Spatial Advertisement in Political Campaigns

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Abstract

The paper analyzes strategic disclosure of hard information in the context of a stylized voting model. Two parties, fully characterized by the platform they intend to implement, choose what to reveal about their chosen platform to voters located in several communities. In every community, parties can conceal their platform (by disclosing a subset of possible platforms), but not falsify it (the true platform always belongs to the disclosed set). First, we consider a scenario in which communities are segmented, that is, each voter belongs to a single community. We characterize the equilibrium level of information disclosure in every community and relate it to the distribution of voters’ preferences. We then study how strategic disclosure changes when communities are not segmented and voters may belong to more than one community. Finally, we analyze a platform-setting game, identify circumstances in which party platforms diverge, and relate this to the empirical finding that more ideologically homogenous communities have emerged as polarization has been increasing.

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1 Introduction

It is common for politicians to not exactly reveal the positions they stand for but aim to cloud their goals and positions. This has already been noted by Sartori:¹

> Ordinary political language is insincere language, for it is less an instrument for expressing thought than a means ‘for concealing or preventing thought.’ Actually, except in the case of some intellectuals, political discourse is more often than not either a device for achieving or maintaining power or a device for stimulation action....I do not wish to sound cynical. I am only trying to point out the special difficulties that a student of politics is bound to encounter. For he has to work in a field where is may be useful to becloud problems, and definitely not useful to clarify them. Indeed, an important part of politics may be described as the art of confusing political issues (and this is not necessarily the wicked part).²

In particular, politicians are selective about which issues they provide information and they adjust their focus to the characteristics of their specific audience. In this spirit, Barack Obama’s presidential campaign in 2012 targeted single women by emphasizing that he is pro-choice and that contraception should be covered by health insurance. Additionally, a higher minimum wage was suggested by Obama’s campaign which benefits in particular unmarried women.³ This highlights how women were targeted, a group that is ideologically close to Democrats, and how the campaign tried to cater to their specific interests.

However, it cannot be expected that the audience will be the only relevant group that hears about the politician’s position. Rather, there will be spill overs and other voters will learn from the information that has been disseminated among a certain target audience. It has been argued that exactly these spillovers cost Mitt Romney the win in the 2012 election. Voters felt that he sold himself as a conservative during the primaries, but after that implied that he was actually moderate. This resulted in voters getting the impression that he was too vague and suppressed turnout on his behalf.⁴

But how do politicians decide what policies to advertise? How does this depend on the characteristics of the target group? What happens if there are other groups that can observe the revealed information?

We address these questions in a model of strategic disclosure of hard information on a network. We consider two parties that first choose their platform, which lies in a unidimensional

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¹This quote is used in Shepsle (1972), who then proceeds to show that with risk-averse voters, ambiguity in the platform is never optimal, which seems at odds with his initial observation.
²Sartori (1962, p. 207)
³See http://www.theguardian.com/world/2012/nov/07/why-obama-won-womens-vote.
policy space and then decide what to reveal about this platform.\textsuperscript{5} The revealed interval has to contain the true platform as politicians cannot lie. Politicians can reveal different intervals in various outlets, though. These outlets can be thought of as news outlets, but can also be thought of as unions or churches or simply a market place where a politician gives a speech, that is they are simply a \textit{pool} of individuals. The information revealed in the pools can be observed by different \textit{groups} of voters. An example of this is given in Figure 2. There are three groups of voters that are connected to two pools. Politicians can disclose information in the two pools. Groups 1 and 2 only observe the interval of pools 1 and 2, respectively. Group 3, however, observed information in both pools. Thus, we allow for groups of voters that can observe the disclosed interval in exactly one or several pools.

Based on the observed intervals voters decide whether to turn out or not. Voters turn out if the observed intervals contain their ideal point, that is if they feel represented by the party \cite*{Brennan:1984, Schuessler:2000b, Kan:2001}. But voters also take into account how precise the revealed intervals are. The larger the intervals, that is the more imprecise the information about the true platform, the lower the likelihood of a given voter to turn out.

We first consider the case of only one politician and characterize the optimal disclosure of information if groups of voters are identical. We show that for any number of pools and groups of voters, the politician discloses the same interval in all pools.

In a next step, we allow for different distributions in voters. In particular, we distinguish between groups that are (i) more or less ideologically homogeneous, (ii) more left or more right. Last, we ask how the presence of a group that can observe disclosure in more than one pool changes the disclosure strategy.

**Related Literature** Our work is related to Galeotti and Mattozzi \cite{Galeotti:2011}, who consider a network of voters and are interested in how the network and homophily affects information dissemination by parties as well as polarization. Different from our model, they consider a model of word

\textsuperscript{5}Alesina and Holden \cite{Alesina:2008} show when politicians reveal intervals instead of an optimal point.
of mouth communication, in which voters reveal information obtained from parties truthfully to each other. There are two parties that can disclose the true policy platform, which can be one of two types, moderate or extremists. Policy space is unidimensional. In Galeotti and Mattozzi (2011)’s setting parties do not target certain individuals. Parties are policy motivated and prefer extreme policies to moderate ones. They show that when there is more word of mouth communication, then there is a higher probability of an extreme candidate selected, which is interpreted as more communication leading to higher polarization. Further, if individuals exhibit a higher degree of homophily, then again there will be more polarization.

Generally, our approach is connected to the targeting in networks literature such as Goyal et al. (2014). Different from this literature our model allows for heterogeneous groups and is to the best of our knowledge the first paper to do so in a general way.

We build on the assumption that parties are vague about their true policy platform, an observation which has been formalized in Alesina and Holden (2008). In their model, politicians need to converge to the median, but at the same time they need to raise contributions that flow from interest groups positioned at the extreme of the ideological spectrum. Other work on ambiguity in politics is Alesina and Cukierman (1990), Aragones and Postlewaite (2002) and Callander and Wilkie (2007).

We abstract from pivotal voting, and build on work that emphasizes that voting is expressive and serves a consumption purpose (Riker and Ordeshook (1968)). Or as Brennan and Buchanan (1984) put it, voting is like cheering a sports team. But in our model voters do not just turn out always, rather they follow the cues of parties, that is they turn out if the parties provide information to them. That parties exert effort to mobilize their voters which results in higher turnout has been established empirically in Shachar and Nalebuff (1999). Further, voters in our model turn out if they feel represented by the party, which is in line with the empirical finding of Kan and Yang (2001).

2 Model

We consider a model in which two politicians, $A$ and $B$ who select a policy platform $\hat{x}^k \in [0, 1]$, $k \in \{A, B\}$, in order to maximize their voter turnout.

Voters are split up in different groups and can belong to different pools. Groups may represent different towns in a given district, whereas pools can be thought of as places where members of group congregate, such as churches or unions. We assume that each group belongs to at least one pool. Overall there are $n$ groups and $m$ pools. The distribution of voters in each group is denoted by $G_i$, $i \in \{1, 2, \ldots, n\}$, the distribution in each pool by $P_j$, $j \in \{1, 2, \ldots, m\}$.
We assume that the probability distribution function in each group is single peaked, that is for each pdf \(g_i(\cdot)\) there exists a \(x_i^M\), s.t. \(g_i'(x^M) = 0\) and \(g_i'(x) > 0\) for \(x < x_i^M\) and \(g_i'(x) < 0\) for \(x > x_i^M\). Additionally, \(g_i(x) > 0\) for all \(x \in [0, 1]\).

A group of voters belongs to a pool if \(ij = 1\). If \(ij = 0\), then the group doesn’t belong to the pool. The relationship between the groups and the pools is given by a bipartite graph \(\Gamma \subset \{ij = 1\mid i \in \{1, 2, \ldots, n\} \land j \in \{1, 2, \ldots, m\}\}\). The groups belonging to a pool \(j\) are then given by the neighborhood of \(j\), \(N_j\).

Politicians do not have to show their true platform, but can decide to reveal an interval \([x^k_j, x^k_j]\). This interval can be different for each pool in which the party discloses and will depend on the distribution of the voters in a given pool. It has to be the case that \(\hat{x} \in [x^k_j, x^k_j]\). This implies that the true policy has to lie within the interval that is revealed. It is possible to hide it, but it is not possible to lie about it.\(^6\) The interval parties reveal affects their turnout and there are two counteracting forces at play. First, only individuals whose ideal policy lies within the interval will turn out and so a larger interval increases turnout as it caters to more voters an. On the other hand, a larger interval, that is clouding the true platform comes at a cost. As the interval increases, voters realize that their preferred policy is less likely to be the true platform and this reduces turnout. This motivates the following function for turnout in each group:

\[
    v^k_i = G_i(y^k_i) - G_i(y^k_i) - c_i(y^k_i - y^k_i)
\]

where \(y^k_i = \min_{j \in N_i}\{x^k_j\}\) and \(y^k_i = \max_{j \in N_i}\{x^k_j\}\). This implies that groups that belong to different pools in which the disclosed intervals differ take only the intersection of all these intervals into account, \([y^k_i, y^k_i] = \bigcap_{j \in N_i}[x^k_j, x^k_j]\). Note further that the intervals can either be included in each other or that they are overlapping. We refer to the intervals as being included if for two pools, \(j, l\), \([x^k_j, x^k_l] \subset [x^k_j, x^k_l]\). Intervals are overlapping if \([x^k_j, x^k_j] \cap [x^k_j, x^k_l] = [x^k_j, x^k_j]\). But this implies then, that for a group observing different intervals, only the maximum of the lower bounds as well as the minimum of the upper bounds is relevant, as the true policy will lie between these two. Intuitively, voters learn from the disclosure of the parties and take into account that politicians cannot lie, which ultimately allows them to restrict the interval in which the true policy lies.

We assume that costs are convex, that there are no fixed costs \((c_i(0) = 0)\) and that the marginal cost of disclosing the true platform equals zero, \(c_i'(0) = 0\). Note that we could easily adjust the cost function such that it incorporates the measure \(G_i(x^k_j) - G_i(x^k_j)\). This implies that

\(^6\)This assumption could derive from a repeated game where politicians will get punished in case they lie, but not if they are being merely vague.
our results are not affected by the fact that the cost function only depends on the length of the interval disclosed, but not on the on measure of voters that are lie in the disclosed interval. This is shown in the Appendix. What is crucial for our set up, though, is the assumption that the turnout function is separable in the measure of voters and the length of the disclosed interval.

We then consider two different cases, namely first the case where there is a one to one mapping between groups and pools and then the case where groups can belong to different pools.

3 Segmented Groups

If each group belongs to exactly one pool, then politicians care to maximize their voter turnout for each group. We are interested in what intervals they disclose in each group and the turnout associated with them. We analyze how the assumption that the disclosed interval has to contain the true platform impacts both the disclosure decision and voter turnout. In a last step, we analyze how the group composition affects disclosure and turnout.

In order to address these questions, we assume for now that the politicians have the same objective function and drop the superscript $k$. Further, to simplify notation, we drop the index $j$. The maximization problem is then given by

$$\max_{\underline{x}, \overline{x}} G(\overline{x}) - G(\underline{x}) - c(\overline{x} - \underline{x}),$$

$$\text{s.t.} \quad \underline{x} < \hat{x} < \overline{x}$$

(2)

The first order conditions combined with the complementary slackness conditions then can be written as

$$\left(c'(\overline{x} - \underline{x}) - g(\overline{x})\right)(\overline{x} - \hat{x}) = 0$$

(3)

$$\left(c'(\overline{x} - \underline{x}) - g(\underline{x})\right)(\hat{x} - \underline{x}) = 0$$

(4)

Given our assumptions, the revealed interval always has to be strictly positive.

Remark 1 (Positive Interval). \text{It is never optimal to fully reveal the true platform.}

To see this note that the first order condition is given by

$$g(\overline{x}) + \lambda_1 = c'(\overline{x} - \underline{x}) = g(\underline{x}) + \lambda_2$$

(5)

where $\lambda_1$ and $\lambda_2$ are the positive multipliers associated with the two constraints. We know that
\( g(\hat{x}) > c'(0) = 0 \), and so disclosing the true policy cannot be an equilibrium as this violates the first order necessary conditions. Therefore, at most one of the boundary conditions binds.

We then show that an optimal interval, that is a maximum, exists and turn next to a characterization of the optimum.

**Proposition 2 (Optimal Interval).** A maximum exists.

1. If the optimal interval is interior, it is unique and the mode is contained in the disclosed interval \( x^M \in [\underline{x}, \overline{x}] \).

2. If the optimal interval is such that one boundary equals the true policy, then the maximum is unique if \( \max |g'| \leq \min c'' \).

A maximum exists which follows from the continuity of the objective function and the fact that the constraints define a compact set. If the optimal interval is such the constraints do not bind, then the interval is unique and the mode is always contained in the optimal interval. This has to be the case as at the optimum, \( g(\overline{x}) = g(\underline{x}) \). Suppose this is not the case, that is the density at one boundary is higher than at the other boundary. Then moving the interval in the direction where the density is higher increases payoffs by more than is lost at the other end, where the density is lower. Therefore, this cannot be an equilibrium. As we have a single-peaked pdf, it also has to be the case that the lower bound lies below the mode and the upper bound above it.

If one of the boundary conditions binds, uniqueness is no longer given for general functions. There can be several local maxima (and minima), depending on the shapes of the cost function and the pdf. In order to avoid this multiplicity we impose the additional assumption, namely that \( \max |g'| \leq \min c'' \). Further, it does no longer have to be the case that the mode is contained in the disclosed interval.\(^7\)

Based on this we can compare how the disclosure strategy differs depending on where the true platform lies. We denote by \( [\underline{z}, \overline{z}] \) the optimal interval, if the politician can always choose what is best for him without taking the constraints into account. Put differently, this is the interval if the politician is allowed to lie. Then, whenever \( \hat{x} \in [\underline{z}, \overline{z}] \) the disclosed interval \([x, \overline{x}]\) is the same as \([x, \overline{x}]\). We focus therefore on the case when \([x, \overline{x}]\) and \([\underline{z}, \overline{z}]\) differ.

**Proposition 3 (Disclosure under Constraint).**

Let \( \hat{x} \notin [\underline{z}, \overline{z}] \). The disclosed interval if the politician is allowed to lie, \([\underline{z}, \overline{z}]\), differs from the one where politicians are not allowed to do so, \([x, \overline{x}]\), in the following way:

1. \( \hat{x} > \overline{x} \) if and only if \( \underline{z} < x, x < \overline{x}, \hat{x} < \overline{x} \) if and only if \( \underline{z} < x < \overline{x} \): the interval is shifted away from the mode if the boundary constraint binds.

\(^7\)Note that it can never be the case that both constraints bind, as this would be equivalent to revealing the true platform and we have just established that this cannot be the case.
2. If $\hat{x}$ is sufficiently close to either $z$ or $\overline{z}$, then $\overline{x} - x > z - \overline{z}$.

3. Assume that $[x, \overline{x}]$ is unique and suppose $\overline{x} = \hat{x}$. Then, $\overline{x} - x$ is larger than $\overline{z} - z$ if $\overline{z} - z > \hat{x} - \overline{z}$ and smaller if $\overline{z} - z < \hat{x} - \overline{z}$.

Proposition 3 tells us that if politicians are not allowed to lie, both the upper and the lower bound shift in the direction of the true policy. If $\overline{x} = \hat{x}$, then the lower bound, $x$, lies above $\overline{z}$, that is $x > \overline{z}$. Additionally, the length of the disclosed interval changes. If the true policy platform is very far away from $\overline{z}$, then the politician will disclose a small interval, which gives a precise notion of the policy he is interested in implementing. If, on the other hand, the true policy is close to $\overline{z}$, then the politician aims to cater to even more individuals in the group and therefore he increases the interval. Note that this is also related to the costs of disclosure. If the cost of being vague, that is the cost of choosing a large interval is low, then the true policy can never be sufficiently far enough from the upper bound of the unconstrained problem to justify disclosing a smaller interval.

We then consider differences in turn out between the two cases.

**Remark 4** (Turn Out under Constraint). The voter turnout is always higher if the politician is allowed to lie.

If the politician is allowed to lie, he chooses the interval that maximizes the turnout. This might not always be possible if the true platform has to lie in the disclosed interval and thus the politician will choose an interval that maximizes turnout conditional on this constraint. The constraint restricts the choice of the politician and if it binds it reduces turnout.

This then also shed light on which pools or in the segmented case, groups, the politician prefer. Everything else equal he will prefer a group that is more aligned with his true policy.

But even when the constraint binds, a politician will disclose in this pool.

**Proposition 5** (Disclosure in Every Pool). A politician discloses in every pool.

This result is driven by the fact that voter turnout is strictly positive in each pool and as the politician is interested in maximizing turnout, he will collect the votes in every pool.

The next results look at how disclosure is affected by changed in pool composition. In order to do so, we define the concepts of spread and shift.

**Definition 1** (Spread). The function $g_S(x)$ is a spread of $g(x)$ if they have the same mode and if there exist $a < b$, $a, b \in (0, 1)$, with $g_S(a) = g_S(b)$ and

\[
g(x) > g_S(x) \iff x \in (a, b) \quad \text{and} \quad g(x) < g_S(x) \iff x \notin [a, b] \quad (6)
\]
A spread simply spreads mass to the boundaries of the interval, keeping the mode fixed. It also shifts mass symmetrically, such that the points of intersection, $a, b$ have the same density for both distributions. This definition allows us to consider the effects of voters being more or less homogenous.

**Definition 2 (Shift).** A function $g_L(x)$ is a left-shift of a function $g_R(x)$ if there exists a unique $a$ s.t. $g_L(a) = g_R(a)$, $g'_L(a) < 0$ and $g'_R(a) > 0$.

A shift allows us to compare disclosure policies depending on whether voters are more centrist or extreme. Further, we can discuss disclosure policies depending on whether voters are closer or further away from the true platform of voters.

**Proposition 6 (Spread).** Assume the solution is interior. Then, let $g_S(\cdot)$ be a spread of $g(\cdot)$. The disclosed intervals differ as follows

- $c'(b - a) < g(a)$ if and only if $[x(g), \bar{x}(g)] \subset [x(g_S), \bar{x}(g_S)]$
- $c'(b - a) > g(a)$ if and only if $[x(g_S), \bar{x}(g_S)] \subset [x(g), \bar{x}(g)]$.

The turnout under $g$ is strictly higher than the turnout under $g_S$.

If costs are high, then the interval that is revealed is larger under the distribution that is more concentrated. In this case, both intervals are small and an increase at the margin leads to a greater increase in voter mass in the more concentrated distribution. The opposite holds true if costs are low. Then the interval disclosed is larger for the spread. In this region, the pdf of the spread lies strictly above the pdf of the original distribution and therefore a marginal increase in the interval increase voter mass more under the spread.

The turnout is higher under $g$ is higher than under $g_S$. This is due to the fact that for any interval that is chosen under the spread, the voter mass under the more concentrated pdf is higher. This implies that the politician could choose the exact same interval as under the spread. If he does not do it, then the interval he chooses must yield him a higher turnout.

We then turn to disclosure under shift.

**Proposition 7 (Shift).** Assume an interior solution and let $g_L(x)$ be a left-shift of $g_R(x)$. Then, $\underline{x}(g_L) < \underline{x}(g_R), \bar{x}(g_L) \,< \bar{x}(g_R)$.

In case of a shift, the disclosed intervals shift as well. Note that given our assumptions, we cannot say anything about the length of the disclosed interval and therefore, turnout. If the distributions only differ with respect to their modes, then the turnout is exactly the same. But generally, we cannot derive any conclusion with respect to turnout as this depends among other factors on how much mass of the distribution is close to the mode.

We turn next to the case in which groups can belong to several pools.
4 Non-Segmented Groups

There are two sets of questions we are interested in addressing. The first part deals with how disclosure changes when there are groups that can observe disclosure in several pools. How is the disclosure affected when groups are more or less ideologically heterogeneous? How is it affected by the groups being left or right leaning? We then turn to the issue of how many pools should be targeted and how this depends on the features of the groups that belong to them. In particular, we ask whether it is still optimal to target all pools. We then consider the case where there is a restriction on how many pools can be targeted and characterize how pools will be selected.

4.1 Disclosure with Non-Segmented Groups

In case there are non-segmented groups the maximization problem becomes slightly more involved. We therefore start out assuming an interior solution.

We consider the case of two pools and three groups. Group 1 only belongs to pool 1, group 2 only belongs to pool 2 and group 3 belongs to both pools. This implies that group 3 knows what parties disclose in both pools, which affects what they think is the true policy.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{Example: 3 Groups in 2 Pools}
\end{figure}

Let the disclosed interval in pool 1 be \([x_1, x_1]\) and the equivalent in pool 2 is given by \([x_2, x_2]\). As defined previously, the relevant interval for group 3 is then given by the intersection of the intervals in pools 1 and 2, namely \([x_1, x_1] \cap [x_2, x_2]\).

We distinguish between the inclusion and overlap of intervals. Intervals are included in each other if we can order pools such that intervals are included within each other. Formally,

\begin{equation}
x_{j+1} \geq x_j, \quad x_j \geq x_{j+1}
\end{equation}

Intervals are overlapping if they are not included in each other. In this case, we can order the intervals from left to right such that for the groups observing more than one pool the constraints
are given by

\[ x_j \leq x_{j+1} \quad \bar{x}_j \leq \bar{x}_{j+1} \quad (8) \]

Based on this we need to solve the maximization problem taking into account the different possible constraint for each group that observes the disclosure in more than one pool. So, in our example we need to solve the problem for each possible constraint and then compare the values of the problem. Given that we are interested in comparing spreads and shift, this problem can be simplified.

4.1.1 Spreads

If all groups are spreads of each other, then the intervals are included in each other.

**Proposition 8** (Included Intervals). If for all groups \( i, l \), s.t. either \( g_i \) is a spread of \( g_l \) or \( g_l \) is a spread of \( g_i \), then the disclosed intervals in all pools are included in each other.

Based on this the maximization problem for the included interval becomes

\[
\max_{\bar{z}, \bar{x}} \sum_{i=1}^{n} (G_i(\bar{y}_i) - G_i(y_i) - c_i(\bar{y}_i - y_i)) \quad s.t. \quad \bar{x}_{j+1} \geq \bar{x}_j \quad \bar{z}_j \geq \bar{z}_{j+1} \quad (9)
\]

From this we can then calculate the disclosure under the spread for our example of 3 groups and 2 pools. We restrict attention to the case where groups can be distributed according to two distributions, with distribution \( g_S \) being a spread of \( g \). Further, we call costs high if \( c'(b - a) > g(a) \) and low if \( c'(b - a) < g(a) \).

**Proposition 9** (Disclosure Spread). .

1. If groups 1 and 2 have distribution \( g \) and group 3 is a spread, then
   
   (a) if costs are high, the disclosed intervals are strictly included in each other.
   (b) if costs are low, the disclosed intervals are identical.

2. If groups 1 and 2 are spreads of group 3, then
   
   (a) if costs are high, the disclosed intervals are identical.
   (b) if costs are low, the disclosed intervals are strictly included in each other.

3. If groups 1 and 2 have different distributions and
   
   (a) costs are high, then the disclosed interval is larger in the pool to which the less spread out group is connected

   (b) If costs are low, then the disclosed interval is larger in the pool to which the more spread out group is connected.
Groups 1 and 2 can have the same distribution or they can have different distributions. If they have the same distribution, but the distribution of group 3 differs, then depending on costs the interval disclosed in pools 1 and 2 are identical or they are contained in each other. To see the intuition behind this result more clearly, consider the case when groups 1 and 2 have distribution $g$ and group 3 is a spread. Then, if costs are high, the politician would like to choose a smaller interval under $g_S$ than under $g$. Therefore, he adjusts the interval in one pool, choosing a smaller interval, whereas the interval in the other pool is the optimal interval for distribution $g$. If costs are low, the politician wants to disclose a larger interval under $g_S$ than under $g$. If he chooses the interval that is optimal for $g$ in both pools, then the interval that group 3 observes is too small. Therefore, the politician increases the intervals in both pools in order to account for the distribution in group 3. Note that he cannot only increase the interval in one pool, as for group 3 this would not be the relevant pool, as group 3 only takes the smallest interval among the two pools into account. This results in the politician disclosing the same in both pools.

Next, if groups 1 and 2 have different distributions, then the disclosed intervals are always strictly contained in each other. Assume without loss of generality that the distribution in group 2 is a spread of that in group 1. Then, if costs are high, the disclosed interval in pool 1 is larger than that in pool 2, if costs are low, the reverse is true. This result does not depend on whether $g_3 = g$ or $g_3 = g_S$. If costs are high, then for group 3 only the interval in pool 2 is relevant and so the distribution faced in pool 2 is always more spread out than the distribution in pool 1.

We then turn to shifts.

4.1.2 Shifts

There are two concerns that can arise in case of a shift. If we consider a pool that is connected to more than one group, then what matters for the disclosure in the pools is the combination of the groups that belong to a given pool and the resulting distribution over all members of a pool. This distribution might no longer be unimodal.

In order to address this issue, we define the concept of strong unimodality.

**Definition 3** (Strong Unimodality). Two distribution functions are strong unimodal, if a convex combination of their distribution functions is unimodal.

For an example of when this holds, consider a triangular distribution. The modes of two triangular distributions are denoted by $c_1$ and $c_2$. Then, the convex combination of the distri-
The convex combination of two triangular distributions is given by

\[ g(x) = \begin{cases} 
\frac{\alpha}{c_1} \frac{x}{c_1} + (1 - \alpha) \frac{x}{c_2} & \text{if } x < c_1 \\
\frac{\alpha}{1-c_1} \frac{2(1-x)}{1-c_1} + (1 - \alpha) \frac{2x}{c_2} & \text{if } c_1 \leq x < c_2 \\
\frac{a}{1-c_1} \frac{2(1-x)}{1-c_1} + (1 - \alpha) \frac{2(1-x)}{1-c_2} & \text{if } x > c_2 
\end{cases} \]

The first part is increasing in \( x \), the third part is decreasing in \( x \) and so it depends on the second part of whether the mode of the convex combination is \( c_1 \) or \( c_2 \). The second part is increasing if and only if

\[
(1 - \alpha)(1 - c_1) - \alpha c_2 > 0
\]

For \( \alpha = \frac{1}{2} \), the distributions are strong unimodal, as long as \( c_1 \neq 1 - c_2 \).

As a further example, consider two normal distributions with the same variance. Intuitively, they are strong unimodal if their means are sufficiently close together. We denote their means by \( \mu_1 \) and \( \mu_2 \), their variance by \( \sigma \). If the standard deviation is greater than \( \frac{1}{2} \), then the mixture of two normal distribution is unimodal. Generally, the mixture between two normal pdfs if the weights are equal is unimodal, if

\[
0 \geq 2 \log(d - \sqrt{d^2 - 1}) + 2d \sqrt{d^2 - 1} \tag{10}
\]

where \( d = \frac{\mu_1 - \mu_2}{2\sigma} \). The larger the distance between the modes of the normal distribution, the larger does the variance have to be in order to ensure uni-modality. On the other hand if modes are close together, then the variance can be smaller. \(^8\)

In what follows, we restrict attention to functions whose convex combination is unimodal.

**Assumption 1.** Any two functions are strong unimodal.

If uni-modality is ensured, we can turn to optimal disclosure under shifts. We focus again on the previous example with 3 groups and 2 pools as this highlights the main workings.

We further assume that there is a left-right ordering in terms of group distributions. Group 1 is the left group with distribution \( g_L \), group 2 is right, with \( g_R \) and group 3 is moderate and distributed according to \( g_M \). We assume that \( g_M \) and \( g_R \) are right-shifts of \( g_L \) and \( g_R \) is a right-shift of \( g_M \). We denote the point of intersection between \( g_L \) and \( g_R \) by \( a_{LR} \), that between \( g_L \) and \( g_M \) as \( a_{LM} \). As \( g_R \) is a right-shift of \( g_M \), \( a_{LM} < a_{LR} \).

We are interested in comparing two cases, namely the case where the third group is not

---

\(^8\)Note that what we are doing here should not to be confused with the sum of two normally distributed random variables that normally distributed. This is radically different from the mixture model we have here.
present to where it is and the case where the third group belongs to the other groups and is then separate. Formally, we compare

\[ g_L(x_1) + g_M(x_1) = c'(x_1 - x_1) + c'(x_1 - x_2) \]

\[ g_L(x_1) = c'(x_1 - x_1) \]

\[ g_R(x_2) = c'(x_2 - x_2) \]

\[ g_M(x_2) + g_R(x_2) = c'(x_2 - x_2) + c'(x_1 - x_2) \]

to Benchmark I

\[ g_L(x_S^1) = c'(x_S^1 - x_S^1) = g_L(x_S^1) \]

\[ g_R(x_S^2) = c'(x_S^2 - x_S^2) = g_R(x_S^2) \]

as well as to Benchmark II

\[ \frac{2}{3} g_L(x_S^1) + \frac{1}{3} g_M(x_S^1) = \frac{2}{3} g_L(x_S^1) + \frac{1}{3} g_M(x_S^1) \]

\[ \frac{2}{3} g_R(x_S^2) + \frac{1}{3} g_M(x_S^2) = \frac{2}{3} g_R(x_S^2) + \frac{1}{3} g_M(x_S^2). \]

As we have assumed an interior solution, this implies that without the third group the optimal disclosed intervals are overlapping. A sufficient condition for an interior solution is that \( a_{LR} \), the point of intersection between \( g_L \) and \( g_R \), is contained in both intervals, \( [x_S^1, x_S^1] \) and \( [x_S^2, x_S^2] \).

Based on this we can show the following.

**Proposition 10 (Disclosure Shift).** Let the solution be interior. Then, the disclosed intervals in the pools if group 3 is present compared to Benchmark I are larger and more moderate, that is

\[ x_1 > x_S^1, \quad x_1 > x_S^1, \quad x_2 < x_S^2, \quad x_2 < x_S^2 \]

If we compare the disclosed intervals in the pools if group 3 is present to Benchmark II, then the disclosed interval becomes strictly larger with \( x_1 \geq x_S^1 > x_S^1 \geq x_S^1 \).

The party adjusts its disclosure if group 3 is present compared to the case where only group 1 is connected to pool 1 and group 2 to pool 2. The presence of the third group, which is

\[ \text{It is clear that at least one of the intervals has to contain} \ a_{LR} \ \text{as otherwise there cannot be an overlap between the two intervals and this contradicts the assumption that the true policy is contained in every disclosed interval. There can be cases, in which only one interval contains} \ a_{LR}, \text{though.} \]
more moderate than groups 1 and 2 induces the party to choose a larger in interval in both pools. Additionally, the boundaries of the interval in pool 1 both move to the right, whereas the boundaries in pool 2 move both to the left, resulting in more moderate intervals.

But the presence of group 3 does not only have an effect due to the additional number of moderates. It also matters that the moderates form their own group. This is what the comparison to Benchmark II illustrates where the moderates belong to groups 1 and 2. Compared to Benchmark II, the intervals become larger and at least one of the boundaries changes. But unlike the previous case, the disclosed intervals do not necessarily become more moderate.

4.2 Targeting of Pools

We first outline the optimal strategy when all groups are have the same mass, distributions and costs. The number of groups that can be reached is given by the neighborhood of each pool, $N_j$. A politician has an incentive to maximize the number of groups he can reach. He can reach at most $n$ groups, and thus $n = \bigcup_j N_j$. If there exists a bound on how many pools he can target, he has to choose a restricted subset, which we refer to as the coverage $C_R = \{\bigcup_{r=1}^{R} N_r | N_r \in \{N_1, \ldots, N_j\}\}$. Then, the maximal target set is given by $c^*_R$ such that $|c^*_R| \geq |c_R|$ for any $c_R \in C_R$.

**Proposition 11** (Maximization of Coverage). *If all groups have same mass, distribution and costs and there is a bound on how many pools can be targeted, then parties maximize their coverage.*

Why do we care about coverage? This concept is clearly related to degree and in the case where parties can only target one pool, the two concepts are the same. Generally, however, the degree will not be crucial. To see this, consider Figure (3).

![Coverage vs Degree](image)

Here, most groups belong to pool 2, that is pool 2 has the highest degree. However, if politicians can target at most two pools, then they are better off targeting pool 1 and pool 3 instead of pools 2 and one of the other pools. Targeting for pool 2 and another one gives
access to 5 groups, whereas targeting pools 1 and 2 provides access to 6 pools. As targeting an additional group always leads to higher payoffs, it is advantageous to target as many groups as possible.

What this also implies is that when groups are identical, politicians will target all groups if they can do so.
References


Appendix

Alternative Cost Function

Consider the following cost function, that also depends on \( G_i(x_k^i) - G_i(x_k^k) \):

\[
h(G_i(x_k^i) - G_i(x_k^k)) + c_i(x_k^i - x_k^k)
\]

with

\[
h(G_i(x_k^i) - G(x_k^k)) = a(G_i(x_k^i) - G_i(x_k^k))
\]

Then the marginal costs are simply scaled by a constant and so the results remain unchanged.

Proof of Proposition 2: Characterization of Optimal Solution

A maximum exists as the objective function is continuous and the set is compact.

Interior Solution If the constraints do not bind, the first order conditions simplify to

\[
g(x) = c'(x - \bar{x}) = g(x)
\]

This implies that \( \bar{x} \) lies to the left of the mode and \( \bar{x} \) to the right of it. Put differently, the mode is always contained in the interval. To see this, suppose to the contrary that \( x^M \) is not contained in the interval. Then there must exist \( \bar{x}, \bar{x} \) such that \( \bar{x}, \bar{x} < x^M \). But by assumption, \( g'(x) > 0 \) for \( x < x^M \), which gives the contradiction.

The maximum is unique as

\[
a_{11} = -g'(x) - c''(\bar{x} - x) < 0
\]

\[
a_{11}a_{22} - a_{12}^2 = (-g'(x) + c''(\bar{x} - x))(g'(\bar{x}) - c''(\bar{x} - x)) - [c''(\bar{x} - x)]^2
\]

\[
= -g'(x)g'(\bar{x}) - g'(x)c''(\bar{x} - x) + g'(x)c''(\bar{x} - x) > 0
\]

which implies that the Hessian is negative definite. Due to the fact that the density is strictly increasing to the left of the mode and strictly decreasing to the right of it and the convexity of the cost function, the maximum is unique.

Boundary Solution If the determinant of the bordered Hessian is positive we have a local
maximum. The bordered Hessian is given by
\[
\begin{pmatrix}
0 & 0 & 1 \\
0 & a_{11} & a_{12} \\
1 & a_{12} & a_{22}
\end{pmatrix}
\]
and its determinant is \(-a_{11}\). If the determinant is positive we have found a local maximum, if it is negative, we have found a local minimum.

Given that \(g(\hat{x}) > c'(0) = 0\), the marginal cost functions starts below the pdf. Then at the intersection point, it has to be the case that \(c''(\hat{x} - x) > -g'(\hat{x})\).

**Proof of Proposition 3: Disclosure under Constraint**

We first show that the disclosed interval is shifted towards the true policy if the constraint binds. Again, we assume that the upper bound of the interval is equal to the true policy. It has to hold that

\[
g(z) = c'(\hat{x} - \hat{z})
\]

(14)

Given that the constraint binds (strictly) we also know that \(\hat{x} > \hat{z}\). Given the convexity of the cost function,

\[
c'(\hat{x} - \hat{z}) > c'(\hat{z} - \hat{z}) = g(\hat{z})
\]

(15)

As the LHS is decreasing in \(\hat{z}\) and the RHS is increasing (at least up to a point), it has to be the case that \(\hat{z} < \hat{x}\).

We show that for \(\hat{x}\) sufficiently close to \(\hat{z}\), the disclosed interval if the boundary condition is larger than in the unconstrained problem. Let \(\delta = \hat{x} - \hat{z}\), where \(\delta\) is arbitrarily small. We first show that there does not exist a \(\tilde{x} \in (\hat{z}, \hat{x})\) that fulfills the first order conditions. As \(\delta\) is arbitrarily small, it follows that \(\epsilon = \hat{x} - \tilde{x} < \delta\). Then for this to be an equilibrium it has to hold that \(c'(\epsilon) = g(\tilde{x}) = g(\hat{x} - \epsilon)\).

\[
c'(\epsilon) < c'(\delta) < g(\hat{x}) < g(\hat{x} - \epsilon)
\]

(16)

But there exists always a \(\delta\) such that costs are arbitrarily close to zero and we know that \(g(x) > 0\) for any \(x\). Then any intersection between the pdf and the marginal costs has to be where \(g(x) > g(\hat{z})\). This implies that the disclosed interval has to be strictly larger.
We then show that in the case where the intersection is unique how the length of the disclosed interval changes. We show that if the true platform is close to $z$, then the disclosed interval increases. Suppose, by contradiction, that $z - z > \hat{x} - \overline{x}$ implies that $z - z > \hat{x} - x$. Consider a $\tilde{x}$ s.t. $\hat{x} - \tilde{x} = z - \overline{x}$. Then, $c'(\hat{x} - \tilde{x}) = c'(\overline{x} - \overline{z})$, but $f(\hat{x}) > f(\overline{x})$. Then it is optimal to increase $\hat{x} - \tilde{x}$ such that $\hat{x} - \tilde{x} > z - z$. The other case works along the same lines and is therefore omitted.

Proof of Remark 4: Turn Out under Constraint

The maximal turnout is decreasing if $\hat{x}$ increases, that is

$$\frac{\partial (G(\overline{x}) - G(x(\hat{x}))) - c(\overline{x} - x(\hat{x}))}{\partial \hat{x}} < 0 \quad (17)$$

To see this note that

$$\frac{\partial (G(\overline{x}) - G(x(\hat{x}))) - c(\overline{x} - x(\hat{x}))}{\partial \hat{x}} = g(\overline{x}) \frac{\partial \overline{x}}{\partial \hat{x}} - g(x) \frac{\partial x}{\partial \hat{x}} - c'(\overline{x} - x) \left( \frac{\partial \overline{x}}{\partial \hat{x}} - \frac{\partial x}{\partial \hat{x}} \right) \quad (18)$$

$$= -\lambda_1 \frac{\partial \overline{x}}{\partial \hat{x}} + (g(\overline{x}) - c'(\overline{x} - x)) \left( \frac{\partial \overline{x}}{\partial \hat{x}} - \frac{\partial x}{\partial \hat{x}} \right) \quad (19)$$

$$= -\lambda_1 < 0 \quad (20)$$

This implies that the voter turnout is strictly larger in the unconstrained problem.

Proof of Proposition 5: Positive Voter Turnout in Each Pool

Voter turnout has to be strictly positive in each pool. Recall that $c(0) = 0$. Consider first the case, where the boundary condition doesn’t bind. Then,

$$g(\overline{x}) = c'(\overline{x} - \overline{x}) = g(\overline{x}) \quad (21)$$

Given that we have an interior solution, $g(x) > g(\overline{x})$ for all $x \in (\overline{x}, \overline{x})$, but $c'(y) < c'(\overline{x} - \overline{x})$ for all $y < \overline{x} - \overline{x}$. Therefore, integrating yields

$$\int_{\overline{x}}^{\overline{x}} g(x) dx > \int_{0}^{\overline{x} - \overline{x}} c'(y) dy \quad (22)$$

$$\Leftrightarrow G(\overline{x}) - G(x) > c(\overline{x} - x) \quad (23)$$

$$\Leftrightarrow G(\overline{x}) - G(\overline{x}) - c(\overline{x} - \overline{x}) > 0 \quad (24)$$
Consider next the case where the boundary condition binds. Consider without loss of generality that the upper bound binds.

\[ g(\bar{x}) + \lambda_1 = g(x) = c'(x - \bar{x}) \]  

(25)

Then, there exists at least one maximum in which \( g(x) > c'(\hat{x} - x) \) for any \( x \in (\bar{x}, \hat{x}] \). For this maximum it holds that

\[ \int_{\bar{x}}^{\hat{x}} g(x)dx > \int_{0}^{\hat{x} - \bar{x}} c'(y)dy \]  

(26)

\[ \Leftrightarrow G(\hat{x}) - G(x) > c(\hat{x} - x) \]  

(27)

\[ \Leftrightarrow G(\hat{x}) - G(x) - c(\hat{x} - x) > 0 \]  

(28)

Therefore, also if the boundary condition binds, there exists a maximum in which the voter turnout is strictly positive.

**Proof of Proposition 6: Disclosure and Turnout under Spread**

**Disclosure**

Let \( c'(b - a) < g(a) \). Let \( \underline{x}_G \equiv \underline{x}(g) \) and \( \underline{x}_{GS} \equiv \underline{x}(g_S) \).

The costs imply that \( \underline{x}_G < a \) and \( \bar{x}_G > b \). Then, \( g(\bar{x}_G) < g_S(\bar{x}_G) \) and \( g(\underline{x}_G) < g_S(\underline{x}_G) \). Therefore, as \( c'' > 0 \), \( [\underline{x}_G, \bar{x}_G] \subset [\underline{x}_{GS}, \bar{x}_{GS}] \). The argument for \( [\underline{x}_{GS}, \bar{x}_{GS}] \subset [\underline{x}_G, \bar{x}_G] \) follows exactly the same line and is therefore omitted.

**Turnout** Let \( [\underline{x}_S, \bar{x}_S] \) be the optimal interval under the spread. If the interval is contained in \( [a, b] \), then by the definition of a spread

\[ \int_{\underline{x}_S}^{\bar{x}_S} g(x)dx > \int_{\underline{x}_S}^{\bar{x}_S} g_S(x)dx \]  

(29)

On the other hand, if \( [a, b] \) is contained in the optimal interval, then

\[ 1 - \int_{0}^{\underline{x}_S} g(x)dx - \int_{\underline{x}_S}^{1} g(x)dx > 1 - \int_{0}^{\underline{x}_S} g_S(x)dx - \int_{\underline{x}_S}^{1} g_S(x)dx \]  

(30)

\[ \Leftrightarrow \int_{0}^{\underline{x}_S} g_S(x)dx + \int_{\underline{x}_S}^{1} g_S(x)dx > \int_{0}^{\underline{x}_S} g(x)dx + \int_{\underline{x}_S}^{1} g(x)dx \]  

(31)

which is again true, by the definition of a spread.
Proof of Proposition 7: Disclosure under Shift

In order to show this we consider the possible constellations of intervals and simplify notation such that $x(g_L) = x_L$ etc.

1. $[x_L, x_L] \cap [x_R, x_L] = \emptyset$. This can never be the case as the true policy must lie in both sets.
2. $[x_L, x_L] \cap [x_R, x_R] = [x_L, x_R]$. It holds that $g_R(x_R) < g_L(x_R) < g_L(x_L)$ and $g_R(x_R) > g_L(x_R)$. For $g_L(x_L) > g_R(x_R)$ it holds that $x_L < x_R$, which is a contradiction.
3. $[x_R, x_R] \subset [x_L, x_L]$: Now, it holds that $g_L(x_R) > g_R(x_R) = c'(x_R - x_R) > g_L(x_L)$. For $x_L < x_R$ and $x_L > x_R$,

$$c'(x_L - x_R) > c'(x_R - x_R) > g_L(x_L)$$

we have the contradiction.

Proof of Proposition 8: Included Intervals

If every group has a distribution $g_i$ that is some spread of an underlying distribution $g_1$ then the distribution in each pool is given by some average of these spreads and is therefore a spread itself. Suppose that there are two pools, pools 1 and 2, in which $x_1 < x_2$, $x_1 < x_2$. Note that here we are only interested in the case where the inequalities are strict. Otherwise, the intervals are weakly contained in each other. We assume The maximization problem (again assuming that the true policy is contained in the interval) for these two pools is then given by the following Lagrangian

$$\mathcal{L} = \sum_{i=1}^{n_1} (G_i(x_1) - G_i(x_1) - c_i(x_1 - x_1)) + \sum_{i=1}^{n_2} (G_i(x_2) - G_i(x_2) - c_i(x_2 - x_2)) + \lambda_1 [x_2 - x_1] + \lambda_2 [x_2 - x_1]$$

(34)

The first order conditions can be simplified to

$$\sum_{i=1}^{n_1} g_i(x_1) - \lambda_1 = \sum_{i=1}^{n_1} c_i'(x_1 - x_1) = \sum_{i=1}^{n_1} g_i(x_1) + \lambda_2$$

$$\sum_{i=1}^{n_2} g_i(x_2) + \lambda_1 = \sum_{i=1}^{n_2} c_i'(x_2 - x_2) = \sum_{i=1}^{n_2} g_i(x_2) - \lambda_2$$

(35) (36)

Assuming that all distributions have the same mass and costs are same, we can then define
a \( g_1 \) and \( g_2 \) which are the averages of the distributions in pool 1 and 2, respectively. It can either be the case that \( g_1 \) and \( g_2 \) are the same, that \( g_1 \) is a spread of \( g_2 \) or vice versa. The problem then simplifies to

\[
\begin{align*}
g_1(\bar{x}_1) - \lambda_1 &= c'(\bar{x}_1 - \bar{x}_1) = g_1(\bar{x}_1) + \lambda_2 \\
g_2(\bar{x}_2) + \lambda_1 &= c'(\bar{x}_2 - \bar{x}_2) = g_2(\bar{x}_2) - \lambda_2
\end{align*}
\]

(37) \hspace{1cm} (38)

We are only interested in the case where \( \lambda_1, \lambda_2 > 0 \). This cannot be the case as already established and therefore the intervals have to be contained in each other.

**Proof of Proposition 9: Disclosure under Spread, Non-Segmented Pools**

We first establish that the intervals are strictly contained in each other. In order to do so, we let \( g_1 \) be the less spread out distribution, \( g_2 \) a spread of \( g_1 \). Groups 1 and 2 only access pools 1 and 2, respectively, 3 has access to both and is distributed according to \( g_3 \). The unconstrained maximization if \([x_1, x_1] \subset [x_2, x_2]\) yields the following first order conditions

\[
\begin{align*}
g_1(\bar{x}_1) + g_3(\bar{x}_1) &= c'_1(\bar{x}_1 - \bar{x}_1) + c'_3(\bar{x}_1 - \bar{x}_1) = g_1(\bar{x}_1) + g_3(\bar{x}_1) \\
g_2(\bar{x}_2) &= c'_2(\bar{x}_2 - \bar{x}_2) = g_2(\bar{x}_1)
\end{align*}
\]

(39) \hspace{1cm} (40)

Assuming that costs are identical, we have that \( c'(\bar{x}_1 - \bar{x}_1) < c'(\bar{x}_2 - \bar{x}_2) \) and thus

\[2g_2(\bar{x}_2) > g_1(\bar{x}_1) + g_3(\bar{x}_1)\]

(41)

The unconstrained maximization if \([x_2, x_2] \subset [x_1, x_1]\) gives the following first order conditions

\[
\begin{align*}
g_2(\bar{x}_2) + g_3(\bar{x}_2) &= c'_2(\bar{x}_2 - \bar{x}_2) + c'_3(\bar{x}_2 - \bar{x}_2) = g_2(\bar{x}_2) + g_3(\bar{x}_2) \\
g_1(\bar{x}_1) &= c'_1(\bar{x}_1 - \bar{x}_1) = g_1(\bar{x}_1)
\end{align*}
\]

(42) \hspace{1cm} (43)

Here, \( c'(\bar{x}_1 - \bar{x}_1) > c'(\bar{x}_2 - \bar{x}_2) \), from which it follows that

\[2g_1(\bar{x}_1) > g_2(\bar{x}_2) + g_3(\bar{x}_2)\]

(44)

**Proof of Proposition 9**

We first consider the case if groups 1 and 2 are identical and suppose the pdf is \( g \).
Then, the first order conditions are given by

\[ \begin{align*}
  g(\overline{x}_1) + g_S(\overline{x}_1) - \lambda_1 &= 2c'(\Delta_{11}) = g(\underline{x}_1) + g_S(\underline{x}_1) + \lambda_2 \quad (45) \\
  g(\overline{x}_2) + \lambda_1 &= c'(\Delta_{22}) = g(\underline{x}_2) - \lambda_2 
\end{align*} \]

where \( \Delta_{ij} = \overline{x}_i - \underline{x}_j \). An unconstrained optimum is only possible if costs are high. We have previously excluded that only one boundary binds and thus in the case of low costs, both constraints have to bind.

The reverse holds if the baseline groups have pdf \( g_S \). Then for low costs the intervals are strictly contained in each other, for high costs, they are the same.

If the baseline groups have a different distribution, then the disclosed interval in the pool with the more spread out group is larger if costs are low and the reverse if costs are high (costs high or low is again defined with respect to \( c'(b - a) \) being larger or smaller than \( g(a) \)).

**Proof of Proposition 10**

We are first interested to show what the optimal disclosure in presence of group 3 looks like and when we have a maximum. In order to see this, we consider the Hessian, which is a symmetric \( 4 \times 4 \) matrix in this case.

The elements are given as follows

\[ \begin{align*}
  a_{11} &= g'_L(\overline{x}_1) + g'_M(\overline{x}_1) - c''(\overline{x}_1 - \underline{x}_1) - c''(\overline{x}_1 - \overline{x}_2) \\
  a_{12} &= c''(\overline{x}_1 - \underline{x}_1) \\
  a_{14} &= c''(\overline{x}_1 - \overline{x}_2) \\
  a_{22} &= -g'_L(\underline{x}_1) - c''(\overline{x}_1 - \underline{x}_1) \\
  a_{33} &= g'_R(\overline{x}_2) - c''(\overline{x}_2 - \underline{x}_2) \\
  a_{34} &= c''(\overline{x}_2 - \underline{x}_2) \\
  a_{44} &= -g'_M(\underline{x}_2) - g'_R(\underline{x}_2) - c''(\overline{x}_2 - \underline{x}_2) - c''(\overline{x}_1 - \overline{x}_2) 
\end{align*} \]

It is then straightforward to establish that in a maximum \( \overline{x}_1 < \underline{x}_M \). Similarly, \( \overline{x}_1 \) is larger then the mode of the resulting mixture distribution.

**Benchmark I** We first establish that \( \overline{x}_1 > \overline{x}_S \). Suppose by contradiction that \( \overline{x}_1 \leq \overline{x}_S \).
1. 

\[ x_1 = x_1^S \quad \Rightarrow \quad g_L(x_1) = g_L(x_1^S) \]
\[ \Rightarrow \quad c'(x_1 - x_1) = c'(x_1^S - x_1^S) \]
\[ \Rightarrow \quad x_1 = x_1^S \]

Further, \( c'(x_1 - x) < c'(x_1 - x_1) = g_L(x_1) \). Then it has to be the case that \( g_M(x_1) < g_L(x_1) \).

But for any \( x \geq a_{LM}, g_M(x) \geq g_L(x) \). Thus, it has to be the case that \( x < a_{LM} \).

We can rewrite the first order condition as

\[ g_L(x_1) + g_M(x_1) = (1 + k)c'(x_1 - x_1), \]

where \( c'(x_1 - x_2) = kc'(x_1 - x_1) \) and \( k < 1 \). Equivalently,

\[ \frac{1}{1 + k}g_L(x_1) + \frac{1}{1 + k}g_M(x_1) = c'(x_1 - x_1) = g_L(x_1) \]

We know that if \( x_1 \) lies to the right of the mode of the distribution \( \frac{1}{1+k}g_L(x_1) + \frac{1}{1+k}g_M(x_1) \), then we have a maximum. We also know that the mode of this mixture distribution has to lie between the modes of the distributions of \( g_L \) and \( g_M \). If the mode is to the right of \( a_{LM} \), then we already have a contradiction. If it is to the left of \( a_{LM} \), then \( \frac{1}{1+k}g_L(x_1) + \frac{1}{1+k}g_M(x_1) > \frac{1}{2}g_L(x_1) + \frac{1}{2}g_M(x_1) > g_L(a_{LM}) > g_L(x_1) \), which yields the contradiction.

2. 

\[ x_1 < x_1^S \quad \Rightarrow \quad g_L(x_1) < g_L(x_1^S) \]
\[ \Rightarrow \quad c'(x_1 - x_1) < c'(x_1^S - x_1^S) \]
\[ \Rightarrow \quad x_1 < x_1^S \]

The difference in marginal costs implies that the disclosed interval in the non-segmented case is smaller and thus \( x_1 < x_1^S \). This implies

\[ g_L(x_1) > g_L(x_1^S) = g_L(x_1^S) > g_L(x_1) = c'(x_1 - x_1) \]

Then,

\[ g_M(x_1) < c'(x_1 - x_2) \]
for the first order condition to hold. Thus,

\[ g_M(x_1) < c'(x_1 - x_2) < c'(x_1 - x_1) < g_L(x_1). \]

It follows that \( x_1 < a_{LM} \) and by the same logic as in 1. we get a contradiction.

**Benchmark II** It has to be the case that \( x_1^S > a_{LM} > x_1^S \), then intervals increase, similar to the logic of a spread as the distribution the emerges from the two groups lies above that of Benchmark II.

**Proof of Proposition 11: Maximization of Coverage**

If each group has exactly the same distribution, then the disclosed interval in every pool is the same. Therefore, there is no incentive for the parties to not go to a pool. As each group yields a positive turnout, parties aim to maximize the number of groups they can target, which justifies the concept of coverage.