Endogenous Corruption in A Dynamic Monetary Model
(Preliminary Version)

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Abstract

There are significant empirical studies analysing the impact of public sector corruption on macroeconomic indicators. In contrast, there are few theoretical explorations of the endogenous emergence of corruption and why this might be correlated with inflation. This explores the link between endogenized corruption and inflation driven by the seigniorage choice of a welfare-maximizing government. Individuals can choose private or public employment. Public employees can either be honest or corrupt. The payoff to a worker is a function of the wage level, the effort required in work, and the level of bribes received if corrupt. Corruption leads to a loss of self-esteem because of a social sanction on corruption. Bribes are paid by a representative firm to corrupt officials in an exchange for a reduced rate of tax, with the division of the gains determined by Nash bargaining. The results demonstrate that a higher tax rate leads to a lower wage level in the private sector and in the public sector, an increase in the number of corrupt individuals, but the proportion of corrupt in the public sector is lower. For a given level of the social sanction, a welfare-maximizing government chooses a higher tax rate when monetary expansion is lower. The chosen tax rate is initially increasing in the social sanction but eventually decreases. The chosen growth rate of money is higher when there is a lower social sanction on corruption. The lower social sanction also causes a higher proportion of corrupt individual in the public sector. As a consequence, both a higher optimal inflation rate and greater corruption are associated with a lower social sanction. The model predicts the observed correlation between inflation and corruption across countries, with the correlation driven by the level of the social sanction.

Keywords: Corruption, Bargain, Bribe, Seigniorage

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1 Introduction

Evidence has shown that there are some officials who choose to act corruptly in public sector employment. It is therefore unrealistic to assume in models of economic policy design that there is no opportunity to be corrupt, and it is far from reality to consider all doors to corruption are quite under control and closed. Corruption seems to always be able to find a way to penetrate, and throughout history people have always been tempted by the potential for personal gain arising from corruption. This study aims to model corruption as an option to choose in a monetary framework in which it is possible to explore the correlation between corruption and seigniorage, and then inflation.

Empirical studies by Al-Marhubi (2000), Abed and Davoodi (2000), Smith-Hilman (2007) and Samimi et al. (2012) confirmed that the link between corruption and inflation is significant and positive. While these empirical investigations show that corruption could be an explanation for inflation, conversely there is some evidence showing that inflation motivates corruption as well. Studies by Braun and Di Tella (2000), Gerring and Thacker (2005), and Akca et al. (2012) explain that inflation increases auditing and monitoring costs, and this makes some level of corruption to be condoned and more demanded. Of course, the demand for corruption could be influenced by other economic characteristics and social norms. However, if inflation is a motive for corruption this leads to an implication that corruption could lead to further corruption due to inflation.

Myles and Yousefi (2014) showed that increased inflation may be a negative side effect of corruption if the government compensates for lost revenue by exploiting seigniorage and increasing the rate of monetary expansion. That analysis was limited by the fact that corruption was exogenous: some individuals were corrupt by nature and did not have an option to choose to be honest. In practice, the choice to be corrupt is endogenously determined by various factors including individual characteristics and the social setting. A more compelling model needs corruption to be explained as an endogenous outcome that reacts to changes in the economic environment. Existing models with endogenous corruption include Hindriks et al. (1999) who study the details of the interaction between a taxpayer and a tax inspector and Blackburn et al. (2010) and Blackburn and Forges-Puccio (2010) who model public sector bureaucrats who collude with households in bribery and tax evasion. Our model extends these papers by setting the analysis within a monetary model that incorporates occupational choice, a decision by each individual on whether to act corruptly in public sector employment, and the endogenous allocation of the benefits from corruption.

In the model individuals choose employment in the public sector or in the private sector taking all components of the reward offered by the work contract into account. This includes the formal parts of the contract, such as wage levels and required work effort, and the non-contractual benefits that can arise from corruption. If employment in the public sector is chosen, officials have to weigh the gains from abusing their power and acting corruptly against a utility cost of engaging in corruption. It is assumed that public sector employees are
tempted by additional income in the form of bribes which provide the incentive for them to engage in corruption. The cost is a loss in self-esteem due to a general social sanction and objection to corruption in society. This endogenized corruption choice is embedded within a dynamic monetary model to analyze the correlation between corruption and inflation. The major finding is that the model generates the observed positive correlation, with the correlation driven by the level of the social sanction. The empirical observation of a positive correlation can therefore be explained by different economies having different attitudes towards corruption.

Section 2 describes the monetary overlapping generations model and the forms of corruption. The welfare function and the measure of seigniorage are constructed in section 3. The process for sharing the benefits of corruption is described in section 4. The equilibrium of the model is described in section 5 and a simulation analysis of the equilibrium is conducted in section 6. Welfare maximization is analyzed in section 6.1. Section 7 provides concluding comments.

2 Dynamic Model

Table 1 summarizes the growth and distribution of money in the economy. We define \( M(t) \) as the nominal money base in year \( t \). The government issues new money in each period at (gross) rate \( \mu \). If \( \mu > 1 \) then there is monetary expansion. Assume that a share of new issued money (\( \varphi \)) is directly stolen by corrupt officials before it is introduced into the economy by the government. Shares \( \gamma_1 \geq 0, \gamma_2 \geq 0 \) and \( \gamma_3 \geq 0 (\gamma_1 + \gamma_2 + \gamma_3 = 1) \) of the remainder of the newly printed money are respectively sold to the young, given to the young, and given to the old by the government.

<table>
<thead>
<tr>
<th>Period</th>
<th>Equation</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>( M(t) = M(t-1) + (\mu - 1)M(t-1) )</td>
</tr>
<tr>
<td></td>
<td>( M(t-1) ) purchased by young from old</td>
</tr>
<tr>
<td></td>
<td>( \varphi(\mu - 1)M(t-1) ) stolen by corrupt official</td>
</tr>
<tr>
<td></td>
<td>( (1 - \varphi)\gamma_1(\mu - 1)M(t-1) ) sold to young by government</td>
</tr>
<tr>
<td></td>
<td>( (1 - \varphi)\gamma_2(\mu - 1)M(t-1) ) given to young by government</td>
</tr>
<tr>
<td></td>
<td>( (1 - \varphi)\gamma_3(\mu - 1)M(t-1) ) given to old by government</td>
</tr>
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Table 1. Money distribution

Assume that each consumer has one unit of labour to supply when young and none when old. Consumers can grant (or accept) consumption loans and can purchase money. They choose whether to work in the private or public sector, and if employment in the public sector is chosen they have a decision on whether to be honest or corrupt. The number of consumers in each generation is \( N \), so total population size is \( 2N \). Let \( n^c \), \( n^h \) and \( n^p \) be the population at time \( t \) who respectively work in the public sector as corrupt officials, work in the public sector as honest employees and the individuals working in the private sector.
First, consider the payoff for the corrupt officials. If acting corruptly in the public sector is chosen, then there is an access to a bribe as an alternative source of revenue besides the wage level. The wage at time $t$ is given by $w^g(t)$ and the bribe per person level by $b(t)$. The budget constraints facing a consumer who has chosen to be corrupt are

$$C^c_t = \omega^g(t) + b(t) - \ell^c(t) - p^m(t)m^c(t) + p^m(t)\varphi(\mu-1)\frac{M(t-1)}{n^c} + p^m(t)(1-\varphi)\gamma_2(\mu-1)\frac{M(t-1)}{N},$$

and

$$C^c_{t+1} = r(t)\ell^c(t) + p^m(t+1)m^c(t) + p^m(t+1)(1-\varphi)\gamma_3(\mu-1)\frac{M(t-1)}{N}.$$ 

$\ell^c(t)$ is the quantity of consumption loans granted at $t$, $p^m(t)$ is the price of money in units of commodity, and $m^c(t)$ is the quantity of money carried from $t$ to $t+1$ that is bought from old individuals and from government. $r(t)$ is the gross interest earned on consumption loans. $\varphi(\mu-1)\frac{M(t-1)}{n^c}$ is the amount of money stolen directly from the new issued money and $(1-\varphi)\gamma_2(\mu-1)\frac{M(t-1)}{N}$ and $(1-\varphi)\gamma_3(\mu-1)\frac{M(t-1)}{N}$ are respectively the money given to each young and old individual by government. By defining the net money demand, $\tilde{m}^c(t)$, and net money supply, $\tilde{m}^c(t)$, where

$$\tilde{m}^c(t) = m^c(t) - \varphi(\mu-1)\frac{M(t-1)}{n^c} - (1-\varphi)\gamma_2(\mu-1)\frac{M(t-1)}{N},$$  

and

$$\tilde{m}^c(t) = m^c(t) + (1-\varphi)\gamma_3(\mu-1)\frac{M(t-1)}{N}.$$  

The two budget constraints can then be combined into a lifetime budget constraint

$$C^c_t(t) + \frac{C^c_{t+1}}{r(t)} = \omega^g(t) + b(t) - p^m(t)m^c(t) + p^m(t+1)\frac{M(t-1)}{N}\tilde{m}^c(t).$$

The no-arbitrage condition that prevents gains from arbitraging money across periods is

$$p^m(t) - \frac{p^m(t+1)}{r(t)} = 0.$$  

Using (3) the lifetime budget constraint can be simplified to

$$C^c_t(t) + \frac{C^c_{t+1}}{r(t)} = \omega^g(t) + b(t) - p^m(t)\tilde{m}^c(t) - \tilde{m}^c(t).$$  

The same process for the honest officials and for individuals working in the private sector can be followed. If an individual chooses to act honestly in the
public sector, there is no bribe as an additional source of revenue, so $\omega_i^g(t)$ is the only income. Hence, the budget constraints for an honest individual are

$$C^h_t(t) = \omega^g(t) - \ell^h(t) - p^m(t)m^h(t) + p^m(t)(1 - \varphi)\gamma_2(\mu - 1)\frac{M(t - 1)}{N},$$

and

$$C^h_t(t + 1) = r(t)\ell^h(t) + p^m(t + 1)m^h(t) + p^m(t + 1)(1 - \varphi)\gamma_3(\mu - 1)\frac{M(t - 1)}{N}.$$ 

Define the net monetary quantities by:

$$\tilde{m}^h(t) = m^h(t) - (1 - \varphi)\gamma_2(\mu - 1)\frac{M(t - 1)}{N}, \quad (5)$$

$$\tilde{m}^h(t) = m^h(t) + (1 - \varphi)\gamma_3(\mu - 1)\frac{M(t - 1)}{N}. \quad (6)$$

Combining the two budget constraints into a lifetime constraint using (3) gives

$$C^h_t(t) + \frac{C^h_t(t + 1)}{r(t)} = \omega^g(t) - p^m(t)[\tilde{m}^h(t) - \tilde{m}^h(t)]. \quad (7)$$

Finally, let $\omega^p(t)$ be the wage level given by private sector at time $t$, so the lifetime budget constraint for a private sector worker becomes

$$C^p_t(t) + \frac{C^p_t(t + 1)}{r(t)} = \omega^p(t) - p^m(t)[\tilde{m}^p(t) - \tilde{m}^p(t)]. \quad (8)$$

With the net monetary quantities

$$\tilde{m}^p(t) = m^p(t) - (1 - \varphi)\gamma_2(\mu - 1)\frac{M(t - 1)}{N}, \quad (9)$$

$$\tilde{m}^p(t) = m^p(t) + (1 - \varphi)\gamma_3(\mu - 1)\frac{M(t - 1)}{N}. \quad (10)$$

The lifetime budget constraint of the three types of individual in the model are determined by (4), (7) and (8). Now assume that the lifetime utility functions for the three types of individual for $i = c, h, p$ is

$$U^i = [C^i_t(t)]^\alpha + \delta [C^i_t(t + 1)]^\alpha + [G(t)]^\alpha + \delta [G(t + 1)]^\alpha - v^i e^j(t) - q \chi^i k,$$

In which consumers are differentiated with respect to the valuation of the social sanction ($\chi$) and with respect to their disutility of effort ($v$).

Define the indicator variable, $q$, where $q = 1$ when $i = c$, otherwise $q = 0$ (if corruption is chosen, there is a loss in the self esteem only for individuals acting corruptly).

Let $j = g$ when $i = c, h$ and $j = p$ when $i = p$. The superscript denotes a corrupt public sector worker ($c$), an honest public sector worker ($h$), and
a private sector worker \((p)\). \(v^i\) and \(\chi^i\) are parameters that differ across the population. \(k\) is the same for all people and is the social sanction on corruption. A low value of \(k\) means a lower economy-wide sanction on corruption. The product \(\chi^i k\) is the loss of self esteem from acting in a corrupt way. \(G(t)\) is the public good provided by the government, and \(c(t)\) is the effort required for the job.

In every case, the maximization of utility leads to the necessary condition
\[
C_i^t (t + 1) = (r\delta)^{\frac{1}{1-\alpha}} C_i^t (t), \quad i = c, h, p. \tag{11}
\]

By writing \(\beta = r^{\frac{1}{1-\alpha}} \delta^{\frac{1}{1-\gamma}}\), the demand functions will take the forms below
\[
C_i^t (t) = \left(\frac{1}{1+\beta}\right) \left[\omega^i (t) + q [b(t) - p^m(t)[\tilde{m}^i(t) - \tilde{m}^i(t)]]\right], \tag{12}
\]
and
\[
C_i^t (t + 1) = \left(\frac{r}{1+\beta}\right) \left[\omega^i (t) + q [b(t) - p^m(t)[\tilde{m}^i(t) - \tilde{m}^i(t)]]\right]. \tag{13}
\]

The indirect utility function can then be written as
\[
U^i = A \left[\omega^i (t) + q [b(t) - p^m(t)[\tilde{m}^i(t) + \tilde{m}^i(t)]]\right]^{\alpha} + [G(t)]^{\alpha} + \delta [G(t + 1)]^{\alpha} - v^i e^i (t) - q [\chi^i k], \tag{14}
\]
where,
\[
A = (1 + \beta)^{-\alpha} + \delta \left(\frac{1+\beta}{r}\right)^{-\alpha}. \tag{15}
\]

### 3 Welfare, Savings, and Seigniorage

In the model the government is assumed to maximize a welfare function that is a weighted sum of the individual utilities (14). The weights may differ according to employment status and engagement in corruption. \(\xi^c\), \(\xi^h\) and \(\xi^p\) are respectively the welfare weights of corrupt, honest and private employees. The government chooses the tax rate and the growth rate of money supply to maximize the welfare function.

Let individual utility functions of the three types be simplified in the form of
\[
U^c = u_1 (w^c(t) + b(t), G(t)) - ve^c(t) - \chi k,
\]
\[
U^h = u_2 (w^h(t), G(t)) - ve^h(t),
\]
and
\[
U^p = u_3 (w^p(t), G(t)) - ve^p(t).
\]

For individuals \(c\), \(h\), and \(p\) respectively:
\[
u_1 (w^c(t) + b(t), G(t)) = A \left[\omega^c (t) + b(t) - p^m(t)[\tilde{m}^c(t) + \tilde{m}^c(t)]]\right]^{\alpha} + [G(t)]^{\alpha} + \delta [G(t + 1)]^{\alpha}, \tag{16}
\]
Figure 1: Figure 1. Distribution of occupation

\[ u_2(w^g(t), G(t)) = A \left[ \omega_2^g(t) - p^m(t)[\bar{m}^h(t) + \bar{m}^h(t)] \right]^\alpha + [G(t)]^\alpha + \delta [G(t + 1)]^\alpha, \]

(17)

and

\[ u_3(w^p(t), G(t)) = A \left[ \omega_3^p(t) - p^m(t)[\bar{m}^p(t) + \bar{m}^p(t)] \right]^\alpha + [G(t)]^\alpha + \delta [G(t + 1)]^\alpha. \]

(18)

The critical values that separate the types are

\[ v^* = \frac{u_3 - u_2}{e^p(t) - e^q(t)}, \]

\[ \chi^* = \frac{u_1 - u_2}{k}, \]

and

\[ v(\chi) = \frac{\chi^k}{e^p(t) - e^q(t)} + \frac{u_3 - u_1}{e^p(t) - e^q(t)}. \]

The number in each occupation can then be found by integrating the areas in the set \( \Theta \) as illustrated in figure 1.

The welfare function is then given by

\[
W = \int_0^{\chi^*} \int_{\max(0, v(\chi))}^1 \xi^c U^c N g(\chi, v) \, dv \, d\chi + \int_{\chi^*}^1 \int_{v^*}^1 \xi^h U^h N g(\chi, v) \, dv \, d\chi \\
+ \left[ \int_{\chi^*}^{v^*} \int_0^{\max(0, v(\chi))} \xi^p U^p N g(\chi, v) \, dv \, d\chi + \int_{v^*}^1 \int_0^v \xi^p U^p N g(\chi, v) \, dv \, d\chi \right]
\]
Assume that the individual characteristics are uniformly distributed on the unit square so \( g(\chi, v) = 1 \). The welfare function can be written in detail as

\[
W = \xi^c N \left[ u_1 - \frac{e^g}{2} - u_1 \left[ \frac{u_3 - u_1}{e^p - e^g} \right] + \left[ \frac{e^g}{2} \right] \left[ \frac{u_3 - u_1}{e^p - e^g} \right]^2 \right] \left[ \frac{u_1 - u_2}{k} \right]
+ \xi^b N \left[ \frac{1}{2} - u_1 \left[ \frac{1}{2(e^p - e^g)} \right] + \frac{1}{2} \left[ \frac{u_3 - u_1}{e^p - e^g} \right] + e^g \left[ \frac{1}{2(e^p - e^g)} \right] \left[ \frac{u_3 - u_1}{e^p - e^g} \right] \right] \left[ \frac{u_1 - u_2}{k} \right]
+ \xi^p N \left[ \frac{1}{3(e^p - e^g)} \right] \left[ \frac{1}{3(e^p - e^g)^2} \right] \left[ (u_1 - u_2)^3 \right] \left[ \frac{1}{k} \right]
+ \xi^p N \left[ u_2 - \frac{e^p}{2} - u_2 \left[ \frac{u_3 - u_2}{e^p - e^g} \right] + \left[ \frac{e^p}{2} \right] \left[ \frac{u_3 - u_2}{e^p - e^g} \right]^2 \right] \left[ 1 - \frac{u_1 - u_2}{k} \right]
+ \xi^p N \left[ u_3 \left[ \frac{(u_1 - u_2)^2}{2k(e^p - e^g)} \right] + u_3 \left[ \frac{u_3 - u_1}{e^p - e^g} \right] \left[ \frac{u_1 - u_2}{k} \right] - \frac{e^p}{2} \left[ \frac{(u_1 - u_2)^3}{3k(e^p - e^g)^2} \right] \right]
+ \xi^p N \left[ \frac{e^p}{2} \left[ \frac{u_3 - u_1}{e^p - e^g} \right] - e^p \left[ \frac{(u_1 - u_2)^2}{2k(e^p - e^g)} \right] \left[ \frac{u_3 - u_1}{e^p - e^g} \right] \right]
+ \xi^p N \left[ u_3 \left[ \frac{u_3 - u_2}{e^p - e^g} \right] - \frac{e^p}{2} \left[ \frac{u_3 - u_2}{e^p - e^g} \right]^2 \right] \left[ 1 - \frac{u_1 - u_2}{k} \right]
\]

The welfare function depends on the endogenous variables \( w^p, w^g, b \) and \( s \) which are determined in the equilibrium through the terms \( u_1, u_2, \) and \( u_3 \).

The income received by individuals in their first period of life can be consumed, granted as a loan, or held in the form of money. The saving function is the income left after consumption expenditure and granting of loans

\[
S^f_1(t) = \omega^f(t) + q[b(t)] - C^f_1(t) - \ell^f(t).
\]

By substituting the demand functions for the first period of life (12) with the net monetary quantities respectively [(1), (2)], [(5), (6)] and [(9), (10)] into the saving function, the saving of a corrupt individual is given by

\[
S^c_1(t) = \left( \frac{\beta}{1 + \beta} \right) [\omega^g(t) + b(t)] - \left( \frac{1}{1 + \beta} \right) p^m(t)(\varphi(\mu - 1) \frac{M(t - 1)}{n^e} + (1 - \varphi)\gamma_2(\mu - 1) \frac{M(t - 1)}{N}) + (1 - \varphi)\gamma_3(\mu - 1) \frac{M(t - 1)}{N}) - \ell^c(t).
\]

As \( M(t - 1) = (1/\mu) M(t) \)

\[
S^c_1(t) = \left( \frac{\beta}{1 + \beta} \right) [\omega^g(t) + b(t)] - \left( \frac{1}{1 + \beta} \right) p^m(t) M(t - 1) \left( \frac{\varphi}{n^e} + \frac{(1 - \varphi)(\gamma_2 + \gamma_3)}{N} \right) - \ell^c(t).
\]
The same process for the individuals of type h and p gives

\[ S^h(t) = \left( \frac{\beta}{1 + \beta} \right) \omega^g(t) - \left( \frac{1}{1 + \beta} \right) p^m(t)M(t)(1 - \frac{1}{\mu})(1 - \varphi)(\gamma_2 + \gamma_3)(\frac{1}{N}) - \ell^h(t), \]

and

\[ S^p(t) = \left( \frac{\beta}{1 + \beta} \right) \omega^p(t) - \left( \frac{1}{1 + \beta} \right) p^m(t)M(t)(1 - \frac{1}{\mu})(1 - \varphi)(\gamma_2 + \gamma_3)(\frac{1}{N}) - \ell^p(t). \]

The aggregate saving function is the sum of individual savings

\[ S(t) = \int S^c(t) + \int S^h(t) + \int S^p(t), \]

where

\[ \int S^c(t) = \left( \frac{\beta.n^c}{1 + \beta} \right) \omega^g(t) + \left( \frac{\beta.n^c.b}{1 + \beta} \right) - \left( \frac{1}{1 + \beta} \right) p^m(t)M(t) \left( 1 - \frac{1}{\mu} \right) \left[ \varphi + \left( \frac{n^c}{N} \right)(1 - \varphi)(\gamma_2 + \gamma_3) \right] - \int \ell^c(t), \]

\[ \int S^h(t) = \left( \frac{\beta.n^h}{1 + \beta} \right) \omega^g(t) - \left( \frac{1}{1 + \beta} \right) p^m(t)M(t)(1 - \frac{1}{\mu})(1 - \varphi)(\gamma_2 + \gamma_3)(\frac{n^h}{N}) - \int \ell^h(t), \]

and

\[ \int S^p(t) = \left( \frac{\beta.n^p}{1 + \beta} \right) \omega^p(t) - \left( \frac{1}{1 + \beta} \right) p^m(t)M(t)(1 - \frac{1}{\mu})(1 - \varphi)(\gamma_2 + \gamma_3)(\frac{n^p}{N}) - \int \ell^p(t). \]

As \( \int \ell^C(t) + \int \ell^h(t) + \int \ell^p(t) = 0 \) it follows that aggregate saving can be written compactly as

\[ S(t) = \beta \left[ \left( \frac{n^c + n^h}{1 + \beta} \right) \omega^g(t) + \left( \frac{n^p}{1 + \beta} \right) \omega^p(t) + \left( \frac{n^c.b}{1 + \beta} \right) \right] - \left( \frac{1}{1 + \beta} \right) p^m(t)M(t)(1 - \frac{1}{\mu})[\varphi + (1 - \varphi)(\gamma_2 + \gamma_3)]. \]

As already noted, money is the only outside asset. The money market is therefore in equilibrium when aggregate saving is equal to the value of the money supply

\[ S(t) = p^m(t)M(t). \quad (19) \]

In a stationary monetary equilibrium, the saving of generation \( t \) is the same as the saving of generation \( t + 1 \), so the amount of goods spent on purchasing money (as the only tool of saving) at time \( t \) equals the quantity of time \( t + 1 \) goods that are given up to buy money.

\[ S(t) = S(t + 1). \]
Hence,
\[ p^m(t)M(t) = p^m(t + 1)M(t + 1). \]
Money supply creation occurs at the gross growth rate of \( \mu \)
\[ M(t + 1) = \mu M(t). \]
This gives
\[ \dot{p}^m(t) = \mu \dot{p}^m(t + 1), \]
with the no-arbitrage condition (3) implying
\[ r(t) = \frac{p^m(t + 1)}{\dot{p}^m(t)} = \frac{1}{\mu}. \] (20)
Therefore, the aggregate saving function is
\[ S(t) = n^c + n^h! g(t) + n^p! p(t) + n^c:b \]
\[ + \left( 1 + \frac{1}{\mu} \right) \left[ \varphi + (1 - \varphi)(\gamma_2 + \gamma_3) \right]. \] (21)

The next step is to clarify the measure of seigniorage in the model.\(^1\) By
definition, the level of government revenue obtained from seigniorage is the
share of the value of newly issued money sold to the young. Selling money
transfers consumption goods from the individuals who purchase the money to
the government. Following this reasoning seigniorage is given by
\[ V(t) = p^m(t) \left[ (1 - \varphi)\gamma_1(\mu - 1)M(t - 1) \right]. \] (22)
Since \( M(t - 1) = (1/\mu)M(t) \) it follows that
\[ V(t) = (1 - \varphi)\gamma_1(1 - \frac{1}{\mu})p^m(t)M(t). \]
The aggregate saving function (21) can be substituted in to replace \( p^m(t)M(t) \)
giving the level of seigniorage as
\[ V(t) = (1 - \varphi)\gamma_1(1 - \frac{1}{\mu}) \left[ \beta \left[ \left( n^c + n^h \right) \omega^g(t) + n^p\omega^p(t) + n^c.b \right] \right. \]
\[ \left. \frac{1 + \beta + (1 - \frac{1}{\mu})[\varphi + (1 - \varphi)(\gamma_2 + \gamma_3)]}{1 + \beta + (1 - \frac{1}{\mu})[\varphi + (1 - \varphi)(\gamma_2 + \gamma_3)]} \right]. \] (23)

To this point the analysis has been phrased in terms of \( p^m(t) \), the price of
money. However, this is defined as the quantity of consumption good that has
to be given up to buy one unit of money. Consequently, the amount of money
that has to be given up to purchase one unit of consumption good is \( 1/p^m(t) \).
This is the price of goods at time \( t \) in the model. Inflation occurs when there is
an increase in the price of goods or a decline in the price of money. Therefore,
the inflation rate in period \( t \) is defined by
\[ \inf(t) = \frac{1/\dot{p}^m(t) - 1/\dot{p}^m(t-1)}{1/\dot{p}^m(t-1)}. \]
\(^1\)Myles and Yousefi (2014) explore the appropriate choice of seigniorage measure in greater
detail.
Using (20), the rate of inflation is determined by the net rate of money supply growth

\[ \text{Inf}(t) = \mu - 1. \]  

(24)

4 Nash Bargaining

Corrupt officials have the opportunity to inter in a bargain with the representative firm in the model to reduce the effective cooperation tax exchanged with a bribe. This bribe is one of the equilibrium conditions of the model. The premise is that the share of the proceeds from corruption are shared between the firm and the corrupt tax officials using a generalization of the Nash (1950) bargaining model (see also Harsanyi, 1977). The component parts of the bargain are described in Table 2 where the value of \( \lambda \) is the proportion of tax officials that are honest. \( f(n^P) \) is the total production of the firm. The firm should pay \( \tau f(n^P) \) in tax but actually pays \( \lambda \tau f(n^P) \), so the surplus to be shared between the firm and the collective officials is \( [1 - \lambda] \tau f(n^P) \). The share of the surplus the officials receive is denoted \( s \), so the firm receives share \( 1 - s \). The powers in the bargain over the share, \( s \), depend on the proportion of corrupt tax officials so that the more corrupt officials there are, then the larger is the power of the officials in the bargain. The bargain happens at time \( t \) and to abbreviate the calculation the sign of \( \text{Inf}(t) \) has been eliminated in this section.

\[
\begin{array}{|l|c|}
\hline
f(n^P) &= [1 - \tau] f(n^P) + \lambda \tau f(n^P) + s [1 - \lambda] \tau f(n^P) + [1 - s] [1 - \lambda] \tau f(n^P) \\
[1 - \tau] f(n^P) &= \text{Amount directly accruing to the firm} \\
\lambda \tau f(n^P) &= \text{Reduced tax take going to the government} \\
s [1 - \lambda] \tau f(n^P) &= \text{Share to corrupt tax collectors after bargaining} \\
[1 - s] [1 - \lambda] \tau f(n^P) &= \text{Share to the firm after bargaining} \\
\hline
\end{array}
\]

Table 2. Distribution of the firm’s total product (\( 0 < \tau < 1 \), \( 0 < \lambda < 1 \) and \( 0 < s < 1 \))

Therefore, the profit of the firm taking the outcome of the bargain into account is

\[
\pi = [1 - \tau] f(n^P) + [1 - s] [1 - \lambda] \tau f(n^P) - w^P n^P. \tag{25}
\]

The payoff and the threat point for the firm, in the bargain are

\[
\begin{align*}
U^f &= [1 - \tau] f(n^P) + [1 - s] [1 - \lambda] \tau f(n^P) - w^P n^P, \\
U_0^f &= [1 - \tau] f(n^P) - w^P n^P,
\end{align*}
\]

where \( U_0^f \) is the utility obtained if the bargain is unsuccessful. Combining these expressions

\[
U^f - U_0^f = [1 - s] [1 - \lambda] \tau f(n^P).
\]

There is a potential difficulty at this point that can be easily circumvented. If the values of \( v \) and \( \chi \) enter into the bargain the fact that these differ among the corrupt officials means that there is no simple way to write the bargain.
The resolution is to assume that the social sanction is incurred even in the case that no agreement is reached in bargaining. The interpretation of this is that the social sanction is incurred through the act of offering to be corrupt, even if corruption does not take place. Under this assumption the payoff of a corrupt official in the event of a successful bargain is

\[ U^c = u(w^g + \frac{s[1-\lambda] \tau f(n^p)}{n^c}, G) - \omega e^g - \chi k, \]

and the threat point is

\[ U^c_0 = u(w^g, G) - \omega e^g - \chi k. \]

The difference between these is given by

\[ U^c - U^c_0 = u(w^g + \frac{s[1-\lambda] \tau f(n^p)}{n^c}, G) - u(w^g, G). \]

In the generalized Nash bargain the value of \( s \) is chosen to solve

\[
\begin{align*}
\max_{\{s\}} X & = (U^f - U^f_0)^{1-\omega} (U^c - U^c_0) ^\omega \\
& = (\lim_{n \to 0} (1-s)(1-\lambda) \tau f(n^p))^{1-\omega} \left(u(w^g + \frac{s[1-\lambda] \tau f(n^p)}{n^c}, G) - u(w^g)\right) ^\omega,
\end{align*}
\]

where \( \omega \) is the power of the corrupt officials in the bargain. It is assumed that \( \omega \) is the share of corrupt officials in the public sector, so

\[
\omega = \frac{n^c(w^p, w^g, b)}{n^h(w^p, w^g, b) + n^c(w^p, w^g, b)},
\]

and \( \lambda \) is assumed to be the share of honest tax collectors who do not offer a reduced rate of tax

\[
\lambda = \frac{n^h(w^p, w^g, b)}{n^h(w^p, w^g, b) + n^c(w^p, w^g, b)}.
\]

The solution of the bargain gives the share of \( s \) as a function of \( \lambda \) and \( \omega \)

\[ s = s(\lambda, \omega). \]

The values of \( \lambda \) and \( \omega \) are, in turn, determined by the variables \( w^p, w^g \) and \( b \).

5 Equilibrium

It now becomes possible to define and analyze the equilibrium of the model. There is a representative firm in the dynamic model offers a work contract to the young individuals in each period. This contract states the wage level \( w^p \) and
the required work effort $e^p$. The firm deals with the government tax officials (honest and corrupt). With Constant Return to Scale (CRS) the maximization condition derived from firm’s profit (30) specifies the wage level paid by the private sector $w^p$. Using this information individuals decide whether to join the firm to work or join the government which offers a contract with wage $w^g$ and effort $e^g$. Public sector employment also offers the opportunity to earn a bribe, $b$, from acting corruptly.

\[
\begin{align*}
b &= \frac{s \left( \lambda(n^h, n^c), \omega(n^h, n^c) \right) \left[ 1 - \lambda(n^h, n^c) \right] \tau fn^p}{\left[ 1 - \tau(s(n^h, n^c) + \lambda(n^h, n^c) - s(n^h, n^c)\lambda(n^h, n^c)) \right] f}, \\
w^p &= \frac{n^h \tau fn^p + V^5 \left[ n^h + n^c \right] - G \left[ n^h + n^c \right]}{[n^h + n^c]^2}, \\
w^g &= \frac{n^h \tau fn^p}{n^h + n^c} + V^5 \left[ n^h + n^c \right] - G \left[ n^h + n^c \right].
\end{align*}
\]

As already discussed, there is a bargain between the firm and the corrupt officials to reduce the effective tax rate from $\tau$ to $\lambda \tau$. $\lambda$ represents the share of honest officials in the public sector (28). Table 2 shows the distribution of the firm’s total product. The benefit from the reduced tax rate is $(1 - \lambda) \tau$ which is distributed between the firm and the corrupt officials. The Nash bargaining solution determines the shares according to (26). The share that goes to corrupt officials is distributed equally as a bribe and is determined by (29). The government’s budget constraint is (31). The government is responsible for providing public goods ($G$) and paying a total wages $w^g (n^c + n^h)$ to public officials financed by tax collected and seigniorage. The government neither borrows nor lends, so is assumed to balance the budget in every period.

The division of the population could be determined by (1). From the figure it can be read that

\[
n^h = \int_{\chi} \int_{v^*}^1 Ng(\chi, v) dv \, d\chi,
\]

and

\[
n^c = \int_{0}^{\chi^*} \int_{\max\{0, v(\chi)\}}^1 Ng(\chi, v) dv \, d\chi,
\]

where $g(\chi, v)$ is the distribution of characteristics in the population, and by definition

\[
n^p = N - n^c - n^h.
\]

The integrals can be written explicitly by assuming a uniform distribution for $\chi$ and $v$, so over the unit square $g(\chi, v) = 1$. First, take $n^h$. Writing the integral in full gives,

\[
n^h = N \left( 1 - \frac{u_3(n^p; G) - u_2(w^g; G)}{e^p - e^g} \right) \left( 1 - \frac{u_1(w^g + b; G) - u_2(w^g; G)}{k} \right).
\]
The solution for $n^c$ depends on whether $v(\chi) \leq 0$. The simpler case occurs when $v(\chi) > 0$, so

$$n^c = N \int_0^{u_1(w^g + b, G)} \frac{1}{v(\chi)} d\chi$$

$$= N \left( \frac{u_1(w^g + b, G) - u_2(w^g, G)}{k} \right) \times \left[ 1 - \frac{u_3(w^p, G) - u_1(w^g + b, G)}{e^p - e^g} - \frac{u_1(w^g + b, G) - u_2(w^g, G)}{2(e^p - e^g)} \right]$$

(34)

Similar solutions can be given for the other two cases.

The division of the population determined by (32), (33) and (34) as well as assumptions (27) and (28) are the final components of the system. This equilibrium system can be solved for the endogenous variables.

6 Simulation Analysis

The purpose of constructing the model is to explore the link between government actions and the level of corruption. The government controls the tax rate, the level of public good provision, and the rate of monetary expansion. The interesting question is how the endogenous variables - particularly the level of corruption - change as these choices are varied. This section considers how the equilibrium depends on the key underlying parameters by conducting numerical simulations of a range of scenarios. The next section considers optimization of the choice variables.

To begin the analysis of equilibrium the full set of conditions that have been specified so far are stated. There are eight conditions in the system that jointly determine the endogenous variables ($w^p$, $w^g$, $b$ and $s$) and the division of the population between occupations. The first three conditions are the indirect utility functions derived from consumer optimization (14). The indirect utilities can be written as

$$U^c = A \left[ w^g(t) + b(t) - p^m(t)M(t)(1 - \frac{1}{\mu})\left[ \frac{\varphi}{n^c} + (1 - \varphi)(\gamma_2 + \gamma_3)\frac{1}{N} \right] \right]^\alpha + [G(t)]^\alpha + \delta [G(t + 1)]^\alpha - ve^g - \chi k,$$

$$U^h = A \left[ w^g(t) - p^m(t)M(t)(1 - \frac{1}{\mu})(1 - \varphi)(\gamma_2 + \gamma_3)\frac{1}{N} \right]^\alpha + [G(t)]^\alpha + \delta [G(t + 1)]^\alpha - ve^g,$$

and

$$U^p = A \left[ w^p(t) - p^m(t)M(t)(1 - \frac{1}{\mu})(1 - \varphi)(\gamma_2 + \gamma_3)\frac{1}{N} \right]^\alpha + [G(t)]^\alpha + \delta [G(t + 1)]^\alpha - ve^p.$$ 

These expressions have used the fact that aggregate saving is given by $p^m(t)M(t)$ from (21).
The indirect utility functions enter into the determination of the endogenized employment levels \( n^h \) and \( n^c \) using (33) and (34) which are the fourth and fifth equilibrium conditions

\[
n^h = N(1 - \frac{u_3 - u_2}{e^p - e^g})(1 - \frac{u_1 - u}{k}),
\]

\[
n^c = N \left( \frac{u_1 - u_2}{k} \right) \left[ 1 - \frac{u_3 - u_1}{e^p - e^g} - \frac{u_1 - u_2}{2(e^p - e^g)} \right],
\]

where, in the dynamic case, \( u_1, u_2 \) and \( u_3 \) are given by

\[
u_1(w^g(t) + b(t), G(t)) = A \left[ \omega^g(t) + b(t) - p^m(t)M(t)(1 - \frac{1}{\mu}) \right] + (1 - \varphi) \left( \gamma_2 + \gamma_3 \right) \frac{1}{N} \right]^\alpha + [G(t)]^\alpha + \delta |G(t + 1)]^\alpha,
\]

\[
u_2(w^g(t), G(t)) = A \left[ \omega^g(t) - p^m(t)M(t)(1 - \frac{1}{\mu})(1 - \varphi) \right] + [G(t)]^\alpha + \delta |G(t + 1)]^\alpha,
\]

and

\[
u_3(w^p(t), G(t)) = A \left[ \omega^p(t) - p^m(t)M(t)(1 - \frac{1}{\mu})(1 - \varphi) \right] + [G(t)]^\alpha + \delta |G(t + 1)]^\alpha.
\]

The next three equilibrium conditions are the bribe per person (29), the firm maximization condition (assuming CRS) (30), and the government budget constraint (31). Finally, \( s \) is determined by the Nash bargain solution (26).

Table 3 presents the effect of a change in the rate of tax holding public good provision and the rate of monetary expansion constant. The basic parameter values chosen for the simulation are given in Appendix 2. Government revenue at time \( t, \text{rev}(t) \), defined as the sum of taxes and seigniorage is increasing when the tax rate \( \tau(t) \) is increased. The use of seigniorage \( V_5(t) \) as an alternative source of revenue correspondingly falls as the tax rate increases even though the rate of monetary expansion is constant because the level of seigniorage (23) depends on the endogenous variables in the model \( (w^p, w^g \text{ and } b) \). Moreover, when the tax rate increases the wage rates \( w^g(t) \) and \( w^p(t) \) both decrease.

<table>
<thead>
<tr>
<th>( \tau(t) )</th>
<th>( s(t) )</th>
<th>( b(t) )</th>
<th>( w^g(t) )</th>
<th>( w^p(t) )</th>
<th>( \text{rev}(t) )</th>
<th>( V_5(t) )</th>
</tr>
</thead>
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<td>8.764551544</td>
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<td>0.6249737687</td>
<td>141.5362395</td>
<td>6.540450402</td>
</tr>
</tbody>
</table>

Table 3. Effects of a tax increase
The relationship between the tax rate and the level of the bribe is more interesting. Table 5.2 illustrates that there is a non-monotonic relationship with the bribe per person reaching a maximum at a tax rate of $\tau = 0.25$ and then decreasing as the tax rate increases further. This relationship is shown in more detail in figure 5.1. An increase in the tax rate increases the potential gain to the firm and the officials from engaging in corruption. The non-monotonic relationship is explained by the fact the $b$ is the bribe received by each corrupt official. As the tax rate increase the number of corrupt as the tax rates increases, and wages decrease. This makes a given size of bribe relatively more attractive.

Figure 5.2 shows the distribution of the three types of individuals for different tax rates. It can be seen that employment in the public sector increases when the tax rate increases ($np$ falls). This can be explained by the decreasing wage offered by the firm (table 5.2) that motivates the workforce towards public sector employment. Moreover, the number of corrupt officials also increases as the tax rate increases, and increases even faster as a proportion of workers in the private sector.

Further insight into how the division of the population between types is affected by the tax rate is given in figure 5.3. The proportion of corrupt in the population is increasing as the tax rate increases. This is not surprising since the private wage is falling and there is a greater surplus to be shared as bribes. What seems surprising is that the proportion of public sector workers that are corrupt falls as the tax rate increases. This is explained by the influx of former private sector workers into the public sector as the tax rate increases, who were formerly in the private sector partially because of their reluctance to engage in
corruption.

It has been assumed that the social sanction on corruption is a determinant of each consumer’s behavior. The consumers who work in the public sector choose to be corrupt based on their attitude towards the social sanction level and how the sanction affects their self-esteem. Figure 5.4 displays the simulated effect of increasing the social sanction upon the proportion of public employers that are corrupt. The figure demonstrates that an economy with a higher social sanction on corruption will experience a lower level of corruption.

As the government controls the tax rate, the level of public good provision, and the rate of monetary expansion, the next step is to analyze how the endogenous variables change if the rate of money expansion is changed by the government (given the tax rate and the level of public good remain constant). Table 4 shows that a higher rate of monetary expansion is associated with a higher number of corrupt officials working in the public sector but a lower bribe per person and a lower share goes to corrupt officials in the bargain. The level of seigniorage and government revenue are higher when $\mu$ increases.
Figure 4: Proportion of corrupt officials

Figure 5: Social sanction and corruption
Figure 6 shows the effect of monetary expansion on the wage rates in the private and public sectors. When there is a higher rate of monetary expansion (the government sells more money to the population as a source of revenue), the wage rate in both public and private sector initially increase. The explanation could be a flow to the public sector when the public revenue increases. However, there is a point for both at which the wage rates $w^g(t)$ and $w^p(t)$ begin to decrease with further monetary expansion.

### 6.1 Welfare Effects

The dependence of social welfare upon the tax rate and the rate of monetary expansion is now considered. These welfare results lead into an analysis of
optimal policy.

Table 5 displays how social welfare changes as the rate of monetary growth increases. The idea is that a welfare-maximizing government will have an incentive to rely on seigniorage to compensate for the reduction in tax revenue that results from corruption. However, seigniorage causes inflation which affects the intertemporal distribution of resources. The table assumes a tax rate of 20% (\(\tau = 0.2\)) and the value of the social sanction on corruption is assumed to be \(1/2 (k = 0.5)\). The values of other parameters used in this simulation are the same as given in Appendix 1.

<table>
<thead>
<tr>
<th>(\mu(t))</th>
<th>1.3</th>
<th>1.31</th>
<th>1.32</th>
<th>1.33</th>
<th>1.34</th>
<th>1.35</th>
<th>1.36</th>
<th>1.37</th>
<th>1.38</th>
<th>1.39</th>
</tr>
</thead>
<tbody>
<tr>
<td>(W(t))</td>
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<td>375.40</td>
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<td>375.49</td>
<td>375.52</td>
<td>375.53</td>
<td>375.54</td>
<td>375.53</td>
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<td>375.49</td>
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<td>(n^c)</td>
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<td>122.27</td>
<td>122.42</td>
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<td>122.70</td>
<td>122.83</td>
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<tr>
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</table>

Table 5. Welfare and the growth rate of money supply

Figure 7 show that higher monetary expansion raises the number of corrupt individuals and their share in the public sector. (while the rate of tax, public goods and the level of social sanction on corruption are constant)

For the base tax rate of 20% the maximum level of welfare is achieved when money supply growth is at the net rate of 36%. The key observation is that there is an optimum rate of monetary expansion and, hence, a limit to the beneficial amount of seigniorage. Equation (24) indicated that the inflation rate is equal to the net rate of money supply growth. Therefore, at this rate of monetary expansion the inflation rate is also 36%. The number of corrupt official is 122 individuals out of 1000 of population and their share in public sector employment is 51%. To explore whether the welfare function generally has a maximum
with respect to monetary expansion the simulation was repeated for different values of $k$ and $\tau$, and table 6 reports the values of the optimal growth rate of money supply ($\tilde{\mu}$) as $\tau$ and $k$ are varied. The graphs of the welfare functions are nicely concave with a clear optimum point in each case (see an example plot in Appendix 3).

It has already been noted that an economy with a lower social sanction experiences higher corruption in the public sector. The table also shows that when the social sanction increases, the optimal value of monetary expansion, $\tilde{\mu}$, that is chosen by the government declines. Therefore, a higher level of corruption is associated with the government choosing a higher growth rate of money supply. This is one of the central conclusions from the analysis: for any given tax rate, increased corruption and higher monetary expansion will occur together in equilibrium.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\tilde{\mu}$</th>
<th>$W$</th>
<th>$\mu$</th>
<th>$W$</th>
<th>$\mu$</th>
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<td>1.10</td>
<td>419.2</td>
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</tbody>
</table>

Table 6. Optimum rate of money supply and social sanction on corruption

The optimal growth rate of the money supply, $\tilde{\mu}$, declines as the social sanctions on corruption increases (and the proportion of corrupt public sector officials also declines) no matter what the tax rate is. This is shown in figure 8. Hence, seigniorage (22) and inflation (24) are lower in a society with less corruption. What is important is that there is no causality of one variable on the other. Both are endogenous and are determined simultaneously in equilibrium.

The parameter $\varphi$ captures the direct appropriation of the newly issued money by corrupt officials. It is assumed to be exogenous in this model even though the number of corrupt officials has been endogenized. Table 7 shows that when a greater proportion of the new money supply is stolen the optimal growth rate of money approaches zero, and the level of welfare is decreasing with higher $\varphi$. This is a consequence of seigniorage becoming less effective as more money is stolen, so there becomes less incentive for the government to inflate the money supply.

<table>
<thead>
<tr>
<th>$\varphi$</th>
<th>$\tilde{\mu}$</th>
<th>$W$</th>
<th>$\mu$</th>
<th>$W$</th>
<th>$\mu$</th>
<th>$W$</th>
<th>$\mu$</th>
<th>$W$</th>
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<tr>
<td>0.1</td>
<td>1.36</td>
<td>375.54</td>
<td>1.30</td>
<td>308.91</td>
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<td>302.64</td>
<td>1.12</td>
<td>337.20</td>
</tr>
<tr>
<td>0.15</td>
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<td>375.54</td>
<td>1.30</td>
<td>308.91</td>
<td>1.24</td>
<td>302.64</td>
<td>1.12</td>
<td>337.20</td>
</tr>
</tbody>
</table>

Table 7. Exogenous appropriation of newly issued money and the monetary expansion
Figure 8: Optimal growth rate of money supply and social sanction on corruption
In addition, the optimal tax rate ($\hat{\tau}$) that maximizes social welfare is also affected by the value of the social sanction. Table 7 shows that $\hat{\tau}$ increases with a higher social sanctions until the point $k = 0.8$, and then declines after this point ($\mu$ is assumed to be 1.1 in this table). However, the maximum welfare level is still higher in an economy with a higher social sanction (even with lower $\hat{\tau}$). When the social sanction is higher we saw that government chooses lower seigniorage ($\hat{\tau}$) but uses a higher tax rate as a source of revenue (table 8) where at some point of social sanction upwards ($k = 0.8$ in this simulation) the optimal tax rate levied is lower. This means that lower seigniorage and lower tax rate are associated with higher level of social sanction on corruption.

Table 8. Optimal tax rate for different social sanctions

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\hat{\tau}$</th>
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<tr>
<td>0.5</td>
<td>1.12 572.9</td>
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<tr>
<td>0.6</td>
<td>1.12073 404.9</td>
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<tr>
<td>0.7</td>
<td>1.1362 425.4</td>
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<tr>
<td>0.8</td>
<td>1.1409 440.8</td>
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<tr>
<td>0.9</td>
<td>1.1406 453.5</td>
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<tr>
<td>1.0</td>
<td>1.1379 444.3</td>
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<tr>
<td>2.0</td>
<td>1.0985 522.7</td>
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</table>

Table 9. Optimal tax rate and growth rate of money supply

7 Conclusions

The paper has analysed a dynamic monetary economy with endogenous determination of the level of corruption. The government engages in seigniorage to generate revenue but this causes inflation. Individuals can choose to be honest or corrupt and the benefits from corruption are shared through a generalized Nash bargain. The equilibrium conditions for the model were derived and a numerical simulation was used to explore the correlation between inflation and corruption.

The numerical results demonstrated that a higher tax rate led to a lower wage level in the private sector and in the public sector and an increase in the number of corrupt individuals. However, the proportion of corrupt in the public sector is lower when the tax rate is higher. An increase in the social sanction on corruption reduces the number of corrupt officials in the public sector. For a given level of the social sanction, the welfare-maximizing government chose.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
$\mu$ & 1.1 & 1.15 & 1.2 & 1.25 & 1.3 & 1.35 & 1.4 \\
\hline
$\hat{\tau}$ & 1.12 & 0.06 & 0.06 & 0.06 & 0.02 & 0.02 & 0.02 \\
\hline
\end{tabular}
\caption{Optimal tax rate and growth rate of money supply}
\end{table}
a higher tax rate when monetary expansion in lower. The chosen tax rate is initially increasing in the social sanction but eventually decreases. The growth rate of money supply chosen by the government is higher when there is a lower social sanction on corruption. The lower social sanction also causes a higher proportion of corrupt individual in the public sector. As a consequence, both a higher optimal inflation rate and greater corruption are associated with a lower social sanction. The model predicts the observed correlation between inflation and corruption across countries, with the correlation driven by the level of the social sanction. Surprisingly, a welfare-maximizing government that can optimize both the tax rate and seigniorage should finance all public spending by seigniorage alone.

The message of the paper is that inflation does not cause corruption, nor does corruption cause inflation. Instead, the two are simultaneously determined. In the model, as in the data, the two are positive correlated with the correlating factor being the level of social sanction. The model supports the argument that the observed empirical correlation is a consequence of different societies having different social sanctions. The policy implication is that encouraging a stronger social antipathy to corruption should reduce both corruption and inflation.

(Chapter head:) Appendix
Appendix 1 - The baseline values of the parameters for the simulation

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$f$</th>
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<th>$e^p$</th>
<th>$e^g$</th>
<th>$\delta$</th>
<th>$\tau$</th>
<th>$k$</th>
<th>$G$</th>
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</thead>
<tbody>
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</table>

Appendix 2 - The baseline values of the parameters for the simulation

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<th>$e^g$</th>
<th>$\delta$</th>
<th>$\mu$</th>
<th>$k$</th>
<th>$G$</th>
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</thead>
<tbody>
<tr>
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<td>1000</td>
<td>1</td>
<td>0.5</td>
<td>0.5</td>
<td>1.2</td>
<td>0.5</td>
<td>10</td>
</tr>
</tbody>
</table>

References


Figure 9: Appendix 3. The plot of the simulated welfare function \((k = 0.7 \text{ and } \tau = 0.15)\)

Figure 10: Appendix 4. The plot of the simulated welfare function \((k = 0.5 \text{ and } \mu = 0.15)\)


