Endogenous leadership in tax competition models with market power: Capital distribution matters.

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Abstract

Hindricks and Nishimura (J. Pub. Econ. 98, 2014) address the issue of leadership in a two-region economy where regions have differentiated market power, defining the large region as the one with the highest market power. They prove that, using the risk-dominance criterion, the large region leads in the tax competition race, thus reversing the result obtained by Kempf and Rota-Graziosi (J. Pub. Econ. 94: 768-776, 2010). We modify the model of Hindricks and Nishimura by taking into account capital distribution among regions and thus introducing the distribution of profits as an argument in the payoff functions of fiscal authorities. We prove that if the share of capital characterizing the small region is sufficiently large, the initial result of Kempf and Rota-Graziosi is restored: the small region endorses leadership. This result provides an explanation of the importance of tax heavens in the tax competition race.

JEL Codes: H30, H87, C72.

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1 Introduction.

The issue of leadership in tax competition models is contentious. Kempf and Rota-Graziosi (2010, hereafter KRG) conclude that not only the Stackelberg outcomes are the PSNEs of a timing game (see Hamilton and Slutsky, 1990), but also that the risk dominant equilibrium in line with Harsanyi and Selten corresponds to the Stackelberg outcome where the smaller country leads. These results have been challenged by Hindriks and Nishimura (2014, hereafter HN). HN focus on the quadratic specification proposed in KRG to apply the risk dominance criteria. They modify the source of heterogeneity and rather than differentiating the slope of the linear demand function, assume that its intercepts vary across regions. They make the point that it corresponds to different market powers and thus is more consistent with what we intuitively relate to the “size” region. Using an otherwise similar model and the same methodology as KRG, they show that the result is reversed: the large region endogenously endorses the leader role contrarily to the claim made by KRG.

In this paper we address the same issue: The identity of the leading country at the risk dominant SPNE. We modify the model used by HN on a single point: The inclusion of capital ownership. HN assume as KRG that capital owners are “absentees” in the following sense: Capital income does not enter the welfare functions of fiscal authorities. Here we relax this assumption and extend these functions so as to include the share of capital and its income repatriated in either region. The whole capital may be fully owned by residents in one or the other region. In this case, the sum of these shares equals 1.\(^1\)

Using this more general specification, we are able to prove that the result initially obtained by KRG may be reached even if the source of heterogeneity is market power as defined by HN. This is due to the fact that capital ownership may modify the sign of tax spillover (from positive to negative) and the strategic property (from strategic complementarity of tax rates towards their strategic substitutability). In particular if the small region has a large share of capital, it is likely to assume leadership. This sheds some light on

\(^1\)Ogawa (2012) introduces an equal distribution of capital among regions, using the original specification of asymmetry made by KRG and establishes that the outcome of the simultaneous move game is the SPNE of the endogenous timing game. KRG (2014) relax the assumption of equal distribution of capital and find configurations which support their previous results.
the phenomenon of “offshore” regions (countries), i.e. regions which are charac-
terized by a share of capital ownership not in line with their productive
capacities.\textsuperscript{2}

2 The tax competition model with capital own-
ership.

2.1 The set-up.

We closely follow the specification developed by HN (using their notation
to make the reading easier). In particular we retain their assumption with
respect to the heterogeneity of the two regions. The only difference is in-
troduced in the welfare functions of the fiscal authorities. We consider a
two-region economy where capital is mobile and the two fiscal authorities set
capital tax rates. A single homogeneous private good is produced locally.
This good can either be consumed or used as an input into the provision of
the local public good. The fiscal authority in region $i$ ($i = A, B$) sets taxes
on capital, with a proportionate rate denoted by $t_i$.

The production function used in region $i$ ($i = A, B$), denoted by $F_i(K_i)$,
where $K_i$ is the capital (per capita) in the region, with the usual assumptions
$F_i'(K_i) > 0 > F_i''(K_i)$. The total amount of capital available in the economy
is fixed and denoted by $\bar{K}$.

Given the perfect mobility of capital, the market clearing conditions are:

\[
\begin{align*}
F_i'(K_i) - t_i &= r \\
K_A + K_B &= \bar{K}
\end{align*}
\]  

(1)

We shall use the quadratic specification of the production function used
by Hindricks and Nishimura (hereafter HN):

\[
F_i(K_i) = (a_i - bK_i)K_i
\]

(2)

\textsuperscript{2}We use the notion of “offshore” regions (countries) rather than “tax heavens”. Tax
heavens are likely to prosper under secrecy measures such as tax rulings, recently under
the spotlights in Europe. Here we do not address the issue of the endogeneity of “offshore”
regions. It is likely that tax breaks, either openly advertised or not, contribute heavily
to the concentration of capital in one country. Delaware in the U.S., Luxemburg in the
European Union, the Virgin islands are examples of an offshore region as we understand
it here.
(and we shall make the same assumption that $a_A - a_B < 4bK$. Hence the heterogeneity between regions is switched from the slope to the gradient of the demand function.

From the market clearing conditions (1), using this specification, we get:

$$K_i(t_i, t_j) = \frac{1}{4} t_j - t_i + \frac{\Delta_i}{b} \quad (i, j = A, B, j \neq i).$$

where $\Delta_i = a_i - a_j + 2bK$. As in HN, we assume that $a_A > a_B$: region $A$ is the “large” region as it displays the largest market power due to its highest vertical intercept.

Capital may be distributed between both countries completely or not ($\theta_A + \theta_B \leq 1$); when $\theta_A + \theta_B = 0$, capital owners are foreign to both regions and are not taken into account in region’s objective function as in KRG; when $\theta_A + \theta_B = 1$, all the capital is distributed among the two jurisdictions. When $\theta_A$ is equal to 1, it is entirely owned by residents of region $A$; when it is equal to zero, it belongs to region $B$; when $\theta_A = \theta_B = 1/2$, capital endowment is equally shared between the two countries.

Public consumption $g_i$ is equal to tax revenues collected in region $i$. Since we may have negative capital tax rates (capital subsidies), we assume that a lump sum tax, denoted by $T$, is raised in each region on some immobile factors so that the quantity of public good always remains positive. We have:

$$g_i = t_i k_i + T \geq 0, \forall t_i \in ]-1,1[, i = A, B.$$

The objective function for government in region $i$, denoted by $W^i (c_i, g_i)$, which is linear in private and public consumption:

$$U^i (c_i, g_i) = c_i + (1 + \lambda) g_i$$

where $1 + \lambda$ denotes the marginal rate of substitution between private and public consumptions. $\lambda$ is assumed to be identical among countries ($\lambda \geq -1$). Private consumption ($c_i$) corresponds to the total income of the representative resident in region $i$, that is the sum of his labor $(F_i (K_i) - K_i F_i' (K_i))$ and capital $(r\theta_i K)$ incomes, where the parameter $\theta_i$ represents his share of capital.

Given the market clearing conditions (1), we can express the objective function of each region as a function of the strategic variables $t_i$ and $t_j$. Denoting the payoff function by $W^i (t_i, t_j)$, we have:

$$W^i (t_i, t_j) = F_i (K_i) - K_i F_i' (K_i) + r\theta_i K + (1 + \lambda) (t_i K_i + T). \quad (3)$$
Remark that this formulation generalizes the specification used by HN by taking into account capital incomes of residents in region $i$ and by assuming that public and private consumption are not necessarily perfect substitutes. In other words, HN make the assumption that $\theta_i = 0$, $\forall i = A, B$, $T = 0$, and $\lambda = 0$. The assumption that capital owners are foreign to both regions, i.e. that capital incomes are not taken into account by the fiscal authorities of either region is also made by Kempf and Rota-Graziosi.

The reaction function of region $i$ is given by

$$ t_i(t_j) = \frac{(1 + 2\lambda) (\Delta_i + t_j) - 4\theta_i bK}{3 + 4\lambda} \quad i = A, B; j \neq i. $$

The Second Order Conditions are

$$ \frac{\partial^2 W^i(t_i, t_j)}{\partial t_i^2} = -\frac{3 + 4\lambda}{8b} < 0 \Leftrightarrow \lambda > -3/4. $$

Strategic complementarity of tax rates holds if and only if

$$ \frac{\partial^2 W^i(t_i, t_j)}{\partial t_i \partial t_j} = \frac{1 + 2\lambda}{8b} > 0 \Leftrightarrow \lambda > -1/2. $$

### 2.2 Equilibria

Here we study three games: the Nash simultaneous game $G^N$, the Stackelberg game where $A$ is leader $G^A$, the Stackelberg game where $B$ is leader $G^B$. A game $G^l, l = N, A, B$, is characterized by a 4-uple $(t^l_A, W^l_A, t^l_B, W^l_B)$.

We get the following results (corresponding to Table 1 in Hindricks and Nishimura):

- At the Nash equilibrium,

$$ t^N_i = \frac{1 + 2\lambda}{2 (2 + 3\lambda)} (a_i - a_j) - \frac{bK [(3 + 4\lambda) \theta_i + (1 + 2\lambda) \theta_j]}{(1 + \lambda) (2 + 3\lambda)} + bK \frac{1 + 2\lambda}{1 + \lambda} $$

- At the Stackelberg equilibrium, where region $i$ leads and region $j$ follows, we get:

$$ t^L_i = \frac{1 + 2\lambda}{5 + 8\lambda} (a_j - a_i) + bK \frac{2 (2 + 3\lambda) (3 + 4\lambda) \theta_i + 2 (1 + \lambda) (1 + 2\lambda) \theta_j}{(1 + \lambda)^2 (5 + 8\lambda)} $$
\[
t^F_j = \frac{1 + 2\lambda}{5 + 8\lambda} (a_j - a_i) - 2bK \frac{(2 + 7\lambda + 6\lambda^2) \theta_i + 2 (1 + \lambda) (2 + 3\lambda) \theta_j}{(1 + \lambda)^2 (5 + 8\lambda)} + 2bK \frac{3 + 11\lambda + 10\lambda^2}{(1 + \lambda) (5 + 8\lambda)}
\]

Proposition 2 of Kempf and Rota-Graziosi (2010a) applies, so \( G^A \) and \( G^B \) are the 2 SPNE. To select one, we resort to the risk-dominance criterion given by equation (3) in HN. The issue is whether the large region (with the highest market power) leads as claimed by HN.

In order to be closer to HN, we assume that \( \lambda = 0 \) and \( T = 0 \). Then we get the following values for the payoffs obtained by the two fiscal authorities in the three games under consideration (see Appendix). For the Nash game, we get:

\[
W^N_i = \frac{3}{64b} (a_i - a_j + 2bK (2 - 3\theta_i - \theta_j))^2 + \theta_i K (a_i - b\theta_i K).
\]

Similarly, we establish (after some tedious calculations):

\[
W^L_i = \frac{1}{20b} (a_i - a_j + 2bK (2 - 3\theta_i - \theta_j))^2 + \theta_i K (a_i - b\theta_i K),
\]

and

\[
W^F_i = \frac{3}{100b} (a_i - a_j + 2bK (3 - 4\theta_i - 2\theta_j))^2 + \theta_i K (a_i - b\theta_i K).
\]

From these equations, we get the following expression for \( \Pi \):

\[
\Pi \equiv (W^L_A - W^N_A) (W^F_B - W^N_B) - (W^F_A - W^N_A) (W^L_B - W^N_B)
\]

\[
= \frac{3K (\theta_A + \theta_B - 1) (a_A - a_B)^3}{4000b} (1 - \chi) (1 - \psi_1) (1 - \psi_2)
\]

where \( \chi = 2bK \frac{\theta_A - \theta_B}{a_A - a_B} \), \( \psi_1 = (1 - \sqrt{6}) \chi + \frac{2\sqrt{6}bK}{a_A - a_B} \), and \( \psi_2 = (1 + \sqrt{6}) \chi - \frac{2\sqrt{6}bK}{a_A - a_B} \).

The difference \((\theta_A - \theta_B)\) is a measure of relative financial power between the two regions whereas the difference \((a_A - a_B)\) is a measure of relative
market power. Therefore $\Delta$ measures the strength of the financial power of region $A$ with respect to region $B$, for a given difference in market power. This ratio varies between $-\infty$ and $\infty$. When it is low, $B$ captures a high fraction of capital income even though it is economically “weak”, i.e. does not benefit from a high market power.

When the distribution of capital is taken into account, two dimensions operate to determine which country would lead at the risk dominant SPNE: the market power of each region and the financial power linked to the distribution of capital. HN only consider the first dimension of power, here we study both dimensions and their interplay. The richer in capital a region, the higher the capital income argument in its payoff function. Let us consider the extreme case of an economy with an “offshore” region, characterized by a very low market power compared to the other “mainland” region. Its welfare function is almost entirely driven by capital income. Any increase in tax rate hurts capital owners by decreasing the worldwide net return of capital. Since tax rates remain strategic complements given the quadratic production functions, this country has a huge incentive to move first in order to signal its preference for a low tax rate and promote moderate equilibrium tax rates. The consequence is that production in the offshore region is negligible, almost all capital is moved to the “mainland” region which produces most of production and almost the entire capital income is repatriated in the offshore region.

Using these notations, we immediately prove the following

**Proposition 1**  \textit{Assuming $\lambda = 0$},

1. If $\theta_A + \theta_B = 1$, $\Pi = 0$; 
2. If $\theta_A + \theta_B < 1$,
   \[ \Pi < 0 \iff 1 > \psi_1 \text{ or } \chi > 1 > \psi_2 \]
   with $\psi_1 = (1 - \sqrt{6}) \chi + (1 - 2\theta_B) \frac{2\sqrt{6} bK}{a_A - a_B}$, $\psi_2 = (1 + \sqrt{6}) \chi - (1 - 2\theta_B) \frac{2\sqrt{6} bK}{a_A - a_B}$.
   and $\chi = 2bK \frac{\theta_A - \theta_B}{a_A - a_B}$.

\textbf{Proof:} See Appendix.

This proposition makes clear that the relative importance of the financial power with respect to the market power plays a crucial role in the endogenous determination of leadership. No region is unambiguously leading. Depending on the parameter values, one region or the other is endogenously selected by
means of the risk-dominance criterion. However there is no ambiguity in this selection process. Notice that when \( \theta_A = \theta_B = 0 \) (included in case (a)), the result of HN is restored: \( \chi = 0 \) and \( \Pi > 0 \), i.e. the region, which displays the higher market power leads in the risk dominant SPNE. This is consistent with the fact that HN’s specification is nested into ours. In the appendix (Appendix A.1) we give the conditions on parameters such that \( \Pi < 0 \).

This proposition allows us to see the importance of capital endowment. Let us first assume that \( \theta_A - \theta_B > 0 \) (which implies \( \frac{1}{2} > \theta_B \)).

If \( \theta_B \) sufficiently close to \( 1/2 \) (and therefore to \( \theta_A \), \( 1 > \psi_1 \).

If not and therefore \( \psi_1 > 1 \), if \( (\theta_A - \theta_B) \) is sufficiently large, \( \chi > 1 \), and if \( \theta_B \) sufficiently small, \( \psi_2 < 1 \).

Now we investigate the opposite case where \( \theta_B - \theta_A > 0 \) (thus \( \chi < 0 \)). It implies:

\[
\psi_1 = \phi \left[ \left( \sqrt{6} \right) (\theta_A - \theta_B) + \sqrt{6} (1 - 2\theta_B) \right] > \phi \sqrt{6} (1 - 2\theta_B)
\]

If \( \theta_B \) is bigger than \( 1/2 \), in order to get \( 1 > \psi_1 \), we must have:

\[
1 > \phi \left[ \left( \sqrt{6} \right) \theta_A - \left( 1 + \sqrt{6} \right) \theta_B + \sqrt{6} \right]
\]

that is:

\[
\theta_B > \frac{(1 - \sqrt{6}) \theta_A + \sqrt{6} - \frac{1}{\phi}}{(1 + \sqrt{6})}
\]

which is true if \( \phi \) is sufficiently small.

If \( \theta_B \) is less than \( 1/2 \), if \( \phi \) is sufficiently small, \( 1 > \psi_1 \).

In brief, it is possible to reverse the result obtained by HN when the distribution of capital is taken into account: if region \( B \) is such that its financial index is large enough, or close enough to the financial index of \( A \), it leads the tax competition race (\( \Pi \) is negative). However when \( B \) is financially weak (a low \( \theta_B \)), it still may be induced to lead if its financial index is low enough, in the case where \( \chi \) is bigger than 1.

Hence, contrary to HN’s result obtained when profits are not taken into account, \( A \) (the “large” region due to its larger market power) does not systematically lead. This is due to the unequal distribution of capital and thus, the distribution of capital incomes. For instance, if \( \theta_A \) is sufficiently low, \( A \) prefers to follow and \( B \) prefers to lead. \( B \) hosts a large part of capital owners and may be considered as an “offshore region” in our framework. By moving
First, it is able to avoid equilibrium higher tax rates, in particular in region $A$ which would decrease the worldwide net return of capital and would hurt its residents. It is less able to finance public consumption and transfers but repatriated profits from the other region are higher. This bolsters welfare and thus more than compensate the negative impact on welfare of the low amount of collected taxes. In other words the offshore region by moving first behaves as a “tax breaker”. On the other hand, the fiscal authority of the “mainland” region faces a dilemma: By leading, it is able to exert its market power and setting a high tax rate leads to equilibrium higher tax rates. However the gap between the two rates increases with $\theta_A$ and decreases with $\theta_B$. Hence the leading advantage vanishes the more capital owners live in the small region. If so, region $A$ on the whole may prefer attracting more capital and thus increase its output by following rather than leading.

Another interesting result is obtained when the capital is perfectly distributed among the two regions: $\theta_A + \theta_B = 1$. The two Stackelberg outcomes are strictly equal and the risk dominance criteria is useless, i.e. does not allow to select one SPNE since $\Pi = 0$.

In order to further illustrate the claim that the region endowed with the highest market power does not systematically lead when profit distribution (i.e. capital ownership) is taken into account, we relax the assumption that the public good is a pure transfer and assume $\lambda \neq 0$, retaining the assumption that capital is fully owned in the two-region economy: $\theta_A + \theta_B = 1$, departing from the HN specification. We consider the following specification of the parameters, which remains sufficiently general to display several PSEs:

$$\theta_B = 1 - \theta_A \text{ and } \theta_A = \theta.$$ $\hspace{1cm}$

$$a_A > a_B = 1, \quad b = 1.$$ $\hspace{1cm}$

$$T = 1, \quad \lambda = 1/2, \quad K = 1.$$ $\hspace{1cm}$

Thus a priori $\Pi \neq 0$. Fixing $\lambda, b, K$ and $T$, this leaves two unfixed parameters $\theta$ and $a_A$. Varying the values taken by these parameters modifies the values of the welfare levels obtained in the three games and therefore, the value of $\Pi$ (see Appendix A.2 for the parameter intervals for which $\Pi > (<)0$). The following figure illustrates the sensitivity of $\Pi$ to values of $\theta$ and $a_A$. The value of $\Pi$ is read on the vertical axis, the value of $a_A$ on the upper horizontal axis (varying between 1 and 2), and the value of $\theta$ on the lower horizontal axis (varying between 0 and 1). The curved surface represents the locus such that $\Pi$ is null. Above (below) it, $\Pi$ is positive (negative).
We can easily display similar figures for other restrictions of the parameter space. On the whole, the result obtained by HN crucially depends on the neglect of capital distribution among regions, and can be reversed when profit distribution is taken into account as well as the nature of the public good.

3 Conclusion.

Hindricks and Nishimura (2014) establish that when defining a large region as a region with the highest market power it is induced to prefer the leader role when playing a timing game. This contradicts a result previously obtained by Kempf and Rota-Graziosi (2010b) who differentiated regions according to the elasticity of capital productivity to capital and defined a large region as a region with the lowest elasticity. Otherwise HN keep the set of assumptions made by KRG, in particular the fact that capital owners are absentees, that is do not reside in any of the two competing regions. In the present paper we show that taking into account the distribution of capital across regions when the specification used by HN is used restores the result obtained by KRG, namely the small region with the lowest market power endogenously exerts tax leadership when it is endowed with a high share in the distribution of capital. More generally the leading region happens to be the one with the highest relative financial power. One implication is that physical size, as
measured by production capacity, is not relevant to determine leadership in capital taxation. What matters is the power ratio, that is the ratio of financial capacity to production capacity.

This result contributes to the understanding of the phenomenon of “offshore regions”. An offshore region is a small fiscally autonomous territory, apparently without much capacity or market power, which attracts a lot of capital and thus exerts a major influence on foreign tax policies. Our result makes the link between the large capital ownership and the impact on tax policies worldwide. The large capital ownership despite the smallness of the territory allows it to assume the leading role in tax competition and thus have a large impact of tax policies of other nations.

It also shows how sensitive the issue of endogenous leadership in a tax competition model is to the set of assumptions characterizing the model. We should not be too confident in attributing the leading role to a region characterized by a unique property. Given the diversity of results obtained under various specifications of the heterogeneity between regions, the concluding remark at this stage of collective research should be that we do have simple rules and criteria which allow us to easily understand and assess leadership in international tax competition configurations with the possible help of simulation techniques.

References


A Appendix.

A.1 Proof of Proposition 1.

Using the expressions obtained for the equilibrium values of tax rates for the Nash game, we get:

\[ W_i^N = \frac{1}{64b} \left\{ 3a_i^2 + 3a_j^2 - 6a_i a_j + 4\left(bK\right)^2 \left(11\theta_i^2 + 18\theta_i\theta_j - 36\theta_i + 12 + 3\theta_j^2 - 12\theta_j\right) \right. \]

\[ + 4a_i bK \left(6 + 7\theta_i - 3\theta_j\right) + 12a_j bK \left(-2 + 3\theta_i + \theta_j\right) \}\]

Given that \((2 - 3\theta_i - \theta_j)^2 = 4 - 12\theta_i - 4\theta_j + 6\theta_j\theta_i + 9\theta_i^2 + \theta_j^2\), we get:

\[ W_i^N = \frac{1}{64b} \left\{ 3(a_i - a_j)^2 + 3\left(2bK\right)^2 \left(2 - 3\theta_i - \theta_j\right)^2 - 64\theta_i^2 \left(bK\right)^2 \right. \]

\[ + 12(a_i - a_j)bK \left(2 + 3\theta_i - \theta_j\right) + 64a_i\theta_i bK \}\]

\[ W_i^N = \frac{3}{64b} \left( a_i - a_j + 2bK \left(2 - 3\theta_i - \theta_j\right) \right)^2 + \theta_i K \left( a_i - b\theta_i K \right) \]

Similarly we get for the game where \(i\) leads:

\[ W_i^L = \frac{1}{20b} \left\{ a_i^2 + a_j^2 + 4\left(bK\right)^2 \left(4\theta_i^2 + \theta_j^2 + 6\theta_i\theta_j - 12\theta_i - 4\theta_j + 4\right) \right. \]

\[ - 2a_i \left( a_j - 2bK \left(2 + 2\theta_i - \theta_j\right) \right) + 4a_j bK \left(-2 + 3\theta_i + \theta_j\right) \}\]

\[ = \frac{1}{20b} \left( a_i - a_j + 2bK \left(2 - 3\theta_i - \theta_j\right) \right)^2 + \theta_i K \left( a_i - b\theta_i K \right) \]

\[ W_i^F = \frac{1}{100b} \left\{ 3a_i^2 + 3a_j^2 + 4\left(bK\right)^2 \left(23\theta_i^2 + 12\theta_j^2 + 48\theta_i\theta_j - 72\theta_i - 36\theta_j + 27\right) \right. \]

\[ + a_i \left( -6a_j + 4bK \left(9 + 13\theta_i - 6\theta_j\right) \right) + 12a_j bK \left(-3 + 4\theta_i + 2\theta_j\right) \}\]
Using these expressions, we get

\[
\Pi = \frac{3K (\theta_A + \theta_B - 1) (a_A - a_B - 2bK (\theta_A - \theta_B))}{4000b}.
\]

\[
[(a_A - a_B)^2 - 4bK (a_A - a_B) (\theta_A - \theta_B) - (2bK)^2 (6 + 5\theta_A^2 + 5\theta_B^2 + 14\theta_A\theta_B - 12\theta_B - 12\theta_A)]
\]
\[
\Pi = \frac{3K (\theta_A + \theta_B - 1) (a_A - a_B - 2bK (\theta_A - \theta_B))}{4000b}.
\]

\[
[(a_A - a_B - 2bK (\theta_A - \theta_B))^2 - 6 (2bK)^2 (1 + \theta_A^2 + \theta_B^2 + 2\theta_A\theta_B - 2\theta_B - 2\theta_A)]
\]

Since \((1 - \theta_A - \theta_B)^2 = 1 + \theta_A^2 + \theta_B^2 + 2\theta_A\theta_B - 2\theta_B - 2\theta_A\), we obtain:

\[
\Pi = \frac{3K (\theta_A + \theta_B - 1) (a_A - a_B - 2bK (\theta_A - \theta_B))}{4000b} [(a_A - a_B - 2bK (\theta_A - \theta_B))^2 - 6 (2bK)^2 (1 - \theta_B)]
\]
or equivalently,

\[
\Pi = \frac{3K (\theta_A + \theta_B - 1) (a_A - a_B - 2bK (\theta_A - \theta_B))}{4000b} [a_A - a_B - 2 \left( 1 + \sqrt{6} \right) bK \theta_A + 2 \left( 1 - \sqrt{6} \right) bK \theta_B]
\]

\[
\left[ a_A - a_B - 2 \left( 1 - \sqrt{6} \right) bK \theta_A + 2 \left( 1 + \sqrt{6} \right) bK \theta_B - 2\sqrt{6}bK \right]
\]

Let us define \(\chi = 2bK \frac{\theta_A - \theta_B}{a_A - a_B}\), we obtain:

\[
\Pi = \frac{3K (\theta_A + \theta_B - 1) (a_A - a_B)^3}{4000b} (1 - \chi) \left[ 1 - \left( 1 - \sqrt{6} \right) \chi - (1 - 2\theta_B) \frac{2\sqrt{6}bK}{a_A - a_B} \right] \left[ 1 - \left( 1 + \sqrt{6} \right) \chi \right]
\]

Defining \(\psi_1 = (1 - \sqrt{6}) \chi + (1 - 2\theta_B) \frac{2\sqrt{6}bK}{a_A - a_B}\) and \(\psi_2 = (1 + \sqrt{6}) \chi - (1 - 2\theta_B) \frac{2\sqrt{6}bK}{a_A - a_B}\), we obtain

\[
\Pi = \frac{3K (\theta_A + \theta_B - 1) (a_A - a_B)^3}{4000b} (1 - \chi) (1 - \psi_1)(1 - \psi_2)
\]

We note that

\[
\psi_1 - \chi = -\sqrt{6} \chi + (1 - 2\theta_B) \frac{2\sqrt{6}bK}{a_A - a_B} = (2\sqrt{6}bK \frac{\theta_A - \theta_B}{a_A - a_B}) (1 - (\theta_A + \theta_B)) > 0
\]
\[
\psi_2 - \chi = \sqrt{6} - (1 - 2\theta_B) \frac{2\sqrt{6}\theta A}{a_A - a_B}
\]

\[
= - \left( -\sqrt{6} + (1 - 2\theta_B) \frac{2\sqrt{6}\theta A}{a_A - a_B} \right) = - (\psi_1 - \chi) < 0
\]

and deduce the following ranking

\[
\psi_2 < \chi < \psi_1.
\]

Thus, we have

\[
\text{Sign} (\Pi) = -\text{Sign} (1 - \chi) (1 - \psi_1) (1 - \psi_2). \quad (4)
\]

We also get:

\[
\psi_1 - \chi = -\sqrt{6} + (1 - 2\theta_B) \frac{2\sqrt{6}\theta A}{a_A - a_B} = -\sqrt{6} + (1 - 2\theta_B) \frac{2\sqrt{6}\theta A}{a_A - a_B} = \left( \frac{2\sqrt{6}\theta A}{a_A - a_B} \right) (1 - (\theta_A + \theta_B)) > 0
\]

\[
\psi_2 - \chi = \sqrt{6} - (1 - 2\theta_B) \frac{2\sqrt{6}\theta A}{a_A - a_B} = - \left( -\sqrt{6} + (1 - 2\theta_B) \frac{2\sqrt{6}\theta A}{a_A - a_B} \right) = -(\psi_1 - \chi) < 0.
\]

\[
\psi_1 + \psi_2 = (\psi_1 - \chi) + (\psi_2 - \chi) + 2\chi = 2\chi.
\]

Hence we rank:

\[
\psi_1 > \chi > \psi_2
\]

and from (4) we deduce:

\[
\Pi < 0 \iff 1 > \psi_1 \text{ or } \chi > 1 > \psi_2
\]

Defining \( a_A = a, a_B = \alpha a, \theta_A = \theta_B, \) and \( \theta_B = \beta \theta \), with \( 1 > \alpha \geq 0 \) and \( \beta \geq 0 \), we obtain:

\[
1 > \psi_1 \quad \text{if}
\]

\[
\left\{ \begin{array}{l}
\beta > 1 \\
\frac{\sqrt{6}}{\sqrt{6}(1+\beta)+\beta-1} < \theta < \frac{1}{\beta+1}
\end{array} \right.
\]

or

\[
\left\{ \begin{array}{l}
\frac{\sqrt{6}-2}{(\sqrt{6}+1)\beta+\sqrt{6}-1} < \theta < \frac{\sqrt{6}}{\sqrt{6}(1+\beta)+\beta-1} \\
b < b_1
\end{array} \right.
\]

or

\[
\left\{ \begin{array}{l}
0 < \beta \leq 1 \\
\frac{\sqrt{6}-2}{(\sqrt{6}+1)\beta+\sqrt{6}-1} < \theta < \frac{1}{\beta+1} \\
b < b_1
\end{array} \right.
\]

where \( b_1 = \frac{(1-\alpha)a}{2\sqrt{\theta-(1+\sqrt{6})\beta-(\sqrt{6}-1)\theta}} \).
\[ \chi > 1 > \psi_2 \text{ if } \]
\[
\left\{ \begin{array}{l}
0 < \beta < 1 \\
0 < \theta \leq \frac{\sqrt{6}}{1+\sqrt{6}+\beta(\sqrt{6}-1)} \\
b > \frac{(1-\alpha)a}{2K\theta(1-\beta)}
\end{array} \right.
\]

or
\[
\left\{ \begin{array}{l}
0 < \beta < 1 \\
\frac{\sqrt{6}}{1+\sqrt{6}+\beta(\sqrt{6}-1)} < \theta < \frac{1}{\beta+1} \\
\frac{(1-\alpha)a}{2K\theta(1-\beta)} < b < b_1
\end{array} \right.
\]

A.2 The case of \( \lambda \neq 0 \).

Using the parameter specification given above we obtain:

\[ t_A^N = \frac{2}{21} (7 + 3a_A - 6\theta), \quad t_B^N = \frac{2}{21} (7 - 3a_A + 6\theta), \]
\[ t_A^L = \frac{4}{81} (21 + 9a_A - 23\theta), \quad t_B^L = \frac{2}{81} (33 - 9a_A + 14\theta), \]
\[ t_A^F = \frac{2}{81} (29 + 9a_A - 14\theta), \quad t_B^F = \frac{4}{81} (16 - 9a_A + 23\theta). \]

We establish that

\[ \Pi = 0 \text{ for } a_A^1 = \frac{1}{9} (32\theta - 7), \quad a_A^2 = \frac{1}{9} \left( 32\theta - 7 - 2\sqrt{77} \right), \quad \text{and } a_A^3 = \frac{1}{9} \left( 32\theta - 7 + 2\sqrt{77} \right). \]

We have:

\[ \Pi > 0 \Leftrightarrow \left\{ \begin{array}{l}
1 < a_A < \frac{137}{8} \\
\frac{1}{32} (3 + 36a_A) < \theta < \frac{1}{32} (7 + 9a_A)
\end{array} \right. \]

\[ \Pi < 0 \Leftrightarrow \left\{ \begin{array}{l}
1 < a_A \leq \frac{137}{81} \\
\frac{1}{32} (7 + 9a_A) < \theta < \frac{1}{32} (17 + 36a_A) \\
\frac{137}{81} < a_A \leq \frac{25}{12} \\
\frac{1}{32} (3 + 36a_A) < \theta < \frac{1}{32} (17 + 36a_A) \\
\frac{25}{12} < a_A < \frac{89}{36} \\
\frac{1}{32} (3 + 36a_A) < \theta < 1
\end{array} \right. \]