Public debt competition and policy coordination

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Abstract
This paper analyzes the consequences of debt policies in a two-period/two-country model. Whether or not public debt competition results in less efficient resource allocation between private and public consumption than under policy cooperation is ambiguous a priori. The greater debt will result in smaller public goods provision and smaller private capital, although deficit-financed lump-sum transfers benefit agents through greater current consumption. Assuming log-linear utility and Cobb-Douglas production functions, we can show that public debt competition will lead to greater public debts and greater marginal rates of substitution between public and private consumption in each country than under policy cooperation.

Keywords: fiscal competition; public debt; international capital mobility
JEL classification: E62; F34; H63

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1. Introduction

Since the world financial crisis in 2007 to 2008, most of the developed countries have increased their public debt/GDP ratios simultaneously. Even those of most EU countries have far exceeded the precondition of at most 60% of the debt/GDP ratio for joining the European Monetary Union according to the Maastricht Treaty (see Figure 1).\(^1\) This is in contrast to the relatively stationary public debt/GDP ratios of the countries before the crisis. Since the debt/GDP ratios have risen in many, but not all, developed countries simultaneously, there has seemed to be no agreement on public debt policy between countries in response to the crisis.\(^2\) These facts raise an intriguing question: If the countries start to increase their public debts independently, were the levels of public debt greater than, equal to or smaller than the ones which would be obtained under policy coordination?

Governments may try to increase the social welfare by issuing public bonds and increasing current income of the domestic agents if public debt is not neutral in the Barro’s (1974) sense. Roubini and Sachs (1989) pointed out that Barro’s hypothesis cannot be applied to European and other countries, and they emphasized the importance of a political factor, that is, the degree of cohesion in the government.\(^3\) It has been nowadays well perceived that national fiscal deficits create negative externalities to other nations, for example, through higher international interest rates, thereby raising the cost of debt finance for many other governments (e.g., Huizinga and Nielsen, 1998; Blanchard and Summers, 1984). These government behaviors may lead to the so-called public debt competition. Chang (1990) showed that the international externalities associated with public debt competition between countries result in inefficiently large fiscal deficits, although assuming away physical capital stocks. More recently, Persson and Tabellini (1995) showed in a two-country model that policy cooperation leaves both countries better off, calling for a larger surplus than non-cooperative policymaking, and also indicated that under certain conditions, both

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1 In 2005, the Stability and Growth Pact was revised so as to be flexibly applicable and, therefore, the EU countries have been admitted to have the budget deficit/GDP ratio higher than 3% after the revision.
2 In fact, the primary balances of countries such as France, Germany, U.S. and Canada showed a surplus in 2000, maybe as a result of reducing public debt in the 1990s. Norway and Sweden are exceptions among these countries.
3 However, Edin and Ohlsson (1991) showed that the result of the greatest budget deficits under multiparty coalition governments found by Roubini and Sachs (1989) is mainly due to minority government, whereas De Haan and Sturm (1997) did not find even this fact later. Woo (2003) obtained the results similar to those in De Haan and Strum (1997) in a comprehensive test using a panel of 57 countries over the period from 1970 to 1990.
countries may have an incentive to even unilaterally deviate to their Nash strategies.\footnote{Fiscal coordination in a monetary union has also been discussed (see, for example, Levine and Brociner, 1994; Beetsma and Jensen, 2005; Okano, 2014).}

Assuming a two-period/two-country model, the present paper analyzes the consequence of public debt competition on the debt levels and compares them with those of policy coordination between countries. Although the marginal cost of public consumption is shown to be always higher in Nash competition than under policy cooperation at any public debt level, the marginal rate of substitution between public and private consumption, i.e., the marginal benefit of public consumption, may not be greater under cooperative optimum, as is well known in the literature. By specifying the utility and production functions, it is shown that public debt (Nash) competition tends to lead countries to more excessive public debt than under the policy coordination. The greater debt will result in a smaller public goods provision in the future, although greater current consumption benefits agents through deficit-financed lump-sum transfers.

The next section introduces the model, which is similar to those in Wildasin’s (1988) and Yakita’s (2014) fiscal competition models. The consequences of Nash competition and policy coordination will be examined and compared in Section 3. The final section provides remarks.

2. Model

There are two countries in the world. Each country is assumed to be inhabited by a single immobile household that has some unspecified region-fixed factors, while capital moves freely across the border. We also assume that the time horizon of the world is divided into two periods, the first being the “present” and the second the “future.” The initial resource endowment in each country at the beginning of the first period is denoted as $R^i (i=1,2)$. The initial resources are held by the households in each country. The household allocates the initial disposable resources between present consumption and savings for the future. The production of private goods is undertaken only in the second period, using homogeneous capital and the country-fixed factors. The government in each country issues public bonds to finance lump-sum transfers to the domestic household in the first period, and provides public goods and repays its debt, financing them by unit taxes on capital, in the second period. Since the interest on public debt is paid at the market interest rate, the aggregate savings in the world are equalized to the sum of the aggregate capital and public debt in the world as a whole in
equilibrium. Capital is paid its marginal product and the remainder is paid to the local residence as returns on the fixed factors. Focusing on the debt policy of each government, we assume that the tax rates in the second period are declared with the government’s commitment in the first period.

2.1 Households

Optimization of the household in country \( i \) can be written as

\[
\max_{c_i^1, c_i^2} u_i'(c_i^1) + \rho [u_2'(c_i^1) + \nu'(g^i)]
\]

subject to \( R^i + z^i = c_i^1 + s^i \) and \( c_i^2 = rs^i + \pi^i \),

where \( u_i'(c_i^1) + \rho [u_2'(c_i^1) + \nu'(g^i)] \) is the intertemporal utility function of the household in country \( i \), \( c_h^i \) is consumption of country \( i \) in \( h \)-th period \((h = 1, 2)\), \( g^i \) is public goods provided in the second period, \( s^i \) denotes savings in country \( i \), \( z^i \) denotes lump-sum transfer from the government in country \( i \) in the second period, \( r \) is the worldwide return rate on savings (i.e., one plus the interest rate) in the second period, and \( \pi^i \) stands for the return on the fixed factors as profits from the production in country \( i \). \( \rho \) denotes the time preference factor, \( 0 < \rho < 1 \).

The first-order condition for the utility maximization gives the following condition:

\[
-u_i''(c_i^1) + \rho u_2''(c_i^1) + \nu''(g^i) = 0.
\]

The optimal saving plan can be written as

\[
s^i = s^i (r, \pi^i, z^i) \text{ where } s^i_r > 0, s^i_\pi < 0 \text{ and } s^i_z > 0.
\]

2.2 Production

Profit maximization of the private goods producer in country \( i \) can be written as

\[
\max_k \pi^i \equiv f_i(k^i) - (r + t^i)k^i
\]

where the production function \( f_i(k^i) \) is strictly concave in capital stock \( k^i \). \( (r + t^i)k^i \) is the user cost of capital and \( t^i \) denotes the unit tax on capital in country \( i \) in the second period \((0 < t^i < 1)\). The first-order condition for profit maximization is:

\[
f_i'(k^i) - (r + t^i) = 0.
\]

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5 The subscript represents the partial derivative with respect to the variable, e.g., \( s^i_r = \partial s^i / \partial r \).
The optimal demand plan for capital is given as:

\[ k^i = k^i(r, t^i). \]  

(7)

Thus, the maximized profit is written as:

\[ \pi^i = f_i(k^i(r, t^i)) - k^i(r, t^i) \cdot f_i'(k^i(r, t^i)) \equiv \pi^i(k^i(r, t^i)) \]  

(8)

where \( \pi^i(k^i) = -k^i f_i'' > 0. \)

2.3 Government

Each government chooses debt policy to maximize the welfare of its own country’s inhabitant, declaring the tax rate to be adopted in the second period. The sum of the repayment of the public bonds issued in the first period and public goods should be financed by the tax revenue. Therefore, public debt policy will affect the public goods provision in the second period, depending upon changes in the worldwide interest rate. In other words, each government may choose its own debt policy in response to changes in the other country’s debt policy. Focusing on the debt policy, we assume that governments do not provide public goods in the first period and that governments set the capital tax rates at certain levels \((t^1, t^2).^6\)

The budget constraint of the government in country \(i\) in the first period is

\[ z^i = b^i \]  

(9)

where \(b^i\) is the amount of public bond issue. The second-period budget constraint can be written as:

\[ g^i = t^i k^i - rb^i. \]  

(10)

For a given tax rate, since the government can independently choose only one of the two policy variables, the amount of deficit-financed income transfer \((z^i)\) and public goods provision \((g^i)\), the public goods provision is determined so as to satisfy the budget constraints when public debt is chosen by the government.

2.4 Capital market equilibrium

The equilibrium condition in the world capital market can be written as:

\[ s^1 + s^2 = k^1 + k^2 + b^1 + b^2. \]  

(11)

The total savings must be equal to the sum of capital (investments) and public debts of both countries. Savings and investment are not necessarily equal in each country and capital stock in a country may generally be partly owned by the foreign household.

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^6 Yakita (2014) analyzed tax competition in a similar model, assuming an endogenous savings-capital formation. However, he did not consider public debt.
2.5 Equilibrium of the world

When the governments choose debt policy \((b^1, b^2)\), we have a market equilibrium satisfying the following conditions:

\[
f_i'(k^i) - (r + i') = 0 \quad (i = 1, 2)
\]

\[
s^1(r, \pi^1(k^1), b^1) + s^2(r, \pi^2(k^2), b^2) = k^1 + k^2 + b^1 + b^2.
\]

The endogenous variables are \(k^i\) and \(r\) for policy variables \(b^i\) \((i = 1, 2)\), i.e., \(k^i(b^1, b^2)\) and \(r(b^1, b^2)\). Public goods provisions are also determined endogenously as a function of \(b^1\) and \(b^2\) so as to satisfy the budget constraint (10). From (12) and (13), we obtain

\[
\frac{\partial k^i}{\partial b^i} = D^{-1}(1 - s'_z)f_j''
\]

\[
\frac{\partial k^j}{\partial b^i} = D^{-1}(1 - s'_z)f_i''
\]

\[
\frac{\partial r}{\partial b^i} = D^{-1}(1 - s'_z)f''_i f''_j
\]

where \(D \equiv (s'_i + s'_j)f_i''f_j'' - (1 - s'_z\pi')f_j'' - (1 - s'_z\pi')f_i''\) \((i, j = 1, 2; i \neq j)\). For the stability of the market, we assume that \(D > 0\), with which we have \(\partial k^i / \partial b^i < 0\) and \(\partial r / \partial b^i > 0\); otherwise, more debt induces more capital inflows.

3. Nash equilibrium and cooperative debt policy

In this section we first analyze the Nash equilibrium in debt competition and the optimal cooperative debt policy between the countries and then compare them from the viewpoint of whether or not the world agreement on debt policy decisions between countries is desirable for the future welfare of the countries.8

3.1 Nash equilibrium

The government in country \(i\) chooses debt policy so as to maximize the intertemporal welfare of the household in its own country, taking the debt policy in the

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7 From (10), \(g^i = t^i k^i(b^1, b^2) - r(b^1, b^2)b^i = g^i(b^1, b^2)\) for \(i = 1, 2\).

8 The present model assumes that the time horizons of households and governments are the same as that of the economy, i.e., a dynasty-type model. Therefore, the social objective function consists of present and future welfare.
other country \( j \), \( b^j \), as given. The maximization problem for the government in
country \( i \) can be written as:

\[
\begin{align*}
Max_{b^i} & \quad u'_i(R^i + b^i - s^i(r, \pi^i(k^i), b^i)) \\
& \quad + \rho[u^i_z(rs^i(r, \pi^i(k^i), b^i) + \pi^i(k^i)) + v^i(t^i k^i - r b^i)]
\end{align*}
\]

(17)

for given \( b^j \) (and \( t^j \)). From the first-order condition and making use of the habitant’s
utility maximization condition, we obtain the following condition:

\[
\frac{\nu_{i^i}}{u^i_2} = \frac{r + s^i \frac{\partial r}{\partial b^i} + \pi^i \frac{\partial k^i}{\partial b^i}}{r - t^i \frac{\partial k^i}{\partial b^i} + b^i \frac{\partial r}{\partial b^i}} \quad (i = 1, 2)
\]

(18)

where the left-hand side is the marginal rate of substitution between public goods and
second-period consumption, i.e., the marginal benefit from an additional supply of
public goods in terms of the second-period consumption, and the right-hand side is the
marginal cost. From (14) and (16), we can show the denominator on the right-hand
side is greater than \( r \).

For the sake of expositional simplicity we assume that the countries are symmetric
in all aspects, setting \( R^i = R^j, b^i = b^j, s^i = s^j, t^i = t^j (\equiv t) \) and \( k^i = k^j \) \((i, j = 1, 2; i \neq j)\). In a symmetric equilibrium in which no capital moves between countries, i.e.,
\( s^i = k^i \) \((i = 1, 2)\), we can show that

\[
s^i(\frac{\partial r}{\partial b^i}) + \pi^i(\frac{\partial k^i}{\partial b^i}) = 0.
\]

(19)

Making use of (19), (18) can be rewritten as:

\[
\frac{\nu_{i^i}}{u^i_2} = \frac{r}{[r - t^i \frac{\partial k^i}{\partial b^i} + b^i \frac{\partial r}{\partial b^i}]} < 1.
\]

(20)

The condition (20) implies that public goods are excessively supplied relative to private
(second-period) consumption, since, as is well known, the first-best condition requires
\( \nu^i / u^i_2 = 1 \), i.e., the Samuelson rule. This can be interpreted as follows. Each
government wants to take benefits by choosing greater public debt since it raises the
interest burden in Nash competition less than that would be in the closed economy.
The higher interest rate tends to increase the repayment cost of public debt in the
second period, reducing public expenditure. On the other hand, public debt crowds out
private capital formation and thereby decreases profits to the country’s fixed-factors,
reducing private second-period consumption. Condition (20) shows that the
deficit-financed transfers in the first period decreases private second-period
consumption more than public goods provision in the presence of unit taxation on capital.
3.2 Policy coordination

Next, we examine the debt policy under an international agreement on debt policy (hereafter, termed "cooperative debt policy"). When debt policy are coordinated to be the same between symmetric countries, home savings will equal foreign savings: that is, all home savings will be invested in the home country, and all the foreign savings will be invested in the foreign country, respectively. Thus, following Kehoe (1989), the social welfare maximization problem of governments resembles that of a government in a closed economy. Therefore, in examining the cooperative debt policy, we consider the optimal debt policy in a closed economy, where savings are equal to capital formation, i.e., \( s(r, \pi(k), b) = k \), while the government budget constraints in the two periods are \( z = b \) and \( g = t k - rb \), respectively. The socially optimal condition is still given by (18). However, in the closed optimum, the effects of a change in debt policy on capital and the return rate are given as:

\[
\frac{\partial k}{\partial b} = \tilde{D}^{-1}(1 - s_z) \\
\frac{\partial r}{\partial b} = \tilde{D}^{-1}(1 - s_z) f''
\]

where \( \tilde{D} \equiv s_z f'' - (1 - s_z \pi') \). From a reason similar to that in the previous case of open economies, we assume that \( \tilde{D} < 0 \). Then, we have \( \partial k / \partial b < 0 \) and \( \partial r / \partial b > 0 \). Since we can also show that \( s(\partial r / \partial b) + \pi'(\partial k / \partial b) = 0 \), the optimum condition can be written as:

\[
\frac{u'}{u''} = r / [r - t \frac{\partial k}{\partial b} + b \frac{\partial r}{\partial b}] < 1.
\]

That is, even in the closed economy case, the government will not achieve the first best optimum. This is because public goods are financed by the revenue from distortional unit capital taxation, less the cost of repayment of public debt. That is, public debt is not neutral. Therefore, a change in public debt alters the optimal intertemporal resource allocation.

3.3 Comparison between Nash equilibrium and cooperative optimum and the policy implications

Now we compare the levels of public debt in the Nash equilibrium and cooperative optimum. As can be seen from (14)-(16) and (21)-(22), the macroeconomic variables take different values in the two situations. Therefore, we examine whether the
cooperative optimum level of public debt is greater than, equal to or smaller than the
level at the Nash equilibrium by evaluating the Nash equilibrium condition in terms of
the values at the cooperative optimum. Attaching the subscripts $\text{OpenNash}$ for the
Nash equilibrium and $\text{Cooperation}$ for Cooperative optimum, respectively, and
making use of (14)-(16) and (21)-(22), we can show that

$$\frac{v_i^{1/1}}{u_i^{1/1}} \bigg|_{\text{Cooperation}} = \frac{r}{r + (b' \frac{\partial r}{\partial b'} - t \frac{\partial k^i}{\partial b'})_{\text{Cooperation}}} > \frac{r}{r + (b' \frac{\partial r}{\partial b'} - t \frac{\partial k^i}{\partial b'})_{\text{OpenNash}}}$$

where both sides are evaluated by the cooperative optimum values (see Appendix A).
Therefore, the Nash equilibrium and the cooperative optimum do not coincide. The
marginal cost of public goods in terms of private second-period consumption in the Nash
equilibrium is always lower than under the cooperation for any $b^j$ ($i \neq j$). Since the
marginal cost should be equal to the marginal benefit (i.e., the marginal rate of
substitution between private and public consumptions), the inequality seems to imply
that $1 > \frac{v_i^{1/1}}{u_i^{1/1}} \bigg|_{\text{Cooperation}} > \frac{v_i^{1/1}}{u_i^{1/1}} \bigg|_{\text{OpenNash}}$ holds. However, it is not necessarily
the case since the marginal benefit and cost of the Nash competition will not coincide at
the cooperative optimum. As is well known, we cannot say a priori whether the level of
public debt in the Nash equilibrium is greater than, equal to, or smaller than the
optimum level under cooperative debt policy (e.g., Atkinson and Stern, 1974). Public
debt crowds out private capital formation, raising the interest rate. The increased
interest rate will be smaller in an open Nash equilibrium than under the cooperation
(see (16) and (22)). When $s^i_\gamma$ is not great (or negatively large), the negative effect of
the higher interest rate on capital formation through changes in savings will be greater
under the cooperation. This in turn tends to raise the interest rate more. While the
higher interest rate affects positively private second-period consumption, the smaller
private capital reduces the second-period consumption through changes in profits. On
the other hand, the high interest rate together with the reduced private capital
decreases public goods provision. Thus, whether the marginal rate of substitution
between public and private consumption in a Nash equilibrium is greater than, equal to
or smaller than that under policy cooperation depends on the respective effect of debt
issue on the public and private consumptions in the respective situation.

Thus, we next examine the levels of public debt in Nash equilibrium and under
cooperation by specifying the utility function as a log-linear function of lifecycle consumption and public goods, i.e., \( \ln c_1 + \rho[\ln c_2 + \varepsilon \ln g] \) where \( \rho \) denotes the time discount factor, and the production function as a Cobb-Douglas production function, i.e., 
\[ f(k) = Ak^\alpha \] where \( 0 < \alpha < 1 \). With these specifications, we obtain the Nash equilibrium condition:
\[
\frac{\varepsilon \rho}{1 + \rho} \frac{r(R + b) + \pi}{tk - rb} = 1/\left[1 + \frac{t - bf^n}{2[(1 + \rho)r - \pi']}ight]
\] (25a)
and the cooperative optimum condition:
\[
\frac{\varepsilon \rho}{1 + \rho} \frac{r(R + b) + \pi}{tk - rb} = 1/\left[1 + \frac{t - bf^n}{[(1 + \rho)r - \pi']}ight]
\] (25b)
where \( \pi = (1 - \alpha)Ak^\alpha \) and \( r = \alpha Ak^{\alpha - 1} - t \). From these conditions, we can obtain the following result:

Proposition Assume a log-linear utility function of private and public consumption and a Cobb-Douglas production function. In a Nash equilibrium each government will have greater public debt and provide less public goods than under a cooperative debt policy.

Proof: See Appendix B. □

Since public debt crowds out private capital formation, thereby reducing the tax revenue, and the repayment of the debt reduces the proportion of the tax revenue spent on public goods provision, a greater public debt leads to relatively less public goods in a Nash equilibrium. Therefore, as can be seen by differentiating the left-hand side of (25) with respect to the public debt level, we can show that the marginal rate of substitution between public and private consumption is greater in a Nash equilibrium than under the cooperative equilibrium in these specifications of functions, 
\[ 1 > u_i' / u_2' \left|_{\text{Nash}} \right. > u_i' / u_2' \left|_{\text{Cooperation}} \right. \]
It should be noted that although the inequality seems to imply that the resource allocation between public and private consumption in the second period is more efficient in the Nash equilibrium, the countries have unit capital taxes which cause distortions in the intertemporal resource allocation, i.e., in the second-best optimum setting. Thus, it does not necessarily imply the welfare of countries is higher in the Nash equilibrium.

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9 Considering the case of symmetric countries, we omit the superscripts.
At this stage we consider a many-country case briefly. We can show that when the number of countries goes to infinity, we will have the Samuelson rule for private and public consumption in the second period in each country in Nash equilibrium (see Appendix B). However, the rule in such a Nash equilibrium will not be the intertemporal worldwide (first-best) optimum since countries will have more public debt when the number of countries increases. In fact, even if the number of countries were to reach infinity, the cooperative optimum would not satisfy the Samuelson rule. It should be noted that this holds even still under more general utility and production functions (see Appendix A). This situation is reminiscent of a prisoner’s dilemma.10

4. Concluding remarks

We have shown that countries may have greater public debts in Nash equilibrium than under policy coordination. This result can be intuitively interpreted as follows. A rise in the interest rate due to an increase in public debt is considered to be lower in Nash equilibrium than under cooperation by each government since the shock will be absorbed by the world capital market, that is, the crowding-out effect is considered to be spread over countries. On the other hand, the crowding out effect in a country will affect the economy under the cooperative policy between symmetric countries as if each economy is closed.

As a final remark, we briefly consider the welfare effect of debt policy. Deficit-financed income transfers in a country do not lower the intertemporal social welfare of the economy. Although the second-period consumption and public goods provision decrease, the first-period consumption will increase. The intertemporal welfare effect depends on the relative magnitudes of these two effects. However, if a country has greater public debt in the present period, the welfare of the economy in the future will be lower. In this sense, Nash public debt competition shifts the heavier burden of deficit-financed expenditure in the present to the future.

Appendix A
From (14) – (16), we have

10 Wildasin (1988) cautioned that symmetric Nash equilibria may not be the only ones possible.
\[
(b^i \frac{\partial r}{\partial b^i} - t \frac{\partial k^i}{\partial b^i})_{\text{Open Nash}} = \frac{(1 - s^i_z) f^i_j (b^i f''_i - t)}{(s^i_r + s^i_r) f^i_j f''_i (1 - s^i_z \pi''_i) (1 - s^i_z \pi''_i) f^i_j} > 0
\]

(A1)

and, from (21) – (22)
\[
(b^i \frac{\partial r}{\partial b^i} - t \frac{\partial k^i}{\partial b^i})_{\text{Cooperation}} = \frac{(1 - s^i_z)(b^i f''_i - t)}{s^i_r f''_i (1 - s^i_z \pi''_i)} > 0.
\]

(A2)

Since both sides of (A1) and (A2) are evaluated at cooperative optimum, we have
\[
(b^i \frac{\partial r}{\partial b^i} - t \frac{\partial k^i}{\partial b^i})_{\text{Cooperation}} - (b^i \frac{\partial r}{\partial b^i} - t \frac{\partial k^i}{\partial b^i})_{\text{Open Nash}} = \frac{(1 - s^i_z)(b^i f''_i - t)}{[s^i_r f''_i (1 - s^i_z \pi''_i)][(s^i_r + s^i_r) f''_i (1 - s^i_z \pi''_i) f''_i (1 - s^i_z \pi''_i) f''_i]}
\]
\[\times f''_i [s^i_r f''_i (1 - s^i_z \pi''_i)] < 0
\]

(A3)

where we assume that \( D > 0 \) and \( \tilde{D} < 0 \) always hold. In a symmetric country case, (A3) becomes
\[
(b^i \frac{\partial r}{\partial b^i} - t \frac{\partial k^i}{\partial b^i})_{\text{Cooperation}} - (b^i \frac{\partial r}{\partial b^i} - t \frac{\partial k^i}{\partial b^i})_{\text{Open Nash}} = \frac{(1 - s^i_z)(b^i f''_i - t)}{2[s^i_r (f''_i)^2 - (1 - s^i_z \pi') (f''_i)]} < 0.
\]

(A3')

If \( s_r \) is positive but sufficiently small or if it is negative, the difference on the right-hand side of (A3') will be great.

Appendix B

First we define a function:
\[
\phi(k, b, n) = \frac{b \alpha (1 - \alpha) A k^{a-2} + t}{n[(1 + \rho)(\alpha Ak^{a-1} - t) - \alpha (1 - \alpha) Ak^{a-1}]} = \phi(k, b, n),
\]

(A4)

which is the second term of the denominator on the right-hand side of (25a) when \( n = 2 \) and that of (25b) when \( n = 1 \). Then the system of the equations
\[
\frac{\varepsilon \rho}{1 + \rho} \frac{r(R + b) + \pi}{tk - rb} = \frac{1}{1 + \phi(k, b, n)}
\]

(A5)
\[
s(r, \pi(k), b) = k
\]

(A6)
gives the solution \((k, b)\) for a given \( n \) (\( n = 1, 2 \)), where
The sign of (A7) is ambiguous. In order to examine the sign of (A7), we examine the effect of an increase in the initial resources on Nash equilibrium capital stock, \( \frac{dR}{dK} \).

From (A5) and (A6) we obtain

\[
\begin{pmatrix}
H_{11} & H_{12} \\
H_{21} & 1
\end{pmatrix}
\begin{pmatrix}
\frac{dK}{dN} \\
\frac{dB}{dN}
\end{pmatrix}
= \begin{pmatrix} B \\ 0 \end{pmatrix}
\] (A10)

where

\[
H_{11} = \frac{\varepsilon \rho}{1 + \rho (tk - rb)^2} ((tk - rb)(R + b) \frac{dr}{dk} + \pi') - [(R + b) + \pi](t - b \frac{dr}{dk})
\]

\[+ \frac{1}{(1 + \phi)^2} \frac{\partial \phi}{\partial k} \] (A11)

\[
H_{12} = (tk - rb) + [(R + b) + \pi] r + \frac{1}{(1 + \phi)^2} \frac{\partial \phi}{\partial b} > 0
\] (A12)

\[
B = -\frac{\varepsilon \rho}{1 + \rho tk - rb} < 0
\] (A13)

\[
H_{21} = \alpha(1 - \alpha)Ak^{\alpha - 1} + (1 + \rho) > 0.
\] (A14)

From (A10) and assuming \( H \equiv H_{11} - H_{12} \cdot H_{21} \neq 0 \), we obtain

\[
\frac{dK}{dR} = H^{-1} \cdot B.
\] (A15)

It is plausible that an increase in the initial resources increases the equilibrium capital both in Nash equilibrium and in cooperative optimum (i.e., for \( n = 1, 2 \)). Assuming this case and taking (A13) into consideration, we can assume that

\[
H < 0.
\] (A16)

From (A5) and (A6), we have

\[
\begin{pmatrix}
H_{11} & H_{12} \\
H_{21} & 1
\end{pmatrix}
\begin{pmatrix}
\frac{dK}{dN} \\
\frac{dB}{dN}
\end{pmatrix}
= \begin{pmatrix} H_{13} \\ 0 \end{pmatrix}
\] (A17)

where

\[
H_{13} = 1 - \frac{1}{(1 + \phi)^2} \frac{\partial \phi}{\partial n} > 0.
\] (A18)
Therefore, we obtain
\[
\frac{db}{dn} = H^{-1}(H_{12} \cdot H_{21}) > 0 \quad \text{(A19)}
\]
\[
\frac{dk}{dn} = H^{-1}(H_{11}) < 0 \quad \text{(A20)}
\]

From (A19) and (A20), public debt is greater in a Nash equilibrium than under policy cooperation while private capital formation is smaller in the Nash equilibrium.

The variable \( n \) in (A4) can be seen as the number of countries in the case of a many-country Nash equilibrium as can be seen from the analyses in Wildasin (1988) and Yakita (2014). Therefore, the result (A19) implies that as the number of countries increases in Nash equilibrium, the public debt of each country will be greater.
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References
Figure 1. Public debt/GDP ratios

Source: OECD (2012) OECD Economic Outlook No. 91: Statistics and Projections