Why academic quality of higher education declines

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Abstract

We investigate the choice of quality, or academic content, in higher education in a two-sector model. Each sector works with only one type of labor, skilled or unskilled, and individuals are differentiated according to their cost of acquiring human capital. A higher academic quality increases productivity upon training, but is also associated with higher cost of acquiring skill. We consider both an American-type university system in which quality is tailored to the individual need, and a European-type system in which a uniform quality is politically determined. The former yields a higher income dispersion. Average quality decreases under both systems when the skill premium increases. Moving from a single stage to a two-stage scheme reduces quality in the first stage and increases quality in the second stage.


Keywords: Higher education, enrollment, quality, two-sector model.

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1 Introduction

Is quality of university education declining, and if so, why? Over the last decades, many countries have experienced a secular increase in the number of workers holding an academic degree. On OECD average, today around 40 per cent of a birth cohort graduate from a theory-based program of tertiary education, with rates exceeding even 50% in Poland, Iceland, New Zealand and the United Kingdom (OECD, 2013). This evolution can be traced back to high and even increasing skill premia in terms of lifetime income for people holding academic degrees in recent decades (Acemoglu, 2002, Mitchell, 2005), also in many developing countries (Ripoll, 2005, Goldberg and Pavcnik 2007). There are different approaches to deal with the strong increase in enrollment. In the US, the top universities remain as selective as ever without any reduction in academic quality. At the same time, many colleges spread all over the place targeting at the needs of the bulk of students. Employers know the ranking of universities and pay their skilled workers accordingly. In Europe, concerns of policy are centered around enhancing the international mobility of students and graduates. This has led, among others, to the Bologna initiative (Bologna declaration, 1999), aiming at standardizing academic curricula across the European Union, making exams of different universities roughly comparable.

In our model, we consider both European and US type systems. There is endogenous sorting into the different sectors of the economy. As more talented individuals display a stronger productivity response to increasing academic quality, the most preferred quality of higher education increases in ability. While the American system with its tailored programs will lead to higher enrollment and higher dispersion of income, the uniform quality in the European system reflects a compromise of the interests of the student population. Academic quality is higher in the American system at the top universities than in Europe, while the opposite holds for the lowest quality colleges in the market. In our benchmark model, the American system Pareto dominates the European system. Increasing the skill premium reduces average quality in both systems, though through different mechanisms. Under the American framework, additional enrollment will be met by an increase of comparatively low quality colleges, while higher quality universities will not adapt their curriculum. In the
European system, the interest of the new marginal group within higher education induces a pressure to cut quality for all students.

Another consequence of the Bologna initiative consists in replacing one-stage (diploma) systems by two-stage schemes with bachelor and master degrees. Compared to the single stage European scheme, this reform will bring about higher quality in the second (master) stage as medium ability students have already completed their studies. At the same time it reduces quality in the first stage allowing for a higher overall enrollment.

Our paper is related to the literature on educational standards (Costrell, 1994, 1997, Betts 1998) in which the early contributions focused on alternative political objectives when determining standards and competition in standards across jurisdictions with mobile workers. Centralizing standards in an asymmetric information environment with student mobility can be welfare-enhancing as higher local standards are associated with a positive externality, thus reducing equilibrium standards in a competitive market (Costrell, 1997) - which could be interpreted as argument in favor of the European system. According to Mechтенberg and Strausz (2008), the Bologna initiative in itself increases quality due to enhanced competition, but a free-rider effect works in the opposite direction. Analyzing the market for universities, Epple et al. (2003, 2006) argue that stratified qualities will be an equilibrium outcome, where universities with a higher endowment choose a higher quality so as to exert market power. If peer effects are important in educational production, a market outcome will generate stratification in ability and income. In the absence of market imperfections, perfect sorting can be achieved also by just relying on tuition fees (Gary-Bobo and Trannoy, 2008). Su et al. (2012) argue that an increase in enrollment will lead private elite colleges to increase their quality, while public colleges decrease their standard to allow for more students.

The remainder of the paper is organized as follows. Section 2 introduces the model, and Section 3 discusses the consequences of uniform quality. Section 4 is concerned with a comparison of the Eurean and the American system, while Section 5 analyzes the move from a one-stage to a two stage scheme within a European framework. The final Section 6 concludes and indicates directions for future research.
2 The Basic Model

2.1 Costs and returns of quality

Each individual lives for one period. Upon learning her ability type, she chooses whether or not to enroll in higher education. All university students graduate and work in the skilled sector, the other individuals work in the unskilled sector. Individuals are heterogeneous in ability $\alpha$. For simplicity, let ability $\alpha$ be distributed on $[0,1]$. Wages reflect productivity differences proportionally. In the unskilled sector, the income of an individual of ability level $\alpha$ is given by $y_u(\alpha) = w_u \alpha$, where $w_u$ is a standard wage in the unskilled sector that would be paid to an individual with the highest ability $\alpha = 1$. In the skilled sector, a worker of ability $\alpha$ having completed higher education of quality $q$ earns $y_s(\alpha, q) = w_s ag(q, \alpha)$. Higher quality increases productivity in the skilled sector, though at a diminishing rate, $g(q, \alpha) > 0 > g_q(q, \alpha)$, with $g(0,0) > 0$. The impact of given quality on productivity is type-specific and stronger at higher initial ability, $g_q(q, \alpha) > 0$. Also the marginal impact of quality increases in quality, $g_{qq}(q, \alpha) > 0$. Finally, we assume $g_{qq}(q, \alpha) > g_q(q^*, \alpha)$. Hence the marginal return to quality reacts more sensitive to an increase in ability than the factor $q$ itself.

To keep the analysis tractable, utility is assumed to be logarithmic in income, $U(y) = \log(y)$. Acquiring skills is associated with a utility cost $C'(\alpha) = \log(h(q)/\alpha)$. Utility cost is increasing and convex in quality, $h_q(q) > 0$ and $h_{qq}(q) > 0$ with $h(0) > 0$. Individuals with the highest ability have utility cost of zero, and individuals with the lowest ability level will face an infinite cost. This ensures that the endogenous ability threshold that separates the skilled workers from the unskilled is interior whenever $w_s > w_u$. In order to guarantee existence of optimal qualities as interior solutions, we impose $\lim_{q \to 0} h_q(q, \alpha) = \infty$, $\lim_{q \to \infty} g_q(q, \alpha) = 0$, $\lim_{q \to 0} h_q(q) = 0$, $\lim_{q \to \infty} h_q(q) = \infty$. Individuals possess perfect foresight with respect to their prospective wage. An individual of ability $\alpha$ enrolls in education of quality $q$ when net utility from doing so exceeds utility from remaining unskilled, that is, if $\log(w_u g(q, \alpha) a) \log(h(q)/\alpha) > \log(w_u a)$ holds. This implies that an agent will enroll if ability $\alpha$ exceeds the threshold
level $a^*$, with
\[
a^* = \frac{h(q)w_u}{g(q, a^*)w_s}.
\] \hfill (1)

2.2 Individual optimal quality

Optimum quality at the level of the individual is determined by maximizing utility as skilled with respect to quality, that is to maximize
\[
X = \log[w_sg(q, a)] - \log[h(q)/a]
\] \hfill (2)

This yields as first-order condition
\[
\frac{dX}{dq} = \frac{g_q(q^*, a)}{g(q^*, a)} - \frac{h_q(q^*)}{h(q^*)} = 0
\] \hfill (3)

The first-order condition reveals that optimum quality is independent of standard wage rate in the skilled sector. This is a consequence of the specification of the utility function where substitution and income effects cancel out. On the one hand, a higher return on quality calls for a higher quality. At the same time, any target utility level can then be achieved by cutting quality.

Proposition 1 is concerned with the pattern of ability-specific optimal qualities.

**Proposition 1** A sufficient condition for uniqueness of optimal quality at the individual level is $h_{qq}h - (h_q)^2 > 0$, or $h_{qq}/h_q > h_q/h$. Given that uniqueness holds, most preferred quality levels increase with rising ability.

**Proof.** See Appendix A. \hfill \square

Figure 1 illustrates the proposition where optimal quality increases in ability. This property is due to the fact that marginal benefits of quality increase in ability. Should the function $g$ be independent of ability $a$, as in Meier and Schiopu (2015), all types would share the same most preferred optimal quality.

Insert Figure 1 about here
Notice that we have \( g_q/g > h_q/h \) below the type-specific optimal quality and 
\( g_q/g < h_q/h \) above that level.

We consider two systems of higher education. In the American-style framework, 
quality is tailored according to individual need. Prospective students can choose be-
tween a continuum of qualities. By contrast, the European-style system has uniform 
quality that is determined through some political process. As the process balances 
the interests of high ability students and students from the middle of the ability dis-
tribution, chosen quality will typically exceed optimal quality of the marginal student 
who is just indifferent between enrolling and not enrolling.

3 Uniform quality

With uniform quality for all students enrolling in higher education, an interior equi-
librium is defined by a market enrollment threshold \( a_m = a^* \) satisfying equation 
(1). Thus, the market enrollment threshold is determined by \( Z = a^* \). Proposition 2 shows sufficient conditions under which a unique in-
terior market enrollment rate always exists.

**Proposition 2** Assume \( w_s > h_q/g(q, 1) \). An interior market enrollment thresh-
old \( a_m \) always exists and is unique.

**Proof.** See Appendix B. \( \Box \)

Proposition 2 considers some fixed quality level at which, according to the condition 
\( w_s > h_q/g(q, 1) \), the skill premium is high enough so that most able type is 
willing to enroll. As the least able type never enrolls and enrollment incentives 
are increasing in ability, an interior enrollment threshold exists. The same line of 
reasoning applies to the American framework with variable qualities increasing in 
ability.

Proposition 3 deals with the impact of changing the uniform quality level on 
enrollment.

**Proposition 3** A higher quality \( q \) increases the market enrollment threshold \( a^* \) if the 
quality lies weakly above the optimal quality of the marginal student. Should quality
lie below optimal quality of the ability type at the enrollment threshold, increasing that quality marginally reduces the market enrollment threshold.

**Proof.** Differenting $a^*(q)$ with respect to $q$ yields

$$\frac{\partial a^*}{\partial q} = -\frac{\partial Z/\partial q}{\partial Z/\partial a^*} - 1$$

(4)

Since $\partial Z/\partial a^* < 1$ holds at any interior enrollment threshold, the sign of the fraction is determined by the numerator, $\text{sgn} [\partial a^*/\partial q] = \text{sgn} [\partial Z/\partial q]$. Evaluating the latter gives

$$\partial Z/\partial q = \frac{w_u h_q(q)g(q,a^*) - g_q(q,a^*)h(q)}{w_s [g(q,a^*)]^2}$$

(5)

Thus, $\text{sgn} \left[ \frac{\partial a^*}{\partial q} \right] = \text{sgn} \left[ \frac{h_q(q)}{h(q)} - \frac{g_q(q,a^*)}{g(q,a^*)} \right]$. Denote by $q_m$ the optimal quality for the individual with ability $a_m$. If $q > q_m$, then $\frac{g_q(q,a^*)}{g(q,a^*)} - \frac{h_q(q)}{h(q)} < 0$, so $\frac{\partial a^*}{\partial q} > 0$, i.e. increasing quality will decrease enrollment. The opposite holds if $q < q_m$. \qed

As it will be shown, the relevant case in any European model is the first, in which the chosen quality level is optimal for some medium skilled. At the same time, it exceeds the optimal quality of the marginal ability type being indifferent between enrolling or staying unskilled. Since a higher quality would make becoming skilled less attractive for this marginal type, increasing quality reduces enrollment. If, for some reason, the chosen quality would be set below the optimum of the marginal individual, increasing quality has a positive impact on utility of the marginal individual, inducing a further increase in enrollment by ability types slightly below the previous margin.

Proposition 4 shows that skill-biased technological change, which is here expressed as rising relative standard wage of skilled workers, increases enrollment.

**Proposition 4** Increasing the skill premium $w_s/w_u$ reduces the market enrollment threshold at any given quality.
Proof. The market enrollment threshold is determined by \( Z - a^* = 0 \) with \( Z \) being defined as above. According to the implicit function theorem, \( \frac{\partial a_m}{\partial (w_s/w_u)} = -\frac{\partial Z}{\partial (w_s/w_u)} \). Since the \( \frac{\partial Z}{\partial a^*} \) is always less than 0, it follows that

\[
\text{sgn} \left[ \frac{\partial a_m}{\partial (w_s/w_u)} \right] = \text{sgn} \left[ \frac{\partial Z}{\partial (w_s/w_u)} \right] \Rightarrow \frac{\partial a_m}{\partial (w_s/w_u)} < 0. \tag{6}
\]

The comparative static properties are easily understood. Raising the skill premium through a higher \( w_s/w_u \) increases the enrollment incentives. This in turn leads to a lower enrollment threshold \( a_m \) and a higher enrollment rate.

Corollary. Increasing the skill premium reduces the market enrollment threshold in the American system.

Proof. Denote the market enrollment threshold in the American system as \( \tilde{a} \), defined by \( \tilde{Z} - \tilde{a} = 0 \) with \( \tilde{Z} = \frac{h(q^*(\tilde{a}))w_u}{g(q^*(\tilde{a}), a)w_s} \). Since \( \frac{\partial \tilde{Z}}{\partial (w_s/w_u)} < 0 \), we obtain

\[
\frac{\partial a}{\partial (w_s/w_u)} < 0. \tag{7}
\]

In principle, changing the skill premium could have an impact on individual optimal quality. In our framework, there is no such effect. However, a higher skill premium makes becoming skilled more attractive. This will induce more individuals to become skilled in the American system, reducing average quality there. In the European system, more individuals find it attractive to enroll at the previously set quality. As their interest is taken into consideration, the compromise quality will decline, increasing incentives to enroll further.

4 Optimal uniform quality

Suppose that the uniform quality in the European framework is chosen so as to maximize welfare. Welfare \( W \) is represented by a Benthamite utilitarian welfare function, aggregating utility from wage income minus utility losses due to acquiring
human capital:

\[ W = \int_0^{\alpha^*(q)} \log(w_u a) f(a) da + \int_{\alpha^*(q)}^{1} \log(w_s g(q, a) a) f(a) da \]

Thus, aggregate welfare is derived by adding utility from income of the unskilled, \( \int_0^{\alpha^*(q)} \log(w_u a) f(a) da \), to utility from income of the skilled, \( \int_{\alpha^*(q)}^{1} \log(w_s g(q, a) a) f(a) da \), net of the aggregate utility cost of acquiring higher education, \( \int_{\alpha^*(q)}^{1} \log(h(q) / a) f(a) da \).

The social planner maximizes welfare with respect to the choice of a uniform quality standard. This can be interpreted as representing the outcome of a probabilistic voting process with two parties choosing a political platform and voters whose choice is governed additionally by ideological concerns. Considering the standard scenario in which all voters have identical political power, this framework has a unique equilibrium in which both parties converge to the same platform - the one that maximizes the Benthamite social welfare function (Coughlin and Nitzan, 1981, Persson and Tabellini, 2000). Rewriting welfare yields

\[ W = \alpha^*(q) \log(w_u) + \int_0^{\alpha^*(q)} \log(a) f(a) da + (1 - \alpha^*(q)) \log(w_s) \]

\[ + \int_{\alpha^*(q)}^{1} \log(g(q, a)) f(a) da + \int_{\alpha^*(q)}^{1} \log(a) f(a) da \]

\[ - \int_{\alpha^*(q)}^{1} \log(h(q)) f(a) da + \int_{\alpha^*(q)}^{1} \log(a) f(a) da. \]
Taking into account the market equilibrium condition (1), the derivative of the welfare function with respect to \( q \) can be simplified as follows:

\[
\frac{\partial W}{\partial q} = \frac{\partial a^*(q)}{\partial q} \log \left( \frac{w_u}{w_x} \right) - \frac{\partial a^*(q)}{\partial q} \log(a^*(q)) + \int_{a^*(q)}^{1} \frac{g}{g} f(a) da \\
- \log(g(q, a^*(q))) \frac{\partial a^*(q)}{\partial q} - \int_{a^*(q)}^{1} \frac{h'}{h} f(a) da + \log(h(q)) \frac{\partial a^*(q)}{\partial q} \\
= \int_{a^*(q)}^{1} \left( \frac{g(q, a)}{g(q, a)} - \frac{h_q(q)}{h(q)} \right) f(a) da
\]  

(9)

The uniform quality \( q^u \) is the solution of the \( \frac{\partial W}{\partial q}(a^*(q^u)) = 0 \). Recalling the structurally similar condition determining ability-specific optimal qualities (3), the last line shows that quality in the European model reflects a compromise. Top ability individual will consider the uniform quality level as being too low, while marginal students are to some extent deterred by a quality that looks too high from their vantage point.

The second-order condition reads

\[
\frac{\partial^2 W}{\partial q^2} = \int_{a^*(q)}^{1} \left( \frac{g_{qq}g - (g_q)^2}{g^2(q^*, a)} f(a) da - \frac{g_q(q, a^*(q))}{g(q, a^*(q))} \right) \frac{\partial a^*(q)}{\partial q} \\
- \int_{a^*(q)}^{1} \frac{h_{qq}h - (h_q)^2}{h^2} f(a) da + \frac{h_q(q)}{h(q)} \frac{\partial a^*(q)}{\partial q} \\
< 0.
\]  

(10)

When substituting \( \frac{\partial a^*(q)}{\partial q} \), it transpires that

\[
\frac{\partial^2 W}{\partial q^2} = \int_{a^*(q)}^{1} \left[ \frac{g_{qq}g - (g_q)^2}{g^2(q^*, a)} - \frac{h_{qq}h - (h_q)^2}{h^2(q^*)} \right] f(a) da + \left[ \frac{h_q(q)}{h(q)} - \frac{g_q(q, a^*(q))}{g(q, a^*(q))} \right]^2 a^*
\]  

(11)
The first term in (11) is negative under the assumptions we made so far, but the second term is positive. In principle it could make sense to have multiple solutions. Increasing the quality will lead to a smaller enrollment rate, which in itself yields a welfare loss. At the same time, those remaining enrolled are served better on average by the higher quality as gains of the highly talented outweigh losses for the now smaller group of medium ability types. However, as we would like to conduct comparative statics, we exclude multiple solutions.

Proposition 5 compares the American to the European system in terms of enrollment, quality, and income dispersion.

**Proposition 5** The American (tailored) system exhibits a higher quality at the top and a lower quality at the bottom than the European uniform system. Moreover, it displays a higher enrollment and a wider dispersion of income.

**Proof.** Recalling Proposition 1 stating that most preferred quality increases in ability, a higher quality at the top in the American system is immediate from comparing (3) and (9). Since utility as skilled worker is at least as high in the American system as in the European system, enrollment in the American system must be weakly higher than under the European frame. As condition (9) together with type-specific most preferred quality increasing in ability ensures that the marginal student under the European frame is not at his or her optimal quality level, utility in the American system is higher at the European marginal type. This in turn ensures that some types slightly below the European margin find it optimal to enroll in the American system. Thus, total enrollment is higher with the American frame. Since quality in the European system is already higher than in the American scheme at the European margin, quality at the bottom of the American system is even lower. Higher quality at the top ability type translates into higher income at the top of the American system than in the European system. Since at the same time lowest incomes of unskilled individuals are identical, income dispersion in the American system is higher.

Notice that the European scheme is not efficient. This can be demonstrated as follows. Consider any optimal allocation under a European framework. The
chosen quality will lie above the optimum of the marginal individual being indifferent whether or not to enroll. Offering some tailored lower quality to an individual with lower ability may induce this individual to enroll without harming anybody.

Since the American system simply Pareto dominates the European system, the issue arises why the European scheme is nevertheless chosen by many countries. One obvious possibility lies in egalitarian preferences. If an income inequality measure enters the choice of the social planner between the American system and the European system, the latter will be preferred given a sufficiently high weight on the inequality measure. From a political economy point of view, this can be supported by ability types from the middle of the spectrum who achieve a higher income under the European frame due to higher quality and potentially suffer from relative deprivation. With such preferences, they are interested in limiting quality of high ability types if no other instrument of redistribution is available. An interesting perspective comes from an income maximizing social planner. If costs of education are ignored, say due to being psychic in nature and irrelevant for purposes of tax generating, the European system can generate higher income. Such a surprising can occur if the dominating effect comes from a high share of middle ability types being forced into higher quality under the European frame, offsetting both lower incomes at the top and lower income due to lower enrollment. At the same time, lower enrollment need not occur if the ability distribution is discrete.

**Proposition 6** Increasing the skill premium reduces quality under the European scheme and reduces average quality under the American scheme.

**Proof.** Equation (3) shows that individualized optimal qualities are independent of wages. As enrollment increases with a higher skill premium and optimal qualities rise with ability according to Proposition 1, all additionally enrolled types have lower optimal qualities than those already enrolling at a lower skill premium. This reduces both chosen quality in the European system according to (9) and average quality in the American scheme. For the European system, totally differentiating (9) yields
\[
\frac{\partial q}{\partial w_s/w_a} = -\frac{\partial^2 W/\partial q \partial (w_s/w_u)}{\partial^2 W/\partial q^2} = sgn\left[\frac{\partial a^*(q)}{\partial (w_s/w_u)} \left(\frac{g_q(q,a^*(q))}{g(q,a^*(q))} - \frac{h_q(q)}{h(q)}\right)\right] < 0
\]

since \(\frac{\partial a^*(q)}{\partial (w_s/w_u)} < 0\) and \(\frac{g_q(q,a^*(q))}{g(q,a^*(q))} - \frac{h_q(q)}{h(q)} < 0\) at the European optimum. \(\square\)

5 Two-stage higher education

Another main feature of higher education reform following the Bologna initiative in many European countries in the early 2000s has been moving to a two-stage higher education scheme with bachelor and master, replacing the older one-stage diploma. We show that this reform leads to higher enrollment as it allows to reduce quality at the first (bachelor) stage. At the same time, quality in the second stage can be increased as only a fraction of all students - and less than the share of the students under the diploma system – is expected to enroll into master studies. At the same type, the master students are harmed relative to a system in which two quality types were employed already at the outset. This can be formalized as follows. While the returns to quality are not altered, the cost function changes such that \(h(q)\) stays relevant for students completing only the first stage, while it is replaced by \(\phi(q_1,q_2)\) for students obtaining both grades, with \(q_1\) denoting quality at the first stage and \(q_2\) representing quality at the second stage, which will typically exceed the former. As there may be a jump in quality, an adaptation cost may arise such that \(\phi(q_1,q_2) \geq h(q_2)\) where \(\frac{\partial \phi}{\partial q_1} < 0\) for any \(q_1 < q_2\). Should, for whatever reason, quality in the first stage be higher than in the second, we would have \(\phi(q_1,q_2) \geq h(q_2)\) with \(\frac{\partial \phi}{\partial q_1} > 0\) for any \(q_1 > q_2\). In that event, the effort demanded at the first stage is too high given the ultimate target quality. Put differently, considering a variation in \(q_1\), the cost function \(\phi(q_1,q_2)\) assumes a minimum at \(q_1 = q_2\). Assume moreover identical marginal cost terms at the ultimate quality, \(\frac{\partial \phi}{\partial q_2} = h_q(q_2)\). This ensures that the only modification of the cost term is an adaptation cost, which will be zero at \(q_1 = q_2\), thus \(\phi(q_1,q_1) = h(q_1)\). Finally, let the adaptation cost be sufficiently small,
that is \( \frac{\partial \phi}{\partial q_1} / \frac{\partial \phi}{\partial q_2} \) remains close to zero.

The problem of the social planner is then modified as follows:

\[
\max_{q_1, q_2} W = \int_0^{a^*(q_1)} \log(w_u a) f(a) da + \int_{a^*(q_1)}^{a^*(q_2, q_1)} \log(w_s g(q_1, a) a) f(a) da \\
+ \int_{a^*(q_2, q_1)}^{1} \log(w_s g(q_2, a) a) f(a) da \\
- \int_{a^*(q_1)}^{1} \log(h(q_1) / a) f(a) da - \int_{a^*(q_2, q_1)}^{1} \log(\phi(q_1, q_2) / a) f(a) da
\]

In this problem, the social planner chooses qualities in the first stage \( q_1 \) and in the second stage, \( q_2 \), where \( a^*(q_1) \) is the overall enrollment threshold and \( a^*(q_2, q_1) \) denotes the ability threshold for the second stage.

With qualities \( q_1 \) and \( q_2 \) given, the related market enrollment thresholds \( a_1 \) and \( a_2 \) then satisfy

\[
a_1 = \frac{h(q_1) w_u}{g(q_1, a_1) w_s}, \\
\frac{\phi(q_1, q_2)}{h(q_1)} = \frac{g(q_2, a_2)}{g(q_1, a_2)}.
\]

The latter condition refers to the ability level \( a_2 \) at which an individual is indifferent between entering the second stage and completing studies after the first stage. Proposition 7 shows that a unique interior enrollment threshold for the second stage at given quality exists under mild conditions.

**Proposition 7** If \( \lim_{a \to 0} \frac{g(q_2, a)}{g(q_1, a)} \) < \( \frac{\phi(q_1, q_2)}{h(q_1)} \) < \( \frac{g(q_2, 1)}{g(q_1, 1)} \) and \( q_2 > q_1 \), the enrollment threshold \( a_2 \) is interior and unique.

**Proof.** See Appendix C.
Uniqueness of the market enrollment threshold in the second stage requires that the elasticity of productivity with respect to ability is increasing in the quality, which is, however, an immediate consequence of our assumption that the marginal impact of quality reacts more sensitive to changes in ability than the productivity factor $g$ itself.

Rewriting welfare as

$$W = \log(w_a) \int_0^{a_1(q_1)} f(a)da + \log(w_a) \int_{a_1(q_1)}^{1} f(a)da + \int_{a_1(q_1)}^{a_2(q_2,q_1)} \log(g(q_1,a)) f(a)da$$

(16)

$$+ \int_0^1 \log(a) f(a)da \int_{a_1(q_1)}^{a_2(q_2,q_1)} \log g(q_2,a) f(a)da + \int_0^1 \log(a) f(a)da$$

$$- \int_{a_1(q_1)}^{a_2(q_2,q_1)} \log(h(q_1)) f(a)da - \int_{a_2(q_2,q_1)}^{1} \log(\phi(q_1,q_2)) f(a)da$$

the first-order conditions to the optimization problem of the social planner are

$$\frac{\partial W}{\partial q_1} = \int_{a_1(q_1)}^{a_2(q_2,q_1)} \left[ \frac{g_q(q_1,a)}{g(q_1,a)} - \frac{h_q(q_1)}{h(q_1)} \right] f(a)da - \int_{a_1(q_1)}^{a_2(q_2,q_1)} \frac{\phi_{q_1}(q_1,q_2)}{\phi(q_1,q_2)} f(a)da$$

(17)

$$+ \frac{\partial W}{\partial a_1} \frac{\partial a_1}{\partial q_1} + \frac{\partial W}{\partial a_2} \frac{\partial a_2}{\partial q_1}$$

$$= \int_{a_1(q_1)}^{a_2(q_2,q_1)} \left[ \frac{g_q(q_1,a)}{g(q_1,a)} - \frac{h_q(q_1)}{h(q_1)} \right] f(a)da - \int_{a_1(q_1)}^{a_2(q_2,q_1)} \frac{\phi_{q_1}(q_1,q_2)}{\phi(q_1,q_2)} f(a)da$$

$$= 0,$$

$$\frac{\partial W}{\partial q_2} = \int_{a_1(q_1)}^{a_2(q_2,q_1)} \left[ \frac{g_q(q_2,a)}{g(q_2,a)} - \frac{\phi_{q_2}(q_1,q_2)}{\phi(q_1,q_2)} \right] f(a)da + \frac{\partial W}{\partial a_2} \frac{\partial a_2}{\partial q_2}$$

(18)

$$= \int_{a_1(q_1)}^{a_2(q_2,q_1)} \left[ \frac{g_q(q_2,a)}{g(q_2,a)} - \frac{\phi_{q_2}(q_1,q_2)}{\phi(q_1,q_2)} \right] f(a)da$$

$$= 0.$$
because \( \frac{\partial W}{\partial a_1} = 0 \) owing to the market equilibrium condition (14) and \( \frac{\partial W}{\partial a_2} = 0 \) due to the other market equilibrium condition (15). The first-order conditions can be interpreted as follows. While (18) again indicates that the chosen quality level in the second stage is a compromise between optimal quality levels of high ability individuals and marginal ability types entering the second stage, the optimality condition for the first stage (17) shows according to the second line that the compromise quality of those leaving the university system after their bachelor is distorted upward to dampen the negative impact on costs of acquiring education of the high ability types going through both stages.

Similar to the analysis of quality choice for single-stage studies above, multiple solutions to the welfare optimization problem may exist. We assume uniqueness also here so as being able to compare the two-stage system to the uniform European-type scheme, where Proposition 8 summarizes the outcomes.

**Proposition 8** Compared to the European single stage system, the two-stage scheme yields (i) higher overall enrollment, (ii) lower quality at the first stage, (iii) higher quality at the second stage.

**Proof.** Recall Proposition 1 stating that individualized optimal qualities increase in ability. This is also true for qualities in the second stage, noting that marginal cost terms stay unaffected by assumption. It can never be optimal to have a lower quality at the second stage, \( q_2 < q_1 \), because nobody is enrolling as the additional cost of obtaining the second degree is positive while the marginal return is negative. It has to be demonstrated that the solution does not simply coincide with the single-stage uniform optimal quality \( q_u \). When \( q_1 \) coincides with the solution from one-stage higher education \( q_u \), choosing some \( q_2 \in (q_1, q^*(1)) \) for the second stage will be taken up by some top ability students, improving their utility without harming anybody. Thus, there will be two stages exhibiting \( q_2 > q_1 \).

Next, we demonstrate that \( q_2 > q_u \). Using the fact that \( \frac{\partial \phi}{\partial q_2} = h'(q_2) \) we rewrite
(18) as

\[
\frac{\partial W}{\partial q_2} = \int_{a_2(q_2,q_1)}^{1} \left[ \frac{g_q(q_2,a)}{g(q_2,a)} - \frac{h_q(q_2)}{h(q_2)} \right] f(a) da \\
+ \int_{a_2(q_2,q_1)}^{1} h_q(q_2) \left[ \frac{1}{h(q_2)} - \frac{1}{\phi(q_1,q_2)} \right] f(a) da \\
= 0.
\]

Since \( q_2 > q_1 \), we have \( \phi(q_1,q_2) > h(q_2) \). Evaluating \( \frac{\partial W}{\partial q_2} \) at \( q_u \) yields \( \frac{\partial W}{\partial q_2}(q_1,q_u) > 0 \), as \( a_2(q_2,q_1) > a_1(q_1) \) when \( q_2 > q_1 \). Consequently, when \( q_2 > q_1, q_2 > q_u \).

Consider now (17), the first-order condition with respect to \( q_1 \). With \( q_2 > q_1 \) and \( q_2 > q_u \), we obtain for the first term

\[
\int_{a_1(q_u)}^{a_2(q_2,q_u)} \left[ \frac{g_q(q_u,a)}{g(q_u,a)} - \frac{h_q(q_u)}{h(q_u)} \right] f(a) da < 0
\]

as long as \( a_2(q_2,q_u) < 1 \), indicating that a uniform quality that would maximize welfare of agents with abilities between \( a_1 \) and \( a_2 \) is lower than \( q_u \), which is the result of an optimization that takes into account the upper portion of the ability distribution (from \( a_2 \) to 1). The second term of \( \frac{\partial W}{\partial q_1}(q_u,q_2) \),

\[
\int_{a_2^*(q_2,q_u)}^{1} \frac{\phi(q_u,q_2)}{\phi(q_1,q_2)} f(a) da
\]

is positive as \( \frac{\partial \phi}{\partial q_1} < 0 \) for any \( q_1 < q_2 \). As this adjustment cost is comparatively small by assumption, the first term dominates and \( \frac{\partial W}{\partial q_1}(q_u,q_2) < 0 \) and \( q_1 < q_u \). Thus, overall enrollment is higher under the two-stage system.

Proposition 8 can be interpreted as follows. Unsurprisingly, the additional political option to differentiate academic qualities will be employed. This is true because
the differentiated scheme can be implemented in a Pareto improving fashion. As nobody would enroll in the second stage otherwise, quality in the masters program will exceed quality of the bachelor stage. The second stage exhibits a higher quality than the uniform program because the adjustment cost calls for a higher marginal benefit of the compromise quality at the second stage. This can only be achieved by raising the quality above the level of the single-stage system. As high ability students proceed to the second stage, quality in the first stage can be reduced below the level from the uniform system to adapt to the preferences of students completing studies after the first stage. Since marginal students are now served better, this in turn also induces a higher overall enrollment.

It is obvious that the two-stage system is dominated by a one-stage system with the same two qualities $q_1$ and $q_2$. While such a reform would leave overall enrollment unchanged, all people taking the higher track benefit, either due to saving the adaptation cost $[h(q_2) - \phi(q_1, q_2)]/a$ if they would opt for the higher quality anyway, or by being offered a more attractive higher track, allowing to switching away from just completing a bachelor-type degree.

6 Concluding discussion

We have seen that both differentiated American-type schemes and uniform European-type schemes will respond to a higher skill premium by cutting quality in higher education - the American by extending the share of comparatively low-quality colleges, and the European by accommodating to the needs to the incoming marginal students. One puzzling issue results from welfare considerations, which is unambiguously in favor of the American system due to Pareto dominance. Unlike the American system, the European system is not efficient. With egalitarian preferences the European system could be preferred though being dominated by the American system. The European system generates a somewhat more compressed income distribution among the skilled and a higher share of the unskilled. At the same time, the jump in income at the margin will typically be higher. A possible other reason may consist in high costs of creating transparency with respect to ranking of universities in terms of quality - at least when it comes to paying according to quality, yielding fears of adverse selection. An alternative argument of changing the picture in favor of the European
system stems from effort choices not discussed by now. When a university ranking exists, competition between students attending different colleges is hampered, which may lead to smaller effort.

It might also happen that political economy forces bring down quality in the European system below the Benthamite optimum, as this could benefit an important group of voters with below average talent at the expense of a smaller group of top ability types. With a typical skewed ability distribution and those remaining unskilled anyway abstaining from the vote, using a median voter framework instead of probabilistic voting would generate a lower quality. This is mainly to be traced back to voters preferring low qualities though not entering the university system in the political equilibrium. At the same time, working in the same direction, the probabilistic framework takes into account higher net gains from increasing quality for high ability students while the median voter scenario does not. The political outcome may also change if the unskilled are affected by skilled workers being trained at a higher quality.

Finally, in terms of international competition, the American system is particularly attractive for highly talented individuals. Hence, one prediction of the model is that there will be net migration from countries with European-style systems to the top American-style universities. While the migration incentive exists for almost all ability types, any model of imperfect mobility would suggest selectivity in migration such that migrating students exhibit disproportional high abilities.
References


Appendix

A: Proof of Proposition 1

The first claim is immediate from observing the sufficient second-order condition

\[ X_{qq} = \frac{g_{qq}g - (g_q)^2}{g^2(q^*, a)} - \frac{h_{qq}h - (h_q)^2}{h^2(q^*)} < 0. \]  

The implicit function theorem then yields:

\[ \frac{dq^*}{da} = -\frac{X_{qq}(q^*, a)}{X_{qq}(q^*, a)} \]  

Since \( X_{qq}(q^*, a) < 0 \) holds due to second-order condition, we have \( sgn \left[ \frac{dq^*}{da} \right] = sgn \left[ X_{qq}(q^*, a) \right] \). Calculating this cross-derivative gives

\[ X_{qq}(q^*, a) = \frac{g_{qq}(q^*, a)g(q^*, a) - g_q(q^*, a)g_a(q^*, a)}{[g(q^*, a)]^2} > 0. \]  

where the sign is determined by our assumption \( \frac{g_{qq}(q^*, a)}{g_q(q^*, a)} > \frac{g_a(q^*, a)}{g(q^*, a)} \).

B: Proof of Proposition 2

Differentiating \( Z \) with respect to \( a^* \) yields \( \frac{\partial Z}{\partial a^*} = -\frac{w_s}{w_s} \frac{g_a h}{g^2} < 0 \) as \( g_a > 0 \). In order to guarantee uniqueness we need \( Z(1) < 1 \), i.e. \( \frac{w_s}{w_s} > \frac{h(q)}{g(q, 1)} \). Assuming \( g(0, 0) > 0 \) and \( h(0) > 0 \) ensures \( Z > 0 \) at \( a^* = 0 \). Since \( \lim_{a^* \to 0} Z(a^*) > 0 \) and \( \frac{\partial Z}{\partial a^*} < 0 \), the additional condition \( Z(1) < 1 \) indeed suffices to arrive at a unique interior threshold.

C: Proof of Proposition 7

Differentiating the right-hand side of equation (15) yields a positive derivative iff

\[ \frac{g_a(q_2, a)g_q(q_1, a) - g(q_2, a)g_a(q_1, a)}{g^2(q_1, a)} > 0, \]  

being equivalent to \( \frac{g_a(q_2, a)}{g(q_2, a)} > \frac{g_a(q_1, a)}{g(q_1, a)} \). The latter condition holds if

\[ \frac{\partial [g_a(q, a)/g(q, a)]}{\partial q} = \frac{g(q, a)g_{aq}(q, a) - g_a(q, a)g_q(q, a)}{g^2(q, a)} > 0, \]  

which is satisfied according to the assumption \( \frac{g_{aq}(q, a)}{g_q(q, a)} > \frac{g_a(q, a)}{g(q, a)} \).
Fig. 1. Type-specific optimal qualities