Abstract: The aim of the paper is to investigate if child allowances are a useful means to raise the number of children and to increase the pension benefits of a pay as you go (PAYG) pension system in an endogenous growth model in which parents determine the number and level of education of their children endogenously. For our purposes, we take the most widely accepted assumptions regarding the modelling of endogenous fertility and education choices into account to analyze the subject. In the corresponding literature - which widely ignores human capital building - child allowances are a means to generate a Pareto improvement in the presence of a PAYG system. Unfortunately, we have to conclude that this result is no longer valid, neither for a small open economy nor for a closed economy, if we consider Gary Becker’s quality-quantity trade-off between quality and the number of children. Additionally, our analysis raises concern if child allowances are a means to increase the fertility rate in general.

Keywords: OLG model, PAYG pension system, child allowances, endogenous growth, fertility, human capital

JEL: D10, E62, H23, H55, J13, O15, O41

1 Introduction

In many developed countries the pension system is organized as a pay-as-you-go (PAYG) pension system. In the view of many economists and policy-makers this kind of pension is under pressure because of a non-sustainable population growth rate and population ageing caused by an
increasing life expectancy. The obvious fear is that on one hand the pension benefits will dramatically decrease, or, on the other hand, the contribution rates must increase dramatically to avoid pension benefits, which are not sufficient to finance a however defined living standard. The problem of a PAYG system is the following: once set up, the society is locked in the system, because a PAYG pension system is efficient when it is introduced (see for instance Breyer (1989) or Verbon (1988)). In other words: if the extent of the PAYG pension system is to be reduced or if it is to be abolished, at least one generation will be worse off.\footnote{Fanti and Gori (2010) state the contrary, but their results are irrelevant because they only compare different steady states, while they ignore the transition from one steady-state to the other. Surprisingly, they did not recognize that Verbon\’s (1988) efficiency argument is based on the finding that a Pareto-neutral reduction of the contribution rate is impossible.}

In the light of this fact, many researchers extended the original model of Diamond (1965) either by including leisure as an argument in the utility function (Homburg, 1990), or by including positive externalities generated by the physical capital stock (Corneo and Marquardt (2000) or by including endogenous fertility decisions (van Groezen et al (2005), van Groezen and Meijdam (2008), Fanti and Gori (2008a) or - very similar - Fenge and Meier (2005, 2008)).

Especially, the idea of child allowances has gained some recognition in politics, because Homburg\’s (1990) argument is only valid in an economy with full employment and the problem with positive externalities created by the capital stock is that they are difficult to measure.

However, the idea that child allowances are a means to increase the number of children leading to a “more secure” PAYG pension system is politically popular and, by science, politicians are provided with a normative reasoning to finance such expenditures. Additionally, child allowances are mostly positively welcomed by voters.

To our knowledge only Fanti and Gori (2008) extended the approach of van Groezen et al (2003) by integrating the quality of children in the model. A critical point of Fanti and Gori (2008) is their assumption that pure child-raising costs play an additional role that takes effect beyond their character as costs. Namely, the pure child-raising costs raise the quality of a child and thus the utility of parents. One implication of this assumption is that an increase of the child-rearing costs has no effect on the equilibrium utility. This implication seems to be somewhat strange, because usually we would think that higher child-rearing costs affect the utility negatively.

Because of this context we extend the model of van Groezen et al (2003) by integrating human capital of children as an argument in the utility function like it is done by Galor and Weil, (1999) or de la Croix and Doepke (2003, 2004). The interpretation is that human capital and the number of children shall represent Becker\’s (1960) quantity-quality trade-off. It is a widely accepted fact that human capital accumulation is a main driver of economic growth in developed countries. Hence it makes sense to integrate this factor of production in the model and to analyze furthermore, if the results of van Groezen et al (2003), Groezen and Meijdam (2008), and Meier and Fenge (2005, 2008) regarding child allowances still hold.

The remainder of the paper is organized as follows: in the next section we introduce the model of a small open economy; in the third section we analyze the effects of child allowances
on the equilibrium values and in the fourth section we repeat the analysis in the framework of a closed economy. We finally conclude the results in the fifth section.

2 The model in a small open economy

At first we introduce the human capital production function, which is - without loss of generality - a special case of the one used by de la Croix and Doepke (2003, 2004):

\[ h_{t+1} = Bh_t q_t^e, \] (1)

where \( B > 0 \) and \( e \in [0,1] \). The variable \( q_t \geq 0 \) represents parents’ investments in the education per child. It should be noted that we waive the possibility of no investment in education, because we want to concentrate on the effects of human capital. However, it is no problem to extend (1) so that the human capital remains constant over time even if no investments are made. This simplification only reduces the paperwork and does not influence the results qualitatively.

Human capital serves for two purposes in this paper; firstly, it is the driver of growth like in the models of Uzawa (1965), Lucas (1988) or Azariadis and Drazen (1990). Secondly, human capital is seen as a synonym of the quality of a child in the sense of Becker (1960). Therefore, this approach differs from the one chosen for instance by Fanti and Gori (2008a, 2008b) or Strulik (2003, 2004a, 2004b), who assume that all expenditures for children represent their quality.

We use a simple OLG model in the line of Diamond (1965). Children do not make any decisions in their first period of life. Later, as parents, they supply labor inelastically, bring birth to a number of children \( n_t \), pay for their education \( q_t \bar{w}_t \), consume \( c_t^1 \) units and save a part of their income \( s_t \). In the third period of their life they are retired and consume their savings plus interest: \( c_{t+1}^2 = s_t R_{t+1} \). The representative agent derives her utility from the consumption in both periods, the number of children (their quantity) and the level of the children’s human capital stock (their quality).

As van Groezen et al (2003) or Fanti and Gori (2008a, 2008b), we assume that the government finances a child allowance of \( \beta \bar{w}_t \) per child. To balance its budget, the government imposes a proportional labor income tax with tax rate \( \tau_t^C \):

\[ \beta n_t \bar{w}_t = \tau_t^C \bar{w}_t, \] (2)

where \( \bar{w}_t \) is the wage rate per capita in period \( t \).

Beside the child allowance, we assume that our model economy applies a pay-as-you-go pension (PAYG) system. To keep the PAYG system in balance, every worker has to contribute

\footnote{We forgo considering positive externalities in the human capital production to make our arguments stronger.}
the share $\tau_t^p$ of her labor income and every retired person receives a pension benefit of $P_t$. The budget constraint of the PAYG system is then given by:

$$P_t = \tau_t^p \bar{w}_t n_{t-1}. \quad (3)$$

Regarding the utility function, we follow de la Croix and Doepke (2003, 2004). The only difference is that parents have non-identical preferences with respect to the quality and quantity of children. From all these assumptions the following maximization problem of a representative agent born in period $t-1$ results:

$$\max_{\{c_t^1, c_{t+1}^2, n_t, q_t\}} U_t(c_t^1, c_{t+1}^2, n_t, q_t) = \ln(c_t^1) + \chi \ln(c_{t+1}^2) + \phi \ln(n_t) + \mu \ln(Bh_t q_t^x), \quad (4)$$

subject to

$$c_t^1 + \frac{c_{t+1}^2}{R_{t+1}} = \bar{w}_t (1 - \tau_t^p - \tau_t^c - (q_t + e - \beta)n_t) + \frac{P_{t+1}}{R_{t+1}}. \quad (5)$$

We assume the subjective discount factor $\chi \in [0,1]$, the preference parameter for the quantity of children $\phi \in [0,1]$ and the preference parameter for the quality of children $\mu \in [0,1]$. The labor time is normalized to one and $e\bar{w}_t$ represents the pure child raising costs per child. Construction of a Lagrange function with (4) as the objective, (5) as the constraint and maximization leads to the following first order conditions:

$$\frac{c_{t+1}^2}{c_t^1} \chi = R_{t+1}, \quad (FOC1)$$

$$\frac{\phi}{n_t} = \frac{\bar{w}_t(q_t + e - \beta)}{c_t^1}, \quad (FOC2)$$

$$\frac{\mu e}{q_t} = \frac{\bar{w}_tn_t}{c_t^1}. \quad (FOC3)$$

If we use FOC2 and FOC3 we can solve directly for the optimal investments in human capital $q_t$:

$$q^* = \frac{\mu e(e - \beta)}{\phi - \mu e}. \quad (6)$$

This solution is only meaningful, if $\phi > \mu e$ and $e > \beta$. The last condition only requires that the child allowance must not exceed the pure child rearing costs.

Given the optimal investments in education, we can determine the human capital stock per capita in period $t+1$ by substituting (6) in (1):
\[ h_{t+1} = Bh_t \left( \frac{\mu e^{-\beta}}{\phi - \mu e} \right)^e. \] (7)

Reformulation delivers the equilibrium growth factor of human capital per capita:

\[ G^h = \frac{h_{t+1}}{h_t} = B \left( \frac{\mu e^{-\beta}}{\phi - \mu e} \right)^e. \] (8)

To avoid pathological cases, we assume that \( B > \frac{1}{\left( \frac{\phi - \mu e}{\mu e e^{-\beta}} \right)^e} \) is fulfilled.

**Proposition 1:** The introduction or an increase of the child allowance results in less investments in human capital and in a reduction of the growth of human capital.

**Proof:**

Differentiation of (6) and (8) results in:

\[ \frac{\partial q^*}{\partial \beta} = -\frac{\mu e}{\phi - \mu e} < 0 \quad \text{and} \quad \frac{\partial G^h}{\partial \beta} = -\frac{e B}{(e^{-\beta})} \left( \frac{\mu e^{-\beta}}{\phi - \mu e} \right)^e = -\frac{\varepsilon}{(e^{-\beta})} G^h < 0. \] Q.e.d.

Because of the fact that we refer to a small open economy, we can rewrite the wage rate per capita as \( \bar{w}_t = w h_t \), the interest factor as \( R_{t+1} = R \) and the pension payout as \( P_{t+1} = \tau^p w h_t G^h n_t \). With the help of the results above, we can solve for the remaining dependent variables in a next step, using the FOCs and the budget constraint:

\[ c^1_t = \frac{(1-\tau^p)(e^{-\beta})Rw h_t}{R[e(1+\chi + \phi) - \beta(1+\chi + \mu e)] - G^h \tau^p(\phi - \mu e)} \] (9)

\[ c^2_{t+1} = \frac{\chi(1-\tau^p)(e^{-\beta})R^2 w h_t}{R[e(1+\chi + \phi) - \beta(1+\chi + \mu e)] - G^h \tau^p(\phi - \mu e)} \] (10)

\[ n^*_t = \frac{(1-\tau^p)\chi R(\phi - \mu e)}{R[e(1+\chi + \phi) - \beta(1+\chi + \mu e)] - G^h \tau^p(\phi - \mu e)} \] (11)

\[ s^*_t = \frac{\chi R(e^{-\beta})G^h \tau^p(\phi - \mu e)(1-\tau^p)w h_t}{R[e(1+\chi + \phi) - \beta(1+\chi + \mu e)] - G^h \tau^p(\phi - \mu e)} \] (12)

As we see, the consumption in both periods of life and the savings depend linearly on the stock of human capital per capita. Therefore, we can conclude that these variables grow with the same speed as the human capital per capita.

Additionally, for our purposes it is useful to calculate the equilibrium pension payout:
\[ P_{t+1}^* = \frac{\tau P(\phi - \mu e)(1 - \tau P)h\chi_{wh}}{R[\epsilon(1 + \chi + \phi) - \beta(1 + \chi + \mu e)] - G^h\tau P(\phi - \mu e)}. \]  

(13)

3. The effects of a child allowance

We can now analyze if the results of van Groezen et al. (2003) still hold after our extension of their model with human capital and therefore endogenous growth. From proposition 1 we know that the growth rate of human capital will decline with an increasing child allowance.

**Proposition 2:** The introduction or an increase of the child allowance leads to an increase in the number of children if \( \frac{R}{G^h} > \frac{\tau P(\phi - \mu e)}{(e - \beta)(1 + \chi + \mu e)}. \)

**Proof:**

\[
\frac{\partial n_t^*}{\partial \beta} = \frac{(1 - \tau P)R(\phi - \mu e)[R(e - \beta)(1 + \chi + \mu e) - \tau P(\phi - \mu e)e^h]}{(e - \beta)[G^h\tau P(\phi - \mu e) - R(e(1 + \chi + \phi) - \beta(1 + \chi + \mu e))]^2} > 0, \text{ if } \frac{R}{G^h} > \frac{\tau P(\phi - \mu e)}{(e - \beta)(1 + \chi + \mu e)}. 
\]

This condition is fulfilled in the case that no PAYG exists (\( \tau P = 0 \)); the number of children will increase with an increasing child allowance. This result remains valid as long as the contribution rate \( \tau P \) is sufficiently small. However, it is remarkable that we cannot exclude the possibility that an increasing child allowance decreases the number of children.

The intuition behind the results regarding the investments in education and the number of children is as follows: the negative impact of a child allowance on the investments in education is caused by the fact that the marginal costs to increase the investments of education are equal to \( \bar{w}_t n_t \). These marginal costs are independent from the child allowance. The marginal costs of an additional child are \( \bar{w}_t (q_t + e - \beta) \) and depend negatively on the child allowance. That means it becomes relatively cheaper to get an additional child than to invest in human capital. Additionally, the remaining income is reduced by the tax to finance the allowance. Solely, the reduction of investments in human capital caused by the child allowance is unambiguous. The effect on the number of children depends on the relative prices of current and future consumption. If the price of future consumption \( R \) is relatively high, the representative agent increases the number of children.

In a small open economy a change of the savings only has an effect on the net lending/borrowing position of the economy, which is not relevant in this kind of model, but is an important indicator in reality. Therefore, we should consider the change of the savings per capita.

**Proposition 3:** If \( \frac{R}{G^h} > \frac{\tau P((1 + \phi) - \beta(1 + \mu e))[1 - (\mu(1 - \epsilon))] - \epsilon(1 - \beta)}{\chi(1 - \beta)} \) and a child allowance is introduced or increased, the savings per capita decrease, and therefore the net lending position of the whole economy worsens.
Proof:
\[
\frac{\partial \tau}{\partial \beta} = \frac{(1-t^P)R\psi t(\phi-\mu)e^{\phi t}(1+\phi-\mu(1-\varepsilon))e^{-(\varepsilon-\beta)} - R\chi e^{(e-\beta)}}{(e-\beta)G^e(\phi-\mu)e^{\phi t} - R[e(1+\phi-\mu(1-\varepsilon))e^{-(\varepsilon-\beta)}] - R\chi e^{(e-\beta)}} < 0,
\]

if \( \frac{R}{G^e} \left( (1+\phi-\mu)e^{\phi t}(1+\mu(1-\varepsilon))e^{-(\varepsilon-\beta)} \right) \chi e^{(e-\beta)} \).

Only if no PAYG pension exists, the savings decline for sure. However, given the existence of a PAYG pension system it is in general far from clear, whether the savings per capita increase or decrease, when a child allowance is introduced or increased.

Our next step is to investigate the reaction of the pension benefits, when the child allowance is introduced or increased. We analyze the effect of an increase of the child allowance on the pension payout in \( t+1 \). We can rewrite the pension payout (3) as a function of the child allowance:
\[
P_{t+1}(\beta) = \tau^p t^p \psi t^p(\phi)(\beta) n_t(\beta).
\]

Differentiation yields:
\[
\frac{\partial P_{t+1}(\beta)}{\partial \beta} = \frac{P_{t+1}(\beta) (\eta_{G^h,\beta} + \eta_{n^e,\beta})}{\beta} \left( \eta_{n^e,\beta} - \frac{e^\beta}{e-\beta} \right).
\]

The term \( \eta_{G^h,\beta} = -\frac{e^\beta}{e-\beta} \) represents the elasticity of the growth factor with respect to the child allowance and the term \( \eta_{n^e,\beta} \) represents the elasticity of the fertility factor with respect to the child allowance. The elasticity of the growth factor is negative and the sign of the elasticity of the fertility factor is ambitious in general. Therefore, it is possible that the pension in the period following the introduction or increase of the child allowance also increases. This is the case if the elasticity of the fertility factor exceeds the absolute value of the elasticity of the growth factor.

**Proposition 4**: Assuming an introduction or increase of child allowances, then, in the short run, the pension payouts increase, if the elasticity of the population growth factor with respect to the child allowance exceeds the absolute value of the elasticity of the human capital growth factor with respect to the child allowance. In the long run the pension payouts will be lower with an introduced or increased child allowance than the pension payouts without the introduction or increase of the child allowance.

The proposition compares the development paths of pension payouts, the pension payouts with an introduced or increased child allowance and the pension payouts without this change. It should be noted that the pension payouts grow over time in both cases.
Proof: To derive the long-run effect of the introduction or of an increase of a child allowance, we have to take a modified version of (14) into account:

\[ P(t) = \tau t^\beta w h_0 n_{t-1}(\beta)[G^h(\beta)]^t. \]  
\[ (14') \]

In this case the derivative becomes:

\[ \frac{\partial P(t)}{\partial \beta} = \frac{P(t)}{\beta} \left( \eta_{nt-1,\beta} - t \frac{\varepsilon \beta}{\varepsilon - \beta} \right) < 0, \text{ for } \forall t > \bar{\ell}. \quad \text{Q.e.d.} \]

The number \( \bar{\ell} \) is the smallest non-negative integer which fulfills:

\[ \bar{\ell} > \frac{(e-\beta)\eta_{nt-1,\beta}}{\varepsilon \beta}. \]

In the long run, the negative effect on the growth rate of human capital and on the growth rate of wage incomes respectively will exceed the potentially positive effect on the number of children. The reason is: the child allowance induces a potentially positive level effect on the pension payouts, but at the same time it induces also a negative effect on the growth rate of the per capita income. The impact of the negative growth effect increases over time and at least, after a finite number of periods, the pension payouts in an economy with a child allowance will be less than the pension payouts in an economy without a child allowance.

From propositions 1-4 we have to conclude that the most important effect of child allowances is the negative growth effect, which makes any further welfare analysis redundant, because it is for sure that at maximum only a few generations can gain from the introduction of a child allowance, while all other generations will suffer. What cannot be denied is the fact that, under specific circumstances, the number of children increases, but the costs to realize this objective with child allowances seem to be excessively high. Additionally, the hope of policy makers that the pension payouts of a PAYG pension system can be increased by increasing child allowances in the long run must be rejected.

4. Closed Economy

Although the analysis of closed economies is not as relevant as in the past because of an increasing globalization, it can deliver useful insights. To gain these, we look at an economy which has access to a usual neoclassical production function:

\[ Y_t = F(K_t, L_t), \]
\[ (15) \]

where \( Y_t \) represents the aggregate production, \( K_t \) the aggregate physical capital stock, and \( L_t \) the total labor force, which is the product of human capital units times the number of workers: \( L_t = h_t N_t \). The production function exhibits the usual diminishing marginal productivities in each
input factor, fulfills the Inada conditions and is linear homogenous. Because of this assumption, we can rewrite it as the production per human capital unit: \(^3\)

\[ y_t = f(k_t), \] (16)

where \( y_t = \frac{y_t}{h_tN_t} \) and \( k_t = \frac{k_t}{h_tN_t} \).

Without loss of generality, we assume that the depreciation rate of physical capital is 100 per cent per period. Additionally, we assume that the goods and factor markets are perfectly competitive. As is well-known under these circumstances, the wage rate per human capital unit and the interest factor become to:

\[ w_t = w(k_t) = f(k_t) - f'(k_t)k_t, \] (17)

\[ R_t = f'(k_t). \] (18)

Before we can analyze the capital market clearing condition, we have to take into account that now the wage rate per human capital unit is no longer constant. Therefore, we should rewrite the savings function (12) and the optimal number of children (11) as:

\[ n_t = \frac{\left(1 - \tau^P\right)w_t h_t R_{t+1} + \bar{p}_{t+1}(\phi - \mu \varepsilon)}{R_{t+1} w_t h_t [e(1 + \chi + \phi) - \beta(1 + \chi + \mu \varepsilon)]}, \] (19)

\[ s_t = \frac{\chi(e - \beta) \left[1 - \tau^P\right] w_t h_t R_{t+1} + \bar{p}_{t+1}}{R_{t+1} [e(1 + \chi + \phi) - \beta(1 + \chi + \mu \varepsilon)]} - \frac{\bar{p}_{t+1}}{R_{t+1}}. \] (20)

By inserting the pension payout (3) into (19) and solving for the optimal number of children, we get:

\[ n_t^{**} = \frac{\left(1 - \tau^P\right) w_t R_{t+1} [\phi - \mu \varepsilon]}{R_{t+1} w_t [e(1 + \chi + \phi) - \beta(1 + \chi + \mu \varepsilon)] - \tau^P G^h w_{t+1} [\phi - \mu \varepsilon]}. \] (21)

Now we can insert (21) into (3) and the resulting expression in (20) and then into the capital market clearing condition \( s_t = n_t^{**} k_{t+1} h_{t+1} \). After some simplifications we get:

\[ f'(k_{t+1}) w(k_t) \chi(e - \beta) - \left[ \tau^P w(k_{t+1}) + f'(k_{t+1}) k_{t+1} \right] G^h (\phi - \mu \varepsilon) = 0. \] (22)

---

\(^3\)Expressed in per human capital units the production function becomes to \( f(k_t) = F \frac{k_t}{h_t}, \). We assume that the corresponding Inada conditions hold: \( f(0) = 0; f(\infty) = \infty; f'(\infty) = 0 \) and \( f''(0) = \infty. \)
Let us assume that an equilibrium $k^*$ exists\(^4\). Then the stability of an interior equilibrium is guaranteed, if at the point $k_t = k^*$ the following holds:

\[
\frac{dk_{t+1}}{dk_t} = \frac{-f^{''}(k_t)k_t f'(k_{t+1}) \chi(e^{-\beta})}{-f^{''}(k_{t+1})w(k_t) \chi(e^{-\beta}) - G^H(\phi - \mu \varepsilon)(\tau^P f^{'''}(k_{t+1})k_{t+1} - (\eta_{R,k} + 1)f'(k_{t+1}))} < 1,
\]

where $\eta_{R,k} = \frac{f^{''}(k_{t+1})k_{t+1}}{f'(k_{t+1})}$.

A sufficient, but not necessary condition for the local stability of the interior steady-state equilibrium is that the capital income share is less than $\frac{1}{2}$ and that the elasticity of the interest rate with respect to capital is less than one.

We can determine how the capital per human capital unit reacts on a change of the child allowance by applying the implicit function theorem on the capital market equilibrium condition at the equilibrium value $k^*$:

\[
\frac{dk^*}{d\beta} = \frac{(\phi - \mu \varepsilon)\frac{\tau^P w(k^*) + k^* f'(k^*)}{\chi f'(k^*) w(k^*)}}{[k^* f'(k^*) - w(k^*)]f^{'''}(k^*) \chi(e^{-\beta}) - G^H(\phi - \mu \varepsilon)(\tau^P f^{'''}(k^*)k^* + (\eta_{R,k} + 1)f'(k^*))} \leq 0. \tag{23}
\]

The denominator is positive because of the stability condition, and the sign of the numerator can be either positive or negative. The sign of the numerator is positive, if

\[
\frac{(\phi - \mu \varepsilon)\frac{\tau^P}{(e^{-\beta}) \chi}}{f(k^*)} (\tau^P + \omega) > \frac{f(k^*)}{G^H},
\]

where $\omega = \frac{f(k^*)}{w(k^*)} f'(k^*)$ represents the ratio between the capital income share and the labor income share.

**Proposition 5:** The physical capital per human capital unit increases (declines), if the following inequality holds (does not hold): $\frac{(\phi - \mu \varepsilon)\frac{\tau^P}{(e^{-\beta}) \chi}}{G^H} (\tau^P + \omega) > \frac{f(k^*)}{G^H}$.

We know that an increase of the child allowance or its introduction lowers the growth rate of human capital. If the child allowance lowers the physical capital per human capital unit, the wage incomes will decline. Anyways, it must be concluded that the wage incomes per capita on the equilibrium growth path will be lower in the long run after a child allowance is introduced or increased, because the growth rate of human capital declines (proposition 1) and this effect will

\(^4\)An equilibrium exists as long as the contribution rate $\tau^P$ is not too huge, because a too huge contribution rate leads to huge pension payouts and this can lead to a negative or zero savings.
additionally exceed a possible positive effect caused by more physical capital per human capital unit.

Nevertheless, it is useful to analyze the effect of a child allowance on the number of children, because most policy-makers intend to increase the fertility rate by subsidizing children. The number of children (21) can be rewritten as a function depending on the child allowance: \( n^* = n(\beta, k^*(\beta)) \).

**Proposition 6**: The introduction of a child allowance or an increase of it leads to an increase of the number of children, if \( \frac{\tau^p + \omega}{x} > \frac{R^*}{G^h} \frac{(e - \beta)}{(\phi - \mu) \varepsilon} > \frac{\tau^p}{1 + x + \mu} \) holds.

Differentiating the function \( n(\beta, k^*(\beta)) \) with respect to \( \beta \) leads to the result:

\[
\frac{dn^*}{d\beta} = \frac{\partial n^*}{\partial \beta} + \frac{\partial n^*}{\partial R^*} \frac{\partial R^*}{\partial \beta} \frac{\partial k^*}{\partial \beta} \leq 0. \tag{24}
\]

In general the sign is unclear, because neither the direct effect of the child allowance on the number of children is unique nor is it the effect on the physical capital per human capital unit. If we take the necessary condition from proposition (2) and combine it with the necessary condition of proposition (5), then it is guaranteed that the direct effect is positive and that the physical capital per human capital unit increases, which leads also to a positive indirect effect. Thus, given both conditions’ validity, the number of children will rise, if the child allowance is introduced of increased.

The opposite of the condition of proposition 6 cannot be fulfilled, because the LHS of the inequality always exceeds the RHS. If either the RHS or LHS of the condition is violated, the overall effect on the fertility rate is still unclear. Therefore, we cannot predict the direction of the change of fertility behavior in general. Consequently, we have to state:

**Proposition 7**: The effect of an increase of the child allowance on the pension is negative in the long run and not unique in the short run.

**Proof**: If we differentiate the pension benefits represented by \( P(t) = \tau^p h_0 n(\beta, k^*(\beta)) w(k^*(\beta))[G^h(\beta)]^t \), the result is:

\[
\frac{dp(t)}{d\beta} = \tau^p h_0 (G^h)^t \left[ \left( \frac{\partial n^*}{\partial \beta} + \frac{\partial n^*}{\partial R^*} \frac{\partial R^*}{\partial \beta} \frac{\partial k^*}{\partial \beta} \right) w^* + \left( \frac{\partial w^*}{\partial k^*} \frac{\partial k^*}{\partial \beta} \right) n^* - t \frac{\tau^p}{e - \beta} w^* n^* \right].
\]
The decisive term is the third one in the brackets: it is negative and its absolute value increases from period to period. That means: even if the number of children and the wage rate per human capital unit increases, from some point of time on and thereafter the pension benefits will be less than the pension benefits without the increase of the child allowance. However, if the number of children decline as a consequence, the pension payouts will decrease in the short run as well.

Consequently, given our model, the hope that child allowances are a means to increase the welfare or the pensions must be rejected.

5 Conclusions

In this paper we use an OLG model, where parents decide about the number and the extent of education of their children. One major assumption is that parents use the level of education or the amount of human capital of their children as an indicator for the quality of children. In this framework we have reconsidered the idea to use a child allowance to increase the number of children and to increase the pension payouts of a PAYG pension system. In contradiction to the results derived from models in which only the number of children and not their human capital enter the utility function of the parents, we have to conclude that child allowances are never a means to enhance the welfare in the sense of Pareto in a growing economy. We show that child allowances can increase the number of children under specific circumstances, but even then this effect does not provide any positive welfare effect for future generations. The main reason for this result is that the pure-child raising costs are the price for giving birth of a child, and this relative price determines the investments in education positively. Thus lowering the price of an additional child leads at best on the one hand to more children but on the other hand to less education and hence human capital per capita. The consequence is that all subsequent generations have lower incomes and consequently a lower level of utility. Our model leads to the conclusion that instead of subsidizing children a taxation of children is probably more target-aimed.

If the results derived in the model are valid in reality, we have to conclude: while European governments are spending billions of Euros with the intention to increase fertility and to keep the existing PAYG pension systems in balance, the outcome will be the contrary in the long-run.

References


