Regional Infrastructure and Economic Growth

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We examine the effect of changes in the number of regions on regional economic growth and welfare, incorporating productive public goods as the growth engine, into a simple overlapping generations model. We demonstrate that the regional welfare-maximizing tax is smaller than the tax revenue of own regions.

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1. Introduction

Since the seminal works of Aschauer (1989) and Iwamoto (1990), the impact of infrastructure on productivity has attracted much attention from economists. It is well known empirically that the output elasticity of infrastructure is much smaller at the regional level than that at the aggregate level (Munnell, 1990). As suggested Moomaw et al (1995), one plausible explanation may be the regional spillover effect that infrastructure in one region affects other regions as well as its own production\(^1\). There is much empirical analysis of this issue and the relevance of regional spillovers (see, for example, Pereira (2013) for an excellent survey). Cohen and Paul (2004) among others suggested a positive relationship between regional infrastructure and output in their empirical analysis, although this effect still seems empirically controversial. The first objective of this paper is to examine the effect of changes in the number of regions on economic growth and welfare.

Theoretically, in endogenous growth models with infrastructure, most of the literature has focused on the growth and welfare effects of fiscal policies. As shown in Barro (1990) and among others, infrastructure increases the marginal product of private capital and, thus, leads to economic growth using a representative agent framework (e.g., Futagami et al., 1993; Turnovsky and Fisher, 1995). To our knowledge, there is little theoretical research attempted to explain the relationship between inter-regional infrastructure and economic growth. Figuieres et al (2013) is the only theoretical study the effects of infrastructure produced externalities using two-countries model. However, the above authors did not consider the regional growth effect. In this study we analyze the effect of changes in the number of regions on economic growth and welfare, incorporating inter-regional infrastructure, into a simple overlapping generations model.

The analysis of tax competition were pioneered by Wilson (1986), Zodrow and Miezkowski (1986), and has been developed by many authors (see, for example,\footnote{Another explanation is the endogeneity issue. See for example, Cadot et al. (2006).}
Wildasin and Wilson, 2004). In the static model of tax competition, they emphasized horizontal externalities arising from mobility of the tax base between regions. Since horizontal externalities tend to leave taxes too low, one solution is to offer matching grants to regions to lower the cost of the public service to them. Alternatively, Hoyt (1991) proposed that the competition for capital is reduced by decreasing the number of regions, i.e., market power.

Our model has two main features. First, and most importantly, we assume that the growth engine is productive public goods to consider four cases such as maximizing the share of capital of own region, the tax revenue of own region, the income of own region and the regional welfare. Second, we assume overlapping-generations economy that consists of $n$ regions.

The remainder of this paper is organized as follows. In Section 2 a model is presented, and decentralized equilibrium is characterized in Section 3. In Section 4 the relation between decentralized and centrally planned economy are examined. Section 5 offers some conclusions.

2. Model

Consider an overlapping-generations economy that consist of $n$ regions denoted by subscript numbers $i = 1, 2, \cdots, n$. Time is discrete and indexed by $t$. The population of region $i$ is constant over periods and denoted by $L_i$. Each region has a representative immobile resident who lives for two periods. The capital market is integrated in the economy, but the labor market is not integrated and its participants are composed of workers live in own region. We assume that the population of the economy is normalized to unity, i.e., $L = \sum_{i=1}^{n} L_i = 1$. Let aggregate private capital as $K = \sum_{i=1}^{n} K_i (t)$.

Each resident works only in the first period, and allocates the wage income between consumption and savings for future consumption. In the second period, he/she spends
all his/her savings and accrued interest on consumption $d_i(t) = R(t+1)s_i(t)$. The
preference of resident in region $i$ at period $t$ is assumed to be log-linear:
$$U_i(t) = \beta_i \ln [w_i(t) - s_i(t)] + (1 - \beta_i) \ln [R(t+1)s_i(t)],$$
where $s_i(t)$ is the saving in region $i$, and $R(t+1)$ the interest factor at period $t+1$.

Solving the utility maximization problem such as $\max_{s_i(t)} U_i(t)$, we obtain the following
saving function:
$$s_i(t) = (1 - \beta_i)w_i(t). \quad (1)$$

We assume that private goods are produced in each region by perfectly competitive
firms using by the capital and labor input. The production function in region $i$ is
$$Y_i(t) = A_i \left[ K_i(t) \right]^{\alpha_i} \left[ G(t)L_i(t) \right]^{-\alpha_i}, \quad (2)$$
where $A_i > 0$ is the productivity in region $i$, $G(t)$ the productive public goods
($0 < \alpha_i < 1$). They are provided using by the following CES production technology:
$$G(t) = \left[ \sum_{i=1}^{n} \left[ H_i(t) \right]^{\varepsilon} \right]^{-\frac{1}{\varepsilon}}, \quad (3)$$
where $H_i(t)$ is the tax revenue of region $i$ and $\varepsilon > -1$. Then, we obtain the
elasticity of substitution $\sigma = 1/1 + \varepsilon. 2$

The regional government taxes on the capital input and uses the tax revenue for the
productive public goods. The budget constraint of region $i$ becomes
$$H_i(t) = \tau_i K_i(t), \quad (4)$$
where $\tau_i$ denote the capital tax rate in region $i$.

The profit maximization conditions of firms are
$$r_i(t) = \alpha_i A_i \left[ K_i(t) \right]^{\alpha_i-1} \left[ G(t)L_i(t) \right]^{-\alpha_i} - (1 + \tau_i), \quad (5)$$
$$w_i(t) = (1 - \alpha_i)A_i \left[ K_i(t) \right]^{\alpha_i} \left[ G(t)L_i(t) \right]^{-\alpha_i}, \quad (6)$$

$2$ If $\sigma \to \infty$, we have $G(t) = \sum_{i=1}^{n} H_i(t)$. 

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where \( r_i(t) \) is the interest rate in region \( i \) at period \( t \) and \( w_i(t) \) the wage rate in region \( i \) at period \( t \), respectively.

Since the capital market is integrated, the factor price equalization holds: \( r_i(t) = r_j(t) \) (\( i, j = 1, 2, \ldots, n \) and \( i \neq j \)). In other words, we have

\[
\alpha_i A_i \left[ K_i(t) \right]^{\kappa_i} \left[ G(t) L_i(t) \right]^{-\alpha_i} - \tau_i = \alpha_j A_j \left[ K_j(t) \right]^{\kappa_j} \left[ G(t) L_j(t) \right]^{-\alpha_j} - \tau_j.
\]

Then, \( R(t + 1) = 1 + r_i(t + 1) \) holds for \( i, j = 1, 2, \ldots, n \).

Let \( k_i := K_i(t)/K(t) \) and \( \lambda_i := L_i(t)/L \). Note that \( \sum_{i=1}^{n} k_i = 1 \) and \( \sum_{i=1}^{n} \lambda_i = 1 \) hold. Using equations (3) and (4), the ratio of productive public goods to private capital is given by

\[
\frac{G(t)}{K(t)} = \left[ \sum_{i=1}^{n} (\tau_i, k_i) \right]_{\sigma-1}^{\sigma} \sigma^{-1}.
\]

Furthermore, the condition for factor price equalization (7) is rewritten as

\[
\alpha_i A_i \kappa_i^{-1} \lambda_i^{-1} \left[ \frac{G(t)}{K(t)} \right]^{\sigma} - \tau_i = \alpha_j A_j \kappa_j^{-1} \lambda_j^{-1} \left[ \frac{G(t)}{K(t)} \right]^{\sigma} - \tau_j.
\]

Equations (7) and (8) yield

\[
\alpha_i A_i \kappa_i^{-1} \lambda_i^{-1} \left[ \sum_{i=1}^{n} (\tau_i, k_i) \right]_{\sigma-1}^{\sigma} \sigma^{-1} \sigma^{-1} - \tau_i = \alpha_j A_j \kappa_j^{-1} \lambda_j^{-1} \left[ \sum_{i=1}^{n} (\tau_i, k_i) \right]_{\sigma-1}^{\sigma} \sigma^{-1} \sigma^{-1} - \tau_j.
\]

Using equations (2) and (8), the production function in equilibrium is

\[
Y_i(t) = A_i \kappa_i^{-1} \lambda_i^{-1} \left[ \sum_{i=1}^{n} (\tau_i, k_i) \right]_{\sigma-1}^{\sigma} \sigma^{-1} \sigma^{-1} K_i(t).
\]

Equations (1), (6), (8) and (11), the dynamic equation of capital accumulation is

\[
K_i(t + 1) = \sum_{i=1}^{n} s_i(t) L_i = (1 - \alpha)(1 - \beta) \sum_{i=1}^{n} Y_i(t).
\]
Then, by equations (11) and (12), the growth factor becomes

\[
1 + \gamma = \frac{K(t + 1)}{K(t)} = (1 - \alpha)(1 - \beta) \sum_{i=1}^{n} A_i \kappa_i^{\sigma} \lambda_i^{1-\alpha_i} \left[ \sum_{i=1}^{n} (\tau_i, \kappa_i) \frac{\sigma-1}{\sigma} \right] \frac{(1-\alpha_i)^{1-\alpha_i}}{\alpha_i}. \tag{13}
\]

The indirect utility of the generation \( t \) of region \( i \) is

\[
V_i(t) = \beta_i \ln \beta_i + (1 - \beta_i) \ln(1 - \beta_i) + \ln w_i(t) + (1 - \beta_i) \ln R(t + 1). \tag{14}
\]

Hereafter, we omit the script \( t \) except for calling readers’ attention.

### 3. Decentralized equilibrium

#### 3.1. The behavior of regional government and economic outcomes

If the regional government ignores the effects of a change in \( \tau_i \) on the average and other regions’ economic variables, we have

\[
\tau_i \frac{\partial \kappa_i}{\partial \tau_i} = \frac{(1 - \alpha_i) \lambda_i \kappa_i^{\sigma-1} \lambda_i^{1-\alpha_i} \left[ \sum_{i=1}^{n} (\tau_i, \kappa_i) \frac{\sigma-1}{\sigma} \right] \frac{(1-\alpha_i)^{1-\alpha_i}}{\alpha_i}}{\sum_{i=1}^{n} (\tau_i, \kappa_i) \frac{\sigma-1}{\sigma}} - \tau_i
\]

\[
(1-\alpha_i) \frac{\alpha_i}{A_i} \frac{\partial \kappa_i}{\partial \tau_i} \frac{\partial A_i}{\partial \tau_i} = \frac{(1 - \alpha_i) \lambda_i \kappa_i^{\sigma-1} \lambda_i^{1-\alpha_i} \left[ \sum_{i=1}^{n} (\tau_i, \kappa_i) \frac{\sigma-1}{\sigma} \right] \frac{(1-\alpha_i)^{1-\alpha_i}}{\alpha_i}}{\sum_{i=1}^{n} (\tau_i, \kappa_i) \frac{\sigma-1}{\sigma}} - \tau_i
\]

\[
(1 - \alpha_i)(1 - \theta_i) = \frac{(1 - \alpha_i)(1 - \theta_i) \lambda_i \kappa_i^{\sigma-1} \lambda_i^{1-\alpha_i} \left[ \sum_{i=1}^{n} (\tau_i, \kappa_i) \frac{\sigma-1}{\sigma} \right] \frac{(1-\alpha_i)^{1-\alpha_i}}{\alpha_i}}{\sum_{i=1}^{n} (\tau_i, \kappa_i) \frac{\sigma-1}{\sigma}} - \tau_i
\]

where \( \theta_i := \left( \frac{\tau_i}{\kappa_i} \right) \frac{\sigma-1}{\sigma} \left[ \sum_{i=1}^{n} (\tau_i, \kappa_i) \frac{\sigma-1}{\sigma} \right]. \)

The regional government chooses the capital tax rate of each region on the basis of
its policy target. As a consequence, different policies make different equilibria. We now consider four cases such as maximizing the share of capital of own region, the tax revenue of own region, the income of own region and the regional welfare of generation $t$.

Case A: The regional government has a concern for maximizing the share of capital of own region. The maximizing condition is given as

$$\frac{\tau_i}{\kappa_i} \frac{\partial \kappa_i}{\partial \tau_i} = 0.$$  

Let the tax rate that satisfies the above equation as

$$\tau_{iA} = (1 - \alpha_i) \alpha_i A \theta_i \kappa_i^{\alpha_i - 1} \lambda_i^{\alpha_i - 1} \left[ \sum_{i=1}^{n} (\tau_i \kappa_i) \right]^{\frac{\sigma-1}{\sigma}}.$$  

(16)

Case B: The regional government has a concern for maximizing the tax revenue of own region. Note that $K(t)$ is given at period $t$. Then, the differentiation of (4) with respect to $\tau_i$ yields

$$\frac{\partial (\tau_i \kappa_i)}{\partial \tau_i} = \left[ 1 + \frac{\tau_i}{\kappa_i} \frac{\partial \kappa_i}{\partial \tau_i} \right] \kappa_i.$$  

By the above equation, we arrive at

$$\frac{\tau_i}{\kappa_i} \frac{\partial \kappa_i}{\partial \tau_i} = -1, \quad \tau_{iA} < \tau_{iB}.$$  

(17)

The leviathan government chooses the tax rate, which is higher than the capital share maximizing tax rate, to raise its revenue.

Case C: The regional government has a concern for maximizing the income of own region. The partial derivative of (11) with respect to $\tau_i$ derives

$$\frac{\tau_i}{Y_i} \frac{\partial Y_i}{\partial \tau_i} = (1 - \alpha_i) \theta_i + \left[ \alpha_i + (1 - \alpha_i) \theta_i \right] \frac{\tau_i}{\kappa_i} \frac{\partial \kappa_i}{\partial \tau_i}.$$  

Therefore, we obtain the following condition:
\[
\frac{\tau_i}{\kappa_i} \frac{\partial \kappa_i}{\partial \tau_i} = -\frac{(1-\alpha) \theta_i}{\alpha + (1-\alpha) \theta_i} > -1. \tag{18}
\]

As a result, we obtain \( \tau_{il} < \tau_{ic} < \tau_{ib} \).

Case D: The regional government has a concern for maximizing regional welfare of generation \( t \) subject to \( \partial R / \partial \tau_i = 0 \). Note that we have the following formulas for regional governments:

\[
\frac{\tau_i}{w_i} \frac{\partial w_i}{\partial \tau_i} = \tau_i \frac{\partial Y_i}{Y_i} \frac{\partial \tau_i}{\partial \tau_i},
\]

\[
\frac{\tau_i}{R} \frac{\partial R}{\partial \tau_i} = 0.
\]

Then, the partial derivative of equation (14) with respect to \( \tau_i \) and evaluating at \( \partial R / \partial \tau_i = 0 \) yields

\[
\left. \frac{\partial V_j}{\partial \tau_i} \right|_{\tau_i, R} = \frac{1}{w_i} \frac{\partial w_i}{\partial \tau_i} + (1-\beta_i) \frac{1}{R} \frac{\partial R}{\partial \tau_i} = \frac{1}{w_i} \frac{\partial w_i}{\partial \tau_i} = \frac{1}{Y_i} \frac{\partial Y_i}{\partial \tau_i}.
\]

This equation implies that the welfare-maximizing is equivalent to the income-maximizing. Then, we have

\[
\frac{\tau_i}{\kappa_i} \frac{\partial \kappa_i}{\partial \tau_i} = -\frac{(1-\alpha_i) \theta_i}{\alpha_i + (1-\alpha_i) \theta_i}, \quad \tau_{id} = \tau_{ic}. \tag{19}
\]

To solve the model, we assume a symmetric equilibrium. In a symmetric equilibrium, we have \( A_i = A \), \( \alpha_i = \alpha \), \( \beta_i = \beta \), and \( \kappa_i = \lambda_i = 1/n \). Then, equations (15)-(19) provide

\[
\tau_A = \left(1 - \alpha A \right) \left[ \frac{\tau - \alpha \tau}{\tau} \right]^{\frac{2-\alpha-\sigma}{\alpha}} n^{\frac{\alpha(\sigma-1)}{\sigma}}.
\]

3 The social welfare function the government cares about both young and old generations are considerable. However, under \( \partial R / \partial \tau_i = 0 \), this class of social welfare function provides same result.

4 With identical jurisdictions, in equilibrium all jurisdictions set the same tax rate. Then as the number of jurisdictions decreases, or analogously the market power of jurisdictions increases. See for Hoyt (1991).
\[ \tau_B = \left[(1-\alpha)\alpha A \right]^\frac{1}{n} n^{\frac{1-\alpha}{\alpha (\sigma-1)}}. \quad (21) \]

\[ \tau_C = \left[\frac{(1-\alpha)\alpha A}{1+(n-1)\alpha} \right] n^{\frac{1-\alpha}{\alpha (\sigma-1)}}. \quad (22) \]

The analyses of the Case A-D and equations (20)-(22) provide the following result:

**Proposition 1.** Note that \( \tau_C = \tau_D \) and \( Y_C = Y_D \) hold. Then, in a symmetric equilibrium, the following relations are true for same level of the capital stock:

\[ \tau_A < \tau_C < \tau_B, \]
\[ Y_A < Y_C < Y_B. \]

**Proof** \( \tau_C = \tau_D, \ Y_C = Y_D \) and \( \tau_A < \tau_C < \tau_B \) are obvious from the main text. Since \( k_i = \lambda_i = 1/n \), equation (11) becomes

\[ Y_X(t) = A n^{\frac{\alpha-\sigma}{\sigma-1}} (\tau_X)^{1-\alpha} K(t). \quad (23) \]

The regional income in the symmetric equilibrium is an increasing function with respect to the capital tax rate for given \( K(t) \). Therefore, we obtain \( Y_A < Y_C < Y_B \) for the same level of capital stock.

Total differentiation of equations (20)-(22) provide

\[ \frac{n}{\tau_A} \frac{\partial \tau_A}{\partial n} = \frac{2-\alpha - \sigma}{\alpha (\sigma-1)}, \quad (24) \]
\[ \frac{n}{\tau_B} \frac{\partial \tau_B}{\partial n} = \frac{1-\alpha}{\alpha (\sigma-1)}, \quad (25) \]
\[ \frac{n}{\tau_C} \frac{\partial \tau_C}{\partial n} = \frac{(1-\alpha)^2 + (2-\alpha - \sigma)\alpha n}{[1+(n-1)\alpha]\alpha (\sigma-1)} \quad (26) \]

### 3.2. Growth and government size

In this subsection, we consider the relation among the economic growth rate,
government size and market power. In a symmetric equilibrium, the equilibrium growth rate becomes

$$\gamma_X = (1 - \alpha)(1 - \beta) An^{\frac{\alpha - \sigma}{\pi - 1}} \left( \tau_X \right)^{1 - \sigma} - 1. \quad \text{(27)}$$

Equation (27) shows that the equilibrium growth rate is increasing with respect to the capital tax rate. We define the capital tax burden as the ratio of capital tax to capital income: $\chi_i := \tau_i / (\alpha \omega_i)$, where $\omega_i := Y_i / K_i$. In a symmetric equilibrium, equations (16)-(19) yield

$$\chi_A = \frac{1 - \alpha}{n}, \quad \text{(28)}$$

$$\chi_B = 1 - \alpha, \quad \text{(29)}$$

$$\chi_C = \frac{1 - \alpha}{1 + (n-1)\alpha}. \quad \text{(30)}$$

Since the regional government size is defined as $g_i := H_i / Y_i$, using by equations (28)-(30), the government size is $g_X := \alpha \chi_X \ (X = A,B,C,D)$.

**Proposition 3.** In a symmetric equilibrium, the government size and economic growth rate in each case satisfy

$$g_A < g_C < g_B,$$

$$\gamma_A < \gamma_C < \gamma_B.$$

**(Proof)** Using (27) and **Proposition 1**, we get $\gamma_A < \gamma_C < \gamma_B$. By equations (28) and (29), we have

$$\chi_C - \chi_A = \frac{1 - \alpha}{1 + (n-1)\alpha} - \frac{1 - \alpha}{n} = \frac{(n-1)(1 - \alpha)^2}{1 + (n-1)\alpha} > 0.$$

Since $\chi_C < \chi_B$, we obtain $\chi_A < \chi_C < \chi_B$. Then, by $g_X := \alpha \chi_X$, we arrive at $g_A < g_C < g_B$. 

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Total differentiation of equation (27) yields

\[
\frac{n}{1 + \gamma_X} \frac{d\gamma_X}{dn} = \frac{\alpha - \sigma}{\sigma - 1} + \frac{n}{\tau_X} \frac{d\tau_X}{dn}.
\]

(31)

Taking into account equations (24)-(26), equation (31) becomes

\[
\frac{n}{1 + \gamma_A} \frac{d\gamma_A}{dn} = \frac{(1 - \alpha)(1 - 2\alpha) - (\sigma - 1)}{\alpha(\sigma - 1)},
\]

(32)

\[
\frac{n}{1 + \gamma_B} \frac{d\gamma_B}{dn} = \frac{(1 - \alpha)(1 - 2\alpha) - \alpha(\sigma - 1)}{\alpha(\sigma - 1)},
\]

(33)

\[
\frac{n}{1 + \gamma_C} \frac{d\gamma_C}{dn} = \frac{1}{\sigma - 1} \left\{ \alpha + \frac{(1 - \alpha)\left[(1 - \alpha)^2 + 2\alpha(2 - \alpha)n\right]}{\alpha[1 + (n - 1)\alpha]} - \frac{(2 - \alpha)\sigma}{1 + (n - 1)\alpha} \right\}.
\]

(34)

4. Centrally planned economy

In this section, we try to analyze the relation between decentralized and centrally planned economy. We assume that the centrally determined tax rate \( \tau \) is common with all regions. Then, the condition for factor price equalization becomes

\[
\alpha A \kappa_i^{\sigma - 1} \lambda_i^{1 - \sigma} \left[ \sum_{i=1}^{n} (\kappa_i) \right]^{\frac{1 - (1 - \alpha)\sigma}{\sigma - 1}} = \alpha A \kappa_j^{\sigma - 1} \lambda_j^{1 - \sigma} \left[ \sum_{j=1}^{n} (\kappa_j) \right]^{\frac{1 - (1 - \alpha)\sigma}{\sigma - 1}}.
\]

(35)

Equation (35) implies that \( \kappa_i \) is independent of the capital tax rate. Equation (35) yields

\[
R = \alpha A \phi^{1 - \alpha} \left[ \sum_{i=1}^{n} (\kappa_i) \right]^{\frac{1 - (1 - \alpha)\sigma}{\sigma - 1}} - \tau.
\]

(36)

Case E: The central government chooses the capital tax rate to maximize the average indirect utility, which is given by

\[
\bar{V}(t) = \frac{1}{n} \sum_{i=1}^{n} V_i(t).
\]

(37)
Differentiating the indirect utility of residents in region \( i \) with respect to \( \tau \), we obtain

\[
\frac{dV_i}{d\tau} = \frac{1}{w_i} \frac{dw_i}{d\tau} + (1 - \beta_i) \frac{1}{R} \frac{dR}{d\tau},
\]

where

\[
\frac{1}{w_i} \frac{dw_i}{d\tau} = 1 - \alpha_i,
\]

and

\[
\frac{\tau}{R} \frac{\partial R}{\partial \tau} = \frac{(1 - \alpha_i) \alpha_i A_i \kappa^a_i \lambda_i^{1-a_i} \tau^{1-a_i} \left[ \sum_{i=1}^n (\kappa_i)_{\sigma} \sigma^{-1} \right]^{(1 - \alpha_i)_{\sigma} \sigma^{-1}} - \tau}{\alpha_i A_i \kappa^a_i \lambda_i^{1-a_i} \tau^{1-a_i} \left[ \sum_{i=1}^n (\kappa_i)_{\sigma} \sigma^{-1} \right]^{(1 - \alpha_i)_{\sigma} \sigma^{-1}} - \tau}.
\]

Differentiating equation (37) with respect to \( \tau \) and applying equations (38)-(40), we arrive at

\[
\frac{d\bar{V}}{d\tau} = \frac{1}{n} \sum_{i=1}^n \frac{1}{w_i} \frac{dw_i}{d\tau} + (1 - \beta_i) \frac{1}{R} \frac{dR}{d\tau}
\]

\[
= \left[ (1 - \alpha_i) + (1 - \beta_i) \right] \left[ (1 - \alpha_i) \alpha_i A_i \kappa^a_i \lambda_i^{1-a_i} \tau^{1-a_i} \left[ \sum_{i=1}^n (\kappa_i)_{\sigma} \sigma^{-1} \right]^{(1 - \alpha_i)_{\sigma} \sigma^{-1}} - \tau \right] \frac{1}{\tau}
\]

Then, we obtain the social welfare-maximizing tax rate such as

\[
\frac{d\bar{V}}{d\tau} = 0, \quad \tau = \frac{(1 - \alpha)(2 - \beta)}{2 - \alpha - \beta} \alpha \omega_E.
\]

Then, equation (41) leads to

\[
\chi_E := \frac{\tau}{\alpha \omega_E} = \frac{(1 - \alpha)(2 - \beta)}{2 - \alpha - \beta},
\]
The comparison among equations (28)-(30) and (42) establish the following proposition:

**Proposition 4.** The relation among the capital tax burden satisfies

\[ \chi_A < \chi_C < \chi_B < \chi_E. \]

We now investigate the welfare comparison among five cases of Case A-E. Starting from same amount of \( K(0) \), we have

\[ V_Z - V_X = \ln \frac{W_Z}{W_X} + (1 - \beta) \ln \frac{R_Z}{R_X} \]

\[
\begin{align*}
&= \ln \left( \frac{1}{\chi_Z} \right) + \ln \left( \frac{1}{\chi_X} \right) + t \ln \left( \frac{1}{\chi_Z} \right) + t \ln \left( \frac{1}{\chi_X} \right) \\
&\quad + (1 - \beta) \left( \frac{1}{\chi_Z} + \frac{1}{\chi_X} \right)
\end{align*}
\]

\[ + (1 - \beta) \left\{ \ln \left( \frac{1}{\chi_Z} \right) + \ln \left( \frac{1}{\chi_X} \right) \right\}. \]

where \( X = A, B, C, D, E \) and \( Z = A, B, C, D, E \ (X \neq Z) \). If \( t \to \infty \), the second term in the right hand side of equation (33) is only important to determine the sign of equation (33). Its term corresponds to growth effect. Therefore, the growth-maximizing policy is important to maximize the long-run regional welfare although it might hurt the welfare of the household born at early period.

Finally, we consider the welfare comparison between the decentralized and centrally planned economy. In a similar way to derive (33), we obtain
\[ \bar{V}_E - V_X = \ln \frac{W_{IE}}{w_{IX}} + (1 - \beta) \ln \frac{R_E}{R_X} \]

\[
\kappa^\tau_{IE} \left[ \sum_{i=1}^{n} (\tau_{i} \kappa_{iE}) \frac{\sigma}{\sigma-1} \right]^{(1-\alpha)\sigma} \frac{1}{\sigma-1} + t \ln \left[ \sum_{i=1}^{n} \kappa^\tau_{i} \lambda^{1-\alpha}_{i} \right]^{(1-\alpha)\sigma} \frac{1}{\sigma-1} \]

\[ + (1 - \beta) \left[ \ln \left( \frac{1 - \chi_{E}}{1 - \chi_{X}} \right) \right] + \ln \left[ \kappa^\tau_{IX} \left[ \sum_{i=1}^{n} (\tau_{i} \kappa_{iX}) \frac{\sigma}{\sigma-1} \right]^{(1-\alpha)\sigma} \frac{1}{\sigma-1} \right] \]

In a symmetric equilibrium, above equation is reduced to

\[ \bar{V}_E - V_X = (1 - \alpha)(2 - \beta + t) \ln \left( \frac{\tau_{E}}{\tau_{X}} \right) + (1 - \beta) \ln \left( \frac{1 - \chi_{E}}{1 - \chi_{X}} \right). \]  

**Proposition 5.** Suppose that the initial economy of each of Case A-E starts from same amount of \( K(0) \) and all regions are homogeneous. In the long-run, the welfare of a centrally planned economy is higher than the welfare of a decentralized economy.

**Proof** The first term in the right hand side of equation (45) is a key determinant of \( \bar{V}_E - V_X \) in the long-run. Using equations (28)-(30) and (42), we get

\[ \frac{\tau_{E}}{\tau_{X}} = \frac{\alpha \chi_{E} \omega_{E}}{\alpha \chi_{X} \omega_{X}} = \frac{\alpha \chi_{E} \tau_{E}^{1-\alpha}}{\alpha \chi_{X} \tau_{X}^{1-\alpha}}, \]

\[ \frac{\tau_{E}}{\tau_{X}} = \left( \frac{\chi_{E}}{\chi_{X}} \right)^{\frac{1}{\alpha}}. \]

By equation (44), we have \( \chi_{A} < \chi_{C} < \chi_{B} < \chi_{E} \). Then, \( \tau_{X} < \tau_{E} \) holds. Since
\ln r_E - \ln r_X > 0$, the first term in the right hand side of equation (35) is positive.

Therefore, we obtain \( \lim_{t \to \infty} [V_E(t) - V_X(t)] = +\infty \).

5. Concluding remarks

In this paper, we consider four cases such as maximizing the share of capital of own region, the tax revenue of own region, the income of own region and the regional welfare. To examine the effect of changes in the number of regions on regional economic growth and welfare, incorporating productive public goods as the growth engine, into a simple overlapping generations model.
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References


