Optimal Implicit Taxes and Subsidies

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Abstract

This paper uses consumer and producer pricing schedules to implement Pareto optimal allocations when there is an external cost or benefit framed by a public good. A higher (smaller) price for marginal than for inframarginal consumption units leads to an implicit tax on (subsidy to) a good creating an external cost (benefit), for example, carbon emissions-generating energy such as coal energy (research generating public knowledge). One of the multiple implicit tax policies efficiently controlling local or global emissions funds a subsidy to emissions-free energy such as solar energy. An optimal policy is characterized in a dynamic general equilibrium model with an externality from using fossil energy on the climate. (i) Quadratic investment cost and (ii) full depreciation of capital used in energy production imply optimal carbon policy that depends on the relative cost and market share of fossil and renewable energy, in a form that does not depend on the specification of the utility function, climate feedback, or the carbon cycle.

Keywords: External cost, external benefit, climate policy, renewable energy subsidy, dynamic provision of public good.

JEL classification numbers: D62, D78, H23, Q31, Q43, Q48, Q58.

1. Introduction

Pigou (1920) posited a tax (subsidy) to implement a Pareto optimum in the presence of an external cost (benefit) such as in an environment with a public good. Dales (1968)

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argued tradeable certificates bounding the production of a public good implement a Pareto optimum. Common to both policies, government revenue (i) is created that is transferred to the income of households (through rebates, or indirectly through subsidies to households or industry, or reduction in capital or labor taxes) or (ii) needs to be created to fund a subsidy. One does not need to grow attached to these basic policies. This paper shows an alternative policy correcting a public good externality: an implicit tax and subsidy system affecting only consumer and producer budgets, and therefore lacking income redistribution through transfers or the need to raise government revenue somewhere else. The class of policy is general and could be applied to various externality problems, notably climate change, assuming consumers or producers respond to marginal prices. Within this widened space of policies that implement a Pareto optimum, an externality-neutral good such as carbon emissions-free solar and wind energy, may be subsidized solely motivated by controlling an external cost created by another good such as through emissions from coal and petroleum energy in the climate problem. This paper helps to better understand currently practiced policy targeted toward greater deployment of renewable energy technologies such as solar or wind energy from a welfare point of view.

Renewable energy subsidy schemes such as specific tariffs, specific premia, quota-fulfilling output certificates, and purchasing agreements with levies as described, for example, in Council of European Energy Regulators (2013), have been implemented in various countries in the recent two decades to mitigate climate change or foster learning in renewable energy technology. Table 1 documents the more widespread use of these policies—with the example of a tariff, than a carbon tax.\(^1\) The renewable energy support schemes are government budget-neutral, because the support in form of a subsidy is fully-funded affecting the economy through changing relative prices and the direct promotion of a specific technology.

There is theoretical and empirical evidence that the use of state revenues from an alternative carbon tax or permit auction might be key for the political support of the renewable

\(^1\)The German \textit{Renewable Energy Sources Act} amended 2014 states, among other purposes, external cost and development of electricity-generating technologies, as reasons for the feed-in tariffs (§1 EEG 2014).
Table 1: Countries with carbon tax or feed-in tariff 2015.

<table>
<thead>
<tr>
<th>Carbon Tax</th>
<th>Both Carbon Tax and Feed-in Tariff</th>
<th>Feed-in Tariff</th>
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<tbody>
<tr>
<td>13</td>
<td>10</td>
<td>77</td>
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Notes: Sources for carbon tax World Bank (2014) and feed-in tariff Renewable Energy Policy Network for the 21st Century (2015). A carbon tax uses a denomination in carbon units, for example, ton CO2e. A carbon tax and feed-in tariff may be implemented in different regions of a country, for example, in the Canadian provinces British Columbia and Ontario, respectively.

energy policies above—at current and future cost.

1. **Economic Theory.**—Tax policy issues in using environmental tax revenue might deter legislators from adopting environmental taxes. Legislators may regard all other taxes as necessary and cannot resolve the redistribution in reasonable time because of conflicting redistributive goals of electorate and government or within government. Or, counteracting the incidence of a carbon tax in reducing other, distortionary taxes, bears political conflict. For example, a carbon tax might disproportionately levy the burden of carbon taxation on low-income households, and the most efficient tax cuts elsewhere have the greatest benefit for high-income households (Marron & Toder 2014). Permits raise the same concerns about tradeoffs of efficiency and equity from recycling revenue. A budget-neutral policy lacks such tradeoffs, which can explain their widespread use.

2. **Empirical Evidence.**—Amdur, Rabe & Borick (2014) find that more surveyed U.S. residents support a carbon tax if “revenues from the tax were used to fund research and development for renewable energy programs” than when funding is unspecified—for example, 70 percent versus 47 percent Democrat voters and 51 percent versus 15 percent Republican voters. In addition, Democrat, Republican, and Independent voters each prefer renewable energy subsidies over tax rebate checks, and each the majority of voters opposes a carbon tax with federal deficit reduction. The willingness to support research and development suggests a role of policy for learning.
The paper contributes to three strands in the literature. First, it provides a novel policy for implementing an optimum when there is a public good such as the climate or knowledge. Second, it examines the current practice of environmental policy with green energy output subsidies. Third, it applies the novel policy to mitigate carbon emissions in dynamic general equilibrium.

(i) I develop a framework with competitive markets to examine the allocative effects of well-motivated government policy. There are many agents who each identically value private goods and a public good and in making decisions take the public good quantity as given.\footnote{Uniform policy variants for all agents may not implement a Pareto optimum when agents identically value private goods and differently value the public good. But there exist nonuniform policy variants that do implement a Pareto optimum in this case under perfect information.}

The missing market for the public good is counterbalanced by pricing schedules that differ for buyers and sellers. That subsidies are fully funded by taxes makes the system neutral to the government budget. A surcharge forms an implicit tax in the presence of an external cost such as emissions, because the surcharge paid on some consumption units funds a subsidy in the form of exemptions from paying the full surcharge on other consumption units. Likewise, a surcharge funds implicit subsidies in the form of exemption in the presence of an external benefit such as knowledge spillover. I choose the name implicit tax rather than indirect tax on an external-cost creating good to contrast the policy to an indirect tax in the sense of a tax on a related good Wijkander (1985) and others assessed or a deposit-refund system on a related market Böhm (1981) discussed.

I will refer to a schedule as the gross payment or receipt for all traded units and a scheme as the per unit payment or receipt. The optimal policy can be applied on the buyer or seller side. A pricing schedule of a private good creating an external cost reducing a public good such as emissions reducing environmental quality is progressive (regressive) for buyers (sellers)—implying an implicit tax on the private good. A pricing schedule of a private good creating an external benefit increasing the amount of a public good such as research increasing knowledge is regressive (progressive) for buyers (sellers)—implying an implicit
subsidy to the private good. I consider both policies motivated by the environment and learning in examples.

Policy is characterized by multiplicity. Infinitely many individually uniform pricing schedules implement an optimum. A two-block pricing schedule is discussed in detail. A two-part tariff, that is, a fixed charge (deduction) in the presence of an external cost (benefit) and a variable payment, is an example of an optimal pricing schedule. A representation with rebate differs from the tax-rebate system in Fullerton & Wolverton (2000), which raises revenue on net. While implementing an optimum, more than the goods-tax equivalent of the Pigouvian emissions tax is charged to all output and rebated for clean output. In contrast, here a subsidy accompanies an implicit tax, or a subsidy to clean output is fully-funded by fees which are differentiated among total consumption units to internalize an external cost (see below). Infinitely many individually different payment schemes with uniform marginal price implement an optimum.³

A widely held view is that market failures additional to an emissions externality are required to rationalize government intervention directed to emissions-free technology. In contrast, I show that some amount of a surcharge that yields an implicit tax may fund a technology-specific subsidy to emissions-free renewable energy with the sole motive of an emissions externality, exploiting the multiplicity of optimal policies. This provides scope for environmental policy to be technology-specific, which seems important because of the political economy obstacles described above. The subsidy is not necessary to implement an optimum, likely because the model abstracts from political incentives. An explicit green energy research subsidy has appeared in Acemoglu, Aghion, Bursztyn & Hemous (2012) correcting a knowledge externality. This subsidy can be mimicked by an implicit subsidy according to my results.

(ii) The present paper sheds light on the efficiency of existing renewable energy output subsidies. Current practice of renewable energy support exhibiting a uniform fee is not

³A pricing schedule with individually varying marginal prices implements an optimum in an economy with identical households when factors are nontradable across sectors.
well-designed to mitigate carbon emissions. A uniform fee fully-funding a subsidy to an emissions-free renewable energy technology paid by all consumers (households and firms) leads to overprovision of both emissions-intensive fossil energy and emissions-free renewable energy when their market shares are optimal. Such underprovision of the public good results because funding with uniform fee distorts the market value of green energy above its socially optimal level. Feed-in tariffs for solar and wind electricity are thus right on track, however in current practice excluding some demand by households from paying the surcharge funding the tariff which can lead to efficient allocations is missing. Canton & Johannesson Lindén (2010) acknowledge that a fully-funded feed-in tariff distorts the price of renewable energy, yet do not derive outcomes. The literature has considered green energy subsidies mostly in interaction with other policy instruments. Pethig & Wittlich (2009) examine welfare changes compared to status quo attained through various pairs of emissions tax and green energy subsidy. In contrast, I analyze welfare relative to optimum under a policy without emissions tax and with uniformly funded green energy subsidy.

(iii) I develop a dynamic general equilibrium model with investment in fossil and renewable energy considering fossil and renewable energy as close substitutes. Specializing the production structure with quadratic investment cost and full depreciation of capital used in energy production helps to characterize the implicit tax policy that implements an optimum. The optimal percentage surcharge and the fraction of demand excluded from paying the surcharge likely increase on a path with increasing market portion of renewable energy in a given sector at an optimum. The optimal policy depends on the relative cost of fossil and renewable energy—expressed by technology parameters and growth rate of energy production capital—and their market share, and the interest rate at an efficient allocation, yet does not depend on the specification of the utility function, feedback of climate on society, or the carbon cycle. This obtains by focusing on the wedge that the social cost of polluting is equal to when a polluting technology is used. The literature has specialized terms in the

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4Firms with a large cost share of electricity are exempted from paying the surcharge on electricity in Germany. The effect of this exemption on allocations in the model is not fully worked out yet.
cost of polluting to derive a carbon tax proportional to output at an optimum. Golosov, Hassler, Krusell & Tsyvinski (2014) have made assumptions on the savings rate, feedback of climate on society, and carbon cycle, that gives a carbon tax formula which depends on the parameterized feedback of climate on society and the carbon cycle. The multiple energy services in the model as in Nordhaus (1973) can lead to simultaneous use of fossil and renewable energy when the assumption of strictly convex investment cost is relaxed.

Plan of paper.—The rest of the paper is organized as follows. Section 2 presents implicit taxes and subsidies. Section 3 develops a dynamic general equilibrium model with climate policy involving implicit taxes including funding of a specific subsidy. Section 4 concludes.

2. The role of implicit taxes and subsidies for public good provision

This section shows first that a government budget-neutral policy implements an optimum in the presence of an external cost or benefit.

Consider the budget balance condition

\[ \Lambda(x) = p_S x \]

for commodity trade \( x > 0 \). Consumers pay \( \Lambda(x) \) units of account in total in exchange for a private good amount \( x \). The function \( \Lambda : \mathbb{R}_+ \rightarrow \mathbb{R} \) giving this payment is increasing and right-differentiable, \( \lim_{x \to +x^*} [\Lambda(x) - \Lambda(x^*)]/[x - x^*] \) at \( x = x^* > 0 \) exists. Sellers receive \( p_S > 0 \) units of account per commodity unit. In particular, consider the two-block pricing schedule \( \Lambda(x) = (p_B - \tau)x \) if \( x \in [0, x_e) \) and \( \Lambda(x) = (p_B - \tau)x_e + p_B(x - x_e) \) if \( x_e \leq x \). This yields the budget balance condition

\[ (p_B - \tau)e + p_B(1 - e) = p_S \]  

of quantity \( x > 0 \) traded with \( 0 \leq e \equiv x_e/x < 1 \). Consumers pay \( (p_B - \tau) \) (if positive) or
receive one minus times it (if negative) for each of the first \(x_e\) units of consumption, and pay the marginal consumer price \(p_B > 0\) for each further unit of consumption. Thus, either (i) the fraction of purchases \(e\) is exempted from paying the surcharge \(\tau > 0\) so that the fraction \((1 - e)\) is included in paying the surcharge in addition to the base price \(p_B - \tau\), or (ii) the fraction \(e\) is included to be paid the surcharge of minus one times \(\tau < 0\) units in addition to the marginal consumer price \(p_B\) so that the portion \((1 - e)\) is exempted from paying the surcharge. The two-block pricing schedule is implied by the scheme of marginal price \(\lambda(x) = p_B - \tau\) if \(x \in [0, x_e)\) and \(\lambda(x) = p_B\) if \(0 \leq x_e \leq x\). This completely characterizes the buyer-sided policy which I will examine in partial and general equilibrium. In Section 2.3, I will instead consider a payment function for sellers.

2.1. Partial equilibrium

Consider a static economy with each one private and public good.

There is a unit mass of households with identical utility function

\[
U(x, y) = B(x, y) - C(x, y),
\]

where \(B : \mathbb{R}_+ \times \mathbb{R} \rightarrow \mathbb{R}\) and \(C : \mathbb{R}_+ \times \mathbb{R} \rightarrow \mathbb{R}\) represent preferences for a private good \(x \geq 0\) and a public good \(y\). The benefit function \(B\) is increasing in the private good, nondecreasing in the public good, and continuously differentiable, \(B_x > 0\) and \(B_y \geq 0\) are continuous. The cost function \(C\) is increasing in the private good and continuously differentiable, \(C_x > 0\) and \(C_y\) are continuous in both arguments.\(^5\) Two further assumptions will be used, next to differentiable marginal benefit and cost.

\begin{assumption}
The marginal benefit is strictly decreasing, \(B_{xx} < 0\).
\end{assumption}

\begin{assumption}
The marginal cost is nondecreasing, \(C_{xx} \geq 0\).
\end{assumption}

\(^5\)Throughout the paper, partial differential of function \(F\) with respect to \(x\) is denoted \(F_x \equiv \partial F/\partial x\), and the second partial differential of \(F\) with respect to \(x\) is defined \(F_{xx} \equiv (\partial^2 F/\partial x^2)\).
Households are indexed $i$. The public good is produced from the aggregate private good $x = \int_0^1 x_i \, di$ according to

$$T(x, y) = 0 \tag{3}$$

where $T : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$ is continuously differentiable, $T_x$ and $T_y \neq 0$ are continuous, and $dy/dx = -T_x/T_y$. The production of the public good occurs through natural processes such as the transformation of material into harmful states or spillover of knowledge.

It is now possible to define and characterize an optimum for this economy. An optimum is private and public good amounts $(x, y)$ that maximize utility given by (2) subject to (3). At an interior optimum, $x > 0$,

$$B_x + B_y(dy/dx) = C_x + C_y(dy/dx). \tag{4}$$

The marginal social benefit, the left side, equals the marginal social cost, the right side, in terms of the private good. To examine government policy, I will decentralize decision-making and define an equilibrium. At an optimum, $B_y \neq C_y$, and $T_x \neq 0$, motivates government policy, so that the difference of marginal cost and benefit of the public good in terms of the private good $(C_y - B_y)(dy/dx)$, and hence the wedge between marginal benefit and cost of the private good $w \equiv B_x - C_x$, is nonzero. For simplicity, I assume that an optimum has an interior value $x > 0$.

Equilibrium.—Decisions will now be decentralized in partial equilibrium, with unspecified factors used in the production of the private good. Each household is composed of a consumer and a producer. The consumer chooses the quantity of the private good $x$ so as to maximize benefit net of payments in solving

$$\max_x \left\{ B(x, y) - \Lambda(x) \right\}$$

The socially optimal allocation will be the same with the planner’s objective being the sum of equally weighted individual utility, because there is a unit mass of households. An allocation maximizing welfare defined with nonuniform individual welfare weights would only be implementable in the decentralized equilibrium below if some producers were paid different marginal prices.
taking as given both the tariff $\Lambda(x)$ which gives the payment in exchange for the private good and the public good amount $y$. The producer chooses output $z$ so as to maximize profit in solving

$$\max_z \{p_S z - C(z, y)\}$$

taking as given the price $p_S$ and public good amount $y$.\footnote{Alternatively, each household has the inverse demand function $D^{-1}(x, y)$. Households are consumers of $(x, y) = D(p)$ units of the private and public good dependent on the marginal price of the private good $p$, where $D$ is a multi-valued function.} Government sets the terms of the pricing function for a buyer $\Lambda$, for example, surcharge $\tau$ and exemption/inclusion rate $e$, exogenous to the consumers and producer problems. A laissez-faire equilibrium has $\Lambda(x) = p_B x$, and can be interpreted such that government policy is absent.

An equilibrium is an allocation of consumption and production of the private good and the public good $(x, z, y)$ and buyer pricing function and seller unit receipt $(\Lambda, p_S)$, such that households solve their problems taking as given $(\Lambda, p_S, y)$, the market clears, $x = z$, buyer payments equal seller receipts, $\Lambda(x) = p_S x$, and condition (3) determines $y$.

An interior equilibrium, $x > 0$, will now be characterized. In equilibrium,

$$p_B = B_x,$$

$$C_x = p_S.$$  \hspace{1cm} (5) \hspace{1cm} (6)

Consumers set marginal price equal to marginal benefit. Producers set marginal cost equal to marginal price. None of the decisions underlying these outcomes takes care of the external effect of the private good on the public good which is expressed by the transformation frontier (3) and dependency of benefit or cost on the public good.

\textit{Implementation of Optimum}.—A pricing function $\Lambda$ is to be found implementing an optimum. As an example, the two-block pricing schedule described above is characterized by the surcharge $\tau$ and the exclusion/inclusion portion $e$. This will be used now.
Proposition 1. The policy of surcharge and exemption/inclusion rate \((\tau, e)\) satisfying \(\tau e = (B_y - C_y)(-dy/dx)\) and \(e \in (0, 1)\) always implements a Pareto optimum.

Proof of Proposition 1

i. The social optimality condition (4) must hold subject to the private optimality conditions (5)-(6). Thus,

\[ p_B - (B_y - C_y)(-dy/dx) = p_S. \]

The budget balance condition (1) then implies the desired result. ii. “Always.” Show that the policy implements only an optimal allocation. The wedge \(B_x - C_x\) is a continuous function of \(x\). The budget balance condition (1) implies that \(p_B - p_S = \tau e\) is equal to the wedge evaluated at an optimum. Now the conditions (5)-(6) hold marginal consumer and producer price equal to the marginal benefit \(B_x\) and private marginal cost \(C_x\), respectively. The marginal benefit is strictly decreasing by Assumption 1, the marginal cost is nondecreasing by Assumption 2, and both are continuous functions of \(x\), determining a unique implementable allocation for every wedge value. Thus, \(\tau e\) at the level stated implies that the allocation is optimal. QED

The difference of marginal cost and benefit of the public good in terms of the private good, \((C_y - B_y)(dy/dx)\), can be seen as the wedge between the private marginal benefit equal to the marginal buyer price \(p_B\) paid by the consumer and the private marginal cost equal to the price \(p_S\) received by the seller in an efficient equilibrium. The policy applies to external cost, \((B_y - C_y)(-dy/dx) > 0\) implying \(\tau > 0\), and external benefit, \((B_y - C_y)(-dy/dx) < 0\), implying \(\tau < 0\). An external cost demands that some buyer pays more than the marginal unit receipt of a seller which is held constant. However, what one market side spends equals what the other market side receives in equilibrium. Both the wedge in marginal prices of buyers and sellers and budget balance hold by paying the base price for initial units and the base price plus an implicit tax in the form of a surcharge on other units amounting to the portion \((1 - e)\) of demand of the externality-creating good. An external benefit requires
that to buyers the order of units paid more and less for is reversed, and thus the fraction of units \((1 - e)\) is implicitly subsidized. I will illustrate graphically optimal policy in the examples in the next subsection.

Implementing only Pareto optima is not shared with a Pigouvian tax when the cost function \(C\) is linear in the private good amount, because a Pigouvian tax does not determine the wedge between the private marginal benefit and marginal cost.

2.2. General equilibrium

In this subsection, I show that the implementation result in Proposition 1 carries over to a general equilibrium setting. In the present context, there are two groups of private goods, public good producing indexed \(1, 2, \ldots, I_F - 1\) and externality-free indexed \(I_F, I_F + 1, \ldots, I\). There is no loss of generality when there is only one of each of them, as I shall assume now.

Households on the unit continuum have identical preferences for private goods \(x = (x^1, x^2)\) and a public good \(y\) expressed by the real-valued utility function \(U(x, y)\) on \(\mathbb{R}_+^2 \times \mathbb{R}\) which has standard properties: continuously differentiable in each \(x\) component, \(U_{x^1}\) and \(U_{x^2}\) are continuous, and continuously differentiable in \(y\), and nondecreasing in \(y\); \(0 \leq U_y\). I will specify the shape of utility regarding the private goods in each example.

There is a process through which the aggregate amount of the private good \(x^1\) affects the public good according to

\[
T(x^1, y) = 0
\]  

(7)

where \(T : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}\) is continuously differentiable, \(T_{x^1} = \partial T / \partial x^1\) and \(T_y = \partial T / \partial y > 0\) are continuous.\(^8\) There are two private goods so that one good can be an input and one other good can be an output. The following domain \(D\) and function \(Y\) capture this idea without using too much structure while permitting to conveniently characterize an optimum. The

\(^8\)The marginal rate of transformation along the function \(T\) and the valuation of \(x^1\) in utility designates the type of externality, which for \(dy/dx^1 = -T_{x^1}/T_y > 0\) and positive valuation of \(x^1\) in utility is a cost and for \(-T_{x^1}/T_y < 0\) and positive valuation of \(x^1\) in utility is a benefit.
aggregate production possibilities of the goods quantities are in the set

\[ Y(x, y) \]

\[ = \{ x \in D \subseteq \mathbb{R}_+^2, y \in \mathbb{R} | Y(x, y) \geq 0 \} \tag{8} \]

where the function \( Y : \mathbb{R}_+^2 \times \mathbb{R} \to \mathbb{R} \) is continuously differentiable and nondecreasing in \( y \),

that is, \( Y_{x1}, Y_{x2}, \) and \( Y_y \geq 0 \) are continuous.

Defining and characterizing an optimum will now be carried out. A social planner determines private and public good amounts \((x, y)\) so as to maximize utility \( U(x, y) \) subject to (7)-(8). The planner problem is solved at an allocation with interior private good amounts \((x^1, x^2)\) only if

\[ \frac{U_{x1}}{U_{x2}} = \theta \left( -\frac{dy}{dx^1} \right) = \frac{Y_{x1}}{Y_{x2}}, \tag{9} \]

where

\[ \theta \equiv \frac{U_y + \left( -\frac{Y_y}{Y_{x2}} \right) U_{x2}}{U_{x2}} \]

gives the externality size \( \theta \).\(^9\) Equation (9) uses the externality scale \((-dy/dx^1) = T_{x1}/T_y\), and equates the marginal social rate of substitution, the left side, and marginal rate of transformation of the private goods, the right side.\(^{10}\) In the following, I assume a role for government policy in the economy, that is, at an optimum, \( U_y + Y_y > 0, U_{x2} \) and \( Y_{x2} \) have the opposite sign if \( Y_y > 0 \), and \( T_{x1} \neq 0 \), so that the marginal valuation of the public good \( \theta(T_{x1}/T_y) \) is nonzero. The private marginal rate of substitution and transformation

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\(^9\)The Samuelson condition for the optimal provision of the public good amounts to (9), because each households’ utility has received the same weight in the planner’s objective function. Results are the same with the aggregate private good amount \( x^1 = \int_0^1 x^1(i)di \) in the constraints (7) and (8).

\(^{10}\)Equation (9) follows after elimination of the Lagrange multipliers \( \mu_Y \) and \( \mu_T \) on the constraints (7)-(8), respectively, from the necessary first-order conditions for an interior optimum:

\[
\begin{align*}
U_{x1} + \mu_Y Y_{x1} + \mu_T T_{x1} &= 0 \\
U_{x2} + \mu_Y Y_{x2} &= 0 \\
U_y + \mu_Y Y_y + \mu_T T_y &= 0.
\end{align*}
\]
$U_{x_1}/U_{x_2}$ and $Y_{x_1}/Y_{x_2}$ are normalized by the marginal utility $U_{x_2}$ and marginal product $Y_{x_2}$, respectively, explaining the sign correction in the wedge

$$w \equiv \text{sign}(U_{x_2}) \left[ \frac{U_{x_1}}{U_{x_2}} - \frac{Y_{x_1}}{Y_{x_2}} \right].$$  

(10)

In addition, I assume that an optimum is in the interior of the feasibility set.

**Equilibrium.**—There are two ways to decentralize decision-making with policy on the purchase of the externality-creating good $x^1$: (I) households buy $x^1$ and sell the externality-free good $x^2$, (II) households sell $x^1$ and buy $x^2$. I will use I and II to prove implementation of an optimum. This is useful to show that decentralizing does not affect optimal policy. I will provide each one example.\(^\text{11}\) There is a unit mass of identical firms whose profit households equally share.\(^\text{12}\)

(I) Households demand consumption and supply input $x = (x^1, x^2)$ so as to maximize utility in solving $\max_{x \in D} U(x, y)$ subject to the budget constraint

$$\Lambda(x^1) = \Pi^{(I)} + x^2$$

taking the pricing function $\Lambda$, firm profit $\Pi^{(I)}$, and the public good amount $y$ as given. Each firm supplies output and demands input $z = (z^1, z^2)$ so as to maximize profit

$$\Pi^{(I)} = \max_{z^1, z^2} \{ p_S z^1 - z^2 \}$$

on the production possibility frontier $Y(z^1, z^2, y) = 0$ taking the price $p_S$ and the public good amount $y$ as given. The price of the input is normalized to one.

(II) Households determine factor supply and consumption $x = (x_1, x_2)$ so as to maximize

\(^{11}\)I omit a third variant—households buy $x^1$ and $x^2$, because it is irrelevant for the examples.

\(^{12}\)Alternatively, households produce output and supply it on markets in any of the decentralized economies.
utility in solving $\max_{x \in D} U(x, y)$ subject to the budget constraint

$$x^2 = p_S x^1 + \Pi^{(II)}$$

taking the price $p_S$, firm profit $\Pi^{(II)}$, and the public good amount $y$ as given. Each firm chooses factor demand and output $(z^1, z^2)$ so as to maximize profit

$$\Pi^{(II)} = \max_{z^1, z^2} \{ z^2 - \Lambda(z^1) \}$$

on the production possibility frontier $Y(z^1, z^2, y) = 0$ taking pricing function $\Lambda$ and the public good amount $y$ as given. The price of the output is normalized to one.

An equilibrium will now be defined including both decentralized economies I and II in which the terms of the buyer pricing function $\Lambda$ are exogenous. An equilibrium is an allocation $(x, z, y)$, buyer pricing function $\Lambda$, and seller unit receipt $p_S$, such that households solve their problem taking as given $\Lambda$ (in I) and $p_S$ (in II), and firms solve their problem taking as given $p_S$ (in I) and $\Lambda$ (in II), given public good $y$, the markets clear, $x^1 = z^1$, $x^2 = z^2$, and the transformation expressed by condition (7) governs the public good.\\

In the following, I consider an interior equilibrium, $x$ in the interior of $D$. In equilibrium I,

$$p_B = \frac{U_{x^1}}{U_{x^2}},$$

$$-\frac{Y_{x^1}}{Y_{x^2}} = p_S.$$  \hspace{1cm}(11,12)

In equilibrium II,\footnote{That buyer payments equal seller receipts, $\Lambda(x^1) = p_S x^1$ using the budget balance condition (1) at good one, follows from the budget constraint of the consumer, the definition of profit, and market clearing, in general equilibrium with pricing function $\Lambda$ on one market.}

$$\frac{X^1}{X^2} = p_S.$$  \hspace{1cm}(13)
\[ 1/p_S = -\frac{U_{x2}}{U_{x1}}, \quad (13) \]

\[ -\frac{Y_{x2}}{Y_{x1}} = 1/p_B. \quad (14) \]

Households equate the relative price and the marginal rate of substitution. Firms equate the marginal rate of transformation (the marginal rate of technical substitution) and the output/input price ratio.\(^\text{14}\) In the equilibrium without government policy, that is, \(p_B = p_S\), the private marginal rate of substitution is equalized to the marginal rate of transformation, \(U_{x1}/U_{x2} = Y_{x1}/Y_{x2}\). The resulting allocation is Pareto inefficient, because the households and firms disregard external cost or benefit in their decision-making.

**Implementation of Optimum.**—Policy needs to be set with the same principle as in Proposition 1 so that the equilibrium allocation is optimal.

**Proposition 2.** The policy of surcharge and exemption/inclusion rate \((\tau, e)\) satisfying \(\tau e = \text{sign}(U_{x2}) \left[ (U_{x1}/U_{x2}) - (Y_{x1}/Y_{x2}) \right]\) and \(e \in (0, 1)\) implements a Pareto optimum.

**Proof of Proposition 2**

The social optimality condition (9) needs to hold subject to the private optimality conditions (11)-(12) and (13)-(14). Substituting the expressions for relative prices yields \(p_S = p_B + \xi (\text{I})\) and \(p_S = p_B - \xi (\text{II})\), where \(\xi = \theta(T_{x1}/T_y)\) is the product of externality size \(\theta\) and scale \(T_{x1}/T_y\). Upon substitution of (1) and (10), \(\tau e = p_B - p_S = w \) (I and II). QED

The surcharge level and the exemption/inclusion rate multiply to the social cost or minus one times the social benefit. The surcharge is positive when there is an external cost, \(w > 0\). Then \(\tau > 0\) so that \(\tau e \in (0, \tau)\). The surcharge is negative, if there is an external benefit, \(w < 0\). Then \(\tau < 0\) so that \(\tau e \in (-\tau, 0)\). One example each illustrates these cases. I use these examples to discuss the relation of the policy in Proposition 2 to Pigouvian tax and

\(^{14}\text{Profit is nonnegative, if the production function implied by } Y = 0 \text{ is concave.}\)
subsidy. Optimal policy with an implicit tax when there is an external cost will be shown first.

Example I (Pollution). Households are endowed with one unit of time which they can spend on leisure or work. Utility depends positively on consumption \( c = x^1 \) and environmental quality \( y \) and decreases in labor supply \( n = x^2 \in [0, 1] \), that is, \( \partial U / \partial x^1 > 0 > \partial U / \partial x^2 \) and \( \partial U / \partial y > 0 \). Production of the consumption good reduces environmental quality one-by-one,

\[
T(c, y) = 1 - c - y.
\]

Output \( c \) is produced requiring labor \( n \) according to

\[
Y(c, n) = An - c, \quad A > 0.
\]

The environmental quality does not affect productivity.\(^{15}\)

An interior optimum \((c, n, y)\) demands that the social marginal rate of substitution of consumption and leisure equals the marginal cost of turning work time into consumption,

\[
\frac{U_c}{-U_n} - \frac{U_y}{-U_n} = \frac{1}{A},
\]

a specialization of the social optimality condition (9). In equilibrium, all output is consumed, \( c = \bar{y} \), where \( \bar{y} = z^1 \) is output choice. The labor market clears, \( n = \tilde{n} \), where \( \tilde{n} = z^2 \) is labor demand choice. Households set \( U_c / (-U_n) = p_B \). Firms set \( p_S = 1/A \). Trade is budget-neutral to the government if

\[
(1 - e)p_B + e(p_B - \tau) = 1/A,
\]

where \( e \in (0, 1) \). An optimum with equal consumption for all households is implemented.

\(^{15}\)This economy can be generalized with the intermediate good energy \( x^1 \) produced from the input labor \( x^3 \) to produce consumption \( x^2 \). Section 3 views a similar economy with multiple technologies producing energy, where energy production uses an investment good instead of labor. The domain restriction is \( x^2 \in [0, 1] \).
if \( \tau e = U_y/(-U_n) \). The left panel in Figure 1 illustrates both the optimum and the laissez-faire equilibrium in which government policy is absent. The marginal social benefit \( B' = U_c/(-U_n) - U_y/(-U_n) \) lies below the marginal private benefit \( B^0 = U_c/(-U_n) \). Their intersection with the marginal rate of transformation \( C' = 1/A \) gives the efficient and laissez-faire equilibrium output level, respectively. Therefore, the optimal output level \( x^* \) is smaller than the laissez-faire equilibrium level \( x^0 \), as shown in the graph. Standard policy in the form of the Pigouvian tax \( \bar{\tau} = U_y/(-U_n) \) would be applied uniformly to all units purchased or sold. The buyer-sided policy here exempts some demand from paying the surcharge \( \tau \), which is thus greater than the Pigouvian tax \( \bar{\tau} \), and equilibrium trade is optimal. The shaded area is the buyer payment for consumption, which consists of the area of the payment for demand from zero up to \( e x^* \) and the area of the payment for demand from there up to \( x^* \). The buyers pay the same marginal price and the sellers receive the same price as under the Pigouvian tax, \( p_B - \tau = p_S \), but because some fraction of demand is exempted from the surcharge, the buyers pay more per unit than the Pigouvian tax for units charged more than the base price \( p_B - \tau \). The taxation of the production activity that creates the external cost is implicit in that the marginal price exceeds marginal cost, \( p_B > 1/A \). Optimal policy with an implicit subsidy when there is an external benefit will be shown in the next example.

Example II (Learning from Others). Utility increases in consumption \( c = x^2 \) and decreases in labor supply \( n = x^1 \in [0, 1] \), that is, \( \partial U/\partial x^2 > 0 > \partial U/\partial x^1 \). The public good knowledge does not affect utility, \( \partial U/\partial y = 0 \). In this static example of learning, using labor \( n = x^1 \) to produce output \( c \) in one firm raises productivity in any given firm through the externality inherited in

\[
T(n, y) = n - y,
\]

and
\[ Y(n, c, y) = An^{1-d}y^d - c, \quad A > 0, 1 > d > 0, \]

where \( d \) is the degree of externality.\(^{16}\)

At an interior optimum \((n, c, y)\), the social marginal rate of substitution of leisure and consumption equals the marginal cost of reducing consumption to increase leisure,

\[
\frac{(-U_n)}{U_c} - dA = (1 - d)A,
\]

corresponding to the social optimality condition (9). In equilibrium, consumption equals output \(\bar{y} = z^2\), that is, \(c = \bar{y}\). Labor demand \(\tilde{n} = z^1\) equals labor supply, \(\tilde{n} = n\). Households set \((-U_n)/U_c = p_S\). Firms set \(p_B = (1 - d)A\). Trade is budget-neutral to the government if

\[
(1 - e)p_B + e(p_B - \tau) = A,
\]

where \(e \in (0, 1)\). An optimum with equal consumption for all households is implemented, if \(\tau e = -dA < 0\), which signals that the seller unit receipt is greater than the marginal buyer price in an efficient equilibrium. This is so because knowledge spillover creates an external benefit.\(^{17}\) The right panel in Figure 1 depicts both the optimum and the laissez-faire equilibrium. The marginal social benefit \(B' = A\) lies above the marginal private benefit \(B^0 = (1 - d)A\). Their intersection with the marginal rate of substitution \(C' = (-U_n)/U_c\) gives the efficient and laissez-faire equilibrium input level, respectively. Therefore, the optimal input level \(x^*\) is greater than the laissez-faire equilibrium level \(x^0\), as shown in the graph. The implicit surcharge \(\tau < 0\) is greater in magnitude than the Pigouvian subsidy \(s = p_S - p_B\), so that a fraction of output \((1 - e)\) is paid less than the marginal social benefit, \(p_B < A\). This implies implicit subsidization of the factor demand that creates the external benefit. The shaded area is the buyer payment for labor, which consists of the area of the

\(^{16}\)The domain restriction is \(x^1 \in [0, 1]\).

\(^{17}\)The wedge \(w = -d\bar{y}/y\) is evaluated at the equilibrium public good amount \(y = n = \bar{y}/A\).
payment for demand from zero up to $ex^*$ and the area of the payment for demand from there up to $x^*$. The policy here drives the same wedge between the uniform seller unit receipt and marginal buyer price as the Pigouvian subsidy $s = dA$ does.

In the examples, buyer-sided policy implements an optimum. It easy to deduce how seller-sided policy can implement an optimum.

2.3. Buyer-sided policy and seller-sided policy

Consider the budget balance condition

$$p_B x = \Psi(x)$$

with function $\Psi : \mathbb{R}_+ \to \mathbb{R}$ giving the receipt by sellers in commodity trade $x > 0$. The seller pricing function $\Psi : \mathbb{R}_+ \to \mathbb{R}$ is increasing and right-differentiable, like the buyer pricing function $\Lambda$. In particular, the two-block pricing schedule $\Psi(x) = (p_S + \tau)x$ if $0 \leq x < x_e$, 


Figure 1: **Buyer-sided policy on output market (I) and input market (II).**
and $\Psi(x) = (p_S + \tau)x_e + p_S(x - x_e)$ if $0 \leq x_e < x$ arises from the price scheme $\psi(x) = p_S + \tau$ if $0 \leq x < x_e$, and $\psi = p_S$ if $0 \leq x_e < x$. The budget balance condition in buyer-sided policy (1) and the one resulting in seller-sided policy

$$p_B = e(p_S + \tau) + (1 - e)p_S$$

are equivalent for same $(\tau, e)$. Now it appears that payment for a buyer in the presence of an external cost is progressive in the sense that the price for the first unit is smaller than the price for the last unit, $\tau > 0$, provided that $e \in (0, 1)$. This levels up the ratio of marginal buyer price to constant per unit price for a seller to the efficient level. This is one of four possible cases for uniform and weakly monotone payment schemes. In proving the progressive versus regressive shape of payment schemes, it is helpful to note that a payment scheme $\sigma \in \{\lambda, \psi\}$ is supported by the unit interval and consumption space and is weakly monotone if either (i) $\sigma(x) \leq \sigma(z)$ or (ii) $\sigma(x) \geq \sigma(z)$, all $x < z$, and define terms of shape:

**Definition 1.** A weakly monotone payment scheme $\sigma(z) \in \{\lambda(z), \psi(z)\}$ with $z \in [0, 1]$ is progressive (regressive), if $\sigma(0) < (>)\sigma(1)$.

**Proposition 3.** A weakly monotone uniform payment scheme for all buyers $\lambda$ correcting an external cost, $w > 0$ (benefit, $w < 0$), is progressive (regressive). A weakly monotone uniform payment scheme for all sellers $\psi$ correcting an external cost, $w > 0$ (benefit, $w < 0$), is regressive (progressive).

**Proof of Proposition 3**

Marginal prices are in the same relation in an efficient partial and general equilibrium,

$$p_B - w = p_S.$$

i. Buyer-sided policy. The budget balance condition $\int_0^1 \lambda(z)xdz = \int_0^x \lambda(z)dz = \Lambda(x) = p_Sx$, weak monotonocity of $\lambda$ on $[0, 1]$, and $\lambda(1) = p_B$ imply that $\lambda(0) < \lambda(1)$ if $w > 0$ and
Table 2: Buyer-sided and seller-sided policy.

<table>
<thead>
<tr>
<th></th>
<th>Buyer-sided policy</th>
<th>Seller-sided policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>External cost, $w &gt; 0$</td>
<td>Progressive</td>
<td>Regressive</td>
</tr>
<tr>
<td>External benefit, $w &lt; 0$</td>
<td>Regressive</td>
<td>Progressive</td>
</tr>
</tbody>
</table>

$\lambda(0) > \lambda(1)$ if $w < 0$. ii. Seller-sided policy. The budget balance condition $p_B x = \Psi(x) = \int_0^x \psi(z) dz = \int_0^1 \psi(z) x dz$, weak monotonicity of $\psi$ on $[0,1]$, and $\psi(1) = p_S$ imply that $\psi(0) > \psi(1)$ if $w > 0$ and $\psi(0) < \psi(1)$ if $w < 0$. QED

Receipts for a seller in the presence of an external cost are regressive in the sense that the price for the first unit is greater than the price for the last unit, because this affords the greater price for a buyer in an efficient equilibrium compared to an equilibrium absent policy. In contrast, payment for a seller in the presence of an external benefit is progressive, so that the uniform buyer price is suppressed to the efficient level. Payment for a buyer in the presence of an external benefit is regressive, because this implicitly subsidizes the good while maintaining a constant per unit price for a seller. Proposition 3 holds for general integrable payment schemes, not only one giving rise to the two-block pricing schedule, and is summarized in Table 2.

I conclude this section with categorizing optimal policy.

(i) **Two-block Pricing.** This was examined in this section. Within the class of two-block tariff, $e \in (0,1)$ in Propositions 1 and 2 can be varied yielding multiple policies. A simple way of implementing an optimum is making buyer-sided policy when there is an external cost and making seller-sided policy when there is an external benefit. Set $\tau = p_B > 0$ to exclude from the surcharge the initial portion $e = (p_B - p_S)/p_B$ when there is an external cost, and $\tau = -p_S < 0$ to exclude from the surcharge the initial portion $e = (p_S - p_B)/p_S$ when there is an external benefit. This way, the buyers spent nothing for this initial portion in return for getting a good that creates an external cost, and the sellers receive nothing for
this initial portion in return for delivering a good that creates an external benefit.

(ii) Multiple-block Pricing. As an extension to two-block tariff, the payment function $\Lambda$ or $\Psi$ may be piecewise linear with multiple finitely many kinks. In the limit, as kinks become close to each other, payment schemes with continuously varying values exist. Practically, there is no problem with this continuity because the assumption that the good is divisible is relevant to both discretely-valued and continuously-valued payment schemes. Energy in most cases seems a divisible good.

(iii) Two-part Pricing. Buyers of a good creating an external cost receive a lump-sum rebate, which leads to an average price sellers receive below the marginal price buyers pay. Buyers of a good creating an external benefit pay a lump-sum amount, which creates an average price sellers receive above the marginal price buyers pay. This rebate is entangled with a transaction in the market prescribed by government, whereas the lump-sum (or any other form of) rebate of a Pigou tax is conducted by government.

I pause to note an important policy design issue. Setting the exclusion/inclusion portion $e$, or similarly defined portions for a piecewise linear payment function, does not guarantee that agents consider the marginal price of the payment scheme at the optimal allocation. Setting the absolute kink point $x_e$ in a two-block pricing schedule, or multiple kink points in an $N$-block tariff, $N > 2$, will be required. Alternatively, a two-part tariff induces marginal price of the payment scheme at the optimal allocation.

The general equilibrium theory presented can be extended to multiple imperfectly substitutable goods without further insights. Instead, Section 3 considers perfectly substitutable goods, of which at least one creates an external cost and one is externality-free, and buyer-sided policy, both which helps analyzing the current practice of renewable energy support policies with a motive to control pollution.\(^{18}\)

\(^{18}\) However, an optimum can be implemented with a seller-sided policy.
3. The role of specific tariff and implicit tax for public good provision

The first scheme supporting the use of particular technology listed in the introduction is now presented as it will be embedded in a model below. Payment by consumers \( \Lambda(x^F + x^G) \) for fossil energy \( x^F \) and green energy \( x^G \) equals the sum of unit receipts times specific output,

\[
\Lambda(x^F + x^G) = p^F x^F + p^G x^G.
\]

Fossil energy creates an external cost in the model. It is helpful to define the unit payment 
\( \lambda = p - \lambda^*(1) + \lambda^* \) using the surcharge function \( \lambda^* : [0, 1] \rightarrow R_+ \) so that

\[
p - \lambda^*(1) + \int_0^1 \lambda^*(z) dz = (1 - \eta)p^F + \eta p^G
\]

given market portion of green energy \( \eta \), and marginal price \( p \) and base price \( p - \lambda^*(1) \) for a buyer. The additional feature of the policy now, which underlies current practice of green energy support, comprises a rule that distributes the surcharge to green energy producers,

\[
\int_0^1 \lambda^*(z) dz = \eta p^G.
\]

The implicit tax funds an explicit specific subsidy \( p^G \). The government may set the tariff \( p^G \) or the portion \( \eta \) of type-G output aimed at supporting the use of technology G.

**Uniform Fee.**—A special case of the policy is a uniform surcharge \( \tau \) levied on all output, \( \tau = \eta p^G \), in the legal requirement (16) with \( \lambda^* = \tau \). The surcharge is included in the unit price \( p \). Funds equal revenue, if the economy-wide spending on type-F output equals the price net of the surcharge, \( (1 - \eta)p^F = p - \tau \), given none of the demand is excluded from paying the surcharge. This results from the budget balance condition (15) which reads

\[
p = (1 - \eta)p^F + \eta p^G.
\]
Nonuniform Fee.—A two-block pricing schedule with surcharge $\tau$ and portion of demand excluded from paying the surcharge $e$ implies that the unit price $p^G$ for type-$G$ output is funded according to the legal requirement $(1 - e)\tau = \eta p^G$. Buyers are exempted from paying a surcharge for the initial mass $e \in [0, 1)$ of output, $\lambda^* = 0$ on $[0, e]$. The base price $p - \tau$ exhausts unit receipts of type-$F$ output, $(1 - \eta)p^F = p - \tau$, as with uniform fee. The weighted average of unit receipts is the maximum unit price $p$ in the budget balance condition

$$p = (1 - \eta)p^F + \frac{\eta}{1 - e}p^G. \quad (18)$$

Presumably, the price $p^G$ making green energy competitive with fossil energy at Pareto optimal scale is greater than the price of fossil energy $p^F$. This motivates environmental policy. Now subsidizing expensive technology raises the price of output. An increase in the price reduces fossil energy use and makes the deployment of more expensive cleaner technology economically viable to private agents, with the outcome of emissions reduction per unit of output.

In practice, households pay an equal surcharge per unit of energy consumption to stimulate investment in renewable energy (Uniform Fee). This poses a problem, since an optimum in the presence of an external cost from using fossil energy cannot be implemented using such a payment scheme. I will show this, and subsequently I will characterize policy that overcomes this problem (Nonuniform Fee) in a dynamic climate-economy model.

3.1. A theory of energy use

This subsection uses a model to discuss taxes on fossil energy and subsidies to renewable energy. Time is discrete and continues forever, $t \in \{0, 1, \ldots \}$.

The Environment.—(i) Preferences. Households on the unit continuum have identical
preferences for consumption $x$ and environmental quality $y$ expressed by the utility function

$$\sum_{t=0}^{\infty} \beta^t U(x_t, y_t)$$

where the period utility function $U : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$ has the same standard properties as in Section 3 and in addition is increasing and strictly concave in $x$: $\partial U / \partial x > 0$ and $\lambda U(x_1, y) + (1 - \lambda) U(x_2, y) < U(\lambda x_1 + (1 - \lambda) x_2, y)$ all $x_1, x_2 > 0$, $x_1 \neq x_2$, $\lambda \in (0, 1)$ and $y \in \mathbb{R}$. Households are endowed with one unit of time.

(ii) Carbon Cycle. The public good is reduced by carbon emissions $E$ along a transformation frontier expressed by

$$y_t = \Phi(E_{t-T}, \ldots, E_{t-1}, y_{t-T}, t)$$

so that emissions decrease environmental quality, $\partial \Phi(\cdot, t+j)/\partial E_t < 0$ all $j > 0$. Emissions $E = \sum_i \varphi^B x_i^B + \varphi^K x_i^K$ occur when brown energy $x_s^B$ or black energy $x_s^K > 0$ is produced, some sector $s$. Here $\varphi^B$ and $\varphi^K$ express the emission intensity of brown energy and black energy, respectively. Green energy $x_s^G$ does not create emissions.\(^{19}\)

(iii) Production Structure. Production of the consumption good is undertaken in accordance with

$$\bar{y} = G(k_0, e_1, e_2, \ldots, e_S, n_0, y)$$

where $G : \mathbb{R}^{S+2} \to \mathbb{R}$ is continuously differentiable and increasing, $\partial G / \partial k > 0$, $\partial G / \partial e_s > 0$, $s = 1, 2, \ldots, S$, $\partial G / \partial n_0 > 0$, and $\partial G / \partial y \geq 0$ are continuous. Capital is required to produce output $\bar{y}$ and energy $(e_1, e_2, \ldots, e_S)$. Capital is used in $N \geq 1$ periods after emission from investment, such as from primary steel making, are assumed to be captured and stored, or fully avoided through the use of technology such as electronic arch furnace or direct reduction in steel-making and increased use of other building materials.

\(^{19}\)Explicit carbon capture $\varphi^B_s < 0$, some technology $j$, may be straightforwardly modeled. For simplicity, emissions from investment, such as from primary steel making, are assumed to be captured and stored, or fully avoided through the use of technology such as electronic arch furnace or direct reduction in steel-making and increased use of other building materials.
installation,

\[ k_{0,t} = \sum_{\tau=1}^{N} q_{0,t-\tau} i_{0,t-\tau} \tag{21} \]

and

\[ k_{s,t} = \sum_{\tau=1}^{N} q_{s,t-\tau} i_{s,t-\tau} \], \quad s \in \{1, 2, \ldots, S\}, \quad j \in \{B, G, K\}. \tag{22} \]

Fossil energy capital is less costly than green energy capital, \(q^j_s > q^G_s > 0\), \(s \in \{1, 2, \ldots, S\}\), \(j \in \{B, K\}\). The intermediate goods brown, green, and black energy are produced according to \(x^j_s = F^j_s(k^j_s, n^j_s), s \in \{1, 2, \ldots, S\}, j \in \{B, G, K\}\). The production function \(F^j_s\) is continuously differentiable, increasing in capital, and nondecreasing in labor, \(\frac{\partial F^j_s}{\partial k^j_s} > 0\) and \(\frac{\partial F^j_s}{\partial n^j_s} \geq 0\) are continuous in capital and labor. The technologies \(B, G\), and \(K\), produce perfectly substitutable output so that

\[ e^j_s = \sum_{j \in \{B, G, K\}} F^j_s(k^j_s, n^j_s), \quad s \in \{1, 2, \ldots, S\}. \tag{23} \]

Infinitely elastic substitution of specific energy in producing output is assumed to suit the context of energy generation with factors utilizing different resources, for example, solar radiation and coal. Brown and black energy deplete the resource stocks \(S^B(0) \in (0, \infty)\) and \(S^K(0) \in (0, \infty)\),

\[ S_{t+1}^j = S_t^j - \sum_s x^j_s, \quad j \in \{B, K\}. \tag{24} \]

Output can be used for consumption \(x\) and expenditure for investment in capital of consumption good production \(\phi^j_0(i_0)\), and for investment in capital of energy production \(\phi^j_s(i^j_s)\), \(j \in \{B, G, K\}, s \in \{1, 2, \ldots, S\}\),

\[ x + \phi^j_0(i_0) + s \sum_{j \in \{B, G, K\}} \phi^j_s(i^j_s) = \bar{y} \tag{25} \]
where the cost functions $\phi_0$ and $\phi_s^j(i)$ are increasing and strictly convex and satisfy $\phi_0(0) = \phi_s^j(0) = 0$, $j \in \{B, G, K\}$, $s \in \{1, 2, \ldots, S\}$. Labor input in the production of the consumption good and energy exhausts labor supply,

$$n_0 + \sum_{s} \sum_{j \in \{B, G, K\}} n_s^j = 1. \tag{26}$$

Define energy input $e = (e_1, e_2, \ldots, e_S)$, energy output $\bar{x} = (x_s^j)_{s,j}$, investment $i = (i_0, (i_s^j)_{s,j})$, and labor $n = (n_0, (n_s^j)_{s,j})$.

(iv) Climate Feedback. The marginal utility or marginal output of the quality of the natural environment is positive, $\partial U / \partial y > 0$ or $\partial G / \partial y > 0$, at an optimum.

The Planning Problem.—A social planner chooses paths of consumption $(x_t)$, energy input $(e_t)$, specific energy output $(\bar{x}_t)$, investment $(i_t)$, and labor $(n_t)$, so as to maximize utility (19) subject to (20), (21), (22), (23), (24), (25), and (26).\(^{20}\)

This problem has necessary first-order conditions that I will use in analyzing implicit taxes with the marginal cost of investment $c_s^j \equiv (\partial \phi_s^j / \partial i_s^j) / q_s^j$, $j \in \{B, G, K\}$ and letting $\mu_t^j$ be the Lagrange multiplier on the constraint (24). It is helpful to define the social cost of carbon

$$\theta_t \equiv \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \left[ \frac{U_{y,\tau}}{U_{x,\tau}} + \frac{U_{x,\tau}}{U_{x,\tau}} \left( \frac{\partial G}{\partial y} \right)_\tau \right] \left( \frac{\partial \Phi_{\tau}}{\partial E_{\tau}} \right) \tag{27}$$

which makes up the wedge between private marginal rate of substitution and private marginal rate of transformation of date $t$ consumption and emissions.\(^{21}\) Investment in capital used

\(^{20}\)Welfare set equal to the utility with the argument of aggregate consumption quantity, welfare equal to the utility of a representative household, or welfare defined as the sum of equally weighted individual utility each with the argument of individual consumption quantity as the planner’s objective yield the same allocation, because there is a unit mass of households.

\(^{21}\)The wedge will be defined shortly. I abstract from effects of carbon on the production of energy. Considering such effect would add the term $\sum_s (\partial G / \partial e_s)_\tau \sum_j (\partial F_s^j / \partial y)_\tau$ to $(\partial G / \partial y)_\tau$ in (27).
in energy production satisfies
\[
\sum_{\tau=1}^{N} \beta^{\tau - t} \left[ \frac{U_{x,\tau}}{U_{x,t}} \left( \frac{\partial F_{s,j}^{G}}{\partial k_{s,j}^{G}} \right)_{\tau} \left( \frac{\partial G}{\partial e_{s}} \right)_{\tau} - \frac{\mu_{s,j}^{G}}{U_{x,\tau}} \right] \\
- \frac{U_{x,\tau}}{U_{x,t}} \left( \frac{\partial F_{s,j}^{G}}{\partial k_{s,j}^{G}} \right)_{\tau} \varphi^{j} \theta_{\tau} \right] \leq c_{s,t,j}^{G},
\]
\[
= \text{if } i_{s,t,j} > 0, \quad j \in \{B, K\},
\]
\[
\sum_{\tau=1}^{N} \beta^{\tau - t} \left[ \frac{U_{x,\tau}}{U_{x,t}} \left( \frac{\partial F_{s,j}^{G}}{\partial k_{s,j}^{G}} \right)_{\tau} \left( \frac{\partial G}{\partial e_{s}} \right)_{\tau} \right] \leq c_{s,t,j}^{G},
\]
\[
= \text{if } i_{s,t}^{G} > 0.
\]

The wedge of technology \( j \in \{B, K\} \) between private marginal rate of substitution and private marginal rate of transformation of date \( t \) consumption and emissions is defined as the discounted sum of the first term in the outer brackets on the left side, the marginal benefit of consuming, less the term on the right side in (28), the marginal cost, provided that investment occurs, \( i_{s,j} > 0 \). The wedge equals the discounted sum of terms \( (\partial F_{s,j}^{G}/\partial k_{s,j}^{G})\varphi^{j} \theta \).

Conditions (28)-(29) have utilized the choice of consumption and the choice of energy input. In addition,
\[
\beta \mu_{t+1,j}^{G} = \mu_{t,j}^{G}
\]
holds if some but not all of the resource \( S^{j} \) is used at date \( t \).

As a prerequisite for characterizing (i) the outcome of some implicit taxes and (ii) optimal implicit taxes themselves, it is helpful to show that there is simultaneous use of fossil and renewable energy in the optimum.

**Remark 1.** *In the social optimum, there is a phase with fossil energy use and green energy use, \( 0 < x_{s}^{G} < e_{s} \), which is succeeded by a phase with exclusive use of green energy, \( 0 < x_{s}^{G} = e_{s} \), and a unique balanced growth path will be approached in the long-run.*
This can be seen as follows. i. Phases. Black and brown energy will be used and their resources will be exhausted, because fossil energy is less costly than green energy, $q_s^j > q_s^G$, $s \in \{1, 2, \ldots, S\}, j \in \{B, K\}$. Both fossil and green energy will be used during a phase of fossil energy use, as the marginal benefit of investment is finite and the marginal cost of investment is increasing and becomes zero as investment approaches zero. Thus, fossil and green energy are simultaneously used, if there are black or brown energy resources, $S^B(t) > 0$ or $S^K(t) > 0$, and only green energy will be used, when the black and brown energy resources are exhausted, $S^B(t) = S^K(t) = 0$. Black and brown energy resources will be exhausted in finite time, because investment in capital of black and brown energy technology forever, which would demand that (30) holds and the shadow price $\mu^j$ goes to infinity, is inconsistent with the green growth potential, which means that the marginal utility of consumption $U_x$ can be made decreasing through growth in green energy (see ii).

ii. Existence and Uniqueness of Balanced Growth Path. In notes available upon request from the author, there is a green growth path, which is a saddle point in detrended variables, when the environmental quality is constant. iii. Approach of Balanced Growth Path. This can be derived with absorption of carbon that continues forever so that atmospheric carbon will reach levels that do not affect the marginal utility of consumption and the production function approximately in the long-run or from some date onward or cumulative carbon that does not return to such levels.

Equilibrium.—The above description of the economy is now completed with a competitive equilibrium in which government policy can implement an optimum. An equilibrium with a sequence of markets given initial conditions will be defined.\textsuperscript{22} Equilibrium is with perfect foresight about production technology including the carbon cycle and effects from climate on society.\textsuperscript{23} Each household chooses consumption $x_t$ and bond holdings $b_{t+1}$ so as to maximize

\textsuperscript{22}For an Arrow-Debreu equilibrium with the climate, see Golosov et al. (2014).

\textsuperscript{23}The equilibrium can be straightforwardly formulated with agents taking rational expectations with respect to exogenous stochastic variables, so that an allocation and price system is the set of contingent time paths of quantities and prices. The first-order necessary conditions of such an equilibrium seemingly do not allow to relate prices of specific energy in the way it will be useful to derive my results.
utility in solving

\[ V_t = \max_{x_t, b_{t+1}} \{ U(x_t, y_t) + \beta V_{t+1} \} \]  \hspace{1cm} \text{P(1)}

subject to the sequence of budget constraints

\[ x_t + \tilde{q}_t b_{t+1} = \tilde{q}_t b_t + w_t \]

taking as given the bond price \( \tilde{q}_t \) and wage \( w_t \).

A representative firm on the unit continuum demands energy \( e_t \), supplies energy \( \tilde{x}_t \) and nonrenewable resources \( (S_t^B - S_{t+1}^B, S_t^K - S_{t+1}^K) \), invests the quantities \( i_t \), and hires labor \( n_t \), so as to maximize profit in solving

\[
Q_t = \max_{e_{t,\tilde{e}_t}, S_{t^B} - S_{t+1}^B, S_{t^K} - S_{t+1}^K, n_t, \tilde{e}_t} \left\{ \right.
\]
\[
G(i_{0,t-N}, \ldots, i_{0,t-1}, e_{1,t}, e_{2,t}, \ldots, e_{S,t}, n_{0,t}, y_t) - \sum_s \Lambda_s(e_{s,t})
\]
\[
+ \sum_s \sum_j p_s^j F_s^j(i_{s,t-N}^j, \ldots, i_{s,t-1}^j, n_{s,t}^j)
\]
\[
- \sum_{j \in \{B,K\}} \psi_t^j \sum_s F_s^j(i_{s,t-N}^j, \ldots, i_{s,t-1}^j, n_{s,t}^j)
\]
\[
- \phi_0(i_{0,t}) - \sum_s \sum_j \phi_s^j(i_{s,t}^j) - w_t \left[ n_{0,t} + \sum_s \sum_j n_{s,t}^j \right]
\]
\[
+ \sum_j \psi_t^j [S_t^j - S_{t+1}^j] + \frac{\tilde{q}_t}{q_{t+1}} Q_{t+1} \left\} \right.
\]  \hspace{1cm} \text{P(2)}

taking as given the pricing functions \( \Lambda_s \), specific energy prices \( p_s^B \), \( p_s^K \), and \( p_s^G \), \( s \in \{1, 2, \ldots, S\} \), nonrenewable resource prices \( \psi_s^B \) and \( \psi_s^K \), and the wage \( w \). Define \( \Lambda = (\Lambda_s)_s \) and \( \tilde{p} = (p_s^B, p_s^K, p_s^G)_s \).

Again, the government policy is exogenous. An equilibrium is given by paths of consumption \( (x_t) \), energy \( (e_t, \tilde{e}_t) \), labor \( (n_t) \), investment \( (i_t) \), bond price \( (\tilde{q}_t) \), wage \( (w_t) \), buyer pricing functions \( (\Lambda_t) \), energy prices \( (\tilde{p}_t) \), and resource prices \( (\psi_t^B, \psi_t^K) \) such that households solve problem P(1) taking as given the price path \( (\tilde{q}_t, w_t) \), firms solve problem P(2).
taking as given the paths of pricing functions \((\Lambda_t)\) and prices \((\tilde{p}_t, \psi_t^B, \psi_t^K, w_t)\), the resource constraint for the consumption good holds in the form

\[
x_t + \phi_0(i_{0,t}) + \sum_s \sum_j \phi_s^j(i_{s,t}^j) = G(i_{0, t-N}, \ldots, i_{0, t-1}, e_{1,t}, e_{2,t}, \ldots, e_{S,t}, n_{0,t}, y_t),
\]

the markets for energy clear,

\[
e_s = \sum_{j \in \{B,G,K\}} x_s^j, \quad j \in \{B, G, K\},
\]

buyer payments equal seller receipts, \(\Lambda_s(e_s) = \sum_{j \in \{B,G,K\}} \psi_s^j x_s^j, \quad s \in \{1, 2, \ldots, S\}\), and \(Q = \tilde{q} b\).\(^{24}\)

To continue, necessary first-order conditions with respect to investment will be stated:

\[
\begin{align*}
\sum_{\tau=t+1}^{t+N} \beta^{\tau-t} \left( \frac{U_{x,\tau}}{U_{x,t}} \right) \left( \frac{\partial F_s^i}{\partial k_s^j} \right)_\tau [p_s,\tau^j - \psi_s^j] \\
\leq c_{s,t}^j, \quad \text{if} \ i_{s,t}^j > 0, \quad j \in \{B, K\}, \quad (31)
\end{align*}
\]

\[
\sum_{\tau=t+1}^{t+N} \beta^{\tau-t} \left( \frac{U_{x,\tau}}{U_{x,t}} \right) \left( \frac{\partial F_s^G}{\partial k_s^G} \right)_\tau p_s,\tau^G \\
\leq c_{s,t}^G, \quad \text{if} \ i_{s,t}^G > 0. \quad (32)
\]

Conditions (31)-(32) have utilized the household equilibrium condition for asset holdings \(\beta U_{x,t+1} R_{t+1} = U_{x,t}\) with interest rate \(R_{t+1} = \tilde{q}_{t+1}/\tilde{q}_t\) and the firm equilibrium condition for energy demand \(\partial G/\partial e_s = p_s\) with the marginal buyer price \(p_s = \partial \Lambda_s/\partial e_s\). In addition,

\(^{24}\)Specific energy prices for black and brown energy \(p_s^B\) and \(p_s^K\), respectively, form the fossil energy price \(p_s^F = p_s^B x_s^B/(x_s^B + x_s^K) + p_s^K x_s^K/(x_s^B + x_s^K)\) used above.
firms choose the resource stocks so that Hotelling’s rule
\[
\frac{1}{R_{t+1}} \psi_{t+1}^j = \psi_t^j
\]  
holds if some but not all of the resource \( S_t^j \) is used at date \( t \).

It is helpful to define the market share of specific energy in a given sector.

\textbf{Definition 2.} The market share of energy type \( j \in \{B,G,K\} \) in sector \( s \in \{1, 2, \ldots, S\} \) is given by \( \eta_s^j \equiv x_s^j / \sum_{j \in \{B,G,K\}} x_s^j \).

\textit{Inefficient Allocation with Uniform Surcharge.}—The basic idea of funding green energy support entirely on the market will now be scrutinized when consumers, which are firms in the model, pay a uniform surcharge on all energy output in a given sector.

\textbf{Proposition 4.} Fossil and green energy are overused relative to the optimum (i) if the optimal relative use of green energy \( \eta_s^G \in (0, 1) \), and optimal capital \( k_0 \) and labor \( (n_0, n_1, \ldots, n_S) \) at some date \( t > 0 \) are implemented by a uniform surcharge funding a green energy tariff, \( \tau_s = \eta_s^G p_s^G \) at the same date, and (ii) given optimal allocation at all other dates.

\textbf{Proof of Proposition 4}

i. A uniform surcharge does not implement an optimum when fossil energy is used, \( \eta_s^B + \eta_s^K > 0 \). In an efficient equilibrium with fossil energy use of type \( j \in \{B,K\} \), the buyers’ marginal price of energy is greater than the seller’s price of energy creating an external cost, \( p_s > p_s^j \), using the social optimality condition (28) and equilibrium condition (31). A uniform surcharge \( \tau_s \) is levied on all demand, \( e_s = 0 \). Now, for \( e_s = 0 \), the left side is greater than the right side in the budget balance condition (17), given by \( p_s - \tau_s e_s = \sum_{j \in \{B,G,K\}} \eta_s^j p_s^j \) using \( \eta_s^G \equiv x_s^G / \sum_{j \in \{B,G,K\}} x_s^j \), a contradiction. ii. In an equilibrium with uniform surcharge, the buyer price of energy relative to the seller unit receipt of green energy, \( p_s / p_s^G \), is smaller than in an efficient equilibrium to accommodate budget balance. Suppose that energy use is lower than optimally. This means that consumption at date
$t - 1$ is greater and consumption at date $t$ is smaller compared to an optimum. Thus, the interest rate $\tilde{q}_t/\tilde{q}_{t-1}$ is smaller. In addition, the marginal product relative to the marginal cost of energy production capital will be greater, and the marginal product of energy will be weakly greater. All this is inconsistent with the necessary conditions (32) and $\partial G/\partial e_s = p_s$ and the lower buyer price of energy relative to the seller price of green energy. Thus, energy use is greater than optimally. QED

An equilibrium with uniform fee and optimal relative use of energy technologies exhibits too much output of both emissions-intensive and emissions-free technology compared to the optimum, because the budget balance condition (17) distorts the seller unit receipt of green energy above the buyer price of energy.$^{25}$

**Implementation of Optimum.**—Policy with implicit taxes on brown and black energy implementing an optimum will now be characterized. Such policy overcomes the problem of overused fossil and green energy relative to the optimum under a uniform surcharge which is redistributed to green energy producers identified in Proposition 4. The social optimality conditions (28)-(29) and the private equilibrium conditions (31)-(32) coincide if energy is valued according to

$$p_s - \varphi^j \theta = p_s^j$$

and fossil resources are valued with $\psi^j = \mu^j/U_x$, $j \in \{B, K\}$. The marginal buyer price of energy $p$ is corrected above the seller unit receipt $p^j$ of energy type $j$ by the external cost $\varphi^j \theta$ in terms of energy—with indexing in each sector $s$. I will use (34) and focus on a two-block pricing schedule.

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$^{25}$In a static model with a tradable factor used in energy production and in another sector, the result in Proposition 4 is easily shown without the qualification of optimal use of capital in the other sector, labor in all sectors, and optimal resource use at the other dates. In an economy in which the multiple technologies of discussion only use a nontradable factor, a uniform surcharge trivially implements an optimum. Agents do not evaluate this factor's use against the use of any other good.
Proposition 5. A surcharge and exemption rate \((\tau_s, e_s)\) satisfying

\[
\tau_s e_s = \left[ \sum_{j \in \{B,G,K\}} x_{s}^B \phi_{s}^B x_{s}^j + \sum_{j \in \{B,G,K\}} x_{s}^K \phi_{s}^K x_{s}^j \right] \theta \in [0, \tau_s)
\]

if fossil energy is used, \(x_{s}^B + x_{s}^K > 0\), and in addition the rule \(p_{s}^K - p_{s}^B = (\phi_{s}^B - \phi_{s}^K)\theta\), if both brown and black energy are used, \(x_{s}^B > 0\) and \(x_{s}^K > 0\), in sector \(s \in \{1, 2, \ldots, S\}\), implement an optimum.

Proof of Proposition 5

Equation (34) and the budget balance condition (17) imply that

\[
\begin{align*}
p_{s} - \tau_s e_s &= \frac{x_{s}^B}{\sum_{j \in \{B,G,K\}} x_{s}^j} [p_{s} - \phi_{s}^B \theta] \\
&\quad + \frac{x_{s}^K}{\sum_{j \in \{B,G,K\}} x_{s}^j} [p_{s} - \phi_{s}^K \theta] \\
&\quad + \frac{x_{s}^G}{\sum_{j \in \{B,G,K\}} x_{s}^j} p_{s}.
\end{align*}
\]

Rewriting this condition yields the desired result. QED

The product of surcharge rate and exclusion portion sums the weighted social cost of polluting, where the weights are the output shares of externality-generating energy. Clearly, policy in Proposition 5 is similar to policy in Proposition 3, if one of brown or black energy is unused, \(x_{s}^B = 0\) or \(x_{s}^K = 0\), at an optimum.\(^{26}\) The policy rates of surcharge and exemption rate in Proposition 5 admit inefficient equilibria if the emission intensities of brown and black energy differ, \(\phi_{s}^B \neq \phi_{s}^K\), unless another condition is stated that directs revenue to brown and black energy if both of them are used at an optimum, because receipts are pooled for specific perfectly substitutable outputs. For example, \(p_{s}^K - p_{s}^B = (\phi_{s}^B - \phi_{s}^K)\theta\) qualifies as such a condition, ensuring efficiency.\(^{27}\) Different emission intensities mean the

\(^{26}\) The contemporaneous ‘wedge’ between buyer price \(p_{s}^j\) and seller unit receipt \(p_{s}\) denoted \(w_{s}^j\) equals \(\phi_{s}^j \theta\) for technology \(j \in \{B, K\}\). Precisely, only the discounted sum of the terms \(w_{s}^j\) is a wedge of private marginal benefit and cost.

\(^{27}\) The specific surcharges and exemption rates \((\tau_{s}^B, e_{s}^B, \tau_{s}^K, e_{s}^K)\) satisfying \(\tau_{s}^j e_{s}^j = x_{s}^j \phi_{s}^j \theta / (x_{s}^K + \ldots)\)
technology with smaller emission intensity has a greater market value. The ad valorem tax rates on specific energy implementing an optimum with simultaneous use differ because of this reason—they differ when the market values $p_s^B$ and $p_s^K$ are unequal, and these are unequal because of different emission intensities of technologies $B$ and $K$.

In one case, a uniform surcharge on all types of energy consumed implements an optimum. Proposition 4 has hinted on the case when only green energy is used. This result is stated now together with the result used to prove Proposition 4 that uniform surcharges do not implement an optimum with fossil energy use.

**Corollary 1.** A uniform fee $\tau_s$ levied on energy fully funding green energy receipts, $\tau_s(x_s^G + x_s^B + x_s^K) = p_s^G x_s^G$, implements an optimum if and only if green energy is exclusively used, $x_s^B = x_s^K = 0$, in sector $s \in \{1, 2, \ldots, S\}$.

**Proof of Corollary 1**

"if." Set $x_s^B = x_s^K = 0$ in the requirement given to obtain $\tau_s = p_s^G$. "only if." This statement is the contrapositive to the first result in the proof of Proposition 4. QED

Green energy does not create an externality here. Thus an optimum that is prescribed by exclusive use of this type of energy can be implemented with a uniform surcharge in the referring sector. The surcharge fully funds green energy revenue, so that this is all consumers pay for energy, that is, the fee equals the price, $\tau_s = p_s$.

A percentage surcharge can be expressed along with an exemption rate of the policy in Proposition 5.

**Corollary 2.** The percentage surcharge $\tau_s/(p_s - \tau_s) = (\tau_s e_s)/[(\partial G/\partial e_s) e_s - (\tau_s e_s)] > 0$ and the fraction of energy demand exempted from paying the surcharge $e_s \in ((\tau_s e_s)/(\partial G/\partial e_s), 1)$ with $(\tau_s e_s)$ given in Proposition 5, if fossil energy is used, $x_s^B + x_s^K > 0$, and in addition the rule $p_s^K - p_s^B = (\varphi^B - \varphi^K)\theta$ if both brown and black energy are used, $x_s^B > 0$ and $x_s^K > 0$, in sector $s \in \{1, 2, \ldots, S\}$, implement a Pareto optimum.
Proof of Corollary 2

The result follows by rewriting the expression for \((\tau_s e_s)\) and using \(\partial G / \partial e_s = p_s\). The percentage surcharge is positive, because \(p_s = (\partial G / \partial e_s)\) and \(p_s > \tau_s\) holds when \(e_s > (\tau_s e_s) / p_s\). QED

Among those two-block pricing schedules in Proposition 5 implementing optimal brown or black energy use, one tariff fully funds green energy revenue. This tariff—which inherits the basic idea of funding green energy support entirely on the market, as in a feed-in tariff for solar or wind electricity in many countries—will now be formalized. The surcharge is expressed as fraction of the base price.

**Proposition 6.** The percentage surcharge

\[
\frac{\tau_s}{p_s - \tau_s} = \frac{1 - \xi_s}{\xi_s} > 0
\]

and the fraction of energy demand exempted from paying the surcharge

\[
e_s = \frac{1 - \eta_s^G - \xi_s}{1 - \xi_s} \in (0, 1)
\]

fully-funding green energy receipts, \((1 - e_s)\tau_s = \eta_s^G p_s^G > 0\), if fossil energy is used, \(x_s^B + x_s^K > 0\), and in addition the rule \(p_s^K - p_s^B = (\varphi^B - \varphi^K)\theta\) if both brown and black energy are used, \(x_s^B > 0\) and \(x_s^K > 0\), imposed upon the market in sector \(s \in \{1, 2, \ldots, S\}\), implement a Pareto optimum, where \(\xi_s \equiv \eta_s^B \left[1 - \varphi^B \theta / (\partial G / \partial e_s)\right] + \eta_s^K \left[1 - \varphi^K \theta / (\partial G / \partial e_s)\right]\).

Proof of Proposition 6

The result is obtained by solving the two equations \((1 - e)\tau = \eta p^G\) and the budget balance condition (18) for the two unknowns \(\tau\) and \(e\), and utilizing \(p = p^G\), where the subscript for the sector \(s \in \{1, 2, \ldots, S\}\) is omitted. The numerator of \(\tau / (p - \tau)\) and the denominator of \(e\) are positive, if the numerator of \(e\) is positive. The numerator of \(e\) is positive, because \(1 - \eta)p > \eta^B(p - \varphi^B \theta) + \eta^K(p - \varphi^K \theta)\) is equivalent to \(0 < (\eta^B \varphi^B + \eta^K \varphi^K)\theta\), which is
true. The exemption rate is smaller than one given positive numerator and denominator. Therefore, $\tau/(p - \tau) > 0$ and $e \in (0, 1)$. QED

The policy in Proposition 6 exploits the multiplicity of optimal policies and the fact that within the range of optimal policy levels an implicit tax on fossil energy provides sufficient funds to cover green energy revenue.

The expressions for policy rates in Proposition 6 are helpful to determine their path dependent on the market portion of green energy. The percentage surcharge $\tau_s/(p_s - \tau_s)$ directly increases, while the exclusion portion from the surcharge $e$ directly decreases, in the market share of green energy $(1 - \eta_s B - \eta_s K)$. However, both policy instruments percentage surcharge and exclusion portion are positively related to the relative market value of green energy, $p_s^G/p_s^B$ and $p_s^G/p_s^K$. This results because $\xi_s = \eta_s B (p_s^B/p_s^G) + \eta_s K (p_s^K/p_s^G)$ in Proposition 6 along a path with fossil energy use, $\eta_s B + \eta_s K > 0$. How do the market values depend on the market share of green energy?

3.2. Specializing the economy

The following assumptions allow characterizing policy in Proposition 6 in terms of the supply side of the economy.

**Assumption 3.** *Capital used in energy production fully depreciates, $N = 1$.***

**Assumption 4.** *The production function of energy is linear in capital, $F_s^j = k_s^j$.***

**Assumption 5.** *The cost of investment in capital used in energy production is quadratic, $\phi(i) = (\xi q/2)(i/k)^2 k$.***

Combining the necessary equilibrium condition (31) under Assumptions 3-5 and Hotelling’s rule (33) implies a first-order difference equation in the relative market value of fossil energy
given investment occurs in technology $j \in \{B, K\}$ at dates $t$ and $t+1$, that is, $i_{s,t,j} > 0$ and $i_{s,t+1,j} > 0$, where $c_{s,t}^{j} = q_{s,t}^{j}$ and

$$r_{t+1,j} = \left( \frac{1 - \eta_{s,t}^{B} - \eta_{s,t}^{K}}{\eta_{s,t}^{j}} \right)_{t+1} / \left( \frac{1 - \eta_{s,t}^{B} - \eta_{s,t}^{K}}{\eta_{s,t}^{j}} \right)_{t}.$$  

Equation (35) reveals that the relative market value of brown or black energy $p_{s,t}^{B}/p_{s,t}^{G}$ or $p_{s,t}^{K}/p_{s,t}^{G}$ increases when brown or black energy, respectively, is used relatively less compared to green energy. Proposition 6 and equation (35) suggest a positive relationship between each policy rate $\tau_{s}/(p_{s} - \tau_{s})$ and $e_{s}$, which are set together, and the portion of green energy in the market.

The path of the policy $(\tau/(p - \tau), e)$ is thus shaped much like the path of the carbon tax $\tilde{\tau} = \theta$. Condition (34) implies that the carbon tax increases if the relative market value of green energy increases, thus along a path with increasing portion of green energy in the market.

**Limitation.**—Implementing an optimum with the tax and subsidy policies may require cooperation among multiple policy-setting governments. First, as with voluntary Pigouvian taxes, voluntary implicit taxes are subject to free-riding, which may result in a too low number of cooperating policy jurisdictions. Consider an interregional pollution externality such that emissions of any of two regions impact both regions, and policy is set in each region. Second, Pigouvian and implicit taxes will yield the same leakage. Consider a region which wants to control its emissions and that is affected by emissions created in other regions. Depending on the tradability of goods, there may be leakage increasing emissions.
4. Conclusion

This paper has characterized government policy involving pricing functions which implement the socially optimal allocation including of a privately produced public good. The class of policy is widely applicable, for external cost and external benefit. Some evaluations of nonlinear electricity pricing schedules, including Ito (2014), find that households instead respond to average prices, which speaks against the policy’s efficiency in settings with purchase of an externality-creating good by households and tradeable factors. However, I show that there is always a seller-sided policy to a buyer-sided policy implementing the same allocation. Firms in the role of buyers or sellers may respond to marginal prices. Implicit taxes provide scope for subsidies to renewable energy such as solar and wind energy fully-funded by fees in the form of price surcharges used around the world in schemes such as feed-in tariffs, green quotas, and renewable electricity purchasing agreements, to efficiently control the climate. In a dynamic general equilibrium model, the percentage surcharge and exemption rate from the surcharge for energy which is produced from fossil resources such as coal and petroleum and renewable energy sources, are characterized. Using (i) quadratic investment cost and (ii) full depreciation of capital used in energy production, optimal carbon policy is found that depends on the relative cost and market share of fossil and renewable energy, and whose form does not depend on the specification of the utility function, climate feedback on society, or the carbon cycle.

References


