Abstract

This study presents an overlapping generations model to capture the nature of the competition between generations regarding two redistribution policies, public education and public pensions. In addition, we investigate the effects of population aging on these policies and economic growth from a political economy viewpoint. We show that two aging factors, longevity and the political power of the old, have opposite effects on redistribution policies and economic growth. The relative strength between the two factors is negative for pensions, but hump-shaped patterns appear for public education and economic growth.

- Keywords: economic growth; population aging; public education; public pensions
- JEL Classification: D78, E24, H55
1 Introduction

Redistribution policy preferences are dictated largely by age. Working adults, who are parents with altruistic concern for their children, are more likely to support public education because they can benefit from highly educated children, which makes expenditure on education more attractive for these parents. Aged retirees are less likely to support public education because they cannot obtain the benefits of education directly. Instead, they support public pension expenditure that compensates for loss of earnings in their retirement. The age-dependent difference in policy preferences suggests that the recent trend of aging in developed countries, which increases the political power of the old, may induce governments to shift expenditure from education to pensions. Some recent studies support this view (Poterba, 1997, 1998; Fernandez and Rogerson, 2001; Harris, Evans, and Schwab, 2001; Casey et al., 2003).

However, such predictions are not always possible when forward-looking decision making by adults is considered. Higher productivity of children may result in a larger tax base, and thus, larger pension benefits for adults in their retirement. Adults may vote on public education today, taking account of such an intertemporal effect in a forward-looking manner. This raises the following two questions. How does a conflict of interest between adults and the old affect the conflicting redistribution policy, that is, public education that benefits adults and public pensions that benefit the old? In turn, how does the conflict affect economic growth in the long run? This study aims to answer these questions from a political economy viewpoint.

For analysis, this study presents a politico-economic model to characterize the nature of the competition between generations, and in addition, examines the effect of population aging on redistribution policy and economic growth. To capture generational conflict, the model economy contains an infinite sequence of overlapping generations in which each is comprised of many identical individuals who lived over three periods, namely, children, adults, and the old. The adults are faced with uncertain lifetimes and are endowed with altruism toward children. Although individuals’ preferences over lifetime consumption and bequests are identical, policy disagreements arise owing to the longer planning horizon of the adults. In addition, the model contains physical and human capital accumulation through saving and educational investment, which enable us to demonstrate the effect of redistribution policy on economic growth.

Within this framework, we consider probabilistic voting a la Lindbeck and Weibull (1987). In each period, the adults and the old participate in voting; children are not enfranchised. The government in power maximizes a political objective function of the weighted sum of the utilities of the adults and the old, taking account of the impact on the adults’ economic decisions (for applications of the probabilistic voting for overlapping-
generations models, see, e.g., Grossman and Helpman, 1998; Hassler et al., 2005; and Song, Storesletten, and Zilibotti, 2012). In this voting environment, the redistribution preferences of the adults are state-dependent. In other words, successive generations are linked through physical and human capital accumulation. The state dependence of policy preferences gives the adult voters the means to influence the policy outcome when they are old. Their incentive to manipulate policy preferences stems from their desire to obtain large pension benefits in old age. We capture this forward-looking behavior of the adults by focusing on Markov-perfect political equilibria, in which voters condition their strategies on payoff-relevant state variables (i.e., physical and human capital in the present model).

The Markov-perfect political equilibrium is affected by the two factors representing population aging: the probability of living in old age (i.e., longevity), and the political weight of the old that reflects their share of voting. We show that greater longevity results in a lower pension-to-GDP ratio, a higher public education-to-GDP ratio, and a higher growth rate. The first two effects arise because of the abovementioned forward-looking behavior. Greater longevity implies a larger weight of the adults’ utility of their old-age consumption. This incentivizes the government to save and invest more for adults’ old-age consumption by cutting the current pension benefits and spending more on education. A shift of government expenditure from pension to education promotes human capital formation, and thus, increases economic growth. In addition, we show that a larger political weight of the old produces opposite effects to those of longevity. A larger weight of the old incentivizes the government to spend more on the current old population through the provision of pension benefits, which works to crowd out physical and human capital accumulation.

To consider the relative strength between the two factors, we investigate their overall effect, and show that the net effect on the pension-to-GDP ratio is negative. However, the public education-to-GDP ratio and the growth rate may show a hump-shaped pattern in response to the joint increase in longevity and the political weight of the old. The hump-shaped pattern in public education may provide one possible explanation for the conflicting findings on the relationship between aging and redistribution spending (e.g., Cattaneo and Wolter, 2009, and references therein). In addition, the hump-shaped pattern in economic growth provides a possible theoretical foundation for explaining the nonlinear relationship between aging and economic growth suggested by An and Jeon (2006) and Kunze (2014).

The remainder of this paper is organized as follows. We first present a literature review. Thereafter, Section 2 presents the model and characterizes economic equilibrium. Section 3 describes political equilibrium. Section 4 investigates how the two aging factors
affect government expenditure on education and pensions. Section 5 analyzes the effects of the two aging factors on economic growth. Section 6 provides concluding remarks.

1.1 Literature Review

The present study is related to the literature on the political economy of public education and pensions by Bearse, Glomm, and Janeba (2001), Soares (2006), Iturbe-Ormaetxe and Valera (2012), Kaganovich and Meier (2012), Kaganovich and Zilcha (2012), and Naito (2012). A common feature of these studies is that the two-dimensional voting aspect is reduced to one dimension for simplicity of analysis. In other words, they consider a vote over public education for a given pension benefit, or a vote over the allocation of tax revenue for a given tax rate. Therefore, these studies do not indicate how the size of the government (i.e., the tax rate) and the allocation of government spending between education and pensions are jointly determined through voting in the presence of generational conflict.

This problem is resolved by introducing two-dimensional voting (Rangel, 2003; Levy, 2005; Poutvaara, 2006; and Arawatari and Ono, 2014). However, these studies abstract from physical and/or human capital formation, and thus, say nothing about the interaction between policy and capital formation. Capital formation is introduced by Kemnitz (2000), Gradstein and Kaganovich (2004), Holz-Eakin, Lovely, and Tosun (2004), Tosun (2008), and Bernasconi and Profeta (2012). These studies assume myopic voting, in which the current voters take future policy as given. In other words, the forward-looking decision of voters is absent in the analysis of these studies. Therefore, they abstract from the feedback mechanism between current and future redistribution policies through physical and/or human capital accumulation, which plays a crucial role in shaping redistribution policies.

The feedback mechanism is demonstrated by Beauchemin (1998), Forni (2005), Bassettto (2008), Gonzalez-Eiras and Niepelt (2008), Song (2011), and Chen and Song (2014). In particular, the present study is closely related to Gonzalez-Eiras and Niepelt (2012), Rancia and Russo (2013), and Ono (2014), who analyze the politics of public education and pensions in overlapping generations models. However, these studies differ from ours in that: (1) there is no altruism toward children (Gonzalez-Eiras and Niepelt, 2012), (2) their focus is on a normative implication rather than the aging implication for the political equilibrium outcome (Lancia and Russo, 2013), (3) there is no physical capital accumulation that affects economic growth through redistribution policy (Ono, 2014). By contrast, the present study contributes to the literature by demonstrating the overall effect of the two aging factors, that is, longevity and the political power of the old, on redistribution policies and economic growth in the presence of altruism toward children.
Apart from the studies mentioned above, the present study is related to Lamrecht, Michel, and Vidal (2005) and Kunze (2014), who investigate the growth effect of redistribution policy in overlapping generations models in which altruistic parents finance the education of their children. However, both of these studies focus on a single policy issue: public pensions in the case of Lamrecht, Michel, and Vidal (2005), and public education in the case of Kunze (2014). The present study differs from theirs in that we consider the two policy issues, investigate how they are shaped by population aging, and in turn, analyze how they affect economic growth.

2 Model

The model is based on that presented by Lamrecht, Michel, and Vidal (2005) and Kunze (2014). The economy starts at period 0 and consists of overlapping generations with a constant population that is normalized to one. Individuals are identical within a generation, live at most for three periods, namely, childhood, adulthood, and old age. They are faced with uncertain lifetimes in the third period of life. The probability of living in old age is $\pi \in [0, 1]$. This is idiosyncratic for all individuals and is constant across periods.

2.1 Individuals

The economic behavior of individuals over the life cycle is as follows. During childhood, individuals make no economic decisions; they receive education financed by parents as well as by the government. In adulthood, individuals work, receive a market wage, and make a tax payment. They use after-tax income for consumption, saving, and educational investment for their children. In old age, individuals are retired. They receive the returns from saving and pension benefits, and consume both.

Consider an individual born in period $t - 1$. In period $t$, he is an adult and is endowed with $h_t$ units of human capital. He supplies it inelastically in the labor market, and obtains the wage $w_t h_t$, where $w_t$ is the wage rate per efficiency unit of labor in period $t$. After paying the tax $\tau_t w_t h_t$ where $\tau_t \in (0, 1)$ the period-$t$ income tax rate, the individual distributes his after-tax income into consumption, $c_t$, savings held as an annuity and invested in physical capital, $s_t$, and educational investment for his children, $e_t$. Therefore, the period-$t$ budget constraint for an adult becomes as follows:

$$c_t + e_t + s_t \leq (1 - \tau_t)w_t h_t.$$ 

The period-$t + 1$ budget constraint in old age is

$$d_{t+1} \leq \frac{R_{t+1}}{\pi} s_t + p_{t+1},$$
where $d_{t+1}$ is consumption in old age, $R_{t+1} (> 0)$ is the gross return from investment in capital, $R_{t+1} s_t / \pi$ is the return from saving, and $p_{t+1}$ is the pension benefit. If an individual dies at the end of adulthood, his annuitized wealth is transferred to the individuals who live throughout old age via annuity markets. Therefore, the return from saving becomes $R_{t+1} / \pi$ under the assumption of perfect annuity markets.

A period-$t$ adult individual cares about his child’s human capital in period $t+1$, $h_{t+1}$. This is a function of the parent’s private educational spending, $e_t$, government spending for public education, $x_t$, and the parent’s human capital, $h_t$. In particular, $h_{t+1}$ is formulated by the following equation:

$$h_{t+1} = D(e_t)^\delta (x_t)^\eta (h_t)^{1-\delta-\eta},$$

where $D(> 0)$ is a scale factor, and $\delta \in (0, 1)$ and $\eta \in (0, 1)$ with $\delta + \eta < 1$ denote the elasticity of education technology with respect to private and public education spending.

We assume that parents are altruistic toward their children and are concerned about the disposable income of their children in adulthood, $(1 - \tau_{t+1}) w_{t+1} h_{t+1}$. The preferences of an individual born in period $t-1$ are specified by the following expected utility function of the logarithmic form:

$$U_t = \ln c_t + \pi \ln d_{t+1} + \gamma \ln (1 - \tau_{t+1}) w_{t+1} h_{t+1},$$

where $\gamma (> 0)$ denotes the intergenerational degree of altruism. We substitute the budget constraints and the human capital production function into the utility function to write the following unconstrained maximization problem:

$$\max_{\{s_t, e_t\}} \ln [(1 - \tau_t) w_t h_t - e_t - s_t] + \pi \ln \left[ \frac{R_{t+1}}{\pi} s_t + \frac{p_{t+1}}{R_{t+1}} \right] + \gamma \ln (1 - \tau_{t+1}) w_{t+1} D(e_t)^\delta (x_t)^\eta (h_t)^{1-\delta-\eta}.$$

By solving the problem, we obtain the following saving, education, and consumption functions:

$$s_t = \frac{\pi}{1 + \pi + \gamma \delta} \cdot \left[ (1 - \tau_t) w_t h_t - (1 + \gamma \delta) \frac{p_{t+1}}{R_{t+1}} \right], \quad (1)$$

$$e_t = \frac{\gamma \delta}{1 + \pi + \gamma \delta} \cdot \left[ (1 - \tau_t) w_t h_t + \frac{\pi p_{t+1}}{R_{t+1}} \right], \quad (2)$$

$$c_t = \frac{1}{1 + \pi + \gamma \delta} \cdot \left[ (1 - \tau_t) w_t h_t + \frac{\pi p_{t+1}}{R_{t+1}} \right]. \quad (3)$$

The saving function in (1) states that a higher level of after-tax wage, $(1 - \tau_t) w_t$, implies higher savings, whereas a higher level of pension, $p_{t+1}$, implies lower savings. The education and consumption functions in (2) and (3) state that a higher lifetime income, $(1 - \tau_t) w_t h_t + \pi p_{t+1}/R_{t+1}$, results in larger spending on education and consumption, respectively.
2.2 Firms

In each period, there is a continuum of identical firms that are perfectly competitive profit maximizers. According to Cobb–Douglas technology, they produce a final good $Y_t$ using two inputs, physical capital $K_t$ and human capital $H_t$. The aggregate output is given by

$$Y_t = A (K_t)\alpha (H_t)^{1-\alpha},$$

where $A(>0)$ is a scale parameter and $\alpha \in (0, 1)$ denotes the capital share.

Let $k_t \equiv K_t/H_t$ denote the physical to human capital ratio. The first-order conditions for profit maximization with respect to $H_t$ and $K_t$ are as follows:

$$w_t = (1 - \alpha)A (k_t)\alpha, \quad (4)$$
$$\rho_t = \alpha A (k_t)^{\alpha-1}, \quad (5)$$

where $w_t$ and $\rho_t$ are the wage and the rental price of capital, respectively. The conditions state that firms hire human and physical capital until the marginal products are equal to the factor prices.

2.3 Government Budget Constraint

Pension and public education are financed by the tax on labor income. Because the population of each generation is normalized to one, the aggregate human capital, $H_t$, is equal to the individual human capital, $h_t$. Given this property, the government budget constraint in period $t$ is expressed as follows:

$$\pi p_t + x_t = \tau_t \rho_t h_t.\quad (6)$$

The left-hand side shows the expenditure on pensions, $\pi p_t$, and public education, $x_t$, and the right-hand side shows the revenue from taxing labor income.

2.4 Economic Equilibrium

The market clearing condition for capital is $K_{t+1} = s_t$, which expresses the equality of total savings by the adult in period $t$, $s_t$, to the stock of aggregate physical capital at the beginning of period $t + 1$, $K_{t+1}$. With the use of $k_{t+1} \equiv K_{t+1}/H_{t+1}$ and $H_{t+1} = h_{t+1}$, the condition is rewritten as

$$k_{t+1}h_{t+1} = s_t.$$

The economic equilibrium in the present model is defined as follows.

**Definition 1.** Given a sequence of policies, $\{\tau_t, x_t, p_t\}_{t=0}^\infty$, an *economic equilibrium* is a sequence of allocations $\{c_t, d_t, s_t, k_{t+1}, h_{t+1}\}_{t=0}^\infty$ and prices $\{w_t, \rho_t, R_t\}_{t=0}^\infty$ with the initial conditions $k_0(>0)$ and $h_0(>0)$ such that (i) given $(w_t, R_{t+1}, \tau_t, x_t, p_{t+1})$, $(c_t, c_{t+1}^y, s_t)$
solves the utility maximization problem; (ii) given \((w_t, \rho_t), k_t\) solves the profit maximization problem of a firm; (iii) given \((w_t, h_t, k_t), (\tau_t, x_t, p_t)\) satisfies the government budget constraint; (iv) \(\rho_t = R_t\) holds; (v) the capital market clears: \(k_{t+1}h_{t+1} = s_t\).

In economic equilibrium, the indirect utility function of the adults in period \(t\), \(V^A_t\), and that of the old in period \(t\), \(V^o_t\), can be expressed as functions of government policy and physical and human capital as follows:

\[
V^A(k_t, h_t, \tau_t, k_{t+1}, h_{t+1}, \tau_{t+1}, p_{t+1}) = \ln \left( \frac{1}{1 + \pi + \gamma \delta} \left( (1 - \tau_t)(1 - \alpha)A(k_t)^\alpha h_t + \frac{\pi p_{t+1}}{\alpha A(k_{t+1})^{\alpha-1}} \right) \right) \\
+ \pi \ln \left( \frac{\alpha A(k_{t+1})^{\alpha-1}}{\pi} k_{t+1}h_{t+1} + p_{t+1} \right) \\
+ \gamma \ln (1 - \tau_{t+1})(1 - \alpha)A(k_{t+1})^{\alpha} h_{t+1},
\]

\( (6) \)

\[
V^o(k_t, h_t, \tau_t, p_t) = \ln \left( \frac{\alpha A(k_t)^{\alpha-1}}{\pi} k_t h_t + p_t \right) + \gamma \ln (1 - \tau_t)(1 - \alpha)A(k_t)^{\alpha} h_t,
\]

\( (7) \)

where some irrelevant terms are omitted from the expressions. The first and second terms in \((6)\) correspond to the utility of consumption in adulthood and in old age, respectively, and the third term shows the utility from the disposable income of their adult children. The first term of the old indirect utility corresponds to the utility of old-age consumption and the second term shows the utility from the disposable income of their adult children.

### 3 Political Equilibrium

The present study assumes probabilistic voting developed by Lindbeck and Weibull (1987) for demonstrating the political mechanism. In each period, the government in power maximizes a political objective function. Formally, the political objective function in period \(t\) is given by

\[
\Omega_t = \omega V^o(k_t, h_t, \tau_t, p_t) + (1 - \omega) V^A(k_t, h_t, \tau_t, k_{t+1}, h_{t+1}, \tau_{t+1}, p_{t+1}),
\]

where \(\omega \in (0, 1)\) and \(1 - \omega\) are the relative weights of the old and the adult, respectively. The government problem in period \(t\) is to maximize \(\Omega_t\) subject to the government budget constraint, given the state variables, \(k_t\) and \(h_t\). An explicit microfoundation for this modeling is explained in Persson and Tabellini (2000, Chapter 3) and Acemoglu and Robinson (2005, Appendix).

In this study, we restrict our attention to a stationary Markov-perfect equilibrium. In the present framework, Markov perfectness implies that outcomes depend only on the payoff-relevant state variables, that is, physical and human capital, \(k\) and \(h\).
stationary property implies that our focus is on equilibrium policy rules that are not dependent on time. Therefore, the expected levels of tax and public pension for the next period, \( \tau_{t+1} \) and \( p_{t+1} \), are given by functions of the next period stock of physical and human capital, \( \tau_{t+1} = T(k_{t+1}, h_{t+1}) \) and \( p_{t+1} = P(k_{t+1}, h_{t+1}) \), respectively. By using recursive notation with \( z' \) denoting the next period \( z \), we can define stationary Markov-perfect political equilibrium as follows.

**Definition 2.**

A stationary Markov-perfect political equilibrium is a set of functions, \( \langle T, X, P \rangle \), where 

\[
T : \mathbb{R}_{++} \times \mathbb{R}_{++} \rightarrow [0, 1] \text{ is a tax rule, } \tau = T(k, h),
\]

\[
X : \mathbb{R}_{++} \times \mathbb{R}_{++} \rightarrow \mathbb{R}_{++} \text{ is a public education expenditure rule, } x = X(k, h), \text{ and}
\]

\[
P : \mathbb{R}_{++} \times \mathbb{R}_{++} \rightarrow \mathbb{R}_{++} \text{ is a public pension expenditure rule, } p = P(k, h), \text{ such that the following conditions are satisfied:}
\]

(i) the capital market clears,

\[
k' h' = \frac{\pi}{1 + \pi + \gamma \delta} \left[ (1 - T(k, h))(1 - \alpha)A(k)^{\alpha} h - (1 + \gamma \delta) \frac{P(k', h')}{\alpha A(k')^{\alpha-1}} \right], \tag{8}
\]

(ii) given \( k \) and \( h \), \( \langle T(k, h), X(k, h), P(k, h) \rangle = \arg \max \Omega(k, h, \tau, x, p, p') \) subject to 

\[
p' = P(k', h'), \text{ the capital market clearing condition in (8), the government budget constraint,}
\]

\[
\pi P(k, h) + X(k, h) = T(k, h)(1 - \alpha)A(k)^{\alpha} h,
\]

and the human capital production function,

\[
h' = D \left[ \frac{\gamma \delta}{1 + \pi + \gamma \delta} \left\{ (1 - T(k, h))(1 - \alpha)A(k)^{\alpha} h + \frac{\pi P(k', h')}{\alpha A(k')^{\alpha-1}} \right\} \right] (X(k, h))^{\eta} (h)^{1 - \delta - \eta},
\]

where \( \Omega \) is defined by \( \Omega(k, h, \tau, x, p, p') \equiv \omega V^o(k, h, \tau, p) + (1 - \omega) V^A(k, h, \tau, k', h', \tau', p') \).

In order to obtain a set of functions in Definition 2, we conjecture the following linear functions:

\[
\begin{align*}
\{ p' &= \bar{P} \cdot A(k')^{\alpha} h', \\
x' &= \bar{X} \cdot A(k')^{\alpha} h', \}
\end{align*}
\tag{9}
\]

where \( \bar{P}(> 0) \) and \( \bar{X}(> 0) \) are constant parameters. By using this conjecture and the constraints in Definition 2(ii), we can obtain the political objective function as follows:

\[
\Omega = \omega \ln \left( \frac{\alpha A(k)^{\alpha-1}}{\pi} kh + p \right) + \phi \ln \left( (1 - \alpha)A(k)^{\alpha} h - \pi p - x \right) + (1 - \omega) \eta (\pi + \gamma)(1 - \alpha) \ln x, \tag{10}
\]

where the terms unrelated to policy are omitted from the expression. The derivation of (10) is provided in Appendix A.1.
The term $\phi$ in the expression of (10) is defined by

$$\phi \equiv \omega \gamma + (1 - \omega) \{1 + (\alpha(1 - \delta) + \delta)(\pi + \gamma)\}.$$ 

This includes several factors that may affect political decision making. First, the term $\omega \gamma$ is the weight of the utility of the old. In particular, the parameter $\omega$ is the political weight of the old; and the parameter $\gamma$ is the weight of the utility from the disposable income of the child.

Second, the term $(1 - \omega) \{1 + (\alpha(1 - \delta) + \delta)(\pi + \gamma)\}$ is the economic weight of the utility of the adult, $(1 + (\alpha(1 - \delta) + \delta)(\pi + \gamma))$, multiplied by the political weight of the adult, $1 - \omega$. In particular, the three terms within the braces, $1$, $\pi$, and $\gamma$, show the weight of the utility of consumption in adulthood, the weight of the utility of consumption in old age, and the weight of the utility of the disposable income of the child, respectively.

We solve the problem of maximizing $\Omega$. The first-order conditions with respect to $p$ and $x$ are

$$p : \frac{\omega}{1 - \alpha} A(k)^{\alpha} h + p \leq \frac{\phi \pi}{(1 - \alpha) A(k)^{\alpha} h - \pi p - x},$$

$$x : \frac{\phi}{(1 - \alpha) A(k)^{\alpha} h - \pi p - x} = \frac{(1 - \omega) \eta (\pi + \gamma) (1 - \alpha)}{x}. $$

A strict inequality holds in the first condition if $p = 0$. By using these conditions, we obtain the following result.

**Proposition 1.** There exists a stationary Markov-perfect political equilibrium distinguished by $p > 0$ if $\omega > \alpha \{\omega + (1 - \omega) \eta (\pi + \gamma) (1 - \alpha) + \phi\}$, and $p = 0$ otherwise. The corresponding policy functions are as follows:

$$P(k, h) = \max \left\{0, \bar{P}_{p>0} \cdot A(k)^{\alpha} h \right\},$$

$$X(k, h) = \max \left\{\bar{X}_{p=0} \cdot A(k)^{\alpha} h, \bar{X}_{p>0} \cdot A(k)^{\alpha} h \right\},$$

$$T(k, h) = \max \left\{\frac{\omega + (1 - \omega) \eta (\pi + \gamma) (1 - \alpha) - \alpha \phi / (1 - \alpha)}{\phi + \omega + (1 - \omega) \eta (\pi + \gamma) (1 - \alpha)}, \frac{(1 - \omega) \eta (\pi + \gamma)}{\phi + \omega + (1 - \omega) \eta (\pi + \gamma) (1 - \alpha)} \right\},$$

where

$$\bar{P}_{p>0} \equiv \frac{\omega - \alpha \{\phi + \omega + (1 - \omega) \eta (\pi + \gamma)(1 - \alpha)\}}{\pi \{\phi + \omega + (1 - \omega) \eta (\pi + \gamma)(1 - \alpha)\}},$$

$$\bar{X}_{p=0} \equiv \frac{(1 - \omega) \eta (\pi + \gamma) (1 - \alpha)^2}{\phi + (1 - \omega) \eta (\pi + \gamma) (1 - \alpha)},$$

$$\bar{X}_{p>0} \equiv \frac{(1 - \omega) \eta (\pi + \gamma) (1 - \alpha)}{\phi + \omega + (1 - \omega) \eta (\pi + \gamma) (1 - \alpha)}.$$

**Proof.** See Appendix A.2.
The condition for $p > 0$ is reformulated as follows:

$$\frac{1}{\alpha} > \left( \frac{1}{\omega} + \gamma \right) + \left( \frac{1}{\omega} - 1 \right) (\pi + \gamma) [(\eta + \delta) + (1 - (\eta + \delta)) \alpha].$$

The right-hand side of the expression is decreasing in $\omega$ and is increasing in $\pi$. That is, the condition indicates that public pensions are more likely to be provided in political equilibrium if longevity is lower and/or the old have larger political power. The two parameters, $\pi$ and $\omega$, representing population aging, have opposite implications for the provision of public pensions.

To understand the reasons for these conflicting results, recall the political objective function

$$
\Omega = \omega \ln \left( \frac{1}{\pi} \alpha A (k)^\alpha h + p \right) \\
+ \left\{ \frac{\omega \gamma + (1 - \omega)}{(\pi.2)} \left\{ 1 + \left( \alpha(1 - \delta) + \delta \right) \left( \frac{\pi}{(\pi.2)} + \gamma \right) \right\} \ln \left( (1 - \alpha) A (k)^\alpha h - \pi p - x \right) \right\} \\
+ \left\{ (1 - \omega) \eta (\pi + \gamma) (1 - \alpha) \ln x. \right\}
$$

The function indicates that longevity, denoted by $\pi$, affects the provision of public pensions through the following four factors: the term $(\pi.1)$, representing the return from saving; the terms $(\pi.2)$ and $(\pi.4)$, representing the weight of the utility of the adults for their consumption in old age; and the term $(\pi.3)$, representing the tax burden on the adults for the provision of public pensions.

Longevity affects the provision of public pensions through these four terms in the following way. First, the term $(\pi.1)$ shows a positive effect of longevity on public pensions. Greater longevity results in a lower return from saving. To compensate for this loss of saving, the government chooses a higher level of public pensions.

Second, the terms $(\pi.2)$ and $(\pi.4)$ show a negative effect of longevity on public pensions. Greater longevity implies a larger weight of the adult’s utility of their old-age
consumption. This incentivizes the government to save more for their old-age consumption by cutting the pension burden in adulthood. This negative effect on public pensions is observed by the term \(\pi_2\).

In addition, greater longevity incentivizes the government to increase pension benefits for adults in their old age. For this purpose, the government shifts the allocation of tax revenue from pensions for the aged to public education for children. This shift expands the tax base in the future by improving human capital of the children, and thus, increases pension benefits for adults in their old age. This negative effect on current pension provision is observed by the term \(\pi_4\).

Finally, the term \(\pi_3\) shows a negative effect of longevity on public pension. Greater longevity implies a larger burden on adults for the provision of public pensions. This incentivizes the government to cut the public pension burden from the viewpoint of maintaining consumption levels of adults. In summary, longevity has a positive effect on pensions through the term \(\pi_1\), but there are three negative effects through the terms \(\pi_2\), \(\pi_3\), and \(\pi_4\), which outweigh the positive effect. Therefore, greater longevity results in a lower level of public pensions.

The political objective function indicates that the political power of the old, denoted by \(\omega\), affects the provision of public pensions through the following four factors: the term \(\omega_1\), representing the weight of the utility of consumption for the old; the term \(\omega_2\), representing the weight of the utility from the disposable income of the children; and the terms \(\omega_3\) and \(\omega_4\), representing the weight of the utility of the adult.

These four terms have the following implications for the provision of public pensions. First, the term \(\omega_1\) shows a positive effect on public pensions. Greater power of the old implies a larger weight of the utility of consumption for the old. This incentivizes the government to allocate more tax revenue to public pensions. Second, the term \(\omega_2\) shows a negative effect on public pensions. Greater power of the old implies a larger weight of their utility from adults’ disposable income. To improve this utility, the government cuts the pension burden of adults, thereby resulting in a lower level of public pensions.

Third, the terms \(\omega_3\) and \(\omega_4\) show a positive effect on public pensions. Greater power of the old is equivalent to a smaller weight of the utility of the adults. In particular, it implies a smaller weight of the utility of consumption for the adults via the term \(\omega_3\) and a smaller weight of the utility from the disposable income of the children via the term \(\omega_4\). These effects imply that the government shifts the tax revenue from public education to public pensions. In summary, the political power of the old has a negative effect on public pension through the term \(\omega_2\), but this is outweighed by the three positive effects through the terms \(\omega_1\), \(\omega_3\), and \(\omega_4\). Therefore, greater power of the old results in a higher level of public pensions.
4 Pensions and Education

The results established in Section 3 indicate that pensions and public pensions are affected by two aging factors, $\pi$, representing longevity, and $\omega$, representing the political power of the old. To consider their effects on pensions and education, we focus on pension-to-GDP and public education-to-GDP ratios, $\pi_p/y$ and $x/y$, respectively, and analyze the effects of increases in $\pi$ and $\omega$ on these ratios (Subsection 4.1). Then, we consider a special case of $\pi = \omega$ and evaluate the overall effects of these factors (Subsection 4.2).

4.1 Effects of Population Aging on Pensions and Education

The following proposition demonstrates the effects of $\pi$ and $\omega$ on $\pi_p/y$.

**Proposition 2.** Suppose that $\omega > \alpha \{\omega + (1 - \omega)\eta (\pi + \gamma) (1 - \alpha) + \phi\}$ holds: there exists a stationary Markov-perfect political equilibrium with $p > 0$. The pension-to-GDP ratio, $\pi_p/y$, increases with greater political power of the old, and decreases with greater longevity: $\partial (\pi_p/y)/\partial \omega > 0$ and $\partial (\pi_p/y)/\partial \pi < 0$.

**Proof.** See Appendix A.3.

To confirm the statement in Proposition 2, we compute the pension-to-GDP ratio as follows:

$$\pi_p/y = \frac{1}{(\frac{1}{\omega} + \gamma) + (\frac{1}{\omega} - 1) (\pi + \gamma) (\alpha + (\delta + \eta) (1 - \alpha))} - \alpha. \quad (12)$$

The derivation of Eq. (12) is given in Appendix A.3. The parameter $\pi$ in Eq. (12) shows the negative effect of longevity on $\pi_p/y$ through the terms $(\pi.2)$ and $(\pi.4)$ in the political objective function; and the parameter $\omega$ shows the positive effect of the political power of the old through the term $(\omega.1)$ in the political objective functions. Therefore, the two aging factors have opposite implications for the pension-to-GDP ratio. The overall effect of aging on pension expenditure depends on the relative strength of the two factors. This issue is analyzed in Subsection 4.2.

Given the result in Proposition 2 and the government budgetary constraint, it is natural to conjecture that the ratio $x/y$ decreases as $\omega$ increases, and it increases as $\pi$ increases, as follows.

**Proposition 3.** The public education-to-GDP ratio decreases with greater political power of the old and increases with greater longevity: $\partial (x/y)/\partial \omega < 0$ and $\partial (x/y)/\partial \pi > 0$.

**Proof.** See Appendix A.4.

To confirm the statement in Proposition 3, we first compute the $x/y$ ratio when $p > 0,$
\[ x \bigg| \frac{y}{y} \bigg|_{p>0} = \frac{1}{\eta(1-\alpha)} \left\{ \frac{1}{\pi + \gamma} \cdot \frac{1 + \omega \gamma}{1 - \omega} + (\alpha (1-\delta) + \delta) \right\} + 1 \right]^{-1}, \tag{13} \]

\[ x \bigg| \frac{y}{y} \bigg|_{p=0} = \eta(1-\alpha)^2 \cdot \left\{ \frac{(\omega.2) \omega \gamma}{(1-\omega)(\pi + \gamma)} + \frac{1}{\pi + \gamma} + (\alpha(1-\delta) + \delta) + \eta(1-\alpha) \right\}^{-1}. \tag{14} \]

The terms \((\pi.4), (\omega.2)\) and \((\omega.4)\) in the expression corresponds to those in the political objective function. The effects of \(\pi\) and/or \(\omega\) through the other terms do not appear in the expression because they cancel each other out.

The term \((\pi.4)\) shows a positive effect of longevity on the \(x/y\) ratio. Greater longevity implies a larger weight of the adults’ utility of their old-age consumption. This incentivizes the government to increase pension benefits for the adults in their old age. To increase the future pension benefits, the government aims to expand the future tax base by shifting the allocation of the current tax revenue from pensions for the old to education for children. Therefore, greater longevity leads to an increase in the ratio \(x/y\) through the term \((\pi.4)\).

Greater power of the old has the following two effects via the terms \((\omega.2)\) and \((\omega.4)\). First, greater power of the old implies a larger weight of their utility from adults’ disposable income. This incentivizes the government to reduce the tax burden of the adults by cutting expenditure on education. This effect is observed by the term \((\omega.2)\). Second, greater power of the old implies a smaller weight of adults’ utility from children’s disposable income. This gives the government a weaker incentive to improve human capital through public education. This effect is observed by the term \((\omega.4)\). Because of these two effects, greater power of the old results in a decrease in the ratio \(x/y\).

### 4.2 Relative Strength between Longevity and Political Power of the Old

This subsection considers the overall effect of the two aging factors on pension and education expenditure. For this purpose, we focus on a special case of \(\omega = \pi\), in which the political weight of the old, \(\omega\), fairly reflects the share of the old in the population. We investigate how an increase in \(\omega\) (i.e., \(\pi\)) affects the ratios of \(\pi p/y\) and \(x/y\).

First, we consider the ratio of \(\pi p/y\) in Eq. (12). When \(\omega = \pi\), the ratio is written as follows:

\[ \pi \frac{p}{y} = \frac{1}{\left( \frac{1}{\omega} + \gamma \right) + (1 + \frac{2}{\omega} - \omega - \gamma) \left( \alpha + (\delta + \eta) (1-\alpha) \right)} - \alpha. \]
From the expression, we can immediately find that \( \partial (\pi p/y) / \partial \omega > 0 \) holds. The positive effect of \( \omega \) outweighs the negative effect of \( \pi \). An aging population results in an increase in the pension-to-GDP ratio when \( \omega = \pi \).

Next, we consider the ratio \( x/y \) in Eqs. (13) and (14). When \( \omega = \pi \), the ratio is written as

\[
x/y \begin{cases} \frac{e^{\omega}}{y} & p > 0 \\
\eta(1 - \alpha)^2 \left[ \frac{\omega^7}{(1-\omega)(\omega+\gamma)} + \frac{1}{\omega+\gamma} + (\alpha(1 - \delta) + \delta) + \eta(1 - \alpha) \right]^{-1} & p = 0.
\end{cases}
\]

The expression indicates that the political power of the old has a negative effect on the ratio through the terms \( \omega \gamma \) and \( 1 - \omega \), and a positive effect through the term \( \omega + \gamma \) for both cases of \( p > 0 \) and \( p = 0 \). The relative strength of these three effects depends on the initial distribution of political power. Therefore, the model may demonstrate a hump-shaped pattern in the ratio.

To investigate the effect further, we first ignore the threshold value of \( \omega \) that distinguishes the status of pension provision, and analyze the effect of \( \omega \) on the ratio for the two cases, \( p > 0 \) and \( p = 0 \). We obtain the following results: (a) for \( \gamma < (-1 + \sqrt{5})/2 \), the ratio shows a hump-shaped pattern in both cases; (b) for \( \gamma \in [(-1 + \sqrt{5})/2, 1) \), the ratio shows a hump-shaped pattern in the case of \( p > 0 \) and decreases with an increase in \( \omega \) in the case of \( p = 0 \); (c) for \( 1 \leq \gamma \), the ratio decreases with an increase in \( \omega \) in both cases. The proof of this statement is provided in Appendix A.5.

Next, we consider the effect of \( \omega \) on the ratio, taking account of a change in the status of pension provision around the threshold value of \( \omega \). Based on the result mentioned above, we can predict that (i) for \( \gamma < 1 \), the ratio shows a hump-shaped pattern; (ii) for \( 1 \leq \gamma \), the ratio decreases with \( \omega \), regardless of whether \( p > 0 \) or \( p = 0 \). The numerical result in Figure 1 supports this prediction.

To understand the mechanism behind the result, recall the ratio \( x/y \) in Eqs. (13) and (14), including the three terms that capture population aging effects. First, the term \( \omega^2 \) in the equations is the weight of the utility of the old from the disposable income of their children. Greater political power of the old implies a larger weight of that utility, thereby, giving the government an incentive to reduce the tax burden on adults. A reduction in tax revenue has a negative effect on public education expenditure. This negative effect is enlarged as the weight of the utility, represented by \( \omega \), becomes higher.

Second, the term \( \omega^4 \) in the equations is the weight of the utility of the adults from the disposable income of their children. Greater power of the old implies a lower weight of that utility, thereby giving the government a weaker incentive to improve human capital.
through public education expenditure. Third, the term \((\pi.4)\) in the equations represents the longevity of the adults. Greater longevity implies a larger weight of the utility of the adults. Thus, the government aims to increase the consumption of adults in their old age by raising pension benefits. The government finances increased pension payments by improving human capital through public education.

In summary, there are two negative effects through the terms \((\omega.2)\) and \((\omega.4)\), and one positive effect through the term \((\pi.4)\). When \(\gamma\) is high, such that \(\gamma \geq 1\), the negative effect through the term \((\omega.2)\) has a major impact, and thus, the two negative effects dominate the positive effect through the term \((\pi.4)\). However, when \(\gamma < 1\), the relative strength of the two opposing effects depends on the initial distribution of political power; the ratio shows a hump-shaped pattern in response to an increase in \(\omega\).

5 Economic Growth

Based on the results established thus far, we derive the growth rate of the economy, and investigate how it is affected by population aging. For the presentation of the analysis, recall that per capita capital, \(k\), is defined by

\[
k = \frac{K}{H} = \frac{K}{h},
\]

where \(K\) is aggregate capital and \(H\) is aggregate human capital. The second equality holds because the size of the working generation is unity for all \(t, L_t = 1\). Then the growth rate of aggregate capital is

\[
\frac{K'}{K} = \frac{k'}{k} \frac{h'}{h}.
\]

In the steady state with \(k' = k\), the growth rate of aggregate physical capital \(K'/K\) is equal to the growth rate of human capital, \(h'/h\). In what follows, we focus on the steady-state growth rate.

The following analysis proceeds as follows. First, we consider the cases of \(p > 0\) and \(p = 0\). We show that per capita capital stably converges to a unique steady state for each case, and that at the steady state, the growth rate of human capital remains constant across periods. Second, we undertake numerical analysis to investigate the overall effects of increases in \(\omega\) and \(\pi\) on the growth rate at the steady state.

5.1 Steady-state Growth Rate

Recall the capital market clearing condition and the human capital production function in Definition 2. With the use of the policy functions derived in Proposition 1, we can
reformulate these two equations in the following physical and human capital formation functions:

\[
k' = \psi_k(\bar{P}) \cdot [(1 - \alpha)A(k)^\alpha h - \pi p - x]^{1-\delta} (x)^{-\eta} (h)^{-(1-\delta-\eta)},
\]

\[
h' = \psi_h(\bar{P}) \cdot [(1 - \alpha)A(k)^\alpha h - \pi p - x]^{\delta} (x)^{\eta} (h)^{(1-\delta-\eta)}. \tag{15}
\]

The terms \(\psi_k(\cdot)\) and \(\psi_h(\cdot)\) are constant and dependent on \(\bar{P}\), where \(\bar{P}\) is the coefficient of the policy function, \(p = \bar{P} \cdot A(k)^\alpha h\), and is presented in Proposition 1. The derivation of these equations and the definition of \(\psi_k(\cdot)\) and \(\psi_h(\cdot)\) are provided in Appendix A.1.

Suppose that \(p > 0\). The policy functions are given by

\[
P(k, h) = \bar{P}_{p>0} \cdot A(k)^\alpha h, \]

\[
X(k, h) = \bar{X}_{p>0} \cdot A(k)^\alpha h.
\]

Substituting these functions into the physical capital formation function in (15), we obtain the law of motion of physical capital when \(p > 0\) as follows:

\[
k' = \psi_k(\bar{P}_{p>0}) \cdot [(1 - \alpha) - \pi \bar{P}_{p>0} - \bar{X}_{p>0}]^{1-\delta} \cdot (\bar{X}_{p>0})^{-\eta} \cdot (A(k)^\alpha)^{(1-\delta-\eta)}. \tag{17}
\]

The equation implies that there exists a unique and nontrivial steady state; and that for any initial condition \(k > 0\), the sequence of \(k\) stably converges to the unique steady state. From (17), we can compute the steady-state level of \(k\) when \(p > 0\), denoted by \(\bar{k}_{p>0}\), as follows:

\[
\bar{k}_{p>0} = \left[\psi_k(\bar{P}_{p>0}) \cdot \{(1 - \alpha) - \pi \bar{P}_{p>0} - \bar{X}_{p>0}\}^{1-\delta} \cdot (\bar{X}_{p>0})^{-\eta} \cdot (A(\bar{k}_{p>0})^\alpha)^{(1-\delta-\eta)^{1/(1-\alpha(1-\delta-\eta))}}\right]. \tag{18}
\]

Using \(\bar{k}_{p>0}\) in (18) and the policy functions in Proposition 1, we can write the law of motion of human capital when \(p > 0\) as follows:

\[
\frac{h'}{h}_{p>0} = \psi_h(\bar{P}_{p>0}) \cdot [(1 - \alpha) - \pi \bar{P}_{p>0} - \bar{X}_{p>0}]^{1-\delta} \cdot (\bar{X}_{p>0})^{-\eta} \cdot (A(\bar{k}_{p>0})^\alpha)^{\delta+\eta}. \tag{19}
\]

The equation says that the growth rate is constant across periods at the steady state. Following the same procedure, we obtain the growth rate when \(p = 0\) as follows:

\[
\frac{h'}{h}_{p=0} = \psi_h(\bar{P}_{p=0}) \cdot [(1 - \alpha) - \pi \bar{P}_{p=0} - \bar{X}_{p=0}]^{\delta} \cdot (\bar{X}_{p=0})^{-\eta} \cdot (A(\bar{k}_{p=0})^\alpha)^{\delta+\eta}. \tag{20}
\]

### 5.2 Numerical Examples

To investigate the effect of increases in \(\pi\) and \(\omega\) on economic growth, we undertake numerical analysis and obtain the following results, as illustrated in Figure 2. First, an increase in \(\pi\) leads to an increase in the growth rate, as depicted in Panel (a); and an increase in \(\omega\) leads to a decrease in the growth rate, as depicted in Panel (b). The two parameters, \(\omega\) and \(\pi\), representing population aging, have opposite effects on economic growth.
To understand the mechanism behind the results, recall the growth rate of human capital in Eqs. (19) and (20). The growth rate is affected by $\pi$ and/or $\omega$ through the following four factors: $\psi_h$, representing the productivity of human capital, $\pi \bar{P}$, representing the pension burden, $\bar{X}$, representing the public education burden and expenditure, and $\bar{k}$, representing steady-state capital. The parameters $\pi$ and $\omega$ have effects on the growth rate through these four factors, but the second and the third factors are crucial in determining the growth rate. Therefore, we focus on these two factors in interpreting the result.

An increase in $\pi$ results in a decrease in the pension burden (Proposition 2) and an increase in the public education burden (Proposition 3). The former implies a positive income effect on economic growth and the latter implies a negative income effect on economic growth. However, the latter effect is outweighed by the positive effect of public education expenditure, observed by the term $(\bar{X})^\eta$ in Eqs. (19) and (20). Therefore, an increase in $\pi$ leads to an increase in the economic growth rate. The opposite result holds when we are concerned with an increase in $\omega$ because the parameter $\omega$ has an opposite effect on pension and education expenditure, as demonstrated in Propositions 2 and 3.

To examine the total effect, we assume $\pi = \omega$ and investigate the effect of $\omega$ on economic growth. We find that there is a hump-shaped pattern between $\omega$ and the growth rate, as depicted in Figure 3. When $\omega = \pi$, a joint increase in $\omega$ and $\pi$ has two opposing effects on the growth rate: a positive effect, as demonstrated above, and a negative effect via the political power of the old. There is a critical value of $\omega$ that balances these two effects. The positive effect is greater (less) than the negative effect below (above) the critical value. Therefore, the model shows a hump-shaped pattern between $\omega$ and the economic growth rate.

6 Summary and Conclusion

How does the conflict of interest between generations affect the two redistribution policies, namely, public education and public pensions? In turn, how does the conflict affect economic growth? The present study is an attempt to answer these questions from a political economy point of view.

We consider the two factors representing population aging: longevity of agents and the political weight of the old. We show that greater longevity results in a lower pension-to-GDP ratio, a higher public education-to-GDP ratio, and a higher economic growth rate. Moreover, greater political power of the old produces the opposite effects to those of longevity. In addition, we consider the relative strength of the two factors, and find that
the net effect is negative on the pension-to-GDP ratio. However, the public education-to-
GDP ratio and the economic growth rate show hump-shaped patterns in response to joint
increases in longevity and the political power of the old. These patterns may provide a
possible explanation for the conflicting findings regarding the effects of population aging
on public education expenditure and economic growth.
A Proofs

A.1 Derivation of (10)

To derive (10), we first use the government budget constraint to replace $\tau$ in the indirect utility functions by $x$ and $p$. Then, we substitute the conjectures in (9) into the political objective function $\Omega$. Finally, we replace $k'$ and $h'$ with $k, h, x$ and $p$, respectively, by using the capital market clearing condition and the human capital production function. Next, we provide the details of the calculation step by step.

**Step 1.**

Recall the government budget constraint in Definition 2(ii), which is rewritten as follows:

$$1 - \tau = \frac{(1 - \alpha)A(k)^\alpha h - \pi p - x}{(1 - \alpha)A(k)^\alpha h}.$$ 

Plugging this into the indirect utility functions in (6) and (7), we obtain

$$V^A = \ln \frac{1}{1 + \pi + \gamma \delta} \left[ (1 - \alpha)A(k)^\alpha h - \pi p - x + \frac{\pi p'}{\alpha A(k')^{\alpha-1}} \right]$$

$$+ \pi \ln \left( \frac{\alpha A(k')^\alpha h'}{\pi} + p' \right) + \gamma \ln \left[ (1 - \alpha)A(k')^\alpha h' - \pi p' - x' \right],$$

$$V^o = \ln \left( \frac{\alpha A(k)^\alpha h}{\pi} + p \right) + \gamma \ln \left[ (1 - \alpha)A(k)^\alpha h - \pi p - x \right].$$  

(21)

**Step 2.**

We substitute the conjecture of the policy functions in (9) into $V^A$ to obtain

$$V^A = \ln \frac{1}{1 + \pi + \gamma \delta} \left[ (1 - \alpha)A(k)^\alpha h - \pi p - x + \frac{\pi p'}{\alpha A(k')^{\alpha-1}} \right]$$

$$+ \pi \ln \left( \frac{\alpha A(k')^\alpha h'}{\pi} + p' \right) + \gamma \ln \left[ (1 - \alpha)A(k')^\alpha h' - \pi p' - x' \right],$$

$$= \ln \left[ (1 - \alpha)A(k)^\alpha h - \pi p - x + \frac{\pi p'}{\alpha A(k')^{\alpha-1}} \right] \cdot (x)^{\rho} \cdot (h)^{1-\delta-\eta}.$$  

(22)

where the terms unrelated to political decision are omitted from the expression.

**Step 3.**

To replace $k'$ and $h'$ in (22) with $k, h, p$ and $x$, we first recall the human capital production function,

$$h' = D \cdot \left[ \frac{\gamma \delta}{1 + \pi + \gamma \delta} \left\{ (1 - \tau) w h + \frac{\pi p'}{R'} \right\} \right]^\delta \cdot (x)^{\eta} \cdot (h)^{1-\delta-\eta}$$

$$= D \cdot \left( \frac{\gamma \delta}{1 + \pi + \gamma \delta} \right)^\delta \cdot \left( w h - \pi p - x + \frac{\pi p'}{R'} \right) \cdot (x)^{\eta} \cdot (h)^{1-\delta-\eta}$$

$$= D \cdot \left( \frac{\gamma \delta}{1 + \pi + \gamma \delta} \right)^\delta \cdot \left\{ (1 - \alpha)A(k)^\alpha h - \pi p - x + \frac{\pi p A(k')^\alpha h'}{\alpha A(k')^{\alpha-1}} \right\} \cdot (x)^{\eta} \cdot (h)^{1-\delta-\eta},$$
where the first line comes from the private education function in (2), the second line from the government budget constraint, and the third line from the profit maximization conditions in (4) and (5) and the conjecture of the policy function \( p' \) in (9). The abovementioned expression is reduced to

\[
h' = D \cdot \left( \frac{\gamma \delta}{1 + \pi + \gamma \delta} \right)^\delta \left[ (1 - \alpha)A (k)^\alpha h - \pi p - x + \frac{\pi}{\alpha} \bar{P} k'h' \right] \cdot (x)^\eta \cdot (h)^{1 - \delta - \eta}. \tag{23}\]

The expression in (23) includes \( k' \) on the right-hand side. To replace \( k' \) by \( k \) and \( h \), recall the capital market clearing condition,

\[
k'h' = \frac{\pi}{1 + \pi + \gamma \delta} \left[ (1 - \tau) w h - (1 + \gamma \delta) \frac{p'}{R} \right]
\[
= \frac{\pi}{1 + \pi + \gamma \delta} \left[ (1 - \alpha)A (k)^\alpha h - \pi p - x - (1 + \gamma \delta) \frac{\bar{P} A (k')^\alpha h'}{\alpha A (k')^{\gamma - 1}} \right],
\]

where the first line comes from the capital market clearing condition, \( k'h' = s \) with the saving function in (1), and the second line comes from the profit maximization conditions in (4) and (5) and the conjecture of the policy function \( p' \) in (9). After rearranging the terms, we can rewrite the abovementioned expression as follows:

\[
k'h' = \frac{\pi}{1 + \pi + \gamma \delta} \cdot \frac{1 + \pi}{\alpha} \cdot \frac{1 + \pi}{\alpha} \cdot \left[ (1 - \alpha)A (k)^\alpha h - \pi p - x \right]. \tag{24}\]

We substitute (24) into (23) to obtain

\[
h' = \psi_h (\bar{P}) \cdot [(1 - \alpha)A (k)^\alpha h - \pi p - x]^\delta \cdot (x)^\eta \cdot (h)^{1 - \delta - \eta}, \tag{25}\]

and we substitute (25) into (24) to obtain

\[
k' = \psi_k (\bar{P}) \cdot [(1 - \alpha)A (k)^\alpha h - \pi p - x]^{1 - \delta} \cdot (x)^{-\eta} \cdot (h)^{-1 + \delta - \eta}, \tag{26}\]

where \( \psi_h (\bar{P}) \) and \( \psi_k (\bar{P}) \) are defined as follows:

\[
\psi_h (\bar{P}) = D \cdot \left( \frac{\gamma \delta}{1 + \pi + \gamma \delta} \right)^\delta \left(1 + \frac{\pi}{\alpha} \bar{P} \cdot \frac{1}{1 + \pi \gamma \delta} \cdot \frac{1}{1 + \pi + \gamma \delta} \cdot \frac{1}{\alpha} \right)^\delta,
\]

\[
\psi_k (\bar{P}) = \left( \frac{1 + \pi \gamma \delta}{1 + \pi + \gamma \delta} \cdot \bar{P} \cdot \frac{1}{1 + \pi \gamma \delta} \cdot \frac{1}{1 + \pi + \gamma \delta} \cdot \frac{1}{\alpha} \right)^{-1}.
\]

By using (24), (25) and (26), we can rewrite \( V^A \) in (22) as

\[
V^A = \left\{ 1 + (\alpha (1 - \delta) + \delta) \cdot (\pi + \gamma) \right\} \ln [(1 - \alpha)A (k)^\alpha h - \pi p - x] + \eta (\pi + \gamma) (1 - \alpha) \ln x, \tag{27}\]

where constant terms are omitted from the expression. With \( V^o \) in (21) and \( V^A \) in (27), we can write the political objective function \( \Omega = \omega V^o + (1 - \omega)V^A \) as expressed in (10).
A.2 Proof of Proposition 1

Suppose that public pensions are provided in the next period, \( p' > 0 \). Because preferences are specified by the logarithmic utility function, we conjecture linear policy functions of public education and public pensions for the next period, \( x' = X_{p>0} \cdot A(k')^\alpha h' \) and \( p' = P_{p>0} \cdot A(k')^\alpha h' \), where \( X_{p>0}(>0) \) and \( P_{p>0}(>0) \) are constant parameters. Under this conjecture, the solution to the problem becomes as follows:

\[
X(k, h) = \frac{(1 - \omega)\eta(\pi + \gamma)(1 - \alpha)}{\phi + \omega + (1 - \omega)\eta(\pi + \gamma)(1 - \alpha)} A(k)^\alpha h, \\
P(k, h) = \frac{\omega - \alpha \{ \phi + \omega + (1 - \omega)\eta(\pi + \gamma)(1 - \alpha) \}}{\pi \{ \phi + \omega + (1 - \omega)\eta(\pi + \gamma)(1 - \alpha) \}} A(k)^\alpha h,
\]

where \( P(k, h) > 0 \) holds if and only if \( \omega > \alpha \{ \phi + \omega + (1 - \omega)\eta(\pi + \gamma)(1 - \alpha) \} \) holds. Given that \( \omega > \alpha \{ \phi + \omega + (1 - \omega)\eta(\pi + \gamma)(1 - \alpha) \} \) holds, the abovementioned solution constitutes a stationary Markov-perfect political equilibrium if

\[
\begin{align*}
X_{p=0} & \equiv \frac{(1 - \omega)\eta(\pi + \gamma)(1 - \alpha)}{\phi + \omega + (1 - \omega)\eta(\pi + \gamma)(1 - \alpha)}, \\
P_{p=0} & \equiv \frac{\omega - \alpha \{ \phi + \omega + (1 - \omega)\eta(\pi + \gamma)(1 - \alpha) \}}{\pi \{ \phi + \omega + (1 - \omega)\eta(\pi + \gamma)(1 - \alpha) \}}.
\end{align*}
\]  

(28)

Alternatively, suppose that \( p' = 0 \) holds; that is, public pensions are not provided in the next period. Considering the estimation of policy functions as \( x' = X_{p=0} \cdot A(k')^\alpha h' \) and \( p' = P_{p=0} \cdot A(k')^\alpha h' \), where \( P_{p=0} = 0 \), the solution to the problem becomes as follows:

\[
X(k, h) = \frac{(1 - \omega)\eta(\pi + \gamma)(1 - \alpha)^2}{\phi + (1 - \omega)\eta(\pi + \gamma)(1 - \alpha)} A(k)^\alpha h, \\
P(k, h) = 0 \text{ if } \omega \leq \alpha \{ \phi + \omega + (1 - \omega)\eta(\pi + \gamma)(1 - \alpha) \}.
\]

Given that \( \omega \leq \alpha \{ \phi + \omega + (1 - \omega)\eta(\pi + \gamma)(1 - \alpha) \} \) holds, this solution constitutes a stationary Markov-perfect political equilibrium if

\[
X_{p=0} \equiv \frac{(1 - \omega)\eta(\pi + \gamma)(1 - \alpha)^2}{\phi + (1 - \omega)\eta(\pi + \gamma)(1 - \alpha)} \text{ and } P_{p=0} \equiv 0.
\]  

(29)

The tax rates for \( p > 0 \) and \( p = 0 \) are obtained by substituting the corresponding policy functions \( X \) and \( P \) into the government budget constraint.

\[ \blacksquare \]

A.3 Proof of Proposition 2

The pension-to-GDP ratio is

\[
\frac{\pi p}{y} = \frac{\omega - \alpha \{ \phi + \omega + (1 - \omega)\eta(\pi + \gamma)(1 - \alpha) \}}{\{ \phi + \omega + (1 - \omega)\eta(\pi + \gamma)(1 - \alpha) \} \omega} = \frac{\phi\gamma + (1 - \omega) \{ 1 + (\alpha (1 - \delta) + \delta) \cdot (\pi + \gamma) \} + \omega + (1 - \omega)\eta(\pi + \gamma)(1 - \alpha) - \alpha}{\phi\gamma + (1 - \omega) \{ 1 + (\alpha (1 - \delta) + \delta) \cdot (\pi + \gamma) \} + \omega + (1 - \omega)\eta(\pi + \gamma)(1 - \alpha) - \alpha},
\]
where the second line comes from the definition of $\phi$. After rearranging the terms, we obtain
\[
\frac{\pi p}{y} = \frac{1}{\frac{1}{\omega} + \gamma + \frac{1}{\omega} - 1} \left( \pi + \gamma \right) \left( \alpha + \left( \delta + \eta \right) \left( 1 - \alpha \right) \right) - \alpha,
\]
indicating that $\pi p/y$ is increasing in $\omega$ and is decreasing in $\pi$.

\[\text{A.4 Proof of Proposition 3}\]

Suppose that $p > 0$ holds. The ratio, denoted by $x/y|_{p>0}$, is
\[
\frac{x}{y} \bigg|_{p>0} = \frac{\left( 1 - \omega \right) \eta \left( \pi + \gamma \right) \left( 1 - \alpha \right)}{\phi + \omega + \left( 1 - \omega \right) \eta \left( \pi + \gamma \right) \left( 1 - \alpha \right)} \frac{1}{\frac{\phi + \omega}{\left( 1 - \omega \right) \eta \left( \pi + \gamma \right) \left( 1 - \alpha \right)} + 1} = \frac{1}{\frac{\omega \gamma + \left( 1 - \omega \right) \left( 1 + \left( \alpha \left( 1 - \delta \right) + \delta \right) \left( \pi + \gamma \right) \right) + \omega \eta \left( \pi + \gamma \right) \left( 1 - \alpha \right)}{\left( 1 - \omega \right) \eta \left( \pi + \gamma \right) \left( 1 - \alpha \right)} + 1}.
\]
The expression in the third line is rewritten as
\[
\frac{x}{y} \bigg|_{p>0} = \frac{1}{\frac{1}{\omega} - \frac{\omega + \frac{\phi + \omega}{\eta \left( 1 - \alpha \right)}}{1 + \frac{\omega \gamma + \left( 1 - \omega \right) \left( 1 + \left( \alpha \left( 1 - \delta \right) + \delta \right) \left( \pi + \gamma \right) \right) + \omega \eta \left( \pi + \gamma \right) \left( 1 - \alpha \right)}{\eta \left( 1 - \alpha \right)} + 1},
\]
indicating that $x/y|_{p>0}$ is decreasing in $\omega$ and is increasing in $\pi$.

Next, suppose that $p = 0$. The ratio, denoted by $x/y|_{p=0}$, is
\[
\frac{x}{y} \bigg|_{p=0} = \frac{\left( 1 - \omega \right) \eta \left( \pi + \gamma \right) \left( 1 - \alpha \right)^2}{\phi + \left( 1 - \omega \right) \eta \left( \pi + \gamma \right) \left( 1 - \alpha \right)} = \frac{\left( 1 - \omega \right) \eta \left( \pi + \gamma \right) \left( 1 - \alpha \right)^2}{\omega \gamma + \left( 1 - \omega \right) \left( 1 + \left( \alpha \left( 1 - \delta \right) + \delta \right) \left( \pi + \gamma \right) \right) + \left( 1 - \omega \right) \eta \left( \pi + \gamma \right) \left( 1 - \alpha \right)} = \frac{\eta \left( 1 - \alpha \right)^2}{\frac{\omega \gamma}{\left( 1 - \omega \right) \left( \pi + \gamma \right)} + \left\{ \frac{1}{\pi + \gamma} + \left( \alpha \left( 1 - \delta \right) + \delta \right) \right\} + \eta \left( 1 - \alpha \right)}. \quad (32)
\]
Therefore, $x/y|_{p=0}$ is decreasing in $\omega$ and is increasing in $\pi$.

\[\text{A.5 Proof of the Statement in Section 4.2}\]

First, suppose that $p > 0$ holds. When $\omega = \pi$, the ratio is
\[
\frac{x}{y} \bigg|_{p>0} = \left[ \frac{1}{\eta \left( 1 - \alpha \right)} \left\{ \frac{1}{\omega + \gamma} \cdot \frac{1 + \omega \gamma}{1 - \omega} + \left( \alpha \left( 1 - \delta \right) + \delta \right) \right\} + 1 \right]^{-1}.
\]
The differentiation of the term \( \frac{1}{\omega+\gamma} \cdot \frac{1+\omega\gamma}{1-\omega} \) with respect to \( \omega \) is

\[
(\omega + \gamma)^2 (1 - \omega)^2 \partial \left[ \frac{1}{\omega + \gamma} \cdot \frac{1+\omega\gamma}{1-\omega} \right] / \partial \omega = f(\omega) \equiv \gamma \cdot (\omega)^2 + 2\omega + ((\gamma)^2 + \gamma - 1),
\]

implying that \( f(\omega) \geq 0 \iff \partial \left( x/y |_{p>0} \right) / \partial \omega \leq 0 \). The function \( f \) has the following property:

\[
\begin{align*}
\{ & f(0) \geq 0 \iff \gamma \geq -\frac{1+\sqrt{5}}{2}, \\
& f'(0) = 2 > 0, \\
& f(1) = (\gamma)^2 + 2\gamma + 1 > 0.
\end{align*}
\]

Therefore, we obtain the following result, as illustrated in Figure 4.

(i) If \( \gamma \geq -\frac{1+\sqrt{5}}{2} \), then \( f(\omega) > 0 \), that is, \( \partial \left( x/y |_{p>0} \right) / \partial \omega < 0 \) holds \( \forall \omega \in (0, 1) \);

(ii) If \( \gamma < -\frac{1+\sqrt{5}}{2} \), then there is a critical value of \( \omega \), denoted by \( \hat{\omega}_{p>0} \in (0, 1) \), such that \( f(\omega) \leq 0 \iff \partial \left( x/y |_{p>0} \right) / \partial \omega \geq 0 \) if and only if \( \omega \leq \hat{\omega}_{p>0} \).

[Figure 4 here.]

Next, suppose that \( p = 0 \) holds. When \( \omega = \pi \), the ratio \( x/y |_{p=0} \) in (32) is

\[
\left. \frac{x}{y} \right|_{p=0} = \eta(1 - \alpha)^2 \cdot \left[ \frac{\omega\gamma}{(1-\omega)(\omega + \gamma)} + \frac{1}{\omega + \gamma} + (\alpha(1 - \delta) + \delta) + \eta(1 - \alpha) \right]^{-1}.
\]

The differentiation of the term \( \frac{\omega\gamma}{(1-\omega)(\omega + \gamma)} + \frac{1}{\omega + \gamma} \) with respect to \( \omega \) is

\[
\frac{\partial}{\partial \omega} \left[ \frac{\omega\gamma}{(1-\omega)(\omega + \gamma)} + \frac{1}{\omega + \gamma} \right] = \frac{g(\omega)}{[(1-\omega)(\omega + \gamma)]^2},
\]

where \( g(\omega) \) is defined by \( g(\omega) \equiv -(1 - \gamma)(\omega)^2 + 2\omega + ((\gamma)^2 - 1) \) and has the following property:

\[
\begin{align*}
g(0) \geq 0 \iff \gamma \geq 1, \\
g(1) = \gamma(1 + \gamma) > 0.
\end{align*}
\]

Therefore, we obtain the following result, as illustrated in Figure 5.

(i) If \( \gamma \geq 1 \), then \( g(\omega) > 0 \iff \partial \left( x/y |_{p=0} \right) / \partial \omega < 0 \forall \omega \in (0, 1) \);

(ii) If \( \gamma < 1 \), then there is a critical value of \( \omega \), denoted by \( \hat{\omega}_{p=0} \in (0, 1) \), such that \( g(\omega) \leq 0 \iff \partial \left( x/y |_{p=0} \right) / \partial \omega \geq 0 \) if and only if \( \omega \leq \hat{\omega}_{p=0} \).

[Figure 5 here.]
References


Figure 1: The figure demonstrates the effect of an increase in $\omega$ on the ratio $x/y$ when $\omega = \pi$. The horizontal axis takes the value of $\omega$; the vertical axis shows the ratio $x/y$. The parameter $\gamma$ is set to be 0.5, 0.8, and 1.1 for panels (a), (b), and (c), respectively. The other parameters are set as $\delta = 1/3$ and $\eta = 0.1$, which are common for these panels. The solid curve is the ratio $x/y$ when $p > 0$; the dotted curve is the ratio $x/y$ when $p = 0$. The vertical line shows the threshold value of $\omega$ that distinguishes the two cases, $p > 0$ and $p = 0$. 
Figure 2: The figure illustrates the growth effect of an increase in $\pi$ in Panel (a) and that of an increase in $\omega$ in Panel (b). The horizontal axis takes the value of $\pi$ in Panel (a) and the value of $\omega$ in Panel (b). The vertical axis shows the growth rate, $h'/h$, in both panels. Parameters are set at $A = 4.8$ and $D = 5.0$ in both panels; $\omega = 0.5$ in Panel (a); $\pi = 0.7$ in Panel (b). The solid and dotted curves are the growth rates when $p > 0$ and $p = 0$, respectively. The vertical lines in Panels (a) and (b) show the threshold values of $\pi$ and $\omega$, respectively.
Figure 3: The figure demonstrates the effect of an increase in $\omega$ on the growth rate when $\omega = \pi$. The setting is identical to that in Figure 2, except that $\omega$ is equal to $\pi$. 
Figure 4: Panel (a) is the case of $\gamma \geq -\frac{1+\sqrt{5}}{2}$; Panel (b) is the case of $\gamma < -\frac{1+\sqrt{5}}{2}$.
Figure 5: Panel (a) is the case of $\gamma \geq 1$; Panel (b) is the case of $\gamma < 1$. 