Good Jobs and Bad Jobs in the Process of Rural–Urban Migration

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Abstract. In this paper, we examine the process of economic development: the irreversible transformation that a country embarks on as its urban centres gradually economically eclipse the initially dominant agricultural sector. We examine the induced rural–urban labor market flows through the conceptual lens of the good-jobs–bad jobs framework proposed by Acemoglu (2001) and Albrecht and Vroman (2002). The paper makes four principal contributions. First, the model helps explain why some economies stubbornly fail to develop. Second, in addition for accommodating equilibrium unemployment in the wake of a great migration, the model sheds light on the causes of upgrading of the industrial base to relatively higher-tech production methods. Third, the framework is flexible enough to allow us to quantify the effects of a number of policies, such as export subsidies and household registration (hokou) schemes, that have been used to stem migratory flows. Finally, the paper offers a contribution to the pure theory of search: we provide a complete characterization of the equilibrium outcomes that arise in the good-jobs–bad jobs framework.

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1 Introduction

A major feature of the industrialization process that swept developed countries and is underway in several developing countries is a significant rural-urban migration, see Lucas (2004). Equally important, the urban sector in several developed and developing countries displayed an increasing wage inequality between and within skill groups—see Bound and Johnson (1992), Machin (1996), Acemoglu (1999), Galor and Moav (2000), and Harrison, McLaren, and McMillan (2011). Surprisingly, there is little research about the relationship between rural-urban migration and wage inequality in the urban sector. This question is important on the grounds of the expected rural-urban wage differential (Harris and Todaro, 1970) being one of the key determinants of migration, and such differential is affected by the urban wage inequality, more specifically by the urban job composition.

In this paper we propose a model that addresses this gap in the literature by embedding the essence of the Harris and Todaro (1970) migration model in a job search friction model. In our model, workers have heterogeneous skills and jobs have different skill requirements that are endogenously chosen by firms. We assume that a technology with higher skill requirement presents a higher productivity only if the vacancy is filled by a high skill worker, whereas a technology with a low skill requirement, produces a smaller output but the firms find a suitable worker more easily since any worker fits the job. Thus, the firms’ decision regarding the type of job offered will be governed by tradeoff posed by relative scarcity of high-skill workers and the higher productivity of a good job. Changes in parameters that affect this decision will impact the the proportion of high productivity jobs, which by its turn alter the expected value of urban income flows. Therefore, migration takes place to offset the difference between the rural and the urban expected wages.

Our two-sector model consists of a rural and an urban sector. The rural sector is comprised of perfectly competitive farms (firms) exhibiting constant returns to scale. Workers are ex ante identical and have the same productivity in farming activities. In the city, newborns and migrants undergo compulsory schooling to acquire skills, which are a random draw from an (at first) exogenous distribution and revealed at the end of the schooling period. ¹ Urban (or manufacturing) firms face a perfectly competitive market for their output. The urban labor market is characterized by search frictions that imply a time consuming matching process between a worker and a job vacancy. Firms can post job vacancies that require either a low

¹This skill definition follows Vroman (1987), and is also used by Albrecht and Vroman (2002). A similar definition is used by Mortensen and Pissarides (1999).
or a high minimum level of worker skill, also called bad and good jobs respectively. This nomenclature follows the literature notion that a good job is more productive than a bad job. We assume that job search is undirected, thus an open vacancy (regardless of its type) meets workers from the unemployment pool with the same likelihood, and workers regardless of their skill find vacancies with uniform probability. This set-up has the advantage of generating within group wage inequality because high skill workers can take good or bad jobs. Furthermore, it allows for the important phenomenon of over-qualified workers, i.e. mismatch between high skill workers and bad jobs, as stressed by Dolado, Jansen, and Jimeno (2009). As pointed out by Albrecht and Vroman (2002), this skill mismatch affects both low skill workers and bad job firms because the larger the share of bad job vacancies due to high skill workers refusal, the higher are the low skill workers’ welfare and the firms’ profits from offering bad jobs.

Provided that the agents take contact rates as given, the model will have a block recursive structure in which the worker-firm and the firm-worker contact rates are the two key endogenous variables that can be obtained by means of the free entry condition of each type of vacancy and an equation that determines for each worker-firm contact rate the unique firm-worker contact rate that is consistent with the matching technology adopted. Once these variables are determined, the equilibrium value of the share of good job vacancies for each worker-firm contact rate equilibrium value can be solved for. Once wages are calculated, the expected income flow of living in the city is obtained. The latter is then used to determine the urban and rural populations by making the expected values of the income flows in rural and urban areas to be equal.

Our model displays two types of equilibrium. In the cross-skill match equilibrium high skill workers accept both good and bad jobs, while in the ex post segmentation equilibrium high skill workers only accept good jobs. The liquidity(represented by product between the worker-firm contact rate and the share of good job vacancies)-productivity (flow value of the firms share of gains due to productivity differential) tradeoff faced by firms is what determines the type of equilibrium. If either the share of high skill workers or the productivity differential is high, the economy will be at an ex post segmentation equilibrium. Conversely, the cross-skill match equilibrium arises. We also determined the parameter ranges in which both equilibrium types and also the mixed strategy equilibrium occur. Most important, the larger the high skill share of workers or the productivity differential the smaller the range of worker-firm contact rates that support cross-skill match equilibrium. If sufficiently large, such changes can move the economy from a cross-skill match to an ex post segmentation equilibrium.
An interesting result of our model is that the share of good jobs in the city will impact migration flows by means of altering the expected income flow a migrant can expect in the city. An implication of this is that improvements in the educational system that leads to a higher share of skilled workers will increase the expected value of moving to the city. As a result, rural population shrinks while the city gets larger, and the rural wage increases. Similarly, increases in the productivity differential—for instance, due to skill biased technological improvement—lead to increased urbanization and agricultural wages while reducing low skill urban wages.

To illustrate our major theoretical contributions, we conducted some numerical simulations in which we characterized all possible equilibria of our model and provided the parameter range in which they are supported. Moreover, we investigated the effects of agricultural policies on the urban wage inequality. Such policies were adopted in several developing countries in the midst of their urbanization process. To do so, we employ an extension of our model in which the productivity differential is increasing (at a decreasing rate) in the size of the urban population. This can interpreted as human capital having external effects as in Lucas (2004). We find that policies that reduce the price of agricultural goods—export tariffs for a small open economy or price ceilings, for instance—lead to higher migration to the cities and therefore a higher productivity differential. Consequently, the share of good job vacancies and the high skill worker wage increases. In contrast, the low skill wage decreases, thus the between skill wage inequality grows. A side effect of this policy is an increase in urban unemployment that can be magnified if the economy were a cross-skill matching equilibrium and moves to a *ex post segmentation* equilibrium. At the end of the day, despite the reduction in the price of the agricultural good, the smaller population in rural areas lead to higher wages in agriculture.

Our model departs from Albrecht and Vroman (2002) in the following ways. First, we added a rural sector to the model to generate migration. Second, to create the skill dispersion across workers, we assume that city newborns and current migrants have to attend school in order to learn their skill level. Third, we generalized their model by allowing workers to have their skill level drawn from an exogenous (at first) continuous distribution. This required us to determine the respective endogenous skill cutoffs used by firms in their good and bad job vacancies. Most important, this framework can potentially be used for more than just two types of jobs. Finally, we completely characterize all types of equilibrium generated by the

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2 Their model is built upon earlier contributions to the heterogeneous workers and heterogenous jobs search literature such as McKenna (1996) and Gautier (1999).
model by providing conditions on the parameters and contact rates needed for each type of equilibrium, and this includes the the parameter range in which multiple equilibria can happen.

More generally, our paper is connected to the literature about search frictions with heterogeneous workers and jobs. Besides Albrecht and Vroman (2002), the closest paper to ours is Acemoglu’s (1999) model that endogenize the creation of good and bad jobs, but contact rates are assumed to be constant. In this vein, Uren and Virag’s (2011) model also allow for heterogeneous workers and firms, and workers meet firms at an exogenous Poisson arrival rate. Acemoglu’s (2001) model differs from ours because the workers are homogeneous and the firms endogenous choice of good and bad vacancies hinges on their different vacancy creation costs. To a lesser extent, our paper is also connected to the directed search literature like Mortensen and Pissarides (1999) and Shi (2002), but in these models the high skill workers do not affect low skill workers directly by potentially accepting bad jobs, and thus does not generate the within group wage heterogeneity discussed earlier. Finally, our paper is also related to the migration and human capital accumulation literature like Lucas (2004). His model differs from ours by having endogenous human capital accumulation choice and a perfectly competitive labor market. Nevertheless, according to Lucas (2004) an important aspect that a migration model must exhibit is that some of the migrants attracted to the city by a higher expected income flow may end up having low skill being stuck in bad jobs, and our model is able to generate this result by introducing heterogeneous jobs and search friction in the urban market.
3 The Environment

We consider a small open economy comprising two separate spheres of economic activity: an agricultural (rural) sector, indexed $A$, and an urban sector, indexed $B$. The agricultural sector produces food and is perfectly competitive, whereas the urban sector produces a homogeneous manufactured good but is characterized by labor market search frictions. The small-economy assumption implies that agents are price takers in world markets and hence treat the competitive world interest rate, $r_0 > 0$, the price of the agricultural good, $p_A \equiv p$, and the price of the manufactured good, $p_B = 1$, as givens, where, for convenience, the latter serves as the numeraire.

In the remainder of this section, we describe the population, specify agents’ preferences, and delineate the principal features of the two sectors.

3.1 The Primitives

The Population

The size of the population is time invariant and its mass is normalized to unity. Each individual faces a constant hazard of death, $d \geq 0$. This and the posited unit mass population imply $d \geq 0$ must also equal the flow of new births. With a slight abuse of notation, we use the two sectoral indices, $A$ and $B$, also to connote population masses. Hence, at any given point in time, $t$, the disposition of the population is

$$1 \equiv A(t) + B(t),$$

where $A(t)$ and $B(t)$ are the respective sizes of the agricultural and urban populations.

We assume migration from one sector to the other is costless and can be effected instantaneously. Nevertheless (as elaborated shortly), finding urban employment calls for time consuming and so costly search. We require that workers physically must be present in the urban sector to acquire a job there, which rules out arms-length search. The main force of this latter assumption, however, is simple: it dichotomizes the two labor markets, thereby allowing us to identify the size of the working population in each sector with the size of its resident population.

Preferences

We assume that all agents are risk neutral and the pure rate of time preference equals

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3 As a practical matter, clearly migration often is associated with substantial pecuniary and psychological costs. Our results would hold a fortiori were these costs formally incorporated. We do not, however, to better isolate the role of search frictions in equilibrating rural–urban migratory flows.
the world interest rate, \( r_0 > 0 \). Given the death rate is \( d \), individuals discount the future at the constant effective rate \( r \equiv r_0 + d > 0 \). In what follows, we refer to \( r > 0 \) simply as the discount rate.

Each individual is endowed with a unit of labor, which he or she supplies inelastically without disutility, and derives instantaneous utility, \( u(t) \), by consuming the agricultural product (food), \( c(t)_A \), and the manufactured good, \( c(t)_B \), which are procured at the given world prices \( p \equiv p_A \) and \( p_B = 1 \). Instantaneous utility is given by the following constant elasticity of substitution (CES) function:

\[
 u(t) = \left[ c(t)_A^{\frac{\sigma-1}{\sigma}} + c(t)_B^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{2}
\]

where \( \sigma > 1 \) is the elasticity of substitution between the two goods.

At each point in time, \( t \), individuals maximize (2) by using their flow incomes, \( \iota(t) \), to purchase an affordable basket of goods, containing the optimal mix of the two goods.\(^4\) Routine calculations reveal the solution to the problem of maximizing (2) subject to the instantaneous budget constraint \( \iota(t) \geq p \cdot c(t)_A + c(t)_B \) is given by \( c(t)_A^* = \iota(t) \cdot \left( p^{-\sigma} / \hat{P} \right) \) and \( c(t)_B^* = \iota(t) / \hat{P} \), where \( \hat{P} \equiv 1 + p^{-(\sigma-1)} \) is the CES price aggregator, which properly accounts for the consumer’s ability to substitute between the two goods.

Substituting the optimal consumption levels into (2) yields the consumer’s maximized flow utility, \( v^*(p, \iota(t)) \):

\[
v^*(p, \iota(t)) \equiv c^*(p) \cdot \iota(t), \tag{3}
\]

where \( c^*(p) \equiv \left[ p^{(\sigma-1)} + 1 \right]^{-\frac{1}{\sigma-1}} \). We interpret \( c^*(p) \) as a consumption index, since it equals the individual’s maximized instantaneous utility per dollar of expenditure on a consumption basket containing the optimal mix of the two goods. This index plays a limited role in the development of the model’s equilibrium properties; individuals are risk neutral and hence, for each price, \( p \), it just scales their flow incomes by \( c^*(p) \). It does, however, play a more substantive role in the welfare analysis because subsequently we explore the effects of the imposition of import tariffs and export subsidies on agricultural goods, which affect the prices perceived by domestic producers and consumers. Numerous developing economies have used these and other related policies in an attempt to manage their internal rural–urban migratory flows.

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\(^4\) More precisely, given any level of flow expenditures, agents choose \( c(t)_A \) and \( c(t)_B \) to maximize their instantaneous utilities. Yet, since individuals neither borrow nor lend in equilibrium, flow expenditures equal flow incomes.
3.2 The Agricultural Sector

The agricultural sector is competitive, consisting of many profit maximizing producers (farms) that are negligible in the continuum. Aggregate agricultural output, \( Y(t)_A \), is produced using only two homogeneous factors of production: labor, \( A \), and land, \( \Lambda \). As previously noted, agricultural goods are sold on competitive world markets at the given world price \( p \). Factor markets are competitive, and each farm can hire labor at the given flow wage \( w(t)_A \) and rent the land it uses at the given flow rental rate of \( R(t)_\Lambda \) per acre.

Since workers supply their labor inelastically and without disutility from effort, the supply of agricultural labor equals the size of the rural population, \( A(t) \). We assume that all individuals are equally productive whenever they are employed as agricultural labor. The supply of arable land, denoted \( \Lambda_0 \), is exogenously given, used only in the agricultural sector, and is of uniform quality. We assume, that both the ownership of arable land and the ownership shares of agricultural producers are distributed uniformly across the unit mass population.\(^5\) The instant an agent dies, his ownership shares are transferred to a newly born non-filial agent. The rental market for land is competitive and land ownership is alienable, hence a land owner can rent his land to an agricultural producer at the competitively determined rental rate \( R(t)_\Lambda \) per acre or sell it at the price of \( p(t)_\Lambda \) per acre.

Each producer has access to an identical production technology that exhibits constant returns to scale (CRS) in land and labor. The flow of aggregate agricultural output is given by the following production function: \( Y(t)_A = F(A(t), \Lambda(t)) \), where, \( \Lambda(t) \) is the land used in agriculture and, for the moment, we suppose (but later verify) the supply of labor, \( A(t) \), equals total employment. The CRS assumption implies that \( F(\cdot) \) is homogenous of degree one. We assume that \( F(\cdot) \) is strictly concave and satisfies the boundary condition \( F(A(t), \Lambda(t)) = 0 \) whenever \( A(t) \cdot \Lambda(t) = 0 \). In addition, we impose Assumption 3.1 to preclude \textit{a priori} an assortment of economically uninteresting pathological boundary equilibria, such as non-existent urban or agricultural sectors.

\(^5\) Agricultural producers earn zero profits in equilibrium, so the distribution of ownership is inconsequential (provided it is consistent with the posited competitive environment). In practice, land ownership often is highly concentrated. Yet, as we shall see, the ownership distribution affects neither the flow of aggregate rental income, the sectoral allocation of resources, nor aggregate welfare. Hence the posited uniform ownership distribution just is an analytic convenience.
Assumption 3.1 (Agricultural Technology)

The agricultural production technology, $F(\cdot)$, possesses the following properties:

(a) $\lim_{A \to 0} F_A = \lim_{A \to 0} F_{A} = \infty$, 
(b) $\lim_{A \to 1} F_A = 0$, and  
(c) $\lim_{A \to A_0} F_A > 0$, 

where the subscripts on $F(\cdot)$ connote partial derivatives.

Assumption 3.1(a) implies farms hire positive measures of land and labor, thereby ensuring a nontrivial agricultural sector in any putative equilibrium. Assumption 3.1(b) performs a similar function but for the urban sector. Finally, Assumption 3.1(c) ensures that all of the available land is used in competitive equilibrium. Next, we describe the primitives pertinent to the urban sector.

3.3 The Urban Sector

The urban labor market is characterized by search frictions, implying it takes time for unemployed workers and open vacancies to consummate employment matches. In this section, we describe the urban population, present the production technologies available to urban firms, and describe the employment relation. In the one that follows, we describe the process of job search.

The Urban Population

At any given instant, $t$, each member of the urban workforce, $B(t)$, belongs to one of the three possible states: either he is acquiring human capital (schooling), $S$; employed, $E$; or unemployed, $U$. Using these indices also to represent population measures, the disposition of the urban workforce is

$$B(t) \equiv S(t) + E(t) + U(t).$$

The urban labor force, $L(t) = E(t) + U(t)$, consists of those members of the workforce, $B(t)$, who either are employed or unemployed. All workers are equally productive as agricultural labor, but, as described next, they differ in their abilities, $a \in A \subset \mathbb{R}^+$, to perform urban jobs.

Human Capital Accumulation

All individuals who plan to work in the urban sector—both agricultural migrants and those born there—first must pass through a human-capital accumulation or schooling state, $S$, that equips them with the skills required for employment there. The flow cost of education is normalized to zero. We assume all workers are ho-

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6 The educational process has several interpretations. For example, it might entail enrolling in a formal institution of education, such as a school or college. Alternatively, it might represent the investment of time to build social capital, such as informal network of contacts that provides valuable information about prospective job opportunities.
mogeneous, *ex ante*, when they enter the schooling state. However, as described in Assumption 3.2, the schooling process generates *ex post* productive heterogeneity.

**Assumption 3.2 (Human Capital Accumulation)**

(a) Each member of the schooling population, \( S(t) \), acquires the skills necessary for urban employment with constant flow probability \( \lambda > 0 \). At the point of ripening the worker’s ability (or skill level), \( a \in A \equiv \{a_{\ell}, a_{h}\} \subset \mathbb{R}_+ \), is publicly revealed. We impose \( a_h > a_{\ell} \geq 0 \), and, in what follows, refer to \( a_h \) as a *high ability*, to \( a_{\ell} \) as a *low ability*, and to \( i \in I \equiv \{\ell, h\} \) as the worker’s (ability) type.

(b) We assume each worker’s realized ability is an *i.i.d.* draw from a common (discrete) probability distribution, where \( \gamma(a)_0 \in (0, 1) \) is the probability he or she draws \( a \in A \). For the types pertinent to our model, we set \( \gamma_{0i} \equiv \gamma(a_i)_0 \).

**Remarks**

(i) The stochastic ripening process is well-suited to our continuous-time birth–death framework. We equally well could posit a deterministic time horizon, \( s > 0 \), and modify (but also encumber) the value functions by including \( \exp[-r \cdot s] \) to properly discount them for the fixed horizon.

(ii) Here, skill-acquisition is treated as a somewhat mechanical process that churns out workers with differing *ex post* abilities. Indeed, the only substantive function of the current setup (in comparison to one that simply imposes *ex ante* productive heterogeneity from the outset) is that it ensures agricultural migrants face a costly time delay before they can enter the urban labor market. Since explicit migration costs are zero, this feature of the model serves to check the flow of workers between the two sectors.

The motivation for the current structure is found later, in Section ??, where we assume each individual chooses a costly schooling effort level, \( s \), that determines the distribution of realized *ex post* abilities. This richer environment allows us to explore the endogenous feedback effects that exist between skill acquisition and job quality.

Assumption 3.2 is consistent with several different interpretations of the way in which the acquisition of human capital can induce *ex post* productive heterogeneity. At one extreme, workers indeed might be homogenous *ex ante*, but because of different educational experiences—resulting from, for example, heterogeneous school qualities—their skills could differ *ex post*. At the other, they might possess heterogenous but unknown given abilities *ex ante*, and the educational system might then serve just as a sorting device that publicly certifies their innate abilities *ex post*.

After they complete the educational phase, individuals enter the urban labor market in search of employment.\(^7\) The sampling process, described in Assumption 3.2 (b), implies \( \gamma(a)_0 \) equals the proportion of type \( a \in A \) workers in the labor force. Suppressing the time index to ease the notational burden, we denote the respective masses of unemployed and employed type \( a \in A \) workers by \( U(a) \) and \( E(a) \). The (endogenous) proportion of this worker type who are unemployed is

\(^7\) We establish that, in equilibrium, they do not migrate to the agricultural sector once their skills are realized.
The $\gamma(a)$'s, play an important role in the subsequent analysis, since each of them equals the probability that a worker selected at random from the unemployment pool is type $a \in A$. Again, for the discrete types pertinent to our model, we set $U_i \equiv U(a_i)$, $E_i \equiv E(a_i)$, and $\gamma_i \equiv \gamma(a_i)$.

**Technology**

The urban sector is populated by firms that produce the manufactured good and, in the process, generate job opportunities for urban workers. The firms belonging to this sector are owned by households and, inconsequentially, we assume ownership is distributed uniformly among the unit-mass population. Each job employs at most one worker, so at any given instant, $t$, either it is filled, $E$, or vacant, $V$—a filled position necessarily employs a worker, so the index $E$ fulfills a dual role without ambiguity. For added variety, we use the terms *firm*, *job*, and *employer* interchangeably.

An urban firm can produce the manufactured good by adopting one of two different available production technologies, indexed $k \in K \equiv \{b, g\}$. Here $k = b$ refers to a technology that, in equilibrium, is associated with the creation of *bad jobs* and $k = g$ to one associated with the creation of *good jobs*. These designations clearly are tendentious and are based on those features, described in Assumption 3.3, that inhere in each of the technologies.

**Assumption 3.3 (The Manufacturing Technology)**

At the point of creating a new vacancy and entering the labor market, each firm (irrevocably) chooses a specific technology indexed $k \in K \equiv \{b, g\}$.

(a) A firm that adopts a type $k \in K \equiv \{b, g\}$ technology and employs a worker whose ability is $a \in A \subset \mathbb{R}_+$ produces a constant flow output, $y(a)_k$, given by

\[
\begin{align*}
  y(a)_b &= y_0, \quad \forall a \in A \\
  y(a)_g &= a \quad \text{if} \quad a \geq \xi > 0 \quad \text{and} \quad y(a)_g = 0 \quad \text{otherwise},
\end{align*}
\]

where the parameter $\xi > 0$ is an exogenous skills-requirement threshold.

(b) Firms freely and instantly can enter or exit the labor market. Each firm incurs the constant flow cost $\nu > 0$ of maintaining an open vacancy, while searching for a worker.

(c) Because of idiosyncratic technology shocks, each firm shuts down with a constant flow probability $\delta > 0$. In this event, displaced workers reenter the labor market in search of alternative employers.

Part (a) of Assumption 3.3 implies bad jobs can hire any type of worker but good

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8 In general, $\gamma(a) \neq \gamma(a)_0$ because the escape rate from unemployment generally varies with ability.

9 The aggregate flow of profits is zero in equilibrium, so the posited ownership distribution is a convenience. Just as for the agricultural sector, we assume that if an agent dies, his shares are allocated uniformly across the inflow of newly born non-filial agents.
jobs only are productive if they hire those with abilities \( a \geq \xi > 0 \). In what follows, we characterize the economically relevant attributes of a type \( a \in \mathcal{A} \) worker by \( y(a) \equiv (y_0, y(a)_g) \), which equals the flow output he or she produces in bad and good jobs respectively.

In Assumption 3.3(c), we envisage that, at the point of entry, a firm creates a job by irrevocably choosing a type \( k \in K \) technology and interpret \( \nu > 0 \) as the flow cost of its subsequent recruitment efforts. As will become clear, the fact that both good and bad jobs incur the same flow cost, \( \nu \), is theoretically harmless but pedagogically useful, since it permits a sharper focus on the effects of the productivity–liquidity effects just described. The free entry assumption implies the equilibrium value to a firm of creating a vacancy and entering the labor market in search of a prospective employee is non-positive for each technology, \( k \in K \).

In practice, employment matches end for a variety of reasons, including idiosyncratic technology shocks (see, for example, [?]), preference shifts, and shocks to households’ home-production technologies. In the interests of simplicity, we model these separations by assuming, in part \( (d) \), that firms dissolve at the constant exogenous rate \( \delta > 0 \). Employment clearly ends also with the employee’s death, which occurs with flow probability \( d > 0 \). To simplify the subsequent arithmetic, we assume, inconsequentially, the firm also exits the market in this event. Together job destruction and worker deaths imply extant employment matches dissolve at the constant flow rate \( \delta_0 \equiv (\delta + d) \). These features of the model ensure the labor market always is populated by positive measures of unemployed workers and unfilled vacancies.

The respective masses of open vacancies and filled jobs are denoted \( V(t) \) and \( E(t) \). We denote the mass of type \( k \in K \) jobs currently filled by type \( i \in I \) workers by \( E(t)_{ik} \) and use the shorthand \( y_{ik} \equiv y(a_{i})_k \) to represent the output generated from such a match. With an obvious extension of this notation, the mass of open type \( k \in K \) vacancies is \( V(t)_k \). The (endogenous) proportion of open vacancies offering type \( k \in K \) jobs is defined by \( \phi(t)_k \equiv V(t)_k/V(t) \in [0, 1] \).

**Good Jobs and Bad Jobs.** Next we characterize the relative productivity differentials that exist between the two technologies for the high and low worker ability

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10 As we shall see, the value of an open searching vacancy is zero, implying each firm is willing to hire any worker type at a wage of zero. To preclude certain pathological cases, we assume workers never accept employment at non-positive wages. This can be formally justified by assuming they derive some positive value, \( \varepsilon > 0 \), from leisure.

11 Alternatively, we could posit that, at the point of entry, the firm acquires a unit of capital that is specific to each technology, \( k \in K \). In this case, \( \nu \) is the rental rate of capital and, in Assumption 3.3, \( y(a)_k \equiv \hat{y}(a)_k - \nu \) would represent the flow of output net of capital expenditures—the circumflex connoting the implicit gross flow.
types that are pertinent to this model. Consider Assumption 3.4.

**Assumption 3.4 (Workers and Jobs)**

(a) We assume that for high and low ability workers

\[ y(a_h)_g = a_h \geq \xi > y_0 \geq y(a_\ell)_g = a_\ell. \]

(b) Define the net productive advantage of good jobs, \( \Delta > 0 \), by \( y(a_h)_g / y(a_h)_b \equiv (1 + \Delta) \). We assume \( \Delta < \Gamma_0 \), where \( \Gamma_0 \equiv \gamma_0\ell / \gamma_0h \) is the proportion of low ability workers in the population at large.

Condition (5), presented in part (a), is apt for our two-by-two framework and is imposed to avoid the need for a somewhat tedious encyclopedic analysis. In particular, the condition \( a_h \geq \xi > a_\ell \) ensures high ability workers are employable in both types of job, but low ability workers only can work in bad jobs. Furthermore, the fact that \( y(a_h)_g > y_0 \) is the technological foundation of the designation good job: upon hiring a suitable worker, a firm that adopts the technology \( k = g \) unequivocally is more productive than one that adopts \( k = b \).

As defined in (b), \( \Delta > 0 \) is the net productive advantage of good jobs vis-à-vis bad ones. Although these firms enjoy this productive edge, bad jobs face a more liquid labor market; they can employ workers of either ability, whereas good jobs only can employ high ability workers. It follows that it potentially is less costly for bad jobs to fill their vacancies than it is for good ones. Indeed, the fundamental properties of the good jobs–bad jobs framework hinge on the ramifications of this productivity–liquidity tradeoff because it means that neither technology is unambiguously dominant, so both may be adopted in equilibrium.

Nevertheless, if the productivity differential, \( \Delta > 0 \), were sufficiently large and if high ability workers were sufficiently abundant in the overall worker population, then good jobs would not only be more productive but also find it relatively easy to fill their vacancies. As a result, they might economically eclipse the bad-job technology altogether, implying it would never be adopted in any putative equilibrium. Accordingly, in (b) we impose the restriction \( \Gamma_0 > \Delta \), which is sufficient to preclude this possibility. Intuitively, \( \Gamma_0 \) governs the relative abundance of low ability workers in the labor force and, as a corollary, the relative scarcity of high ability workers. In this light, the assumption just ensures the scarcity of high ability workers is enough of counterweight to the productive edge of the good-job technology.

\[ \text{Indeed, were } a_h < y_0, \text{ then supposed good jobs would be technologically inferior to bad ones; they would be not only less productive but also encumbered with more restrictive hiring requirements.} \]
to ensure the both are potentially viable.

**The Employment Relation.** All agents are risk neutral, information is perfect, there are no idiosyncratic shocks that disrupt firms (as opposed to destroying them), and the level of flow output is constant, which implicitly precludes on-the-job human capital investments. If we add to this list the facts that workers supply their labor inelastically and do not value non-wage job attributes, it follows the nature of the employment relation must be quite rudimentary.\(^{13}\) Indeed, in this rather Spartan setting, workers are concerned only with their flow wages and employers only with their implied flow profits that accrue over the course of the match.

Accordingly, let \(w(a)_k\) and \(\pi(a)_k \equiv y(a)_k - w(a)_k\) denote the flow wage and level of flow profits that arise from the employment of a type \(a \in A\) worker by a type \(k \in K\) firm. We assume \(w(a)_k\) is determined via a symmetric Nash bargain—nothing of substance hinges on the posited symmetry—and is continuously renegotiated over the course of the match. Furthermore, we develop the properties of the model under the assumption that \(w(a)_k\) is constant for the duration of each match, leaving to the appendix the task of verifying that this property arises endogenously. Finally, throughout we impose \(w(a)_k \in [0, y(a)_k]\), thereby restricting our attention to non-negative wages and non-negative profits.\(^{14}\)

### 4 Search, Matching, and Steady State Populations

Search frictions inhere in the urban labor market, impeding the formation of employment matches. In this section, we describe the nature of these frictions and derive the model’s steady-state populations. In what follows, we suppress the time index, \(t\), whenever possible, to simplify the notation.

#### 4.1 Job Search

We assume that search is undirected, in the sense that each open vacancy, regardless of its type, \(k \in K\), has a uniform chance of meeting any of the unemployed workers who belong to the unemployment pool, \(U\). Likewise, each unemployed worker, regardless of his or her type, \(i \in I\), has a uniform chance of meeting any of the open vacancies belonging to the population \(V\). Unemployed workers encounter

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\(^{13}\) For instance, neither severance pay nor performance bonds play any useful role in this environment.

\(^{14}\) The restriction is innocuous because workers and firms can assure themselves payoffs of zero by dropping out the labor force and exiting the market respectively.
unfilled vacancies at the flow rate $\mu$ and, on the other side of the market, vacancies encounter unemployed workers at the flow rate $\eta$.\textsuperscript{15} The contact rates $\mu$ and $\eta$ are determined in equilibrium but treated by all agents as givens when formulating their individually optimal actions.

Whenever a searching worker encounters an open vacancy, the vacancy necessarily encounters the worker. Hence the following flow identity holds at all points in time:

$$m \equiv \eta \cdot V \equiv \mu \cdot U,$$

where $m$ is the flow of contacts between unfilled vacancies and unemployed job seekers. Most important, as described later, there is no presumption the contact between a job seeker and an unfilled vacancy necessarily leads to the formation of an employment relationship.

**Matching**

Assumption 4.5 presents the properties of what now might be deemed a standard matching technology governing the flow of worker–firm contacts, $m$.

**Assumption 4.5 (The Matching Technology)**

The flow of worker–firm contacts, $m$, is governed by the matching technology,

$$m = M(U, V; m_0),$$

where $m_0 > 0$ and $M(\cdot; m_0) < 0$. The function $M(\cdot)$ is strictly concave, homogeneous of degree one in $U$ and $V$, satisfies the boundary condition $M(U, V) = 0$ if $U \cdot V = 0$, and the Inada conditions.\textsuperscript{16}

In equation (7), the parameter $m_0 > 0$ captures the severity of the search frictions that are inherent in the urban labor market: for any given populations of searchers, $U$ and $V$, an increase in $m_0$ reduces the rate of contacts, $m$, between them. It is included so later we can better understand the effects of these frictions on equilibrium outcomes. The boundary and Inada conditions rule out several economically uninteresting cases. Thus matches occur only if both sides of the market are active, since $M(U, V) = 0$ whenever $U \cdot V = 0$, and the Inada conditions steer

\textsuperscript{15}For simplicity, employed workers are assumed to devote their entire unit time endowment to production activities, thereby precluding on-the-job search.

\textsuperscript{16}Specifically, if $U \cdot V > 0$, then $\lim_{U \to 0} M_U \to \infty$, $\lim_{V \to 0} M_V \to \infty$, $\lim_{U \to \infty} M_U \to 0$, and $\lim_{V \to \infty} M_V \to 0$. 
the market away from the $U \cdot V = 0$ boundaries.

The Steady-State Matching (SS) Locus

Together, the identity (6) and the matching technology, described in Assumption 4.5, impose considerable structure on the locus of admissible contact rates, $\mu$, and $\eta$. Thus, (6) implies $U/V \equiv \eta/\mu$ and (7) implies $\eta = M(U, V ; m_0)/V = M(U/V, 1; m_0)$ because $M(\cdot)$ is homogeneous of degree 1. Together these facts imply $\eta = M(\eta/\mu, 1; m_0) \equiv m(\eta/\mu; m_0)$, which implicitly defines the steady-state matching locus—or SS locus for short:

$$\eta = \eta^{SS}(\mu; m_0)$$

(8)

The SS locus determines, for each value of $\mu$, the unique firm–worker contact rate, $\eta$, that is consistent with the matching technology. The properties of this function follow directly from Assumption 4.5 and they are summarized in Lemma 4.1.

Lemma 4.1 (The SS Locus)

Assumption 4.5 implies the SS locus $\eta = \eta^{SS}(\mu; m_0)$ possesses the following properties:

$$\eta^{SS}(\cdot)_\mu < 0, \quad \eta^{SS}(\cdot)_{\mu\mu} > 0, \quad \eta^{SS}(\cdot)_{m_0} < 0, \quad \lim_{\mu \to 0} \eta^{SS}(\cdot) \to \infty, \quad \text{and} \quad \lim_{\mu \to \infty} \eta^{SS}(\cdot) \to 0,$$

where the subscripts on $\eta^{SS}(\cdot)$ denote first- and second-order partial derivatives.

In essence, the SS locus (see Figure ?? on page ??) corresponds to the urban labor market’s Beveridge curve; however, it has been cast in the contact rate space, $(\mu, \eta)$, rather than the more usual unemployment–vacancy space, $(U, V)$. It is one of the model’s principal partial equilibrium constructs. (In anticipation, the other is an equilibrium entry, $EE$, locus that governs the $(\mu, \eta)$ pairs consistent with the free entry assumption. Together these loci determine the equilibrium contact rates.)

Employment Accession

Not all worker–firm meetings result in employment. For instance, already we have indicated low ability workers do not work in good jobs. Furthermore, high ability worker may reject the job offers made by one of the technologies because they are productive in good and bad jobs.

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17 Using $\eta = m(\mu/\eta; m_0)$, it is possible to write the contact rates as $\eta = m(\theta; m_0)$ and $\mu = \theta \cdot m(\mu; m_0)$, where $\theta \equiv U/V \equiv \eta/\mu$ denotes the tightness of the labor market. This formulation is intuitively appealing and has gained considerable traction in the search literature. Nevertheless, as we will see, it is simpler to develop the equilibrium properties of this model using (8) instead.
To capture this flexibility, suppose, at the instant of contact between a type \( k \in K \) vacancy and a type \( a \in \mathcal{A} \) job seeker—in the interests of brevity, henceforth, *an \((a, k)\) match or meeting*—the firm offers to employ the worker at the wage that is agreed under the symmetric Nash bargaining procedure.\(^{18}\) Assume the *average* worker in this group accepts the job offer with probability \( Q(a)_k \in [0, 1] \) and a *particular* worker optimally chooses his acceptance probability, \( q(a)_k \in [0, 1] \). If the offer is rejected, the match dissolves and the two parties reenter the labor market in search of alternative partners.\(^{19}\)

The acceptance probabilities of the average worker, \( Q(a)_k \), are aggregate objects that agents treat as parametric when formulating their individually optimal actions. They can be used to derive workers and firm’s *employment accession rates*—the flow probabilities, denoted \( \mu(a) \), at which unemployed type \( a \in \mathcal{A} \) workers enter the employment state. The accession rates govern the evolution of the masses of active searchers over time and hence are integral to the determination of the model’s steady-state populations.

The employment accession rate of a type \( a \in \mathcal{A} \) worker, \( \mu(a) \), is given by

\[
\mu(a) = \mu \cdot E_k \left[ Q(a)_k \right],
\]

where \( E_k \left[ \cdot \right] \equiv \sum_{k \in K} \{ \cdot \} \phi_k \). Equation (9) has a simple interpretation: \( \mu \) is the flow probability the worker encounters prospective employers, and the expectation equals the probability he is employed in this event.\(^{20}\) For the types pertinent to our model, we set \( \mu_{\ell} \equiv \mu(a_{\ell}) \) and \( \mu_{h} \equiv \mu(a_{h}) \).

### 4.2 The Steady-State Populations

At each point in time, \( t \), the unit mass of agents is divided between the rural and urban sectors. The latter population is further subdivided into those who are acquiring human capital, employed, or unemployed. The membership of each of these groups is in constant flux, as new people are born and as others die, leave school, find work, or their extant jobs dissolve. We focus on steady-states in which all populations and contact rates are time invariant.

#### The Evolution Equations

The evolution of each population depends on birth–death rates, the accession rates,
and the population proportions. For instance, the equation of motion describing the evolution of the mass of unemployed high ability workers, $U_h$, is
\[ \dot{U}_h = \lambda \cdot S \cdot \gamma_{0h} + \delta \cdot E_h - (\mu_h + d) \cdot U_h, \]
where $\dot{U}_h$ is the instantaneous rate of change of $U_h$. The differential equation just given has a standard inflow–outflow interpretation: high ability workers enter the unemployment state after they lose their current jobs, or after they complete the schooling phase of their lives, their abilities are revealed, and they enter the labor market in search of work.\(^{21}\) They exit this state after they find work or, less propitiously, die.

The evolution equations for the others have analogous forms and interpretations and are presented in the Appendix. Lemma 4.2 uses them to derive the model’s principal steady-state populations.

**Lemma 4.2 (The Steady-State Populations)**

Given the urban workforce, $B \in [0, 1]$, the steady-state population masses are

(a) (The Labor Force) $L = B - S = U + E$

(b) (The Schooling Population) $S = \left( \frac{d}{d + \lambda} \right) \cdot B$.

(c) (Unemployment) $U = U_\ell + U_h$, where

\[ U_\ell = \left( \frac{\delta_0}{\delta_0 + \mu_\ell} \right) \cdot \gamma_{0\ell} \cdot L, \quad \text{and} \quad U_h = \left( \frac{\delta_0}{\delta_0 + \mu_h} \right) \cdot \gamma_{0h} \cdot L, \]  

(10)

where $\mu_h$ and $\mu_\ell$ are the respective employment accession rates of high and low ability workers.

(d) (Employment) $E \equiv E_\ell + E_h = L - U$, where $E_\ell = \gamma_{0\ell} \cdot L - U_\ell$ and $E_h = \gamma_{0h} \cdot L - U_h$.

In (10), recall $\delta_0 \equiv (\delta + d)$ equals the rate at which extant employment matches dissolve—employers exit the market at the rate $\delta$, workers die at the rate $d$, and in both cases current employment matches necessarily end. All else equal, an increase in $\delta_0$ raises the steady state unemployment level. Notice too, for each $i \in I$, the size of the population $U_i$ depends negatively on the corresponding employment accession rate, $\mu_i$, since it governs the ease with which unemployed workers escape this state by forming employment matches.

The fraction of unemployed workers who have low abilities, $\gamma_\ell = U_\ell / U$, is read-

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\(^{21}\) The instantaneous flow of workers who enter the labor market in search of work from the schooling sector is $\lambda \cdot S$. A fraction $\gamma_{0h}$ of them possess high abilities. Hence the flow of high ability workers who leave schooling and enter the labor force is $\gamma_{0h} \cdot (\lambda \cdot S)$. 

Page 18
Corollary 4.1 The Composition of the Unemployment Pool
The steady-state population proportions, $\gamma_\ell$ and $\gamma_h$, are given by

$$
\gamma_\ell = \gamma_0 \cdot \left[ \frac{\delta_0 + \mu_h}{\delta_0 + \gamma_0 h \cdot \mu \ell + \gamma_0 \ell \cdot \mu_h} \right] \quad \text{and} \quad \gamma_h = \gamma_0 h \cdot \left[ \frac{\delta_0 + \mu_\ell}{\delta_0 + \gamma_0 h \cdot \mu \ell + \gamma_0 \ell \cdot \mu_h} \right].
$$  \hspace{1cm} (11)

Since search is undirected, $\gamma_\ell$ equals the probability that a searching firm encounters a low-ability worker in any given worker–firm meeting. However, low ability workers are not productive in good jobs. Hence $\gamma_\ell$ is a principal determinant of the relative value to a firm of adopting either of the two available technologies. Because of this property, we have good reason to consider subsequently the ratio of low to high ability workers in the unemployment pool, $\Gamma \equiv \gamma_\ell/\gamma_h$. It is immediate from Corollary 4.1 that $\Gamma$ is given by

$$
\Gamma = \Gamma_0 \cdot \left[ \frac{\delta_0 + \mu_h}{\delta_0 + \mu_\ell} \right],
$$  \hspace{1cm} (12)

where, as described earlier, $\Gamma_0 \equiv \gamma_0 \ell/\gamma_0 h$ equals the ratio of low to high ability workers in the population at large.

Equation (12) both is simple and intuitive. For example, all else equal, an increase in the employment accession rate of high ability workers, $\mu_h$, lowers $\Gamma$, since there are fewer unemployed high ability workers and the mass of unemployed low ability workers remains unchanged—see equations (10). Similar but opposite remarks apply to the effects of an increase in $\mu_\ell$. From a given firm’s vantage point, the population ratio $\Gamma$ measures the relative abundance of low ability labor or, equivalently, the relative scarcity of high ability workers. Under either interpretation, $\Gamma$ is of fundamental importance to our analysis.

5 Values, Wages, and Accession Rates

In this and the next section, we develop and characterize certain important partial equilibrium relationships that we later use, in Section ??, to establish the existence of an equilibrium and characterize its properties. The following offers a road map of our approach:

1.) First, we derive the properties of the competitive agricultural equilibrium, and, in the urban sector, the wages agreed under the Nash bargain and workers’ optimal employment decisions conditional on given values of the model’s
aggregates—the contact rates $\mu$ and $\phi$, the populations, and the population proportions $\phi_k$ and $\gamma(a)$.

2.) Again treating $\mu$, $\eta$, and $\phi_k$ as given, we use the optimal (*) behavior of workers and the wages derived in step 1. to determine the steady-state populations, $U^*_i$ and $E^*_i$, the population proportions, $\gamma(a)^*$, and population ratio, $\Gamma^*$, using Lemma 4.2, Corollary 4.1, and equation (12) respectively.

3.) In this step, we treat the worker–firm contact rate, $\mu$, as given and derive the proportion of each job type, $\phi_k^*$, in the population of active vacancies, $V$, that is consistent with the free entry assumption and the behavior determined in the first two.

4.) The first three steps just described culminate in the construction of a partial equilibrium relationship—the equilibrium entry (EE) locus—that governs the admissible contact rates, $\mu$ and $\eta$, that are consistent with worker’s optimal employment choices, the wages agreed under the Nash bargain, the implied steady state populations and population proportions, and the posited free-entry assumption.

In Section ??, we use the EE and SS loci to determine the equilibrium values of the contact rates, $\mu^*$ and $\eta^*$. Once they have been determined, we then can exploit the model’s recursive structure to educe the general equilibrium properties of all of the model’s remaining endogenous variables. Our first order of business, however, is determining agents’ expected present discounted values—values for short—because they govern their optimal actions.

Values

An agent’s current value depends on both his type and the state he occupies—a type is immutable but the state can change. Hence, in the context of this model, a worker’s type corresponds to his ability, $a \in A$, and a firm’s to the technology it has adopted, $k \in K$. As for the state, workers either are rural, $A$, or urban, $B$, residents, and, in this latter case, either they are employed, $E$, unemployed, $U$, or acquiring human capital, $S$. Likewise urban jobs either are filled, $E$, or vacant, $V$.

In what follows, we represent workers’ and firms’ respective values by $J$ and $\Pi$, suitably indexed by the state and type. Definition 5.1 presents the notation used to

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22 Under this taxonomy, a type just is an absorbing state—one that an agent enters but cannot exit.
represent the complete array of values pertinent to the model.

**Definition 5.1** (Values)
Workers’ values correspond to agricultural employment, $J_A$; the acquisition of human capital, $J_S$; and for each worker type $a \in A$, unemployment, $J(a)_U$ and employment by a type $k \in K$ firm, $J(a)_E_k$. For each type, $k \in K$, firms’ values correspond to those of an unfilled vacancy that is searching for a worker, $\Pi_{V_k}$, or to a position currently occupied by a type $a \in A$ worker, $\Pi_{E_k}$.

Next, we characterize the properties of the agricultural sector and determine the value to a worker of agricultural employment, $J_A$.

### 5.1 The Agricultural Sector

In this section, we develop those properties that necessarily characterize any competitive agricultural equilibrium, at each instant in time, conditional on a given constant rural population (and so rural labor supply), $A > 0$.

**The Competitive Equilibrium**

Each farm optimally hires labor and land to maximize its profits, taking the product price, $p$, the flow wage, $w_A$, and land rental rate, $R_A$, as givens. Since the technology exhibits CRS, the optimal scale of each farm is indeterminate. Despite this micro-level indeterminacy, however, given the factor supplies, $A$ and $\Lambda_0$, the model’s ag-

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23 It is sufficient to consider $A > 0$, for if $A = 0$, then the farming sector is inactive because $Y_A = 0$ for all $\Lambda > 0$. 

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aggregate properties are well defined. Lemma 5.3 describes them.

**Lemma 5.3 (The Agricultural Sector)**

Given the available supplies of agricultural labor, \( A \in (0, 1] \), and arable land, \( \Lambda_0 > 0 \),

(a) There is full employment of both land and labor. Each agricultural producer earns zero profits and the equilibrium land rental rate, \( R^*_\Lambda \), and land price, \( p^*_\Lambda \), satisfy

\[
R^*_\Lambda = p^*_\Lambda \cdot r.
\]

(b) The equilibrium flow wage, \( w^*_A \), and rental price of arable land, \( R^*_\Lambda \), are given by

\[
w^*_A = \omega(A, p) \quad \text{and} \quad R^*_\Lambda = \rho(A, p)
\]

where \( \omega(A, p) \equiv p \cdot F_A(A, \Lambda_0) \) and \( \rho(A, p) \equiv p \cdot F_\Lambda(A, \Lambda_0) \).

(c) The functions \( \omega(\cdot) \) and \( \rho(\cdot) \) possess the following properties:

(i) \( \frac{\partial \omega(\cdot)}{\partial A} < 0 \), \( \lim_{A \to 0} \omega(\cdot) = \infty \), \( \lim_{A \to 1} \omega(\cdot) = 0 \) and \( \frac{\partial \omega(\cdot)}{\partial p} > 0 \).

(ii) \( \frac{\partial \rho(A, \Lambda_0)}{\partial A} > 0 \) and \( \frac{\partial \rho(A, \Lambda_0)}{\partial p} > 0 \).

**Proof of Lemma 5.3.** All proofs are presented in the Appendix.||

The full employment result reported in Lemma 5.3(a) follows from the limiting properties of \( F(\cdot) \) given in Assumption 3.1. The fact that producers earn zero profits in equilibrium is a direct consequence of Euler’s theorem in general and the CRS assumption in particular. Finally, \( R^*_\Lambda = p^*_\Lambda \cdot r \), is a no-arbitrage condition that reflects the alienability of land. Thus a land owner either can keep his land (thereby reaping the flow rental income of \( R_\Lambda \) per acre), or he can sell it for \( p_\Lambda \) per acre (thereby accruing a flow income \( r \cdot p_\Lambda \) by holding other assets). In competitive equilibrium, these two returns must be equal, for then all land is held and no agent has an incentive to change his or her current land holdings.

From one vantage point, equations (13) just are the familiar marginal benefit equals marginal cost conditions that would govern the interior aggregate profit-maximizing demands for the two inputs given any competitive prices \( p \), \( w_A \), and \( R_\Lambda \). From another (and the one that is pertinent for the Lemma), they pinpoint the equilibrium market-clearing factor prices that ensure the demand for each input exactly equals its positive (and thus necessarily interior) supply.

It follows immediately from (b) that, given a constant level of agricultural employment, the value a worker derives from agricultural employment, \( J_A \), just is the present discounted value of the constant flow wage, \( J_A = \omega(A, p) / r \). Furthermore, the normalized unit population and the posited uniform distribution of land ownership, imply the aggregate flow of rental income, \( \rho(A, p) \cdot \Lambda_0 \), equals the individual flow of rental income.\(^{24}\)

\(^{24}\) As we will see, the income generated from land does not influence the fundamental properties of the equilibrium—i.e.,
Lemma 5.3(c) is the most important for the development of the model’s general equilibrium properties. In particular, (ci) establishes that—because of the posited diminishing marginal returns to labor—the agricultural wage, \( w_A^* \), is strictly decreasing in employment, \( A \). Furthermore, the limiting properties of \( \omega(\cdot) \) ensure each sector remains active (i.e., populated) in any putative equilibrium. Indeed, since \( w_A^* \in (0, \infty) \), it follows that for any given value an individual achieves by living and working in the urban sector there is a unique population, \( A \), that ensures he achieves precisely the same value by living in the rural one, \( J_A \). Although we have yet to delineate the values of urban residents, it is sufficient for now to note that this property drives the equilibrium sectoral allocation of labor, since a population, \( A \), always can be found that ensures no agent has an incentive to switch sectors. Finally, the result that \( \partial \omega(\cdot)/\partial p > 0 \) and those reported in Lemma 5.3(cii) are used in subsequent comparative static exercises.

5.2 The Urban Sector

In this section, we derive the values, wages, and accession probabilities that are pertinent to the urban sector, conditional on given populations, population proportions, and contact rates. As is usual in this type of model, agents’ values are derived as the solution to a system of Bellman equations.

The Bellman Equations

For each ability, \( a \in \mathcal{A} \), and each technology, \( k \in K \), agents’ steady-state values are the sectoral allocation of resources and wages. This income source, however, is an important element of our normative analysis. Thus, any parametric change that induces an adjustment in the equilibrium sectoral allocation of labor generically affects land rents and so welfare.
characterized by the following Bellman equations:

\[ r \cdot J_S = \lambda \cdot \left( \sum_{a \in A} \gamma(a)_0 \cdot [J(a)_U - J_S] \right) \]  
\[ (14a) \]

\[ r \cdot J(a)_{Ek} = w(a)_k + \delta \cdot (J(a)_U - J(a)_{Ek}) \]  
\[ (14b) \]

\[ r \cdot J(a)_U = \mu \cdot \left( \max_{\{q(a)_k\}_{k \in K}} \left[ \sum_{k \in K} \phi_k \cdot q(a)_k \cdot \Delta J(a)_{Ek} \right] \right) \]  
\[ (14c) \]

\[ r \cdot \Pi(a)_{Ek} = \pi(a)_k - \delta \cdot \Pi(a)_{Ek} \]  
\[ (14d) \]

\[ r \cdot \Pi(a)_{Vk} = -\nu + \eta \cdot \left[ \sum_{a \in A} \gamma(a) \cdot Q(a)_k \cdot (\Pi(a)_{Ek} - \Pi(a)_{Vk}) \right] \]  
\[ (14e) \]

\[ \Pi(a)_{Vk} = \max\{0, \Pi(a)_{Vk}\}, \]  
\[ (14f) \]

where \( \Delta J(a)_{Ek} \equiv (J(a)_{Ek} - J(a)_U) \) is the (potentially negative) capital gain accruing to the worker if he accepts the type \( k \) firm’s job offer.

The Bellman equations presented in (14) have familiar forms and so standard interpretations. We consider first the implications for workers.

**Workers.** In (14a), the flow value of schooling, \( r \cdot J_S \), equals the flow probability an agent in this sector acquires the skills necessary for urban employment, \( \lambda \), multiplied by the expected capital gain in this event—the term in parentheses. Here, \( \gamma(a)_0 \) is the probability his realized type is \( a \in A \) and \( J(a)_U - J_S \) is his realized capital gain if it is. Summing the product of these terms across all abilities then equals the individual’s expected capital gain of acquiring human capital and entering the labor market in search of work.

In (14b), the flow value to the worker of employment by a type \( k \in K \) firm equals the flow wage, \( w(a)_k \), plus the capital loss he suffers if his job ends, which occurs with flow probability \( \delta \).\(^{25} \) Rearrangement of (14b) yields the value \( J(a)_{Ek} \):

\[ J(a)_{Ek} = \frac{w(a)_k}{r + \delta} + \left[ \frac{\delta}{r + \delta} \right] \cdot J(a)_U. \]  
\[ (15) \]

The first term on the right-hand side is the present value of the flow wage, \( w(a)_k \), suitably discounted to reflect the fact the current job ends with its destruction by the inclusion of the breakup rate, \( \delta \). The term in square brackets is the time cost of

\(^{25} \) In fact, as written, \( J(a)_{Ek} \) is the worker's value derived under the restriction he does not quit, which would not be optimal were \( J(a)_U > J(a)_{Ek} \). Yet, were this the case, the worker would not have accepted the job in the first place.
waiting for this latter event to happen, at which point the worker reenters the labor market to search for a new job and derives the value $J(a)U$.

According to (14c), the flow value a type $a \in A$ worker derives from job search, $r \cdot J(a)U$, equals the flow probability he meets a potential employer, $\mu$, multiplied by the expected capital gain in this event—the term in round braces. Thus, $\phi_k$ is the probability he meets a type $k \in K$ firm. Conditional on this event, he chooses the probability of accepting the firm’s offer, $q(a)_k$, and he derives the capital gain $\Delta J(a)E_k$ only if he does. Summing the product of these three terms over all employer types, $k \in K$, then equals the capital gain the worker expects if he meets an employer.

**Employers.** Equations (14d)–(14f) govern the values of firms. Rearranging (14d), reveals the value of a type $k \in K$ firm that employs a type $a \in A$ worker, $\Pi(a)E_k$, equals the present discounted value of the firm’s flow profits:

$$\Pi(a)E_k = \frac{\pi(a)_k}{r + \delta} \geq 0,$$

where the inequality follows from the fact that $\pi(a)_k \geq 0$ and the discount factor includes the job destruction rate, $\delta$, reflecting the prospect of the firm’s subsequent demise.

In (14e), $\tilde{\Pi}_V k$ denotes the value of a type $k \in K$ vacancy that searches until its position is filled. The flow value of such a firm equals minus the flow cost of search, $-\nu < 0$, plus the flow probability of encountering a job seeker, $\eta$, multiplied by the expected capital gain in this event (the term in round parentheses). Here, $\gamma(a)$ is the probability the firm encounters an unemployed type $a \in A$ worker, $Q(a)_k$ is the probability the average worker accepts its offer, and $\Pi(a)E_k - \tilde{\Pi}_V k$ is the capital gain that would accrue to the firm were its offer accepted. Summing the product of these three terms across all worker types, $a \in A$, then equals the capital gain the firm expects once it meets a worker.

Because firms incur positive search costs, the value $\tilde{\Pi}_V k$ can be negative—to see this, simply set $\eta = 0$ in (14e). In this event, however, the firm is strictly better off disbanding because it then assures itself a value of zero. Accordingly, (14f) takes the possibility of optimal exit into account and so ensures the value of an active type $k \in K$ vacancy, $\Pi_V k$, must be non-negative. Next, in Lemma 5.4, we establish
it cannot be strictly positive.

**Lemma 5.4 (Free Entry, Free Exit, and Optimality)**

Given the population of vacancies, \( V = \sum_{k \in K} V_k \), extant firms and prospective new entrants are behaving rationally if and only if \( \forall k \in K \) (a) \( V_k \geq 0 \) (b) \( \Pi_{V_k} \leq 0 \), and (c) \( \Pi_{V_k} \cdot V_k = 0 \). This and equation (14f) imply \( \Pi_{V_k} = 0 \).

Clearly, (a) is satisfied trivially. It is included to acknowledge explicitly the possibility that firms may choose not to adopt one of the technologies, \( k \in K \), which would imply \( V_k = 0 \). According to the lemma, rational behavior on the part of new entrants requires \( \tilde{\Pi}_{V_k} \leq 0 \) for every \( k \in K \). Intuitively, if for some \( k \in K \), it were true that \( \tilde{\Pi}_k > 0 \), then so many vacancies populate the labor market that by definition, the firm–worker contact rate, \( \eta \), must approach zero, which, from (14e), yields the contradiction \( \tilde{\Pi}_{V_k} = -\frac{\nu}{r} < 0 \). Notice too, the complementary slackness condition (c), implies that if \( \tilde{\Pi}_{V_k} < 0 \), then \( V_k = 0 \) and, if \( V_k > 0 \), then \( \tilde{\Pi}_{V_k} = \Pi_{V_k} = 0 \).

**Wages and Employment Accession Probabilities**

Next, we determine the wage offer that arises under the Nash bargaining agreement and characterize workers’ optimal job-acceptance probabilities.

To this end, consider a particular \( (a, k) \) worker–firm match, in which the firm offers the wage \( \hat{w}(a)_k \) and the worker accepts it with probability \( \hat{q}(a)_k \). The (possibly negative) quasi-rent or economic surplus, \( \tilde{\Omega}(a)_k \) generated from employment equals the joint benefits to the two parties minus the joint costs they bear—the option value of searching for alternative partners. Since the value of an active vacancy, \( \Pi_{V_k} \), is zero, the economic surplus, \( \tilde{\Omega}(a)_k \), is given by

\[
\tilde{\Omega}(a)_k = \Pi(a)_{E_k} + J(a)_{E_k} - J(a)_{U} \equiv \Pi(a)_{E_k} + \Delta J(a)_{E_k}.
\]

(17)

Clearly, neither the firm nor the worker agree to employment if doing so violates either of their individual rationality (IR) constraints, \( \Pi(a)_{E_k} \geq 0 \) and \( \Delta J(a)_{E_k} \geq 0 \). Yet, as (17) makes apparent, this is precisely what would happen if \( \tilde{\Omega}(a)_k < 0 \) and the worker were employed. It follows that, under these circumstances, either the firm or the worker reject the prospective match and it dissolves immediately. Hence, at the instant of contact, the quasi-rents generated by a worker–firm meeting are in fact \( \Omega(a)_{E_k} = \max\{0, \tilde{\Omega}(a)_k\} \), which accommodates the possibility of its instant dissolution.

In the context of the particular \( a-k \) meeting under consideration, the symmetric Nash bargaining arrangement selects the wage offer, \( \hat{w}(a)_k \), that splits the
surplus, $\Omega(a)_k$, that accrues from the match:

$$\Delta J(a)_E^k \equiv J(a)_E^k - J(a)_U = \Pi(a)_E^k = (1/2) \cdot \Omega(a)_k \geq 0,$$

(18)

where the worker’s outside option, $J(a)_U$, is exogenous from the perspective of this particular match because it depends on future wages and accession probabilities.

As just noted, if $\tilde{\Omega}(a)_k < 0$, the match dissolves instantly because there is no possible wage offer, $\hat{w}(a)_k$, that satisfies even the rudimentary IR requirement. In this case, the worker rejects the offer by setting $\hat{q}(a)_k = 0$, and the match dissolves. In contrast, if $\tilde{\Omega}(a)_k \geq 0$, then (18) implies there is at least one wage offer that satisfies the IR condition. Indeed, it is immediate from (18) that if $\tilde{\Omega}(a)_k > 0$, then it is strictly optimal for the worker to pick $\hat{q}(a)_k = 1$, thereby accepting the wage offer determined under the Nash bargain. Alternatively, if $\tilde{\Omega}(a)_k = 0$, the worker is indifferent between accepting or rejecting the wage offer, so any $\hat{q}(a)_k \in [0, 1]$ is weakly optimal.\(^{26}\)

Thus far, the value of the outside option, $J(a)_U$, has been treated as given from the perspective of the particular match under consideration because it depends on the wages agreed, $w(a)_k$, and acceptance probabilities, $q(a)_k$, determined in future matches. Nevertheless, at the instant of contact, any such future match is precisely a particular match of the sort just described. Hence the wage offers agreed under the Nash bargain, $w(a)_k^* \in [0, y(a)_k]$, and workers’ associated optimal accession probabilities, $q(a)_k^* \in [0, 1]$, must satisfy not only the Bellman equations, (14b)–(14d) and the equal division rule, (18), but also the following intertemporal consistency condition:\(^{27}\)

$$\forall (a, k) \in \mathcal{A} \times K, w(a)_k^* \equiv \hat{w}(a)_k \text{ and } q(a)_k^* \equiv \hat{q}(a)_k$$

(19)

Our next task is to characterize the properties of the wage offers, $w(a)_k^*$, and acceptance probabilities, $q(a)_k^*$, that satisfy the conditions just laid out. As we shall see, it is extremely fruitful to accomplish this goal by embedding our two-agent framework in a more general setting. Accordingly, we relax temporarily the skills-requirement condition $a \geq \xi$, given in Assumption 3.3(a), and consider the generic worker $y(a) = (y(a)_b y(a)_g) = (y_0, a)$, where $y_0$ is his productivity in the bad job, and $y(a) = a$ is his productivity in the good one, which now takes any value $a \in \mathcal{A} \equiv [0, \infty)$. As we will see in Proposition 5.1, we then can determine the wages

\(^{26}\) Notice if $\Omega(a)_k = 0$, then (18) implies $\hat{w}(a)_k = y(a)_k$.

\(^{27}\) There also is the individual optimality–aggregate consistency condition, $q(a)_k^* = Q(a)_k$.
Proposition 5.1 (Accession Probabilities and the Wage Agreements)

Define the following two critical abilities $\hat{a}_L$ and $\hat{a}_H$:

$$
\hat{a}_L \equiv \left[ \frac{(1 - \phi) \cdot \mu}{(1 - \phi) \cdot \mu + 2 \cdot \hat{r}} \right] \cdot y_0 < \hat{a}_H \equiv \left[ \frac{\phi \cdot \mu + 2 \cdot \hat{r}}{\phi \cdot \mu} \right] \cdot y_0,
$$

where $\hat{r} \equiv r + \delta$ and $\phi \equiv \phi_g$ is the proportion of good jobs. Form a disjoint partition of $[0, \infty)$ by constructing the intervals: $A_L = [0, \hat{a}_L]$, $A_M = (\hat{a}_L, \hat{a}_H)$, and $A_H = [\hat{a}_H, \infty)$—low, medium, and high.

Next, consider the generic worker, $y(a) = (y_0, a)$. The job acceptance probabilities, $q(a)^*_k \in [0, 1]$, and wages agreed under the Nash bargain, $w(a)^*_k \in [0, y(a)_k]$, that satisfy the Bellman equations, (14b)–(14d), the equal division rule, (18), and the intertemporal consistency condition (19), are as follows:

(a) The optimal accession probabilities, $q(a)^*_k$, are given by

- if $a \in A_L$, then $q(a)^*_y = 1$, and if $a = \hat{a}_L$, then $q(a)^*_y \in [0, 1]$, otherwise $q(a)^*_y = 0$;
- if $a \in A_M$, then $q(a)^*_y = q(a)^*_g = 1$;
- if $a \in A_H$, then $q(a)^*_y = 1$, and if $a = \hat{a}_H$, then $q(a)^*_y \in [0, 1]$, otherwise $q(a)^*_y = 0$.

(b) For each type $a \in [0, \infty)$, the implied employment accession rate, $\mu(a)^*$, is

$$
\mu(a)^* = \mu \cdot \left[ \phi \cdot q(a)^*_y + (1 - \phi) \cdot q(a)^*_g \right]
$$

(c) The wages offered by bad, $w(a)^*_b$, and good jobs, $w(a)^*_g$, are continuous in $a \in [0, \infty)$. They are

\[
\begin{align*}
    w(a)^*_b &= \begin{cases} 
        \left( \frac{\hat{r} + (1 - \phi) \cdot \mu}{2 \hat{r} + (1 - \phi) \cdot \mu} \right) \cdot y_0, & \text{if } a \in A_L \\
        (1/2) \cdot y_0 + \left[ \frac{\mu}{2 \hat{r} + \mu} \right] \cdot \left( \phi a + (1 - \phi) y_0 \right), & \text{if } a \in A_M \\
        y_0, & \text{if } a \in A_H
    \end{cases} \\
\end{align*}
\]

\[
\begin{align*}
    w(a)^*_g &= \begin{cases} 
        a, & \text{if } a \in A_L \\
        (1/2) \cdot a + \left[ \frac{\mu}{2 \hat{r} + \mu} \right] \cdot \left( \phi a + (1 - \phi) y_0 \right), & \text{if } a \in A_M \\
        \left( \frac{\hat{r} + \phi \mu}{2 \hat{r} + \phi \mu} \right) a, & \text{if } a \in A_H
    \end{cases}
\end{align*}
\]

(d) Consider any $\mu > 0$ and $\phi \in (0, 1)$. The wage $w(a)^*_b$ is strictly increasing in $y_0$, and is increasing in $a$ and $\mu$ (with strict inequality iff $a \in A_L \cup A_M$); and $w(a)^*_g$, is strictly increasing in $a \in [0, \infty)$ and is increasing in $y_0$ and $\mu$ (with strict inequality iff $a \in A_M \cup A_H$). Next consider the effect of the proportion of good jobs, $\phi$. Letting subscripts denote partial derivatives, we show: (i) if $a \in A_L$, then $w^*_{b\phi} < 0$ and $w^*_{g\phi} = 0$, (ii) if $a \in A_H$, then $w^*_{b\phi} = 0$ and $w^*_{g\phi} > 0$, and (iii) if $a \in A_M$, then $w^*_{b\phi}$ and $w^*_{g\phi}$ take the sign of $a - y_0$.

We define two critical abilities $\hat{a}_L$ and $\hat{a}_H$ that partition $\mathcal{A} \equiv [0, \infty)$ into three

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28The value of $\hat{a}_H$ is finite (and so well defined) provided $\mu \cdot \phi > 0$. We allow for the possibility that $\mu \cdot \phi = 0$ by setting $\hat{a}_H = \infty$ and letting $A_H = \emptyset$. Here, $A_L = [0, \hat{a}_L]$ and $A_M \equiv (\hat{a}_L, \infty)$ form a disjoint partition of $[0, \infty)$, and the statements made in the Proposition about $a \in A_H$ remain valid because they are vacuously true.
disjoint intervals that determine the principal characteristics of the economic outcomes of the workers who belong to them. Most important, notice that the values of \( \hat{a}_L \) and \( \hat{a}_H \) (and so the three intervals) depend on the proportion of vacant good jobs, \( \phi \), and the worker–firm contact rate, \( \mu \). Hence the membership of each these sets is not immutable but instead varies with economic circumstances. The significance of this observation, is that the proposition provides sharp boundaries that delimit the membership of each set (and so the behavior of each worker ability) in terms of \( \phi \) and \( \mu \).

Part (a) of the proposition characterizes the optimal accession probabilities, \( q(a)^*_k \), for every ability, \( a \geq 0 \). Those who belong to the low ability group, \( A_L \), always accept the job offers made by bad jobs. Moreover, with the exception of the particular ability \( \hat{a} = a_L \)—who is indifferent—reject the offers made by good ones, so they can continue searching for a bad job. Clearly, a good job is not good from the perspective of these workers because their productivity is so lacklustre if employed by one. The behavior of those in the high ability group, \( A_H \), mirrors the \( A_L \) group. This time they always accept the offers made by good jobs and—with the exception of the particular type \( a = \hat{a}_H \) (who is indifferent)—reject offers made by bad ones. Finally, those belonging to the medium ability group, \( A_M \), accept both offers. Their abilities neither are so high nor so low that they reject one type of offer so they may search for the other. Part (b) just restates the definition of the flow employment accession rate for a type \( a \in A \) worker, \( \mu(a) \), when there are only two job types and each workers optimally chooses his job acceptance probabilities, \( q(a)^*_k \).

Parts (c) and (d) of the proposition present and characterize the wages agreed under the Nash bargaining agreement. It is immediate from (22) that \( w(a)^*_b \) is strictly increasing in \( y_0 \) and from (23) that \( w(a)^*_g \) is strictly increasing in \( a \in [0, \infty) \). This is a direct consequence of the fact that the size of the economic surplus that is split between the worker and firm is strictly increasing in \( y_0 \), in the case of bad jobs, and \( a \in [0, \infty) \), in the case of good ones.

The cross effects of \( a \in A \) on \( w(a)^*_b \) and \( y_0 \) on \( w(a)^*_g \) are more complicated because they depend on workers’ optimal accession choices. For example, consider the wages offered by bad jobs. For a worker who belongs to the low ability group, his wage, \( w(a)^*_b \), is independent of his ability level \( a \in A_L \). It has to be. He chooses to reject the offers made by good jobs who are precisely those employers where his ability gains some economic traction. In contrast, \( w(a)^*_b \) increases with the abilities of those in the middle ability group, \( A_M \). These workers accept offers from both jobs and \( w(a)^*_g \) is increasing in \( a \). Hence the greater their ability, the greater their leverage when bargaining with type \( k = b \) employers over the wage. Finally, those in the high ability group, \( A_H \), already receive the highest feasible wage this technology
can offer, \( w(a)_h^* = y_0 \), so it cannot increase further with \( a \geq \hat{a}_H \). The cross effects of \( y_0 \) on the wage offers made by good jobs, \( w(a)_g^* \), mirror those just described.

The other results reported in (d) are straightforward. Thus an increase in the worker–firm contact rate, \( \mu \), means it is easier for workers to generate alternative job offers. This increases the value of continued search and, by providing additional leverage, the wage offers. Finally, the effects of an increase in the proportion of good jobs, \( \phi \), depend on whether the worker resembles more a lower ability worker \( (a < y_0) \), or a higher ability worker \( (a > y_0) \). In the former case, an increase in \( \phi \) reduces both wage offers by assigning a greater weight to those firms where he has a relative productive disadvantage; the latter case is just the opposite. (If \( a = y_0 \), then good and bad jobs are, in essence, economically indistinguishable.)

The significance of Proposition 5.1 is that it allows us to characterize completely the behavior of the low and high ability workers pertinent to our model. Consider Lemma 5.5.

**Lemma 5.5 (Wages, Optimality, and Steady-State Populations)**

Consider the low, \( a_\ell \), and high ability, \( a_h \), workers described in Assumption 3.4, where \( 0 \leq a_\ell < \xi \) and \( a_h = (1 + \Delta) \cdot y_0 \). Let the worker–firm contact rate, \( \mu \), and the proportion of vacant good jobs, \( \phi \), be given. Proposition 5.1 implies the optimal behavior (\( * \)) of each type satisfies the following properties:

(a) Low ability workers, \( a_\ell \), belong to \( A_L \) and \( \mu_\ell^* = \mu \cdot (1 - \phi) \).

(b) For high ability workers, \( a_h \), there is a critical hazard rate, \( \rho \equiv 2\hat{r}/\Delta \) such that

i.) if \( \phi \cdot \mu < \rho \), then \( a_h \in A_M \) and \( \mu_h^* = \mu \).

ii.) if \( \phi \cdot \mu = \rho \), then \( a_h = \hat{a}_H \in A_H \), and \( \mu_h^* = \mu \cdot (1 - \phi) \cdot q(a_h)_h^* \), where \( q(a)_h^* \in [0, 1] \).

iii.) if \( \phi \cdot \mu > \rho \), then \( \hat{a}_H < a_h \in A_H \) and \( \mu_h^* = \mu \cdot (\phi + (1 - \phi) \cdot q(a_h)_h^*) \).

(c) Given the optimal accession rates in (a) and (b), the endogenous proportion of unemployed workers who have low abilities, \( \gamma^* \equiv \gamma(a_\ell)_\ell^* \), the ratio of low- to high-ability workers in the unemployment pool, \( \Gamma^* \equiv \gamma^*/(1 - \gamma^*) \), and the steady populations, \( U_i^* \) and \( E_i^* \), follow directly from Corollary 4.1, equation (12), and Lemma 4.2 respectively.

Low ability workers always belong to the low ability group, \( A_L \); however, as is apparent from (b) high ability workers can belong to either the middle ability, \( A_M \), or high ability, \( A_H \), groups. The intuition underlying part (bi) is that if \( \phi \cdot \mu < \hat{r}/(\Delta/2) \equiv \rho \), then high ability workers opt \( A_M \) because they find it too costly to belong to \( A_H \). More specifically, if \( \phi \cdot \mu \cdot (\Delta/2) \) is sufficiently small, either it is too difficult to find a good job (\( \phi \cdot \mu \) is low) or even if one is found, the terms of employment are not particularly lucrative at the margin (\( \Delta/2 \) is small). Similarly, if \( \hat{r} \) is large, workers are impatient and so disinclined to pass up any job offer they have in hand—the intuition underlying parts (ii) and (iii) is analogous. (The wage offers follow directly from part (c) of the proposition and so are not reported.)
first two parts of the Lemma also report the optimally determined accession rates, $\mu_i^*$, for each worker type. The result reported in part (c) of the Lemma is of great importance for the further development of the model’s properties. In particular, notice, after accommodating workers’ optimal employment accession choices, the steady-state populations and population proportions are determined completely by $\phi$ and $\mu$.

Parts (a) and (b) of the Lemma imply that for, given $\phi$ and $\mu$, the nature of the resulting economic outcome can be classified under one of three separate categories. More specifically, if $\phi \cdot \mu > \rho$, the outcome is separating, since high and low ability workers are employed exclusively in different jobs. Alternatively, if $\phi \cdot \mu < \rho$, it is pooling because high ability workers work in both good and bad jobs. (The case in which $\phi \cdot \mu = \rho$ straddles the two just described; it is the mixed case in which high ability workers accept the offers made by bad jobs with some yet-to-be-determined probability $q(a_h)_b^* \in [0, 1]$.) For future reference, the features of each of these outcomes are summarized in Definition 5.2.

**Definition 5.2 (Economic Outcomes)**

There are three potential economic outcomes, $j \in J \equiv \{1, 2, 3\}$, that depend on the sign of $\phi \cdot \mu - \rho$. Thus,

**Case 1. (Separating)** If $\phi \cdot \mu > \rho$, then low ability workers are employed only by bad jobs and high ability workers by good ones.

**Case 2. (Pooling)** If $\phi \cdot \mu < \rho$, then low and high ability workers accept offers from bad jobs and only high ability workers accept them from good ones.

**Case 3. (Mixed)** If $\phi \cdot \mu = \rho$, then the features of Case 2 hold, with the exception that high ability workers optimally accept offers from bad jobs with probability $q(a_h)_b^* \in [0, 1]$.

As is usual in this sort of model, the analysis is conducted on a case-by-case basis. Hence frequently we will refer both to the descriptions presented in Definition 5.2 and the index, $j \in J$.

Next we will show that, conditional on a given worker–firm contact rate, $\mu$, we can isolate the proportion of good jobs, $\phi$, that is consistent with the free entry assumption.