Bidding Wars for Multi-Establishment Firms: An Auction-Theoretical Approach to the Inter-Regional Competition for Firms

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Abstract: We propose a model of competition between local governments to attract new investment. The main contribution of this paper is to consider firms as multi-establishment firms, an oversight in the current literature. The model shows that it is always in the firm’s best interest to split her investment in two asymmetric plants (or establishments).

Keywords: Fiscal competition, Auctions, Firm’s location choice.

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1 Introduction

In recent years, multimedia giant Apple, Inc. built (or is building) a number of new data centres to expand its cloud storage capacity. In at least three cases (North Carolina, Nevada, and Oregon), the company received large subsidies from local governments for choosing that location. Tax incentives such as these subsidies represent an appreciable amount of government spending each year. In the United States alone, state and local governments award approximately $80 billion in tax incentives each year to companies.¹ These subsidies are also often the result of bidding wars between many governments.

Owing to their importance, economists have investigated the reasoning of local governments participating in these bidding wars. However, they have generally considered a single firm opening a single establishment. The example above shows that when it’s a large firm, there is potentially more than one new plant up for grabs. In fact, the firms running these contests are frequently multinational, or at least multi-establishment companies. For example, between 2007 and 2012, Boeing received at least $327 million in incentives from 11 US states. In the same period, Procter & Gamble received at least $128 million from 10 states.²

Some may see these bidding wars as wasteful, but it can also play an important role in eliciting private information and improving allocation efficiency (Menezes, 2003). In fact, despite paying subsidies to the firm, the winning region may also benefit from the presence of the new plant. For example, Greenstone and Moretti (2003) compare the outcomes for winning and losing counties in contests for “million dollar plants”, and find that winning counties experience greater increases in land value as well as in the total wage bill of other firms in the industry of the new plant.

Many authors have previously highlighted the resemblance between this type of competition and open ascending auctions (e.g., King, McAfee and Welling, 1993; Menezes, 2003; Ferrett and Wooton, 2010a, 2010b). As suggested by Klemperer (2004, Chapter 2), these papers illustrate how auction theory can provide a rich set of tools to study location contests. Black and Hoyt (1989) and King, McAfee and Welling (1993) were among the first to explicitly apply auction models to the firms’ location choice problem. Later authors applied other models in auction theory to study specific aspects of that problem. For example, Haaparanta (1996) uses

²Other examples are available from the New York Times, at the following URL: http://www.nytimes.com/interactive/2012/12/01/us/government-incentives.html.
a menu auction model to analyse a firm setting up establishments in multiple regions, while Martin (2000) applies auctions with favouritism to study these contests when firms have explicit preferences for a region.

In this paper, we will analyse a situation in which the firm can decide to split her production in two different regions, instead of the usual single establishment of the previous literature. If the firm decides to invest in many establishments, then the location contest more closely resembles a multi-unit auction. We propose such a model in which the firm can invest in more than one establishment. Our model shows that the firm faces an important trade-off. By differentiating her production sites, the firm may be able to gain larger subsidies from the governments in competition. However, doing so may reduce her operating profits.

2 Related Literature

Social scientists have been exploring the location decision of firms (or more generally production) at least since Marshall (1890). Our goal here is not to review the whole literature on the subject before or since then. Instead, we focus on models of firm location that specifically include some bargaining game between governments and firms. In other words, we focus on the fiscal competition literature involving discussions between governments and a specific firm.

2.1 Bargaining Games between Firms and Governments

Doyle and van Wijnbergen (1994), in a paper first published in 1984, raise the issue that economists until then had ignored the issue of “the bargaining game that may arise between a multinational and the host country for any given double-taxation agreement.” They build a dynamic model in which the government and a multinational bargain over the tax rate. In their solution, the host government initially sets a low tax rate, but gradually increases it until it reaches a limit. The government has some bargaining power due to the fact that the multinational must incur a positive cost if it relocates to a new location. Oechssler (1994) considers a similar model, but with a firm already installed in one location and threatening to move if taxes are not lowered.

Doyle and van Wijnbergen (1994) assumed that firms negotiate with a single government at a time. However, firms have no reason not to negotiate simultaneously with multiple governments. Recognising this fact, Bond and Samuelson (1986) investigate a situation in which a firm has to
decide between two locations. In their model, tax holidays are used as a signal of productivity by the governments. An important feature of their model is information asymmetry. It allows for the presence of tax holidays even if there are no fixed costs, in opposition to Doyle and Van Wijnbergen (1984).

Black and Hoyt (1989) were, to our knowledge, the first to explicitly model the firm’s location choice as an auction. They also highlight the fact that this competition need not be a zero-sum game; the bids offered by government can promote the efficient location of production. In their model, they also consider that smaller, already established firms may move once a new firm is opened in one of the regions. Indeed, small firms will relocate to the winning region, thus increasing its potential gains. This multiplier effect can explain why regions may seem to “overbid” for the large firm.

Black and Hoyt (1989) had highlighted some caveats to their analysis. One caveat was the lack of dynamic considerations. King and Welling (1992) explore the consequences of allowing the firm to relocate in later periods. They consider a two-period model, in which the firm conducts an auction to decide on its location in each period. They find that when players cannot commit to second-period actions, the firm can re-locate to the region that lost in the first period. This possibility modifies the first-period bids, thus changing the outcome even if the relocation threat is not materialised. The authors also show that the firm would prefer a world with commitment, but that without commitment, total social welfare is higher.

King, McAfee and Welling (1993) generalise the model of King and Welling (1992), but with a continuum of local productivities. They also consider an extension in which regions can invest in infrastructure in a previous stage, thus increasing their productivity potential. They find that in equilibrium, regions tend to choose different levels of infrastructure, thus endogenously creating the productivity continuum described in their main model.

Taylor (1992) also investigated the role of infrastructure in the competition for investment. He built a model in which jurisdictions compete for new firms by investing in new infrastructure, in a way that resembles an all-pay auction. He finds that the investment race can potentially be wasteful, and that regions with a lower initial stock of infrastructure may be less willing to enter the competition. In this situation, infrastructure inequality between regions could rise over time.

Realising the vast array of auction models available to researchers, Menezes (2003) describes the basic competition for investment under several auction mechanisms, and shows that the
expected amount paid to the firm is the same. He also describes how to implement the \textit{ex ante} optimal mechanism. However, his results rely on the traditional assumptions in auction theory that may not be satisfied in most fiscal competition models. Furusawa, Hori and Wooton (2010), for example, show that English auctions lead to more aggressive bidding, or to a “race beyond the bottom,” compared to bidding in sealed bid auctions.

Some authors have recently studied the question of tax incentives in relation with other aspects regarding the firm and its production. For example, Ferrett and Wooton (2010a) analyse the question of firm ownership. Intuition may have us believe that regional governments would offer higher subsidies to firms owned in part by shareholders residing locally. However, their model shows that the tax or subsidy offers are independent of the ownership structure of the firm. In another paper, the same authors (Ferrett and Wooton, 2010b) show that if the regions are competing for a duopoly, the two firms will locate in separate countries. Indeed, with trade costs, both firms can enjoy monopoly power in serving the local market. Regional governments can take advantage of this situation, and set positive tax rates.

2.2 Other Factors Affecting the Location of Production

We mentioned above that firms take factors other than government subsidies into account when deciding on a location. These factors, however, may also interact with these subsidies. One of these factors is specific preferences for a location. For example, the CEO may enjoy the weather in a city, and wishes to build new headquarters there. This type of preference may be taken into account in the auction process. Martin (2000) builds a model that analyses this question. He uses an asymmetric auction model, in which the firm handicaps the regions that are not her favourite. Equivalently, she adds a “bonus” to bid of her favourite region. In other words, the favourite region can win and pay a bid that is not the largest. In his model, the favourite region is the one with lower costs. Martin (2000) notes that the firm can fully handicap the regions (i.e. by the full amount of the cost difference) or only partially. By partially handicapping some regions, the firm can induce higher bids from the favourite region, but runs the risk of letting the high-cost region win the auction. She thus faces a trade-off between higher subsidies (bids) and lower production costs.

Another factor that can influence the firm is the size of the region, or more explicitly the size of the internal market. Producing close to a larger market reduces transportation costs for the firm. Haufler and Wooton (1999) consider such costs explicitly in a tax competition model.
They do not take tax breaks as such into account, instead analysing competition on tax rates. However, they do find that large countries are advantaged when trying to lure companies to their territory. Indeed, they are able to set a tax rate higher than smaller countries while still attracting the investment. In other words, the race to the bottom often invoked in the fiscal competition literature does not take place.

Economists have long been interested by such geographical concepts. The New Economic Geography literature, for example, explains the clustering and dispersion of firms by highlighting the trade-off between agglomeration economies (e.g., technological spillovers from one firm to another) and transportation costs. Venables (2005) offers a good summary of that literature. Besley and Seabright (1999) are among the few that consider a multi-firm model. They use a menu auction model to study the dynamic implications of a second firm choosing to locate in the same set of countries in a second period as the first firm did in the first period. They find that lack of commitment from the governments lead to inefficient outcomes.

2.3 Multi-Establishment Firms

Although we do not dwell on the geographical aspects of production in our model, we do consider firms that are located in multiple locations. In fact, it is the main contribution of this paper. Multi-establishment firms, although prevalent around the world and studied extensively by economists and other social scientists, have made only limited appearances in the tax competition literature, especially in the subset of papers interested in tax breaks. Haaparanta (1996) is an exception. This author considers a firm wishing to make a divisible investment in potentially many countries. In his paper, he uses a menu auction model in which every country submits a full schedule of subsidies to the firm. He finds that even high-wage countries are able to attract investment, even if all other countries also use subsidies. Our motivation is similar to Haaparanta’s: firms produce in many locations.

My analysis differs from his in two aspects. First, the objective of my analysis is to determine whether the subsidy game itself can affect the firm’s location decision. In short, I ask whether the auction will modify the allocation of investment compared to a case without subsidies. Haaparanta instead aims to analyze marginal subsidies and investment, and how they are affected by differences in wages. In my model, the regions are identical from the point of view of the firm. I will show that even under this assumption, the firm may wish to have establishments of different sizes. Second, Haaparanta’s analysis is in complete information, a
feature of menu auction models, while this chapter considers a model in incomplete information. This assumption captures the fact that the firm does not know which region values its presence the highest. Indeed, this lack of information is a justification to use a mechanism similar to an auction in the first place. As the model will show formally, analyzing the question under an open auction instead of a menu auction will reveal new channels through which the subsidy game affects the allocation of investment. In particular, when establishments are asymmetric, infra-marginal competition takes place between the last two remaining bidders, allowing the firm to benefit from higher subsidies (but at the cost of lower profits).

Janeba (2000) also studied the reasons for firms to split in multiple establishments. He argues that one reason for firms to produce in more than one location is to avoid having too high a share of its profits captured by the local governments of the only region in which it produces (an argument similar to the hold-up problem). By splitting production, the firm can install excess capacity everywhere, and credibly threaten to move production elsewhere when the government tries to raise the tax rate. The present paper will show that splitting the production can also enable the firm to extract larger subsidies and tax breaks from local governments.

2.4 Empirical Contributions

So far, the contributions we have discussed in this section have been mostly theoretical in nature. Some authors, however, have estimated econometric models to investigate which factors actually influence the firms’ choice. Wheeler and Mody (1992), for example, use data on FDI by US firms, and find that the most important factors are infrastructure quality, labour costs, market size, and industrialisation. In their analysis, corporate taxation does not influence firms.

Head, Ries and Swenson (1999) provide a more direct analysis of subsidies to attract investment. They consider Japanese FDI in the United States between 1980 and 1992, and conclude that foreign trade zones, lower taxes as well as job-creating subsidies all influence the location of these firms. They also highlight the importance agglomeration effects: if a region is successful in attracting firms early, it may be easier to attract firms later. By using solely Japanese firms, the authors are able to isolate the agglomeration economies enjoyed by these firms. Indeed, Japanese firms often have specific production methods such as “just in time” production, so that they benefit greatly from being surrounded by other Japanese firms. Greenstone, Hornbeck and Moretti (2010) confirm that agglomeration economies exist. Incumbent plants in US
counties that win a large investment following a location contest benefit from an increase in total factor productivity relative to plants in losing counties.

Devereux, Griffith and Simpson (2003) consider, for their part, the interactions between agglomeration effects and fiscal incentives. They consider data from firms applying to a system of grants in the United Kingdom aimed at regional assistance. They find that firms prefer to set up new plants close to other firms in the same industry. However, they find that grants only have a small effect on the location choice. Brühlhart, Jametti and Schmidheiny (2008) come to a similar conclusion. Using data from startups in Swiss municipalities, they find that high taxes deter the entry of firms, but that this effect is smaller in sectors that are more concentrated.

3 The Model

The actors in this model are the firm and the $n$ regional governments, indexed by $i \in 1, \ldots, n$. The firm wants to build new production facilities. To decide the location of these plants, the firm puts the regional governments in competition against each other. The governments submit offers of subsidies to attract the firm to their territory. In contrast to most of the previous literature, however, the firm can divide her production in multiple locations, either in symmetric or asymmetric establishments. For simplicity and tractability, we limit the model to the case of two establishments, indexed by $j \in 1, 2$. Without loss of generality, we label the largest plant by $j = 1$, so that $K_1 \geq K_2$.

The Firm

In each establishment, the firm produces according to the production function $f(K_j, L_j)$, with $K_j$ the capital invested in location $j$, and $L_j$ the labour employed in that establishment. We make the usual assumptions that the production function exhibits decreasing returns to scale in both inputs ($\frac{\partial f(K_j, L_j)}{\partial K_j} > 0$, $\frac{\partial f(K_j, L_j)}{\partial L_j} > 0$ and $\frac{\partial^2 f(K_j, L_j)}{\partial K^2_j} < 0$, $\frac{\partial^2 f(K_j, L_j)}{\partial L^2_j} < 0$). The firm acts as a price-taker, and production costs ($w, r$) and prices ($p$) are the same in every region. Therefore, the firm’s operating profits in each establishment are equal to

$$\pi_j = pf(K_j, L_j) - wL_j - rK_j$$

(3.1)
In addition to the profits from production, the firm also receives subsidies from the regions, so that her total ex-post profits are equal to:

$$\Pi = s_1^* + s_2^* + \Pi_1 + \Pi_2$$  \hspace{1cm} (3.2)

where $s_j^*$ is the equilibrium subsidy for establishment $j$.

**The Regions**

These subsidies depend on the regions’ valuation of the firm’s investments. In particular, if regional government $i$ wins establishment $j$, it receives a payoff equal to

$$V_{ij} = L_j \cdot b_i - s_{ij}$$  \hspace{1cm} (3.3)

where $L_j$ is the number of persons employed by the firm in establishment $j$, $b_i$ is the level of private benefits from hosting the firm for region $i$’s government, and $s_{ij}$ is the subsidy (bid) offered to the firm by region $i$ when winning establishment $j$. The subsidy can be interpreted as a total “fiscal package” offered to the firm.\(^3\)

A region’s private benefits $b_i$ are private knowledge, and they capture, for example, an increase in labour taxation from workers who will be employed by the firm, as well as spillovers to domestic firms, but also the compatibility of the firm for the region. Indeed, if the industry of the firm has a bad reputation in one region, the regional government would put only a small value on the firm’s investment (due to, for example, re-election concerns).\(^4\) The private benefits are identically and independently distributed according to a distribution $g(\cdot)$ on some interval $[\underline{b}, \bar{b}]$ (with $\bar{b} \geq 0$).

**The Auction Process**

The equilibrium subsidies are then determined by an auction in which the firm takes the role of the auctioneer, and the regional governments submit their bids to host the firm’s plants. Since there are two establishments available, the firm conducts a multi-unit auction, with both establishments available simultaneously.

\(^3\)In effect, our model assumes that all regional governments have the same basic tax rate, but differentiate themselves with targeted tax holidays that may differ. This assumption may not be unreasonable in the case of sub-national jurisdictions. Even when considering countries, we are mostly interested in the competition taking place in subsidies, and abstracting from tax competition allows us to focus on our variables of interest.

\(^4\)Martin (2000:6) provides a more thorough list of potential explanation for these benefits.
The formal mechanism is an open ascending auction. More specifically, the firm runs an ascending clock, representing the current price for the lowest-value establishment still available (the one with the lowest investment). Regional governments still in the running are ready to offer a bid equal to the current price. The winning bid is determined from the price on the clock when the previous bidder dropped from the auction. In particular, if the two establishments are still available, then when there are only two regions left bidding, the price for the lowest-valued establishment will be determined from the clock price at which the third-to-last region exit the auction. These two remaining regions will then continue bidding until one of them drops. The clock price at which the second-to-last region dropped will be the price for the highest-valued establishment. Formally, this mechanism is thus a type of second-price auction.

**Timing**

We can summarize the timing of the whole game as follows.

**Stage 0:** Nature picks the set of \( \{b_i\}_{i=1,...,n} \). Regional governments learn their \( b_i \).

**Stage 1:** The firm chooses an allocation of capital \( (K_1, K_2) \), anticipating the subsidies offered by governments resulting from the auction in Stage 2, and the firm’s own profit maximization in the last stage.

**Stage 2:** The multi-unit auction takes place. Winning regions offer \( s_1^* \) and \( s_2^* \), based on their expectation of the labor that will be employed by the firm (from profit maximization in the last stage).

**Stage 3:** The firm invests capital \( K_1 \) and \( K_2 \), as determined in Stage 1, in the winning regions. She then maximizes her profits, taking capital fixed, choosing \( L_1 \) and \( L_2 \).

In the first stage, the firm commits to a certain allocation of capital. One could reasonably argue that the firm has incentives to deviate from that allocation once she receives the subsidies from the region. However, in that case, regions would anticipate these deviations and bid accordingly. To facilitate the analysis, we make the assumption that the firm can credibly commit to her allocation.

We will solve this game by backwards induction.
4 Stage 3: Production

We first solve the last stage of the game, to find the firm’s optimal labour input demand in each firm for each level of capital invested. At this stage of the game, the firm already knows the identity of the winning regions, and invests the capital in these two regions as determined in the first stage. She also knows how the amount of the subsidies conditional on the amount of labour she will employ.

The firm thus maximizes her profits in each plant, choosing $L$ and considering $K$ as fixed. Her maximization problem in each plant is as follows.

$$\max_{L_j} \quad pf(K^*_j, L_j) - wL_j - rK^*_j$$

(4.1)

The first-order condition is

$$pf'(K^*_j, L_j) - w = 0$$

implying that the firm chooses $L_j$ to equalize the marginal product that input, $f'(K^*_j, L_j)$, with the ratio of $w$ and $p$. Therefore, the optimal quantity of $L_j$ will depend on the amount of capital invested, $K^*_j$. We define the function $\hat{L}_j(K_j)$, determining the amount of labour employed for each possible equilibrium level of capital invested in the first stage.

By totally differentiating the first-order condition, we can obtain the sign of $\frac{dL}{dK}$:

$$\frac{dL}{dK} \leq - \frac{\partial^2 f(K^*, L^*)}{\partial K^2} \frac{\partial^2 f(K^*, L^*)}{\partial K \partial L} > 0$$

This derivative is greater than zero as long as the cross partial derivatives in $K$ and $L$ are positive (e.g., increasing capital increases the marginal product of labour).

As an example, take a simple Cobb-Douglas production function $f(K, L) = K^\alpha L^\beta$ with $\alpha + \beta < 1$. At that stage, $K$ is fixed in each establishment and the firm already received the subsidies. Therefore, the firm chooses $L$ in each plant to maximise her operating profits. In that case, for each level of $K$, she chooses an optimal amount of labour $L$ equal to

$$L(K) = \left(\frac{p^\beta}{w}\right)^{\frac{1}{1-\beta}} K^{\alpha/(1-\beta)}$$

In this example, larger investments by the firm translate in more labour employed ($L'(K) > 0$), but at a decreasing rate ($L''(K) < 0$).
5 Stage 2: Auction and Equilibrium Subsidies

In the auction stage, the firm puts up two plants for sale of sizes $K_1$ and $K_2$. The regional governments expect the firm to employ $L(K_1)$ and $L(K_2)$, respectively, and bid according to their valuation functions $V_{ij}$. The following lemma describes the equilibrium subsidies resulting from the auction.

**Lemma 1.** The equilibrium bids for the two establishments will be equal to

\begin{align}
 s^*_2(K_2) &= L^*(K_2) \cdot b_{(3)} \\
 s^*_1(K_1, K_2) &= (L^*(K_1) - L^*(K_2))b_{(2)} + L^*(K_2)b_{(3)}
\end{align}

where $b_{(z)}$ is the $z^{th}$-highest signal among the $n$ regions.

**Proof.** To see why these two bids are optimal, take a region $i$ with private benefits $b_i$ and assume that everyone else bids according to the following strategy: continue bidding until the clock reaches my private valuation. In that case, if the clock reaches $L_2b_i$ and there are still 3 or more regions in the auction, then region $i$ has no incentive to continue bidding. Indeed, if she does, whatever the stop price, she will need to pay more than her valuation if she wins. Therefore, at price $L_2b_i$, she prefers to leave the auction. Now consider prices lower than $L_2b_i$, for example $L_2b_l$. At that clock price, region $i$ has a positive valuation and would like to win. Therefore, she has no incentive to leave the auction. Therefore, the equilibrium bid for the small establishment will be equal to

\[ s^*_2(K_2) = L(K_2) \cdot b_{(3)} \]

where $b_{(3)}$ is the third-highest signal among the $n$ regions.

If the two plants are of symmetric sizes (i.e., $K_1 = K_2$), then the two remaining regions each pay $s^*_2(K_2)$ and each receive the same investment.

However, if the two plants are asymmetric (i.e., $K_1 \neq K_2$), we still have to determine which region receives the largest investment. Both regions know that their possibilities are now to pay $s^*_2(K_2)$ and receive the small establishment, or to pay more and receive the large establishment. The bid for the largest establishment will thus be determined by the infra-marginal competition between the two remaining bidders. Since at that point, the auction becomes a simple second-price auction between two bidders, it is optimal for both regions to simply withdraw once the
clock price reaches their valuation of the large plant. If they continue past that price, they either win and pay a price higher than their valuation, or they lose and pay the price for the second establishment, which was already determined.

Take the decision problem of the region with the second-highest private benefits.\textsuperscript{5} It will be indifferent between the two establishments when

\[ L(K_1)b(2) - s_1^*(K_1, K_2) = L(K_2)b(2) - s_2^*(K_2) \]

By rearranging this equation and substituting the value of \( s_2^*(K_2) \) found earlier, we obtain the value of the highest bid

\[ s_1^*(K_1, K_2) = (L(K_1) - L(K_2))b(2) + L(K_2)b(3) \]

Note that, as expected, if \( K_1 = K_2 \), this equation is equal to \( s_2^*(K_2) \).

In the more interesting case of asymmetric investments, therefore, the two last remaining regions continue to compete for the large establishment. We see, from equation 2.6, that an increase in \( K_1 \) for a given value of \( K_2 \) raises, through the infra-marginal competition, the subsidy offered for the most valuable establishment. A reduction in \( K_2 \) has a similar effect, while also reducing the bid received for the small investment. Cowie \textit{et al.} (2007) previously considered infra-marginal competition in the context of an auction. They analyse how a seller can divide the units for sale in multiple lots in order to receive higher offers from the bidders. They find that differentiating the lots can lead to higher bids due to the infra-marginal competition for the largest lot. We have a similar reasoning in our paper.

6 The Firm’s Optimal Location Choice

In the first stage, the firm’s optimisation problem is the following:

\[ \max_{K_1, K_2} E(s_1^* + s_2^* + \Pi_1 + \Pi_2) \quad (6.1) \]

where \( \Pi_j = pf(K_j, L_j) - wL_j - rK_j \) and \( s_j^* \) are, respectively, the operating profits in each establishment and the equilibrium subsidies as determined in Lemma 2.1. The firm thus chooses

\textsuperscript{5}Given the monotonicity of the valuation function of the regions, for any level of private benefits, regions prefer the largest establishment to the small one.
$K_1$ and $K_2$ to maximise her total expected revenues, anticipating the bids of the regions, as well as her profit maximisation in the last period. The solution to this optimisation problem leads to the following proposition.

**Proposition 1.** When the firm allocates her production units through a multi-unit auction, she always chooses to differentiate the two establishments. In mathematical terms, $K_1 > K_2$, with strict inequality.

**Proof.** The firm does not know the private benefits of the regions in the competition, but knows that they are distributed according to $g(\cdot)$ on the interval $[b, \overline{b}]$. Her objective function can thus be expressed as

$$E(R) = \int_b^{\overline{b}} \int_b^{b(2)} \left[ (L(K_1) - L(K_2))b(2) + 2L(K_2)b(3) 
+ pf(K_1, L(K_1)) - wL(K_1) - rK_1 + pf(K_2, L(K_2)) - wL(K_2) - rK_2 \right] \cdot h(b(2), b(3), n) db(3) db(2) \quad (6.2)$$

where the last part $h(b(2), b(3), n) = n(n-1)(n-2) \cdot \left[ 1 - G(b(2)) \right] \left[ G(b(3)) \right]^{n-3} g(b(2))g(b(3))$ is the joint distribution of $b(2)$ and $b(3)$. We obtain the following first-order conditions:

$$\frac{\partial E(R)}{\partial K_1} = L'(K_1)E(b(2)) + \left[ pf(K_1, L(K_1)) + \frac{\partial f(K_1, L(K_1))}{\partial L(K_1)} \cdot L'(K_1) \right] - wL'(K_1) - r = 0 \quad (6.3)$$

$$\frac{\partial E(R)}{\partial K_2} = -L'(K_2)E(b(2)) + 2L'(K_2)E(b(3)) + \left[ pf(K_2, L(K_2)) + \frac{\partial f(K_2, L(K_2))}{\partial L(K_2)} \cdot L'(K_2) \right] - wL'(K_2) - r = 0 \quad (6.4)$$

Since $L(K)$ represents equilibrium values, the FOCs can be simplified using the Envelope Theorem. We then obtain:

$$\frac{\partial E(R)}{\partial K_1} = L'(K_1)E(b(2)) + \frac{pf(K_1, L(K_1))}{\partial K_1} - wL'(K_1) - r = 0 \quad (6.5)$$

$$\frac{\partial E(R)}{\partial K_2} = -L'(K_2)E(b(2)) + 2L'(K_2)E(b(3)) + \frac{\partial f(K_2, L(K_2))}{\partial K_2} - wL'(K_2) - r = 0 \quad (6.6)$$
Combining the two FOCs, we see that

\[
p \left( \frac{\partial f(K_2, L(K_2))}{\partial K_2} - \frac{\partial f(K_1, L(K_1))}{\partial K_1} \right) = L'(K_2) \left( w + E(b(2)) - 2E(b(3)) \right) - L'(K_1) \left( w + E(b(2)) \right)
\]

We want to show that \( K_1 \neq K_2 \). Let’s first assume that \( E(b(2)) \neq E(b(3)) \) (i.e., we focus on the interesting cases where the firm expects regions to have different valuations). To prove that the firm has to optimally split in asymmetric establishments, we first assume that she does not, and show that it leads to an inconsistency. Indeed, if \( K_1 = K_2 = K \), the previous equation reduces to

\[
0 = 2L'(K) \left( E(b(2)) - E(b(3)) \right)
\]

Since the regions have different expected private benefits, this equation is true only if \( L'(K) = 0 \). However, that derivative is always positive. Therefore, we can conclude that \( K_1 \neq K_2 \).

\[
\square
\]

Note that we can rearrange the first-order conditions as such:

\[
p \frac{\partial f(K_1, L(K_1))}{\partial K_1} = L'(K_1)(w - E(b(2))) + r
\]

\[
p \frac{\partial f(K_2, L(K_2))}{\partial K_2} = L'(K_2)(w + E(b(2)) - 2E(b(3))) + r
\]

This formulation is informative of the trade-offs at play. In each establishment, the firm chooses a level of capital such that marginal revenues are equal to marginal costs. However, the bidding war modifies the marginal costs. Indeed, we can see that increasing capital increases the amount of labour employed (through \( L'(K) \)), each unit costing not \( w \), but salaries adjusted by the private benefits on the regions.

For comparison purposes, without a bidding war, the firm’s revenues are simply equal to \( \Pi_1(K_1) + \Pi_2(K_2) \). The first-order conditions imply that

\[
\frac{\partial \Pi_1}{\partial K_1} = p \cdot \frac{\partial f(K_1, L(K_1))}{\partial K_1} - wL'(K_1) - r = 0
\]

\[
\frac{\partial \Pi_2}{\partial K_2} = p \cdot \frac{\partial f(K_2, L(K_2))}{\partial K_2} - wL'(K_2) - r = 0
\]

Put differently, the firm’s optimal allocation in this case simply results from equating marginal revenues and marginal costs in each establishment. Since the firm has no information about the private benefits of the regions, and since regions are identical in terms of productive capacity,
the firm chooses to invest an equal amount in two regions. She can just choose two regions at random, since her production costs and revenues will be identical with any set of two regions.

Remark 1. Without a bidding war, the firm chooses to invest an equal amount of capital in two random regions.

6.1 A Numerical Illustration

To illustrate, let’s continue with the simple Cobbs-Douglas production function introduced in a previous section: \( f(K, L) = K^\alpha L^\beta \), with \( \alpha + \beta < 1 \). Assume also that the private benefits are distributed according to a uniform distribution on \([0, 1]\). \( L(K) \) is given by the previous equation. With specific functional forms, we can find the optimal investment allocation given a set of parameters \( \{\alpha, \beta, p, w, r, n\} \). The analytical solutions are omitted here, as they are not informative. Instead, we describe graphically how the firm behaves facing different conditions. Note that in Haaparanta’s (1986) model, a Cobbs-Douglas production function implied that investment allocations were identical with and without subsidies. In our model, we find the opposite result.

One interesting question is whether the number of regions in the bidding war affects the firm’s investment choices. As seen in the more general model, when the firm expects the regions to have relatively similar private benefits of hosting the firm, she wants to reduce the differentiation between her two establishments. If the distribution of private benefits is uniform, then, a low number of regions (e.g., 3 regions) will translate in a large difference between the expected private benefits of the regions, while a larger number of regions will translate in lower differences. For that reason, we should see decreasing differentiation with an increasing number of competitors. Figure 1 illustrates this relationship for specific values of the parameters.

This figure raises another interesting question: does the bidding war lead to an increase in total investment and production? We can see that total investment increases as \( n \) increases, in this specific case. However, it would be interesting to compare \( K_1 + K_2 \) in the auction and no-auction cases.

Without an auction, the firm’s problem is simplified to the extreme. She wants to build new facilities, and does so without running a bidding war. Since all regions have identical productive characteristics in our problem, the firm simply chooses one or two regions randomly, and allocates \( K_1 \) and \( K_2 \) to maximize profits. We saw before that in this case, she allocates identical production to two randomly chosen locations (given the assumptions on the technology). Is that
value of investment larger than $K_2$ with a bidding war? We do not provide a general proof, but at least in some cases, the firm actually achieves differentiation only by increasing production in one establishment, and not by decreasing production in the other one. The following figure shows such a case, and how the allocations depend on $n$. 

Figure 1: $K_1$ vs $K_2$

Figure 2: $K_1$ vs $K_2$
7 Welfare

The region

Given the results above, one may wonder if it’s in the regions’ best interests that such a bidding war takes place. Without a bidding war, region \( i \) has the following expected utility:

\[
\frac{2}{n} L(\bar{K}) \cdot b_i \tag{7.1}
\]

where \( \bar{K} \) is the investment from the firm in one establishment, without a bidding war.

With a bidding war, the same region has the following expected utility

\[
\int_{b_i}^{b_i(1)} \int_{b_i}^{b_i(2)} (L(K_i^*)b_i - s_i^*)h(b_{(1),-i}, b_{(2),-i}, n-1)db_{(2),-i}db_{(1)}
+ \int_{b_i}^{\bar{b}_i} \int_{b_i}^{b_i(1)} (L(K_2^*)b_i - s_i^*)h(b_{(1),-i}, b_{(2),-i}, n-1)db_{(2),-i}db_{(1),-i} \tag{7.2}
\]

where for region \( i \), \( db_{(k),-i} \) is the \( k \)-th highest benefit among the \( n-1 \) other regions.

With a Cobb-Douglas production function of parameters \( \alpha = \beta = 1/3 \), a uniform distribution of benefits on \([0, 1]\), and \( p = w = r = 1 \), to illustrate, we find that a given region prefers that the firm uses a bidding war as long as

\[ b_i > 0.227 \]

In other words, regions with higher benefits stand to gain from participating in a bidding war.

A more complete characterization of the conditions for these preferences will be provided later.

**Remark 2.** In some cases, regions have ex ante preferences for a bidding war instead of a random allocation.

The Social Planner

Another interesting question to ask is whether it is beneficial, from the point of view of society as a whole (e.g., the federal government), to allow these bidding wars.

If a social planner knew the regions’ private benefits and had to decide on the allocation of capital, it would allocate capital to the two regions that value it the most. However, the social planner would also allocate identical amounts of capital, to maximize the operating profits of
the firm. This outcome is similar to the one of the bidding war: the regions that value the investment the most get it. However, the firm clearly gets lower total profits, since she receives no subsidy. In exchange, she maximizes her operating profits.

In the case that the social planner does not observe the private benefits of the regions, the implications in terms of welfare are more interesting. Indeed, in that case, it is uncertain that the regions that value the investment the most get it.

8 Conclusion

This paper proposed a model in which a firm wishes to install new production facilities and puts regional governments in competition against each other to decide the location of those facilities. Regional governments submit bids, in the form of tax holidays or other financial packages, and the firm invests in the winning region(s). In contrast to previous models, the firm can split her production in two establishments. This split introduces new trade-offs for the firm, and modifies the bidding behaviour of the regional governments. Indeed, the firm may want to differentiate her plants to take advantage of infra-marginal competition. We find that the firm indeed optimally chooses to differentiate her two investments.
References


