Household characteristics and their impact on child education: bargaining and contribution to public good *

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Abstract

This paper proposes a theoretical framework in which altruistic parents contribute to their child's human capital formation through time spent helping at a primary stage of the education process, while the final educational attainment is the child’s decision. This setting provides a particular case of voluntary subscription to a family public good, in which transfers are not lump sum and the level of public good eventually produced is the result of a third agent’s optimization (the child). This model, while remaining consistent with other stylized facts, shows that despite the lack of coordination, parents tend to specialize in either help or labor, depending on the elasticity of their wage to education.

Keywords: human capital, public good, household, non-cooperative Nash.
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1 Introduction

In the human capital literature so far two categories of models have been used to examine the educational and occupational outcomes achieved by children. On the one hand, we have models in which parents are assumed to have total control over decisions regarding their children’s human capital. In these models a child’s outcome is determined by her initial endowment, her parents’ investments in her human capital, and her, so-called, ‘market luck’. (Becker and Tomes, 1979, 1986; Becker, 1974, 1981) On the other hand, a second type of human capital model focuses on young adults’ own schooling decisions (e.g. whether or not to attend college), given some pre-existing endowment determined by both inherent ability and previous investments by parents. Such an approach has led to numerous regressions of schooling decisions on standardized test scores, family background, and neighborhood and peer characteristics. (Haveman and Wolfe, 1995; Rainey and Murova, 2004; Tobias, 2003)

Three broad classes of household’s decision processes have been introduced in the literature: the unitary approach, a cooperative and a non-cooperative bargaining process. Two models provide the theoretical ground for the unitary approach, the consensus model introduced by Samuelson (1956) and the altruist model elaborated in Becker (1974, 1981). In short, both these models treat the household as if it was a single decision maker. Despite the extensive use of this hypothesis, it is now well-established that this approach fails to describe the observed behaviors of the families, e.g. recent empirical evidence have rejected the assumption of income pooling, finding that earned and unearned income received by the husband or the wife significantly affect demand patterns (Thomas, 1990; Udry, 1996). As an alternative, non-unitary models of the household have thus emerged. In cooperative models, which are dominant in the literature, intra-household interactions lead to Pareto efficient outcomes. A typical cooperative bargaining model of the household consider two members: a husband and a wife, each with an utility function that depends on his or her consumption of private goods. The husband and wife settle their differences by Nash bargaining, and if agreement is not reached, then the payoff received is represented by the utilities associated with a default outcome of divorce or, alternatively, a non-cooperative equilibrium within the marriage as in Lundberg and Pollak (1993). The nowadays popular collective model, originally suggested by Chiappori (1988, 1992), is a generalization of the Nash bargaining model where rather than applying a particular cooperative bargaining model to the household allocation process, it is assumed only that equilibrium allocations are Pareto optimal. A particularly recent strand of the literature that is also worth mentioning, focuses on revealed preference analysis of the decision process, analyzing and comparing collective models and non cooperative Nash equilibrium models (Cherchye et al., 2007, 2011a,b). In a more recent contribution, Gobbi (2013) develops a two-step semi-cooperative decision model, where children enter the household decision-making as a public good. In her setting parents collectively choose the amount of labor to supply and, in a second step, each of them chooses the amount of childcare as the outcome of a Cournot-Nash game. The non-cooperative decision of childcare supplied by each spouse results in an inefficient under-provision of care (in comparison to the collective model).

In a non-cooperative approach each individual in the household behaves as a rational agent, maximizing his/her

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1 Refer to Lundberg and Pollak (2008) for a recent review of household models.
own utility subject to an individual budget constraint, taking as given the decisions of other individuals within
the household. Non-cooperative models of the household relax both the condition regarding Pareto efficiency
and the assumption of binding enforceable agreements, embedded instead in the cooperative/collective models
briefly described above.

In this paper we introduce a theoretical framework in which altruistic parents contribute to their child’s human
capital formation through time spent helping at a primary stage of the education process. While the educational
attainment is the child’s decision, this parental intervention lowers its effort. On the one hand, more educated
parents provide help of higher quality, which increases the child’s incentive to study. On the other hand, this
effect may be counterbalanced by the fact that the opportunity cost of help (i.e. their hourly wage) is higher for
more educated parents. This setting provides a particular case of voluntary subscription to a family public good,
in which transfers are not lump sum and the level of public good eventually produced is the result of a third
agent’s optimization (the child).

Following recent literature on household behavior in the production of family public goods, we adopt here a
non-cooperative approach in the parental decision making. (Rainer, 2008; Del Boca and Flinn, 2012)

The interaction between the household’s agents in choosing their respective effort is based on the concept of
non-cooperative Nash equilibrium, which is thus self-enforcing and stable. A weakness of those non-cooperative
model in the context of the family can be found in the rather unrealistic assumption that individuals care only
about their own well-being. To somehow overcome this shortcoming, in this paper we depart from this assump-
tion, allowing the parents to care about the child well-being and some income sharing among partners.

Despite the lack of coordination, parents tend to specialize in either help or labor, depending on the elasticity of
their wage to education.

The remainder of this paper is organized as follows. In the following section we introduce the basic setting of our
theoretical model of parental contribution. In section 3 we analyze and discuss some comparative statics. Section
4 concludes.

2 Basic Setting

Let us consider as a family consisting of two parents $p \in \{m, f\}$ (mother and father) and a child $c$, whose human
capital formation process can be described as follows.

In the first period, the child, endowed with an innate ability level $a$, is at a basic stage of the education process
(i.e. compulsory education). During this first period, the child builds a basic level of human capital $B(a, P)$,
where $P$ represents the inputs brought by parents to help and educate the child. In the second period, the child
chooses her final level of education $E_c$. In order to reach this level, the student needs to exert some study effort

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2 It should be noted that cooperative and non-cooperative models will produce the same outcomes if all goods are private goods and there
are no externalities (Chiappori and Donni, 2009). However, if there are public goods and/or externalities, non-cooperative models will yield
outcomes that may not be Pareto efficient.
The final level of education in the second generation in this setting is thus the result of two choices: the amount of investment chosen by the parents and the amount of effort chosen by the child herself.

The child’s utility $V$ depends on her consumption in adulthood $c_c$, and study effort $C$:

$$V = v(c_c) - C(E_{c}, B),$$

where $v(.)$ is a strictly increasing and concave function of her consumption $c_c = w(E_{c})$, where $w(E_{c})$ is the child’s income.

The cost of study effort is increasing and convex in the final level of education ($C_{E_{c}, E} > 0, C_{E_{c}, E} > 0$), decreasing and concave in the level of basic human capital ($C_{B} < 0, C_{BB} \geq 0$). Also, we assume that $C_{E_{c}, B} < 0$, which implies that the marginal cost of an additional year of schooling decreases with the child’s basic level of human capital. This $B$ is positively affected by the child’s innate ability ($a$) and by the parents’ input $P$:

$$B = B(P, a),$$

with $B_{P} > 0, B_{PP} \leq 0$ and $B_{a} > 0$. Also, the function $B$ is assumed supermodular: $B_{Pa} \geq 0$. Intuitively, the parent’s help cannot be less efficient at producing $B$ on a more able child. The parents’ input $P$ is a function of both parents’ contributions, the quality (captured by the parent’s education $E_{p}$) and quantity (the amount of time spent $h_p$) of help:

$$P = P(H^{f}(h_{f}, E_{f}), H^{m}(h_{m}, E_{m}))$$

where $P_{H^{f}} > 0, P_{H^{m}H^{f}} \leq 0$, and $P_{H^{m}H^{m}} = 0$. We assume that the function $H^{p}(h_{p}, E_{p})$ is also increasing and concave in each argument and is supermodular ($H^{p}_{h_{p}} > 0, H^{p}_{h_{p}h_{p}} \leq 0, H^{p}_{E_{p}} > 0, H^{p}_{E_{p}E_{p}} \leq 0$, and $H^{p}_{h_{p}E_{p}} > 0$) for both parents.

Parents derive utility from their own consumption and from the utility of their child, $V$. Although the altruism coefficient $\alpha_{p}$ may differ across parents, the child’s utility is a family public good. The utility of parent $p \in \{m, f\}$ is written:

$$U_{p}(c_{p}, V) = u(c_{p}) + \alpha_{p}V,$$

where private consumption $c_{p}$ is based on income sharing between $p$ and $p'$:

$$c_{p} = (1 - \gamma)Y_{p} + \gamma Y_{p'},$$

where $\gamma \in [0; \frac{1}{2}]$ represents the proportion of income that each parent shares with his/her spouse and income $Y_{p}$.

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3 We assume here that there are no cost of higher education.

4 Note that this assumption will not prevent parents of less able children from providing more help if those children are performing poorly at school.
is a decreasing function of help:

\[ Y_p = (T - h_p) w_p (E_p) . \]  

(4)

where \( T \) is the total time endowment of the parent, \( h_p \) is the time spent helping the child and and \( w_p(E_p) \) is the parent’s hourly wage, which is an increasing function of parental education.

Parents are assumed to provide help to the child in a non-cooperative way. Formally, each parent maximizes his/her own utility, taking his/her spouses’ behavior as given. By doing so, they both contribute to the production of a family public good, i.e. the child’s basic level of human capital.

The timing of the model is the following. First, each parent \( p \in \{m, f \} \) chooses the amount of help \( h_p \) so as to maximize \( U_p \). Second, the child chooses the final level of education \( E_c \) in order to maximize \( V \). Solving the model backwards, we first analyze the child’s optimization, taking the parents’ behavior as given. The child’s optimal amount of education \( E_c^* \) maximizes (1) and is implicitly defined by the following first order condition:

\[
\frac{\partial V}{\partial E_c} = v_c'(E_c)w(E_c) - \frac{\partial C(E_c; B)}{\partial E_c} = 0. \tag{5}
\]

In the child’s optimization, \( B \) is given and has to be treated as a parameter. We therefore perform here the comparative statics of \( E_c^* \) with respect to \( B \).

**Lemma 1.** The basic level of human capital of the child \((B)\) has a positive impact on her final level of education \((E_c)\).

**Proof.** Applying the implicit function theorem to (5), we obtain:

\[
\frac{\partial E_c^*}{\partial B} = \frac{\partial^2 V}{\partial E_c \partial E_c} = -\frac{\partial^2 C(E_c; B)}{\partial E_c^2} > 0.
\]

and \( C_{E_c, B} < 0 \) by assumption. \( \square \)

This lemma clearly implies that parental help \( P \), which improves \( B \), has a positive impact on the child’s final level of education.

Let us now analyze the decision-making process of parents. As previously mentioned, they both voluntarily, but non-cooperatively provide their child with help time \( h_p \). The Nash equilibrium is a set of efforts \((h_f^N, h_m^N)\) such that each parent provides a level of help which maximizes his/her own utility, considering as given the other parent’s contribution. Formally, the marginal effect of one parent’s help on his/her utility is:

\[
\frac{\partial U_p}{\partial h_p} = -u_p'(1 - \gamma) w_p + \beta_p \left[ \frac{\partial V}{\partial h_p} + \frac{\partial V}{\partial E_c^*} \frac{dE_c^*}{dh_p} \right],
\]

If \( \gamma = \frac{1}{2} \), there is complete income sharing/pooling, while if \( \gamma = 0 \), there is no pooling of resources.
where
\[
\frac{\partial V}{\partial h_p} = -\frac{\partial C(E^*_p)}{\partial B} \frac{\partial P}{\partial \partial h_p} \frac{\partial H_p}{\partial h_p} > 0,
\]
\[
\frac{\partial V}{\partial E_p} = 0.
\]
Therefore, each parent’s reaction function is determined by the following Kuhn-Tucker condition:
\[
\frac{\partial U_p}{\partial h_p} = -u'_p(1-\gamma)w_p + \beta_p \left(-\frac{\partial C(E^*_p)}{\partial B}\right) \frac{\partial P}{\partial \partial H_p} < 0 \text{ and } h_p > 0,
\]
\[
\frac{\partial U_p}{\partial h_p} = -u'_p(1-\gamma)w_p + \beta_p \left(-\frac{\partial C(E^*_p)}{\partial B}\right) \frac{\partial P}{\partial \partial H_p} \frac{\partial H_p}{\partial h_p} = 0 \text{ and } h_p = 0.
\]
Focusing on the interior solution, the best responses of both parents can be rewritten as
\[
\frac{u'_f w_f}{\alpha_f} \frac{\partial P}{\partial \partial H_f} = \frac{u'_m w_m}{\alpha_m} \frac{\partial P}{\partial \partial H_m} = \left(-\frac{\partial C(E^*_p)}{\partial B}\right) \frac{\partial H_p}{\partial h_p} \frac{\partial B}{B} \frac{1}{1-\gamma},
\]
where \(\frac{-\frac{\partial C(E^*_p)}{\partial B}}{B} \frac{\partial B}{\partial h_p} \frac{1}{1-\gamma}\) is identical to both parents, so that at equilibrium:
\[
\frac{u_f}{u_m} = \frac{\alpha_f w_m}{\alpha_m} \frac{\partial P}{\partial \partial H_f} \frac{\partial H_f}{\partial h_f} \frac{\partial H_m}{\partial h_m}.
\]

In order to be able to derive the equilibrium provision of help let us introduce here some simplifying assumption.
First, let us assume that \(P\) and \(H\) are linear in \(h_p\), so that \(\frac{\partial P}{\partial \partial H_p} \frac{\partial H_p}{\partial h_p}\) can be rewritten as a parameter \(\theta_p\) which does not depend on \(h_p\), but may depend on ability \(a\) and \(E_p\). Second, let us assume that the child’s preferences over consumption exhibit constant elasticity of substitution \(\kappa\): \(v(c) = c^{1-\frac{1}{\kappa}}\), the returns to education are \(w(E_c) = \omega E_c\), and the cost of study effort is \(C(E_c; B) = E_c B\). Third, let us define the production function of basic human capital as
\[
B = \Lambda + \beta P,
\]
\[
P = \theta_m h_m + \theta_f h_f,
\]
where \(\theta_p\), parent \(p\)’s productivity of help, is positively affected by his/her education \(E_p\). Finally, also the parents’ preferences over consumption exhibit constant elasticity of substitution \(\sigma\): \(u(c_p) = c_p^{1-\frac{1}{\sigma}}\). In this setting, we obtain the following result.

**Proposition 1.** The Nash equilibrium of the parents’ game is
\[
h_f^N = T - \frac{1}{\beta \theta_f} \left(\frac{\Lambda + \beta (\theta_f + \theta_m) T}{\alpha_f w_m} + \frac{\theta_m}{\alpha_f}\right) \Psi + 1 + \alpha_f \frac{\gamma}{1-\gamma},
\]
\[
h_m^N = T - \frac{\Psi}{\beta \theta_f} \left(\frac{\Lambda + \beta (\theta_f + \theta_m) T}{\alpha_f w_m} + \frac{\theta_m}{\alpha_f}\right) \Psi + 1 + \alpha_f \frac{\gamma}{1-\gamma}.
\]
with \( \Psi = \frac{w_f \gamma + \Theta \gamma - \Theta f}{w_m \gamma + \Theta \gamma - \Theta} \) and \( \Theta = \left( \frac{\alpha f w_m}{\alpha m w_f} \right)^\sigma \).

Proof. See Appendix A.1

As introduced above, the setting of this paper differs from a common contribution to a public good game in few aspects. Firstly, the contribution here is not lump-sum, as parents are heterogeneous and the productivity of their time differs. Secondly, the agents are sharing resources up to some degree \( \gamma \). Finally, the final public good is the result of a third agent’s choice, the child. Thus, in equilibrium, each parent’s effort is not only the result of their different willingness to pay (given different incomes) but depends on different mechanisms: (1) the heterogeneity in the caring for the child utility, given by the altruistic parameter \( \alpha_f \); (2) the technology of the production of the public good, as \( \Lambda + \beta (\theta_f + \theta_m) T \) represents the maximum possible basic human capital that, given their respective productivity of effort, will be produced if both parents were using all the available time to help the child; (3) the heterogeneity in the cost and the productivity of their respective effort, as captured by \( \left( \frac{\alpha_f w_m + \theta_m}{\alpha_f w_f} \right) \Psi \); (4) the level of income sharing \( \gamma \), which plays a role in both \( \alpha_f \frac{\gamma}{\gamma - 1} \) and \( \Psi \).

Note that if we consider a setting closer to the usual contribution to public good game, in which the agents do not share resources,

\[
h_f^N = T - \frac{1}{\beta \theta_f} \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \left( \frac{\alpha_f}{\alpha_m} \frac{\theta_f}{\theta_m} \right)^\sigma \left( \frac{w_m}{w_f} \right)^{\sigma-1} + 1,
\]

the equilibrium efforts depend on the heterogeneity of the parents in their income, productivity and altruistic parameter together with the factor concerning the technology of the production of the public good. Finally, in case of identical parents, this condition boils down to:

\[
h_f^N = T - \frac{\Lambda + 2T \beta \theta}{\beta \theta (\alpha + 2)},
\]

where the equilibrium effort of each parent is equal to the available time minus the maximum producible child basic human capital divided by the productivity of the effort itself \( (\beta \theta) \) by the weights of the three agents concerned by this effort, namely 1 for each parent and \( \alpha \) (the altruistic parameter) for the child.

Having defined the Nash equilibrium, we can now compute the child’s basic level of human capital at equilibrium.

**Proposition 2.** The equilibrium level of basic human capital is

\[
B^N = \Lambda + \beta \left( \theta_m h_m^N + \theta_f h_f^N \right) = \Pi^N \mathcal{B}, \tag{7}
\]

where

\[
\mathcal{B} = \frac{\Lambda + \beta T (\theta_m + \theta_f)}{\Theta}, \quad \Pi^N = \frac{1 - \gamma}{1 - 2 \gamma} \frac{\Theta (1 - \gamma) (\alpha_f + \alpha_m) - \gamma (\theta_f \alpha_m + \alpha_f)}{\Theta + \Theta (1 - \gamma) (\alpha_f + \alpha_m - \gamma (\theta_f \alpha_m + \alpha_f))} \in [0; 1].
\]

\( \mathcal{B} \) represents the highest feasible \( B \), which corresponds to the case in which both parents spend their whole time
helping their child, and \( \Pi^N \) represents the fraction of \( \overline{B} \) which is produced at the Nash equilibrium. Note that \( \Pi^N \) summarizes the impact of all the strategic interactions between parents on the child’s basic level of human capital.

In other words, the effects of variables such as the parents’ education on the non-cooperative equilibrium level of help is entirely captured in \( \Pi^N \). It is therefore not surprising that it depends on the parents’ characteristics, such as wages, quality of help and altruism. Also, the degree of homogeneity among parents of these characteristics, captured by \( \Theta \), appears crucial. Finally, \( \Pi^N \) depends on the way parents share resources through \( \gamma \). The impact of such key parameters of the model will be analyzed in the next section.

3 Comparative statics

We can now analyze the effects of the parent’s education and the child’s ability on the parent’s help.

Proposition 3. Comparative statics: Effect of the parent’s education on his/her own help:

\[
\frac{\partial h^N_f}{\partial E_f} = \frac{\theta_f'}{\theta_f} \left( T - h^N_f \right) \left( \frac{\Lambda + T\beta_m \theta_m}{\Lambda + T\beta_f + T\beta_m} + \varepsilon_f \right) - \frac{w_f}{w_f} \left( T - h^N_f \right) \varepsilon_f - \phi_E \left( \frac{\alpha_f w_m + \theta_m}{\theta_f} \Psi + 1 + \alpha_f \frac{\gamma}{1-\gamma} \right),
\]

where

\[
\varepsilon_f = \frac{\theta_f'}{\theta_f} \left( \frac{\alpha_f w_m + \theta_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1-\gamma} > 0.
\]

Proof. See Appendix A.2

The incentive for more educated parents to spend more time helping their child depends on the impact of their education on 1) the productivity of their help, 2) their wage (i.e. the opportunity cost of help) and 3) the basic level of education irrespective of \( h_p \). In other words, \( h^N_f \) is more likely to increase with \( E_f \) if \( \frac{w_f}{w_f} \) is large, \( \frac{w_f}{w_f} \) is low, and \( \phi_E \), the effect of the parent’s education on \( B \) for \( h_p = 0 \), is low.

Proposition 4. Comparative statics: Effect of one parent’s education on his/her spouse’s help:

\[
\frac{\partial h^N_f}{\partial E_m} = -\frac{\theta_m'}{\theta_m} \frac{T\theta_m + \left( T - h^N_f \right) \theta^2_f \varepsilon}{\theta_f \left( \left( \alpha_f w_m + \theta_m \right) \Psi + 1 + \alpha_f \frac{\gamma}{1-\gamma} \right)} + \frac{w_m'}{w_m} \beta \theta_f \left( \left( \alpha_f w_m + \theta_m \right) \Psi + 1 + \alpha_f \frac{\gamma}{1-\gamma} \right),
\]

where

\[
\phi_E = \frac{\beta \theta_f}{\left( \left( \alpha_f w_m + \theta_m \right) \Psi + 1 + \alpha_f \frac{\gamma}{1-\gamma} \right)},
\]

Proof. See Appendix A.3

The incentive for a parent with a more educated spouse to spend more time helping their child depends on the impact of their spouse’s education 1) on the productivity of the spouse’s help, 2) on the spouse’s wage and 3) on the basic level of education irrespective of \( h_p \). In other words, \( h^N_f \) is more likely to increase with \( E_m \) if \( \frac{w_m'}{w_m} \) is low, \( \frac{w_m'}{w_m} \) is large, and \( \phi_E \) is low.

This proposition suggest that, despite the lack of cooperation, there is specialization in the activities of the parents: if a parent improves his/her education and earns a larger wage, relative to the improvement in the quality of help,
it is in both parents’ interest to increase the work time of the more educated parent, while the other parent will increase his/her supply of help to the child, or the other way round if the increase in education produce a larger improvement in the quality of help, relative to the increase in wages.

Let us now study the impact of the parent’s education on the final level of education of the child, $E_c$. Based on (7), we can write

$$\frac{\partial E_c}{\partial E_f} = \frac{\partial E_N}{\partial B} \frac{\partial B}{\partial E_f},$$

where by Lemma 1, $\frac{\partial E_N}{\partial B} > 0$, and

$$\frac{\partial B}{\partial E_f} = \frac{\partial \Pi}{\partial E_f} \frac{\partial \Pi}{\partial B} + \frac{\partial B}{\partial E_f} \Pi.$$

The direct, “technological” effect of $E_f$ on $B$ is straightforward:

$$\frac{\partial B}{\partial E_f} = \frac{\partial \Lambda}{\partial E_f} + \beta \theta_f' = \beta \left( \rho_f \phi_e + T \theta_f' \right) > 0.$$

The second, indirect effect of $E_f$ through strategic interactions, $\frac{\partial \Pi}{\partial E_f}$, is less immediate and described in the following proposition.

**Proposition 5.** Comparative statics: Effect of one parent’s education on the Nash equilibrium fraction $\Pi^N$

$$\frac{\partial \Pi^N}{\partial E_f} > 0 \iff \{ \frac{\theta_f'}{\frac{w_f'}{w_f}} > \frac{\theta_f}{\frac{w_f}{w_f}} \text{ and } \frac{\theta_f}{\frac{w_f}{w_f}} > \frac{\theta_m}{\frac{w_m}{w_m}} \frac{\gamma}{1-\gamma} \frac{2 \alpha_m}{\alpha_f + \alpha_m}, \text{ or} \}

\begin{align*}
\quad \frac{\theta_f'}{\frac{w_f'}{w_f}} < \frac{\theta_f}{\frac{w_f}{w_f}} \text{ and } \frac{\theta_f}{\frac{w_f}{w_f}} < \frac{\theta_m}{\frac{w_m}{w_m}} \frac{\gamma}{1-\gamma} \frac{2 \alpha_m}{\alpha_f + \alpha_m}.
\end{align*}

**Proof.** See Appendix A.4

If a parent, say the father, becomes more educated, the Nash equilibrium will affect both parents’ provision of help. The total effect of this increase in $E_f$ will be beneficial to the child through $B$ under a pair of conditions. The first condition naturally pertains to the comparison between the marginal impacts of education on the quality of help, $\theta_f'$, and on the wage, $w_f'$. If the father’s quality of help increases at a faster rate than his wage, he will increase his level of help. This is however not the end of story, since as shown in the previous proposition, the mother might choose to reduce her help to such an extent that $B$ decreases. Therefore, if $\frac{\theta_f'}{\frac{w_f'}{w_f}} > \frac{\theta_f}{\frac{w_f}{w_f}}$, it must also be that the (level of) the help quality (relative to the wage) of the father is larger than the mother’s, up to some component $\frac{\gamma}{1-\gamma} \frac{2 \alpha_m}{\alpha_f + \alpha_m}$. This condition is more easily satisfied if this component is small, that is, if parents share a large fraction of their income, and if the father is more altruistic than the mother, in which case the increase in the father’s help dominates the decrease in the mother’s help.

Combining this Proposition with previous findings, we can conclude with the following corollary.

**Corollary 1.** Comparative statics: Effect of one parent’s education on the child’s final level of education

$$\frac{\partial E_c}{\partial E_f} > 0 \iff \{ \frac{\theta_f'}{\frac{w_f'}{w_f}} > \frac{\theta_f}{\frac{w_f}{w_f}} \text{ and } \frac{\theta_f}{\frac{w_f}{w_f}} > \frac{\theta_m}{\frac{w_m}{w_m}} \frac{\gamma}{1-\gamma} \frac{2 \alpha_m}{\alpha_f + \alpha_m}, \text{ or} \}

\begin{align*}
\quad \frac{\theta_f'}{\frac{w_f'}{w_f}} < \frac{\theta_f}{\frac{w_f}{w_f}} \text{ and } \frac{\theta_f}{\frac{w_f}{w_f}} < \frac{\theta_m}{\frac{w_m}{w_m}} \frac{\gamma}{1-\gamma} \frac{2 \alpha_m}{\alpha_f + \alpha_m}.
\end{align*}

Let us then analyze the role played by the income sharing variable, $\gamma$. 9
Proposition 6. **Comparative statics: Effect of parents’ income sharing on the child’s final level of education**

\[
\frac{\partial E_c}{\partial \gamma} = \frac{\partial E^N_c}{\partial B^N} \frac{\partial \Pi^N}{\partial \gamma} \geq 0.
\]

Proof. See Appendix A.5

Note that the only case for which \( \frac{\partial \Pi}{\partial \gamma} = 0 \) is when parents are identical (\( \Theta = 1 \)) and fully share income (\( \gamma = 0.5 \)). The intuition behind this result is the following. Sharing income among spouses is beneficial to the child, as parents are more inclined to reduce their labor force participation since they can rely more on their spouse’s income.

Finally, let us conclude this section with the analysis of the effect of the child’s innate ability.

Proposition 7. **Comparative statics: Effect of the child’s innate ability on the parent’s help:**

\[
\frac{\partial h^N_f}{\partial a} = \frac{\Lambda \beta_X - \beta_a}{\beta f \left( \left( \frac{\alpha f \theta_m + \theta_a}{\theta_f} \right) \Psi + 1 + \alpha f \frac{\theta_m}{\theta_f} \right)} < 0 \iff \frac{\Lambda}{\beta} < \frac{\beta_a}{\beta_X}.
\]

Proof. See Appendix A.6

This proposition states that for children with lower ability to receive more help from their parents in equilibrium, the complementarity between parental input and ability, captured by \( \epsilon_{BP,a} \) should not be too high, surely lower than 1. Otherwise, the marginal productivity of helping highly able children would be so high that parents would prefer to increase the help for more able children.

Proposition 8. **Comparative statics: Effect of the child’s innate ability on her final level of education**

\[
\frac{\partial E_c}{\partial a} = \frac{\partial E^N_c}{\partial B^N} \frac{\partial B}{\partial a} \Pi^N > 0.
\]

Proof. \( \Pi^N \) does not depend on \( a \), and

\[
\frac{\partial B}{\partial a} = \frac{\partial \Lambda}{\partial a} + T (\theta_m + \theta_f) \frac{\partial \beta}{\partial a} = \beta_a + T (\theta_m + \theta_f) \beta_X > 0.
\]

While it is unclear whether smarter children receive more or less help from their parents, they always end up with a higher final level of education.
4 Concluding remarks

This paper presents an innovative theoretical model of intergenerational transmission of human capital which account for the interactions between household members. Altruistic parents contribute to their child’s human capital formation through time spent helping at a primary stage of the education process. While the educational attainment is ultimately the child’s decision, parents may affect this decision, decreasing her effort-cost of studying through their help. More educated parents provide help of higher quality but they also face a higher opportunity cost (i.e. their wages) of providing this help. Furthermore, our setting provides a particular case of voluntary subscription to a family public good, in which transfers are not lump sum, as parents are heterogeneous and the productivity of their time differ, the agents shares resources up to some degree, and the level of public good eventually produced is the result of a third agent’s optimization (the child).

This model, while remaining consistent with other stylized facts, explains why it may be rational for educated women to stay at home and educate their child, if they face a discriminating labor market, where their level of education in not compensated as much as the one of their spouses.

To conclude, the results of this paper suggests that it may be worth focusing more on gender discrimination on the labor market then on social norms or attitude when analyzing woman labor supply/child caring issues.

References


A Appendix

A.1 Proof of Proposition 1

Equation (5) leads to an explicit solution to $E^*_c$:

$$E^*_c = \omega^{\kappa-1} B^\kappa.$$ 

We can then write the value function of the child as

$$V^* = v(w(E^*_c)) - \frac{E^*_c}{B} = \frac{(\omega B)^{\kappa-1}}{\kappa-1},$$ 

so that

$$\frac{\partial V^*}{\partial B} = \omega^{\kappa-1} B^{\kappa-2}.$$ 

The Nash equilibrium can be rewritten as the solution to the system of parents’ best response functions:

$$\frac{\partial U_p}{\partial h_p} = -(1 - \gamma)w_p u'(c_p) + \alpha_p \frac{\partial V^*}{\partial B} \frac{\partial P^*}{\partial P} = 0 \text{ for } p \in \{m, f\}.$$ 

This equilibrium can be summarized in a single equation by first exploiting (6), which combines both reaction functions, in order to express $h^*_m$ as an explicit function of $h^*_f$. Based on the assumptions made for this Lemma, the first equality in (6) can be rewritten as $\frac{c_m}{c_f} = \Theta$. Combining this equality with (3) and (4), we can express $h^*_m(h^*_f) = T - (T - h^*_f) \Psi$. Plugging this function inside one of the parents’ reaction function completes the definition of the Nash equilibrium $h^*_p$ which is such that:

$$(1 - \gamma)w_f \left( (T - h^*_f) (\gamma w_f + (1 - \gamma) \Psi w_m) \right) - \frac{1}{\beta} = \alpha_f \theta_f \omega^{\kappa-1} \beta \left( \Lambda + \beta \left( \theta_m \left( (1 - \Psi) T + \Psi h^*_f \right) + \theta_f h^*_f \right) \right)^{\kappa-2}.$$ 

Assuming $\sigma = \kappa = 1$, we obtain the formula presented in the proposition.

Finally, note that, substituting in (3) and (4), we obtain:

$$c^*_f = c_f \left( h^*_f, h^*_m(h^*_f) \right) = (T - h^*_f) (\gamma w_f + (1 - \gamma \Psi w_p).$$ 

The same reasoning applies to $E^*_c$ and $P^*_c$.

$$E^*_c = E_c \left( B \left( P^*_c \right) \right),$$ 

$$P^*_c = P \left( H_f \left( h^*_f \right), H_m \left( h^*_m \right) \right).$$ 

A.2 Comparative statics: Proof of Proposition 3

In order to do this, let us rewrite the father’s Nash equilibrium help as

$$f \left( h^*_f; E_f, E_m, a \right) = \Lambda + \beta (\theta_f + \theta_m) T - (T - h^*_f) \beta \theta_f \left( \frac{w_m}{w_f} \frac{\theta_m}{\theta_f} + \Psi + 1 + \alpha_f \frac{\gamma}{1 - \gamma} \right) = 0.$$ 

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Based on these functional forms presented in the section on comparative statics, we have that

\[
\frac{\partial \beta}{\partial E_p} = 0, \\
\frac{\partial \Lambda}{\partial E_p} = \beta \rho_p \phi, \\
\frac{\partial \theta_m}{\partial E_p} = \theta'_m = \rho_p \phi X,
\]

and

\[
\frac{\partial \beta}{\partial a} = \beta X, \\
\frac{\partial \Lambda}{\partial a} = \beta a.
\]

Effect of \( E_f \) on \( h_f \). From the implicit function theorem, we can compute

\[
\frac{\partial h_f^N}{\partial E_f} = - \frac{f_{E_f}}{f_{h_f}} = \frac{-f_{E_f}}{\beta \theta_f \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1-\gamma} \right)},
\]

where

\[
f_{E_f} = \frac{\partial \Lambda}{\partial E_f} + \beta \theta'_f T - (T - h_f^N) \beta \frac{\partial \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1-\gamma} \right)}{\partial E_f},
\]

and

\[
\frac{\partial \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi \right)}{\partial E_f} = \frac{\partial \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi \right)}{\partial w_f} \frac{\partial w_f}{\partial E_f} + \frac{\partial \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi \right)}{\partial \theta_f} \frac{\partial \theta_f}{\partial E_f},
\]

It can be shown that

\[
\frac{\partial \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi \right)}{\partial w_f} = - \frac{\theta_f}{w_f} \frac{\partial \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi \right)}{\partial \theta_f},
\]

where

\[
\frac{\partial \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi \right)}{\partial \theta_f} = \frac{\theta_m}{w_m} \frac{\frac{\gamma}{1-\gamma} \left( \theta_f \alpha_f \frac{w_m}{w_f} - \theta_m \alpha_m \frac{w_f}{w_m} \right)^2 + (1 - 2 \gamma) \theta_f \alpha_f \alpha_m \frac{w_m}{w_f} (2 \gamma \theta_m \frac{w_f}{w_m} + \theta_f \alpha_f \frac{w_m}{w_f})}{\gamma \theta_f \alpha_f \frac{w_m}{w_f} - \theta_m \alpha_m \frac{w_f}{w_m} + \gamma \theta_m \alpha_m \frac{w_f}{w_m}} = \Xi > 0.
\]

Therefore,

\[
\frac{\partial \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi \right)}{\partial E_f} = - \frac{\theta_f}{w_f} \Xi + \Xi \theta'_f
\]

\[
= \theta_f \left( \frac{\theta'_f}{\theta_f} - \frac{w'_f}{w_f} \right) \Xi
\]

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Rearranging,

\[ f_{E_f} = \beta p \phi E + \beta \theta_f T - (T - h_f^N) \beta \left[ \theta_f' \left( \left( \alpha f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi + 1 + \alpha f \frac{\gamma}{1 - \gamma} \right) + \theta_f^2 \left( \frac{\theta_f'}{\theta_f} - \frac{w_f'}{w_f} \right) \right], \]

so that

\[ \frac{\partial h_f^N}{\partial E_f} = -\frac{f_{E_m}}{f_{h_f}} = -\frac{-f_{E_m}}{f_{h_f} \left( \frac{\alpha f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f}}{\theta_f} \right) \Psi + 1 + \alpha f \frac{\gamma}{1 - \gamma}, \]

\[ f \left( h_f^N; E_f, E_m, \alpha \right) = 0. \]

where

\[ f_{E_m} = \frac{\partial A}{\partial E_m} + \beta \theta_f T - (T - h_f^N) \beta \left( \frac{\alpha f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f}}{\theta_f} \right) \Psi, \]

\[ \frac{\partial}{\partial E_m} \left( \frac{\alpha f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f}}{\theta_f} \Psi \right) = \frac{\partial}{\partial w_m} \left( \frac{\alpha f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f}}{\theta_f} \Psi \right) w_m' + \frac{\partial}{\partial \theta_m} \left( \frac{\alpha f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f}}{\theta_f} \Psi \right) \theta_m'. \]

Therefore,

\[ \frac{\partial}{\partial E_m} \left( \frac{\alpha f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f}}{\theta_f} \Psi \right) = \theta_f \left( \frac{w_f'}{w_f} - \frac{\theta_m'}{\theta_m} \right) \Psi. \]
B. It can be shown that

First, note that

Comparative statics: Proof of Proposition 6

A.4

Comparative statics: Proof of Proposition 6

It can be shown that

\[
\frac{\partial}{\partial \theta_f} \left( \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi = -\frac{\theta_f}{w_f} \Xi \Psi_f + \Xi \theta_f
\]

\[
= \theta_f \left( \frac{e'_f}{\theta_f} - \frac{w'_f}{w_f} \right) \Xi
\]

Combining,

\[
I_E = \beta \rho_m \Phi E + \beta \theta'_m T - \left( T - h_f \right) \beta \theta'_f \left( \frac{w'_m}{w_m} - \frac{\theta'_m}{\theta_m} \right) \Xi
\]

so that

\[
\frac{\partial h_N}{\partial E_m} = -\beta \rho_m \phi_E - \beta \theta'_m T + \left( T - h_f \right) \beta \theta'_f \left( \frac{w'_m}{w_m} - \frac{\theta'_m}{\theta_m} \right) \Xi
\]

\[
= -\frac{\beta \rho_m}{\beta \theta_f} \left( \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1 - \gamma}
\]

\[
= -\frac{\beta \rho_m}{\beta \theta_f} \left( \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1 - \gamma}
\]

\[
\frac{\partial h_N}{\partial E_m} = \frac{\partial \Phi E}{\partial h_m} \left( \frac{w'_m}{w_m} - \frac{\theta'_m}{\theta_m} \right) \Xi + \frac{\partial T}{\partial h_m} \left( T - h_f \right) \beta \theta'_f \Xi
\]

A.4 Comparative statics: Proof of Proposition 6

It can be shown that

\[
\frac{\partial \Pi}{\partial E_f} = \left( \frac{\theta_f}{w_f} - \frac{w'_f}{w_f} \right) \Phi_f
\]

where

\[
F = \frac{\Theta \left( 1 - \gamma (\alpha_f + \alpha_m) - 2 \alpha_f \right)}{\Theta + \gamma (\alpha_f + \alpha_m) - \left( \Theta \alpha_f + \alpha_m \right) \frac{\gamma}{1 - \gamma}}.
\]

If \( \frac{\theta_f}{w_f} > \frac{w'_f}{w_f} \), then \( \frac{\partial \Pi}{\partial E_f} > 0 \) if and only if \( F > 0 \):

\[
F > 0 \iff \Theta \left( \frac{\alpha_f w_m \theta_f}{\alpha_m \theta_m w_f} \right) > \frac{\gamma}{1 - \gamma} \frac{2 \alpha_f}{\alpha_f + \alpha_m}
\]

If \( \frac{\theta_f}{w_f} < \frac{w'_f}{w_f} \), then \( \frac{\partial \Pi}{\partial E_f} > 0 \) if and only if \( F < 0 \), that is \( \frac{\theta_f}{w_f} < \frac{\alpha_f w_m}{\alpha_m} \frac{\gamma}{1 - \gamma} \frac{2 \alpha_m}{\alpha_f + \alpha_m} \).

A.5 Comparative statics: Proof of Proposition 6

First, note that

\[
\frac{\partial E_m}{\partial \gamma} = \frac{\partial E^N}{\partial \gamma} \frac{\partial B^N}{\partial \gamma} = \frac{\partial E^N}{\partial B^N} \frac{\partial \Pi^N}{\partial \gamma} B_f
\]

It can be shown that

\[
\frac{\partial \Pi}{\partial \gamma} = \frac{\alpha_f \alpha_m \Theta^2 \left( \Theta \Theta^2 + \alpha_f \right) \left( 1 - 2 \gamma + 2 \gamma^2 \right) - 2 \Theta \gamma \left( 1 - \gamma \right) \left( \alpha_f + \alpha_m \right)}{\Theta \alpha_f \alpha_m \left( 1 - 2 \gamma \right) \left( \Theta + \gamma (\alpha_f + \alpha_m) - \left( \Theta \alpha_f + \alpha_m \right) \frac{\gamma}{1 - \gamma} \right)^2} \geq 0.
\]

\[
\left( \Theta \alpha_f \alpha_m \left( 1 - 2 \gamma \right) \left( \Theta + \gamma (\alpha_f + \alpha_m) - \left( \Theta \alpha_f + \alpha_m \right) \frac{\gamma}{1 - \gamma} \right)^2 \right) \geq 0.
\]
A.6 Comparative statics: Proof of Proposition 7

\[ f \left( h_f^N; E_f, E_m, a \right) = \Lambda + \beta \left( \theta_f + \theta_m \right) T - \left( T - h_f^N \right) \beta \theta_f \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1 - \gamma} \right) = 0. \]

\[ f_a = \beta_a - \Lambda \frac{\beta_X}{\beta}, \]

\[ \frac{\partial h_f^N}{\partial a} = \frac{\Lambda \frac{\beta_X}{\beta} - \beta_a}{\beta \theta_f \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1 - \gamma} \right)} < 0 \iff \frac{\Lambda}{\beta} < \frac{\beta_a}{\beta_X}. \]

\[ \epsilon_{B_p,a} < \frac{1}{\left( 1 + \frac{\beta_X}{\beta_a} + (\beta_p + \beta_X)(\alpha_f + (\rho_f E_f + \alpha_f \rho_m E_m)) + \rho_m (\rho_f E_f + \alpha_f \rho_m E_m) \right)} < 1 \]