A rent-seeking contest with private information and delegation

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Abstract

This paper demonstrates that asymmetric information about the value of the prize can explain endogenous delegation in a rent-seeking contest. We consider a two-player contest for a prize of common value. One player has private information about the value of the prize. The other player only knows that it is high or low, with given probabilities. The uninformed player can hire a delegate to act on his behalf. We derive the conditions under which delegation occurs in equilibrium. If the uninformed player can choose between an uninformed or informed delegate, only an informed one is hired in equilibrium.

Keywords: rent-seeking contest, private information, delegation.
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1 Introduction

We consider a rent-seeking contest where risk-neutral players can compete for a single prize by exerting nonrefundable efforts. The contest has been introduced in a seminal study by Tullock (1980). It has many applications, including the analysis of lobbying by firms for a monopoly position, R&D patent races between firms, litigation, and sport competitions. Nitzan (1994), Lockard and Tullock (2001), Congleton et al. (2008) and Konrad (2009) give overviews of the rent-seeking literature. We offer a new explanation for the occurrence of endogenous delegation in this contest, namely asymmetric information regarding the value of the prize. Examples of delegation in contests abound: firms hire lobbyists to acquire monopoly rents from the government, and parties hire attorneys to win lawsuits, etc. It is thus interesting to explain why delegation can happen endogenously.

In the standard contest, the value of the prize for each player is given and publicly known. However, we consider the case where the value is uncertain. We investigate a two-player contest for a prize of common value, which is either high or low with given probabilities. One player has private information about the realized value of the prize. The other player is uninformed and only knows the prior distribution. We assume that the uninformed player has the option to hire a risk-neutral delegate who will compete on his behalf. The uninformed player does not observe the effort made by the delegate in the contest. In order to cope with the resulting moral hazard problem, he offers the delegate a contract with a contingent fee, i.e. the delegate receives a deliberately chosen fraction of the prize if she wins the contest and nothing otherwise. If the uninformed player does not hire a delegate, he competes himself with the informed player. In this setup, the goal of our paper is twofold. First, we demonstrate that delegation might arise endogenously in (the sub-game perfect) equilibrium, even if the delegate has the same amount of information as the uninformed player himself. Second, we show that in case the uninformed player can choose between an uninformed or informed delegate, only an informed one will be hired in equilibrium.
In order to accomplish this, we consider three variants of our game. In the first one, the delegate is uninformed and has the same information as the uninformed player. We show that delegation occurs in equilibrium if and only if both the probability of a low prize and the ratio of the high and low value of the prize are large enough. The intuition behind this finding is that the optimal effort of the uninformed player depends on the possible realization of both the high and low prize. As a result, the uninformed player’s payoff conditional on the realization of the low prize is relatively small, in particular if the ratio of the high and low prize is large (because then the player’s effort is relatively large). This is especially relevant for the uninformed player’s ex ante payoff if the low prize occurs with a large probability. Thus, he then prefers to delegate. In the second variant, we have an informed delegate who is equipped with the same information as the informed player. We demonstrate that delegation arises again in equilibrium if and only if both the probability of a low prize and the ratio of the high and low value of the prize are large enough. However, the thresholds under which delegation takes place are different now than in the first variant. In the third variant, the uninformed player can freely decide to hire an uninformed or informed delegate. We show that if the uninformed player delegates in equilibrium, he will hire an informed delegate.

Our analysis applies to a situation where two firms lobby a local government that will award a license to one of them. The license gives the winning firm the monopoly right to provide say a bus service. It then might happen that one of the firms is an incumbent firm having superior information about the market and knowing the value of the new license, whereas the other firm is an entrant on the market and lacks such information. In that case, the uninformed firm might employ a lobbyist to act on its behalf. An uninformed lobbyist has the same information as the firm itself, whereas an informed lobbyist could be an expert in the local market. Another example is a court case where two parties dispute the property right of a good, and one party is well informed about its value (for example, since it owns a similar good), but the other party is not. In that case, the uninformed party might hire an attorney to act on its behalf. An uninformed attorney has no specific knowledge about the good, whereas an informed attorney has
experience from similar cases and is able to determine the value of the disputed right.

A number of papers have studied delegation in two-player rent-seeking contests (assuming that delegates receive a contingent fee). However, in those studies the value of the prize is given and publicly known. In a pioneering paper, Baik and Kim (1997) analyze the case where a delegate has a larger ability than the player himself, i.e. the delegate’s effort has a larger effect on the probability of winning the prize than the same effort would have if exerted by the player himself. Baik and Kim also endogenize the decision to hire a delegate. However, they assume that the payment scheme offered to the delegates is exogenously given, i.e. a delegate receives an exogenous contingent fee, plus a given fixed fee (depending on the delegate’s ability) which is paid regardless of the outcome of the contest. Wärneryd (2000) examines the case where the players and delegates have the same abilities and the contingent fee is set endogenously. He shows that delegation does not occur endogenously in that case. Focusing on the case with an endogenous contingent fee, Schoonbeek (2002) shows that one-sided endogenous delegation occurs if the two players have different risk-attitudes (in equilibrium, a risk-averse player might decide to hire a risk-neutral delegate), while Schoonbeek (2007) demonstrates that one-sided or two-sided endogenous delegation can arise if delegates have two instruments at their disposal while the players can use only one instrument.\footnote{Baik (2007) investigates the case where delegation is compulsory, the players and delegates have identical abilities, and the endogenous payment scheme of the delegates consists of a contingent fee and nonnegative fixed fee. He shows that the fixed fee is zero in equilibrium. Other papers studying compulsory delegation in Tullock contests are Baik (2008) and Baik and Lee (2013).} Our paper offers an alternative interesting explanation of endogenous delegation.

A number of papers have studied rent-seeking contests with private information. However, those studies do not include the option of delegation. Similar to us, Wärneryd (2003) considers a two-player common-value contest, where one player knows the value of the prize, while the other player only knows its prior distribution. Hurley and Shogren (1998a) take a two-player contest with independent valuations of the prize (i.e. each player might have a different value for the prize) and analyze the case where one player has private information about his valuation. Hurley and Shogren (1998b) numerically
analyze a similar case with two-sided private information. Malueg and Yates (2004) study a two-player contest with independent valuations and two-sided private information analytically, assuming that the valuations are high or low, while imposing a simple structure on the probabilities associated with these valuations.\(^2\)

Our paper is organized as follows. Section 2 presents the game and preliminary results. Section 3 gives the equilibrium results for the three variants of the game. We briefly conclude in Section 4. The proofs of the propositions are in the Appendix.

2 The Game

We consider a Tullock contest where risk-neutral players 1 and 2 can compete by exerting nonrefundable efforts for a single prize of common value that will be awarded to one of them. It is common knowledge that the value of the prize is \(V_H\) (high) with probability \(q\) and \(V_L\) (low) with probability \(1 - q\), where \(V_H > V_L > 0\) and \(0 < q < 1\) are given. The game has three stages. In stage 1, Nature draws the true value of the prize from the prior distribution. Player 1 is privately informed about the realized value, player 2 does not receive this information. Player 1 is of type \(t\) if the value is \(V_t\) (\(t = H, L\)).

In stage 2, player 2 can decide to hire a risk-neutral delegate who will compete on his behalf in the contest. We distinguish three possible variants. In the first variant, player 2 can hire an uninformed delegate who is also uninformed about the true value of the prize, i.e. she only knows its prior distribution, just like player 2 himself. Player 2 does not observe the effort put forward by his delegate. He offers the uninformed delegate a contingent fee contract at the beginning of stage 2. According to the contract, the delegate will receive a fraction \(w\) (\(0 \leq w \leq 1\)) of the prize if she wins it, while she receives nothing if she is not successful. Player 2 determines the size of the contingent fee \(w\).\(^3\) The delegate only accepts the offer if the corresponding payoff is nonnegative.

\(^2\)Other related studies are Schoonbeek and Winkel (2006), Fey (2008), Ryvkin (2010), Wärneryd (2012) and Wasser (2013).

\(^3\)We assume that the contingent fee offered to the uninformed delegate is the same for both possible values of the prize. This seems a natural assumption. Moreover, the analysis becomes cumbersome if we allow the contingent fee to depend on the realized value of the prize, without adding much additional insight.
Her payoff equals zero if she does not accept. In the second variant, player 2 can hire an informed delegate who is also privately informed about the true value of the prize in stage 1, and thus has the same information as player 1. We say that the informed delegate has type $t$ if the prize has value $V_t$ ($t = H, L$). Player 2 offers the informed delegate a contingent fee contract, while in exchange the delegate informs player 2 in a credible manner about the true value of the prize. In this case, the contingent fee might depend on the true value of the prize, i.e. the informed delegate of type $t$ is offered a contingent fee $w_t$ ($0 \leq w_t \leq 1$). Each type of the informed delegate accepts the offer if the corresponding payoff is nonnegative, while her payoff is zero if she does not accept. In the third variant, we combine the first two ones. Player 2 then can choose between hiring an uninformed or informed delegate. The rest of the game is the same. In all variants the contract accepted by the delegate is observed by player 1 at the end of stage 2.

In stage 3, the actual contest takes place. We have a contest between players 1 and 2 if player 2 has not hired a delegate. In that case, if the true value of the prize is $V_t$ ($t = H, L$), player 1 wins the prize with probability $e_{1t}/(e_{1t} + e_2)$, where $e_{1t} \geq 0$ and $e_2 \geq 0$ denote the effort of player 1 of type $t$ and player 2, respectively. If $e_{1t} + e_2 = 0$, each player wins with probability $1/2$. Player 1 of type $t$ solves the following problem

\[
\max_{e_{1t}} \frac{e_{1t} V_t}{e_{1t} + e_2} - e_{1t},
\]

while player 2 considers

\[
\max_{e_2} q \left( \frac{e_2 V_H}{e_{1H} + e_2} \right) + (1 - q) \left( \frac{e_2 V_L}{e_{1L} + e_2} \right) - e_2.
\]

If player 2 has hired a delegate, the contest is played between player 1 and the delegate. We will examine the variants with delegation in detail in Section 3. At the end of stage 3, if player 2 has competed himself, then he observes the true value of the prize and payoffs are realized. If player 2 has hired an uninformed delegate, then the same holds for this

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\footnote{In either variant of our game, if all participants in the contest exert zero effort, then each player will win the prize with probability 1/2. From now on we disregard these cases since they will not happen in equilibrium.}
delegate. If player 2 has hired an informed delegate, then payoffs are also realized at the end of stage 3.

We solve the game by using backward induction in order to find the (subgame-perfect) equilibrium. It is convenient to write $V_H = \mu V_L$, with $\mu > 1$ the ratio of the high and low value of the prize, and to analyze equilibrium payoffs as a function of $\mu$.

\section{2.1 Preliminary Results}

In this section we examine stage 3 of the situation where player 2 competes himself. We have to distinguish two cases for the corresponding equilibrium: case (a) where both types of player 1 are active, i.e. exert a positive effort, and case (b) where only the high-type player 1 is active. Denoting the equilibrium values of this situation with a bar, we obtain the following by using Wärneryd (2003). In case (a) we have $\bar{e}_a^H(\mu) = -\bar{e}_a^L(\mu) + \sqrt{V_L \bar{e}_a^L(\mu)}$ ($t = H, L$) and $\bar{e}_2^a(\mu) = (q\sqrt{\mu} + 1 - q)^2 V_L / 4$. The corresponding ex ante payoff of player 2 is

$$\bar{\Pi}_{2}^{n,a}(\mu) = \left(\frac{q\sqrt{\mu} + 1 - q}{4}\right)^2 V_L, \quad (3)$$

where superindex ‘n, a’ denotes no delegation, case (a). Both $\bar{e}_2^a(\mu)$ and $\bar{\Pi}_{2}^{n,a}(\mu)$ are increasing in $\mu$. In case (b), the equilibrium efforts are $\bar{e}_1^bH(\mu) = q\mu V_L / (1 + q)^2$, $\bar{e}_1^bL = 0$ and $\bar{e}_2^b(\mu) = q^2 \mu V_L / (1 + q)^2$. The corresponding ex ante payoff of player 2 is

$$\bar{\Pi}_{2}^{n,b}(\mu) = \left(\frac{q^3 \mu}{(1 + q)^2} + 1 - q\right) V_L. \quad (4)$$

We see that $\bar{e}_2^b(\mu)$ and $\bar{\Pi}_{2}^{n,b}(\mu)$ are increasing in $\mu$. Given $q$, case (a) is relevant if and only if $\bar{e}_2^a(\mu) < V_L$, or equivalently $\mu \in (1, \bar{\mu}(q))$, where $\bar{\mu}(q) = (1 + 1/q)^2$. Hence, case (a) occurs if and only if, given the probability of a prize with a high value, the ratio of the high and low prize is small enough. Case (b) is relevant in the opposite case, i.e. if and only if $\mu \in [\bar{\mu}(q), \infty)$. Note that $\bar{\mu}(q) > 4$ for $q \in (0, 1)$.

It is useful to decompose the payoff of player 2 in terms of two parts which are related to the cases where the prize is respectively high or low. In case (a), we write

$$\bar{\Pi}_{2}^{n,a}(\mu) = q \times W_{2}^{a, H}(\mu) + (1 - q) \times W_{2}^{a, L}(\mu), \quad (5)$$
where

\[ W_H^a(\mu) = \bar{p}_H^a(\mu) \times \mu V_L - \bar{e}_2^a(\mu), \]
\[ W_L^a(\mu) = \bar{p}_L^a(\mu) \times V_L - \bar{e}_2^a(\mu), \]  
and \( \bar{p}_i^a(\mu) \) is the probability that player 2 wins the prize of value \( V_t \) \( (t = H, L) \), conditional on its realization. Clearly, \( \bar{W}_H^a(\mu) \) is player 2’s payoff conditional on the realization of the high prize. Similarly, \( \bar{W}_L^a(\mu) \) is this player’s payoff conditional on the realization of the low prize. Using analogous notation in case (b), we write

\[ \bar{\Pi}^{n,b}_2(\mu) = q \times \bar{W}^b_H(\mu) + (1 - q) \times \bar{W}^b_L(\mu), \]  
where

\[ W_H^b(\mu) = \bar{p}_H^b(\mu) \times \mu V_L - \bar{e}_2^b(\mu), \]
\[ W_L^b(\mu) = 1 \times V_L - \bar{e}_2^b(\mu). \]  

Note that player 2 wins the low prize with certainty once it is realized, see (8).

In case (a), \( \bar{W}_H^a(\mu) \) is positive and increasing in \( \mu \), while \( \bar{W}_L^a(\mu) \) is positive and decreasing in \( \mu \) (the latter is driven by the fact that \( \bar{e}_2^a(\mu) \) increases faster in \( \mu \) than \( \bar{p}_L^a(\mu) \)). In fact, \( \bar{W}_L^a(\mu) \) converges to zero if \( \mu \) approaches \( \bar{\mu}(q) \) from below. Note that \( \bar{W}_H^a(\mu) > \bar{W}_L^a(\mu) \). In case (b), \( \bar{W}_H^b(\mu) \) is positive and increasing in \( \mu \), whereas \( \bar{W}_L^b(\mu) \) is negative and decreasing in \( \mu \) (the latter follows since \( \bar{e}_2^b(\mu) \) is increasing in \( \mu \)). Hence, player 2’s payoff conditional on the realization of the low prize is negative. Stated otherwise, in case (b), player 2 incurs a loss ex post if the true value of the prize is low.

Remark: It is interesting to compare our setup with private information to the full-information benchmark case where both player 1 and player 2 are informed about the true value of the prize at the end of stage 1. Denoting the equilibrium values of the benchmark case with a star, the ex ante payoff of player 2 is \( \pi_2^*(\mu) = (q\mu + 1 - q)V_L/4 \). We see that \( \pi_2^*(\mu) > \pi_2^{n,a}(\mu) \) for \( \mu \in (1, \bar{\mu}(q)) \), and \( \pi_2^*(\mu) > \pi_2^{n,b}(\mu) \) for \( \mu \in [\bar{\mu}(q), \infty) \). Hence, compared to the benchmark case, player 2 is worse off if he is uninformed about the true value of the prize. Player 2’s equilibrium effort in the benchmark case is \( e_{2H}^*(\mu) = \mu V_L/4 \). 

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if the prize is high, and \( e_{2L}^* = V_L/4 \) if it is low. We see that \( e_{2L}^* < \bar{e}_2^2(\mu) < e_{2H}^*(\mu) \) in case (a), and \( 4e_{2L}^* < \bar{e}_2^2(\mu) < e_{2H}^*(\mu) \) in case (b). Thus, in both case (a) and case (b), the effort in the game with private information is some (weighed) average of the benchmark efforts associated with a high and low prize. This explains why player 2 is worse off in the case with private information. □

3 Equilibria with Delegation

We now present the equilibrium results for the three variants of our game. Section 3.1 considers the variant with an uninformed delegate, Section 3.2 the variant with an informed delegate, and Section 3.3 the variant with both kinds of delegates.

3.1 An Uninformed Delegate

In this section we analyze the variant where player 2 has the option to hire an uninformed delegate. Given \( w \), take stage 3 of the situation where player 2 hires such a delegate. Player 1 of type \( t \) \((t = H, L)\) then solves

\[
\max_{x_1} x_1 V_t - x_1, \tag{9}
\]

and the uninformed delegate considers

\[
\max_{x_d} q \left( \frac{x_d w \mu V_L}{x_1 + x_d} \right) + (1 - q) \left( \frac{x_d w V_L}{x_1 + x_d} \right) - x_d, \tag{10}
\]

where \( x_{1t} \geq 0 \) denotes the effort of player 1 of type \( t \) (competing against an uninformed delegate) and \( x_d \geq 0 \) the effort of the uninformed delegate. The FOC of the uninformed delegate is equal to

\[
\frac{q x_{1H} w \mu V_L}{(x_1 + x_d)^2} + \frac{(1 - q) x_{1L} w V_L}{(x_1 + x_d)^2} - 1 = 0. \tag{11}
\]

In equilibrium, we distinguish between case (a) where both types of player 1 are active, and case (b) where only the high-type player 1 is active. First, consider case (a). Using the FOCs pertaining to the two types of player 1, and defining \( \rho_H^a > 0 \) and \( \rho_L^a > 0 \) by \( x_{1H}^a = \rho_H^a x_d^a \) and \( x_{1L}^a = \rho_L^a x_d^a \), respectively (the superindex ‘a’ denotes case (a)), we find

\[
\rho_H^a + 1 = (\rho_L^a + 1) \sqrt{\mu}. \tag{12}
\]
We can rewrite (11) as
\[ w(q\rho_H^a + (1 - q)\rho_L^a) = 1. \] (13)

It follows that
\[ \rho_L^a + 1 = \frac{1 + w}{wq\sqrt{\mu} + w(1 - q)}. \] (14)

Turning to stage 2, player 2 solves
\[ \max_w q \left( \frac{1}{\rho_H^a + 1} \right) (1 - w)\mu V_L + (1 - q) \left( \frac{1}{\rho_L^a + 1} \right) (1 - w)V_L, \] (15)
which, using (12) and (14), can be rewritten as
\[ \max_w \left( \frac{w}{1 + w} \right) (1 - w)(q\sqrt{\mu} + 1 - q)^2 V_L. \] (16)

Using the FOC, the contingent fee in case (a) is \( \tilde{w}^a = \sqrt{2} - 1 \). A tilde denotes an equilibrium value in the situation where player 2 hires an uninformed delegate. It can be verified that the corresponding payoff of the delegate is positive, hence she accepts the offer. Substituting \( \tilde{w}^a \) in (14), we obtain \( \tilde{\rho}_L^a \). Given \( q \), case (a) is relevant if and only if \( \tilde{\rho}_L^a > 0 \), or equivalently \( \mu \in (1, \tilde{\mu}(q)) \), where \( \tilde{\mu}(q) \equiv (1 + 1/q(\sqrt{2} - 1))^2 \). Note that \( \tilde{\mu}(q) > \bar{\mu}(q) \).

Substituting \( \tilde{w}^a \) in (16), the ex ante payoff of player 2 is equal to
\[ \tilde{\pi}_2^{d,a}(\mu) = (\sqrt{2} - 1)^2 (q\sqrt{\mu} + 1 - q)^2 V_L, \] (17)
which is increasing in \( \mu \). We employ a lower case for player 2’s payoff if he hires an uninformed delegate, and the superindex ‘d, a’ refers to delegation, case (a).

Proceeding, we take case (b) with \( \mu \in [\bar{\mu}(q), \infty) \), where only the high-type player 1 is active if player 2 hires an uninformed delegate. Using obvious notation and defining \( \rho_H^b > 0 \) by \( x_{1H}^b = \rho_H^b x_{a}^b \), it follows from the FOCs of the high-type player 1 and uninformed delegate that \( \rho_H^b = 1/(qw_b) \). Using this, player 2 solves
\[ \max_w q \left( \frac{1}{\rho_H^b + 1} \right) (1 - w)\mu V_L + (1 - q)(1 - w)V_L, \]
or equivalently
\[ \max_w (1 - w) \left( \frac{q^2 w\mu}{qw + 1} + 1 - q \right) V_L. \] (18)
Taking the FOC, the equilibrium contingent fee is
\[ \tilde{w}^b = \frac{1}{q} \left( \sqrt{\frac{\mu q(1+q)}{\mu q + 1-q}} - 1 \right). \] (19)

The resulting payoff of the delegate is positive, so she will accept the contract. Substituting \( \tilde{w}^b \) into (18), we find that the corresponding ex ante payoff of player 2 is
\[ \tilde{\pi}^{d,b}_2(\mu) = \left( (q+1)(\mu q + 1-q) + \mu q - 2\sqrt{\mu q(1+q)(\mu q + 1-q)} \right) \frac{V_L}{q}. \] (20)
It can be verified that \( \tilde{\pi}^{d,b}_2(\mu) \) is increasing in \( \mu \).

We now investigate under which conditions we obtain endogenous delegation in the equilibrium of the whole game. Therefore, we have to compare the payoff of player 2 when he competes himself to the payoff when he hires an uninformed delegate. We obtain the following proposition.

**Proposition 1** Consider the game where player 2 has the option to hire an uninformed delegate. We have the following in the equilibrium.

(i) Take \( q \in (0, q_1) \). Then there is a unique \( \mu_1(q) \in (\bar{\mu}(q), \tilde{\mu}(q)) \) such that player 2 does not hire an uninformed delegate if \( \mu \in (1, \mu_1(q)) \), whereas player 2 hires an uninformed delegate if \( \mu \in (\mu_1(q), \infty) \).

(ii) Take \( q \in [q_1, q_2) \). Then there is a unique \( \mu_2(q) \in [\tilde{\mu}(q), \infty) \) such that player 2 does not hire an uninformed delegate if \( \mu \in (1, \mu_2(q)) \), whereas player 2 hires an uninformed delegate if \( \mu \in (\mu_2(q), \infty) \).

(iii) Take \( q \in [q_2, 1) \). Then player 2 does not hire an uninformed delegate.

Here \( q_1 \approx 0.297 \) is the unique \( q \in (0, 1) \) such that \( \tilde{\pi}^{d,a}_2(\tilde{\mu}(q)) = \Pi_2^{n,b}(\hat{\mu}(q)) \), where \( \tilde{\pi}^{d,a}_2(\mu) \) is considered in the limit case with \( \mu = \hat{\mu}(q) \), and \( q_2 \approx 0.420 \) is the unique \( q \in (0, 1) \) such that \( 1 - 4q^2(1+q) = 0 \).

Proposition 1 shows that an uninformed delegate is hired if and only if both the probability of a low prize \( (1 - q) \) and the ratio of the high and low prize \( (\mu) \) are large enough.
In order to understand this intuitively, we make the following observations. First, if player 2 delegates, he will only receive a fraction of the prize if the delegate wins the prize on his behalf. On the other hand, player 2 does not incur costs of efforts now, since these are exerted by the delegate. In turn, the delegate determines her optimal effort taking into account that she receives only a fraction of the prize herself in the event of winning. This will make her less aggressive in the contest than player 2 would be (if this player would compete himself), reducing the probability that player 2 obtains (his share of) the prize. Second, if player 2 competes himself, then his optimal effort takes into account that he may keep the entire prize in case of winning and that the costs of his effort are nonrefundable. Importantly, he further takes into account the possible realization of both a high and low prize. Given these observations, it becomes clear why player 2 hires an uninformed delegate when both $1 - q$ and $\mu$ are large enough, as shown in parts (i) and (ii) of Proposition 1. The intuitive reason is that then player 2’s payoff, if he would compete himself, is largely affected by the part associated with the low prize, i.e. $(1 - q)\bar{W}_L^b(\mu)$ in (7)\textsuperscript{5}, where $1 - q$ is large and the payoff conditional on the realization of this prize ($\bar{W}_L^b(\mu)$) is considerably negative (since player 2’s effort is large if $\mu$ is large). As a result, player 2’s ex ante payoff is lower if he competes himself than under delegation. This intuition is confirmed by numerical calculations done by us.

3.2 An Informed Delegate

Next, we examine the variant where player 2 has the option to hire an informed delegate. Take stage 3 of the situation where player 2 hires such a delegate. If the prize has value $V_t$ ($t = H, L$), player 1 of type $t$ solves

$$\max_{y_{1t}} \frac{y_{1t}V_t}{y_{1t} + y_{dt}} - y_{1t},$$

(21)

while, given $w_t$, the informed delegate of type $t$ solves

$$\max_{y_{dt}} \frac{y_{dt}w_tV_t}{y_{1t} + y_{dt}} - y_{dt},$$

(22)

\textsuperscript{5}Note that $\mu_1(q) > \bar{\mu}(q)$ and $\mu_2(q) > \bar{\mu}(q)$, so (7) is relevant in that case.
where $y_{1t} \geq 0$ and $y_{dt} \geq 0$ denote the effort of player 1 and delegate of type $t$, respectively. We find from the FOCs corresponding to (21) and (22) that $y_{1t} = w_t V_t / (1 + w_t)^2$ and $y_{dt} = (w_t)^2 V_t / (1 + w_t)^2$. Hence, the delegate of type $t$ wins the prize with probability $w_t / (1 + w_t)$ (which is increasing in $w_t$).

Using this, we turn to stage 2, where player 2 solves

$$\max_{w_t} \left( \frac{w_t}{1 + w_t} \right) (1 - w_t) V_t.$$ 

(23)

The FOC yields the contingent fee $\hat{w}_t = \sqrt{2} - 1$, which is independent of $t$ (and the same as $\hat{w}^a$ in Section 3.1). A hat denotes an equilibrium value in the situation where player 2 hires an informed delegate. The equilibrium probability that the delegate wins the prize is $1 - \frac{1}{\sqrt{2}}$, regardless of her type. The equilibrium efforts in stage 3 are $\hat{y}_{1t} = \frac{1}{2} (\sqrt{2} - 1) V_t$ and $\hat{y}_{dt} = \frac{1}{2} (\sqrt{2} - 1)^2 V_t$. The corresponding payoff of the delegate of type $t$ equals $\frac{1}{4} (\sqrt{2} - 1)^3 V_t > 0$, so she accepts the contract. Using $\hat{w}_t$ and (23) for $t = H, L$, we find that the ex ante payoff of player 2 when he hires an informed delegate (denoted by a capital letter and superindex ‘$d$’) is given by

$$\hat{\Pi}_2^d(\mu) = (\sqrt{2} - 1)^2 (q\mu + 1 - q) V_L,$$

(24)

which is increasing in $\mu$.

Next, we analyze when it is optimal for player 2 to hire an informed delegate in the equilibrium of the complete game. We present the following proposition.

**Proposition 2** Consider the game where player 2 has the option to hire an informed delegate. We have the following in the equilibrium.

(i) Take $q \in (0, \frac{1}{2} \sqrt{2})$. Then there is a unique $\mu_3(q) \in (1, \bar{\mu}(q))$ such that player 2 does not hire an informed delegate if $\mu \in (1, \mu_3(q)]$, whereas player 2 hires an informed delegate if $\mu \in (\mu_3(q), \infty)$.

(ii) Take $q \in [\frac{1}{4} \sqrt{2}, \frac{1}{2} \sqrt{2})$. Then there is a unique $\mu_4(q) \in [\bar{\mu}(q), \infty)$ such that player 2 does not hire an informed delegate if $\mu \in (1, \mu_4(q)]$, whereas player 2 hires an informed delegate if $\mu \in (\mu_4(q), \infty)$.
(iii) Take $q \in \left[\frac{1}{2}\sqrt{2}, 1\right)$. Then player 2 does not hire an informed delegate.

We see from Proposition 2 that player 2 hires an informed delegate if and only if both the probability of a low prize and the ratio of the high and low prize are large enough. Qualitatively speaking this corresponds to our findings in Section 3.1 for the variant with an uninformed delegate. However, the thresholds for $1 - q$ and $\mu$ are different in the two variants. For example, part (i) of Proposition 2 shows that we also have equilibria with an informed delegate if $\mu \in (1, \bar{\mu}(q))$, which is not possible with an uninformed delegate (see Proposition 1). The reason is that an informed delegate can choose her effort level contingent on the realization of the prize. This allows the informed delegate to increase (decrease) her effort if she knows that the high (low) prize has been realized, which benefits both herself and player 2. An uninformed delegate is not able to do this, which makes delegation less attractive in that case.

### 3.3 An Uninformed or Informed Delegate

Finally, we consider the third variant of our game, where player 2 can choose between an uninformed or informed delegate. We have the following result.

**Proposition 3** Consider the game where player 2 has the option to hire either an uninformed or informed delegate. Then player 2 will not hire an uninformed delegate in the equilibrium.

Proposition 3 means that if player 2 hires a delegate in equilibrium, then it must be an informed one. The reason is again that an informed delegate is able to set her effort depending on the realization of the prize, which is profitable for player 2. Proposition 3 implies that Proposition 2 also characterizes the conditions under which player 2 hires an informed delegate in the equilibrium of the third variant of our game.

Concluding Section 3, we have seen that the uninformed player might decide to hire a delegate in the equilibrium of each variant of our game with one-sided private information about the value of the prize. It is interesting to compare this to the equilibrium of the full-information benchmark case where both player 1 and player 2 as well as the delegate
are informed about the value of the prize at the end of stage 1, player 2 has the option to hire the delegate in stage 2, and the actual contest is played in stage 3. It can be verified that then in equilibrium the contingent fee is $\sqrt{2} - 1$, while player 2’s payoff equals, respectively, $(q\mu + 1 - q)V_L/4$ if player 2 competes himself, and $(\sqrt{2} - 1)^2(q\mu + 1 - q)V_L$ if he hires a delegate. As a result, player 2 will not hire a delegate in equilibrium (cf. Wärneryd, 2000). In the benchmark case, delegation has a positive effect for player 2, since he does not incur any costs of effort. However, it also has two negative effects. The effort of the delegate is lower than the effort put forward by player 2 if he would compete himself, which lowers the probability that the prize is won for player 2. Moreover, player 2 only receives a fraction $2 - \sqrt{2}$ of the prize if the delegate is successful. In equilibrium, the two negative effects dominate the positive effect. We conclude that the equilibria with endogenous delegation in the three variants of our game arise due to the presence of asymmetric information about the value of the prize.

4 Conclusion

This paper has demonstrated that endogenous delegation can occur in Tullock rent-seeking contests if there is asymmetric information regarding the value of the prize. We have studied a two-player contest with a common but uncertain value of the prize. The value is either high or low with given probabilities. One player is privately informed about the true value of the prize, while the other player only knows its prior distribution. The uninformed player has the option to hire a delegate who will compete on his behalf. We have considered three variants of our game which differ in terms of the amount of information available to the delegates about the value of the prize. In the first variant the delegate is uninformed, in the second variant she is privately informed. In either variant endogenous delegation arises if and only if both the probability of a low prize and the ratio of the high and low prize are large enough (although the relevant thresholds differ in the two variants). In the third variant, the uninformed player can select between an uninformed or informed delegate. It turns out that then only an informed delegate will be hired in equilibrium. Hence, the uninformed player prefers to hire a delegate with
better information.

In future work, it is interesting to extend our analysis to the cases where the valuation of the prize might differ for both players or we have two-sided private information.
Appendix: Proofs of the Propositions

In this Appendix we present a number of useful lemmas. Proposition 1 follows directly from Lemmas 1-3. Proposition 2 follows directly from Lemmas 4 and 5. The Appendix concludes with the proof of Proposition 3.

Lemma 1 Consider the game where player 2 has the option to hire an uninformed delegate. Let \( \mu \in (1, \bar{\mu}(q)) \) for \( q \in (0,1) \). Then player 2 will not hire an uninformed delegate in the equilibrium.

Proof. Take \( \mu \in (1, \bar{\mu}(q)) \) for \( q \in (0,1) \). Using \( \bar{\mu}(q) < \bar{\mu}(q) \), \( \bar{\Pi}^{n,a}(\mu) \) is relevant if player 2 competes himself, while \( \tilde{\pi}^{d,a}(\mu) \) is relevant if he hires an uninformed delegate. The proof follows since \( \bar{\Pi}^{n,a}(\mu) > \tilde{\pi}^{d,a}(\mu) \) if \( \mu \in (1, \bar{\mu}(q)) \). ■

Lemma 2 Consider the game where player 2 has the option to hire an uninformed delegate. Let \( \mu \in [\bar{\mu}(q), \tilde{\mu}(q)) \) for \( q \in (0,1) \). We have the following in the equilibrium.

(i) Take \( q \in (0, q_1) \). Then there is a unique \( \mu_1(q) \in (\bar{\mu}(q), \tilde{\mu}(q)) \) such that player 2 does not hire an uninformed delegate if \( \mu \in [\bar{\mu}(q), \mu_1(q)] \), whereas player 2 hires an uninformed delegate if \( \mu \in (\mu_1(q), \tilde{\mu}(q)) \).

(ii) Take \( q \in (q_1, 1) \). Then player 2 does not hire an uninformed delegate.

Here \( q_1 \approx 0.297 \) is the unique \( q \in (0,1) \) such that \( \tilde{\pi}^{d,a}(\bar{\mu}(q)) = \tilde{\Pi}^{n,b}(\bar{\mu}(q)) \), where both payoffs of player 2 are considered in the limit case with \( \mu = \bar{\mu}(q) \).

Proof. Take \( \mu \in [\bar{\mu}(q), \tilde{\mu}(q)) \) for \( q \in (0,1) \). Then \( \tilde{\Pi}^{n,b}(\mu) \) is relevant if player 2 competes himself, while \( \tilde{\pi}^{d,a}(\mu) \) is relevant if he hires an uninformed delegate. Define \( f(\lambda) \equiv a(q)\lambda^2 + b(q)\lambda + c(q) \) (for \( \lambda \in \mathbb{R} \)), where

\[
\begin{align*}
a(q) & = q^2 \left( (\sqrt{2} - 1)^2 - \frac{q}{(1 + q)^2} \right), \\
b(q) & = 2(\sqrt{2} - 1)^2q(1 - q) > 0, \\
c(q) & = (1 - q) \left( (\sqrt{2} - 1)^2(1 - q) - 1 \right) < 0. 
\end{align*}
\]
Notice that $\tilde{n}_{2}(\mu) > \Pi_{2}(\mu)$ if and only if $f(\sqrt{\mu}) > 0$. Further, $a(q) = 0$ if $q = q^* = (4\sqrt{2} - 5 - \sqrt{8\sqrt{2} - 11})/(2 - \sqrt{2}) \approx 0.282$, $a(q) > 0$ if $q \in (0, q^*)$, and $a(q) < 0$ if $q \in (q^*, 1)$. Straightforward but tedious calculations demonstrate that the discriminant of $f(\lambda)$, i.e. $\text{Discr}(q) = (b(q))^2 - 4a(q)c(q)$, is positive if and only if $q \in (0, \sqrt[4]{2})$. We see that $f(0) < 0$ and $f(\sqrt{\mu(q)}) < 0$ for $q \in (0, 1)$. In order to give the proof, we determine the sign of $f(\sqrt{\mu(q)})$. We consider three cases.

First, take $q \in (0, q^*)$. Then $f(\lambda)$ is an upward opening parabola attaining a negative minimum value for some $\lambda = \lambda(q) < 0$. Tediou manipulations show that $f(\sqrt{\mu(q)}) > 0$ in this case. Using this and $f(\sqrt{\mu(q)}) < 0$, it follows that there exists a unique $\mu_1(q) \in (\bar{\mu}(q), \tilde{\mu}(q))$ implicitly defined by $\tilde{n}_{2}(\mu_1(q)) = \Pi_{2}(\mu_1(q))$. Further, $\tilde{n}_{2}(\mu) < \Pi_{2}(\mu)$ if $\mu \in [\bar{\mu}(q), \mu_1(q)]$, and $\tilde{n}_{2}(\mu) > \Pi_{2}(\mu)$ if $\mu \in (\mu_1(q), \tilde{\mu}(q))$.

Second, take $q = q^*$. Then $f(\sqrt{\mu(q^*)}) = b(q^*) + c(q^*) > 0$. Using $f(\sqrt{\mu(q^*)}) < 0$ and $b(q^*) > 0$, there exists a unique $\mu_1(q^*) \in (\bar{\mu}(q^*), \tilde{\mu}(q^*))$ such that $\tilde{n}_{2}(\mu_1(q^*)) = \Pi_{2}(\mu_1(q^*))$, while $\tilde{n}_{2}(\mu) < \Pi_{2}(\mu)$ if $\mu \in [\bar{\mu}(q^*), \mu_1(q^*)]$, and $\tilde{n}_{2}(\mu) > \Pi_{2}(\mu)$ if $\mu \in (\mu_1(q^*), \tilde{\mu}(q^*))$.

Third, take $q \in (q^*, 1)$. Then $f(\lambda)$ is a downward opening parabola, attaining a maximum value for some $\lambda_{\max} = \lambda_{\max}(q) > 0$. We now distinguish two subcases, depending on the size of $\text{Discr}(q)$. To begin with, suppose that $\text{Discr}(q) > 0$, so $q \in (q^*, \sqrt[4]{2})$. Then $f(\lambda)$ has two positive zeros, $\lambda_1 = \lambda_1(q)$ and $\lambda_2 = \lambda_2(q)$ say, and $f(\lambda) > 0$ for $\lambda \in (\lambda_1, \lambda_2)$. Note that $\sqrt{\mu(q)} < \lambda_1$. Further, the derivative of $f(\lambda)$ evaluated in $\lambda = \sqrt{\mu(q)}$ is positive in this case. As a result, $\sqrt{\mu(q)} < \lambda_{\max}$. Furthermore, (i) $f(\sqrt{\mu(q)})$ is decreasing in $q$ for $q \in (q^*, \sqrt[4]{2})$, (ii) taking the limit case with $q = q^*$, we have $f(\sqrt{\mu(q^*)}) > 0$, and (iii) taking the limit case with $q = \sqrt[4]{2}$, we have $f(\sqrt{\mu(\sqrt[4]{2})}) < 0$. It follows that there exists a unique $q_1 \in (q^*, \sqrt[4]{2})$ such that $f(\sqrt{\mu(q_1)}) = 0$ (or equivalently $\tilde{n}_{2}(\mu_1(q)) = \Pi_{2}(\mu_1(q))$), where we evaluate $\tilde{n}_{2}(\mu)$ and $\Pi_{2}(\mu)$ in the limit case with $\mu = \tilde{\mu}(q)$. Numerical calculations show that $q_1 \approx 0.297$. If $q \in (q^*, q_1)$, then $f(\sqrt{\mu(q)}) > 0$, and there exists a unique $\mu_1(q) \in (\bar{\mu}(q), \tilde{\mu}(q))$ such that $\tilde{n}_{2}(\mu_1(q)) = \Pi_{2}(\mu_1(q))$, while $\tilde{n}_{2}(\mu) < \Pi_{2}(\mu)$ if $\mu \in [\bar{\mu}(q), \mu_1(q)]$, and $\tilde{n}_{2}(\mu) > \Pi_{2}(\mu)$ if $\mu \in (\mu_1(q), \tilde{\mu}(q))$. If $q \in [q_1, \sqrt[4]{2})$, then $f(\sqrt{\mu(q)}) \leq 0$, and
\[ \tilde{\pi}_2^{d, a}(\mu) \leq \tilde{\Pi}_2^{n, b}(\mu) \] for all \( \mu \in [\tilde{\mu}(q), \mu(q)) \). Finally, suppose that \( \text{Discr}(q) \leq 0 \), so \( q \in \left[ \frac{1}{2} \sqrt{2}, 1 \right) \). Then \( f(\sqrt{\mu(q)}) \leq 0 \), and thus \( \tilde{\pi}_2^{d, a}(\mu) \leq \tilde{\Pi}_2^{n, b}(\mu) \) for all \( \mu \in [\tilde{\mu}(q), \mu(q)) \).

Combining results, the lemma follows easily. \[ \blacksquare \]

**Lemma 3** Consider the game where player 2 has the option to hire an uninformed delegate. Let \( \mu \in [\tilde{\mu}(q), \infty) \) for \( q \in (0, 1) \). We have the following in the equilibrium.

(i) Take \( q \in (0, q_1) \). Then player 2 hires an uninformed delegate.

(ii) Take \( q \in [q_1, q_2) \). Then there is a unique \( \mu_2(q) \in [\tilde{\mu}(q), \infty) \) such that player 2 does not hire an uninformed delegate if \( \mu \in [\tilde{\mu}(q), \mu_2(q)] \), whereas player 2 hires an uninformed delegate if \( \mu \in (\mu_2(q), \infty) \).

(iii) Take \( q \in [q_2, 1) \). Then player 2 does not hire an uninformed delegate.

Here \( q_1 \) is defined in Lemma 2, and \( q_2 \approx 0.420 \) is the unique \( q \in (0, 1) \) such that \( 1 - 4q^2(1 + q) = 0 \).

**Proof.** Take \( \mu \in [\tilde{\mu}(q), \infty) \) for \( q \in (0, 1) \). Then \( \tilde{\Pi}_2^{n, b}(\mu) \) is relevant if player 2 competes himself, while \( \tilde{\pi}_2^{d, b}(\mu) \) is relevant if he hires an uninformed delegate. Tened calculations show that \( \tilde{\pi}_2^{d, b}(\mu) > \tilde{\Pi}_2^{n, b}(\mu) \) if and only if \( g(\mu) > 0 \), where \( g(\mu) \equiv \alpha(q)\mu^2 + \beta(q)\mu + \gamma(q) \) (for \( \mu \in \mathbb{R} \)), and

\[
\begin{align*}
\alpha(q) &= \left( \frac{q}{1 + q} \right)^4 \left( 1 - 4q^2(1 + q) \right), \\
\beta(q) &= -2q^2(1 - q) \left( 1 + \left( \frac{q}{1 + q} \right) \right) < 0, \\
\gamma(q) &= (1 - q)^2 > 0.
\end{align*}
\]

Let \( q_2 \approx 0.420 \) be the unique \( q \in (0, 1) \) such that \( \alpha(q) = 0 \), i.e. \( \alpha(q_2) = 0 \). Then \( \alpha(q) > 0 \) if \( q \in (0, q_2) \) and \( \alpha(q) < 0 \) if \( q \in (q_2, 1) \). Note that \( g(0) > 0 \) and \( g(\tilde{\mu}(q)) < 0 \) for \( q \in (0, 1) \).

Lengthy but straightforward manipulations show that \( g(\tilde{\mu}(q)) = 0 \) (with \( q \in (0, 1) \)) can be rewritten as \( \sum_{i=0}^{6} s_i \times q^i = 0 \), with real-valued constants \( s_i > 0 \) for \( i = 0, 1 \).
Lemma 4 Let $g(q)$ see that $q$ in the second subcase we have that $q = \mu(q)$. Using Lemma 2, it thus follows that $q^{**} = q_1$. Notice that $g(\mu(q)) > 0$ for $q \in (0,q_1)$, and $g(\mu(q)) < 0$ for $q \in (q_1,1)$. We now examine three cases.

Take $q \in (0,q_1)$. Then $g(\mu)$ is an upward opening parabola, attaining a negative minimum value for some $\mu = \mu(q) > 0$. We consider two subcases. In the first subcase we have $q \in (0,q_1)$. Using $g(0) > 0$, $g(\mu(q)) < 0$ and $g(\mu(q)) > 0$, part (i) follows directly.

In the second subcase we have $q \in [q_1,q_2)$. Using $g(0) > 0$, $g(\mu(q)) < 0$ and $g(\mu(q)) \leq 0$, part (ii) follows, where $\mu_2(q)$ is implicitly defined by $\tilde{\pi}^{d,b}_2(\mu_2(q)) = \Pi^{n,b}_2(\mu_2(q))$. Note that $\mu_2(q_1) = \tilde{\mu}(q_1)$, while $\mu_2(q) > \mu(q)$ if $q \in (q_1,q_2)$.

Next, take $q = q_2$. Then $g(\mu) = \beta(q_2)\mu + \gamma(q_2)$. Using $g(\mu(q_2)) < 0$ and $\beta(q) < 0$, we see that $g(\mu) < 0$ for $\mu \in [\mu(q),\infty)$. Finally, take $q \in (q_2,1)$. Then $g(\mu)$ is a downward opening parabola attaining a positive maximum value for some $\mu = \mu(q) < 0$. Using $g(0) > 0$ and $g(\mu(q)) < 0$, and combining with the case $q = q_2$, part (iii) follows easily.

\begin{lemma}
Consider the game where player 2 has the option to hire an informed delegate. Let $\mu \in (1,\tilde{\mu}(q))$ for $q \in (0,1)$. We have the following in the equilibrium.

(i) Take $q \in (0,\frac{1}{4}\sqrt{2})$. Then there is a unique $\mu_3(q) \in (1,\tilde{\mu}(q))$ such that player 2 does not hire an informed delegate if $\mu \in (1,\mu_3(q)]$, whereas player 2 hires an informed delegate if $\mu \in (\mu_3(q),\tilde{\mu}(q))$.

(ii) Take $q \in [\frac{1}{4}\sqrt{2},1)$. Then player 2 does not hire an informed delegate.

\end{lemma}

\textbf{Proof.} Take $\mu \in (1,\tilde{\mu}(q))$ for $q \in (0,1)$. Using (3) and (24), $\hat{\Pi}^{d}_2(\mu) > \Pi^{n,a}_2(\mu)$ if and only if $h(\mu) < 4(\sqrt{2} - 1)^2$, where

$$h(\mu) = \frac{(q\sqrt{2} + 1 - q)^2}{\mu q + 1 - q}, \quad \mu \geq 1.$$  

(A.3)
We have (i) \( h(1) = 1 \), (ii) \( h'(\mu) < 0 \) for \( \mu > 1 \), (iii) \( h(\bar{\mu}(q)) = 4q/(3q + 1) \), and (iv) \( \lim_{\mu \to \infty} h(\mu) = q \). The proof follows since \( h(\bar{\mu}(q)) < 4(\sqrt{2} - 1)^2 \) if and only if \( q \in (0, \frac{1}{4}\sqrt{2}) \), while \( \mu_3(q) \) in part (i) is implicitly defined by \( \Pi_2^d(\mu_3(q)) = \Pi_2^{d,a}(\mu_3(q)) \). □

Lemma 5 Consider the game where player 2 has the option to hire an informed delegate. Let \( \mu \in [\bar{\mu}(q), \infty) \) for \( q \in (0, 1) \). We have the following in the equilibrium.

(i) Take \( q \in (0, \frac{1}{4}\sqrt{2}) \). Then player 2 hires an informed delegate.

(ii) Take \( q \in [\frac{1}{4}\sqrt{2}, \frac{1}{4}\sqrt{2}) \). Then there is a unique \( \mu_4(q) \in [\bar{\mu}(q), \infty) \) such that player 2 does not hire an informed delegate if \( \mu \in [\bar{\mu}(q), \mu_4(q)] \), whereas player 2 hires an informed delegate if \( \mu \in (\mu_4(q), \infty) \).

(iii) Take \( q \in [\frac{1}{2}\sqrt{2}, 1) \). Then player 2 will not hire an uninformed delegate.

Proof. Take \( \mu \in [\bar{\mu}(q), \infty) \) for \( q \in (0, 1) \). From (4) and (24), \( \Pi_2^d(\mu) > \Pi_2^{n,b}(\mu) \) if and only if \( m(\mu) > 1 \), where

\[
m(\mu) \equiv \frac{(\sqrt{2} - 1)^2q - \mu q^2}{(1 - q)(1 - (\sqrt{2} - 1)^2)}, \quad \mu \geq 1. \tag{A.4}
\]

We have (i) \( m(\mu) > 0 \) if and only if \( q \in (0, \frac{1}{4}\sqrt{2}) \), (ii) \( m'(\mu) > 0 \) for \( \mu \geq 1 \) if and only if \( q \in (0, \frac{1}{4}\sqrt{2}) \), and (iii) \( m(\bar{\mu}(q)) > 1 \) if and only if \( q \in (0, \frac{1}{4}\sqrt{2}) \). The proof follows directly, by noting that \( \mu_4(q) \) in part (ii) is implicitly defined by \( \Pi_2^d(\mu_4(q)) = \Pi_2^{n,b}(\mu_4(q)) \). □

Proof of Proposition 3

First, take \( \mu \in (1, \bar{\mu}(q)) \). The result then follows from Lemma 1. Second, take \( \mu \in [\bar{\mu}(q), \bar{\mu}(q)) \). Then \( \Pi_2^d(\mu) \) is relevant if player 2 hires an informed delegate, while \( \tilde{\pi}_2^{d,a}(\mu) \) is relevant if he hires an uninformed delegate. Using \( \mu > 1 \), we see that \( \Pi_2^d(\mu) > \tilde{\pi}_2^{d,a}(\mu) \) if \( \mu \in [\bar{\mu}(q), \bar{\mu}(q)) \). So, player 2 will not hire an uninformed delegate in this case.

Third, take \( \mu \in [\bar{\mu}(q), \infty) \). Then \( \Pi_2^d(\mu) \) is relevant if player 2 hires an informed delegate and \( \tilde{\pi}_2^{d,b}(\mu) \) is relevant if he hires an uninformed delegate. We see that \( \Pi_2^d(\mu) > \tilde{\pi}_2^{d,b}(\mu) \) if and only if

\[
(q + 1 - (\sqrt{2} - 1)^2q)(\mu q + 1 - q) + \mu q \leq 4\mu q(1 + q)(\mu q + 1 - q). \tag{A.5}
\]
Tedious calculations show that (A.5) can be rewritten as $n(\mu) < 0$, where $n(\mu) \equiv A(q)\mu^2 + B(q)\mu + C(q)$ (for $\mu \geq 0$), and

\[
\begin{align*}
A(q) &= q^4 \left(1 - (\sqrt{2} - 1)^2\right)^2 - 4q^3(\sqrt{2} - 1)^2 < 0, \\
B(q) &= 2q^2(1 - q) \left(1 + q - 2q(\sqrt{2} - 1)^2 - 3(\sqrt{2} - 1)^2 + q(\sqrt{2} - 1)^4\right), \\
C(q) &= \left(1 + q - q(\sqrt{2} - 1)^2\right)^2 (1 - q)^2 > 0.
\end{align*}
\] (A.6)

Note that $n(\mu)$ is a downward opening parabola with $n(0) > 0$. Lengthy calculations show that $n(\bar{\mu}(q)) < 0$ for all $q \in (0, 1)$. Using this, $n(\mu) < 0$ for $\mu \in [\bar{\mu}(q), \infty)$. The result follows directly. ■
References


