Abstract—The intermittency of renewable energies like wind and solar leads to the following effect: when their supply is high, the price of energy is low, or even zero. Conversely, when price is high, the supply of renewables is low. This constellation reduces the ability of renewables to earn money, and one might expect that this leads to too little investments in renewable capacities. Our paper shows that this intuition is wrong. In the absence of market restrictions - such as price ceilings - markets lead to efficient choices of renewable and fossil capacities. We also investigate some properties of electricity markets with renewable supply patterns, expressed by an availability factor, \( \sigma \), and one of the energy transition are still to come.

I. Model and Methodology

In our model we have two technologies, \( j = r, f \), and one demand period, for which the supply has to satisfy the demand. Technology \( r \) is a renewable technology with an intermittent supply pattern, expressed by an availability factor, \( \sigma \in [a, 1] \), where \( 0 < a < 1 \). Technology \( f \) is a fossil technology with full availability. Variable production costs of renewables are lower than those of fossils, \( b_r < b_f \), and costs of minimal available capacity of renewables are higher than capacity costs of fossils, \( \frac{b_r}{\alpha} > \beta_f \). We are assuming a linear demand function, \( x(p) = A - \gamma p \), and applying this linear demand on a continuous supply approach, whereas the realisation of the availability factor is uncertain. We are assuming a uniform distribution of the availability, \( \sigma \). The described game consists of two stages. In the first stage, a regulator chooses optimal capacities for renewables and fossils based on the unknown realisation of the availability of renewables. In the second stage, the regulator chooses optimal production for renewables and fossils based on the known realisation of the availability of renewables. We are solving the game by backwards induction.

A. Production Decision

In the second stage, production decisions were made and the regulator maximises the social welfare under known availability realisation and already installed capacities over the production decision for renewables and fossils

\[
\max_{q_r(\sigma), q_f(\sigma)} W = \int_0^{x(\sigma)} \int_0^{\tilde{x}(\sigma)} p(\tilde{x}) d\tilde{x} - b_r q_r(\sigma) - b_f q_f(\sigma) \tag{1}
\]

subject to the supply-equals-demand constraint (2), capacity constraints (3, 4), and non-negative constraints (5 to 7)

\[
q_r(\sigma) + q_f(\sigma) - x(\sigma) = 0 \tag{2}
\]

\[
\sigma Q_r - q_r(\sigma) \geq 0 \tag{3}
\]

\[
Q_f - q_f(\sigma) \geq 0 \tag{4}
\]

\[
q_r(\sigma) \geq 0 \tag{5}
\]

\[
q_f(\sigma) \geq 0 \tag{6}
\]

\[
x(\sigma) \geq 0. \tag{7}
\]

B. Capacity Decision

In the first stage, capacity decisions were made and the regulator maximises the social welfare under unknown availability realisation including the capacity costs

\[
\max_{Q_r, Q_f} W = \int_{A}^{1} \left\{ \int_{0}^{x(\sigma)} p(\tilde{x}) d\tilde{x} - b_r q_r - b_f q_f \right\} f(\sigma) d\sigma - \beta_r Q_r - \beta_f Q_f \tag{8}
\]

Solving the first-order conditions

\[
FOC_{Q_r} = (A - \gamma b_f - Q_f - a Q_r)^2 - 2\gamma (1 - a) \beta_f Q_r \tag{9}
\]

\[
FOC_{Q_f} = a^2 Q_r - 3 \left\{ a^2 (A - \gamma b_r - Q_f) - 2\gamma (1 - a) \beta_r \right\} Q_r^2 + (A - \gamma b_f - Q_f)^3 + (A - \gamma b_r)^3 - (A - \gamma b_f)^3 \tag{10}
\]

leads to optimal capacity decisions for renewables and fossils

\[
0 = 2 Q_r^2 - 3 (A - \gamma b_r) Q_r^2 + a^2 \gamma (b_f - b_r) Q_r^2 + 6\gamma (1 - a) (\beta_r - \alpha \beta_f) Q_r^2 - [2\gamma (1 - a) \beta_f Q_r]^{1.5} + (A - \gamma b_f)^3 \tag{11}
\]
II. PRESENT RESULTS

In the second stage, we derived three scenarios: only fossil capacities (scenario F), only renewable capacities (scenario R), and positive capacities of fossils and renewables (scenario F&R), which is the most interesting scenario in this examination. For scenario F&R we can distinguish between four cases. In case 1 renewables will be used only, but not at full capacity (excess capacity of renewables). In case 2 renewables will be fully used, but no fossils will be needed to meet the demand. In case 3 renewables will be fully used as well, but fossils will be needed to meet the demand. In case 4, in turn, renewable and fossil capacities will be fully used. The situation is illustrated in Figure 1. The case distinction based on the realisation of \( \sigma \), and can be split by the borders \( \sigma''' \), \( \sigma'' \), and \( \sigma' \), whereas the upper border is the maximum availability, \( \sigma = 1 \), and the lower border is the minimum availability, \( \sigma = a \).

Figure 1. Price development in dependency of demand and supply

The prices will fall - as pointed out in the intuition of the problem - with higher \( \sigma \) as shown in Figure 2.

From the four cases in scenario F&R we distinguished between four situations. The situation where all four cases occur, \( a < \sigma' < \sigma'' < \sigma''' < 1 \), will be denoted as high renewables. This situation is shown in Figure 1 and 2. In medium renewables only cases 2 to 4 occur, for low renewables only case 2 and 3, and for low capacities there will be only case 4. We add scenario F as situation only fossils and scenario R as situation only renewables (only case 1 and 2). Assuming that the capacity costs of renewables, \( \beta_r \), are the driving factor for the market penetration of renewables, we simulated numerical solutions like in Figure 3 (situation only fossils right, situation only renewables left). We can see an exponential increase of renewable capacity in the situation low renewables. An analytical proof will be shown in the full paper. With growing market penetration of renewables, high increases of renewable capacity will be reached with low decreases in capacity costs of renewables because the renewable capacity will be fully used for all realisations of \( \sigma \) and simultaneously prices setting by fossils are high.

Figure 3. Capacity of renewables and fossils for all situations

Additionally, we can show that optimal capacities can also implement without a regulator in markets with perfect competition.

III. FURTHER RESEARCH

With our model we want to show the effect of taxes on fossil generation, and compare this effect to the effect of subsidies like feed-in tariffs for renewable energies. Additionally, we want to show the effect of price ceilings, which should lead to inefficiencies. We will show how these inefficiencies will change with the level of renewables. As well as we can implement a market solution in perfect competitive markets, we are going to scrutinize market failures in oligopoly and monopoly markets.