Adaptation to climate change: public-private arrangements for multi-stage investment projects

Marco Buso∗, Anne Stenger†

Preliminary draft

Abstract

Climate change impact on economical activities is increasingly becoming an important issue that private and public actors should adequately take into account. In the paper we propose a theoretical model to help the understanding about the optimal public and/or private governance to implement and manage adaptation (or mitigation) investments to face climate changes. Our general setting can be easily applied to the agricultural or forest sectors where the private agent owns the field, while the public institution acts as a regulator.

In our model, adaptation investments generate private and social surpluses (even in terms of lower expected losses from climate changes) We compare the private regime with two types of PPPs: a benefit-sharing regime where the public actor intervention aims at enhancing the private benefit from the investment; a cost-sharing governance where the government pays a part of the investment costs. We conclude that in most cases the private agent under-invests given that social benefits of the two activities are not considered. On the other hand, public intervention can be excessively costly for the society (public finance distortion). We compare the two types of PPPs and we figure out that, when public involvement is relevant, a cost-sharing regime is suggested to limit the government costs and for achieving a higher investment level.

Keywords: climate change, mitigation, adaptation, externality, PPPs
JEL classification: D86, D82, D62, L33, H11, C61

∗marco.buso@nancy.inra.fr LEF (Laboratoire d’Economie Forestière) UMR 356, INRA, AgroParisTech
†anne.stenger@nancy.inra.fr LEF (Laboratoire d’Economie Forestière) UMR 356, INRA, AgroParisTech; BETA, Bureau d’Economie Théorique et Appliquée, UMR 7522 CNRS Université de Strasbourg.
1 Introduction and literature review

With climate change, private land owners like farmers or foresters are more and more confronted to its effects in terms of land degradation and monetary costs; besides, climate change impact on the field conservation entails consequences for the collectivity. A thorough description of climate change effects on the forest ecosystem is provided in the report to the European Commission "Impacts of Climate Change on European Forests and Options for Adaptation" (EFI, BOKU, INRA, IAFS 2007 - thereafter EBII 2007). The analyses initially presents climate change scenarios, thereafter it studies effects of direct and indirect impact factors, i.e., atmospheric CO2 increase, changes in temperature, changes in precipitation, abiotic disturbances (changes in fire occurrence, changes in wind storm frequency and intensity), biotic disturbances (frequency and consequences of pest and disease outbreaks). These weather adjustments substantially alter the ecosystem vulnerability affecting, as a consequence, the field capability to generate revenues for the private owners and social services for the collectivity. For instance, higher frequency of wind storm implies negative effects on the forest management activity (harvesting and transport of timber) as well as on the availability of forest services for the population (the recreation forest expected by walkers, naturalists or inhabitants).

In such a context, the alternative strategy to "laissez faire" is adaptation. According to the IPCC (2007) adaptation is defined as adjustment in natural or human systems in response to actual or expected climatic stimuli or their effects, which moderates harm or exploits beneficial opportunities. In the forest sector adaptation options imply different levels of investment intensity and management implications. Indeed, developing infrastructure and transport (road network, irrigation canals or machine technology to facilitate harvest operations) is initially costly for the forester, but it facilitates adaptive management practices such as: shorter rotation length, forest fire prevention systems, changes in species composition, silvicultural strategies, tending and thinning treatments, forest regeneration and, in general, forest management planning. In the model we are going to present, we integrate these two aspects of adaptation practices by considering a two-stage strategy that includes 1- an initial costly investment that facilitates the following operational phase, 2- forest management activities aiming at practically adapt the forest to climate change.

Adaptation investment allow public and private stakeholders to face with the consequences of climate change. Whether the government participates or not in the investment strategy, what changes is the objective function. If only the private agent is involved, social effects are not considered. On the other hand, the public intervention in the investment strategy can be costly for the society, notably in terms of financing costs. In this paper we test the relevance of public - private interactions (Public Private Partnership - PPP). Different versions of PPPs have already been applied: in the forest sector as a way to restore forest management (Sturla, 2012), and in tourism as a contribution to climate change adaptation (Wong et al. 2012). In general, private adaptation to climate change can benefit to the entire society; this public characteristic raises the necessity to think about the institutional challenges notably in terms of innovative PPP (Tompkins and Eakin, 2012).

The goal of the paper is then to compare private and PPP scenarios for applying adaptation investments. To reach this purpose, we construct a model based on the theory of incentives. This approach allows us to take into account the presence of asymmetric information in a context of multiple actors. In the model we have an initial costly investment \( I \) that generates a positive externality on the management phase in which effort is required from the private agent \( e \). The higher the effort, the lower the size of losses (the higher the benefit) in case of climate change events. For each type of governance, the decision making process is made of two steps: first, the choice concerns the level of investment, then the agent set the optimal effort.

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1 Moreover, especially the forestry sector is really important in terms of climate change impacts: "Forestry can make a very significant contribution to a low-cost global mitigation portfolio that provides synergies with adaptation and sustainable development." (Nabuurs et al., 2007).

2 Distortive social consequences of collecting resources from the population to finance the investment.
We solve the model considering three different scenarios: private, incentive-contract and cost-sharing. In a private regime, the agent chooses the investment level and is in charge of the management activity. In case of an incentive-contract regime, the government intervenes on the investment implementation by setting an incentive transfer depending on final realizations of social surpluses; the owner manages the financing and the management phases. Finally, in case of a cost-sharing regime, investment costs are shared between the two actors. In all scenarios, expenses of the public sector are weighted with a shadow cost of public funds ($\lambda$).

Thanks to our approach, we are not just comparing private regime with PPP, but we are also introducing the possibility of alternative types of public-private governance. From our analysis, we can put forward three main results: i/ when a public-private partnerships is selected both social and private returns are taken into account, nevertheless the shadow cost of public funds distorts downward final outcomes; ii/ from a public perspective, the intervention during the financing rather than the management phase is less costly to foster the investment level and, where the externality parameter is sufficiently high, more effective in enhancing the management effort; iii/ the incentive transfer in favor of the private agent is more effective to foster final outcomes when it is decided before rather than after the financing phase.

When speaking about PPPs, the main applications and research studies concern long-term infrastructure projects to provide public and private services. In such contexts, PPPs commit the government to set a long term contract with a single consortium. Thus, the private contractor is in charge of different connected tasks, such as: building an infrastructure, financing the investment, managing the realized asset for a predetermined period of time. Thanks to this "bundling" mechanism, when the different phases are connected through positive externalities, PPPs are able to create additional incentives (with respect to Traditional Public Procurement) to the private actor, reducing problem of contract incompleteness or asymmetric information (Iossa and Martimort, 2015; Buso, 2014; Martimort and Pouyet, 2008; Hart, 2003). On the other hand, there are issues related to long term lack of flexibility (Martimort and Straub, 2014; Iossa and Martimort 2012). Looking at this strand of literature, our study presents similarities and differences.

Our theoretical model applies to contexts where a private agent owns a field/forest, while the public institution intervenes to provide incentives for adapting the field to climate change events. Moreover, the adaptation investment implies two different phases (financing and management) that are connected through a positive externality. Thus, we are still dealing with multi-stage public/private investment characterized by the presence of positive links among different phases. However, we don’t focus on advantages/disadvantages of the bundling mechanism. In fact, we compare our benchmark scenario (private regime) with different types of public-private governance studying what is the optimal method and time of intervention for a government aiming at positively affecting final outcomes of an uncertain investment; i.e., whether the government has to intervene during the financing or the management phase, and which is the optimal degree of public involvement in the form of incentive transfer or cost sharing.

2 The model

In our model, in the absence of uncertainty, the private agent receives from his own forest a definite level of revenues ($R_0$). Moreover, the field conservation implies a positive externality for the community ($S_0$). When we allow for the possibility of climate changes, the expected private revenues and social surpluses become respectively equal to: $R = R_0 + pR^l$ and $S = S_0 + pS^l$, where $R^l$ and $S^l$ can be positive or negative, while $p$ captures the probability that climate changes occur and is lower than one.

However, the owner can reduce negative (or increase positive) consequences of climate changes by adapting his own field to the expected conditions. In this case, the investment is set up in two phases:

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3 Cost imposed to taxpayer for financing investments.

4 Climate changes with no adaptation normally imply a decreasing of revenues and social surpluses; however, there are cases when the impact is expected as positive.
first, the agent has to choose the level of investment - $I$ (investment phase); second, the owner decides on the optimal effort - $e$ (management phase) for adapting the field.

The monetary cost of the investment is equal to $I^2$, while the management activity implies a non monetary cost equal to $\frac{(e-\delta I)^2}{2}$. The investment cost are assumed as quadratic on $I$. The management cost function is convex on the level of effort; moreover, higher the initial investment level, lower the non-monetary effort cost. In fact, applying adaptation on the field require initial costly investment (machinery, equipment, etc.) that help the management activity enhancing, as a consequence, the probability of success. The positive impact of $I$ on the management phase is captured by the externality parameter $\delta$ whose range of values is between 0 and 1. Final outcomes are uncertain and directly depend on management effort ($e$). More precisely, private returns are equal to $pR^h > pR^l$ with probability $\frac{e}{k_r}$, while social returns correspond to $pS^h > pS^l$ with probability $\frac{e}{k_s}$, $k_r$ and $k_s$ are parameters (higher than 1) measuring the capacity of private effort to influence respectively private revenues and social surpluses. From the previous statements, we implicitly allow $e$ to be between 0 and $\max(k_r, k_s)$. The public regulator cannot verify the effort, but it can observe the final outcome in the form of private and social benefits. The time-line of the investment is reported in the following graph:

![Time-line of the investment](image)

Starting from this setting, our model aims at explaining how governance tasks and responsibilities should be shared between public and private actors in a context characterized by endogenous risks, asymmetry of objectives and public finance distortions. In fact, the public regulator is interested in the investment implications in terms of social welfare, while the private agent aims at enhancing private returns from the two investments. Moreover, whenever the public regulator sustains monetary costs, the shadow cost of public funds ($\lambda$) is considered as a way to capture the distortion imposed to taxpayers for financing the investment.

The following paragraphs are organized as follows. In section 3 we initially study the first best case, thereafter we suppose that the private owner realizes the investment without any public helps and we adopt this scenario as a benchmark. Then, we consider two possible public-private governance where investment and effort levels are jointly determined: in the first case by transferring additional benefits to the private agent contingent to social surpluses realizations; in the second one by sharing investment costs (section 4). In section 5 we propose a comparative statics analysis to determine which scenario is preferred in the form of final outcomes ($I$ and $e$). Finally, in section 6 we conclude and we propose future developments.

### 3 Assumption and Benchmark Scenarios

The model is solved considering that $R_0 = S_0 = 0$. This condition does not affect final results, but it simplifies the notation. Moreover, we assume that the probability of climate change occurrence ($p$) is known from the beginning. This assumption can be too strict in case we introduce risk aversion or we consider the presence of asymmetric information between the public and private agent. On the other hand, in a context of risk neutrality, $p$ can be considered as the expected probability that the climate change event occurs.

In addition, we suppose that, whenever the public intervene in the investment strategy, the ex-post revenues of the private agent in the worst scenario cannot be lower than what he can get by doing the investment alone ($R^l$). This condition represents a limited-liability constraint that protects the private

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5The analysis can be enlarged by considering general cost functions ($c(I)$) that respect the following properties: $c'(I) > 0$, $c''(I) \geq 0$. 


agent from an excessive transfer of risks from the public regulator. It reflects a sort of risk aversion of the private agent against the probability of excessive looses.

In the next two paragraphs: we first compute first best results, then we solve the model under the benchmark case (private regime)

**First Best**

At the first best levels of investment and effort derive from the maximization of the total welfare function that is given by the sum of the private utility ($U$) and the government social function ($W$), as defined in the following formula:

$$U + W = \frac{e}{k_r}pR^h + (1 - \frac{e}{k_r})pR^l + \frac{e}{k_s}pS^h + (1 - \frac{e}{k_s})pS^l - \frac{(e - \delta I)^2}{2} - \frac{I^2}{2} \quad (1)$$

The function consists of several parts:

- First, the expected private revenues: $\frac{e}{k_r}pR^h + (1 - \frac{e}{k_r})pR^l$;
- Second, the expected benefit in terms of social surplus: $\frac{e}{k_s}pS^h + (1 - \frac{e}{k_s})pS^l$;
- Third, the effort cost that decreases with the level of initial investment: $-\frac{(e - \delta I)^2}{2}$;
- Finally, the monetary cost of the investment: $-\frac{I^2}{2}$.

The maximization problem is solved backward.

**Proof.** See Appendix A

First, we report the optimal level of investment:

$$I^{fb} = \frac{p}{k_r} \Delta R + \frac{p}{k_s} \Delta S \delta \quad (2)$$

The value at the equilibrium increase higher is the expected gain in the form of private revenues of social surpluses from doing the adaptation investment. Moreover, higher the externality between investment and effort ($\delta$), higher the optimal investment level.

Second, it’s computed the level of effort that should be implemented during the management phase:

$$e^{fb} = \min\{\frac{p}{k_r} \Delta R + \frac{p}{k_s} \Delta S + \delta I^{fb}, \max(k_r, k_s)\} \quad (3)$$

The optimal value increases higher is the investment level, larger is the potential impact of adaptation on private revenues and social surpluses, and higher is the externality parameter ($\delta$). Effort does not exceed the maximum between $k_r$ and $k_s$ inasmuch as the probabilities to attain positive public or private outcomes by adapting cannot be higher than one.

**Private Regime**

Under the benchmark scenario, the private agent is in charge of financing and managing the investment. Thus, it sets the optimal levels of effort and investment by maximizing his utility function that takes the following form:

$$U_{pr} = \frac{e}{k_r}pR^h + (1 - \frac{e}{k_r})pR^l - \frac{(e - \delta I)^2}{2} - \frac{I^2}{2} \quad (4)$$

Solving the problem backwards, the owner can choose how much he wants to invest and which is the optimal effort during the management process.

**Proof.** See Appendix B
We start by reporting the level of investment:

\[ I^{pr} = (\frac{\Delta R}{k_r})\delta \]  \hspace{1cm} (5)

Differently from the first best, only expected private revenues enter the first best, then there is still the positive effect of the externality parameter (\(\delta\)).

Second, this is the effort level at the optimum:

\[ e^{pr} = \min\{ (\frac{p\Delta R}{k_r}) + \delta I^{pr}, \max(k_r, k_s) \} \]  \hspace{1cm} (6)

Effort depends on: the expected private revenues, the investment level and the externality parameter. Effort is lower than the first best unless the optimal value is higher than the maximum between \(k_r\) and \(k_s\).

4 Public-Private Governance

In the next paragraphs we study different types of public-private governance where the public regulator intervenes affecting investment level and/or management effort to allow social surpluses to be included on private choices. At first, the government can intervene at the initial investment stage by paying part of the investment cost. Second, we allow the public regulator to write incentive transfers contingent on realized management outcomes (social surpluses) that are ex-post verifiable.

4.1 Investment-contract scenario

Incentive contracts could be written if ex-post outcomes will be verifiable. Differently, the government can only intervene at the investment stage by paying part of the investment cost. Indeed, before the agent invests, the public regulator can decide how the investment cost is shared between the public and the private sector (\(\alpha\) percentage is paid by the private, \(1 - \alpha\) percentage is paid by the public). This allows the government to enhance the initial investment and to support, through the externality parameter (\(\delta\)), the management phase. For a better understanding, the following graph summarizes the time of the investment implementation:

\[ \alpha \text{ is chosen} \quad I \text{ is decided} \quad e \text{ is decided} \quad R \text{ and } S \text{ are realized} \]

The private utility and the government function appear, within this case, as in the following formulas:

\[ U_i = \frac{e}{k_r}pR^h + (1 - \frac{e}{k_r})pR^l - (\frac{e - \delta I}{2})^2 - \frac{\alpha I^2}{2} \]  \hspace{1cm} (7)

\[ W_i = \frac{e}{k_s}pS^h + (1 - \frac{e}{k_s})pS^l - (1 + \lambda)(1 - \alpha)(\frac{I^2}{2}) \]  \hspace{1cm} (8)

Under this scenario, there are no transfers, but the monetary cost is shared between the private agent (\(\alpha\) - eq. 7) and the public regulator (\((1 - \alpha)\) - eq. 8).

Given the two functions, optimal values of effort, investment and cost sharing could be determined following a backward induction strategy.

**Proof.** See Appendix C

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6In this case, the government loses any incentives to intervene once the agent has already implemented the investment.
We start by reporting the values of the investment and the cost sharing variable at the equilibrium:

$$\alpha = \min \{ \frac{2 \Delta R}{k_r} (1 + \lambda), 1 \}$$  \hspace{1cm} (9)

$$I^i = \max \{ \delta [p \frac{\Delta S}{(1 + \lambda)k_s} + p \frac{\Delta R}{2k_r}], I^p r \}$$  \hspace{1cm} (10)

The fraction of cost held by the private agent increases, the higher the $\Delta R$ and the $\lambda$ (equation 9). However, the higher the social surpluses, the higher the benefit from a government participation in financing the initial cost. The investment level (equation 10) is higher than the optimal investment under the private regime whenever the following condition is satisfied: $2 \frac{\Delta S}{k_r} > (1 + \lambda) \frac{\Delta R}{k_r}$. Moreover, the value at the equilibrium strictly depend on the externality parameter: the higher the $\delta$, the higher the investment level, and when $\delta$ is equal to zero the optimal investment is also zero.

Looking at the management phase, the following formulas report the optimal effort:

$$e^i = \max \{ \frac{\Delta R}{k_r} + \delta I^i e^p_r \}$$  \hspace{1cm} (11)

Differently than what is stated in equation 3 (first best result), under this scenario the social surpluses is not directly included in the optimal effort, but only through the investment impact. Then, the difference between $e^i$ and $e^p_r$ exactly corresponds to the difference between $I^i$ and $I^p_r$, i.e. $e^i$ is higher than $e^p_r$ if the following condition is satisfied: $2 \frac{\Delta S}{k_r} > (1 + \lambda) \frac{\Delta R}{k_r}$.  

4.2 Management-contract scenario

Within this framework the public regulator intervenes in the decision making process trying to influence the implementation of the adaptation investment (management phase). More precisely, the government can set incentive transfers based on verifiable outcomes; whether social surpluses result equal to $S^h$, the public transfer corresponds to $T^h$, otherwise it is equal to $T^i < T^h$. Thanks to this menu of contingent transfers, the government provides the private agent with additionally incentives to implement effort during the management phase, and to increase the investment during the financing phase\(^7\).

Under this scenario, the private utility and the government function are respectively equal to:

$$U_m = \frac{e}{k_r} p R^h + (1 - \frac{e}{k_r}) p R^i + \frac{e}{k_s} p T^h + (1 - \frac{e}{k_s}) p T^i - \frac{(e - \delta I)^2}{2} - \frac{T^2}{2}$$  \hspace{1cm} (12)

$$W_m = \frac{e}{k_s} p S^h + (1 - \frac{e}{k_s}) p S^i - (1 + \lambda) (\frac{e}{k_r} p T^h + (1 - \frac{e}{k_r}) p T^i)$$  \hspace{1cm} (13)

Equation 12 differs from equation 4 because of the expected transfer that is part of the marginal benefit. On the other side, the government’s objective function (equation 13) is made of the expected social surpluses minus the expected social costs (considering the role of public finance distortions - $\lambda$). In addition, the government in deciding which is the level of the two transfers must take into account the limited liability constraint of the private agent; i.e., the monetary loss in case of bad outcomes ($R^i + T^i - \frac{e}{k_r}$) cannot exceed the corresponding result under the benchmark scenario $R^i - \frac{e}{k_r}$. As a consequence, given that the government aims at extracting rents from the private agent, the minimum level of feasible $T^i$ is equal to zero, meaning that penalties are not feasible when bad outcomes occur.

Incentive transfers levels can be decided before or after the investment is set up. Depending on which scenario is implemented, the time-line of the investment is respectively as follows:

\(^7\)In fact, increasing the investment during the financing phase allows the agent to decrease the management costs and, as a consequence, to enhance the probability to attain higher private revenues and government transfers.
Transfer decided before the investment

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<th>Investment</th>
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<td>( T^h ) and ( T^l ) are chosen</td>
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Transfer decided after the investment

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Considering the private utility and the government’s objective function, the problem is solved backwards to derive the values of effort, investment and transfers at the equilibrium.

**Proof.** See Appendix D

As a first result, we can conclude that the optimal levels of investment and effort are higher when the transfer is decided before rather than after the investment (see appendix D). In the second case the government can just affect the management effort knowing that a fewer transfer is needed the higher the initial investment. The private owner can anticipate the government behavior reducing as a consequence the optimal investment for obtaining, during the second stage, a higher level of transfer. This is a first result that is summarized in the following lemma:

**Lemma n. 1:** The government participation on the management activity results in a different level of optimal investment and effort depending on the time-line of the intervention. When a transfer is decided before the investment choice, a benefit arises in the form of both a higher initial investment and management effort. Moreover, the level of the transfer is higher when it is set at the beginning of the project.

From this point forward we focus on the case where the transfer is decided at the beginning of the project. We start by reporting the optimal levels of transfer and investment

\[
T^h = \max \left\{ k_s \frac{\Delta S}{k_s} - \frac{(1 + \lambda) \Delta R}{2(1 + \lambda)} + 0 \right\} \quad T^l = 0
\]  

\[
I^m = \max \left\{ \left( p \frac{\Delta S}{2(1 + \lambda)k_s} + \frac{\Delta R}{2k_r} \right) e, I^{pr} \right\}
\]  

The incentive transfer in case of good outcomes is given for sufficient levels of social surpluses; precisely, only when \( \frac{\Delta S}{k_s} > (1 + \lambda) \frac{\Delta R}{k_r} \). In such a case, the investment level is higher with respect to the benchmark scenario (private regime), however it is still lower than the first best result. In fact, differences come from the simultaneous effect of the limited liability constraint and the shadow cost of public funds. Because of the limited liability constraint, the government can provide incentives only through positive transfers that are socially costly (\( \lambda > 0 \)).

After the initial investment, the management phase starts and the optimal level of private effort is the following:

\[
e^m = \max \left\{ p \frac{\Delta S}{2(1 + \lambda)k_s} + \frac{\Delta R}{2k_r} + \delta I^m, e^{pr} \right\}
\]

The optimal level of effort increases, the higher the initial investment and the larger the private revenues and social surpluses. The condition to have \( e^m > e^{pr} \) is the same required to have \( I^m > I^{pr} \); however, once this hypothesis is satisfied, the difference between \( e^m \) and \( e^{pr} \) is always higher than the one between \( I^m \) and \( I^{pr} \). Distortions with respect to the first best scenario come from the presence of socially costly public transfers. Previous results are summarized by the following lemma that explains whether and under which conditions a public-private governance is preferred than the private scenario.
Lemma n. 2: The condition to have a public-private governance preferred to the benchmark (private) scenario is less strict in case the government opts for an investment-contract regime. Nevertheless, the preference of the management-contract governance in terms of initial investment implies even a stronger preference concerning the management phase (if $I^m > I^pr$, then $e^m \gg e^pr$).

In the next paragraph we perform a comparative statics analysis to compare the two public-private governance in terms of final outcomes. It is important to highlight how it is not possible to implement simultaneously the two scenarios. In fact, under the management-contract case, due to the presence of the limited liability constraint, the government can set the lower transfer ($T_l$) such that investment costs are completely paid by the private agent whatever is the level of $\alpha$.

5 Comparative Statics Analysis

We start our comparison by looking at the level of investment. The difference between the investment outcome under the investment-contract and the management-contract regime is reported in the following equation:

$$I^i - I^m = \delta (\frac{p \Delta S}{2(1 + \lambda)}) \geq 0$$  \hspace{1cm} (17)

Unless $\delta$ is equal to zero, equation 17 is always positive and the difference is higher, the higher is the externality between the two phases and the more important are social surpluses. On the other hand, the difference concerning the management effort is shown in the following formula.

$$e^i - e^m = p \frac{\Delta R}{2k_r} - p \frac{\Delta S}{2(1 + \lambda)k_s} + p \delta^2 \frac{\Delta S}{2(1 + \lambda)k_s}$$ \hspace{1cm} (18)

Assuming that both public-private governance are preferred than the private regime ($\Delta S > (1 + \lambda) \Delta R$), equation 18 is positive if the following condition is satisfied: $\frac{(\frac{\Delta S}{k_s} - \frac{\Delta R}{k_r})}{(1 + \lambda)} \leq \delta^2$. It means that if the externality parameter is sufficiently high, then the investment-contract regime is more effective than the management-contract in the form of management effort. On the other hand, when the externality is low the management-contract governance is the most effective to enhance the private effort, in the extreme case of $\delta = 0$, the government is not able to affect, through an investment-contract, the management effort that, as a consequence, is equal to the optimal outcome under the private regime ($e^pr$). The dynamics described by equations 17 and 18 are reported in the following graphs.
The previous analysis related to the optimal values of the investment outcomes can be summarized by the following proposition that represents the first finding of our study:

**Proposition n. 1:** When the positive externality between the investment and the management phase is high enough, then the best solution for the public regulator is to construct a partnership from the beginning of the project (investment phase). However, when the two stages of the project are not sufficiently linked, then the government is more effective in enhancing adaptation by contracting the management stage.

Although we already detect differences between the two public-private governance in the form of outcomes, also important are their consequences in terms of final payoffs. In order to perform this analysis, the following paragraphs are organized as follows:

- first, we compute the value functions of the government and the private agent under the different scenarios and we detect their optimal decision;
- second, we derive the comparative statics analysis with respect to our parameter of interest: $\delta$;
- finally, we compare the investment-contract and the management-contract scenarios in terms of variations of the total social payoff (welfare payoff + private utility) with respect to $\delta$.

**Proof.** See Appendix E

From the comparison we can observe that if $\lambda$ is equal to $0$, then the preference of the Investment-contract with respect to the Management-contract scenario increases the higher the $\delta$. Moreover, the same conclusion holds if $\lambda = 1$ and $\frac{\Delta S}{2k_s} > \frac{\Delta R}{k_r}$. It is straightforward to observe that the latter condition is less strict than the one required to have a public-private governance preferred than a private regime. Thus, we can derive the second main finding of our paper that is summarized in the following proposition:

**Proposition n. 2:** When a public-private governance is preferred with respect to the private regime, then the preference of the Investment-contract with respect to the Management-contract scenario is higher in terms of total social surplus (welfare payoff + private utility), the higher the positive externality between the two investment and the management phases.

Previous proposition reinforce the previous finding and emphasize the role of the externality parameter in explaining which timing and investment method the government should select to enhance private and social returns from an adaptation investment. The analysis is developed in a context of complete
contract, whereas it can be the case that contingent transfers based on verifiable outcomes are not possible. In this latter situation, the government is only able to offer a contract based on the final investment that is effective in enhancing effective adaptation only when $\delta$ is sufficiently high, as reported in the next statement:

**Proposition n. 3:** As is described in the PPPs literature, the government intervention from the beginning of the project allows to avoid the problem of contract incompleteness, especially when the positive externality between the two phases is sufficiently high.

### 6 Conclusion and future developments

In the paper we compare private regime with public-private governance for the realization of adaptation (or mitigation) investments to face climate change shocks.

Our finding are related to the optimal method and timing of government participation for supporting the private investment in adaptation. In our paper the investment in adaptation involves two phases (investment and management) that are connected through an externality parameter $\delta$. We find that, the higher the externality, the more the government participation is suggested on the investment costs (from the beginning of the project) rather than on the management activity. Moreover, when a contract based on contractible outcomes is not possible, the investment-contract scenario remains the only option able to obviate the presence of contract incompleteness.

In terms of policy implications, our results show that investments linked to climate change are not only private or public concerns. This statement is intuitively more relevant when both private returns and social surpluses are substantially affected by climate change shocks. We propose as alternative strategies to the private regime, different types of PPP where the governance of adaptation (or mitigation) investments is shared between public and private actors. By comparing two strategies, we discover that the government participation on the initial investment phase is particularly suggested whenever the initial investment in adaptation is relevant and the externality between the two phases is particularly high. In other words, in cases of strong adaptation the government should participate directly on the investment costs (Investment-contract scenario), while in cases of soft adaptation, the government should intervene on the management stage (Management-contract scenario).

This is a first step for understanding what’s the optimal mix of public and private governance to realize and manage adaptation (mitigation) climate change investments. Several complexities can be introduced to enrich and complete this benchmark analysis, like: including risk aversion and transaction costs between public and private partners, considering the role of private financing, or studying the possible impact of an adverse selection issue.
References


Appendix A

Solving the problem through a backward induction strategy, we first derive the optimal level of effort:

\[ \max W + U = \frac{e}{k_e} p R^h + (1 - \frac{e}{k_e}) p R^l + \frac{e}{k_e} p S^h + (1 - \frac{e}{k_e}) p S^l - \frac{\alpha}{2} I^2 - \frac{\delta I}{2} \]

\[ \frac{d(W + U)}{dI} = p \Delta R + p \Delta S - e + \delta I = 0 \implies e^f = \left( \frac{\partial R}{\partial k_e} + \frac{\partial S}{\partial k_e} \right) + \delta I \]

Second, we get the level of investment at the optimum:

\[ \max W + U = \frac{e}{k_e} p R^h + (1 - \frac{e}{k_e}) p R^l + \frac{e}{k_e} p S^h + (1 - \frac{e}{k_e}) p S^l - \frac{\alpha}{2} I^2 - \frac{\delta I}{2} \]

\[ \max W + U = \frac{\partial R}{\partial k_e} + \frac{\partial S}{\partial k_e} = -I + \delta \]

Finally, the public regulator sets the cost sharing variable anticipating what will be the optimal investment variable:

\[ W^f = \max \{0, q(p \Delta R + p \Delta S)\}, e^f = (\frac{\partial R}{\partial k_e} + \frac{\partial S}{\partial k_e})(1 + \delta^2) \]

Appendix B

Solving the problem through a backward induction strategy, we first derive the optimal level of effort:

\[ \max U = \frac{e}{k_e} p R^h + (1 - \frac{e}{k_e}) p R^l - \frac{\alpha}{2} I^2 - \frac{\delta I}{2} \]

\[ \frac{dU}{dI} = \frac{\partial R}{\partial k_e} - e + \delta I = 0 \implies e^p = \frac{\partial R}{\partial k_e} + \delta I \]

Second, we get the level of investment at the optimum:

\[ \max W + U = \frac{e}{k_e} p R^h + (1 - \frac{e}{k_e}) p R^l - \frac{\alpha}{2} I^2 - \frac{\delta I}{2} \]

\[ \max W + U = \frac{\partial R}{\partial k_e} = -I + \delta \]

Final results are as follows:

\[ I^f = q(p \frac{\partial R}{\partial k_e} + \frac{\partial S}{\partial k_e}) \delta, e^p = (\frac{\partial R}{\partial k_e} + \frac{\partial S}{\partial k_e})(1 + \delta^2) \]

Appendix C

Solving the problem through a backward induction strategy, we first derive the optimal level of effort:

\[ \max U = \frac{e}{k_e} p R^h + (1 - \frac{e}{k_e}) p R^l - \frac{\alpha}{2} I^2 - \frac{\delta I}{2} \]

\[ \frac{dU}{dI} = \frac{\partial R}{\partial k_e} - e + \delta I = 0 \implies e^l = \frac{\partial R}{\partial k_e} + \delta I \]

Second, the level of investment is set by maximizing the following objective function of the private agent. In this case, the limited liability constraint is always slack \((R^l - \alpha I^2) \Rightarrow R - I^2\) unless \(\alpha\) is equal to one.

\[ \max U = \frac{e}{k_e} p R^h + (1 - \frac{e}{k_e}) p R^l - \frac{\alpha}{2} I^2 - \frac{\delta I}{2} \]

\[ \max U = \frac{\partial R}{\partial k_e} = -I + \delta \]

Final results are as follows:

\[ I^l = \frac{\partial R}{\partial k_e} \delta - \alpha I = 0 \implies e^l = \frac{\partial R}{\partial k_e} + \delta I, e^l = \left( \frac{\partial R}{\partial k_e} \right) (1 + \delta^2) \]

Finally, the public regulator sets the cost sharing variable anticipating what will be the optimal investment choice of the private agent:

\[ \max W = \frac{e}{k_e} p S^h + (1 - \frac{e}{k_e}) p S^l - (1 + \lambda)(1 - \alpha)(\frac{\partial S}{\partial k_e}) \]

\[ \frac{dW}{dI} = \frac{\partial S}{\partial k_e}(1 + \delta^2) p \frac{\partial S}{\partial k_e} + p S^l - (1 + \lambda)(1 - \alpha)(\frac{\partial^2 S}{\partial k_e}) \]

\[ \frac{dW}{dI} = \frac{\partial S}{\partial k_e}(1 + \lambda) \frac{\partial S}{\partial k_e} + p S^l - (1 + \lambda)(1 - \alpha)(\frac{\partial^2 S}{\partial k_e}) \]

\[ = 0 \]
\[
\begin{align*}
\frac{dW}{da} &= -\Delta S_k - (1 + \lambda) \frac{\Delta R}{k_r} + (1 + \lambda) \frac{\Delta R}{k_e} = 0 \\
\frac{dW}{da} &= \alpha [2 \Delta S_k + (1 + \lambda) \frac{\Delta R}{k_e}] = 2(1 + \lambda)(\frac{\Delta R}{k}) \\
\alpha &= \min\{\frac{2 \Delta S}{k_r} (1 + \lambda), 1\} \\
\text{Final results are as follows:} \\
\alpha &= \min\{\frac{2 \Delta S}{k_r} (1 + \lambda), 1\} \\
I^t &= \max\{\frac{\delta}{2}[2p_r(1 + \lambda)k_e + p_k R_k]I^{pr}\} \\
e^t &= \max\{p \frac{\Delta R}{k} (1 + \frac{\delta}{2} + \frac{2p_r S_k}{2(1 + \lambda)k_e} e^{pr}\}
\end{align*}
\]

**Appendix D**

**Transfers are decided before the investment**  Solving the problem through a backward induction strategy, we first derive the optimal level of effort:

\[
U_m = \frac{\kappa}{k_r} p R^h + (1 - \frac{k_e}{k_r}) p R^l + \kappa p T^h + (1 - \frac{k_e}{k_r}) p T^l - \frac{(\epsilon - \delta I)^2}{2} - \frac{I^2}{2}
\]

\[
\frac{dW}{da} = -e^m + p \frac{\Delta R}{k_r} + \frac{\Delta T}{k} + \delta I = 0 \implies e^m = p \frac{\Delta R}{k_r} + \frac{\Delta T}{k} + \delta I
\]

Second, we get the level of investment at the optimum:

\[
U_m = \frac{\kappa}{k_r} p R^h + (1 - \frac{k_e}{k_r}) p R^l + \kappa p T^h + (1 - \frac{k_e}{k_r}) p T^l - \frac{(\epsilon - \delta I)^2}{2} - \frac{I^2}{2}
\]

\[
\frac{dW}{dt} = \left[p(\frac{\Delta R}{k_r} + \frac{\Delta T}{k_e}) + \delta I \right] \Delta R + p R^l + \frac{\kappa}{k_r} p T^h + \left[p(\frac{\Delta R}{k_e} + \frac{\Delta T}{k_e}) + \delta I \right] \Delta T + p T^l - \frac{p^2(\frac{\Delta R}{k_e} + \frac{\Delta T}{k_e})^2}{2} + \frac{I^2}{2}
\]

Finally, the public regulator sets the transfers anticipating what will be the optimal investment and effort choices of the private agent and considering the limited liability constraint:

\[
(R^l + T^l) - \frac{I^2}{2} = R^l - \frac{I^2}{2}
\]

\[
T^l = 0
\]

Then the government maximizes the welfare function:

\[
\max_{T^l, T^l} W_m = \frac{\kappa}{k_r} p S^h + (1 - \frac{k_e}{k_r}) p S^l - (1 + \lambda)(\frac{k_e}{k_r} p \Delta T + T^l)
\]

s.t. \( (R^l + T^l) - \frac{I^2}{2} = R^l - \frac{I^2}{2} \)

\[
\max_{\Delta T} W_m = \left[p(\frac{\Delta R}{k_r} + \frac{\Delta T}{k_e}) + \Delta S + p S^l - (1 + \lambda)\left(p(\frac{\Delta R}{k_r} + \frac{\Delta T}{k_e}) + \Delta T\right) \right]
\]

\[
\frac{dW}{dT} = \left[p(\frac{\Delta R}{k_r} + \frac{\Delta T}{k_e}) + \delta I\right] \Delta T + \left[p(\frac{\Delta R}{k_e} + \frac{\Delta T}{k_e}) + \delta I\right] \Delta T + p T^l - \frac{p^2(\frac{\Delta R}{k_e} + \frac{\Delta T}{k_e})^2}{2} + \frac{I^2}{2}
\]

\[
\text{Final results are as follows:} \\
T^l = \frac{\kappa}{k_r} \frac{(1 + \lambda) \frac{\Delta R}{k_r}}{2(1 + \lambda)} , \ T^l = 0 \\
I^m = \left(\frac{\Delta S}{k_r} + (1 + \lambda) \frac{\Delta T}{k_e}\right) \delta \\
e^m = \left(\frac{\Delta S}{k_r} + (1 + \lambda) \frac{\Delta T}{k_e}\right) (1 + \delta^2)
\]

**Transfers are decided after the investment**  Solving the problem through a backward induction strategy, we first derive the optimal level of effort:

\[
U_m = \frac{\kappa}{k_r} p R^h + (1 - \frac{k_e}{k_r}) p R^l + \kappa p T^h + (1 - \frac{k_e}{k_r}) p T^l - \frac{(\epsilon - \delta I)^2}{2} - \frac{I^2}{2}
\]

\[
\frac{dW}{da} = -e^m + p \frac{\Delta R}{k_r} + \frac{\Delta T}{k} + \delta I = 0 \implies e^m = p \frac{\Delta R}{k_r} + \frac{\Delta T}{k} + \delta I
\]

Second, the public regulator sets the transfers anticipating what will be the optimal investment and effort choices of the private agent and considering the limited liability constraint:

\[
(R^l + T^l) - \frac{I^2}{2} = R^l - \frac{I^2}{2}
\]

\[
T^l = 0
\]

Then the government maximizes the welfare function:

\[
\max_{T^l, T^l} W_m = \frac{\kappa}{k_r} p S^h + (1 - \frac{k_e}{k_r}) p S^l - (1 + \lambda)(\frac{k_e}{k_r} p \Delta T + T^l)
\]

s.t. \( (R^l + T^l) - \frac{I^2}{2} = R^l - \frac{I^2}{2} \)
\[ \max_{\Delta T} W_m = \frac{[p(\Delta R + \Delta T) + \delta I]}{k_r} p\Delta S + pS^l - (1 + \lambda)(\frac{[p(\Delta R + \Delta T) + \delta I]}{k_r}) p\Delta T \]

\[ \frac{dW_m}{d\Delta T} = \frac{p(\Delta R + \Delta T)}{k_r} - (1 + \lambda) \frac{p\Delta T}{k_r} = (1 + \lambda) \left[ \frac{p(\Delta R + \Delta T) + \delta I}{k_r} \right] p = 0 \]

\[ \frac{dW_m}{d\Delta T} = \frac{p\Delta S}{k_r} - (1 + \lambda) \frac{p\Delta T}{k_r} = (1 + \lambda) \left[ \frac{p(\Delta R + \Delta T) + \delta I}{k_r} \right] = 0 \rightarrow \Delta T = \frac{k_r}{p(\Delta R + \Delta T) + \delta I} \]

Finally, we get the level of investment at the optimum:

\[ U_m = \frac{e}{k_r} pR^h + (1 - \frac{e}{k_r}) pR^l + \frac{e}{k_r} pT^h + (1 - \frac{e}{k_r}) pT^l - \frac{(e - \delta I)^2}{2} - \frac{T^2}{2} \]

\[ U_m = \frac{e}{k_r} \left( \frac{p(\Delta R + \Delta T)}{2(1 + \lambda)} + \delta I \right) \Delta R + pR^l + \frac{e}{k_r} \left( \frac{p(\Delta R + \Delta T)}{2(1 + \lambda)} + \delta I \right) \Delta T + pT^l - \frac{(p(\Delta R + \Delta T))^2}{2} - \frac{T^2}{2} \]

\[ U_m = \frac{e}{k_r} \left( \frac{p(\Delta R + \Delta T)}{2(1 + \lambda)} + \delta I \right) \Delta R + pR^l + \left( \frac{p(\Delta R + \Delta T)}{2(1 + \lambda)} + \delta I \right) \left( \frac{p(\Delta R + \Delta T)}{2(1 + \lambda)} + \delta I \right) + \frac{(p(\Delta R + \Delta T))^2}{2} - \frac{T^2}{2} \]

\[ \frac{dU_m}{dI} = \frac{e}{k_r} \frac{p(\Delta R + \Delta T)}{2(1 + \lambda)} + \delta I - \frac{T^2}{2} \]

\[ \frac{dU_m}{dI} = \frac{e}{k_r} \left( \frac{p(\Delta R + \Delta T)}{2(1 + \lambda)} + \delta I \right) - \frac{T^2}{2} - \frac{(T^2)}{2} - I = 0 \]

\[ \frac{dU_m}{dI} = \frac{e}{k_r} \left( \frac{p(\Delta R + \Delta T)}{2(1 + \lambda)} + \delta I \right) - \frac{T^2}{2} - \frac{(T^2)}{2} - I = 0 \]

\[ \frac{dU_m}{dI} = \frac{e}{k_r} \left( \frac{p(\Delta R + \Delta T)}{2(1 + \lambda)} + \delta I \right) = (1 + \frac{T^2}{2} - I) \rightarrow I^m = \left( \frac{p(\Delta R + \Delta T)}{2(1 + \lambda)} \right) \frac{\delta I}{2} \]

Final results are as follows:

\[ T^h = k_r \frac{p(\Delta R + \Delta T)}{2(1 + \lambda)} \frac{\delta I}{2} \]

\[ T^l = 0 \]

\[ I^m = \left( \frac{p(\Delta R + \Delta T)}{2(1 + \lambda)} \right) \frac{\delta I}{2} \]

\[ e^m = p(\Delta R + \Delta T) + \frac{\delta I}{2} I^m = (\frac{p(\Delta R + \Delta T)}{2(1 + \lambda)}) \frac{\delta I}{2} \]

It is straightforward to observe how final outcomes in terms of investment and effort are lower when the transfer is decided after the investment that when it is decided before.

Appendix E

Welfare Analysis Investment-contract regime

- Welfare

\[ W_i = \frac{e}{k_r} pS^h + (1 - \frac{e}{k_r}) pS^l - (1 + \alpha) \frac{T^2}{2} \]

\[ W_i = \frac{e}{k_r} p\Delta S + pS^l - (1 + \alpha) \frac{T^2}{2} + (1 + \alpha) \frac{\delta I^2}{2} \]

\[ W_i = \frac{e}{k_r} p\Delta S + pS^l - (1 + \alpha) \frac{\delta I^2}{8} [p(\Delta S - \frac{\delta I}{k_r}) + p\Delta R]^2 + (1 + \lambda) \frac{\Delta S}{2(1 + \lambda)k_r} + p\Delta R] \]

\[ W_i = \frac{p(\Delta R + \Delta T)}{k_r} \frac{\delta S}{2} [2p(\Delta S - \frac{\delta I}{k_r}) + p\Delta R]^2 + \frac{\Delta S}{2(1 + \lambda)k_r} + p\Delta R] \]

\[ W_i = p(\Delta R + \Delta T) \frac{\delta S}{2} [2p(\Delta S - \frac{\delta I}{k_r}) + p\Delta R]^2 + \frac{\Delta S}{2(1 + \lambda)k_r} + p\Delta R] + \frac{\Delta S}{2} \frac{p(\Delta R + \Delta T)}{k_r} \]

\[ W_i = p(\Delta R + \Delta T) \frac{\delta S}{2} [2p(\Delta S - \frac{\delta I}{k_r}) + p\Delta R]^2 + \frac{\Delta S}{2(1 + \lambda)k_r} + p\Delta R] + \frac{\Delta S}{2} \frac{p(\Delta R + \Delta T)}{k_r} \]

\[ W_i = p(\Delta R + \Delta T) \frac{\delta S}{2} [2p(\Delta S - \frac{\delta I}{k_r}) + p\Delta R]^2 + \frac{\Delta S}{2(1 + \lambda)k_r} + p\Delta R] + \frac{\Delta S}{2} \frac{p(\Delta R + \Delta T)}{k_r} \]

\[ W_i = p(\Delta R + \Delta T) \frac{\delta S}{2} [2p(\Delta S - \frac{\delta I}{k_r}) + p\Delta R]^2 + \frac{\Delta S}{2(1 + \lambda)k_r} + p\Delta R] + \frac{\Delta S}{2} \frac{p(\Delta R + \Delta T)}{k_r} \]

\[ W_i = p(\Delta R + \Delta T) \frac{\delta S}{2} [2p(\Delta S - \frac{\delta I}{k_r}) + p\Delta R]^2 + \frac{\Delta S}{2(1 + \lambda)k_r} + p\Delta R] + \frac{\Delta S}{2} \frac{p(\Delta R + \Delta T)}{k_r} \]

\[ W_i = p(\Delta R + \Delta T) \frac{\delta S}{2} [2p(\Delta S - \frac{\delta I}{k_r}) + p\Delta R]^2 + \frac{\Delta S}{2(1 + \lambda)k_r} + p\Delta R] + \frac{\Delta S}{2} \frac{p(\Delta R + \Delta T)}{k_r} \]
Utility

\[ U_i = \frac{\epsilon}{k_r} p R^h + (1 - \frac{\epsilon}{k_r}) p R^l - \frac{(e - \delta I)^2}{2} - \alpha^2 \]

\[ U_i = \frac{\epsilon}{k_r} p \Delta R + p R^l - \frac{(e - \delta I)^2}{2} - \alpha^2 \]

\[ U_i = (e - \delta I)(e + \delta I) + p R^l - \alpha^2 \]

\[ U_i = \frac{1}{2}(e^2 - \delta^2 I^2) + p R^l - \alpha^2 \]

\[ U_i = \frac{1}{2}(e^2 - \delta^2 I^2) + p R^l - \frac{2 \Delta R (1 + \lambda)}{2 \kappa_c} \frac{(e - \delta I)^2}{2} + \frac{p \Delta R^2}{2} \]

\[ U_i = \frac{1}{2}(e^2 - \delta^2 I^2) + p R^l - \frac{\Delta S}{\kappa_c} + \frac{\Delta R}{\kappa_c} p \Delta R + p R^l \]

Welfare Analysis Management-contract regime

Utility

\[ W_m = \frac{\epsilon}{k_r} p S^h + (1 - \frac{\epsilon}{k_r}) p S^l - (1 + \lambda)(\frac{\epsilon}{k_r} p \Delta T) \]

\[ W_m = \frac{\epsilon}{k_r} p \Delta S + p S^l - (1 + \lambda)(\frac{\epsilon}{k_r} p \Delta T) \]

\[ W_m = e(p \Delta S - (1 + \lambda)p \frac{\Delta S}{\kappa_c}(1 + \alpha)) + p S^l \]

\[ W_m = e(p \Delta S + p (1 + \lambda) \frac{\Delta R}{\kappa_c}) + p S^l \]

\[ W_m = (1 + \lambda) e(p \frac{\Delta S}{(1 + \lambda) \kappa_c} + p \frac{\Delta R}{\kappa_c}) + p S^l \]

\[ W_m = (1 + \lambda) \frac{e^2}{(1 + \lambda)} + p S^l \]

Utility

\[ U_m = \frac{\epsilon}{k_r} p R^h + (1 - \frac{\epsilon}{k_r}) p R^l + \frac{\epsilon}{k_r} p T^h - \frac{(e - \delta I)^2}{2} - \frac{\pi^2}{2} \]

\[ U_m = \frac{\epsilon}{k_r} p \Delta R + p R^l + \frac{\epsilon}{k_r} p \Delta T - \frac{(e - \delta I)^2}{2} - \frac{\pi^2}{2} \]

\[ U_m = e(p \Delta T - p \frac{\Delta T}{\kappa_c} + p R^l - \frac{(e - \delta I)^2}{2} - \frac{\pi^2}{2} \]

\[ U_m = (e - \delta I)(e - \frac{(e - \delta I)}{2}) + p R^l - \frac{\pi^2}{2} \]

\[ U_m = \frac{1}{2}(e - \delta I)(e - \frac{(e - \delta I)}{2}) + p R^l - \frac{\pi^2}{2} \]

\[ U_m = \frac{e^2}{2} + p R^l - \frac{\pi^2}{2} \]

\[ U_m = \frac{1}{2} \left( \frac{p \Delta S + p \frac{\Delta S}{\kappa_c}}{2(1 + \lambda)} \right)^2 (1 + \delta^2) + p R^l - \frac{1}{2} \left( \frac{p \Delta S + p \frac{\Delta S}{\kappa_c}}{2(1 + \lambda)} \right)^2 \delta^2 (1 + \delta^2) \]

\[ U_m = \frac{1}{2} \left( \frac{p \Delta S + p \frac{\Delta S}{\kappa_c}}{2(1 + \lambda)} \right)^2 (1 + \delta^2) - \frac{\delta^2}{2} + p R^l \]

\[ U_m = \frac{1}{2} \left( \frac{p \Delta S + p \frac{\Delta S}{\kappa_c}}{2(1 + \lambda)} \right)^2 (1 + \delta^2) + p R^l \]

\[ U_m = \frac{\epsilon}{2} \left( \frac{p \Delta S + p \frac{\Delta S}{\kappa_c}}{2(1 + \lambda)} \right)^2 + p R^l \]

\[ U_m = \frac{e^2}{2(1 + \alpha)} + p R^l \]
Comparative Statics Analysis with respect to $\delta$

- Investment-contract scenario

\[
U_i + W_i = \frac{1}{2} p \frac{\Delta R}{r_c} e + p R_l + p \frac{\Delta R}{r_c} p \frac{\Delta S}{k_s} + (1 + \lambda) T^2 + p S_l
\]

\[
\frac{d(U_i + W_i)}{d\delta} = \frac{1}{2} p \frac{\Delta R}{r_c} \delta [2p \frac{\Delta S}{(1+\lambda) k_s} \frac{r_c}{k_r} + 2(1 + \lambda) \frac{1}{2} [2p \frac{\Delta S}{(1+\lambda) k_s} + p \frac{\Delta R}{k_r}]
\]

\[
\frac{d(U_i + W_i)}{d\delta} = \delta [2p \frac{\Delta S}{(1+\lambda) k_s} \frac{r_c}{k_r} + p \frac{\Delta R}{k_r} \frac{r_c}{k_r}] + \frac{1}{2} \delta [2p \frac{\Delta S}{(1+\lambda) k_s} + p \frac{\Delta R}{k_r}]
\]

- Management-contract scenario

\[
U_m + W_m = \frac{e^2}{2(1+\delta^2)} + (1 + \lambda) \frac{e^2}{(1+\delta^2)} + p S_l + p R_l
\]

\[
\frac{d(U_m + W_m)}{d\delta} = \frac{e^2}{2(1+\lambda)} (3 + 2\lambda) + p S_l + p R_l
\]

\[
\frac{d(U_m + W_m)}{d\delta} = \left( p \frac{\Delta S}{2 k_r} (1+\lambda) \right) 2 (1 + \delta^2) \frac{(3+2\lambda)}{2} + p S_l + p R_l
\]

- Difference

\[
\frac{d(U_i + W_i)}{d\delta} - \frac{d(U_m + W_m)}{d\delta} = \delta [2p \frac{\Delta S}{(1+\lambda) k_s} \frac{r_c}{k_r} + p \frac{\Delta R}{k_r} \frac{r_c}{k_r}] (1 + \lambda) \frac{e^2}{(1+\delta^2)} + p S_l + p R_l
\]

\[
\frac{d(U_i + W_i)}{d\delta} - \frac{d(U_m + W_m)}{d\delta} = \left( p \frac{\Delta S}{k_r} (1+\lambda) \right) + p \frac{\Delta R}{k_r} \frac{r_c}{k_r} \left( 2p \frac{\Delta S}{(1+\lambda) k_s} \frac{r_c}{k_r} + p \frac{\Delta R}{k_r} \right) - \left( p \frac{\Delta S}{k_r} (1+\lambda) \right) + p \frac{\Delta R}{k_r} \frac{r_c}{k_r} \left( 2p \frac{\Delta S}{(1+\lambda) k_s} \frac{r_c}{k_r} + p \frac{\Delta R}{k_r} \right)
\]

\[
\frac{d(U_i + W_i)}{d\delta} - \frac{d(U_m + W_m)}{d\delta} = \left( p \frac{\Delta S}{k_r} (1+\lambda) \right) + p \frac{\Delta R}{k_r} \frac{r_c}{k_r} \left( 2p \frac{\Delta S}{(1+\lambda) k_s} \frac{r_c}{k_r} + p \frac{\Delta R}{k_r} \right) - \left( p \frac{\Delta S}{k_r} (1+\lambda) \right) + p \frac{\Delta R}{k_r} \frac{r_c}{k_r} \left( 2p \frac{\Delta S}{(1+\lambda) k_s} \frac{r_c}{k_r} + p \frac{\Delta R}{k_r} \right)
\]