Equality of opportunity, social mobility and inequality*

Montserrat Vilalta-Bufi†

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†Department of Economic Theory and CREB, University of Barcelona, Avinguda Diagonal 696, 08034 Barcelona, Spain. Email: montsevilalta@ub.edu
Abstract

I study social mobility, equality of opportunity and income inequality in a model where parental background matters in education and the labor market. My framework allows for the evaluation of policies on social segregation, early intervention programs and the power of unions among others. Results show the relationship between social mobility, income inequality, overeducation and equality of opportunity under each scenario. I calibrate the model to the US economy and perform counter-factual exercises.

Keywords: Social mobility; Equality of opportunities; Inequality; Intergenerational cultural transmission; Overeducation

JEL Codes: J21, J24, J62, I24.
1 Introduction

Social mobility and equality of opportunity are generally considered ethically good objectives for any society. However, although they are closely related, they have mostly been studied separately\(^1\). On the one hand, there is a vast literature on intergenerational mobility (see reviews of the empirical literature in Solon [1999], Bjorklund and Jantti [2009], Black and Devereux [2011], Corak [2013]). Social mobility is often studied in relationship to the process of development, wealth distribution, inequality and economic growth. These variables are usually linked via individuals’ investment decisions in education.

For instance, the co-existence of financial constraints and ability differences is behind most theoretical explanations linking social mobility to income inequality and economic growth [Banerjee and Newman 1993, Becker and Tomes 1986, Loury 1981, Owen and Weil 1998, Maoz and Moav 1999, Solon 2004].\(^2\) While some empirical literature reports a negative relationship between income inequality and intergenerational mobility [Bjorklund and Jantti 1997, Blanden 2013, Corak 2013], other papers point towards an unclear correlation. Checchi et al. [1999] find that Italy has both lower inequality and lower social mobility than the US. Canada and Australia have been found to be highly mobile, but quite unequal (\()\). Solon [2002] finds that the mapping between inequality and intergenerational mobility is not exact.

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\(^1\)An exception is Corak [2006]. He discusses the relationship between the intergenerational elasticity of income and equality of opportunity.

\(^2\)An exception is Galor and Tsiddon [1997], who study the effect of technological progress on intergenerational mobility under the assumption of perfect capital markets. Technological progress increases incentives to invest in education overtime, leading to higher mobility. I differ from them in that I do not include technological progress, but cultural transmission of education and frictions in the labor market.
Several papers propose redistribution policies as the link between inequality and social mobility (Arawatari and Ono 2013, Alonso-Carrera et al. 2012, Hassler et al. 2007). Lefgren et al. 2012, however, estimate that no more than 37 percent of the income persistence between fathers and sons is attributable to the causal effect of financial resources. Therefore, there is room for other mechanisms to explain the persistence in income across generations.

On the other hand, the literature on inequality of opportunity distinguishes between individuals’ circumstances and effort as determinants of their future outcomes (see a review in Roemer and Trannoy 2015). Differences in outcomes due to differences in effort are ethically acceptable, at least to some extent. However, policies equalizing opportunities for different circumstances- as being born in a poor family- are seen as ethically desirable. Then, redistribution policies, for instance, should improve the equality of opportunity in the case of existence of financial constraints for education. The main discussion within this literature is on distinguishing effort from circumstances. Bjorklund et al. 2012, for instance, find that most of the inequality in Sweden comes from differences in effort rather than circumstances.

In this paper I study the relationship between social mobility- and its several measures- with equality of opportunity in a model without financial constraints. I depart from any capital market imperfections story or wealth transmission since these effects have been already widely analyzed in the literature (Becker and Tomes 1986, Galor and Tsiddon 1997, Loury 1981, Maoz and Moav 1999). Instead, I analyze the so called ‘mechanistic persistence’, which refers to the transmission of human capital independently of the level of financial investment. Understanding the transmission mecha-
isms in place is crucial to develop effective policies to improve equality of opportunities and social mobility. Income redistribution might be useful if financial constraints are binding (Bénabou [2002]), but they might be futile otherwise. Mayer [2008], for instance, assumes the presence of heterogeneous abilities which are transmitted across generations, generating a positive correlation between fathers’ and sons’ incomes via self-selection into education. Early childhood intervention programs to improve ability of children from low-income families would then lead to better results than subsidizing college education. I attempt to answer three questions:

- Is there an optimal level of equality of opportunity and social mobility?
- When are social mobility and equality of opportunity negatively (positively) related? Which mechanisms are behind this relationship?
- Does an increase in social mobility always imply more equality of opportunity?

In this paper I propose a stylized economy where agents differ in their circumstances. Individuals decide how much effort to exert in the education of their kids. The outcome of the kid depends on parental effort and other circumstances. Therefore, kids’ effort and their ability do not play a role. I use this economy to compute different measures of inequality, social mobility (upward mobility, downward mobility, intergenerational income elasticity, intergenerational education elasticity) and equality of opportunity. A comparison of different steady states will reveal the relationship between all these measures. Economies might differ in their degree of social segregation, the
power of unions, the importance of networks in education or in the labor market,... My framework allows for the evaluation of these and other public policies. Roemer and Unveren [2012] propose a model in a similar spirit, although with financial resources as the main mechanism driving equality of opportunity. They find that investment in poor households is necessary to improve equality of opportunity.

I set up two initially independent mechanisms in my model. One refers to the educational attainment of the population and the other explains how individuals get allocated to jobs. Educational attainment is done using a cultural transmission mechanism similar to the one proposed in Bisin and Verdier [2001] and Patacchini and Zenou [2011]. Both, direct parental effort in terms of time devoted to children and socialization with neighbors, affect the probability of the kid to obtain high education. Several papers provide empirical evidence on the importance of early parental attention (Heckman et al. [2013], Restuccia and Urrutia [2004], Guryan et al. [2008]) and neighborhood effects (Bayer et al. [2007], Borjas 1992, 1995, Case and Katz 1991, Cutler and Glaeser 1997, O’Regan and Quigley 1996) on kids’ future educational outcomes. Other papers show the influence of parental background on education decisions through the transmission of risk aversion (Brodaty et al. 2014) and cognitive ability (Anger and Heineck 2010). As in Bowles et al. 2014, I introduce different levels of segregation in the economy that will influence the transmission of education. Cutler and Glaeser 1997 show that segregation has a negative effect on schooling, employment and single parenthood for black individuals.

The second mechanism in the model explains how individuals get allo-
cated to jobs using a simple matching model. Therefore, I combine the cultural transmission of education with the presence of frictions in the labor market. Parental background has also been found to influence the labor outcome of kids either through networks or other (Corak and Piraino 2011, Mulligan 1999, Ioannides and Datcher Loury 2004). In my model, the number of good jobs is endogenously determined as well as the productivity level. Moreover, individuals with parents in a good job are more likely to find a good job than those individuals whose parents have a bad job. The model includes human capital externality since job productivity depends on the level of human capital of the economy. This type of externalities has been found in Moretti 2004.

The paper is organized as follows. In the next section I develop the theoretical model, which comprises of four main parts: the education transmission mechanism, the job allocation process, the individual’s problem and the steady state equilibrium. In section 3, I define the variables of interest: social mobility, equality of opportunity and inequality. Section 4 presents the calibration of the model to the US economy and some comparative statics exercises. Finally, section 5 concludes giving directions for further research.

2 The Model

Consider an overlapping-generations model where each individual lives for two periods. In the first period, individuals receive with some probability an education level, which can be low or high. At the beginning of the second period, individuals look for a job, which can be good or bad. Their level of
education and their parental background limits their labor opportunities. In their second period, they decide how much time to work and how much of their leisure time to devote to their only child. The amount of time they spend with their kid affects the probability of their child to achieve high education. Individuals get utility from their consumption when adults, their leisure time and the expected future income/consumption of their kids. For convenience, I denote each generation by the time period when they are adult.

Within each adult generation there are potentially four types of individuals depending on their level of education (low or high, \( i \in \{L, H\} \)) and their job type (bad or good, \( j \in \{B, G\} \)). Let \( \mu_{i,j}^t \) be the adult population measure of educational type \( i \) working in type \( j \) job for generation \( t \). I assume that all individuals with low education are employed in bad jobs (\( \mu_{L}^{LG} = 0 \)), while individuals with high education may be employed in good or bad jobs. Moreover, I normalize the generation size to 1. Hence, \( 1 = \mu_{H}^{HG} + \mu_{H}^{HB} + \mu_{L}^{LB} \).

Let me now define in detail the education transmission mechanism and the job allocation process of this economy.

### 2.1 Education Transmission

Parents decide how much time to work, how much time to have leisure and how much time to devote for their kids.

Parents transmit their preferences for education to their children a la Bisin and Verdier [2001]. All parents prefer a high education level for their kids. They exert, however, different effort in transmitting this preference.
Parents’ effort is the amount of leisure time they devote to their children \((d_{ij}^t \in [0,1])\). This direct effort increases the probability to transmit high education to their kids. This is called the vertical transmission of education. If they fail to transmit their preferences in this fashion, then the kid might still get high education from the rest of the society (horizontal transmission). The idea is that the interaction with people outside the family also affects the transmission of education.\(^3\)

I allow for different levels of segregation in the economy. I consider the job type of the parent to represent a network type (bad or good, \(j \in \{B,G\}\)). In societies with perfect segregation, kids of parents from the good network do not interact with parents from the bad network and vice versa. In this case, the horizontal socialization occurs strictly within networks. In contrast, if there is no segregation, all kids interact equally with all parents, independently of their network, and horizontal socialization occurs at the economy level. Intermediate levels of segregation allow for different degrees of horizontal socialization across networks. The probability to achieve high education from horizontal socialization is larger the higher the probability to interact with someone with high education.

Let \(\rho_j^t\) indicate the probability to meet an individual with high education if you are in network \(j\). This depends on the proportion of individuals with high education in the population \((\alpha_t = \mu_t^{HB} + \mu_t^{HG})\), the proportion of individuals with high education in each network \((\alpha_t^B = \frac{\mu_t^{HB}}{\mu_t^{HB} + \mu_t^{LB}}\) and \(\alpha_t^G = 1\), and the

\(^3\)Patacchini and Zenou [2011] propose a similar education transmission.
level of segregation of the economy ($I_S \in [0, 1]$).

$$\rho^I_t = \alpha_t(1 - I_S) + \alpha^I_t I_S.$$  \hspace{1cm} (1)

In the case of perfect segregation across networks, $I_S = 1$ and then only the proportion of highly educated individuals in the network matters for socialization. In contrast, for a non-segregated society, the index of segregation of the country $I_S$ is zero and $\rho^I_t$ corresponds to the proportion of highly educated individuals in the whole population. As mentioned above, I also allow for intermediate levels of segregation, $I_S \in [0, 1]$.

The probability of vertical transmission, $f^i(d^i_{jt}, \rho^I_{jt})$, is an increasing and concave function of parental effort $d^i_{jt}$. That is, $f^i_1 > 0$ and $f^i_{11} < 0$. I assume that it also depends positively on the neighborhood quality. Peer effects interact with the direct effort of parents in the vertical transmission of education. Vertical transmission requires parental effort ($f^i(0, \rho^I_t) = 0$). Parents with low education are assumed to have lower quality time than parents with high education. They might have more difficulties in helping their kids learning. I model this as $f^L() = \delta f^H()$, with $\delta < 1$.

The probability of horizontal transmission, $g(\rho^I_t)$, is an increasing function of the probability to interact with someone with high education ($g' > 0$). In contrast to vertical transmission, it is independent of parental effort. Moreover, oblique socialization is never successful with certainty, $g(1) < 1$. This way there is always an open downward mobility channel\footnote{This condition is necessary to obtain a stable equilibrium. See appendix.}.

\hspace{1cm}
Then the probabilities of education transition are the following:

\[ \pi_{ij}^{t+1} = f^i(d_i^j, \rho_i^t) + (1 - f^i(d_i^j, \rho_i^t))g(\rho_i^t), \quad (2) \]

\[ 1 - \pi_{ij}^{t+1} = (1 - f^i(d_i^j, \rho_i^t))(1 - g(\rho_i^t)). \quad (3) \]

\( \pi_{ij}^{t+1} \) is the probability of a parent with education \( i \) and network \( j \) to transmit his/her preference for high education to his/her kid. It has two components, the probability of success in the vertical transmission plus the probability of success in the horizontal transmission if direct transmission fails. \( 1 - \pi_{ij}^{t+1} \) represents the probability of a parent with education \( i \) and network \( j \) to fail to transmit high education to the kid. This happens in the event of failure in vertical and horizontal transmission.

Given these transition probabilities, I can find the measure of young individuals for each type of education and network. Let \( \gamma_{ij}^{t+1} \) be the measure of young individuals with education type \( i \) and parental background type \( j \) just before entering the labor market. Parental background refers to the job type of their parents, which in my framework coincides with their network type.

\[ \gamma_{t+1}^L = (1 - \pi_{t+1}^{LB}) \mu_t^{LB} + (1 - \pi_{t+1}^{HB}) \mu_t^{HB} + (1 - \pi_{t+1}^{HG}) \mu_t^{HG}, \]

\[ \gamma_{t+1}^{HB} = \pi_{t+1}^{LB} \mu_t^{LB} + \pi_{t+1}^{HB} \mu_t^{HB}, \]

\[ \gamma_{t+1}^{HG} = \pi_{t+1}^{HG} \mu_t^{HG}. \]
or in matrix notation:

\[
\begin{bmatrix}
\gamma_{L_t+1} \\
\gamma_{HB_t+1} \\
\gamma_{HG_t+1}
\end{bmatrix} =
\begin{bmatrix}
1 - \pi_{t+1}^{LB} & 1 - \pi_{t+1}^{HB} & 1 - \pi_{t+1}^{HG} \\
\pi_{t+1}^{LB} & \pi_{t+1}^{HB} & 0 \\
0 & 0 & \pi_{t+1}^{HG}
\end{bmatrix}
\begin{bmatrix}
\mu_{t+1}^{LB} \\
\mu_{t+1}^{HB} \\
\mu_{t+1}^{HG}
\end{bmatrix}
\]  

Once the education transmission is completed I observe three types of children. Next I study how these young individuals, given their education and network types, are allocated to jobs.

### 2.2 Job allocation

Let me now describe the labor demand in this economy. As already mentioned above, there are two types of job: good and bad. Both types of job produce the same final good, which is consumed by individuals. A job exists when a suitable worker fills a vacancy created by a firm. Each firm can open only one vacancy. The vacancy cost is \( \kappa > 0 \) for a good job and 0 for a bad job. Firms are expected profit maximizers and there is free entry. Whereas there are search frictions in the market for good jobs, the market for bad jobs is assumed perfectly competitive. Therefore, everybody can instantly find a bad job and there are as many bad jobs as required. On the other hand, individuals must search in order to find a good vacancy. The total number of good jobs created is determined by a matching function \( m(u_t, v_t) \), where \( v_t \) denotes the number of good vacancies opened and \( u_t \) denotes the total efficiency units of search devoted to the good job market (similar to

\[\text{Notice that there is no unemployment in this economy.}\]
The maximum efficiency units of search for a good job that an individual can use include networks (private information and contacts) and formal channels (employment agencies, newspaper adds, internet search, etc.). I assume that individuals’ network channel is directly linked to their parental occupation, while the formal channels are the same for everyone. Young individuals whose parents are in good jobs have better information and contacts than young individuals with parents in bad jobs. Therefore, the former are endowed with a larger fraction of efficiency units of search than the latter. In other words, each parental occupation is associated with different endowments of efficiency units of search for the child; $S$ for those with parents in job type $G$, and $s$ for those with parents in job type $B$, with $1 \geq S > s > 0$. I normalize $S = 1$. Therefore, the total efficiency units of search in the good jobs market is $u_t = \gamma^G_t + s \gamma^B_t$.

I assume that the matching function for the good job, $m(u_t, v_t)$, is increasing in both arguments, continuous and upper bounded by $\min\{u_t, v_t\}$.[6]

I allow the productivity level of the bad job to vary with the level of education of the individual. Denote the productivity per unit of time obtained in a bad job filled by a low-educated worker by $y^L_t$, the productivity per unit of time of a bad job filled by a highly-educated individual by $y^H_t$ and

[6]Ribó and Vilalta-Clotí 2012 analyze which properties are necessary for particular matching functions to be bounded from above by the number of search units and vacancies.
the production in a good job by $y_t^G$. I assume that:

$$y_t^G = A \alpha_t^e = A \left( \mu_t^{HB} + \mu_t^{HG} \right)^e,$$

(4)

$$y_t^{LB} = B (\alpha_t^B)^e = B \left( \frac{\mu_t^{HB}}{1 - \mu_t^{HG}} \right)^e,$$

(5)

$$y_t^{HB} = (1 + e)y_t^{LB},$$

(6)

where $\epsilon < 1$, $e > 0$ and $A > 0$ and $B > 0$ represent the productivity level in each type of job. Notice that I am assuming that the production of a good job depends on the proportion of highly educated individuals in the whole population, meanwhile production of a bad job depends on the proportion of highly educated people in the bad job holders. I assume that $y_t^G > y_t^{LB} \forall t$. There are several papers that find human capital externalities in the aggregate productivity. Moreover, the empirical literature on overeducation finds that, at the micro level, overeducated individuals are more productive that their matched colleagues.

In this setting, the production of the bad job is given entirely to the worker as a wage. In contrast, in the good job, workers and firms negotiate under a Nash bargaining game. The solution is that each player gets a proportion of the total surplus created by a good match, which is $y_t^G - y_t^{HB}$, plus the outside option. Let workers’ bargaining power be represented by $\beta$. Then the wage per unit of time of a worker employed in a good job is $w_t^G = \beta \left( y_t^G - y_t^{HB} \right) + y_t^{HB}$ and the number of realized matches corresponds to the number of individual with high education in a good job, $\mu_t^{HG} = m(u_t, v_t)$.

\footnote{We need the parameter $A$ to be large enough so that the expected gains from opening a good vacancy covers its cost $\kappa$.}
I assume free entry of firms, and hence, firms will open new vacancies in
the good job market until the expected profit of opening a vacancy equals
its cost.

\[ q_t (1 - \beta) E_{t-1} (y_t^G - y_t^{HB}) = \kappa. \quad (7) \]

where \( q_t = \frac{m(u_t,v_t)}{w_t} \in [0, 1] \) is the probability for a firm of getting a vacancy filled.

Let \( p_t = \frac{m(u_t,v_t)}{u_t} \in [0, 1] \) be the probability per efficiency unit of search to
find a good job. Then, the probability of obtaining a good job conditional
on having high education is \( p_t \) for individuals with parents in a good job and
\( sp_t \) for individuals with parents in a bad job.

The dynamics of the population distribution are defined by:

\[ \mu_{LB}^{t+1} = (1 - \pi_{LB}^{t+1}) \mu_{LB}^t + (1 - \pi_{HB}^{t+1}) \mu_{HB}^t + (1 - \pi_{HG}^{t+1}) \mu_{HG}^t, \quad (8) \]

\[ \mu_{HB}^{t+1} = (1 - sp_{t+1}) \left( \pi_{LB}^{t+1} \mu_{LB}^t + \pi_{HB}^{t+1} \mu_{HB}^t \right) + \pi_{HG}^{t+1} (1 - p_{t+1}) \mu_{HG}^t, \quad (9) \]

\[ \mu_{HG}^{t+1} = sp_{t+1}(\pi_{LB}^{t+1} \mu_{LB}^t + \pi_{HB}^{t+1} \mu_{HB}^t) + \pi_{HG}^{t+1} p_{t+1} \mu_{HG}^t, \quad (10) \]

or in matrix form:

\[
\begin{bmatrix}
\mu_{LB}^{t+1} \\
\mu_{HB}^{t+1} \\
\mu_{HG}^{t+1}
\end{bmatrix} =
\begin{bmatrix}
1 - \pi_{LB}^{t+1} & 1 - \pi_{HB}^{t+1} & 1 - \pi_{HG}^{t+1} \\
\pi_{LB}^{t+1}(1 - sp_{t+1}) & \pi_{HB}^{t+1}(1 - sp_{t+1}) & \pi_{HG}^{t+1}(1 - p_{t+1}) \\
\pi_{LB}^{t+1}sp_{t+1} & \pi_{HB}^{t+1}sp_{t+1} & \pi_{HG}^{t+1}p_{t+1}
\end{bmatrix}
\begin{bmatrix}
\mu_{LB}^{t} \\
\mu_{HB}^{t} \\
\mu_{HG}^{t}
\end{bmatrix}
\]

### 2.3 The individual’s problem

Individuals choose how much time to work, their leisure and their effort in
transmitting their education preferences. They maximize a composed utility
function which consists of two parts: their own utility and the expected productivity of their kids. They derive utility from consumption and leisure.

Each individual is endowed with 1 unit of time and has to decide how much of it will be devoted to work, to leisure for him/herself and how much will be devoted to the kid. The time devoted to the kid corresponds to the direct effort of education transmission.

When parents decide how much time to invest in educating their kids \((d_i^j)\), they have rational expectations on the salaries their kids will get. I am assuming that parents do not take into account the effect of their decision in the aggregate outcome of the economy. Then, the problem that parents of education \(i\), having a job \(j\) have to solve is

\[
\max_{c_i^j, l_i^j, d_i^j} u(c_i^j, l_i^j) + \pi_{i+1} y_i^{HB} + sp_{i+1} \beta(y_{i+1} - y_i^{HB}) + (1 - \pi_{i+1}) y_i^L
\]

subject to equation (2),

\[
c_i^j = w_i^j (1 - l_i^j - d_i^j),
\]

where \(l_i^j\) and \(d_i^j\) denote how much time is devoted to leisure and children respectively, and \(u(c_i^j, l_i^j)\) is increasing in both arguments. Moreover, I assume that \(u_{11} < 0\), \(u_{12} \geq 0\) and \(u_{22} < 0\).

\(^8\)Individuals consume all their earnings in my model because I want to abstract from any monetary transfer to the kids, which has already been widely studied in the literature.
The first order conditions are the following:

\[-w_{ij}^{ij}u_1 + u_2 = 0, \quad (11)\]

\[-w_{ij}^{ij}u_1 + f_1^i(d_{ij}^{ij}, \rho_j^i)(1 - g(\rho_j^i))(y_j^{Hi+1} + s\rho_j^i + y_j^{Hi+1}) - y_j^{Li+1}) = 0, \quad (12)\]

for \(i = \{L, H\}\) and \(j = \{B, G\}\).

The economic interpretation of the first order condition is that the marginal cost of spending time with the kid must be equal to the benefit of spending time with the kid (the utility you get from spending time with the kid plus the increase in expected productivity of your kid).

The Hessian matrix is assumed negative definite.

Using the implicit function theorem I can check whether the vertical and horizontal transmission of education are complements or substitutes.

\[\frac{\partial d_{ij}^{ij}}{\partial \rho_j^i} = \frac{[f_{12}(d_{ij}^{ij}, \rho_j^i)(1 - g(\rho_j^i)) - f_1^i(d_{ij}^{ij}, \rho_j^i)g(\rho_j^i)]E_i(w_j^{Hi+1} - w_j^{Li+1})}{u_{11} - 2u_{12} + u_{22} + f_1^i(d_{ij}^{ij}, \rho_j^i)[1 - g(\rho_j^i)]E_i(w_j^{Hi+1} - w_j^{Li+1})}. \quad (13)\]

Therefore, the direct effort depends on the probability of the kid to interact with someone with high education. Whether \(d_{ij}^{ij}(\rho_j^i)\) is increasing or decreasing in \(\rho_j^i\) will tell us whether parental direct effort and network socialization are complements or substitutes. Patacchini and Zenou [2011] estimate this relationship for the UK and find that neighborhood and parental involvement in kids education are complements.

**Proposition 1.** Direct effort and \(\rho_j^i\) are related in the following manner:

- If \(f_{12}(d_{ij}^{ij}, \rho_j^i)[1 - g(\rho_j^i)] > f_1^i(d_{ij}^{ij}, \rho_j^i)g(\rho_j^i)\), then parental direct effort and network socialization are complements. That is, \(\frac{\partial d_{ij}^{ij}}{\partial \rho_j^i} > 0\).
• If \( f_{12}^i(d_{ij}^t, \rho_j^t)[1 - g(\rho_j^t)] < f_1^i(d_{ij}^t, \rho_j^t)g'(\rho_j^t) \), then parental direct effort and network socialization are substitutes. That is, \( \frac{\partial d}{\partial \rho} < 0 \).

When \( f_{12}^i(d_{ij}^t, \rho_j^t) \leq 0 \), direct effort and network socialization are necessarily substitutes. When \( f_{12}^i(d_{ij}^t, \rho_j^t) > 0 \), direct effort and network socialization are complements or substitutes depending on the values of \( d_{ij}^t \) and \( \rho_j^t \). Therefore, in this case, it is an endogenous result of the model.

2.4 Steady-state equilibrium

In this section I characterize the interior long run equilibrium of this economy assuming particular functional forms. The steady state is characterized by a stationary distribution of the population. Therefore, I skip the time subscripts from here onwards.

**Proposition 2.** Assume that \( u(c^{ij}, l^{ij}, d^{ij}) = (c^{ij})^a(l^{ij})^b, m = (u^\theta + v^\theta) \frac{1}{\theta} \), \( f^H(d_{Hj}^t, \rho_j^t) = (d_{Hj}^t)^r(\rho_j^t) \), \( f^L(d_{LB}^t, \rho_B^t) = \delta \ast (d_{LB}^t)^r\rho_B^t \) and \( g(\rho_j^t) = c\rho_j^t \) with \( a, b \in (0, 1), \theta < 0, c < 1 \) and \( r \in (0, 1) \). Then there exists at least a set of values \( \{d^L, d^{HB}, d^{HG}, v, \mu^{LB}, \mu^{HB}, \mu^{HG}, l^L, l^{HB}, l^{HG}\} \) such that satisfies equations (7) and (13), and

\[
\begin{align*}
\mu^{LB} &= 1 - p\pi^{HG}[1 - (1 - s)\pi^{HB}] - \pi^{HB}(1 - sp), \\
\mu^{HB} &= \pi^{LB}[1 - sp - (1 - s)p\pi^{HG}], \\
\mu^{HG} &= sp\pi^{LB},
\end{align*}
\]

(14)  (15)  (16)
where:

\[
\begin{align*}
\pi^{ij} &= f^i(d^{Hj}, \rho^j) + (1 - f^i(d^{Hj}, \rho^j))g(\rho^j), \\
\Upsilon &= 1 - (1 - sp)\pi^{HB} + \pi^{LB} - p\pi^{HG} + p(1 - s)\pi^{HG}(\pi^{HB} - \pi^{LB}).
\end{align*}
\]  

Proof of proposition 2 is in the appendix. Notice that equations (8)-(10) describe a time inhomogeneous transition matrix since it depends on the expected future value of \(\mu_{t+1}^{ij}\). Therefore, uniqueness and stability of equilibrium cannot be established analytically. I proceed to numerical calibration in section 4.

3 Measures of social mobility, income inequality and equality of opportunity

Several measures of social mobility are used in the literature. Most economic studies estimate the intergenerational elasticity of income as a measure of social immobility. My model allows for the measurement of upward mobility (the so called 'American dream'), downward mobility, the intergenerational elasticity of income and the intergenerational elasticity of education. With regards to inequality, I compute the Gini index as well as the relative wage and the coefficient of variation of income.

Measures on inequality of opportunity evaluate the existence of discrimination across groups. I compute the probability to achieve the high-income
group relative to the probability to remain in the high-income group. The closer this relative probability is to 1 the larger the equality of opportunity of the society.

**Social mobility:**

Intergenerational elasticity of income and education: \( \epsilon = \frac{\text{Cov}(X_t, X_{t+1})}{\text{Var}X} \).

Rank-rank slope: in our case it coincides with IGE.

Upward mobility (‘American Dream’): Probability to change from a Bad Network to a Good Network.

\[
UM = \frac{\pi_{t+1}^{LB}sp_{t+1}\mu_{t}^{LB} + \pi_{t+1}^{HB}sp_{t+1}\mu_{t}^{HB}}{\mu_{t}^{LB} + \mu_{t}^{HB}}.
\]

Upward mobility-low education: Probability to change from a family with low educated parent to a good job.

\[
UM_L = \pi_{t+1}^{LB}sp_{t+1}.
\]

Upward mobility-high education: Probability to change from a family with highly educated family with bad network to a good job.

\[
UM_H = \pi_{t+1}^{HB}sp_{t+1}.
\]

Downward mobility: Probability to change from a Good Network to a Bad Network.

\[
DM = (1 - \pi_{t+1}^{HG}) + \pi_{t+1}^{HG}(1 - p_{t+1}) = 1 - p_{t+1}\pi_{t+1}^{HG}.
\]

\(^9\)In the steady state the mean and variance of income and education are time invariant, therefore, the intergenerational elasticity corresponds to the correlation measure.
Equality of opportunity:

\[
\frac{\text{Prob to change from LB or HB to HG}}{\text{Prob of transition from HG to HG}} = \frac{UM}{1 - DM}, \quad (19)
\]

\[
\frac{\text{Prob to change from LB to HG}}{\text{Prob of transition from HG to HG}} = \frac{UM_L}{1 - DM}, \quad (20)
\]

\[
\frac{\text{Prob to change from HB to HG}}{\text{Prob of transition from HG to HG}} = \frac{UM_H}{1 - DM}. \quad (23)
\]

Notice that upward mobility depends on the probability to get high education and the probability to get a good job. In contrast, equality of opportunity depends on the relative probability of getting high education and the relative amount of search units between two groups of individuals. The number of vacancies does not enter directly in the measure of equality of opportunity.

Income inequality:

Relative wage = \( \frac{w^G}{w^B} \).

Gini index = \( \frac{H^G \beta (y^G - y^B)}{y^B + H^G \beta (y^G - y^B)} \).

Coefficient of variation of income: \( CV(w) = \frac{\text{St.Dev.}(w)}{\text{Mean}(w)} \).

Inequality depends on difference in the productivity between good and bad jobs, the amount of people in good jobs and the bargaining power of workers.

\( ^{10} \)I have two levels of income in my model.
4 Calibration to the US

4.1 Data

Next I calibrate the model to the US economy. I use the multi-year American Time Use Survey (ATUS) and data from CHrelaiejg (2015), apart from standard macroeconomic variables.

The multi-year ATUS file contains information of a representative sample of US households from 2003 to 2013. In each household one respondent reports detailed information on his/her daily activities. This includes how much time he/she spends with the kids and with own leisure. There is moreover information on the household composition and job characteristics and education level of the respondent. Using this data I categorize households in four groups according to the occupation and education level of the respondent (one of the parents). The occupation is defined as a good job if more than 75% of individuals in this job have university degree, and as a bad job otherwise. The education level is high if the individual has some university studies, and low otherwise. This classification resembles the model classification. In the model, the group with low education working in a good job is assumed to be zero. In the data, less than 5% of individuals belong to this group (see figure ). In the following we exclude these individuals from the analysis.

"Primary childcare activities include time spent providing physical care; playing with children; reading with children; assistance with homework; attending childrens events; taking care of childrens health care needs; and dropping off, picking up, and waiting for children. Passive childcare done as
a primary activity (such as keeping an eye on my son while he swam in the pool) also is included. A child’s presence during the activity is not enough in itself to classify the activity as childcare. For example, watching television with my child is coded as a leisure activity, not childcare.”

“Socializing, relaxing, and leisure. This category includes face-to-face social communication and hosting or attending social functions. Time spent communicating with others using the telephone, mail, or e-mail is not part of this category. Leisure activities include watching television; reading; relaxing or thinking; playing computer, board, or card games; using a computer or the Internet for personal interest; playing or listening to music; and other activities, such as attending arts, cultural, and entertainment events.

Sports, exercise, and recreation. Participating in as well as attending or watching sports, exercise, and recreational activities, whether team or individual and competitive or noncompetitive, falls into this category. Recreational activities are leisure activities that are active in nature, such as yard games like croquet or horseshoes.”

The next step is to compute the average time each individual spends with the kid and doing leisure activities. This may be affected by the family composition and the labor force status of the parents. Therefore, I classify families by their type in terms of one/two parents, number of kids (1, 2 or 3+), and how many parents work full-time. Then I compute the average time spend in kids and in leisure for each type of family and whether parents have high/low education and bad/good job. In the main calibration I will use the results from a family of 2 parents and 2 kids. Robustness exercises use other types of families.
5 Conclusions

I study the interaction of the cultural transmission of education from parents to kids and the effect of networks in a labor market with frictions to analyze the relationship between social mobility, equality of opportunity and income inequality.
References


Ausias Ribó and Montserrat Vilalta-Bufí. Is the matching function Cobb-


Appendix: Proof of proposition 2

Assume that $u(l_{ij}, d_{ij}) = z \ln(l_{ij}) + (1 - z) \ln(d_{ij})$, $f^i = (d_{HJ})^\theta \rho$, $m = (u^\theta + v^\theta)^{\frac{1}{\theta}}$ and $g(\rho^i) = c(\rho^i)$ with $z \in (0, 1)$, $r \in (0, 1)$, $\theta < 0$ and $c < 1$. The long run equilibrium of this economy is the set of values $\{d^L, d^{HB}, d^{HG}, v, \mu^{LB}, \mu^{HB}, \mu^{HG}\}$ such that satisfy equations (7), (13) and (14)-(16). In this section I prove that at least one equilibrium exists. I split the proof in three parts.

1. Proof that, given $\{v, \mu^{LB}, \mu^{HB}, \mu^{HG}\}$, the decision on effort is interior and unique. Decisions on how much leisure time to devote to kids are taken such that:

$$
\frac{z}{1 - d^{LB}} - \frac{1 - z}{d^{LB}} = \delta r(d^{LB})^{-1} \rho (1 - c \rho^B) sp\beta(y^G - y^B)
$$

$$
\frac{z}{1 - d^{HB}} - \frac{1 - z}{d^{HB}} = r(d^{HB})^{-1} \rho (1 - c \rho^B) sp\beta(y^G - y^B)
$$

$$
\frac{z}{1 - d^{HG}} - \frac{1 - z}{d^{HG}} = r(d^{HG})^{-1} \rho (1 - c \rho^G) p\beta(y^G - y^B)
$$

The RHS of the FOC is decreasing in $d_{ij}$. Moreover, $\lim_{d_{ij} \to 0} RHS = +\infty$ and $\lim_{d_{ij} \to \infty} RHS = 0$.

$$
\frac{\partial RHS}{\partial d} = \delta r(r - 1)d^{-2} \rho (1 - c \rho^B) sp\beta(y^G - y^B) < 0.
$$

The LHS of the FOC is increasing with $d_{ij}$. Moreover, $\lim_{d_{ij} \to 0} LHS = -\infty$ and $\lim_{d_{ij} \to 1} LHS = +\infty$.

$$
\frac{\partial LHS}{\partial d} = \frac{z}{(1 - d_{ij})^2} + \frac{1 - z}{(d_{ij})^2} > 0.
$$

Therefore the RHS and the LHS cross once and only once in the interval $[0, 1]$. This is true for $d^{LB}$, $d^{HB}$ and $d^{HG}$.

2. Proof that, given $\{d^L, d^{HB}, d^{HG}, \mu^{LB}, \mu^{HB}, \mu^{HG}\}$, the number of vacancies to open is positive and unique.

$$
(u^\theta + v^\theta)^{1/\theta} = \frac{v^K}{(1 - \beta)(y^G - y^B)}.
$$

The RHS of equation (25) is a straight line that goes through $(0, 0)$. Moreover,
it is increasing in \( v \). The LHS of (25) is positive, increasing and concave.

\[
\frac{\partial \text{LHS}}{\partial v} = \left( u^\theta + v^\theta \right) \frac{1-\theta}{\theta} v^{\theta-1} > 0 \tag{26}
\]

\[
\frac{\partial^2 \text{LHS}}{\partial v^2} = (1-\theta) \left( u^\theta + v^\theta \right) \frac{1-\theta}{\theta^2} v^{\theta-2} \left[ \frac{v^\theta}{u^\theta + v^\theta} - 1 \right] < 0 \tag{27}
\]

Moreover, \( \lim_{v \to 0} \frac{\partial \text{LHS}}{\partial v} = +\infty \) and \( \lim_{v \to \infty} \frac{\partial \text{RHS}}{\partial v} = 0 \). Therefore, they cross once and only once and \( v \) is positive and bounded from above.

3. Proof that, given \( \{v, d^{LB}, d^{HB}, d^{HG}\} \), the distribution of population is stationary. The population distribution of this economy is characterized by the following Markov chain:

\[
\begin{bmatrix}
\mu_{LB}^{t+1} \\
\mu_{HB}^{t+1} \\
\mu_{HG}^{t+1}
\end{bmatrix}
\begin{bmatrix}
1 - \pi_{LB}^{t+1} \\
\pi_{LB}^{t+1} (1 - sp_{t+1}) \\
\pi_{LB}^{t+1} sp_{t+1}
\end{bmatrix}
\begin{bmatrix}
1 - \pi_{HB}^{t+1} \\
\pi_{HB}^{t+1} (1 - sp_{t+1}) \\
\pi_{HB}^{t+1} sp_{t+1}
\end{bmatrix}
\begin{bmatrix}
1 - \pi_{HG}^{t+1} \\
\pi_{HG}^{t+1} (1 - pt_{t+1}) \\
\pi_{HG}^{t+1} pt_{t+1}
\end{bmatrix}
= \begin{bmatrix}
\mu_{LB}^{t} \\
\mu_{HB}^{t} \\
\mu_{HG}^{t}
\end{bmatrix}
\tag{28}
\]

where

\[
\pi_{ij}^{t} = f^i(d^{ij}, \rho^j) + \left[ 1 - f^i(d^{ij}, \rho^j) \right] cp^j.
\]

The transition matrix \( M \) is a regular Markov matrix if \( M^n \) has only positive entries for some integer \( n \). In my case, the transition matrix is regular if \( \pi_{ij} < 1 \) for all \( i, j \). This will always be true as long as \( c < 1 \), since \( f^i(d^{ij}, \rho^j) < 1 \). Then, 1 is an eigenvalue of \( M \) and every other eigenvalue \( e \) of \( M \) satisfies \( |e| < 1 \). Moreover, the eigenvector associated to the eigenvalue 1 is strictly positive and can be transformed so that it is the long-run solution of the Markov process (Theorem 23.15, p. 618, Mathematics for economists). Therefore, the long-run equilibrium satisfies that \( \mu_{LB} + \mu_{HB} + \mu_{HG} = 1 \) and

\[
\begin{bmatrix}
-\pi_{LB}^{t+1} \\
\pi_{LB}^{t+1} (1 - sp_{t+1}) \\
\pi_{LB}^{t+1} sp_{t+1}
\end{bmatrix}
\begin{bmatrix}
1 - \pi_{HB}^{t+1} \\
\pi_{HB}^{t+1} (1 - sp_{t+1}) - 1 \\
\pi_{HB}^{t+1} sp_{t+1}
\end{bmatrix}
\begin{bmatrix}
1 - \pi_{HG}^{t+1} \\
\pi_{HG}^{t+1} (1 - pt_{t+1}) \\
\pi_{HG}^{t+1} pt_{t+1}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\tag{28}
\]

The solution is the following.

\[
\mu_{LB} = \frac{1 - p\pi_{HG}^{t+1} (1 - s)\pi_{LB}^{t+1} - \pi_{HB}^{t+1} (1 - sp_{t+1})}{\Upsilon}, \tag{28}
\]

\[
\mu_{HB} = \frac{\pi_{LB}^{t+1} (1 - sp_{t+1}) - (1 - s)p\pi_{HG}^{t+1}}{\Upsilon}, \tag{29}
\]

\[
\mu_{HG} = \frac{sp\pi_{LB}^{t+1}}{\Upsilon}, \tag{30}
\]

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where

$$\Upsilon = 1 - (1 - sp)\pi^{HB} + \pi^{LB} - p\pi^{HG} + p(1 - s)\pi^{HG}(\pi^{HB} - \pi^{LB}).$$

Since $p$ and $\pi^{ij}$ are functions of $\mu^{ij}$, there may be multiple solutions.