Rational PhD Glut in Academic Factories

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Abstract
There exists a growing glut of PhDs in academia. In this study, we develop an occupational choice model with rational PhDs to explain the phenomenon. We distinguish two stages of market demand for PhDs. The first stage is the professors’ demand for doctoral students in knowledge production and a training wage is the equilibrium price. The second stage is the future market demand for professors in higher education. Given a low probability of being professors, students will choose academic career if the expected lifetime income, i.e., the training wage plus the discounted professor wage, meets the participation constraint. Consequently, an increase in doctoral student productivity or a growing demand for higher education will raise the expected lifetime income for doctoral students. In those cases, we will have more PhDs but a lower probability of being professors in the equilibrium.

JEL classification: J24, J44, I23
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1. Introduction

There exists oversupply of PhD graduates in some academic fields. A survey of Sauermann and Roach (2012) indicates that most PhD students in science fields prefer academic career in research and teaching. However, Cyranoski, Gilbert, Ledford, Nayar, and Yahia (2011) show that in 2006 only 15% of science PhDs in the US find a tenure-track position within six years after receiving their degrees. Comparing annually PhD graduates in science between 1998 and 2008, Cyranoski et al. (2011) also find that the total number has increased by nearly 40% among OECD countries. Therefore, given such a low rate for graduates to stay in preferred academic career, there exists a growing glut of PhDs in science fields. This phenomenon seems to contradict the traditional economic equilibrium hypothesis such as the engineer-scientist model of Arrow and Capron (1959) which suggests that given the market demand the oversupply should decrease overtime.

In this study, we develop an occupational choice model to justify the aforementioned phenomenon by applying the overlapping generations (OLG) structure of Samuelson (1958) in which doctoral students overlap professors. For doctoral students, our definition includes PhD students and postdocs as well. As Cantwell and Lee (2010) emphasize, postdocs are not faculties because their duty is to do research. A survey of Nerad and Cerny (1999) indicates that the main reasons for choosing postdocs are training and as a necessary step to tenured positions. In addition, Stephan and Ma (2005) find that financial benefits of pension and health have significantly positive contribution to the increased frequency and duration of postdocs in the US. Therefore, in our model, both PhD students and postdocs are defined as a general concept of doctoral students in the training stage and the financial benefit for them plays a crucial role as the equilibrium price.

Doctoral students in our model are assumed to have two distinct sources of lifetime income in their academic career. One is the certain wage for students as being a teaching or research assistant. The other is the expected wage as being a professor later, but students have some risks of finding tenured positions and the market condition for future wages is still unsure. Thus, given an uncertain future income and a low probability of being professors, students would choose to stay in academic career only in cases of having satisfactory wage level for the training stage. That is, their expected lifetime income from the student wage plus the discounted professor wage should meet the participation constraint of academic career. This decomposition of lifetime income provides a plausible explanation for the PhD oversupply in academia.
In fact, we distinguish two stages of market demand for PhDs. The first stage is the current professors’ demand for doctoral students as an input in knowledge production. The second stage is the future market demand for professors as providers of higher education. In contrast, the market supply of PhDs is only determined in the first stage, i.e., the number of students who decide to pursue doctoral degrees. As long as the expected lifetime income meets the participation constraint, we could have more PhD graduates than the market vacancies for professors in the second stage. In this aspect of sequentially demand, the market in the first stage actually do not have excess supply since the number of doctoral students just equals the quantity of demand in knowledge production in which professors are employers.

Previous studies of occupational choice focus on the process of expectation formation. Freeman (1975a, 1975b, 1976) has related studies on professional markets with cobweb-type models of adaptive expectation. In a cobweb model, the inelasticity of labor supply is due to lag responses to demand shocks because individuals take the current wage as the prediction of future wage. On the other hand, Siow (1984) propose a forward-looking model in which individuals have rational expectation on assessing the future wage and response to demand shocks rapidly. Ryoo and Rosen (2004) develop a framework to model different processes of expectation formation and find that the rational expectation one fit the data well in the US engineer market.

In our model, doctoral students also have rational expectation on assessing the future market conditions; however, their occupational choices could be inelastic over time. For instance, given the information of negative demand shock on future enrollment reduction, the number of doctoral students could still be the same in the first stage and results in a glut in the second stage. This is because the effect of demand shocks can be compensated by adjusting current training wages. In other words, tenured professors would provide incentive to attract doctoral students until they have the optimal number of inputs in knowledge production. Indeed, our model shows that professors will do so if the doctoral student productivity is high. The two-stage demand here is the reason for how rational PhDs could slowly response to demand shocks, which contradicts most rational models on occupational choice.

The rest of the paper is organized as follows. Section 2 presents the model to illustrate how students choosing career according to the expected lifetime income. Section 3 solves the model equilibrium in steady state and conducts simulations to show an increase in doctoral
student productively or a growing demand for higher education will raise the expected lifetime income for doctoral students. Section 4 discusses an empirical extension of this study.

2. Model

This is an occupational choice model for academic careers by applying an OLG structure in which doctoral students and professors are overlapped with two stages of market demand. In the first stage, professors hire doctoral students as an input in knowledge production. The doctoral student here is a general concept for individuals in the training stage and hence includes PhD students and postdocs as well. After the training stage, there exists a second-stage demand for professors as providers of knowledge in higher education and in this stage doctoral students will become professors or otherwise become workers.

2.1 Occupational states

Time is infinite, $t = 0, 1, 2, ...$. At generation $t$, there are $n_t$ agents live for two periods, young and old period. Let $g \geq 1$ be the gross rate of population growth and then we have $n_{t+1} = gn_t$. For every generation $t$, $y_i(i) \in \{0, 1\}$ and $o_i(i) \in \{0, 1\}$ are respectively the occupational states of agent $i$ when young and old for $i = 1, 2, ..., n_t$. A young agent can choose to be a worker as $y_i(i) = 0$ or to be a doctoral student as $y_i(i) = 1$. A young worker $y_i(i) = 0$ will surely become an old worker $o_i(i) = 0$ in the next period. A doctoral student $y_i(i) = 1$ will become a professor $o_i(i) = 1$ with a probability of $p_{i+1} \in [0, 1]$ and become an old worker $o_i(i) = 0$ with a probability of $(1 - p_{i+1})$. That is, if doctoral students cannot become professors, they will switch to be workers in the next period.

Let $w_t > 0$ and $E_t[w_{t+1}] > 0$ respectively be the current and expected wage of worker. The expected lifetime income for a worker of generation $t$ is

$$W_t \equiv w_t + \beta E_t[w_{t+1}], \quad (1)$$

where $\beta \in (0, 1)$ is the discount factor. On the other hand, the expected lifetime income for a doctoral student of generation $t$ is

$$D_t \equiv s_t + \beta p_{t+1} E_t[d_{t+1}] + \beta(1 - p_{t+1}) E_t[w_{t+1}], \quad (2)$$

where $s_t \in (0, w_t)$ is the current wage of doctoral student as a teaching assistant or research assistant; $E_t[w_{t+1}] > 0$ is the expected wage as a professor in the next period. A young agent will choose to be a doctoral student if $D_t \geq W_t$, which is the participation constraint of
2.2 Knowledge production

Following Levin and Stephan (1991), the income of professors is generated by knowledge production. In most fields, Wuchty, Jones, and Uzzi (2007) show that the team production of knowledge dominates solo work. In addition, Petersen, Riccaboni, Stanley, and Pammolli (2012) demonstrate decreasing marginal return with team size and emphasize that team productivity is related to group efficiency in knowledge production. Thus, we highlight the collaboration between professors and doctoral students and their productivity in the process of knowledge production.

Let \( t_m \) be the proportion of young agents of generation \( t \) who choose to be doctoral students, i.e., \( \mu_t = \sum_{i=j}^n y_t(i) / n_t \). Thus, at time \( t \), there are \( n_{t-1} + p_t \) professors and \( n_t \mu_t \) doctoral students. We define the knowledge production function of professors and doctoral students as

\[
X_t = A n_t^{1-a} (n_{t-1} + p_t) + \lambda n_t \mu_t, \quad (3)
\]

where \( X_t \) is the output; \( A > 0 \) and \( \alpha \in (0,1) \) are respectively parameters for technology and marginal return; \( \lambda \in (0,1) \) is the doctoral student productivity. The output function implies that professors and students are imperfectly substitutable and one professor is equal to \( 1 / \lambda \) doctoral students. Hence a larger \( \lambda \) means a relatively higher productivity of doctoral students. Moreover, the output is increasing with the number of young generation, which reflects spillover nature of knowledge such as in Jaffe (1989) and Audretsch and Feldman (1996).

For every generation \( t \), there are two stages of market demand for PhDs in knowledge production. In the first stage, professors \( (n_{t-1} + p_t) \) from last generation hire those doctoral students \( (n_t \mu_t) \) to produce knowledge. In the next period, the market has a second-stage demand for new professors \( (n_{t+1} p_{t+1}) \) as providers of knowledge. We assume that every young agent needs to have (consume) knowledge of \( e \). When the knowledge is produced, professors then provide (sell) knowledge to young agents and the total need of knowledge \( e \) constitutes the second-stage demand for professors.

In the first stage, professors as employers try to maximize their benefit after paying wages to doctoral students. Note that (3) is homothetic. By defining \( x_i = X_t / n_t \), we have...
Since all doctoral students earn $s_i n_i \mu_i$ as being teaching or research assistants, the wage for professor is thus

$$d_t = \frac{n_i x_t - s_i n_i \mu_i}{n_i - n_{t-1} \mu_i p_i}.$$  \hspace{1cm} (5)

for $t = 0, 1, 2, \ldots$. Note that $n_i \mu_i$, the number of students who choose to be doctoral students, is not directly controlled by professors; however, professors can set the enrollment limit to maximize (5). Let $m_i$ be the enrollment capacity, i.e., the proportion of young agents $n_i$ who can be enrolled to be doctoral students. By maximizing (5), we can derive the demand function for doctoral students by substituting $\mu_i = m_i$ into (5) and then obtain the first order condition of

$$m_i = \lambda^{\frac{-a}{\alpha}} \left( \frac{\alpha A}{s_i} \right)^{\frac{1}{1-a}} - \frac{\mu_{t-1} p_i}{\lambda g}.$$  \hspace{1cm} (6)

The function of (6) is the first-stage demand for doctoral students. However, if $s_i$ is too low, professors might not have enough students, i.e., $\mu_i < m_i$. Therefore, the supply of doctoral students $\mu_i$ in this stage should also be considered in determining the equilibrium path.

3. Equilibrium

In this section, we solve the model equilibrium in steady state. That is, given parameters, the proportion of doctoral students, the probability of being professors, and the wages of them converge to the equilibrium values over time. Moreover, we conduct simulations on various doctoral student productivities and population growth rates to illustrate the model implication.

3.1 Steady state

Recall the participation constraint that a young agent will choose to be a doctoral student if $D_i \geq W_i$. Thus, $D_i = W_i$ determines the supply function of doctoral students as follows:

$$s_i = w_i + \beta_{t+1} E_t [w_{t+1} - d_{t+1}].$$  \hspace{1cm} (7)
In the equilibrium, demand equals supply and then \( \mu_t = m_t \) for \( t = 0, 1, 2, \ldots \). Substituting (7) into (6) yields the equilibrium condition as

\[
m_t = \lambda^{1-a} \left( \frac{\alpha A}{w_t + \beta p_{t+2} E_t[w_{t+2} - d_{t+1}]} \right)^{\frac{1}{1-a}} - \frac{m_{t+1} p_t}{\lambda g}.
\]

where \( d_{t+1} \) contains \( m_t \), \( m_{t+1} \), and \( s_{t+1} \). Substituting \( \mu_t = m_t \) and \( \mu_{t+1} = m_{t+1} \) into (5) gives

\[
d_{t+1} = \frac{g}{m_t p_{t+1}} \left\{ A \left( \frac{m_{t+1}}{g} + \lambda m_{t+1} \right) - s_{t+1} m_{t+1} \right\}.
\]

Hence (8) is the condition for the equilibrium path of \( \{m_t; t = 1, 2, 3, \ldots\} \). Moreover, we have \( E_t[s_{t+1}] = E_t[w_{t+1} + \beta p_{t+2} E_t[w_{t+2} - d_{t+2}]] \) in (8). Because all agents of generation \( t \) make decisions according to information in period \( t \), we have \( E_t[w_{t+2}] = E_t[E_t[w_{t+2}]] = E_t[w_{t+1}] \) by the law of iterated expectations. In general, for all young agents of generation \( t \), we define \( E_t[w_{t+2+j}] = E_t[E_{t+j}[w_{t+2+j}]] = E_t[w_{t+1}] = \bar{w} \) for \( j = 1, 2, 3, \ldots \).

We focus on the equilibrium quantities and prices of the market. In Proposition 1, we first describe the equilibrium path of \( m_t \), the proportion of young agents who choose to be doctoral students in the first stage, and equilibrium path of \( p_t \), the probability of being professor in the second stage. The equilibrium prices are discussed later in Proposition 3.

**Proposition 1.** In steady state, we have \( \{m_t, p_t\} = \{m_{t+1}, p_{t+1}\} = \{m^*, p^*\} \) such that

\[
m^* = \left( \frac{g}{p^* + \lambda g} \right) \left\{ A \left( \frac{\alpha(1 - \beta g) + \beta (p^* + \lambda g)}{\bar{w} \left( 1 + \beta p^* \right)} \right) \right\}^{\frac{1}{1-a}},
\]

\[
p^* = \frac{A \lambda (1 - \beta g) + \beta g - \bar{w} a}{\beta \left( \bar{w} a - A \right)},
\]

where \( a = (\epsilon / A)^{(1-a)/a} \).

**Proof.** See Appendix 1.

In Proposition 1, we are interested in the marginal effect of student productivity (\( \lambda \)) and population growth (\( g \)) on \( m^* \) and \( p^* \). Note that in steady state \( p^* \) should be in the range of \([0,1]\), this also restricts the valid values of parameters. First, if \( A > \bar{w} e^{1-a} \), we must have \( \beta \left( \bar{w} a - A \right) < 0 \). The denominator of \( p^* \) in Proposition 1 is thus negative and the
numerator of it should also be negative. Specifically, \( \lambda \) and \( g \) should be sufficiently small such that \( \beta (\bar{w}A - A) \leq A\lambda (\alpha (1 - \beta g) + \beta g) - \bar{w}a \leq 0 \) to maintain \( p^* \in [0,1] \). In these cases, we would have \( dp^* / d\lambda < 0 \) and \( dp^* / dg < 0 \) because of \( \alpha \beta < \beta \). Second, if \( A < \bar{w}^\alpha e^{1 - \alpha} \), the denominator of \( p^* \) in Proposition 1 is positive. However, if \( \lambda \) or \( g \) is sufficiently small such that \( \lambda \beta g \leq 1 \), we will have \( \lambda (\alpha (1 - \beta g) + \beta g) \leq 1 \); in such cases, the numerator is negative and hence we cannot have \( p^* \in [0,1] \). We conclude this result in Proposition 2.

**Proposition 2.** If \( \lambda \) or \( g \) is sufficiently small, we have \( p^* \in [0,1] \) and \( p^* \) is a decreasing function of \( \lambda \) and \( g \).

In other words, an increase in the doctoral student productivity or population growth rate will lower the probability of becoming professors. A reasonable conjecture is that higher values of \( \lambda \) or \( g \) will induce more students to choose academic career. For example, given a higher value of \( \lambda \), professors may raise the training wage to attract more students in the optimal knowledge production. However, the implication of changing \( \lambda \) and \( g \) on \( m^* \) in Proposition 1 is complicated because it depends on relative values of parameters. In order to demonstrate such conjecture, we solve the equilibrium wages of students and professors in Proposition 3.

**Proposition 3.** In steady state, we have \( \{s^*, d^*\} = \{s_{t+1}, d_{t+1}\} = \{s^*, d^*\} \) such that

\[
\begin{align*}
    s^* &= \bar{w}(1 + \beta p^*) + \left( \bar{w}(1 + \beta p^*) - A \left( \lambda + \frac{p^*}{g} \right)^\alpha (m^*)^{\alpha - 1} \right) \frac{\beta g}{1 - \beta g}, \\
    d^* &= \frac{gA}{p^*} \left( \lambda + \frac{p^*}{g} \right)^\alpha (m^*)^{\alpha - 1} - \frac{gs^*}{p},
\end{align*}
\]

where \( m^* \) and \( p^* \) are derived from Proposition 1.

**Proof.** See Appendix 2.

Proposition 3 shows that \( s^* \) and \( d^* \) are functions of \( m^* \) and it is also difficult to observe the marginal effect of \( \lambda \) and \( g \) on \( s^* \), \( d^* \), and \( m^* \) directly. We thus simulate the equilibrium results in the next subsection.

### 3.2 Simulation

According to Proposition 1 and Proposition 3, we simulate the equilibrium value of \( m^* \), \( p^* \), \( s^* \), and \( d^* \) on various \( \lambda \) given a parameter set of \( (g, \alpha, \beta, A, w, e) = (1, 0.9, 0.9, 10, 5, 1) \) in Figure 1. It implies that the population is constant; the discount factor is 0.9 between young
and old period; and the opportunity cost of higher education is 1/5 of working income for young generations. As we can see, the number of doctoral student $m^*$ is increasing as the doctoral student productivity $\lambda$ increases. There are two reasons for this increasing trend. First, professors will provide a higher wage $s^*$ to students if students’ productivity increases. Second, the professor wage $d^*$ is dramatically increasing when $\lambda$ increases. These reasons are also shown in Figure 1. Thus, a higher current wage plus a larger future wage is attractive to students in choosing the academic career so that $m^*$ is increasing in $\lambda$. Consequently, there are too many PhDs in the second-stage market and the probability of becoming professors $p^*$ decreases when $\lambda$ increases in Figure 1, which has been proved in Proposition 2.

In addition, we are also interested in how the population growth rate affects the equilibrium results. Figure 2 illustrates the equilibrium value of $m^*$, $p^*$, $s^*$, and $d^*$ on various $g$, given a parameter set of $(\lambda, \alpha, \beta, A, w, c) = (0.33, 0.9, 0.9, 10, 5, 1)$. The setting of $\lambda = 0.33$ means one professor almost equals three doctoral students in productivity. We can observe similar patterns that $m^*$ is an increasing function of $g$ and $p^*$ is a decreasing function of $g$. However, the incentive for attracting more students in choosing the academic career is not due to higher training wages as doctoral students but due to higher expected wages as professors. Figure 2 shows that $s^*$ does not change when $g$ increases. In contrast, $d^*$ is increasing in $g$. That is, a larger population growth rate implies a higher demand for professors and hence a greater professor wage in the second stage. Therefore, there are more students choosing the academic career in the first stage and result in a PhD glut later.

4. Concluding remarks

In an extension of this study, we will test the model by constructing a unique dataset in Taiwan and use panel data among fields to estimate the inflexibility of PhD supply through field-specific shocks. Regarding population (student) growth rates in educational markets, Zarkin (1985) points out that the future demand for primary or secondary school teachers can be explicitly estimated by the number of children who are already born. The future demand for professors essentially could also be estimated by the number of existing students prior to colleges. However, the main distinction is that the distribution of students’ majors might change over time and hence affects the corresponding demand for professors in different fields. Accordingly, our model should be considered a partial description for one from diverse fields. Such distinction provides a good opportunity to conduct empirical test
since we could have panel data of various fields to estimate inflexibility of PhD supply through field-specific shocks.

We will construct a dataset to test the model based on the market of PhDs with local doctoral degrees (local PhDs) in Taiwan. The data consists of the numbers of enrolled PhD students, PhD graduates, estimated training wages, estimated professor vacancies, professor wages, and enrolled undergraduates in Taiwan during 1997-2014. This dataset has several advantages in empirical testing. First, the professor wages in Taiwan are publicly observable and hence the uncertainty of lifetime income is mainly from the probability of finding tenured positions. Second, the supply of local PhDs can be explicitly accounted since most of them will not search academic positions outside Taiwan. Finally, following Bound, Braga, Golden, and Turner (2013), we also count the PhDs with doctoral degrees aboard and then estimate the market vacancies and real demands for local PhDs in different fields by calculating the proportions of newly-hired professors who receive local PhD degrees every year.
Appendix 1. Model equilibrium in steady state

In the steady state, we have \( \{m_t, p_t\} = \{m^*, p^*\} \). Substituting \( m_{t-1} = m_t = m^* \) and \( p_t = p_{t+1} = p^* \) into (8) gives

\[
(1 + \frac{p^*}{\lambda g})m^* = \left( \frac{\alpha A}{s_t} \right)^{\frac{1}{1-\alpha}} = \lambda^{\frac{1}{1-\alpha}} \left( \frac{\alpha A}{w^* \beta E_t[w_{t+1} - d_{t+1}]} \right)^{\frac{1}{1-\alpha}}.
\]

(A1)

Note that \( \beta E_t[w_{t+1} - d_{t+1}] = \beta \tilde{w} - \beta E_t[d_{t+1}] \) and

\[
E_t[d_{t+1}] = \frac{gA}{p^*} \left( \lambda + \frac{p^*}{g} \right) (m^*)^{\gamma-1} - \frac{gE_t[s_{t+1}]}{p^*}.
\]

(A2)

from (9). In the steady state, we also have \( w_t = E_t[w_{t+1}] = \tilde{w} \). Thus (A1) can be rewritten as

\[
(1 + \frac{p^*}{\lambda g})m^* = \lambda^{\frac{1}{1-\alpha}} \left( \frac{\alpha A}{\tilde{w} + \beta p^* \tilde{w} - \beta gA(\lambda + (p^*/g))(m^*)^{\gamma-1} + \beta gE_t[s_{t+1}]} \right)^{\frac{1}{1-\alpha}},
\]

(A3)

where \( E_t[s_{t+1}] = \tilde{w} + \beta p^* \tilde{w} - \beta gA(\lambda + (p^*/g))(m^*)^{\gamma-1} + \beta gE_t[s_{t+1}] \). Substituting the sequence of \( E_t[s_{t+j}] \), \( j = 1, 2, 3, \ldots \), into (A3) yields

\[
(1 + \frac{p^*}{\lambda g})m^* = \lambda^{\frac{1}{1-\alpha}} \left( \frac{\alpha A}{\tilde{w}(1 + \beta p^*) + \tilde{w}(1 + \beta p^*) - A(\lambda + \frac{p^*}{g})(m^*)^{\gamma-1}} \right)^{\frac{1}{1-\alpha}}.
\]

(A4)

Solving (A4) obtains the equilibrium proportion as

\[
m^* = \left( \frac{g}{p^* + \lambda g} \right)^{\frac{1}{1-\alpha}} \left( \frac{A(\lambda \alpha (1 - \beta g) + \beta (p^* + \lambda g))}{\tilde{w}(1 + \beta p^*)} \right)^{\frac{1}{1-\alpha}}.
\]

(A5)

Since all young agents need knowledge of \( e \) provided from the knowledge production, we shall have \( x_t = e \) in the equilibrium. Substituting \( m^* \), \( p^* \), and \( x_t = e \) into (4), we have
\[
\begin{align*}
e &= A \left( \frac{g}{(p^* + \lambda g)} \left[ A \left( \frac{\lambda \alpha (1 - \beta g) + \beta (p^* + \lambda g)}{\bar{\nu} (1 + \beta p^*)} \right) \right] \right)^{-1} \left[ \begin{array}{c} p^* \\ \frac{\bar{\nu}}{\nu} + \lambda \end{array} \right]. \quad \text{(A6)}
\end{align*}
\]

Solving (A6) yields the equilibrium \( p^* \) of the second-stage demand for professors as

\[
\begin{align*}
p^* &= \frac{A \lambda \left( \alpha (1 - \beta g) + \beta g \right) - \bar{\nu} a}{\beta \left( \bar{\nu} a - A \right)} \quad \text{(A7)}
\end{align*}
\]

where \( a = (e / A)^{(1 - \alpha) / \alpha} \).
Appendix 2. Equilibrium wages

From (A1), we know that $w(1 + \beta \rho^*) + \left(\frac{w(1 + \beta \rho^*)}{w(1 + \beta \rho^*)} - A(\lambda + \rho^* / g)^\gamma (m^*)^{\gamma-1}\right) \beta g / (1 - \beta g)$ in (A4) is equal to $s^*$ in the steady state. Hence, substituting $m^*$ and $\rho^*$ of Proposition 1 into $s^*$, we obtain the equilibrium wage of students as

$$s^* = \frac{w(1 + \beta \rho^*)}{w(1 + \beta \rho^*)} + \left(\frac{w(1 + \beta \rho^*)}{w(1 + \beta \rho^*)} - A\left(\lambda + \frac{\rho^*}{g}\right)^\gamma (m^*)^{\gamma-1}\right) \frac{\beta g}{1 - \beta g}. \quad (A8)$$

Similarly, in the steady state, substituting $m^*$, $E[s_{t+1}] = s^*$, and $E[d_{t+1}] = d^*$ into (A2) yields the equilibrium wage of professors as

$$d^* = \frac{gA}{p}\left(\lambda + \frac{\rho^*}{g}\right)^\gamma (m^*)^{\gamma-1} - \frac{gs^*}{p}. \quad (A9)$$
References


Figure 1. Simulations of doctoral student productivities
Figure 2. Simulations of population growth rates