

(23)

Figure 1 presents both human capital production functions.

From (12) and (15), when $h_t^*$ or $\tilde{h}_t^*$ rises (falls), $P_t$ increases (decreases). In this model, $1 - n$ is not influenced by $E_t$; it adversely (affirmatively) affects human capital accumulation under public education when $h_t^*$ or $\tilde{h}_t^*$ rises (falls). Human capital level converges to $h_t^u$ among individuals who choose public education and to $h_t^r$ among individuals who choose private education. When $P_t$ increases (decreases), the number of individuals enrolled in private schools decreases (increases). We assume $h_t^u < h_t^r$—i.e., when $h_t^*$ or $\tilde{h}_t^*$ rises (falls), the number who choose private education decreases (increases). It adversely
Fig. 1. Human Capital Production Functions

(affirmatively) affects nationwide accumulation of human capital. From equation (17), it has positive effect for human capital accumulation under public education that \( h^*_t \) and \( h^*_t \) become lower value. But, from equation (22), they have minimum values.

4 Public Education Policies

We now consider the public education policies that increase investment in public education, embodied in Public Education Policy 1, and the time span allocated to human capital accumulation when public education is provided, embodied in Public Education Policy 2. We assume that government rises the income tax rate \( \tau \) in Public Education Policy 1 and decreases the leisure in public education \( n^u \) in Public Education Policy 2.
4.1 Public Education Policy 1

In examining Public Education Policy 1, we assume that $\tau$ rises to $\tau' (> \tau)$, and $h^u_t$, $h^x_t$, and $h^r_t$ shift to $h'^u_t$, $h'^x_t$, and $h'^r_t$, respectively. From equation (12), (15) and (21), then

$$h^*_t = \left( \frac{\alpha_1 + \alpha_2}{\alpha_1} \right)^{\frac{\alpha_2}{2}} \frac{E^*_t (\alpha_1 + \alpha_2)}{\alpha_2 (1 - \tau)}$$

$$< \left( \frac{\alpha_1 + \alpha_2}{\alpha_1} \right)^{\frac{\alpha_2}{2}} \frac{E^*_t (\alpha_1 + \alpha_2)}{\alpha_2 (1 - \tau)} = h'^*_t \quad (24)$$

$$h^\sim_t = \left( \frac{n^u}{n^r} \right)^{1-\alpha_1-\alpha_2} \left( \frac{\alpha_1 + \alpha_2}{\alpha_1} \right)^{\frac{\alpha_1}{2}} \frac{E^\sim_t (\alpha_1 + \alpha_2)}{\alpha_2 (1 - \tau)}$$

$$< \left( \frac{n^u}{n^r} \right)^{1-\alpha_1-\alpha_2} \left( \frac{\alpha_1 + \alpha_2}{\alpha_1} \right)^{\frac{\alpha_1}{2}} \frac{E^\sim_t (\alpha_1 + \alpha_2)}{\alpha_2 (1 - \tau)} = h'^\sim_t \quad (25)$$

$$h^r_t = (1 - n^r)^{1-\beta - \gamma} \left\{ \frac{\alpha_2 (1 - \tau)}{\alpha_1 + \alpha_2} \right\}^{1-\gamma - \delta}$$

$$> (1 - n^r)^{1-\beta - \gamma} \left\{ \frac{\alpha_2 (1 - \tau)}{\alpha_1 + \alpha_2} \right\}^{1-\gamma - \delta} = h'^r_t \quad (26)$$

From equation (24), (25) and (26), $h^*_t$ and $h^\sim_t$ rises. Then, from equation (13) and (16), $P_t$ increase, which adversely affects all those who choose public education. This paper assumes that $n^u > n^r$ and $h^*_t > h^\sim_t$. Then, from equation (24) and (25), we find that $h'^*_t > h'^\sim_t$. On the other hand, from equation (26), $h^r_t$ declines, which adversely affects all those who choose private education.

Examining Public Policy 1, we find that Public Education Policy 1 has adverse effect on nationwide human capital in the case that parents devote to education of their children than the one that parents do not devote to education of their children, and three instances of $h^u_t$ shifting: $h (n^u, h_{i,t}, E_t)$ and $h (n^r, h_{i,t}, e_{i,t})$ shift to $h' (n^u, h_{i,t}, E_t)$ and $h' (n^r, h_{i,t}, e_{i,t})$, respectively.
4.1.1 When $h^u_t < h^v_t$

Figure 2 illustrates the shift of $h(n^u, h_{i,t}, E_t)$ to $h'(n^u, h_{i,t}, E_t)$ and that of $h(n^r, h_{i,t}, e_{i,t})$ to $h'(n^r, h_{i,t}, e_{i,t})$.

![Graph showing the shift in human capital](image)

Fig. 2. Public Education Policy 1($h^u_t < h^v_t$)

In this circumstance, more individuals choose public education, and the steady-state equilibrium of human capital under public education $h^u_t$ rises. However, fewer individuals choose private education and $h^r_t$ declines. There is no assurance that Public Policy 1 stimulates nationwide human capital accumulation in this case.

4.1.2 When $h^u_t = h^v_t$

Figure 3 illustrates the case in which $h(n^u, h_{i,t}, E_t)$ remains unchanged and $h(n^r, h_{i,t}, e_{i,t})$ shifts to $h'(n^r, h_{i,t}, e_{i,t})$.

In this circumstance, more individuals choose public education, but the steady-state equilibrium of human capital under public education $h^u_t$ does not
rise. Moreover, fewer individuals choose private education and \( h^r_s \) declines. Public Policy 1 constricts nationwide human capital accumulation in this case.

4.1.3 When \( h^u_t > h^{w}_t \)

Figure 4 illustrates the shifts of \( h(n^u, h_{i,t}, E_t) \) to \( h'(n^u, h_{i,t}, E_t) \) and that of \( h(n^r, h_{i,t}, e_{i,t}) \) to \( h'(n^r, h_{i,t}, e_{i,t}) \).

We see that more individuals choose public education, but the steady-state equilibrium of human capital under public education \( h^u_t \) declines. Fewer individuals choose private education and \( h^r_s \) also declines. In this circumstance, Public Policy 1 has a greater adverse effect on nationwide human capital accumulation than when in Section 4.1.2.
4.2 Public Education Policy 2

In examining Public Education Policy 2, we assume that $n^u$ decreases to $n^u( < n^u)$, and $h^*_t$ and $h^u_t$ shift to $h^*_t$ and $h^u_t$, respectively. From equation (15), then,

$$h^*_t = \left( \frac{n^u}{n^r} \right)^{1-\alpha_1-\alpha_2} \left( \frac{\alpha_1 + \alpha_2}{\alpha_1} \right)^{\alpha_1} \frac{E^*_t \left( \alpha_1 + \alpha_2 \right)}{\alpha_2 (1 - \tau)} \quad \text{or} \quad h^*_t = \left( \frac{n^u}{n^r} \right)^{1-\alpha_1-\alpha_2} \left( \frac{\alpha_1 + \alpha_2}{\alpha_1} \right)^{\alpha_1} \frac{E^*_t \left( \alpha_1 + \alpha_2 \right)}{\alpha_2 (1 - \tau)} = h^*_t \quad (27)$$

From equation (12), when $n^u$ changes, $h^*_t$ does not change. But, from equation (15), when $n^u$ decreases, $h^*_t$ also falls. Then, in the case that parents devote to education of their children, from equation (13), $P_t$ does not change. But in the case that parents do not devote to education of their children, from equation (16), $P_t$ decreases. Therefore, Public Education Policy 2 rises $h^u_t$ in both cases. Figure 5 illustrates the shift of $h \left( n^u, h_{i,t}, E_t \right)$ to $h' \left( n^u, h_{i,t}, E_t \right)$.
Examining Public Policy 2, we find that there are three instances that in which cases Public Education Policy 2 has more adverse effect on nationwide human capital accumulation.

**When** $n^u > n^r$

In this case, $h^* > h^r$ and we find that Public Education Policy 2 has adverse effect on nationwide human capital in the case that parents devote to education of their children than in the one that parents do not devote to education of their children. Moreover, it is desirable that the magnitude of $\alpha_1$ becomes smaller and the one of $\alpha_2$ becomes larger.

**When** $n^u = n^r$

In this case, $h^* = h^r$ and we find that the effect of Public Education Policy 2 on nationwide human capital is the same in both cases. Moreover, there is no influence by the magnitude of $\alpha_1$ and $\alpha_2$. 
When \( n^u < n^r \)

In this case, \( h^*_t < h^*_t' \) and we find that Public Education Policy 2 has adverse effect on nationwide human capital in the case that parents do not devote to education of their children than in the one that parents devote to education of their children. Moreover, it is desirable that the magnitude of \( \alpha_1 \) becomes larger and the one of \( \alpha_2 \) becomes smaller.

4.3 Public Education Policies and Private Education

In this model, private school is independent from government and time span to human capital accumulation when private education is provided, \( 1 - n^r \), is not influenced by public education policies. Therefore, it needs to examine the influences of the changes of \( n^r \) on the efficiency on public education policies in both cases. We assume that \( n^r \) decreases to \( n^r'(< n^r) \)

The case that parents devote to education of their children

From equation (12), in this case, when private school changes \( n^r \), \( h^*_t \) does not change. On the other hand, from equation (21), when \( n^r \) increases(decreases), \( h^*_s \) declines(inclines). We find the decrease of \( n^r \) stimulate nationwide human capital accumulation. Especially, it is desirable that \( h^*_s \) inclines to the level which is larger than \( h^*_s \) in Public Education Policy 1. Moreover, in Public Education Policy 1, \( n^u > n^c \) and \( n^u/n^r > 1 \). Then, it is desirable that the magnitude of \( \alpha_1 \) becomes smaller and the one of \( \alpha_2 \) becomes larger.

The case that parents do not devote to education of their children

From equation (15), in this case, when private school increases (decreases)
On the other hand, from equation (21), when \( n^r \) increases (decreases), \( h^r_s \) declines (inelines) which is the same in the case that parents devote to education of their children. Therefore, we find that the decrease of \( n^r \) stimulates nationwide human capital accumulation in Public Education Policy 2 when \( n^u < n^r \). Moreover, it is desirable that the magnitude of \( \alpha_1 \) becomes larger and the one of \( \alpha_2 \) becomes smaller.

5 Concluding Remarks

This study extended the analysis in Cardak (2004a) and suggested that human capital accrues through parental level of human capital and educational expenditure, but also include individuals’ time span allocated to human capital accumulation. In addition, this paper considers two cases that education choice is influenced by the leisure in public and private education or not which embodies the difference in parental sense of values. Moreover, we considered two public education policies—one that increases investment in public education and the other that increases the time span allocated to human capital accumulation when public education is provided. Four primary conclusions and contributions emerged.

(A) When the choice of public or private education is determined by comparing their respective utilities, we find that there is no assurance that the policy to increase investment in public education has a positive effect on nationwide human capital accumulation in case that education choice is not influenced by the leisure in public and private education.

(B) When the choice of public or private education is determined by com-
paring their respective utilities, we find that there is assurance that the policy to increase the time span allocated to human capital accumulation when public education is provided has a positive effect on nationwide human capital accumulation in the case that the decrease of leisure in public education is enough.

(C) When the choice of public or private education is determined by comparing their respective utilities, in the case that education choice is not influenced by the leisure in public and private education, it is desirable that the leisure of private education decreases with the policy to increase investment in public education.

(D) When the choice of public or private education is determined by comparing their respective utilities, in the case that education choice is influenced by the leisure in public and private education, it is desirable that the leisure in private education decreases with the policy to increase the time span allocated to human capital accumulation when public education is provided in the case that the leisure in private education is larger than the one in public education.

References


