Occupational choice and entrepreneurship: effect of R&D subsidy on economic growth

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Abstract

This paper constructs an overlapping-generations model including entrepreneurial innovation and an occupational choice between an entrepreneur and a worker. Taking such occupational choice into account, even a policy intended to encourage the innovation can have a negative effect on economic growth. Especially, we focus on a R&D subsidy as such a policy. While the R&D subsidy promotes R&D activities of each entrepreneur, the subsidy raises worker’s wage by increasing labor demand. Therefore, the subsidy makes it more attractive to be a worker and thus distorts an allocation of work force through the occupational choice. Due to this distortion, the subsidy has a negative effect on the growth as well as a positive one. Because of these two effects, a relation between the subsidy and the growth is inverted-U shaped and there exists a growth-maximizing R&D subsidy rate. However, such growth-maximizing rate is too high to maximize the welfare level of any one of generations. We also show that when individuals are heterogeneous in their abilities, the R&D subsidy reduces intra-generational inequalities.

Keywords: Occupational choice, Entrepreneurship, R&D subsidy

JEL Classification: J24, O31, O41

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1 Introduction

As is generally known, technological innovation is a main determinant of economic growth. Many R&D based growth models have been constructed to explain the growth (Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992)). As pointed out by Wong et al. (2005), however, although these studies recognize some aspects of entrepreneurship by describing the innovation process, most of these studies do not explicitly take account of roles of entrepreneurs who set up business and bring the innovation. In most cases, entrepreneurs are those who could choose other jobs instead of entrepreneurs, that is, entrepreneurs arise from an occupational choice. Therefore, entrepreneurship has its origin in the occupational choice between entrepreneurs and workers.

This paper argues that by explicitly focusing on entrepreneurs and the occupational choice, some policy implications arise. That is, even a policy intended to promote economic growth does not always accelerate the growth. The analysis is conducted on an overlapping-generations model where each individual chooses one’s own occupation, an entrepreneur or a worker, and the engine of economic growth is the entrepreneurial innovation. In this paper, a government gives a R&D subsidy to entrepreneurs. Taking the occupational choice into account, the R&D subsidy has a negative effect on economic growth as well as a positive one. Because of these two effects together, a relation between the subsidy and the growth becomes inverted-U shaped and there exists a growth-maximizing R&D subsidy rate. This is because the subsidy on the one hand encourages the entrepreneurial innovation and on the other hand reduces the number of entrepreneurs by making it more attractive to become a worker. This latter negative effect is not due to taxation for financing the R&D subsidy but due to the R&D subsidy itself and arise from the occupational choice between entrepreneurs and workers. Therefore, too high R&D subsidy rate can be detrimental to the economic growth through the occupational choice. We also study welfare analysis and show that the growth-maximizing R&D subsidy rate is higher than the rate that maximizes the welfare level. Moreover, when individuals are heterogeneous in their abilities, the R&D subsidy decreases intra-generational inequalities.

In terms of the occupational choice between entrepreneurs and workers, this paper can
be related to many studies \(^1\). Especially, this paper closely related to studies including the innovation caused by entrepreneurs (or innovators) as well as the occupational choice, such as Keuschnigg (2004), García-Peñalosa and Wen (2008) and Jaimovich and Rebelo (2012). Keuschnigg (2004) investigates how venture capitalists who finance and advise entrepreneurs boosts the economic growth. García-Peñalosa and Wen (2008) shows that redistributive taxation increases the economic growth rate because it provides insurance to entrepreneurs and makes more agent to engage risky entrepreneurial activities. In these two studies, the innovation takes the form of the Quality-Ladder like Aghion and Howitt (1992). In contrast, Jaimovich and Rebelo (2012) consider the innovation so-called the Variety-Expansion which is in the spirit of Romer (1990) and constructs a model where the effect of the corporate tax on the growth is highly non-linear. In these studies considering the Variety-Expansion type or the Quality-Ladder type of innovation, input for the R&D activities are entrepreneurs (or innovators) themselves and their effort and thus the more entrepreneurs (or innovators), the faster the economic growth is. This study instead assumes what is called in-house R&D such as Young (1998) and by so doing the growth is determined not only by the number of entrepreneurs but also by investment for R&D activities of each entrepreneur. This assumption is crucial for the inverted-U relation between the economic growth and the R&D subsidy since the R&D subsidy increases labor demand used for the R&D activities and makes the workers more attractive than entrepreneurs.

Concerning the in-house R&D, this study is also related to Grossmann (2007) and Grossmann (2009). The former constructs an in-house R&D model that incorporates the occupational choice among unskilled labor, skilled labor for production and skilled labor for R&D activities. In contrast to Grossmann (2007) in which the R&D activities are conducted by firms, this paper investigates under the setting where entrepreneurs are those who conduct the R&D activities. The latter is more closely related to this paper since Grossmann (2009) provides an economic growth model without the scale effect by modeling entrepreneurial in-house R&D. In his settings, individuals live for two period and work as workers in the first period. In the second period of their lives, individuals choose whether become entrepreneurs.

or not. Entrepreneurs found firms, invest in the R&D activities and produce goods whereas those who do not become entrepreneurs only consume their savings and do not supply labor. That is, there does not exist the choice between entrepreneurs and workers. We instead assume individuals should choose workers or entrepreneurs and these two occupations are incompatible. This assumption is also important for the negative effect of the R&D subsidy since the negative effect arises from the occupational choice between entrepreneurs and workers.

This paper is organized as follows. Section 2 presents a setup of the basic model where all individuals in one generation are homogeneous in their abilities. Section 3 solves for the equilibrium and investigates how the R&D subsidy affects economic growth and welfare of each generation. Section 4 extends the model to include heterogeneity of abilities of individuals and shows that such extension does not alter main results. Moreover, we investigate how intra-generational inequalities which arise from the heterogeneity are affected by the subsidy. Finally, section 5 provides some concluding remarks.

2 The model

We consider the following overlapping-generations economy. Time is discrete and indexed by $t$ and the economy is closed. Each generation is populated by $L$ individuals and there is no population growth. Individuals in each generation live for two periods: youth and adulthood. In youth, individuals choose their future occupations between entrepreneurs and workers. When they have grown up, entrepreneurs found intermediate-good firms and workers supply labor. Each intermediate-good firm invests in R&D activities and produces one variety of differentiated goods. There are two sectors in this economy, that is, a final-good sector and an intermediate-good sector. The former is perfectly competitive whereas the latter is monopolistically competitive. In addition, there is a government which subsidizes the R&D activities and imposes income tax.
2.1 Individuals

In youth, each individual chooses whether to become an entrepreneur or a worker. Moreover, each individual is endowed with one unit of time and allocates it to leisure and effort. An individual who decides to become an entrepreneur should make an effort to get an entrepreneurial skill which is necessary to manage a firm and to supervise the R&D activities. It takes \( z \in (0, 1) \) units of time to acquire such skill. Therefore, individuals who want to be entrepreneurs enjoy only \( 1 - z \) units of time as leisure whereas those who will be workers enjoy full time leisure.

In adulthood, skilled individuals become entrepreneurs and earn monopolistic profits by producing differentiated goods. Unskilled ones become workers and get wages by supplying labor. And then all adult individuals use all of their disposal incomes for consumption of final goods. Since they are economically active in their adulthood, we call adults in period \( t \) generation \( t \) or the \( t \)th generation.

Individuals receive utilities from leisure in youth and consumption in adulthood. Therefore, the utility function of an individual \( i \) in \( t \)th generation can be represented as:

\[
u_t(i) = \ln d_{t-1}(i) + \beta \ln c_t(i), \tag{1}\]

\( \beta \in (0, 1) \), where \( d_{t-1}(i) \) and \( c_t(i) \) represent leisure and consumption of individual \( i \), respectively. Since it takes \( z \) units of time for him to be skilled, his leisure is:

\[
d_{t-1}(i) = \begin{cases} 
1 - z, & \text{if } i \text{ is an entrepreneur} \\
1, & \text{otherwise}
\end{cases}. \tag{2}\]

Let \( I_t(i) \) denote his income at time \( t \); his budget constraint can be written as:

\[
p_{c,t}c_t(i) = (1 - \tau_t)I_t(i), \tag{3}\]

where \( p_{c,t} \) and \( \tau_t \in [0, 1) \) are a price of the final goods and an income tax rate in period \( t \), respectively. Let us denote monopolistic profit of firm \( i \) and wage by \( \Pi_t(i) \) and \( w_i \), respectively; therefore, individual \( i \)'s income is:

\[
I_t(i) = \begin{cases} 
\Pi_t(i), & \text{if } i \text{ is an entrepreneur} \\
w_i, & \text{otherwise}
\end{cases}. \tag{4}\]

Each individual chooses his own occupation to maximize (1) subject to (2), (3) and (4).
2.2 Final-good sector

The final-good sector is perfectly competitive and there is one representative firm. The final-good firm combines \( n \) kinds of intermediate goods to produce the final good \( Y_t \) in period \( t \) according to the following constant-returns-to-scale technology:

\[
Y_t = \left[ \int_0^n x_t(j)^{\frac{\eta+1}{\eta}} d j \right]^{\frac{1}{\eta+1}},
\]

\( \eta > 1 \), where \( x_t(j) \) is \( j \)th intermediate-good input. Given the price of the final good \( (p_{c,t}) \) and the ones of intermediate goods \( (p_t(j)) \), the firm maximizes its profit. Because of the perfect competition, the final-good firm earns zero profit, that is, \( p_{c,t}Y_t = \int_0^n p_t(j)x_t(j)dj \). Since the production function is in the spirit of Dixit and Stiglitz (1977), the first order conditions for the profit maximization and the zero-profit condition yield well-known following demand functions:

\[
x_t(j) = \frac{p_t(j)^{\frac{-\eta}{\eta}}}{P_t^{\frac{1}{\eta}}} p_{c,t} Y_t, \forall j \in [0, n].
\]

where \( P_t = \left[ \int_0^n p_t(j)^{1-\eta} d j \right]^{-\frac{1}{\eta}} \) is a price index of the intermediate-goods. We set the nominal GDP as the numeraire, i.e., \( p_{c,t}Y_t = 1 \).

2.3 Intermediate-good sector

The adult who acquired the skill in his youth can become an entrepreneur \(^2\). Each entrepreneur found an intermediate-good firm. The intermediate-good firms first invest in the R&D activities to raise their own productivities and second manufacture the intermediate goods. Each firm produces one variety of differentiated goods and thus the number of entrepreneurs is equivalent to the number of intermediate goods, \( n_t \). Moreover, the market of the intermediate goods is monopolistically competitive.

We assume that the \( j \)th intermediate good is produced by the following linear technology:

\[
x_t(j) = A_t(j) l_t^j(j),
\]

\(^2\)Skilled individuals can also become workers instead entrepreneurs. But in the equilibrium, anyone does not so.
where $A_t(j)$ and $l_t^x(j)$ are productivity and labor input for production of firm $j$, respectively. Let $\bar{A}_t = 1/n_t \int_0^n A_t(j) dj$ denote the average productivity in period $t$.

The productivity, $A_t(j)$, is determined by its R&D activities. Moreover, there is intertemporal knowledge spillover, that is, the more knowledge is accumulated, the more productive the R&D activities are. We specify the R&D technology as:

$$A_t(j) = \theta K_{t-1}[l_t^R(j)]^\gamma,$$

where $\theta > 0$ and $\gamma > 0$, where $K_{t-1}$ is knowledge accumulated by previous R&D activities and $l_t^R(j)$ is labor input for the R&D activities of firm $j$. We specify the knowledge accumulation as $K \equiv \bar{A} n^{1-\varepsilon}$ where $\varepsilon \in [0, 1]$. When $\varepsilon$ is equal to zero, each entrepreneur accumulates completely different knowledge from each other and $K$ is the sum of all productivities. $\int_0^n A_t(j) dj$. In contrast, if $\varepsilon$ is equal to one, each entrepreneur accumulates the same knowledge and $K$ is merely the average productivity, $\bar{A}$. Thus, $\varepsilon$ represents an extent of duplication of knowledge. Moreover, this specification implies that R&D activities create proprietary knowledge for only one period and it will be public one in the next period.

Each intermediate-good firm maximizes its net profit, $\Pi_t$. Since R&D activities are subsidized, a profit maximization problem of intermediate-good firm $j$ is:

$$\Pi_t(j) \equiv \max \left\{ p_t(j)x_t(j) - w_t l_t^x(j) - (1 - \mu_t)w_t^R l_t^R(j) \right\},$$

subject to (6), (7) and (8), where $\mu_t$ is a R&D subsidy rate at period $t$. By solving the problem (see Appendix), we obtain the optimal labor input as:

$$l_t^x = \frac{\eta-1}{n_t w_t} \frac{1}{\eta},$$

$$l_t^R = \frac{\eta-1}{\eta} \frac{\gamma}{1-\mu_t n_t w_t} \frac{1}{\eta}.$$  

Hereafter we assume $1 - \gamma(\eta - 1) > 0$ in order to satisfy the second order condition for maximization. Using (6)–(11), we obtain the following maximum net profits for entrepreneurs:

$$\Pi_t = \frac{1}{\eta m_t} [1 - \gamma(\eta - 1)].$$  

Because of ex ante homogeneity of individuals, all intermediate-good firms behave in the same way. Thus we omit index $j$ whenever this does not lead to confusion.

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3In section 4, we develop the model to include ex ante heterogeneity of individuals and show that qualitative results do not change.
Although the R&D subsidies seem to be beneficial to entrepreneurs, it does not affect the entrepreneurs’ profits. The R&D subsidy increases the labor input for the R&D activities and thus raises productivities of all intermediate-good firms. Each intermediate-good firm enjoys a benefit by increasing its own productivity. At the same time, however, an increase of the productivities of all intermediate-good firms makes the price index of intermediate goods lower and this harms entrepreneurial profits. Therefore the R&D subsidy is harmful as well as beneficial for entrepreneurs. And these two offset each other. This is why the subsidy does not affect the profit. In contrast to the subsidy, the number of the firms decreases the profits. This is simply because an increment of the number of the firms intensifies the competition.

2.4 Government

The government subsidizes the R&D activities and imposes income tax. Its budget balances in every period. Thus, its budget constraint of period $t$ is:

$$
\tau_t \left( \int_0^L L_r(i) \, di \right) = n_t r_w \mu_t I_r^e.
$$

Assume that the government provides a constant rate of subsidy through time, i.e., $\mu_t = \mu$ for all $t$.

3 Equilibrium

This section derives the equilibrium properties. We first describe a labor market clearing condition and individual decisions. These two determine the equilibrium wage rate and the number of entrepreneurs in the equilibrium. Then we compute the equilibrium income tax rate. Using these variables, we derive the economic growth rate and the welfare of one generation. Then we investigate how the R&D subsidy affects the growth rate and the welfare.

3.1 Market equilibrium and occupational choices

Since workers—the adults other than entrepreneurs—inelastically supply one unit of labor, labor supply at period $t$ is $L - n_t$. These labor is demanded by the intermediate-good firms to
invest the R&D activities and to produce the goods. Thus a labor market clearing condition is:

\[ \int_0^{n_t} l_t^j + l_t^R d j = L - n_t. \]  

(14)

Using (10) and (11), (14) can be rewritten as:

\[(L - n_t)w_t = \frac{\eta - 1}{\eta} \left( 1 + \frac{\gamma}{1 - \mu} \right).\]  

(15)

This determines the wage rate, \( w_t \), given the number of entrepreneurs, \( n_t \).

In each period, young individuals choose their own occupation to maximize their own utilities. Thus they want to become entrepreneurs if the utility of an entrepreneur is higher than that of a worker and vice versa. Therefore, the number of entrepreneurs is determined by the following condition:

\[ \ln \left[ \frac{(1 - \tau_t)w_t}{p_{c,j}} \right]^{\beta} = \ln(1 - z) \left[ \frac{(1 - \tau_t)\Pi_t}{p_{c,j}} \right]^{\gamma}. \]  

(16)

The left hand side of (16) is the utility of a worker and the right hand side is one of an entrepreneur. This means that in the equilibrium every individual in one generation obtains the same level of utilities. Substituting (12) into (16) and rearranging it, we obtain:

\[ w_t = (1 - z)^{\frac{1}{\beta}} \frac{1 - \gamma(\eta - 1)}{\eta n_t}. \]  

(17)

This determines the number of entrepreneurs, \( n_t \), given the wage rate, \( w_t \).

(15) and (17) determines the equilibrium wage rate and the number of entrepreneurs as follows:

\[ w_t = \left\{ 1 + Z \left( 1 + \frac{\gamma}{1 - \mu} \right) \right\} \frac{\eta - 1}{\eta Z L}, \]  

(18)

\[ n_t = \left\{ 1 + Z \left( 1 + \frac{\gamma}{1 - \mu} \right) \right\}^{-1} L. \]  

(19)

where

\[ Z = \frac{\eta - 1}{(1 - z)^\beta [1 - \gamma(\eta - 1)]} > 0. \]

\(^4\) In addition to the labor market, there are a final-good market and many intermediate-good markets. Each of the intermediate-good markets is cleared because the intermediate-good firms behave taking the demands for those goods into account. Moreover, if the labor market is cleared, the final-good market is also cleared due to the Walras’ law.
Note that $w_t$ and $n_t$ are time-invariant. The R&D subsidy, $\mu$, raises the wage because it increases labor demand for the R&D activities. In contrast, the R&D subsidy decreases the number of entrepreneurs. The R&D subsidy does not affect the entrepreneurs’ profits, $\Pi$ given the number of entrepreneurs, whereas it increases the wage. Thus, an increment of the R&D subsidy rate makes it less attractive to be an entrepreneur. This is why the number of entrepreneurs declines.

There is one more variable to be determined—income tax rates, $\tau_t$. Using (4), (11), (12), (13) and (15), we derive the tax rates as follows:

$$\tau_t = \frac{\frac{\gamma}{\eta} \frac{\mu}{1-\mu}}{1 + \frac{\gamma_{t-1}}{\eta} \frac{\mu}{1-\mu}}.$$

(20)

Note that $\tau_t$ is also time-invariant as well as other variables.

### 3.2 Economic growth

We have already obtained all of the equilibrium variables in period $t$. Then we define an economic growth rate as a growth rate of output level of the final-good, $g$. Using (5), (7), (8), (10), (11), (18) and (19), the growth rate can be represented by:

$$1 + g_t = \frac{Y_t}{Y_{t-1}} = \frac{\bar{A}_t}{\bar{A}_{t-1}} = \frac{K_t}{K_{t-1}}$$

$$= (1 - \mu)^{-\gamma} \left( 1 + Z \left( 1 + \frac{\gamma}{1 - \mu} \right) \right)^{\frac{1}{1-\varepsilon}} \theta(\gamma Z)^{\varepsilon} L^{1-\varepsilon}.$$

(21)

The growth rate is constant over time as long as the R&D subsidy rate is constant. If $\varepsilon$ is equal to one, the growth rate is independent on population size $L$. Thus this model can replicate the scale-invariant growth.

The economy starts from period 0 where there are adults of the 0th generation and youths of the 1st generation given an initial knowledge level $K_{-1}$. We assume that $n_0$ is determined by (19), that is, individuals of the 0th generation, in their youth, determine their occupation foreseeing the next period. By so doing, the economy jumps to the balanced growth path at period 0 where $Y$, $K$ and $A$ grow at the constant rate $g$ and $w$, $n$ and $\tau$ are constant over time.
Then, we analyze how the R&D subsidy affects the growth. Differentiating (21) with respect to \( \mu \) yields:

\[
\frac{dg}{d\mu} = \frac{\gamma(1 + g)}{1 - \mu} \frac{1 - \mu + Z(\varepsilon + \gamma - \mu)}{1 - \mu + Z(1 + \gamma - \mu)}.
\]

(22) shows that the R&D subsidy does not always promote the growth. We can state the following proposition with respect to the relationship between the R&D subsidy and the growth rate.

**Proposition 1.** Assume \( \varepsilon + \gamma < 1 \). Then the economic growth rate is an inverted-U shaped function of the R&D subsidy rate, and maximized at:

\[
\mu = \hat{\mu} \equiv \frac{1 + Z(\varepsilon + \gamma)}{1 + Z} \in (0, 1).
\]

We can give the following intuition of this inverted-U relation as follows. The determinant of the growth is the intertemporal knowledge spillover represented by \( K \) and this knowledge is accumulated by entrepreneurs as \( K = \bar{A}n^{1-\varepsilon} \). On accumulation of knowledge, the R&D subsidy has two opposite effects: positive and negative effects. The subsidy on the one hand raises average productivity of intermediate-good firms by increasing labor inputs for R&D activities (positive effect) and on the other hand decreases the number of entrepreneurs (negative effect). When the R&D technology exhibits decreasing returns to scale \( (\gamma < 1) \), the marginal productivity of R&D gradually decreases, and this diminishes the positive effect. Meanwhile the negative effect is strong when duplication of knowledge represented by \( \varepsilon \) is sufficiently small. Therefore, if \( \varepsilon + \gamma < 1 \), the positive effect declines as the R&D subsidy rate increases and the negative one overcomes the positive one when the subsidy rate is sufficiently large.

Note that the income tax is neutral on the growth, that is, there is no effect of the R&D subsidy through the income tax on the economic growth. The knowledge spillover—the determinant of the growth—depends on an allocation of work forth, \( n, l^x \) and \( l^R \). Since the income tax are equally imposed on entrepreneurs and workers, it does not distort the occupational choice between entrepreneurs and workers. Moreover, because the firms are not imposed the tax, the tax rate is neutral on firms’ activities, that is, it does not distort an allocation between labor input for production and that for the R&D activities. Consequently, the
tax affects neither the occupational choices nor the firms’ behaviors and thus the growth is independent of it. The negative effect of the subsidy described above is not via the taxation but purely due to the R&D subsidy itself. Therefore, Proposition 1 implies that even policies that intend to encourage the innovation such as the R&D subsidy can be detrimental to the growth. This results arise from the occupational choices of individuals.

3.3 Welfare analysis

We next examine how the R&D subsidy affects on welfare of one generation. Recall that all individuals in one generation receive the same level of utility and there is no population growth. Thus we define the welfare of generation $t$ as an indirect utility of a worker in that generation and denote it as $v_t$. Substituting the equilibrium variables into the worker’s utility $(1 - \tau)w/p_c$ and using a relation $p_{c,t} = Y_t^{-1}$, we can write $v_t$ as:

\[
v_t = \beta \left\{ -\ln \left[ 1 + \gamma \frac{\eta - 1}{\eta} \left( \frac{1}{1 - \mu} \right) \right] \\
- \frac{1}{\eta - 1} \ln \left[ 1 + Z \left( 1 + \frac{\gamma}{1 - \mu} \right) \right] - \gamma \ln(1 - \mu) + t \ln(1 + g) \right\} + \Delta, \tag{24}
\]

where

\[
\Delta \equiv \beta \ln \left\{ \theta K_{-1} \frac{\eta - 1}{\eta} L^{\frac{1}{\eta}} (\gamma Z)^{\gamma} \right\}.
\]

Since we assume the R&D subsidy rate is constant over time, we analyze the effect of perpetual change of the R&D subsidy rate at period 0 on the welfare. Moreover, in order to investigate whether the growth-maximizing R&D subsidy rate is desirable or not, we assume $\varepsilon + \gamma < 1$, that is, the growth-maximizing R&D subsidy rate is less than one. Then, We get a following proposition.

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5If we consider the other way of taxation, the results slightly change. For example, assume that the government imposes a corporate tax on firms’ profits. Then, firms do not change its behaviors but the tax makes it less attractive to be an entrepreneur because it reduces only entrepreneurial incomes. So, an increase of the R&D subsidy rate decreases the number of entrepreneurs through an increment of the tax rate, that is, another negative effect arises. For this new effect, the growth maximizing subsidy rate will be less than $\hat{\mu}$. 

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Proposition 2. Assume $\varepsilon + \gamma < 1$ and let $\mu^*_t$ denote the R&D subsidy rate which maximizes the welfare of generation $t$. Then the welfare-maximizing R&D subsidy rates are:

$$
\mu^*_t = \begin{cases} 
0, & \text{for } t \leq \bar{t} \\
(0, \hat{\mu}), & \text{for } t > \bar{t}
\end{cases}
$$

(25)

where

$$
\bar{t} \equiv \frac{(1 - z)^{-1/\beta} - 1}{(1 + \varepsilon Z + \gamma Z)\eta} > 0.
$$

Proof. See Appendix.

Early generations, at least the initial adults, do not hope the R&D subsidy whereas later generations hope the positive subsidies. This is because the later generations are, the greater benefit they enjoy from the growth. Although the future generations want to make the growth rate higher, however, the growth-maximizing subsidy rate is too high for all generations. Therefore, an optimal R&D subsidy rate which maximizes a discounted sum of welfare of each generations is less than the growth-maximizing R&D subsidy rate.

4 Extensions

The R&D subsidy on the one hand raises productivity of each intermediate-good firm and on the other hand reduces the number of the firms. Because of these two effects, an inverted-U relation between the growth and the R&D subsidy arises. The reason why the number declines is that the R&D subsidy raises only worker’s wage and does not affect entrepreneurial profit, that is, the R&D subsidy makes it less attractive to be an entrepreneur. One may suspect, however, that the entrepreneurial profits (12) are not affected because all entrepreneurs behave in the same way and thus the homogeneity of individuals is crucial for the inverted-U.

In this section, we extend the model to include a heterogeneity of individuals and show that the homogeneity is not crucial for the present results. We consider the heterogeneity of entrepreneurial abilities which are used for managing firms and for supervising the R&D activities. We assume this ability affects efficiency of the R&D activities, that is, the R&D efficiency $\theta$ differs among individuals. We call $\theta_j \in [\theta_L, \theta_H]$ the entrepreneurial ability of individual $j$. Let $F_\theta$ denote the distribution function of $\theta$ and be time-invariant.
This extension alters the intermediate-good firms’ behaviors as well as the individuals’ occupational choices whereas the final-good firm’s behavior does not change. Solving \( j \)th intermediate-good firm’s problem (see Appendix), we obtain the following optimal labor inputs:

\[
\begin{align*}
    l^x(j) &= \frac{\eta - 1}{\eta} \frac{\theta_j^{\frac{\eta-1}{\eta-1}}}{w_t \int_0^{n_j} \theta_k^{\frac{-1}{\eta-1}} dk}, \\
    l^R(j) &= \frac{\eta - 1}{\eta} \frac{\gamma}{1 - \mu} \frac{\theta_j^{\frac{\eta-1}{\eta-1}}}{w_t \int_0^{n_j} \theta_k^{\frac{-1}{\eta-1}} dk},
\end{align*}
\]  

(26)  

(27)

and net profit of entrepreneur \( j \) as follows:

\[
\Pi_j(j) = \frac{1 - \gamma(\eta - 1)}{\eta} \frac{\theta_j^{\frac{\eta-1}{\eta-1}}}{\int_0^{n_j} \theta_k^{\frac{-1}{\eta-1}} dk}.
\]  

(28)

The optimal labor input for the R&D activities is increasing in \( \mu \). Thus the R&D subsidy promotes the productivity of each intermediate-good firm (positive effect). Moreover, the net profit of entrepreneur (28) does not affected by the R&D subsidy as in the section 2.

Since \( \Pi_j(j) \) is increasing in \( \theta_j \), there exists a threshold ability, \( \tilde{\theta} \) such that, individual \( i \) is an entrepreneur if and only if \( \theta_i \geq \tilde{\theta} \). Because a threshold individual are indifferent in becoming an entrepreneur or a worker, \( \tilde{\theta} \) satisfies the following condition:

\[
w_t = \frac{\eta - 1}{\eta ZL} \frac{\tilde{\theta}^{\frac{-1}{\eta-1}}}{\int_{\tilde{\theta}}^{\theta_{\text{hi}}} \theta^{\frac{-1}{\eta-1}} dF_{\theta}(\theta)}.
\]  

(29)

Using the distribution function of entrepreneurial ability, the number of entrepreneurs, \( n_t \), and of workers, \( L - n_t \), can be written as:

\[
n_t = \int_{\tilde{\theta}}^{\theta_{\text{hi}}} dF_{\theta}(\theta)L = [1 - F_{\theta}(\tilde{\theta})]L,
\]  

\[
L - n_t = \int_{\theta_{L}}^{\tilde{\theta}} dF_{\theta}(\theta)L = F_{\theta}(\tilde{\theta})L.
\]  

(30)

By substituting (26) and (27) into the labor market clearing condition (14), one can find that the condition becomes the same as (15).
Using (15), (29) and (30), we obtain:

\[
\tilde{\theta}^{\frac{\nu+1}{\nu+\eta-1}} \frac{F_\theta(\tilde{\theta})}{\int_0^{\tilde{\theta}} \theta^{\frac{\nu+1}{\nu+\eta-1}} dF_\theta(\theta)} = Z \left( 1 + \frac{\gamma}{1 - \mu} \right).
\]  

(31)

This condition determines the threshold ability and the number of entrepreneurs. The left hand side of (31) is increasing in \( \tilde{\theta} \) and the right hand side is increasing in \( \mu \). Thus the threshold ability \( \tilde{\theta} \) is increasing in the R&D subsidy. Therefore the R&D subsidy reduces the number of entrepreneurs (negative effect).

From the above, the positive and negative effects of the R&D subsidy on the growth are still alive even though the model includes heterogeneity, and so do the inverted-U relation between the R&D subsidy and the growth. That is, the homogeneity is not crucial for the inverted-U and the results of Proposition 1.

Introducing the heterogeneity brings inter-generational inequalities hence we can study how the R&D subsidy affect it. From (28) and (29), the difference between utility of entrepreneur with ability \( \theta_j \) and that of a worker is:

\[
\ln(1 - z) + \beta \ln\left( \frac{(1 - \tau_t)\Pi_t(j)}{p_{c,t}} \right) - \beta \ln\left( \frac{(1 - \tau_t)w_t}{p_{c,t}} \right) = \frac{\beta(\eta - 1)}{1 - \gamma(\gamma - 1)} \ln\left( \frac{\theta_j}{\tilde{\theta}} \right).
\]  

(32)

This difference is decreasing in \( \tilde{\theta} \) which is increasing in \( \mu \). Therefore, the R&D subsidy decreases the intra-generational inequality. This is because the R&D subsidy raises the wage by increasing labor demand remaining the profit of intermediate-good firms constant.

5 Conclusion

This paper provides a model that incorporates the entrepreneurial innovation and the occupational choice between entrepreneurs and workers. Taking such occupational choice into account, even a policy intended to encourage the innovation can have a negative effect on economic growth as well as a positive one. This paper considers a R&D subsidy which is financed by income tax. Of course the subsidy makes the R&D activities more active and raises labor demand to input into the activities. This increment of labor demand raises the worker's income and it becomes less attractive to become entrepreneurs. That is, the R&D subsidy, on the one hand, promotes the R&D activities and, on the other hand, reduces the
number of entrepreneurs who found firms and practice the R&D activities. Because of these
two effects, a relation between economic growth and the R&D subsidy becomes inverted-U
shaped and there exists a growth-maximizing R&D subsidy rate. Thus even a policy such as
the R&D subsidy does not always promote growth. We also study welfare analysis. Then, we
find that such growth-maximizing rate is too high at the viewpoint of welfare. Moreover, if
individuals are heterogeneous in their abilities, the R&D subsidy reduces intra-generational
inequalities.

Appendix

Intermediate-good firms’ problems

There are two steps in the intermediate-good firms’ problems. First, the intermediate-good
firms invest the R&D activities and secondly produce the intermediate goods. We solve the
problems backward. In the second step, we maximize an intermediate-good firm’s profit from
the production given its productivity and the R&D outlay. Then we turn back to the first step
and decide how much the intermediate-good firms should invest in the R&D activities.

\[ p_t(j) = \frac{\eta}{\eta - 1} A_t(j), \]

\[ x_t(j) = \frac{\eta}{\eta - 1} \frac{w_t}{A_t(j)} \]

Substituting (A.2) and (A.3) into (A.1) yields the profit function as:

\[ \pi_t(A_t(j)) = \frac{(\eta - 1)^{\eta - 1}}{\eta^\eta} \left[ \frac{A_t(j)}{w_t} \right]^{\eta - 1}. \]
Then turn back to the first step. The object of this step is to maximize the intermediate-good firms’ net profits, $\Pi_i$. Since R&D activities are subsidized, the net profit of $j$th intermediate-good firm is:

$$\Pi_i(j) \equiv \max\{\pi_i(A_i(j)) - (1 - \mu)w_i l_i^R(j)\},$$  \hspace{1cm} (A.5)

subject to (8) and (A.4). The first order condition with respect to $l_i^R(j)$ is:

$$\gamma\left(\frac{\eta - 1}{\eta}\right)\left[\frac{\theta K_{i-1}}{w_i}\right]^{\eta - 1} [l_i^R(j)]^{\eta (\eta - 1) - 1} = (1 - \mu)w_i.$$  \hspace{1cm} (A.6)

Equation (A.6) implies that $l_i^R(j)$ is independent on $j$ and so are $A_i(j)$ and $p_i(j)$. In addition, it is necessary to assume $1 - \gamma(\eta - 1) > 0$ in order to satisfy the second order condition. Since all firms behave in the same way, the price index becomes:

$$P_i = \frac{n_i^{\frac{1}{\eta}}}{\eta - 1} \frac{\eta}{\bar{A}_i} w_i.$$  \hspace{1cm} (A.7)

By substituting (A.7) into (A.6) and rearranging it, we get the optimal level of labor input for R&D activities as (11). Moreover, substituting (A.3) and (A.7) into (7) yields (10).

**Proof of proposition 2**

**Proof.** We have to show that (i) $v_t$ is maximized at $\mu = 0$ for $t \leq \bar{t}$, (ii) $v_t$ is maximized at an internal point $\mu^* \in (0, 1)$ for $t > \bar{t}$, and (iii) such $\mu^*$ is less than the growth-maximizing rate $\hat{\mu}$.

Differentiating (24) with respect to $\mu$ yields:

$$\frac{dv_t}{d\mu} = \frac{\beta\gamma}{1 - \mu}\left\{1 - \frac{\eta - 1}{\eta(1 - \mu) + \gamma(\eta - 1)\mu} - \frac{Z}{(\eta - 1)[1 - \mu + Z(1 + \gamma - \mu)]}\right\} + \beta t \frac{dg/d\mu}{1 + g},$$  \hspace{1cm} (A.8)

$$= \frac{\beta\gamma}{(1 - \mu)[\eta(1 - \mu) + \gamma(\eta - 1)\mu][1 - \mu + Z(1 + \gamma - \mu)]} \Phi_i(\mu),$$  \hspace{1cm} (A.9)

where

$$\Phi_i(\mu) \equiv \phi_{1,i}\mu^2 + \phi_{2,i}\mu + \phi_{3,i},$$  \hspace{1cm} (A.10)

$$\phi_{1,i} \equiv (1 + t)(1 + Z)[\eta - \gamma(\eta - 1)] > 0,$$

$$\phi_{2,i} \equiv -\left\{[\eta - \gamma(\eta - 1)][1 + Z - \frac{1 - \gamma(\eta - 1)}{\eta - 1}Z + (1 + \epsilon Z + \gamma Z)t] + (1 + Z)(1 + \eta t)\right\},$$

$$\phi_{3,i} \equiv 1 - \frac{1 - \gamma(\eta - 1)}{\eta - 1}Z + (1 + \epsilon Z + \gamma Z)\eta t.$$
Note that \( \frac{1}{(1-\mu)[\eta(1-\mu)+\gamma(1-\gamma\mu)][1-\mu+Z(1+\gamma\mu)]} > 0 \) for all \( \mu \in [0, 1] \), that is, the sign of \( dv_\mu/d\mu \) is the same as the one of \( \Phi_\mu \). Note also that \( \Phi_\mu \) is a continuous, quadratic and convex function of \( \mu \).

By substituting \( \mu = 0, 1 \)—the lower and the upper limits of \( \mu \), respectively—into \( \Phi_\mu(\mu) \), we obtain:

\[
\Phi_\mu(0) = 1 - \frac{1 - \gamma(\eta - 1)}{\eta - 1}Z + (1 + \gamma Z + \gamma Z\eta)\eta, \\
\Phi_\mu(1) = -Z\gamma[\eta - \gamma(\eta - 1) + (\eta - 1)\eta] < 0, \ \forall t.
\]

\( \Phi_\mu(0) \) is negative if and only if \( t < \tilde{t} \equiv \frac{(1-\gamma)^{1-\beta}-1}{(1+\gamma Z+\gamma Z\eta)} \) whereas \( \Phi_\mu(1) \) is always negative.

(i) When \( t \) is less than or equal to \( \tilde{t} \), \( \Phi_\mu(0) \) is non-positive and \( \Phi_\mu(1) \) is negative. Since \( \Phi_\mu \) is quadratic and convex, \( \Phi_\mu \) is non-positive for all \( \mu \in [0, 1] \) and so do \( dv_\mu/d\mu \). That is, \( v_\mu \) is a non-increasing function for the interval \( [0, 1] \) and maximized at \( \mu = 0 \).

(ii) When \( t \) is larger than \( \tilde{t} \), on the other hand, \( \Phi_\mu(0) \) is positive whereas \( \Phi_\mu(1) \) is negative. Since \( \Phi_\mu \) is continuous, by the Intermediate value theorem, there exists at least one \( \mu \in (0, 1) \) at which \( \Phi_\mu \) becomes zero. Because of the functional form of \( \Phi_\mu \) (quadratic and convex), such \( \mu \) is unique and \( \Phi_\mu \) is downward-sloping at such \( \mu \). Let \( \mu^*_t \) denote such \( \mu \), then we get \( \Phi_\mu(\mu^*_t) = 0 \) and \( \Phi'_\mu(\mu^*_t) < 0 \). Then, it is found that \( \mu^*_t \) satisfies the first and second order conditions for maximization because:

\[
\frac{dv_\mu}{d\mu}\bigg|_{\mu=\mu^*_t} = \frac{\beta\gamma}{(1-\mu^*_t)[\eta(1-\mu^*_t)+\gamma(\eta-1)\mu^*_t][1-\mu^*_t+Z(1+\gamma-\mu^*_t)]} \Phi_\mu(\mu^*_t) = 0, \\
\frac{d^2v_\mu}{d\mu^2}\bigg|_{\mu=\mu^*_t} = \frac{\beta\gamma}{(1-\mu^*_t)[\eta(1-\mu^*_t)+\gamma(\eta-1)\mu^*_t][1-\mu^*_t+Z(1+\gamma-\mu^*_t)]} \Phi'_\mu(\mu^*_t) < 0.
\]

(iii) Finally, we show that welfare-maximizing rates of R&D subsidy, \( \mu^*_t \), are less than the growth-maximizing one. By evaluating (A.8) at the growth-maximizing R&D subsidy rate, \( \hat{\mu} \), we get:

\[
\frac{dv_\mu}{d\mu}\bigg|_{\mu=\hat{\mu}} = \frac{\beta\gamma}{(1-\hat{\mu})(\eta-1)(1-\epsilon)[\gamma(\eta-1)(1+(\gamma+\epsilon)Z) + (1-\gamma-\epsilon)\eta Z]} \\
\times \left\{ (\eta-1)(1-\epsilon)(1-\gamma)(\eta-1) + [\eta - \gamma(\eta - 1)](\gamma + \epsilon)Z \\
+ \gamma(\eta-1) + (1-\gamma-\epsilon)(1-\gamma(\eta-1)) \right\} < 0.
\]

This inequality holds for any \( t \). Since \( \Phi_\mu \) is negative only for \( \mu > \mu^*_t \), the growth-maximizing rate, \( \hat{\mu} \) is larger than the welfare-maximizing rate, \( \mu^*_t \).
Intermediate-good firms’ problems with heterogeneity

We solve the problems backward as in Section 2. Since the final-good firm’s behavior does not change, the demand functions for the intermediate goods are the same as (6). The production functions of the intermediate goods given the productivities are also the same as (7). Therefore, the second step of the intermediate-good firms’ problem is (A.1) subject to (6) and (7). Thus the optimal price and output and the profit function of firm $j$ given its productivity and price index are (A.2), (A.3) and (A.4), respectively.

The first step of the problem slightly changes because the R&D efficiency differs among entrepreneurs. The object of this step is to maximize the intermediate-good firms’ net profits (A.5) subject to (A.4) and:

\[ A_r(j) = \theta_j K_{r-1} [l^R_r(j)]^\gamma, \]

The first order condition with respect to $l^R_r(j)$ is:

\[ \gamma \left( \frac{\eta - 1}{\eta} \right) \left[ \frac{\eta}{\theta_j K_{r-1}} P_t \right]^{\eta - 1} [l^R_r(j)]^{\gamma(\eta - 1) - 1} = (1 - \mu_t)w_r. \]  

Since there is $\theta_j$ in the left hand side of (A.11), $l^R_r(j)$ differs among the intermediate-good firms and so do $A_r(j)$ and $p_r(j)$. Note that it is necessary to assume $1 - \gamma(\eta - 1) > 0$ in order to satisfy the second order condition. Substituting (A.2) and (A.11) into the definition of the price index of the intermediate goods, $P_t$, we obtain:

\[ P_t = \frac{\eta}{\eta - 1} w_r \left\{ \int_0^{\eta} \theta_j^{\eta - 1} [l^R_r(j)]^{\gamma(\eta - 1)} d\eta \right\}^{\frac{1}{\gamma(\eta - 1)}}. \]  

We define $\Theta_t$ as $\int_0^{\eta} \theta_j^{\eta - 1} [l^R_r(j)]^{\gamma(\eta - 1)} d\eta$. Then, by using (A.13), we obtain:

\[ \Theta_t = \int_0^{\eta} \theta_j^{\eta - 1} [l^R_r(j)]^{\gamma(\eta - 1)} d\eta = \left[ \frac{\eta - 1}{\eta} \frac{P_t}{w_r} K_{r-1} \right]^{1 - \eta}. \]  

Solving (A.12) for $l^R_r(j)$ and rewriting by using $\Theta_t$, we obtain:

\[ l^R_r(j) = \left[ \frac{\gamma}{1 - \mu} \left( \frac{\eta - 1}{\eta w_r} \Theta_t^{-1} \theta_j^{\eta - 1} \right) \right]^{\frac{1}{\gamma(\eta - 1)}}. \]
Raising the both side of (A.15) to the power $\gamma(\eta - 1)$ and multiplying these by $\theta_j^{\eta - 1}$ yields:

$$\theta_j^{\eta - 1} [t_i^R(j)]^{\gamma(\eta - 1)} = \left[ \frac{\gamma}{1 - \mu} \frac{\eta - 1}{\eta w_i} \Theta_j^{-1} \right]^{\frac{\gamma(\eta - 1)}{\eta - 1}} \theta_j^{\frac{\gamma - \eta + 1}{\eta - 1}}. \quad (A.16)$$

Integrating both side of (A.16) with respect to $j \in [0, n_t]$, we obtain:

$$\int_0^{n_t} \theta_j^{\eta - 1} [t_i^R(j)]^{\gamma(\eta - 1)} dj = \left[ \frac{\gamma}{1 - \mu} \frac{\eta - 1}{\eta w_i} \Theta_j^{-1} \right]^{\frac{\gamma(\eta - 1)}{\eta - 1}} \int_0^{n_t} \theta_j^{\frac{\gamma - \eta + 1}{\eta - 1}} dj. \quad (A.17)$$

The left hand side of (A.17) is $\Theta_j$ and thus we can solve for $\Theta_j$ as follows:

$$\Theta_j = \left[ \frac{\gamma}{1 - \mu} \frac{\eta - 1}{\eta w_i} \right]^{\gamma(\eta - 1)} \left[ \int_0^{n_t} \theta_k^{\frac{\eta - 1}{\eta w_i}} dk \right]^{\gamma(\eta - 1) - 1}. \quad (A.18)$$

Substituting (A.18) into (A.15), we obtain the optimal labor input of $j$th intermediate-good firm for the R&D activities as (27). Substituting (27) into (A.11), the productivity of $j$th intermediate-good firm becomes:

$$A_t(j) = K_{t-1} \left( \frac{\eta - 1}{\eta} \right)^{\gamma(\eta - 1)} \theta_j^{\frac{1}{\gamma(\eta - 1)}} \left[ \int_0^{n_t} \theta_k^{\frac{\eta - 1}{\eta w_i}} dk \right]^{\gamma(\eta - 1)}. \quad (A.19)$$

Using (A.13), (A.14) and (A.18), the price index of the intermediate goods can be computed as follows:

$$P_t = \left( \frac{1 - \mu}{\gamma} \right)^{\gamma(\eta - 1)} \left( \frac{\eta w_i}{\eta - 1} \right)^{1 + \gamma} \left[ \int_0^{n_t} \theta_k^{\frac{\eta - 1}{\eta w_i}} dk \right]^{1 - \gamma(\eta - 1)}. \quad (A.20)$$

Using (7), (A.3), (A19) and (A.20), we obtain the optimal labor input of $j$th intermediate-good firm for the production as (26). Finally, substituting (27), (A.4), (A.19) and (A.20) into (A.5), we obtain the net profit of $j$th intermediate-good firm as (28).

**Acknowledgements**

During the course of this study, we talked to many people, and their knowledge and ideas contributed significantly to the analysis in this paper. In particular, we would like to thank Koichi Futagami, Tatsuro Iwaisako, Kazuhiro Yamamoto, Real Arai and Keisuke Kawata for helpful discussions and comments. We also thank the participants in the seminar at the Hiroshima University and the 2014 Spring Annual Meeting of the Japanese Economic Association at the Doshisha University.
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