The direct and indirect effects of corruption on inequality

Ratbek Dzhumashev

Department of Economics, Monash University

Abstract

Empirical studies have established the following regularities in the relationship between corruption and inequality: (i) the nominal amount of bribes paid increases with the income of agents; (ii) the burden of bribes as a share of income decreases; and (iii) at the macro-level, corruption and inequality have a nonlinear relationship; that is, in countries with intermediate levels of corruption, the effect of corruption on inequality is higher than in countries where corruption levels are low or high. This study proposes a theoretical model that jointly explains the aforementioned empirical findings by showing that corruption affects inequality directly, by creating income and productivity disparity across agents, and indirectly, by reducing private productivity through curtailing the positive externality of government spending.

Key words: inequality, corruption, public spending

JEL classification: H54, O41
1. INTRODUCTION

Corruption occurs when public agents, abuse their public positions, seek and extract illicit income from the government or private agents; as a result, it distorts the quality and quantity of the services and infrastructure provided by the government. The services and infrastructure provided by the government are an essential input to private production, (Barro, 1990; Barro and Sala-i-Martin, 1992); hence, by creating inefficiencies in the public sector activities, corruption affects economic growth of the economy as well as the welfare of the society.1 Although, a significant body of research is dedicated to understanding the causes and the growth implications of corruption, much less research has been done to study the factors that may affect the relationship between corruption and income inequality. Analysing the relationship between corruption and inequality is important because it leads to a better understanding of how corruption affects economic growth. There is empirical evidence that indicates existence of negative relationships between inequality, income and economic growth (Barro, 2000; Galor and Zeira, 1993; Galor and Moav, 2004; Persson and Tabellini, 1994; Aleshina and Rodrik, 1994).

The primary objective of this study is to obtain new insights into the relationship between corruption and inequality that help to reconcile analytically several empirical findings about this relationship. Specifically, a few empirical papers that examine this problem at the macro level have found that corruption increases income inequality. For example, Gupta et al. (2002), have presented empirical evidence to support the hypothesis that there is a positive relationship between corruption and income inequality. Alongside Gupta et al. (2002), Li et al. (2000), Gyimah-Brempong (2002) and Gyimah-Brempong and de Camacho (2006) using cross-country and panel data analyses find that corruption has a significant positive effect on inequality.2

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1 See Rajkumar and Swaroop (2002), Mauro (1998), Tanzi and Davoodi (2000), Shleifer and Vishny (1993), Aidt (2003); Alam (1989), Dreher and Gassebner (2007), Méon and Weill (2010), and Dzhumashev (2014a,b). Literature on the effect of corruption on economic growth by lowering incentives to invest (see, for example, Mauro, 1995; Wei, 2000; Lambsdorff, 2003; and Lin and Zhang, 2009), on creating inefficiencies in public spending (see, for example, Mauro, 1998; Tanzi and Davoodi, 2000, and Shleifer and Vishny, 1993), and on imposing burden on firms (see, for example. Kaufmann and Wei, 1999; Guriev, 2004; and Fisman and Svensson, 2007).

2 It should be noted that the relationship between corruption and inequality does not run in just one direction. For example, You and Kharam (2005) highlight that it is not only that corruption leads to higher inequality, but also inequality in turn affects the social norms about corruption and helps it perpetuate. They also find empirical evidence that the adverse effect of inequality on corruption depends on the strength of democracy in a country. Analogously, Alam (1995) reasons that the ability of private agents to resist corruption depends on their income levels as any counter-action is costly, thus greater inequality encourages corruption.
The empirical findings of the corruption-inequality relationship based on micro-economic data are less conclusive. For example, Hunt and Lazlo (2012a) find that the burden of bribes as a share of income decreases with the agent’s income. Dincer and Gunalp (2012), using an objective measure of corruption (the number of government officials convicted of corruption in the US), find robust evidence that income inequality increases with higher corruption. On the other hand, studies based on micro-level data such as Hunt (2007b, 2004), Mocan (2008) and Svensson (2003) find that people with higher incomes bear the brunt of corruption, which appears to be inconsistent with the positive relationship between corruption and inequality established by the above-mentioned studies.

The inequality-corruption nexus is complicated further by the evidence indicating that the relationship between corruption and inequality might be of a non-linear nature. For example, Li et al. (2000) find that the effect of corruption on inequality is inverted U-shaped; that is, the effect of corruption on inequality is less pronounced when the level of corruption is at the two extremes, high or low, but is stronger when corruption incidence is in the intermediate range. The results by Dobson and Ramlogan-Dobson (2010) and Andres and Ramlogan-Dobson (2011) also support the nonlinearity of the relationship between corruption and inequality. They show that lower corruption is associated with higher income inequality for a subset of economies where the corruption level is in the intermediate range.

The main points of these empirical findings can be summarized as follows:

i) at the macro-level corruption and inequality are positively correlated (Gupta et al., 2002; 2002; Gyimah-Brempong and de Camacho, 2006), and this relationship is nonlinear, as the impact of corruption on inequality is higher in countries with an intermediate level of corruption compared to countries with low or high levels of corruption (Li et al., 2000; and Andres and Ramlogan-Dobson, 2011);

ii) at the micro level, the nominal amount of bribes paid increases with the agent’s income (Hunt, 2007b, 2004; Mocan, 2008; and Svensson, 2003);

iii) the burden of bribes as a share of income decreases with the agent’s income (Hunt and Lazlo, 2012a).

Naturally, one may wonder whether there exists a theoretical explanation for all these stylized facts. In the extant literature, there are some theoretical studies that try to obtain insights into how corruption affects inequality. The first strand of these models, relate either the quality of institutions or the level of corruption to the allocation of productive factors,
which in turn leads to income inequality. In particular, Spinesi (2009) develops a Schumpeterian growth model where institutional quality drives inequality and growth. In this model, higher public sector inefficiency diverts a larger share of both skilled and unskilled labour from productive to unproductive activities. This leads to higher wage inequality, and negatively affects the aggregate rate of innovation and growth. In the same manner, in Eicher et al. (2009), corruption affects the decision of private agents in regards to obtaining education, which then drives the aggregate distribution of skilled and unskilled types of production. Similarly to Spinesi (2009), in the model of Mandal and Marjit (2010), corruption diverts resources from productive sectors, which contributes to the wage gap in different sectors. Combining both these mechanisms, Chong and Gradstein (2007a) link the quality of institutions to the rents captured by private agents, which in turn, drive the productivity of private agents. These authors show that the variability of income across agents increases with higher rates of rent-seeking which are in turn fuelled by weak institutions. Even more straightforward, in Alesina and Angeletos’s (2005) paper, corrupt rent-seeking by creating unequal access to income flows directly generates income inequality.

The aforementioned models show that at the macro level, inequality and corruption are positively correlated due to income and productivity effects caused by corruption or institutional quality. However, the mechanisms leading to those effects do not incorporate bribery and the productivity effect of government spending (as in Barro, 1990). It is well known that corruption creates both income and productivity effects through the burden of bribes (Hunt, 2007b, 2004; Mocan, 2008; and Svensson, 2003) and distortions in the productive externality generated by government spending (Mauro, 1998; Tanzi and Davoodi, 2000; and Shleifer and Vishny, 1993). Obviously, without accounting for bribery one cannot reconcile the fact that bribes paid by the private agents increase with their level of income, albeit it does not reduce inequality. Moreover, the existing models cannot explain why the relationship between corruption and inequality is nonlinear at the macro-level (Li et al., 2000). This paper addresses these issues by accounting for the burden of bribes and distortions caused by corruption in the productivity effect stemming from government spending.

In order to incorporate the effects of bribes and productivity effects of government spending into the relationship between corruption and inequality, it is helpful to establish an intuitive link between these factors. First, given that the burden of corruption is a decreasing function of income, it can be postulated that the amount of bribes paid increases with income.
at a decreasing rate. That is, in the inequality-inducing setting, the poor pay more bribes as a share of their income than the rich, although the rich pay more bribes in nominal terms. The latter outcome is quite straightforward as, if we consider this from the efficiency point of view; the demand for public services is greater amongst the agents with a higher level of productivity (Leff, 1964; Huntington, 1968; Lui, 1985; Svensson, 2003; and Mocan, 2008). On the other hand, the fact that the burden of bribes is regressive implies that the relationship between bribes and the agent’s income level must be convex. An explanation for this may be that the gains from corruption on the individual level should be increasing faster than the burden of bribes. Then establishing the mechanism of how these gains from corruption are obtained by the bribe paying individuals will be required. In this regards, it seems reasonable to link corruption outcomes at the individual level to the agent’s level of productivity, as the above-mentioned literature suggests the demand for corrupt transactions increases with the private agent’s level of productivity. Therefore, as the corruption-induced productivity gains increase at a higher rate than the burden of bribes, the relative burden of corruption on the wealthy is falling.

The above rationale still cannot explain why the relationship between corruption and inequality is nonlinear at the macro-level. Specifically, it is still unclear why in case when the incidence of corruption is high (or low) its effect on the level of inequality is weaker than when the incidence of corruption is of an intermediate level (Li et al., 2000). To understand the causes of this nonlinearity in the inequality-corruption nexus, one needs to explain that why the relationship between corruption-induced productivity and the level of corruption is of the concave form. To do that, recall that corruption distorts the positive externality provided by government spending (Barro, 1990) and hence, an increase in the incidence of corruption leads to a decline in the productive externality provided by government spending (see, for example, Mauro, 1998; Tanzi and Davoodi, 2000, Shleifer and Vishny, 1993; and Dzhumashev 2014a). Thus, it follows that as long as with higher levels of corruption, an individual gain from corruption is falling due to increasing decline in the productive externality provided by government, one can expect that the inequality-corruption nexus will be a concave function.

In light of the above discussion, one can conjecture that the non-linearity of dependence between corruption and inequality is driven by the effect of corruption on government-induced productivity in two ways. Specifically, one can postulate that corruption affects the income level of agents through the following channels: (i) the burden of bribes as a share of
income declines with higher corruption-induced productivity gains, (ii) higher corruption leads to increasing negative effects by reducing the average amount of productive public inputs for the private agents. In other words, there is a direct effect of corruption on inequality by creating disparity in terms of the burden of bribes and productivity gains at the individual level. In addition, there is an indirect effect of corruption on inequality as the magnitude of corruption-induced individual gains extracted from the interactions with the public sector is conditional on the overall efficiency of the public sector. Thus, with higher levels of corruption, relative gains from corruption for the rich segments of population fall.

Taking into account the above arguments, this paper develops a model where corrupt bureaucrats and private agents interact, which produces both the direct and indirect effects of corruption on the income levels of private agents. The results of the analysis suggest that as soon as there is disparity in the productivity of agents that stems from public sector involvement, the direct burden of corruption is disproportionately borne by the poor. In addition to the burden of bribes, corruption generates an indirect burden by affecting the productivity of private agents through distortions in the positive externalities provided by public services. The current study demonstrates that when corruption levels are low, the negative impact of the declining average public sector productive externalities is less than the benefit of corruption obtained by an agent. However, with the levels of corruption increasing beyond intermediate levels, the loss of productivity stemming from reduced public externalities becomes high enough to offset most of the benefits in the form of corruption-induced individual productivity gains. Therefore, these findings suggest a theoretical explanation for that why at the macro level the relationship between corruption and inequality is inverted U-shaped (Li et al., 2000) while the burden of bribes is regressive (Hunt and Lazlo, 2012a). Overall, this study adds to the growing literature on corruption by showing the

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3 One can also recall that not only do the productivity gains decrease with higher corruption, but it is possible that the burden of bribes falls as well. For example, Miller (2006), Kingston (2007, 2008), Çule and Fulton (2009), Mishra (2006) and Hunt (2004) find that a higher acceptance of corruption through social connections decreases its burden. For example, Hunt (2004) finds that the development of a bilateral trust between the private and public agent leads to the substitution of an implicit ‘quid pro quo’ for a bribe, thus reducing bribery. In similar vein, Kingston (2008) considers informal relationships among bribe payers that help them to enforce agreements against paying bribes. These arrangements also lead to a reduction in the incidence of bribe payments. However, paying less in bribes, for example to conceal non-compliance to regulations or tax evasion, does not mean a reduction of the incidence of corruption.

4 According to Bardhan et al. (2007), there is also a negative feedback effect from inequality to government spending that contributes to a decrease of productive externality provided. Chong and Gradstein (2007b) also find that a higher inequality results in a larger informal economy, which implies that the tax revenue collected by the government falls, hence, government spending also declines.
importance of both individual productivity differentials (direct effect) and the overall public sector positive externalities (indirect effect) in determining the effect of corruption on inequality.

The rest of the paper is structured as follows: in Section 2, the related literature is discussed in more detail, and in Section 3, the theoretical model is presented and implications are drawn. Section 4 concludes the study.

2. Model of the Economy with Corruption

2.1. The Environment

There are two types of agents: bureaucrats and producers. It is assumed that there is no population growth and no social mobility, so that those who start as a producer, remain a producer. Workers pay tax at a fixed rate, \( \tau \). Tax revenue is used to pay salaries of the bureaucrats that are providing public services. Given that a fraction of population, \( \eta \), is workers and the rest are bureaucrats, the following equality holds:

\[
\tau \bar{y} \eta = w_i (1 - \eta),
\]

where \( \bar{y} \) is the average income of the worker agents and \( w_i \) is the market determined wage rate. It is assumed that the bureaucrats are paid the market wage rate, \( w_i \). This implies that in the absence of corruption, the expected (statutory) amount of services to be provided to each private agent, is given by

\[
\hat{q} = \tau \bar{y}.
\]

In their interactions, the public and private agents play a simple sequential game. The public agent moves first by shirking and providing less than the statutory level of services. The private agent observes the move of the public agent and then moves either by accepting the sub-standard services or by offering a bribe to obtain higher than the initial level of services offered by the corrupt public agent. Then the game ends. That is, by decreasing the supply of services to an admissible minimum, \( q \), the supplier effectively creates rents and captures them. In this respect, the rationale of such an assumption is based on the finding of Guriev (2004), who shows that corruption raises the equilibrium level of red tape; and of Shleifer and Vishny (1993), who demonstrate that higher red tape results in shortages of the services being provided. In the given context, corruption results in the imposition of more red
tape on the producer, which leads to a reduction in the amount of public services being offered to the admissible minimum, $q$.

### 2.2. Workers

The worker-agents, indexed by $i \in [0,1]$, are identical except in their capacity of productivity gains from public services, captured by coefficient $\beta_i$. This coefficient is interpreted as an individual’s ability to increase his or her private productivity using the public sector through corrupt deals with the bureaucrats. It is assumed that this productivity coefficient is normally distributed across the worker agents as $\beta \sim N(\beta, \sigma^2_\beta)$, where $\beta$ is the mean and $\sigma^2_\beta$ is the variance of this distribution.

Each agent $i$ is self-employed and has an identical endowment of initial capital, $k_{i0} = k$, and a unit of time in each period. Thus, initially there is no wealth inequality; hence, any change in inequality will be explained by changes in the productive factors that may be caused by corruption. The producer combines capital and government goods to produce the single final good. In this sense, the production function is similar to one employed in Barro (1990). It is assumed that the output of a self-employed agent $i$, at each date $t$, is given by the following function:

$$y_i = (1-\tau)k^\alpha_a g^{1-\alpha}_a,$$

where $g_a = [\theta q + \beta_i (q_i - q)]\bar{q}$, $\bar{q}$ is the average per worker amount of public services, and $0 < \alpha < 1$. It is assumed that the private agent pays a bribe proportional to the amount of additional services he or she is trying to obtain. That is, $b(q - \bar{q})$, where $b$ is the bribe rate. This constitutes as the response by the producer to the initial move of the bureaucrat.

Since, the valuation of the additional service, $\beta_i$, may differ depending on the individual attributes of the agent and the level of corruption, those who attach a higher value to the services obtained are willing to pay more in bribes to obtain more of the public services. This assumption is justified by the findings of Lui (1985), who has shown that those who value the service to be obtained from the corrupt bureaucrat have a higher willingness to pay bribes on the efficiency grounds. Moreover, empirical studies by Svensson (2004), Mocan (2008) and Hunt (2007) indicate that those who have higher incomes also demand more services.
Supporting this line of reasoning, there is evidence showing that government spending benefits low and middle income groups differently (Roin et al., 2009). By linking together the level of productivity that results in a higher income and the demand for public services supplied by corrupt bureaucrats, allows us to explain why bribe payment increase with income.

The difference in the corruption-induced productivity gains may also stem from having social networks entwined in the public sector operations. The stronger the role of these networks in corruption transactions the higher is the incidence of corruption, as these connections can facilitate enforcement of collusive arrangements (Choe et al., 2011; Kingston, 2007; 2008; and Hunt, 2004). Therefore, in some environments the incidence of corruption is higher because the social norms in place permit corruption, creating greater individual benefits compared to environments where corruption is less widespread (Mishra, 2006; and Miller, 2006). This rationale leads one to postulate that with a rise in corruption, the private benefits in terms of individual productivity gains increase for all private agents. This implies the value of $\beta_i$ is increasing in the overall level of corruption. In other words, if there is an index, $\xi$, that measures the extent of corruption in the economy, then $\frac{\partial \beta}{\partial \xi} > 0$.

However, it may be more realistic to assume that this relationship is concave in the level of corruption, as the gains in individual productivity through corruption are most likely to be diminishing. That is, $\frac{\partial^2 \beta}{\partial \xi^2} < 0$.

### 2.2.1. Workers' optimal choices

Let us consider agent $i$ 's optimal choices. The objective of the worker agent is to maximize their intertemporal utility by choosing their consumption level, $c_i$, and the level of government goods, $q_i$, given their social status and initial capital. That is,

$$
\max_{c,q} u = \int_0^\infty \ln c_i \cdot e^{-\rho t} dt 
$$

s.t.

$$
\dot{k}_i = (1 - \tau)Ak_i^\alpha q_i^{1-\alpha} - b(q_i - \underline{q}) - c_i,
$$

where $\rho$ is the discount rate.
and the initial endowment of capital, \( k_{i0} = k \).

It is assumed that a bribe is only paid if \( q_i > q \). To exclude non-optimal cases, it is assumed that the transversality condition, given as

\[
\lim_{t \to \infty} \{ k_{i0} e^{\rho(t-u)} \} = 0,
\]

holds.

The amount of public services obtained after paying bribes is

\[
g_{n} = \left[ \theta q + \beta(q_i - q) \right] \bar{q}
\]

and without paying bribes is \( g_{n} = \theta q \bar{q} \).

The Hamiltonian of the workers’ problem is given by

\[
H = u(c_{nt}) e^{-\rho t} + \lambda \left[ (1-\tau) Ak_{nt}^{\alpha} g_{q}^{1-\alpha} - b(q_{nt} - q) - c_{nt} \right].
\]

Further, for simplicity and where it does not obscure the presentation, time and agent indices are dropped. The first-order conditions of the above optimal control problem are given by:

\[
\frac{\partial H}{\partial c} = \lambda - u'(c_{t}) e^{-\rho t} = 0,
\]

\[
\frac{\partial H}{\partial q} = \lambda \left[ (1-\alpha)(1-\tau)\beta \bar{q} A \left( \frac{k}{\theta q + \beta(q - q)} \right)^{\alpha} - b \right] = 0,
\]

\[
\dot{\lambda} = -\frac{\partial H}{\partial k} = -\lambda \left[ \alpha(1-\tau)A \left( \frac{\theta q + \beta(q - q)}{k} \right)^{1-\alpha} \right].
\]

From (10), the following equilibrium expression for the amount of public services demanded is obtained:

\[
q = \frac{1}{\beta} \left[ \left( \frac{(1-\alpha)(1-\tau)\beta \bar{q} A}{b} \right)^{\frac{1}{\alpha}} k - (\theta - \beta) q \right].
\]

Combining (9) and (11), one can find the growth rate of consumption,
\[
\frac{\dot{c}}{c} = \alpha(1-\tau)A \left(\frac{g}{k}\right)^{1-\alpha} - \rho .
\]  

(13)

Analysing the equilibrium amount of public services demanded yields the following proposition.

**Proposition 1.** *An agent’s demand for public services rises with wealth, and falls with the bribe rate. That is, \( \frac{\partial q}{\partial k} > 0 \) and \( \frac{\partial q}{\partial b} < 0 \).*

**Proof.** It is straightforward to show that \( \frac{\partial q}{\partial k} > 0 \) and \( \frac{\partial q}{\partial b} < 0 \), by taking the first-order differential of (12) with regards to \( k \) and \( b \):

\[
\frac{\partial q}{\partial k} = \frac{1}{\beta} \left(\frac{(1-\alpha)(1-\tau)\beta qA}{b}\right)^{1-\alpha} > 0
\]

(14)

\[
\frac{\partial q}{\partial b} = -\frac{k}{\beta} \left(\frac{(1-\alpha)(1-\tau)\beta qA}{b^2}\right)^{\frac{1}{\alpha}-1} < 0 .
\]

(15)

The above results are quite intuitive. Wealthier people demand a greater amount of productive public services, which is an empirically observed fact (Svensson, 2004; Mocan, 2008; Hunt and Laszlo, 2012b and Hunt, 2007). On the other hand, the more extortive bureaucrats become through the imposition of higher bribe rates, the lower is the demand for public services, as it increases the marginal cost of obtaining public services for the private agents.

Next, it is important to ascertain how the amount of public services demanded by the private agents depends on the productivity of the individual agent \( i \) induced by the public sector. The following proposition summarizes the link between the productivity of the agents and the amount of public service demanded.

**Proposition 2.**

*The amount of public services is a concave function of the corruption-induced productivity.*

**Proof.** To establish the curvature of the function, let us determine the first-order and the second-order derivatives of \( q_i \) with regards to \( \beta_i \). It can be verified that
\[ \frac{\partial q_i}{\partial \beta_i} = \frac{1}{\alpha} \left( 1 - \alpha \right) \left( 1 - \tau \right) A \left( 1 - \frac{q_i}{\hat{q}} \right) ^{\frac{1}{\alpha}} \frac{\int^1 b}{\beta_i^a} k + \frac{\theta q_i}{\beta_i^3} > 0 \quad \text{and} \]
\[ \frac{\partial^2 q_i}{\partial \beta_i^2} = \frac{1}{\alpha} \left( 1 - \alpha \right) \left( 1 - \tau \right) A \left( 1 - \frac{q_i}{\hat{q}} \right) ^{\frac{1}{\alpha}} \frac{\int^1 b}{\beta_i^a} k - \frac{2\theta q_i}{\beta_i^3} < 0 \quad \text{hold.} \]

Therefore, the amount of \( q_i \) is increasing in \( \beta_i \), at a decreasing rate. ■

The main finding here is that a one unit increase in the corruption-induced productivity measure will lead to less than one unit increase in the productive public services obtained. The finding stated in Proposition 2, allows one to draw a conclusion about the average value of the amount of public services demanded and the average agent’s productivity. This conclusion is stated as the following lemma.

**Lemma 1.** The ratio of the average amount of public services demanded to the average productivity benefit of the private agents is a decreasing function. That is, \( \frac{\Delta q}{\Delta \beta} < 0 \).

**Proof.** Let us consider agent \( i \), who experiences one unit increase in his or her corruption-induced productivity measure, which results in an increase in the public services obtained by less than one unit, due to Proposition 2. That is, \( \Delta q_i < 1 \). Then the incremental increase in the average productivity induced by the public sector will be equal to \( \Delta \bar{\beta} = \frac{1}{n} \), whereas the incremental increase in the average public services will be given by \( \Delta \bar{q} = \frac{\Delta q_i}{n} \). Clearly, \( \Delta \bar{\beta} > \Delta \bar{q} \). Since, this outcome is true for an arbitrary agent, it is also true for all of them simultaneously. ■

### 2.3. The Bureaucrats

For simplicity, it is assumed that the bureaucrats do not save. This assumption does not change the general structure of the analysis being conducted. An individual bureaucrat deals with \( n = \eta \frac{1}{1 - \eta} \) workers, and the average amount of public services the workers require is given by

\[ \bar{q} = \int_{\hat{q}} q dF(q), \quad (16) \]
where $F(q)$ is the cumulative distribution of the level of public services demanded by the private agents. There is a cost to the bureaucrat for being corrupt, which is increasing in the difference between the statutory and minimum amounts of public services offered by the bureaucrat. This cost is specified as $\chi(\hat{q} - q)^2$, where $\chi$ is a cost parameter, $\hat{q}$ is defined as in (2) and $q$ is determined through the optimizing behaviour of the agents.\(^5\)

The bureaucrat maximizes her or his utility given by

$$\max_{c_2, q} \int_0^\infty \ln c_2 \cdot e^{-\rho t} dt$$

s.t. the budget constraint,

$$c_{2t} = w_t + nb(\bar{q} - q) - \chi(\hat{q} - q)^2.$$ \hspace{1cm} (18)

The Hamiltonian of this problem is given by

$$H_2 = u(c_{2t})e^{-\rho t} + \mu \left[ w + nb(\bar{q} - q) - \chi(\hat{q} - q)^2 - c_{2t} \right].$$ \hspace{1cm} (19)

The first-order conditions (FOCs) are given as follows:

$$\frac{\partial H_2}{\partial c_2} = 0 \Rightarrow \mu_t = u'(c_{2t})e^{-\rho t},$$ \hspace{1cm} (20)

$$\frac{\partial H_2}{\partial q} = \mu \left[ -nb + 2\chi(\hat{q} - q) \right] = 0.$$ \hspace{1cm} (21)

Solving for $q$ from (21) we obtain that in equilibrium the bureaucrat sets the minimum level of services at the expected level. That is,

$$q = \hat{q} - \frac{nb}{2\chi}. $$ \hspace{1cm} (22)

By analysing the above result, the following lemma is formulated.

**Lemma 2.** *The minimum level of public services offered falls with an increase in the bribe rate, $b$ and rises with the cost of bureaucratic corruption, $\chi$.*

\(^5\) This assumption follows Chen (2003) and Dzhumashev (2014a,b).
Proof. Given that the expected amount of public services, \( \hat{q} \) and the number of private agents, \( n \) to deal with are given to the bureaucrat, his or her behaviour only alters the second term on the right-hand side of (22). Then, clearly, an increase in the bribe rate, \( b \) results in lower values, whereas an increase in the cost of corruption leads to higher values of the minimum level of public services, \( q \).

In the next section, the equilibrium conditions are stated and the dynamics of the model considered.

2.4. Equilibrium

Equilibrium in the economy is defined as the streams of consumption \( \{c_i(t), c_j(t)\}^\infty_0 \), physical capital, \( \{k_i(t)\}^\infty_0 \), prices, \( \{w(t), r(t)\}^\infty_0 \), the tax rate, \( \tau(t) \), the actual level of public services obtained, \( \{q_i(t)\}^\infty_0 \), the minimum level of public services offered, \( \{q(t)\}^\infty_0 \), and the bribe rates, \( \{b(t)\}^\infty_0 \), such that they satisfy the optimality conditions obtained for the household’s and bureaucrat’s problems, as described above.

Given the production function by equation (3), the productivity of a private agent depends on the actual government services obtained relative to the stock of capital owned. Combining (12) and (7) one can find this ratio as

\[
\bar{g} = \frac{g_{i}}{k_{i}} = \left( \frac{\bar{q}(1-\alpha)(1-\tau)\beta_{i}A}{b} \right)^{\frac{1}{\alpha}}.
\]

Equation (23) leads to the following lemma.

**Lemma 3.** The effective government services for an agent, given by (23) are increasing in the corruption-induced productivity gain, \( \beta_{i} \), and decreasing in the bribe rate, \( b \).

**Proof.** The result is straightforward from (23). ■

The dynamics of this system are described by two differential equations in \( c \) and \( k \) given by (13) and (5) correspondingly. The growth rate of consumption, given by (13) as

\[
\gamma_{c} = \frac{\dot{c}}{c} = \alpha(1-\tau)A \left( \frac{g_{i}}{k_{i}} \right)^{1-\alpha} - \rho
\]

has the following general solution:
\[ c_{it} = c_0 e^{\ell (1-\tau) A \beta_{it} - \rho \ell t}, \]  

(24)

where \( c_0 \) is the exogenously given initial level of consumption.

Then the growth rate of capital, given by equation (5), can be written as

\[ \dot{k}_t = (1-\tau) A \tilde{g}_i k_t - b(q_i - q) - c_0 e^{\ell (1-\tau) A \beta_{it} - \rho \ell t}. \]  

(25)

By accounting for (12), one can further simplify (25) as

\[ \dot{k}_t = \Gamma_i k_t + \frac{bq(\beta_i - 1-\theta)}{\beta_i} - c_0 e^{\ell (1-\tau) A \beta_{it} - \rho \ell t}, \]  

(26)

where \( \Gamma_i \equiv (1-\tau) A \left[ \tilde{g}_i - (1-\alpha) \left( \frac{\beta_i}{b} \right)^{1-\alpha} \right] \).

The general solution of equation (26) is

\[ k_t = \frac{bq(\beta_i - 1-\theta)}{\Gamma_i \beta_i} + c_1 e^{\Gamma_i t} + \frac{c_0}{\varphi_i} e^{\ell (1-\tau) A \beta_{it} - \rho \ell t}, \]  

(27)

where \( \varphi_i = \rho - (1-\tau)(1-\alpha) A \left( \frac{\beta_i}{b} \right)^{1-\alpha} \). Now, substitute for \( k_t \) from (27) into the transversality condition (6) and obtain,

\[ \lim_{t \to \infty} \left[ c_1 + \frac{bq(\beta_i - 1-\theta)}{\Gamma_i \beta_i} e^{-\Gamma_i t} + \frac{c_0}{\varphi_i} e^{-\rho \ell t} \right] = 0. \]

Clearly, with time, the last two terms in the square brackets converge to zero. Then, for the transversality condition to hold, it must be \( C_1 = 0 \).  

Therefore, combining (27) and (24) one can establish that

\[ c_{it} = \varphi_i k_t + \frac{bq(\beta_i - 1-\theta)}{\Gamma_i \beta_i}. \]  

(28)

\[ ^6 \text{See Barro and Sala-i-Martin (2004, p.208).} \]
This implies that consumption and capital grow at the same rate. Therefore, the following lemma is stated.

**Lemma 4.** The growth rates of consumption and capital satisfy $\gamma_k = \gamma_c$.

**Proof.** Due to the equilibrium condition, given by (28), the level of consumption of an agent is proportional to the capital stock he or she owns. This implies that on the balanced growth path, both consumption and capital grow at the same rate. ■

Since, the growth rate of consumption is given as $\frac{\dot{c}}{c} = \alpha(1 - \tau)A\left(\frac{g}{k}\right)^{1-\alpha} - \rho$, it is straightforward to see that this growth rate is constant and depends on the values of parameters reflecting the incidence of corruption, the effect of social status, and the productivity of the economy.\(^7\)

### 3. Evolution of inequality

The next step is to show how the difference in the productivity levels due to the difference in obtaining public services will lead to an increase in inequality. Since, the productivity of each agent depends on the value of the ratio of public services and capital, $\left(\frac{g_u}{k_u}\right)$, the difference in the magnitude of this measure across the agents will drive the level of income and consumption to diverge. This is how inequality evolves between agents of different social statuses and hence, different exposure to corruption and productive public sector externalities.

That is, the growth rate of consumption of agent $i$, $\gamma_i$, is increasing in the public services-to- capital ratio, $\left(\frac{g_u}{k_u}\right)$. This leads to the following lemma.

**Lemma 5.** The growth rate of consumption and wealth of an agent positively depends on the effective productive public input given as the ratio $\left(\frac{g_u}{k_u}\right)$.

---

\(^7\) A constant growth rate is a standard feature of the AK-type growth models. See, for example, Barro (1990) for discussions on this issue.
Proof. Recall the growth rate of consumption, \( \gamma_u = \alpha(1-\tau)A \left( \frac{g_u}{k_u} \right)^{1-\alpha} - \rho \). Clearly, for any given tax rate, higher values of the ratio, \( \frac{g_u}{k_u} \), lead to higher growth rates. ■

The intuition underlying this result is not very different from what has been stated by Barro (1990): output (income) is increasing with productive public inputs, \( g_u \). The only difference, in this case is that the agents are heterogeneous in terms of demand for public inputs, so that those who are more productive will be getting more of the public inputs due to distortions in the delivery of public services as a result of corruption. Based on this intuition one can state the following lemma.

Lemma 6. The ratio \( \frac{g_u}{k_u} \) is increasing in the corruption-induced productivity of agent \( i \), \( \beta_i \).

Proof. Using the definition of \( g_u \), it can be verified that \( \frac{\partial \left( \frac{g_u}{k_u} \right)}{\partial \beta_i} > 0 \). ■

This result demonstrates how the diversity in the corruption-induced productivity gains results in differences in the public services obtained by private agents. The important point here is that the differences in the productivity will not lead to differences in the level of public services obtained unless there is corruption in the process of delivering the public service. Furthermore, the effect of corruption is reflected on the dispersion of growth rates of consumption and wealth across agents, as the following propositions states.

Proposition 3. The growth rate of consumption and wealth is increasing in the corruption-induced productivity of the agent and decreasing in the bribe rate.

Proof. It can be verified that \( \frac{\partial (\gamma_{nu})}{\partial \beta_i} > 0 \) and \( \frac{\partial (\gamma_u)}{\partial b} < 0 \). The details are given in Appendix A1. ■
In light of the above findings, one can state that heterogeneity of private agents in terms social or political status imminently leads to economic inequality. Importantly, the evolution of this inequality is possible only when there is corruption in the delivery of public services.

It is reasonable to expect that inequality will grow in an environment where the agents have different growth rates of capital accumulation. A more rigorous way of showing the evolution of inequality is to demonstrate the dynamics using a measure of inequality. As a measure, one can adopt the logarithm of the ratio of mean to median capital stock per worker, following Bandyopadhyay and Tang (2011), Chong and Gradstein (2007), and Benabou (2002). That is, if capital has a lognormal distribution, then the measure of inequality is expressed as

\[ \Lambda_t \equiv \log \left( \frac{k_t}{k_{\text{median}}} \right) = \frac{1}{2} \var \left[ \log k_t \right]. \] (29)

Combining (24) and (28), and taking a logarithm of both sides of the equation, one obtains

\[ \log k_t = \left[ (1 - \tau) A \tilde{\theta} - \rho \right] t \log \frac{c_0}{\rho}. \] (30)

Clearly, the variance of \( \log k_t \) depends on the variance of public services, \( \tilde{g}_{it} \), and time. This consideration allows one to state the following proposition.

**Proposition 4.** Corruption induced distortions in the distribution of public services increase inequality.

**Proof.** It follows from (30) that

\[ \var \left[ \log k_t \right] \approx \left[ (1 - \tau) A \log \frac{c_0}{\rho} g'_{\beta}(\beta) \right]^2 \var(\beta) \cdot t^2. \] (31)

Thus, with time \( \var \left[ \log k_t \right] \) increases and hence, the level of inequality, \( \Lambda_t \), also rises. The details are in Appendix A2.
Given that \( g(\bar{\beta}) = \left( \frac{\bar{q}(1-\alpha)(1-\tau)\bar{\beta}A}{b} \right)^{1/\alpha} \), from (31) it is evident that an increase in the average benefit of corruption for private agents, \( \bar{\beta} \), results in a higher value of the variance of wealth. Thus, when an increase in the incidence of corruption leads to the larger average productive gain, \( \bar{\beta} \), this would imply that corruption is positively correlated with inequality. However, when the incidence of corruption increases, it results in the reduction of the average level of public services, \( \bar{q} \). This latter effect reduces the magnitude of \( g(\bar{\beta}) \), hence, that of the variance. It is only when the incidence of corruption is in the intermediate range will the average level of public services also be intermediate; hence, the effective public service, \( g(\bar{\beta}) \), as a product of these two elements will be the highest. Therefore, inequality grows faster when corruption is increasing in the intermediate range as the empirical evidence suggests (Li et al., 2000). This intuition leads us to the following proposition.

**Proposition 5.** If an increase in corruption reduces the average level of public services more than it increases the average level of corruption-induced productivity, the relationship between corruption and inequality is of an inverted U-shaped form (i.e. concave).

**Proof.** Based on Proposition 4 and using equation (31), one can see that the impact of the variance of the productivity, induced by the public sector, on inequality is driven by this term:

\[
\left[ (1-\tau)A \log \frac{\tilde{c}_o}{\rho} g'_\rho(\bar{\beta}) \right]^2.
\]

It is straightforward to establish that the variance of the public sector-induced productivity depends on \( \left( g'_\rho(\bar{\beta}) \right)^2 = \frac{1}{\alpha^2} \left[ \bar{q}(1-\alpha)(1-\tau)A \right]^{2/\alpha} \frac{\bar{\beta}^{2(1-\alpha)}}{b^\alpha} \). Since, according to (22) the lower value of public services, \( \bar{q} \), is reduced when corruption levels rise, the average value of public services, \( \bar{q} \), also falls. On the other hand, a higher level of corruption implies a higher level of private benefits; hence, the average level of the corruption benefit, \( \bar{\beta} \), is higher. Since, according to Lemma 1, \( \Delta \bar{q} < \Delta \bar{\beta} \) holds, \( \left( g'_\rho(\bar{\beta}) \right)^2 \) will be of an inverted U-shaped form.

**3.1. Numerical illustration**

Although the findings of Proposition 5 are intuitive, a numerical simulation of this part of the model can be helpful to visualize the concavity of the relationship between the level of
corruption and inequality, and to see if the proposition is confirmed. In particular, ascertaining if the expression by

\[ g'_\alpha(\bar{\beta}) = \frac{1}{\alpha} \left[ \bar{q}(1-\alpha)(1-\tau)A \right]^{\frac{1}{\alpha}} \frac{\bar{\beta}^{(1-\alpha)}}{b^{\alpha}}, \]

exhibits a concave form will also be useful. In the latter expression, one can consider only the part that depends on the level of corruption. Since, \( \tau \), and \( A \) are exogenous parameters, one can disregard their contribution to the non-linear relationship between the incidence of corruption and income volatility, and focus only on the impact of the average value of public services, \( \bar{q} \), the average level of the corruption-induced productivity gains, \( \bar{\beta} \), and the bribe rate, \( b \). This leads to analysing the behaviour of the measure \( \tilde{g} = \frac{1}{\bar{q}^{\alpha}} \frac{\bar{\beta}^{(1-\alpha)}}{b^{\alpha}} \) by changing the level of corruption. Here, following Chen (2003), one can assume that \( \alpha = 0.9 \).

Figure 1. Simulation results for the element of variance of the income distribution that depends on the level of corruption, \( \tilde{g} \)

Recall that both the average level of the corruption-induced productivity gains, \( \bar{\beta} \), and the bribe rate, \( b \), positively depend on the level of corruption in the economy. As aforementioned in the setup of the model, the level of individual benefit induced by corruption is a concave function of the incidence of corruption. For practical purposes, this
function can be assumed to be of the following form: \( \tilde{q} = x^p, 0 < p < 1 \). From Proposition 2, it is known that \( \tilde{q} \) is a concave function of \( \tilde{\beta} \). To capture this idea, we again simply assume that \( \tilde{q} = \beta^c, 0 < \varepsilon < 1 \). In Figure 1 given below, it can be seen that the relationship between income inequality and the level of corruption has an inverted-U shape. In this particular case, the simulation is done assuming that \( \beta = \xi^{0.35}, \tilde{q} = \beta^{0.6} \) and a corruption measure is defined as \( \xi \in [0.1, 0.8] \). Additional calculations show that the variance of the income distribution will have a concave form in the level of corruption as soon as the curvature of the average public inputs and the individual corruption-induced productivity gains are significantly different. In this particular simulation, this means that as long as the parameter values assumed for \( \pi \) and \( \varepsilon \) are given on the opposite sides of point 0.5, one will obtain a variance for the income distribution that is concave in the level of corruption.

4. Conclusion

This study proposes an analytical model that explains some of the empirical evidence that relates corruption to inequality. In particular, it has been shown by several empirical studies that the nominal amount of bribes paid increases with the income of the agents, whereas the burden of bribes as a share of income decreases. Thus, inequality is positively correlated with corruption. However, it has also been demonstrated that at the macro-level, corruption and inequality have a nonlinear relationship: as in countries with intermediate levels of corruption, the effect of corruption on inequality is stronger than in countries where corruption levels are low or high.

To reconcile these stylised facts, this paper develops a model building on the model of Barro (1990) where government services enter the private production function as an input. The extension proposed in the present study is that the effect of public activities affects the private productivity not only through public sector externalities a la Barro (1990), but also through the agents’ individual productivity induced by corruption. The possibility of heterogeneity of private agents in demanding public services depends on their productivity as well as their social status, as highlighted in the literature, so the same rationale is employed in the present paper. Given the costs and benefits of corruption for both private agents and bureaucrats, it has been shown that for any given level of corruption, the existence of heterogeneity across agents in terms of corruption-induced productivity gains leads to an increase in inequality.
In addition, more corruption implies a greater distortion in the delivery of public sector services. Thus, with higher levels of corruption the overall public generated productive externality falls (indirect effect), whereas the individual gains (direct effect) from corruption increase. The indirect effect of corruption tends to reduce, whereas the direct effect increases inequality. Therefore, when corruption is low the individual gain from corruption is also low, while the overall public sector positive externality is high. As the level of corruption increases, the private gains from corruption rise while the public positive externality falls. The latter effect grows stronger than the individual gains from corruption; hence, the variation of income and wealth due to corruption declines after the corruption level reaches some threshold value. This explains the observed non-linearity of the relationship between the corruption level and inequality.

Appendix

A1. Proof of Proposition 3. Given that \( \gamma = \alpha(1-\tau)A(\tilde{g})^{1-\alpha} - \rho \) and
\[
\tilde{g} = \left( \frac{\tilde{g}(1-\alpha)(1-\tau)\beta A}{b} \right)^{\frac{1}{\alpha}},
\]
one can obtain the following:
\[
\frac{\partial \gamma}{\partial \beta} = (1-\alpha)(1-\tau)A(\tilde{g})^{\frac{1-2\alpha}{\alpha}} g'_{\rho}(\beta),
\]
(32)
g'_{\rho}(\beta) is the first-order derivative of \( \tilde{g} \) with respect to \( \beta \). Since \( g'_{\rho}(\beta) > 0 \), it implies that
\[
\frac{\partial \gamma}{\partial \beta} > 0. \]

A2. Proof of Proposition 4. Recall that \( \log k_u = \left[ (1-\tau)A\tilde{g} - \rho \right] t \log \frac{c_0}{\rho} \). For a given point in time, the variance of the right-hand side of this expression is as follows:
\[
\text{var} \left[ \log k_u \right] = \text{var} \left[ \left( (1-\tau)A\tilde{g} - \rho \right) t \log \frac{c_0}{\rho} \right] = \left[ (1-\tau)A t \log \frac{c_0}{\rho} \right]^2 \text{var} (\tilde{g}).
\]
(33)
Now, recall from (23) that 
\[
\tilde{g} = \left( \frac{\tilde{g}(1-\alpha)(1-\tau)\beta A}{b} \right)^{\frac{1}{\alpha}}.
\]
Here, only \( \beta \) is the variable that differs across agents. Using the delta method (see Seber, 1982) the variance of \( \tilde{g} \) can be approximated as

\[
\text{var}(\tilde{g}) \approx \left[ g'_\beta(\bar{\beta}) \right]^2 \text{var}(\beta),
\]

(34)

where \( g'_\beta(\bar{\beta}) = \frac{1}{\alpha} \tilde{g}(1-\alpha)(1-\tau)A \frac{\frac{1}{\alpha} \bar{\beta}^{\frac{1}{\alpha} - 1}}{b^{\frac{1}{\alpha}}} \) is the first-order derivative of \( \tilde{g} \) with respect to \( \beta \), and \( \bar{\beta} \) is the expected value (mean) of \( \beta \). Therefore,

\[
\text{var}[\log k_y] \approx \left[ (1-\tau) A t \log \frac{c_0}{\rho} g'_\beta(\bar{\beta}) \right]^2 \text{var}(\beta).
\]

(35)

This result implies that as soon as the private agent can get unequal private gains from public services, and hence, \( \text{var}(\beta) > 0 \), wealth inequality increases overtime. ■

References


