The Impact of Financial Centres Quality on Stock Markets∗

(Preliminary and Incomplete Version. Please do not Distribute or Cite)

Sarah El Joueidi†

April 14, 2015

Abstract

This article studies how the institutional quality of financial centres impacts stock markets and location of financial firms across countries. We first establish a theoretical model which considers two stock markets (located in different countries) with only financial firms. The stock demand we found is the typical demand function for horizontal differentiated products. Then, we show that consumers increase their demand for stocks when they trust the financial system. We also explore whether firms have an incentive of doing a costly effort to improve the institutional quality. Results prove that firms decide to boost the institutional quality only when the gain induced by the effort is higher than the cost. We have shown that several equilibria exist. In absence of effort to enhance the institutional quality, firms prefer to be listed in the bigger size country. However, even when firms in only one country do the effort, prices and profits increase in the two countries. Then, under some conditions, firms can decide to be listed in a smaller country. Finally, we present the optimal quality regulation planned by an international regulator.

Keywords: Institutional quality, stock market, firm location, competition

JEL Classification: F12, G15, G28, R12

∗The author would like to thank Pierre M. Picard and Chiara Peroni
†CREA, University of Luxembourg (Luxembourg) and Institut national de la statistique et des études économiques du Grand-Duché du Luxembourg (STATEC). E-mail: sarah.eljoueidi@uni.lu
1 Introduction

This paper analyses the effect of a good institutional quality on stock markets and on the location of financial firms. We also study whether or not having more financial regulation has a positive impact on the stock market. Indeed, if people trust financial contracts are being enforced and that the cost of fraudulent behavior is sufficiently high then, presumably, they are more likely to invest (Asgharian, Liu and Lundtofte, 2014). Hossein Asgharian, Lu Liu and Frederik Lundtofte (2014) analyse a large sample of European (SHARE survey) data on households residing in fourteen European countries that have a large variation in institutional quality. They find that institutional quality has a significant effect on trust and that trust (particularly the part that is explained by institutional quality) affects significantly stock market participation. By consequence, financial firms have an incentive to be (listed) in financial centre where the institutional quality is high. Indeed, under some conditions, aggregate profits and prices can increase under those policies.

To the best of our knowledge, this research is the first study that explore and develop a theoretical model which explain the impact of the financial centre quality on stock markets as well as the impact of regulation established by an international institution to improve the quality of the financial system. Indeed, different policies have been created in the last years to protect the international financial system. An important policy developed by an international committee is the Basel accords (I, II, or III). Basel III is a comprehensive set of reform measures, developed by the Basel Committee on Banking Supervision, to strengthen the regulation, supervision and risk management of the banking sector (BIS). These measures aim to (BIS):

1. improve the banking sector’s ability to absorb shocks arising from financial and economic stress, whatever the source.

2. improve risk management.
3. strengthen banks’ transparency and disclosures.

The difficulty of this kind of policy is to maintain sufficient consistency in regulations in order to not create a source of competitive inequality amongst members and non-members of Basel financial centres.

In this paper, we develop a theoretical model which considers two financial centres (located in different countries) with only financial firms (banks, etc...) who compete to attract the higher number of investors (higher demand) and a higher number of financial firms (higher supply). Investors are distributed between the two countries, each agent can decide in which country he prefers to invest. According to the model, higher institutional quality leads to a higher level of trust on the financial system, and by consequence a higher demand.

As we know, firms prefers to be listed in a place where the demand is high and then when the institutional quality is high. However, firms have a cost to comply to this regulation. These costs come from the fact that regulation force them for example to keep a part of the money without investing it to absorb shock (capital requirement) etc,... Firms also need to work more to search and check all information and transcribe them in complete reports. This procedure should be done several times per year.

The first objective of this paper is to highlight the winners and losers of a good institutional quality. Our second objective is to establish the conditions under which the financial firms are enticed to do an effort to enhance the institutional quality. Our final objective is to analyze the efficient regulation of the decision in terms of the effort exerted to improve the institutional quality.

The present paper is organized as follows. Section 2 presents the model. Section 3 derives the equilibrium under compliance and no compliance of financial firms. Section 4 discusses the efficient pressure. Finally Section 5 concludes.
2 Model

In this section, we present the model. Our model studies the consumer’s preference for financial product quality. We consider a two-country two-financial center model: $i$ and $j$. In country $i$ and $j$, we have respectively a proportion $\theta_i$ and $\theta_j$ of investors and $N_i$ and $N_j$ of financial firms. Each agent can decide in which country he prefers to invest: domestic or foreign stocks. In additional to the price, an agent which buy a foreign stock will face to an inter-market cost. This cost captures banking commissions and variable fees, exchange rate transaction costs and possibly information costs. The timing is as follows. Financial firms simultaneously choose their quality (by choosing their effort), second they set their prices and finally consumers buy stocks issued by firms.

2.1 Preferences

In this part, we analyse the choice of investors. To simplify the understanding, we first, consider only one country with $N$ firms. Investors can invest in these $N$ firms. All consumers have identical preferences represented by the same quadratic utility function.

$$U = c_o + c_1 (1 - c_1/2)$$

where $c_o$ is the first period consumption and $c_1$ is the second period consumption. To model the existence of a stock market we assume a random shock between the two periods. The market allows firms $k \in \mathcal{N} \equiv \{1, ..., N\}$ to sell their stocks at an ex-ante stock price $p_k$ in compensation for an ex-post dividend $d_k$ to investors. Individuals purchase a quantity $q_k$ of each stock and hold a portfolio $c_1 = \sum_{k=1}^{N} d_k q_k$. They therefore maximize their expected utility:

$$EU = c_o + Ec_1 (1 - c_1/2)$$
subject to their budget constraint $\sum_{k=1}^{N} p_k q_k + c_o = w$. In this expression, $E$ is the expectation operator and $w$ is each individual’s income. The price of the first period good and the discount rate between time periods are normalized to one.

For the sake of the analysis we assume a stochastic structure of shocks that is made of $N + 1$ states of nature, $\omega \in \Omega \equiv \{0, 1, ..., N\}$. We consider two types of shocks; independent shocks and a macroeconomic (correlated) shock.

In state of nature $\omega = 0$, the shock is perfectly correlated. It is a negative macroeconomic shock. All firms simultaneously pay the same dividend $d_k = 1 - \gamma$ to investors. More $\gamma$ is important, less the dividend is. The maximum dividend in the case of a macroeconomic shock is equal to one. The probability to have a macroeconomic shock is equal to $f_\gamma$.

Whereas, in state of nature $\omega = \{1, ..., N\}$, only the firm $k = \omega$ pays a dividend $d_k = 1 - \beta$ to investors. It is an idiosyncratic negative shock. Indeed, all other firms will have a maximum dividend except the firm $k = \omega$ which will pay a lower dividend (which depend on $\beta$). The probability of each independent shock $\omega = \{1, ..., N\}$ is equal to $f_\beta$. This shock structure is similar to the one discussed in Acemoglu and Zilibotti (1997). Note that $f_\gamma + N f_\beta = 1$.

In order to analyse the stock demand, we have to understand the payoff in each state of nature:

In state of nature $\omega = 0$, all firms pay the same dividends. Thus, the payoff is equal to $\sum_{k=1}^{N}(1 - \gamma)q_k = (1 - \gamma)Q$ where $Q = \sum_{k=1}^{N} q_k$ is the total demand.

In state of nature $\omega = \{1, ..., N\}$, only the firm $k = \omega$ pays a dividend $d_k = 1 - \beta$ to investors. Others firms pay the maximum dividend of 1. Thus, the payoff is $(1 - \beta)q_k + 1 \sum_{j \neq k} q_j = q_k - \beta q_k + \sum_{j \neq k} q_j = Q - \beta q_k$

Given the budget constraint, each individual chooses the optimal investment portfolio
\{q_k\} that maximizes:

\[
EU = Ec_1 (1 - c_1/2) - \sum_{k=1}^{N} p_k q_k + w
\]

\[
= f_\gamma (1 - \gamma)Q[1 - \frac{1}{2}(1 - \gamma)Q] + \sum_{k=1}^{N} f_\beta (Q - \beta q_k)[1 - \frac{1}{2}(Q - \beta q_k)] - \sum_{k=1}^{N} p_k q_k + w
\]

Maximizing the quadratic expected utility with respect to \(q_k\), we get as a first order condition the following equation

\[
f_\gamma (1 - \gamma)[1 - (1 - \gamma)Q] + f_\beta N \beta[\beta^{-1}(1 - (Q - \beta q_k)) + f_\beta N(1 - Q) + f_\beta \beta Q - p_k = 0 \quad (1)
\]

In order to solve for \(q_k\), we have to replace \(Q\) by its value. Thus, we aggregate over all stocks yields:

\[
N f_\gamma (1 - \gamma)Q[1 - (1 - \gamma)Q] + f_\beta N \beta(-1 + 2Q) - \beta^2 f_\beta Q + f_\beta N^2(1 - Q) - P = 0 \quad (2)
\]

where \(P\) is the price index and \(P = \sum_{k=1}^{N} p_k\). Rearranging this equation, we find:

\[
Q = \frac{N f_\gamma (1 - \gamma) + N f_\beta (N - \beta) + -P}{N f_\gamma (1 - \gamma)^2 + f_\beta (N - \beta)^2} \quad (3)
\]

Plugging 3 in the derivation 1 and solving for \(q_k\):

\[
q_k = a - bp_k + cP \quad (4)
\]

where

\[
a = \frac{[(1-\gamma)f_\gamma + (N-\beta)f_\beta]}{[N f_\gamma (1-\gamma)^2 + f_\beta (N-\beta)^2]}
\]

\[
b = \frac{1}{\beta^2 f_\beta}
\]

and

\[
c = \frac{(1-\gamma)^2 f_\gamma - 2\beta f_\beta + N f_\beta}{[(1-\gamma)^2 N f_\gamma + (N-\beta)^2 f_\beta]^2 f_\beta}
\]
Equation 4 is the typical demand function found for horizontal product differentiation.

The parameter $a$ measures the demand size for each stock. It increases with the expected return of independent and correlated dividends (numerator) and falls with larger variance of the independent and correlated shock (denominator is proportional to the variance). The parameter $b$ measures the price sensitivity of stocks. It increases with the probability of having an independent shock or with the loss in dividend due to this microeconomic shock.

The parameter $c$ measures the product substitutability of stocks. In particular, when $c \to 0$ stocks are perfectly differentiated, whereas they become perfect substitutes when $c \to \infty$.

In order to analyse the effect of the (institutional) quality on stock markets and location of financial firms, we have to assume the following assumptions.

First, in this model, we consider that the amplitude of the macroeconomic shock is high. By consequence, in case of a macroeconomic shock, the dividend received by investors is approximately equal to zero.

As we can remark in the demand function (equation 4), only coefficients $a$ and $c$ depend on the payoff of macroeconomic shock $(1-\gamma)$. By approximating coefficients $a$ and $c$ when the payoff of a macroeconomic shock $(1-\gamma)$ approaches zero using a Taylor expansion, we get that the Taylor first order term of $a$ is a positive constant equal to $\frac{\gamma N}{\beta(N-\beta)^2} > 0$ assuming that $N > \beta$. Then, we can conclude that $(1-\gamma)$ is a positive function of $a$. By consequence, an important implication can be detected here, by increasing the dividend in case of macroeconomic shock by having a better institutional quality, we can increase the stock demand (through the demand size).

In contrast, the Taylor first order term of $c(1-\gamma)$ is null. By consequence, we can state that $c$ does not depend on $(1-\gamma)$. Thus, by improving the quality, governments and firms affect only the demand by increasing the demand size.

Second, to reach our purpose, we assume that firms which decide to be listed a good
quality financial centre must do a costly effort. This effort should be done for example to keep a part of the money (capital requirement) without investing it to absorb macroeconomic shock. As explained above, we consider that this effort has a positive impact on the dividend in the case of macroeconomic shock \((1 - \gamma)\) through the demand size. This effort improves the banking sector’s ability to absorb shocks arising from financial crisis. Thus, the dividend in the case of macroeconomic shock \((1 - \gamma)\) is a linear function of the effort done by all firms. Thus, \(a(E)\) is a positive linear function of the correlated dividend and because the latter is a positive linear function of the total effort, we can reinterpreted \(a(E)\) as follows: \(a(E) = a_0 + a_1E\) where \(E\) is the effort done by all financial firms in all countries: \(E = \sum_{k=1}^{N} e_k\). Indeed, we state that an effort to improve the quality has a positive impact on the demand size of the complete financial system (for all stocks).

Considering this demand function, we support the result of Asgharian, Liu and Lundtofte (2014) that institutional quality has a significant effect on trust and that trust affects significantly stock market participation. Thus, we can rewrite the demand function as follows:

\[
q_k = a(E) - bp_k + cP
\]  

(5)

The innovative element in this paper is the function \(a(E)\), that reflects the quality of the stock. \(a(E)\) depends on the effort done by all firms in all countries.

Now, we extend to two countries. We consider a unit mass of investors, with a share \(\theta_i\) located in region \(i\) and \(\theta_j\) located in region \(j\). Agents can invest in country \(i\) by buying a stock \(k\) for a price equal to \(p^i_k\) or invest in country \(j\) and pay \(p^j_k\).

Now, suppose the demand for stock \(k\) sold by a firm listed in country \(i\): 

\[
q^i_k = q^i_{ki} + q^i_{kj} = \theta_i(a(E) - bp^i_k + cP^i) + \theta_j(a(E) - b(p^j_k + t) + cP^j)
\]  

(6)

where \(q^i_{ki}\) is the stock demand for \(k\) by investors from country \(i\) and \(q^i_{kj}\) is the stock
demand for $k$ by investors from country $j$.

$\theta_i$ and $\theta_j$ are the shares of immobile investors in each country ($\theta_i + \theta_j = 1$). The parameter $a$ measures the demand size for each stock. As we described earlier, in our model, this parameter depends positively on the effort level done by all firms in all countries. Thus, $a$ can be reinterpreted as $a = a_0 + a_1 E$ where $E$ is the effort done by all firms in the two countries: $E = \sum_{k=1}^{N} e_i^k + \sum_{k=1}^{N} e_j^k$ where $e_i^k$ is the effort done by the firm $k$ from country $i$ and $e_j^k$ is the effort done by the firm $k$ from country $j$. We assume that $e_i^k$ and $e_j^k$ are defined between $[0, 1]$.

$P_i$ and $P_j$ are price indices in each country. $P_i = \sum_{k=1}^{N_i} p_i^k + \sum_{k=1}^{N_j} (p_j^k + t)$ and $N_i + N_j = N$.

$t$ is the transaction cost which incurs when an investor buys stock from a foreign country. Inter-market transaction costs captures banking commissions and variable fees, exchange rate transaction costs and possibly information costs.

3 Price equilibrium and optimal effort

In this section, we study and derive the equilibrium of the decision of financial firms. Financial firms simultaneously choose their quality (by choosing their effort) and location. Then, they also set their equilibrium prices.

Consider the following oligopoly; financial firms in country $i$ and $j$ set their prices and decide if they want to incur an effort to boost the institutional quality by maximizing their profits.

The profit of firm $k$ which is located in country $i$ can be written as:

$$\pi_i^k = (p_i^k - m_i^k)[\theta_i(a - bp_i^k + cP_i) + \theta_j(a - b(p_j^k + t) + cP_j)]$$

where $\pi_i^k$ is the profit of the firm $k$ from country $i$. $m_i^k$ is the firm’s marginal cost. The marginal cost is a linear function of the effort level. Indeed, firms which are listed in high quality financial centre, have for example to keep a part of the money without investing it
to absorb macroeconomic shock. The amount of money they have to keep depends on the total raised funds (quantity). The marginal cost can be defined as follows: $m_k = m_0 e_k$

The profit of firms $k$ in country $i$ can be split in two part; the first part is the markup times the total quantity sold to home investors and the second part is the markup times the total quantity sold to foreign investors (it includes the transaction cost). Using the definition of $a$ and $m_k$, the profit can be written as follows:

$$\pi_k = (p_k - m_0 e_k)[\theta_i(a_0 + a_1 E - b p_k + c P)_i + \theta_j(a_0 + a_1 E - b (p_k + t) + c P)_j] = (p_k - m_0 e_k)(q_{ki} + q_{kj})$$  \hspace{1cm} (8)

Doing an effort affects firms in two ways. First, the demand size ($a(E)$) increases and consequently the total demand raises. Indeed, when the market is more transparent and accurate, investors trust more the financial market, and then demand more. Second, doing an effort leads to a marginal cost rising. Indeed, there is a cost for financial firms to enhance the institutional quality.

Financial firms in country $i$ and $j$ set their prices and decide if they want to incur an effort by maximizing their profits.

$$\max_{(p_k, e_k)} \pi_k = (p_k - m_0 e_k)[\theta_i(a_0 + a_1 E - b p_k + c P)_i + \theta_j(a_0 + a_1 E - b (p_k + t) + c P)_j]$$  \hspace{1cm} (9)

Where $E = e_i + \sum_{k' \neq k} (e_{k'})^* + E_j^*$;

$$P_i = p_k^* + \sum_{k' \neq k} (p_{k'}^*)^* + \sum_{k=1}^{N_j} (p_{k}^t)^*$$

and

$$P_j = (p_k^t + t) + \sum_{k' \neq k} (p_{k'}^t + t)^* + \sum_{k=1}^{N_j} (p_{k}^t)^*$$

First order conditions are solved in order to find $p_k^*$ and $e_k^*$

$$\text{FOC} e_k^* : -m_0(q_{ki} + q_{kj}) + (p_k^* - m_0 e_k^*)a_1 = 0$$  \hspace{1cm} (10)
Equation (10) can be interpreted as follows. To have an incentive to comply with the regulation, the total cost for the firm \((m_0(q_{ki}^i + q_{kj}^i))\) should be at maximum equal to the increase in demand due to a better institutional quality \(((p_k^i - m_0e_k^i)a_1)\).

From (10) and (11), we get

\[
(e_k^i)^* = \begin{cases} 
0 & \text{if } a_1 < m_0(b - c) \\
1 & \text{if } a_1 \geq m_0(b - c) 
\end{cases}
\]  

As we can note, this optimization problem has corner solutions. The sufficient condition for establishing \((e_k^i)^*\) as a maximum has been checked. Thus, the optimal effort is 0 or 1. Financial firms boost the institutional quality when the gain from increasing demand is higher than the increase in marginal cost. Thus, when the above condition is filled, firms have an advantage to improve the institutional quality. Firms are more likely to do an effort to improve the quality when stocks are more substitutes (higher \(c\)). Indeed, higher the substitution is, lower the cost for firms is important. This effect can be explained in the following way; when firms do an effort, prices are affected and by consequences increase. When prices increase and \(c\) (coefficient of substitution) is high, the demand for stock \(k\) increases. So, the gain of the effort is more substantial. One can also note that when the price sensitivity \((b)\) of stock is high, the cost is higher. Indeed, we can follow the same logic than above, when price raises and \(b\) (price sensitivity) is high, consumers decrease their demand. Then, the gain is reduced.

**Proposition 1.** Firms are more likely to do an effort to improve the quality of their financial centres when the gain in demand raise is more important than the loss in profit due to the increase in cost. (ii) More stock are substitutes, more likely firms improve their financial centre’s quality. (iii) More the price sensitivity
of stock is high, less likely firms boost their quality.

The stock price of the firm $k$ in country $i$ can be derived using equation (11) and the price index $P_i = \sum_{k=1}^{N_i} p_k^i + \sum_{k=1}^{N_j} (p_k^j + t) = P + tN_j$.

$$(p_k^i)^* = m_0(e_k^i)^* \left( \frac{b-c}{2b-c} \right) + \frac{1}{(2b-c)} \left[ a_0 + a_1 \sum_{k=1}^{N_i} e_k^i + \sum_{k=1}^{N_j} e_k^j \right] - \theta_j b t + c(\theta_i P_i + \theta_j P_j) \right]$$

(13)

At given prices, the stock price rises because of a larger market (smaller $\theta_j$). It also rises when $\theta_i P_i + \theta_j P_j = P + t(\theta_i N_j + \theta_j N_i)$ increases. That is, the stock price rises when investors and firms locate in different markets and falls when they locate in the same market.

Aggregating those prices we get

$$P = \sum_{k=1}^{N_i} p_k^i + \sum_{k=1}^{N_j} p_k^j$$

$$= m_0 \left( \frac{b-c}{2b-c} \right) \left( \sum_{k=1}^{N_i} e_k^i + \sum_{k=1}^{N_j} e_k^j \right) + \frac{N}{(2b-c)} \left[ a_0 + a_1 \left( \sum_{k=1}^{N_i} e_k^i + \sum_{k=1}^{N_j} e_k^j \right) + c(\theta_i P_i + \theta_j P_j) \right] - \frac{bt}{2b-c} (\theta_i N_j + \theta_j N_i)$$

Using $P = \sum_{k=1}^{N_i} p_k^i + \sum_{k=1}^{N_j} (p_k^j + t) = P + tN_j$, we have that $\theta_i P_i + \theta_j P_j = P + t(\theta_i N_j + \theta_j N_i)$

Solving for the fixed point yields

$$P = m_0 \left( \frac{b-c}{2b-c - Nc} \right) \left( \sum_{k=1}^{N_i} e_k^i + \sum_{k=1}^{N_j} e_k^j \right) + \frac{N}{(2b-c - Nc)} \left[ a_0 + a_1 \left( \sum_{k=1}^{N_i} e_k^i + \sum_{k=1}^{N_j} e_k^j \right) \right] - \frac{t(b - Nc)}{2b-c - Nc} (\theta_i N_j + \theta_j N_i)$$

The global price index falls when investors and firms locate in different markets and rises when they locate in the same market. This partially compensates the opposite effect on stock price.
The equilibrium stock price is then

\[ p^*_k = m_0 e^{i_k} + cm_0 \frac{(b-c)}{(2b-c)(2b-c-Nc)} \left( \sum_{k=1}^{N_i} e^{i_k} + \sum_{k=1}^{N_j} e^{j_k} \right) \]

\[ + \frac{1}{(2b-c-Nc)} \left[ a_0 + a_1 \left( \sum_{k=1}^{N_i} e^{i_k} + \sum_{k=1}^{N_j} e^{j_k} \right) \right] \]

\[ + \frac{ct(\theta_i N_j + \theta_j N_i)}{(2b-c)(2b-c-Nc)} - \frac{\theta_j b t}{2b-c} \]

(14)

We can remark that the marginal cost due to the effort affects partially the stock price and by consequence consumers. Stock price increases with the total effort done by all countries. It decreases with the number of investors in the other country.

Using equation (11), the profit of firm \( k \) in country \( i \) can be described as follows:

\[ \pi^i_k = (p^*_k - m_0 e^{i_k})^2 (b-c) \]

\[ = \left[ m_0 e^{i_k} + cm_0 \frac{(b-c)}{(2b-c)(2b-c-Nc)} \left( \sum_{k=1}^{N_i} e^{i_k} + \sum_{k=1}^{N_j} e^{j_k} \right) \right] \]

\[ + \frac{1}{(2b-c-Nc)} \left[ a_0 + a_1 \left( \sum_{k=1}^{N_i} e^{i_k} + \sum_{k=1}^{N_j} e^{j_k} \right) \right] \]

\[ + \frac{ct(\theta_i N_j + \theta_j N_i)}{(2b-c)(2b-c-Nc)} - \frac{\theta_j b t}{2b-c} - m_0 e^{i_k})^2 (b-c) \]

Now, we can study the different equilibria. We consider that all firms in a country have the same behavior (effort and prices). First, we suppose the case where firms in the two countries decide to not do an effort to improve the institutional quality. Second, we analyse the case where the two countries perform an effort to enhance the institutional quality and by consequence reduce the macroeconomic shock. Finally, we consider that only firms in one country try to improve the institutional quality.

**Equilibrium 1:** \( e^{i_k}_k = e^{j_k}_k = 0 \)

In this case, we derive the equilibrium price when costs in the two financial centres originating from the effort are higher than the gain. So, firms decide to not enhance the institutional quality.

In equilibrium, \( P = \sum_{k=1}^{N_i} p^i_k + \sum_{k=1}^{N_j} p^j_k \). Replacing the optimal effort by their values, using
\[ P_i = \sum_{k=1}^{N_i} p_i^k + \sum_{k=1}^{N_j} p_j^k + t = P + tN_j \] and solving for the fixed point, we get

\[ P^* = \frac{a_0 N}{2b - c - Nc} - t \frac{b - Nc}{2b - c - Nc} (\theta_i N_j + \theta_j N_i) \] \hspace{1cm} (15)

and the optimal price is then

\[ (p_i^*)^* = \frac{1}{(2b - c - Nc)} a_0 + \frac{ct(\theta_i N_j + \theta_j N_i)(b - c)}{(2b - c)(2b - c - Nc)} - \frac{\theta_j bt}{2b - c} \] \hspace{1cm} (16)

Therefore, the stock price is larger in the larger market (where the number of investors is higher). Indeed, \( p_i^* > p_j^* \), if and only if \( \theta_i > \theta_j \).

The profit is as follows:

\[ \pi_i^* = (p_i^*)^2 (b - c) \]

\[ = \left\{ \frac{a_0}{(2b - c - Nc)} + \frac{ct(\theta_i N_j + \theta_j N_i)(b - c)}{(2b - c)(2b - c - Nc)} - \frac{\theta_j bt}{2b - c} \right\}^2 (b - c) \]

\[ \pi_i^* - \pi_j^* = \frac{2(2b - c - Nc)a_0bt}{(2b - c)} (\theta_i - \theta_j) + 2bt \frac{ct(\theta_i N_j + \theta_j N_i)(b - c)}{(2b - c)^2(2b - c - Nc)} (\theta_i - \theta_j) \]

\[ + \frac{(bt)^2}{(b - c)^2} (\theta_i^2 - \theta_j^2) \] \hspace{1cm} (17)

In absence of effort to improve the institutional quality in the two markets, stock prices are larger in the larger market (higher number of investors) and then, firms earn higher profit in the larger market.

**Proposition 2.** Suppose price making firms in absence of effort and difference of behaviour to improve the quality. (i) Stock prices are larger in the larger market. (ii) Profit is higher in the larger market. (iii) Firms
decide to listed in the bigger country.

Equilibrium 2: $e^*_k = e^*_k = 1$

When marginal costs are the same in the two countries and the gain from the effort is higher that the cost. Firms in the two countries decide to increase the quality.

In equilibrium, $P = \sum_{k=1}^{N_i} p_k^i + \sum_{k=1}^{N_j} p_k^j$. Replacing the optimal effort by their values, using $P_i = \sum_{k=1}^{N_i} p_k^i + \sum_{k=1}^{N_j} (p_k^j + t) = P + tN_j$ and solving for the fixed point, we get

$$P^* = \frac{Nm_o(b-c)}{(2b-c-Nc)} + \frac{N}{2b-c-Nc} [a_0 + a_1N] - t \frac{b-Nc}{2b-c-Nc} (\theta_jN_j + \theta_jN_i)$$

(18)

and the optimal price is then

$$(p_k^i)^* = \frac{m_o(b-c)}{(2b-c-Nc)} + \frac{1}{(2b-c-Nc)} [a_0 + a_1N] + \frac{ct(\theta_iN_j + \theta_jN_i)(b-c)}{(2b-c)(2b-c-Nc)} - \frac{\theta_jbt}{2b-c}$$

(19)

Therefore, the stock price is larger in the larger market (where the number of investors is higher). Indeed, $p_k^i > p_k^j$, if and only if $\theta_i > \theta_j$.

The profit is as follows

$$\pi_k^i = (p_k^i - m_0)^2(b-c)$$

In presence of effort in the two financial markets, prices and profits are higher than in the others equilibria. The cost of the effort is paid by all firms in the world. Stock prices and by consequence profits are larger in the larger market (higher number of investors).

**Proposition 3.** Suppose price making firms in presence of effort to improve the quality. (i) Stock prices and profits are higher than in the others equilibria. (ii) Stock prices are larger in the larger market. (iii) Profit is higher in the larger market.

Now, we analyse the same equilibrium when marginal costs are different in the two countries ($m_k^j = n_0e^j_k$ is the marginal cost in country $j$). This difference can be explained in the following way; some countries are more efficient than others in doing the effort
(faster decision, . . .). However, the gain from the effort is still higher than the cost in $i$ and $j$. Firms in the two countries decide to increase the quality.

In equilibrium, $P = \sum_{k=1}^{N_i} p_k^i + \sum_{k=1}^{N_j} p_k^j$. Replacing the optimal effort by their values, using $P_i = \sum_{k=1}^{N_i} p_k^i + \sum_{k=1}^{N_j} (p_k^i + t) = P + tN_j$ and solving for the fixed point, we get

$$P^* = \frac{N(b-c)(m_0 + n_0)}{2b - c - Nc} + \frac{N}{2b - c - Nc} [a_0 + a_1N - t] \frac{b - Nc}{2b - c - Nc} [\theta_i N_j + \theta_j N_i]$$  \hspace{1cm} (20)$$

and the optimal price is then

$$(p_k^i)^* = \frac{m_0(b - c)}{2b - c} + \frac{cN(b - c)(m_0 + n_0)}{(2b - c - Nc)(2b - c)} + \frac{1}{(2b - c - Nc)} [a_0 + a_1N] + \frac{ct(\theta_i N_j + \theta_j N_i)(b - c)}{(2b - c)(2b - c - Nc)} - \frac{\theta_i bt}{2b - c}$$  \hspace{1cm} (21)$$

$$(p_k^j)^* = \frac{n_0(b - c)}{2b - c} + \frac{cN(b - c)(m_0 + n_0)}{(2b - c - Nc)(2b - c)} + \frac{1}{(2b - c - Nc)} [a_0 + a_1N] + \frac{ct(\theta_i N_j + \theta_j N_i)(b - c)}{(2b - c)(2b - c - Nc)} - \frac{\theta_i bt}{2b - c}$$  \hspace{1cm} (22)$$

Therefore, the stock price raises with the marginal cost in each country and the sum of the marginal costs. It also increases with the effort and the number of firms.

Profit are as follows

$$\pi_k^i = (p_k^i - m_0)^2 (b - c)$$

$$\pi_k^j = (p_k^j - n_0)^2 (b - c)$$

In presence of effort in the two financial markets, prices and profits are higher than in the others equilibria. When a financial centre is more efficient to do an effort even if there are less investors, firms can decide to be listed in the smaller country which are most efficient in terms of effort.

**Proposition 4.** Suppose price making firms in presence of effort to improve the quality. (i) prices and profits are higher than in the others equilibria. (ii) Profits
can be larger in the smaller markets when the efficiency in terms of effort is high enough.

**Equilibrium 3:** $e_k^* = 0$ and $e_k^* = 1$

Now, we study the third possible equilibrium when only firms in country $j$ decide to do an effort to raise the quality. This case is only possible if the marginal cost is different in the two countries ($n_0 < m_0$). In country $i$, the following condition $a_1 \geq m_0(b - c)$ is not filled.

In equilibrium, $P = \sum_{k=1}^{N_i} p_k^i + \sum_{k=1}^{N_j} p_k^j$. Replacing the optimal effort by their values, using $P = \sum_{k=1}^{N_i} p_k^i + \sum_{k=1}^{N_j} (p_k^j + t) = P + tN_j$ and solving for the fixed point, we get

$$P^* = \frac{N_j n_0 (b - c)}{2b - c - Nc} + \frac{N}{2b - c - Nc}[a_0 + a_1 N_j] - t\frac{b - Nc}{2b - c - Nc} (\theta_i N_j + \theta_j N_i)$$

(23)

and the optimal price is then

$$(p_k^i)^* = \frac{cN_j n_0 (b - c)}{(2b - c - Nc)(2b - c)} + \frac{1}{(2b - c - Nc)[a_0 + a_1 N_j]} + \frac{ct(\theta_i N_j + \theta_j N_i)(b - c)}{(2b - c)(2b - c - Nc)} - \frac{\theta_j bt}{2b - c}$$

(24)

$$(p_k^j)^* = \frac{n_0 (b - c)}{(2b - c)} + \frac{cN_j n_0 (b - c)}{(2b - c - Nc)(2b - c)} + \frac{1}{(2b - c - Nc)[a_0 + a_1 N_j]} + \frac{ct(\theta_i N_j + \theta_j N_i)(b - c)}{(2b - c)(2b - c - Nc)} - \frac{bt\theta_i}{2b - c}$$

(25)

$$p_k^i^* - p_k^j^* = (\theta_i - \theta_j)bt - n_0(b - c)$$

Then, $p_k^i^* > p_k^j^*$, if and only if $(\theta_i - \theta_j) > \frac{n_0}{bt}(b - c)$. This means that the price in $i$ is higher if the difference in number of investors is bigger than the cost implied by the effort. Profits are as follows:
\[\pi_k^i = (p_k^i)^2(b - c)\]
\[\pi_k^j = (p_k^j - n_0)^2(b - c)\]

In this equilibrium, prices and profits are higher than in the case of absence of quality. Indeed, consumers trust more the financial system and by consequence the demand for stocks increases. However, the cost is only paid by firms in \(j\).

**Proposition 5.** Suppose price making firms where only firms in country \(j\) are doing an effort to improve the quality. (i) Stock prices and profits depend not only on the number of investors but also on the cost and the gain initiated by the effort. (ii) Prices and profits are higher in this equilibrium than in absence of quality.

4 International regulation

International institutions usually put effort in improving the regulatory compliance of financial centres and in convincing investors to deposit their funds in those centers.

We now discuss the optimal pressure exerted by an international regulator who maximizes an objective that encompasses profits of both countries as follows:

\[
Max_{(p_k^i, p_k^j, e_k^i, e_k^j) \forall k} \prod = \sum_{k=1}^{N_i} \pi_k^i + \sum_{k=1}^{N_j} \pi_k^j
\]  

(26)

4.1 First best

In this section, the planner detects the optimal effort that firms should do in order to improve the institutional quality and get deposits of investors in those centers. In the first best, planner also set prices.
\[
Max_{(p^i_k, p^j_k, e^i_k, e^j_k) \forall k} \prod = \sum_{k=1}^{N_i} (p^i_k - m_0 e^i_k)q^i_k + \sum_{k=1}^{N_j} (p^j_k - m_0 e^j_k)q^j_k
\]  
(27)

\[
\prod = \sum_{k=1}^{N_i} (p^i_k - m_0 e^i_k)\left[\theta_i(a_0 + a_1 E - b p^i_k + c P_i) + \theta_j(a_0 + a_1 E - b(p^i_k + t) + c P_j)\right] 
+ \sum_{k=1}^{N_j} (p^j_k - m_0 e^j_k)\left[\theta_i(a_0 + a_1 E - b(p^j_k + t) + c P_i) + \theta_j(a_0 + a_1 E - b p^j_k + c P_j)\right]
\]  
(28)

Deriving the first order conditions to get prices and optimal effort

\[
\frac{\delta}{\delta p^i_k} = 0
\]

\[
q^i_k - b(p^i_k - m_0 e^i_k) + c[\sum_{k=1}^{N_i} (p^i_k - m_0 e^i_k) + \sum_{k=1}^{N_j} (p^j_k - m_0 e^j_k)] = 0
\]  
(29)

\[
\frac{\delta}{\delta p^j_k} = 0
\]

\[
q^j_k - b(p^j_k - m_0 e^j_k) + c[\sum_{k=1}^{N_i} (p^i_k - m_0 e^i_k) + \sum_{k=1}^{N_j} (p^j_k - m_0 e^j_k)] = 0
\]  
(30)

\[
\frac{\delta}{\delta e^i_k} = 0
\]

\[
-m_0 q^i_k + a_1[\sum_{k=1}^{N_i} (p^i_k - m_0 e^i_k) + \sum_{k=1}^{N_j} (p^j_k - m_0 e^j_k)] = 0
\]  
(31)

\[
\frac{\delta}{\delta e^j_k} = 0
\]

\[
-m_0 q^j_k + a_1[\sum_{k=1}^{N_i} (p^i_k - m_0 e^i_k) + \sum_{k=1}^{N_j} (p^j_k - m_0 e^j_k)] = 0
\]  
(32)

where \(q^i_k = \theta_i(a_0 + a_1 E - b p^i_k + c P_i) + \theta_j(a_0 + a_1 E - b(p^i_k + t) + c P_j)\)

Assuming that all firms in a country are symmetric, we can state that \(q^i_k = q^j, p^i_k = \)
\( p^i \) and \( e_k^i = e^i \). Rearranging equation (31)

\[
\frac{m_0}{a_1} q^i = [\sum_{k=1}^{N_i} (p_k^i - m_0 e_k^i) + \sum_{k=1}^{N_j} (p_k^j - m_0 e_k^j)]
\] (33)

Replacing (33) in (29)

\[
q^i \left( \frac{a_1 + cm_0}{a_1 b} \right) = (p^i - m_0 e^i)
\] (34)

Putting (34) in equation (31), we can rewrite

\[-m_0 q^i + a_1 \left[ N_i q^i \left( \frac{a_1 + cm_0}{a_1 b} \right) + N_j q^j \left( \frac{a_1 + cm_0}{a_1 b} \right) \right] = 0
\] (35)

Using (35) and the first order condition with respect to \( e^j \), we can show that \( q^i = q^j \), then

\[
\frac{m_0}{a_1} q^i = N q^i \left[ \frac{a_1 + cm_0}{a_1 b} \right]
\] (36)

\[
(e_k^i)^* = \begin{cases} 
0 & \text{if } a_1 < \frac{m_0}{N}(b - Nc) \\
1 & \text{if } a_1 \geq \frac{m_0}{N}(b - Nc)
\end{cases}
\] (37)

where \( N_i + N_j = N \)

This optimization problem has also corner solutions. The sufficient condition for establishing \((e_k^i)^*\) as a maximum has been checked. Thus, the optimal effort is 0 or 1.

International institution requires an effort from firms when the gain from demand increasing is higher than the increase in marginal cost. We can remark that the number of firms is a decreasing function of the cost. More firms are listed in those financial centres, more the compliance with the regulation is a winning strategy. One can also note, as before, more substitutes stocks are, more the likelihood of doing an effort is important. Indeed, when firms are perfectly differentiated \((c = 0)\), the cost of doing an effort is higher. Thus, the substitution reduces the cost of the effort. Moreover, more the price sensitivity
is important (high $b$), higher the cost of doing an effort increases. This can be explained in the following way, when firms do an effort the price becomes higher. Indeed, a part of the effort cost affects consumers through prices. Then, the demand of high sensitive in price stocks decreases and by consequence the total cost of the effort becomes more important.

**Proposition 6.** The first best decision of the international planner is as follows:

(i) He decides to regulate when the gain from the regulation is higher than the cost.
(ii) Higher is the number of firms in the two countries, lower is the cost of doing the effort and by consequence higher is the likelihood to establish a regulation to raise the quality. (iii) More the stocks are substitutes, more the regulator decides to improve the quality by regulate the market. (iv) Higher the price sensitivity is important, lower the gain of the effort is substantial.

Now we search for the optimal prices, using equation (31) in (29)

$$q_k^i - b(p_k^i - m_0e_k^i) + cm_0q_k^i = 0$$

(38)

re-arranging, we get

$$p_k^i = m_0e_k^i + \frac{(a_1 + cm_0)}{a_1b}[\theta_i(a_0 + a_1E - bp_k^i + cP_i) + \theta_j(a_0 + a_1E - b(p_k^i + t) + cP_j)]$$

(39)

Using $P_i = \sum_{k=1}^{N_i} p_k^i + \sum_{k=1}^{N_j} (p_k^j + t) = P + tN_j$, we have that $\theta_iP_i + \theta_jP_j = P + t(\theta_iN_j + \theta_jN_i)$

$$p_k^i = m_0e_k^i + \frac{a_1}{2a_1 + cm_0}[a_0 + a_1E - \theta_jbt + cP + ct(\theta_iN_j + \theta_jN_i)]$$

(40)
The price index is derived as follows:

\[ P = \sum_{k=1}^{N_i} p_k^i + \sum_{k=1}^{N_j} p_k^j \]

\[ = m_0 \frac{a_1}{[(2a_1 + cm_0)b - (a_1 + cm_0)Nc]} \left[ \sum_{k=1}^{N_i} e_k^* + \sum_{k=1}^{N_j} e_k^* \right] \]

\[ + N \frac{(a_1 + cm_0)}{[(2a_1 + cm_0)b - (a_1 + cm_0)Nc]} \left[ a_0 + a_1 \left( \sum_{k=1}^{N_i} e_k^* + \sum_{k=1}^{N_j} e_k^* \right) + cP + ct(\theta_i N_j + \theta_j N_i) \right] \]

\[ - bt \frac{(a_1 + cm_0)}{[(2a_1 + cm_0)b - (a_1 + cm_0)Nc]} (\theta_i N_j + \theta_j N_i) \]

The global price index falls when investors and firms locate in different markets and rises when they locate in the same market.

The equilibrium stock price is

\[ p_k^i = m_0 e_k^i \frac{a_1}{2a_1 + cm_0} \left[ \frac{cm_0 a_1 b E}{a_1 (2b - Nc) + cm_0 (b - Nc)} \right] \]

\[ + \frac{(2a_1 + cm_0)b(1 + Nc) - (a_1 - cm_0)Nc}{(2a_1 + cm_0)b[a_1(2b - Nc) + cm_0(b - Nc)]} \left[ a_0 + a_1 E \right] \]

\[ - t(b - c) \frac{(2a_1 + cm_0)b(1 + Nc) - (a_1 - cm_0)Nc}{(2a_1 + cm_0)b[a_1(2b - Nc) + cm_0(b - Nc)]} (\theta_i N_j + \theta_j N_i) \]

[To continue]

### 4.2 Second best

By computing the second best; we analyse here a more realistic case. First, the planner computes the optimal effort level and then, firms choose their prices.

\[ \text{Max}_{\{e_k^i, e_k^j\} \forall k} \quad \Pi \]

where
\[ \Pi = \sum_{k=1}^{N_i} (p_k^i - m_0 e_k^i)[\theta_i(a_0 + a_1 E - b p_k^i + c P_i) + \theta_j(a_0 + a_1 E - b p_k^j + c P_j)] \\
+ \sum_{k=1}^{N_j} (p_k^j - m_0 e_k^j)[\theta_i(a_0 + a_1 E - b p_k^i + c P_i) + \theta_j(a_0 + a_1 E - b p_k^j + c P_j)] \]

\[ (42) \]

\[ s.t \begin{cases} 
    p_k^i = \text{Argmax} \ \pi_k^i \text{ given } p_k^j \text{ and } e_k^j \\
    p_k^j = \text{Argmax} \ \pi_k^j \text{ given } p_k^i \text{ and } e_k^i 
\end{cases} \]

This problem can be solved in two stages using the backward induction method. In the first stage, the international regulator computes the effort that maximizes the sum of all profits (in the two countries). In the second stage, firms set their prices considering the decision of the regulator for the two countries. Then, depending on the effort asked by the regulator, firms decide their prices.

**Stage 2: Firm set its price**

Firm set its price by maximizing its profit.

\[ \text{Max}_{\{p_k^i\}_{i,k}} \pi_k^i = (p_k^i - m_0 e_k^i)[\theta_i(a_0 + a_1 (E_i + E_j) - b p_k^i + c P_i) + \theta_j(a_0 + a_1 (E_i + E_j) - b (p_k^j + t) + c P_j)] \]

\[ (43) \]

where \( E = \sum_{k=1}^{N_i} e_k^i + \sum_{k=1}^{N_j} e_k^j = E_i + E_j \)

We can rewrite the total demand for the firm \( k \) which is located in \( i \) as follows:

\[ [\theta_i(a_0 + a_1 (E_i + E_j) - b p_k^i + c P_i) + \theta_j(a_0 + a_1 (E_i + E_j) - b (p_k^j + t) + c P_j)] = q_k^i \]

First order condition to find the optimal price

\[ \frac{\delta}{\delta p_k^i} = 0 \]

\[ q_k^i - (p_k^i - m_0 e_k^i)(b - c) = 0 \]

\[ (44) \]

\[ (p_k^i - m_0 e_k^i) = \frac{1}{(b - c)} q_k^i \]

\[ (45) \]
\[ p^*_k = m_0 e^*_k + \frac{1}{b-c} \left[ \theta_i(a_0 + a_1(E_i + E_j) - b p^*_k + c P_i) + \theta_j(a_0 + a_1(E_i + E_j) - b(p^*_k + t) + c P_j) \right] \]  

(46)

Aggregating those prices we get

\[
P = \sum_{k=1}^{N_i} p^*_k + \sum_{k=1}^{N_j} p^*_k
\]

\[
= m_0 \frac{(b-c)}{(2b-c)} \left( \sum_{k=1}^{N_i} e^*_k + \sum_{k=1}^{N_j} e^*_k \right) + \frac{N}{(2b-c)} [a_0 + a_1 \left( \sum_{k=1}^{N_i} e^*_k + \sum_{k=1}^{N_j} e^*_k \right) + c(\theta_i P_i + \theta_j P_j)] - \frac{bt}{2b-c} (\theta_i N_j + \theta_j N_i)
\]

Using \( P = \sum_{k=1}^{N_i} p^*_k + \sum_{k=1}^{N_j} (p^*_k + t) = P + tN_j \), we have that \( \theta_i P_i + \theta_j P_j = P + t(\theta_i N_j + \theta_j N_i) \)

\[
P^* = m_0 \frac{(b-c)}{(2b-c - N c)} \left( \sum_{k=1}^{N_i} e^*_k + \sum_{k=1}^{N_j} e^*_k \right) + \frac{N}{(2b-c - N c)} [a_0 + a_1 \left( \sum_{k=1}^{N_i} e^*_k + \sum_{k=1}^{N_j} e^*_k \right)] - \frac{t}{2b-c - N c} (\theta_i N_j + \theta_j N_i)
\]

The global price index falls when investors and firms locate in different markets and rises when they locate in the same market. This partially compensates the opposite effect on stock price.

The equilibrium stock price is then

\[
p^*_k = m_0 e^*_k + \frac{(b-c)}{(2b-c + N c)} a_0 + a_1 \left( \sum_{k=1}^{N_i} e^*_k + \sum_{k=1}^{N_j} e^*_k \right) + \frac{1}{(2b-c + N c)} (a_0 + a_1 \left( \sum_{k=1}^{N_i} e^*_k + \sum_{k=1}^{N_j} e^*_k \right) + c(\theta_i N_j + \theta_j N_i)(b-c) - \frac{\theta_j bt}{2b-c}(47)
\]

Now, we can plug the optimal price in the international institution maximization problem to derive the optimal effort.

**Stage 1:** The international institution fixes the effort
\[
\begin{align*}
\max_{\{e^i_k, e^j_k\}_{k=1}^N} \prod &= \sum_{k=1}^{N_i} (p^i_k - m_0 e^i_k)q^i_k + \sum_{k=1}^{N_j} (p^j_k - m_0 e^j_k)q^j_k \\
&= \sum_{k=1}^{N_i} (p^i_k - m_0 e^i_k)[\theta_i(a_0 + a_1 E - bp^i_k + cP_i) \\
&\quad + \theta_j(a_0 + a_1 E - b(p^i_k + t) + cP_j)] \\
&+ \sum_{k=1}^{N_j} (p^j_k - m_0 e^j_k)[\theta_i(a_0 + a_1 E - b(p^j_k + t) + cP_i) \\
&\quad + \theta_j(a_0 + a_1 E - bp^j_k + cP_j)]
\end{align*}
\]

Replace \(p^i_k\) and \(p^j_k\) by their values in the maximization function.

To simplify the problem and explain the intuition behind this derivation we consider a simplest case: 2 firms in each country (two countries)

Let consider the \(\prod\) function as follows

\[
\begin{align*}
\prod &= \pi^i_1[p^i_1(e^i_1, e^i_2, e^j_1, e^j_2), e^i_1, P_i(e^i_1, e^i_2, e^j_1, e^j_2), P_j(e^i_1, e^j_1, e^j_2), E(e^i_1, e^j_1, e^j_2)] \\
&+ \pi^i_2[p^i_2(e^i_1, e^i_2, e^j_1, e^j_2), e^i_2, P_i(e^i_1, e^i_2, e^j_1, e^j_2), P_j(e^i_1, e^j_1, e^j_2), E(e^i_1, e^j_1, e^j_2)] \\
&+ \pi^j_1[p^j_1(e^i_1, e^i_2, e^j_1, e^j_2), e^j_1, P_i(e^i_1, e^i_2, e^j_1, e^j_2), P_j(e^i_1, e^j_1, e^j_2), E(e^i_1, e^j_1, e^j_2)] \\
&+ \pi^j_2[p^j_2(e^i_1, e^i_2, e^j_1, e^j_2), e^j_2, P_i(e^i_1, e^i_2, e^j_1, e^j_2), P_j(e^i_1, e^j_1, e^j_2), E(e^i_1, e^j_1, e^j_2)]
\end{align*}
\]
Deriving the first order condition $\frac{\delta \Pi}{\delta e^i_1} = 0$ with respect to the effort of firm 1 in country $i$

\[
\begin{align*}
\frac{\delta \Pi}{\delta e^i_1} &= \frac{\delta \pi^1_1 \delta p^i_1}{\delta p^i_1 \delta e^i_1} + \frac{\delta \pi^1_1 \delta P^i_1}{\delta P^i_1 \delta e^i_1} + \frac{\delta \pi^1_1 \delta P^i_j}{\delta P^i_j \delta e^i_1} + \frac{\delta \pi^1_1 \delta E}{\delta E \delta e^i_1} \\
&= 0 \text{ by the envelop theorem} \\
+ &\frac{\delta \pi^1_2 \delta p^i_2}{\delta p^i_2 \delta e^i_1} + \frac{\delta \pi^1_2 \delta P^i_2}{\delta P^i_2 \delta e^i_1} + \frac{\delta \pi^1_2 \delta P^i_j}{\delta P^i_j \delta e^i_1} + \frac{\delta \pi^1_2 \delta E}{\delta E \delta e^i_1} \\
&= 0 \text{ by the envelop theorem} \\
+ &\frac{\delta \pi^1_3 \delta p^i_3}{\delta p^i_3 \delta e^i_1} + \frac{\delta \pi^1_3 \delta P^i_3}{\delta P^i_3 \delta e^i_1} + \frac{\delta \pi^1_3 \delta P^i_j}{\delta P^i_j \delta e^i_1} + \frac{\delta \pi^1_3 \delta E}{\delta E \delta e^i_1} \\
&= 0 \text{ by the envelop theorem} \\
&= 0
\end{align*}
\]

Then, we generalize the first order condition, so

\[
-m_0 q_k^i + a_1 [\sum_{k=1}^{N_i} (p_k^i - m_0 e_k^j) + \sum_{k=1}^{N_j} (p_k^j - m_0 e_k^j)] \\
+ [c m_0 \frac{(b-c)}{2(b-c-Nc)} + \frac{a_1 N_i}{(2b-c-Nc)}] \sum_{k=1}^{N_i} (p_k^i - m_0 e_k^i) + \sum_{k=1}^{N_j} (p_k^j - m_0 e_k^j) = 0
\]

Using (45), we can replace $(p_k^i - m_0 e_k^i)$ by $\frac{1}{(b-c)} q_k^i$ and $(p_k^j - m_0 e_k^j)$ by $\frac{1}{(b-c)} q_k^j$, we get

\[
m_0(b-c)(2b-c-Nc)q_k^i = [cm_0(b-c) + a_1(2b-c)] \sum_{k=1}^{N_i} q_k^i + \sum_{k=1}^{N_j} q_k^j = 0 \quad \text{(49)}
\]

Because we assume that in country all firms have the same effort and have the same price, we can consider that the demand are the same for all firms $q_k^i = q^i$. Then, (49) becomes:

\[
m_0(b-c)(2b-c-Nc)q^i = [cm_0(b-c) + a_1(2b-c)] [N_i q^i + N_j q^j] = 0 \quad \text{(50)}
\]
Re-arranging the equation (50) and replace \( q^j \) by \( q^i \), we get

\[
m_0(b - c)(2b - c - Nc)q^j = (b - c)[cm_0(b - c) + a_1(2b - c)]Nq^i
\]

(51)

The left hand-side determines the cost exhorted by the effort and the right hand side is the gain due to the effort.

\[
m_0(2b - c - Nc)(b - c) = N[cm_0(b - c) + a_1(2b - c)]
\]

(52)

From 52, we get

\[
(e_k^i)^\ast = \begin{cases} 
0 & \text{if } a_1 < \frac{m_0(b - c)(2b - c - 2Nc)}{(2b - c)N} \\
1 & \text{if } a_1 \geq \frac{m_0(b - c)(2b - c - 2Nc)}{(2b - c)N}
\end{cases}
\]

(53)

where \( b > c \)

Here again, we have corner solutions. The sufficient condition for establishing \((e_k^i)^\ast\) as a maximum has been checked. Thus, the optimal effort is 0 or 1. International institution requires an effort from firms when the gain from demand increasing is higher than the increase in marginal cost. We can remark that the number of firms is a decreasing function of the cost. More firms are listed in those financial centres, more the compliance with the regulation is a winning strategy. Comparing with the first best solution, we can note that the cost decrease more strongly with \( N \) in this case. Here, the substitution effect is less clear than in the first best. In the first best, more substitutes are stocks, lower is the cost and the regulator has more tendency to ask financial firms to comply with the effort. Indeed, when firms are perfectly differentiated \((c = 0)\), the cost of doing an effort is higher. Thus, the substitution reduces the cost of the effort. Here the effect is more ambiguous, the substitution still reduces the cost and then, increases the gain of doing an effort, but in less strongly way. This comes from the fact that here the regulator considers only the effort, and does not fix the price in contrast with the first best. So, he cares
less about the substitution effect. In the second best, the effect of the price sensitivity coefficient is less clear also. The price-sensitivity still affects negatively the effort gain but in a lower proportion. As before, here we assume that the regulator considers that firms are able to adjust their prices and price indices in order to keep their demand constant.

**Proposition 7.** The second best decision of the international planner is as follows: (i) He decides to regulate when the gain from the regulation is lower than the cost. (ii) Higher is the number of firms, lower is the cost of doing the effort. The impact of an increase in $N$ is more important in this case. (iii) More the stocks are substitutes, more the regulator decides to improve the quality by regulating the market. This effect is less clear in the second best case. (iv) Higher the price sensitivity is important, lower the gain of the effort is substantial. As for the substitution effect, it is more ambiguous in the second best than in the first best.

5 Concluding remarks

We have shown how the institutional quality of financial centres impacts stock markets and the location of financial firms across countries. Using a theoretical model which considers two stock markets with only financial firms, we found the typical demand function for horizontal differentiated products. Then, we have modeled the institutional quality by considering the demand size as a function of the dividend (payoff) in case of a macroeconomic shock. We have assumed that firms can impact this payoff by doing an effort in institutional quality to absorb shocks arising from financial and economic stress. Then, the demand size is a function of this effort, and by consequence is a function of the institutional quality. We have also explored the firms’ decision. Whether firms, without regulation, have an incentive of doing a costly effort to improve the institutional quality and give more trust to consumers. We have proved that firms decide to boost the institutional quality only when the gain induced by the effort is higher than the cost.
Then, firms’ decision is 0 or 1, doing or not an effort. Results have shown that several equilibria exist. In absence of effort to boost the institutional quality, firms prefers to be listed in the bigger size country. However, even when firms in only one country do the effort, prices and profits increase in the two countries. Then, under some conditions, firms can decide to be listed in a smaller country. Finally, we have presented the optimal quality regulation planned by an international regulator. Such as the firms’ decision, the international regulator regulates the institutional quality only when the gain induced by the effort is higher than the cost. For the regulator, the number of firms is a decreasing function of the cost. More firms are listed in those financial centres, more the regulation is a winning strategy.
6 References


December


