Fertility, market child care, labor supply and redistributive effects of public pensions

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Abstract
Taking account of heterogeneity of individuals in the labor productivities and incorporating the possibility of childlessness and substitutability between parental child care at home and market child care, we examine effects of changes in social security contribution rate on the fertility choices of individuals and population growth rate.

JEL Classification: H55, J13, J18

Keywords: Pay-as-you-go social security, Fertility, Market child care, Beveridgean scheme, Social security sustainability

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1 Introduction

Declining fertility is a commonly observed feature in most developed countries. As is well recognized, it can cause serious problems on sustainability of pay-as-you-go social security systems employed in those countries, and social security reforms have been discussed in many economics literatures. The decrease in the number of children could happen through two channels: (i) the decrease in the number of children a woman has in her lifetime, and (ii) the decrease in the number of women who have some number of children in their lifetime (childlessness). In fact, an increase in the population of women who do not have children in their lifetime has been observed in many developed economies (e.g., OECD Family Database). And, when some people have no children, social security reform could affect the level of population through both the following two channels: (i) effect on whether each individual has children or not (i.e., ‘external margin’) and (ii) if an individual choose to have some number of children, how many (i.e., ‘internal margin’).

However, most theoretical studies have not considered the issue on childlessness and have assumed that all individuals have some number of children in their lifetime. Gobbi (2013) examined the issues on childlessness in a recent paper, but the paper did not deal with problems of social security reform.

In a recent paper, incorporating both possibility of childlessness and heterogeneity in preferences for having children, Hirazawa, Kitaura, and Yakita (2014) examined effects of changes in the size of pay-as-you-go social security system on fertility choices of individuals and population growth with the focus on the intra-generational redistributive effects of pension systems through benefits. But, in the paper, it was assumed that parental time is the only input for rearing children, so the only cost for rearing children is foregone income of parents.
As was indicated in Apps and Rees (2004), if parental time at home and bought-in child care are substitutable, policy effects of that case could be different from the case where parental time is the only input for rearing children.

Hirazawa and Yakita (2009) examined the effects of social security reform on fertility choices and on welfare of individuals incorporating substitutability of parental child cares by market child care into an overlapping-generations model. But the paper did not consider the possibility of childlessness and identical individuals were assumed.

If individuals differ in their productivity, however, time cost of rearing children is different among individuals. And, in that circumstance, through the effects on the substitutability between parental child care and market child care, social security reform could affect different effects on the decisions of whether or not to have children.

For an individual with higher labor productivity, time cost is relatively high while market child care is relatively less. On the other hand, for an individual with lower labor productivity, time cost is relatively low while market child care is relatively expensive. Some individuals with relatively high labor productivity may choose not to have children because the time cost is too high, while some individuals with relatively low labor productivity may choose not to have children because the market child care is costly. If the pension benefits were financed by payroll tax, changes in the tax rate affect the opportunity cost of having children and lifetime income, thereby affecting both the time cost and goods cost for rearing children. Since they are different between individuals with different labor productivity, the pension reform could affect fertility choices of individuals, not only on the number of children they have, but also on whether or not to have children, through the channels stated above. Hence, considering possibility of childlessness and substitutability of child-care at home by child-care purchased from the market together could have some implications on the discussion of social security reform. That is the problem we attack in the present paper.

Our purpose in the present study is to examine effects of changes in social security
contribution rate on the fertility choices of individuals when they differ in the labor productivities and incorporating the possibility of having no children and substitutability between parental child care at home and market child care.

The rest of the paper is organized as follows. The next section introduces the model of a small open economy with heterogeneous individuals and possibility of childlessness, and derives the equilibrium. Section 3 examines effects of a change in the size of PAYG social security system on the fertility choices of individuals concentrating on the system with perfect redistribution within each generation. This study focuses on steady states in order to examine the long-term effects of policy changes. The final section gives some concluding remarks.

2 Model
We consider a small open economy populated by overlapping generations of people who live for three periods: childhood, working period and retirement period. Individuals are reared by their parents in the first period of life, work and “might” have some children in the second, and retire in the third. We call individuals working in period $t$ as generation $t$. We assume that individuals in each generation differ in their labor productivity, which is assumed to be exogenously given for simplicity. The labor productivity of an individual is represented by $\varepsilon \in [\underline{\varepsilon}, \bar{\varepsilon}]$ which is assumed to distribute over $[\underline{\varepsilon}, \bar{\varepsilon}]$ according to the cumulative distribution function $F(\varepsilon) = \int_{\underline{\varepsilon}}^{\varepsilon} f(x)dx$ where $f(x)$ is the density function and $F(\underline{\varepsilon}) = 0$ and $F(\bar{\varepsilon}) = 1$. We assume that the lower bound $\underline{\varepsilon} (> 0)$ is sufficiently low and the upper bound $\bar{\varepsilon}$ is sufficiently high. The distribution is assumed to be the same for every generation, although the size of population may change from period to period. We abstract away from gender differences in the model to focus on the effects of social security on fertility and we call the basic units
of our analysis simply as “individuals”. We assume that the only difference between individuals within each generation is the labor productivity.

The economy is facing the world interest rate, \( r \), which is assumed to be constant over time. Assuming that the production technology is represented by a standard neoclassical constant-returns-to-scale production function with respect to capital and efficiency labor and that perfectly competitive factor markets, the wage rate for efficiency labor, \( w \), is also constant.

2.1 Individuals

Following Balestrino, Cigno and Pettini (2002, 2003) and Apps and Rees (2004), we consider children as goods which are produced at home by combining parental time and goods purchased from the market.\(^1\) We specify the production function of children to be Cobb-Douglas type:

\[
n_t = \phi x_t^\beta z_t^{1-\beta}
\]

where \( n_t \) is the number of children, \( x_t \) is goods input purchased from the market, \( z_t \) is parental time input for rearing children, \( \phi \) and \( \beta \) are parameters with \( \phi > 0 \) and \( 0 < \beta < 1 \). Normalizing time endowment of each individual during the working period to unity, \( z_t \) satisfies \( 0 \leq z_t < 1 \).\(^2\)

The budget constraints of an individual with productivity level \( e \) in the second and third periods of life are given by

\[
(1-\tau)we(1-z_t) = c_t + s_t + x_t
\]

\[
d_{i,t+1} = Rs_t + P_{i,t+1}
\]

respectively, where \( \tau \) is the social security contribution rate, \( n_t \) is the number of

\(^1\) Hirazawa and Yakita (2009) examined effects of social security reform on fertility and welfare of individuals in an overlapping generations model taking account of substitutability between parental time and market child care. However, the paper did not consider the heterogeneity of individuals within each generation and possibility of childlessness.

\(^2\) In the present paper, some individuals may have no children and then the parental time input is zero, i.e., \( z_t = 0 \) can happen in the optimum.
children she has, \( c_t \) and \( d_{t+1} \) are consumptions in the second and the third periods, \( s_t \) is savings, \( R = 1 + r \) is the gross rate of interest, and \( P_{t+1} \) is the social security benefits paid to her in period \( t+1 \). In the present study, we focus on the case of full redistribution within the generation (i.e., a flat rate benefit scheme or the Beveridgean scheme)\(^3\).

Each individual is assumed to derive utility from consumptions in the second and the third periods of life and the number of children the individual has. We incorporate the possibility that some individuals may choose not to have children in order to examine the free-rider problems stated in the introduction. To this end, we assume, following Hirazawa, Kitaura, and Yakita (2014), that, the utility functions of individuals are given by

\[
u_t = \ln c_t + \rho \ln d_{t+1} + \alpha \ln (\theta + n_t) \tag{3}\]

where \( \rho \) is the subjective discount factor, \( \alpha \) represents the preference for children, and \( \theta \) is a preference parameter, which is assumed to be the same for every individual.

We divide optimization problems of individuals into two steps. First, we consider cost minimization problem. We assume that individuals choose time input \( z_t \) and goods input \( x_t \) to minimize cost for rearing given number of children. The minimization problem can be written as follows:

\[
\begin{align*}
\min_{x_t, z_t} & \quad C_t = x_t + (1 - \tau)wz_t \\
\text{s.t.} & \quad n_t = \phi x_t^\beta z_t^{1-\beta}
\end{align*}
\]

The first order conditions for the minimization problem are given by

\(^3\) As discussed in Hirazawa, Kitaura, and Yakita (2014), we could examine cases with imperfect intra-generational redistribution (i.e., cases involving some redistribution within each generation) by modifying this model, that is, replacing the pension benefits in the individuals’ budget constraints by \( \sigma P_{t+1} \) with \( \sigma \) being proportional to the contribution and determining the value of \( \sigma \) endogenously from the budget constraint of the government. But, in the setting of the present paper, it is difficult to determine the value of \( \sigma \), at least analytically. To examine the case with imperfect redistribution (including the case of Bismarckian scheme) is the subject of future research.
\[
\frac{(1-\beta) x_i}{\beta z_i} = (1-\tau)we
\]  
(4)

and (1). From the conditions, we can obtain the optimal parental time input and goods input as follows:

\[
z_i = \frac{1}{\phi} \left( \frac{\beta}{1-\beta} (1-\tau)we \right)^{-\beta} n_i = \hat{x}(e; w, \tau)n_i
\]  
(5)

\[
x_i = \frac{1}{\phi} \left( \frac{\beta}{1-\beta} (1-\tau)we \right)^{1-\beta} n_i = \hat{x}(e; w, \tau)n_i
\]  
(6)

where

\[
\frac{\partial \hat{x}(e; w, \tau)}{\partial e} = -\frac{1-\beta}{\beta} \frac{\hat{x} > 0}{e}, \quad \frac{\partial \hat{x}(e; w, \tau)}{\partial \tau} = -\frac{1-\beta}{1-\tau} \frac{\hat{x} > 0}{1-\tau}
\]

\[
\frac{\partial \hat{z}(e; w, \tau)}{\partial e} = -\frac{1-\beta}{\beta} \frac{\hat{z} < 0}{e}, \quad \frac{\partial \hat{z}(e; w, \tau)}{\partial \tau} = \frac{\beta}{1-\tau} \frac{\hat{z} > 0}{1-\tau}
\]  
(7)

That is, more productive individuals (i.e., individuals with higher \( e \)) rear one children by using more market goods (or services from the market) and less parental time, while less productive individuals rear children by using more parental time and less market goods. An increase in the social security contribution rate reduces the opportunity cost for rearing children, so people come to rear children by using parental time (and to reduce the purchase of market goods for rearing children) as the social security contribution rate rises.

Substituting (5) and (6) into the definition of the cost for child care, we have

\[
C_i = \frac{1}{\beta \phi} \left( \frac{\beta}{1-\beta} (1-\tau)we \right)^{1-\beta} n_i = p(w; \tau, e)n_i
\]  
(8)

where

\[
\frac{\partial}{\partial e} p(w; \tau, e) = -\frac{1-\beta}{\beta} p(w; \tau, e) > 0, \quad \frac{\partial}{\partial \tau} p(w; \tau, e) = -\frac{1-\beta}{1-\tau} p(w; \tau, e) < 0
\]  
(9)
The value of $p(w; \tau, e)$ represent unit cost of a child.

Next we examine the utility maximization problem. Individuals choose consumptions and the number of children so as to maximize the lifetime utility (3) subject to the lifetime budget constraint and the non-negativity constraint on the number of children:

$$\max_{c_t, d_{t+1}, n_t} \ln c_t + \rho \ln d_{t+1} + \alpha \ln(\theta + n_t)$$

subject to

$$c_t + \frac{d_{t+1}}{R} + p(w; \tau, e)n_t = (1 - \tau)we + \frac{P_{t+1}}{R}$$

and

$$n_t \geq 0.$$  

Denoting the Lagrangean multiplier attached to the constraint (6) by $\lambda_t$, the first-order conditions for the optimum are given by (6) and

$$\frac{1}{c_t} - \lambda_t = 0$$  

(12a)

$$\frac{\rho}{d_{t+1}} - \frac{\lambda_t}{R} = 0$$  

(12b)

$$\frac{\alpha}{\theta + n_t} - \lambda_t p(w; \tau, e) \leq 0, \ n_t \left[ \frac{\alpha}{\theta + n_t} - \lambda_t p(w; \tau, e) \right] = 0, \ n_t \geq 0$$  

(12c)

The following two cases can occur:

(i) $n_t > 0$ and $\frac{\alpha}{\theta + n_t} - \lambda p(w; \tau, e) = 0$, or

(ii) $n_t = 0$ and $\frac{\alpha}{\theta + n_t} - \lambda p(w; \tau, e) \leq 0$.

In case (i), the conditions (12a)-(12c) give the solution

$$c_t = \frac{1}{1 + \rho + \alpha} \left[ (1 - \tau)we + p(w; \tau, e) \theta + \frac{P_{t+1}}{R} \right]$$

(13)

$$n_t = \frac{\alpha}{(1 + \rho + \alpha) p(w; \tau, e)} \left[ (1 - \tau)we + \frac{P_{t+1}}{R} \right] - \left( \frac{1 + \rho}{1 + \rho + \alpha} \right) \theta$$

(14)
where

\[
\frac{\partial n_t}{\partial e} = \frac{\alpha}{(1 + \rho + \alpha)p} \left[ \beta(1 - \tau)we - (1 - \beta)\frac{P^{t+1}}{R} \right]
\]

\[
\frac{\partial n_t}{\partial \tau} = \frac{-\alpha}{(1 + \rho + \alpha)p(1 - \tau)} \left[ \beta(1 - \tau)we - (1 - \beta)\frac{P^{t+1}}{R} \right]
\]

\[
\frac{\partial n_t}{\partial P^{t+1}} = \frac{\alpha/R}{(1 + \rho + \alpha)p} > 0
\]  \hspace{1cm} (15)

Notice that the terms in the square brackets in the light-hand sides of \( \partial n_t / \partial e \) and \( \partial n_t / \partial \tau \) are the same, so \( \partial n_t / \partial e \) and \( \partial n_t / \partial \tau \) has the opposite sign.

In case (ii), from the conditions (8a), (8b), (6) and \( n_t = 0 \), we can obtain that

\[
c_t = \frac{1}{1 + \rho} \left[ (1 - \tau)we + \frac{P^{t+1}}{R} \right]
\]  \hspace{1cm} (13)

and the following condition:

\[
\alpha \left[ (1 - \tau)we + \frac{P^{t+1}}{R} \right] \leq \theta(1 + \rho)p(w; \tau, e)
\]  \hspace{1cm} (16)

Notice that the left-hand side of (16) is a linear function of \( e \) with positive intercept, while the right-hand side of (16) is a concave function of \( e \). We can show that, under some conditions, the two functions defined by each side of (16) intersect at two points \( e_L \) and \( e_H (> e_L) \). The values \( e_L \) and \( e_H \) are cut-off levels for having children. Now we can show the following:

(I) If \( e < e < e_L \) or \( e_H < e < \bar{e} \), then individuals with labor productivity \( e \) choose to have some children, and

(II) If \( e_L \leq e \leq e_H \), then individuals with labor productivity \( e \) choose not to have children.

Individuals with intermediate labor productivity levels will not have children. For them, the opportunity cost is high, but child care purchased from the market is expensive as well to substitute the child care at home by the market child care. Therefore, they
prefer not to have children. But, if the payroll tax rate is increased, the opportunity cost for rearing children decreases. In the next section, we examine effects of a change in the payroll tax on the fertility choices of individuals.

2.2 Total Labor supply and population growth

Since the number of children an individual has depends on the labor productivity of each individual, the total population of this economy evolves according to

\[ N_{t+1} = \left[ \int_{e}^{e} n(e) dF(e) \right] N_t \]  (17)

where \( N_t \) is the population of generation \( t \). We define as \( \nu_t = \int_{e}^{e} n(e) dF(e) \) which represents the (gross) growth rate of population.

The total labor supply of this economy in period \( t \) measured by efficiency unit is given as:

\[ L_t = \left[ \int_{e}^{e} (1 - z(e)) dF(e) \right] N_t. \]  (18)

2.3 Social security schemes

The government operates a social security system with pay-as-you-go basis, and we assume that the budget for the system is balanced at each period. In this study, we consider the defined contribution scheme. So, level of the pension benefit at each period is determined so as to satisfy the following budget constraint:

\[ \tau w L_t = P_t N_{t-1} \]

or

\[ P_t = \tau w \left[ \int_{e}^{e} (1 - z(e)) dF(e) \right] \nu_{t-1} = \tau w \left[ \int_{e}^{e} (1 - \dot{z}(e, \tau) n(e, \tau, P)) dF(e) \right] \nu_{t-1} \]  (19)

Under the assumption of full redistribution within each generation, the pension benefit
Effect of contributions on fertility choices of individuals

In this section, we examine the long-run effects of changes in the contribution rate, so we focus on the policy effect on the steady state. (In the following, we drop the time subscript to simplify expressions.)

Differentiating (19) with respect to \( \tau \), we obtain

\[
\frac{dP}{d\tau} = w_l \nu - \nu \int_{\zeta}^{\cE} \left[ \frac{\partial E}{\partial n} \right] dF(e) + \int_{\nu}^{\cE} \left[ \frac{\partial z}{\partial n} \right] dF(e)
- \nu \int_{\zeta}^{\cE} \left[ \frac{\partial E}{\partial \tau} \right] dF(e) + \int_{\nu}^{\cE} \left[ \frac{\partial z}{\partial \tau} \right] dF(e)
- \nu \int_{\zeta}^{\cE} \left[ \frac{\partial E}{\partial \beta} \right] dF(e) + \int_{\nu}^{\cE} \left[ \frac{\partial z}{\partial \beta} \right] dF(e)
+ \nu \int_{\zeta}^{\cE} \left[ \frac{\partial E}{\partial \nu} \right] dF(e) + \int_{\nu}^{\cE} \left[ \frac{\partial z}{\partial \nu} \right] dF(e)
\]

(20)

where

\[
\frac{d\nu}{d\tau} = \int_{\zeta}^{\cE} \left( \frac{\partial n}{\partial \tau} + \frac{\partial n}{\partial \beta} \right) dF(e) + \int_{\nu}^{\cE} \left( \frac{\partial n}{\partial \beta} + \frac{\partial n}{\partial \nu} \right) dF(e)
\]

and \( n(e_L) = n(e_H) = 0 \) are used and where \( l_E = \int_{\zeta}^{\cE} (1 - \hat{z}(e, \tau)n(e; \tau, P)) dF(e) \) is the average labor supply measured by efficiency unit. Rearranging terms in (20), we have

\[
\left( 1 + P \int_{\zeta}^{\cE} \left( \frac{\hat{z}}{l_E} - \frac{1}{\nu} \right) \frac{\partial n}{\partial \tau} dF(e) + \int_{\nu}^{\cE} \left( \frac{\hat{z}}{l_E} - \frac{1}{\nu} \right) \frac{\partial n}{\partial \beta} dF(e) \right) \frac{dP}{d\tau} = \frac{P}{\tau} - P \int_{\zeta}^{\cE} \left[ \frac{\partial z}{\partial \tau} \right] dF(e) + \int_{\nu}^{\cE} \left[ \frac{\partial z}{\partial \beta} \right] dF(e)
\]

(21)

where \( \nu l_E \) is used.

Differentiating the equation determining the cut-off levels with respect to \( \tau \), we have
or making use of (9), we have

$$\frac{1}{e^*} \left( \alpha (1 - \tau) w - \theta (1 + \rho)(1 - \beta) \right) \frac{\partial e^*}{\partial \tau} + \frac{\alpha}{R} \frac{\partial P^*}{\partial \tau} = \alpha we^* + \theta (1 + \rho) \frac{\partial P}{\partial \tau} - \theta (1 + \rho)(1 - \beta) p$$

(22)

Note that the terms in the round brackets of both sides are the same and satisfy that

(I) If \( e = e_L \), then \( \alpha (1 - \tau) w e_L - \theta (1 + \rho)(1 - \beta) p < 0 \), and

(II) If \( e = e_H \), then \( \alpha (1 - \tau) w e_H - \theta (1 + \rho)(1 - \beta) p > 0 \).

Define that

\[ A = \left( 1 + P \right) \left[ \int_{e_L}^{e_H} \left( \frac{\partial e^*}{\partial \tau} - 1 \right) \frac{\partial \bar{n}}{\partial \bar{P}} e dF(e) + \int_{e_H}^{e_L} \left( \frac{\partial e^*}{\partial \tau} - 1 \right) \frac{\partial \bar{n}}{\partial \bar{P}} e dF(e) \right] \text{, and} \]

\[ B = \frac{P}{\tau} \left[ \int_{e_L}^{e_H} \left( \frac{\partial^2 \bar{n}}{\partial \tau \partial \bar{P}} + \frac{\partial \bar{n}}{\partial \bar{e}} \right) e dF(e) + \int_{e_H}^{e_L} \left( \frac{\partial^2 \bar{n}}{\partial \tau \partial \bar{P}} + \frac{\partial \bar{n}}{\partial \bar{e}} \right) e dF(e) \right] \text{.} \]

We can rewrite the relation (21) as \( dP/d\tau = B / A \). In the following, we focus on the case of \( dP/d\tau > 0 \), hence \( \text{sign} A = \text{sign} B \) holds. Substituting the relation into (23), we have

$$\frac{\partial e^*}{\partial \tau} = \frac{\left( 1 - \tau \right) \left( \alpha (1 - \tau) w e^* - \theta (1 + \rho)(1 - \beta) p \right) A - \left( \alpha / R \right) B}{\left( \alpha (1 - \tau) w e^* - \theta (1 + \rho)(1 - \beta) p \right) A / e^*}$$

(24)

where \( e^* = e_L \) or \( e^* = e_H \). From the relation (24), we can show that

$$\frac{\partial e_l}{\partial \tau} > 0$$

That is, the lower cut-off level of labor productivity increases with an increase in the social security contribution rate; more individuals with lower labor productivity will come to have children when the social security contribution rate is raised. On the other hand, individual with the higher cut-off level of the effect of labor productivity might increase or decrease with the rise of the social security contribution rate. If the cut-off level is
sufficiently higher, then \( \frac{\partial e_{\mu}}{\partial \tau} > 0 \) holds, that is, more individuals will choose to have no children with a rise of the social security contribution rate. On the other hand, if the upper cut-off level is relatively low, then \( \frac{\partial e_{\mu}}{\partial \tau} < 0 \) holds, that is, the increase in the social security contribution rate makes more people with higher productivity have children. From (16), we can see that the upper cut-off level \( e_{\mu} \) is higher when the preference for children \( \alpha \) is low, or discount factor \( \rho \) is high, or preference parameter \( \theta \) is high, or the world interest rate \( r \) is high. Summarizing, we have:

**Result**

Under the Beveridgean scheme, an expansion of social security system raises the lower cut-off level, so the more individuals with lower productivity will come to have children, while the upper cut-off level may increase or decrease. If the preference for children \( \alpha \) is high (low), or discount factor \( \rho \) is low (high), or preference parameter \( \theta \) is low (high), or the world interest rate \( r \) is low (high), then the expansion of social security system increases (reduces) the number of individuals who have children.

6 Concluding remarks

Assuming heterogeneity of individuals with respect to labor productivity and incorporating substitutability between child care in home and market child care in an overlapping generations model of a small open economy, we have examined the effect of a change in the size of the PAYG social security system on fertility choices of individuals through two channels: (i) whether an individual has some children or not (external margin), and (ii) if an individual has some number of children, how many (internal margin).
We have only considered the social security scheme with perfect redistribution through pension benefits. But, as was examined in Hirazawa, Kitaura, and Yakita (2014), considering the contribution-related system (i.e., imperfect redistribution within each generation) and comparing the results between the two different schemes might have some implications for the debate on social security reform in societies with declining fertility. And, in the present study, we have not examined welfare effects of the policy changes. These issues are subjects for future research.

References


