1 Introduction

When we evaluate the welfare gains attributed to public intervention the natural inclination is to assume policies will reduce, or alter in some favorable way, existing uncertainty in factors that affect individual well-being. However, there is increasing recognition that new policies are also a source of uncertainty. A policy may be proposed to meet one objective, such as lower the cost of energy, but in the process it creates uncertainty in the quality or availability of non-market, environmental services indirectly affected by the policy. Equally important, regulatory mandates for new pollution control equipment also create uncertainty in costs and, ultimately, prices or the products provided by the regulated sectors. A recent example is the set of rules aimed at reducing CO2 emissions for existing coal-fired electric plants that were upheld by the Supreme Court in April. We argue that even if the cost increases are confined to one sector, a policy can lead to general equilibrium effects that create price uncertainty. This paper considers the properties of welfare measures for both types of policy induced sources of uncertainty. Our extensions are also relevant to recent concerns about the problems with partial equilibrium analysis of policy. Some authors characterize the problem as reflecting of “behavioral” responses to policy. Others suggest the concerns reflect the need to do a general equilibrium analysis.¹ In the latter case it is argued that the analysis needs to include both

¹A notable recent example can be found in the U.S. Environmental Protection Agency’s [2014] announcement that it will form a special panel to inform their Science Advisory Board on “...the technical merits and changes of using economy-wide models to evaluate the social costs, benefits and economic impacts associated with EPA’s air regulations.” p. 6899.
market and non-market adjustments in the general equilibrium.\textsuperscript{2} Hendren’s [2013] recent paper can be interpreted to offer a demonstration that both arguments are making the same point. His goal is to characterize “. . . the parameters required for welfare measurement of marginal changes to government policies.” He describes his measures for the impact of policy as “. . . the difference between the government budget with behavioral responses to the policy and a counterfactual world without any behavioral responses . . .” If we describe models that include actions with direct and indirect influences to market (and non-market) outcomes as the general equilibrium (GE) approach then his analysis outlines situations where multi-market, GE effects can be important even with marginal policies.

2 Context

Applied welfare economics considers the effects of uncertainty in distinctly different ways when it is associated with marketed and non-marketed goods.\textsuperscript{3} In the case of marketed goods, the literature generally follows the Waugh-Oi-Massell tradition in judging how price instability affects welfare.\textsuperscript{4} The analysis assumes incomplete markets and generally relies on assumptions that assure the stochastic factors leading to price instability do not stem from instability in consumers’ incomes.\textsuperscript{5} As a rule, the source for the uncertainty is assumed to be exogenous to the consumer. Factors such as weather shocks that would lead to commodity price uncertainty are often cited in applications to agricultural markets. With price treated as a random variable, the analysis considers the restrictions to preferences as well as to the source of uncertainty that are required to evaluate whether policy interventions enhance welfare. For example, Turnovsky, Shalit, and Schmitz’s [1980] analysis of policies to stabilize a single price developed a sufficient condition for Waugh’s [1944] conclusion that consumers lose from price stabilization. Their result implies that the income elasticity of demand for the good whose price is random must exceed one-half the coefficient of relative risk

\textsuperscript{2}See Carbone and Smith [2008] for discussion in the context of the double dividend. More generally, the EPA [2014] draft charge questions for the proposed special panel focus attention on the concerns raised in Carbone and Smith, asking whether there is sufficient information to implement their proposals. Just [2011] raises a similar concern noting that: “Forcing calibration by means of questionable assumptions has unknown and potentially serious consequences and limits any claims of robustness” p.64. His general observation, while potentially important, misrepresents the issues that are important in general equilibrium analysis that treats nonmarket services as making non-separable contributions to preferences and production functions. However, those concerns are tangential to our objectives here.

\textsuperscript{3}A prominent illustration of this separation can be found with the treatment of uncertainty in Just, Hueth and Schmitz [2004]. In their discussion of stochastic welfare measurement (Chapter 12) they ignore the Turnovsky et al. work and attribute the distinctions between the analysis of price instability and environmental uncertainty to the inclusion of risk aversion. Turnovsky et al. introduce a measure of risk aversion due to price uncertainty and use it to define their sufficient condition for Waugh’s result. The Just et al. distinction overlooks this interpretation.

\textsuperscript{4}It is important to acknowledge that the Waugh-Oi-Massell logic is partial equilibrium. As Samuelson [1972] demonstrated in a general equilibrium context, it is not possible to have this partial bootstrap logic to overcome the effects of price instability.

\textsuperscript{5}There are early notable exceptions such as Anderson [1979], and Chavas, Bishop and Segerson [1986] in the environmental context. However, both of these studies focus on the distinction between ex ante and ex post welfare measures rather than the implications of the uncertainty for policy design.
In an independent example, with somewhat more restrictive assumptions (i.e. additive separability between the price and income), Rogerson [1980] demonstrated that expected surplus for a good with price and income uncertainty correctly signals the consumer’s preference over distributions of values for that price (holding income constant). The assumptions involved in these assessments usually lead to a certainty equivalent interpretation of the Hicksian welfare measure used to evaluate stabilization policy. By contrast, the analysis of policies relevant to uncertainty in the quantities associated with non-market goods has generally followed the Weisbrod-Schmalensee-Graham tradition. Preferences are assumed to be state-dependent or a two period dynamic model is used to introduce the equivalent of this dependency through the resolution of uncertainty over time and its effects on time dated choices of goods and services. The primary focus of the analysis has been on whether the uncertainty affects the amount of the non-market good received by everyone and if there exist individual opportunities to adjust to the risk. Graham’s [1981] summary highlights the key insights from this approach.

(i) “Option Price is the appropriate measure of benefit to situations involving similar individuals and collective risk.

(ii) Expected value calculations (using the Hicksian consumer surplus for each state) are appropriate to situations involving similar individuals and individual risks.

(iii) Whether or not option price exceeds the expected value of surplus is largely irrelevant to the evaluation of risky projects.” (p 716, parenthetical phrase added.)

His last conclusion is a bit misleading. In practice, it is usually not possible to estimate the option price for a policy that alters the quantity or quality of a non-market good. Thus, while the option price may be the appropriate ex ante welfare measure, ex post measures are usually what can be measured. Under these circumstances a set of conditions that allow an assessment of their relationship would be relevant. Most discussions of the properties of expected values for ex post Marshallian consumer surplus measures for environmental policy follow the Weisbrod-Schmalensee-Graham (WSG) tradition. Our analysis of policy induced uncertainty is developed in two parts: an adaptation of the logic used in Turnovsky, Shalit, and Schmitz in the next section and an extension to the first author’s recent results (Schlee [2013]) on when expected consumer surplus measures signal Pareto efficient allocations in the section after that.

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6In the analysis that is our focus here, Turnovsky, Shalit and Schmitz [1980] (pp. 140-141) assume for a subset of the goods consumed, the marginal utility of income is invariant to changes in their prices, or \( V_p \text{im} = 0 \) for \( i = 1 \) to \( k \) with \( k < n \), \( k \) is the number of goods with price-independent marginal utility of income and \( n \) is the total number of goods. The demand for these \( k \) goods will be related to income, but the indirect utility function is restricted to be separable as:

\[
V(P_1, P_2, \ldots, P_k, \ldots P_n, m) = f(P_1, P_2, \ldots P_k) + G(P_{k+1}, \ldots P_n, m)
\]

Thus their income elasticities will be equal.

7Chavas and Mullarkey [2002] extend the insights from Graham, considering the role of information, risk aversion, and the type of policy on the ex ante measures of benefits at the individual level.
3 Quality (or Quantity) Instability and Individual Welfare

We ask: when is it desirable to stabilize a random quality of a good (or random quantity of a public
good); and when is the case that expected aggregate surplus is higher whenever it is desirable to
stabilize? We suppose here that changes in the distribution of quality or quantity do not affect
prices.

3.1 Willingness to pay for quality stabilization

Chavas and Mullarkey [2002] provide the most comprehensive effort to unify the literature on ex
ante welfare measures under uncertainty. Using a two period model that distinguishes both the
timing of decisions and the resolution of uncertainty, these authors describe how the contributions
of information, risk aversion, and policy can be distinguished in ex ante welfare measures consistent
with the WSG formulation. There are important features of each of their concepts. Their ex ante
value of information measures the smallest amount an individual would be willing to accept to
choose private goods in the first of the two periods described by their model without knowledge of
the first period sources of uncertainty. This concept could also be interpreted as a positive value for
flexibility, due to convexity of the maximum operator and Jensen’s inequality. Their risk measure
is a conditional measure of risk aversion. It is the premium that an individual would pay ex ante
to have knowledge of the expected values for both periods’ sources of uncertainty. In this context,
choices could be made under conditions that reflect compensation for resolution of the first period’s
uncertainty, but with only the information a risk neutral decision maker would need for private
choices. Policies are defined so that they focus on changes in services that are not subject to private
choice. These services can influence the choices of private consumption goods and thus would affect
the decomposition of the ex ante compensation into the conditional value of risk and the individual’s
valuation for the policy. Their characterization of ex ante welfare measures generalizes the earlier
decomposition proposed by Chavas, Bishop, and Segerson [1986]. While an important template for
organizing the contributions in the Weisbrod-Schmalensee-Graham framework, it does not offer a
direct approach for describing how operational measures of the compensation could be developed.
Our adaptation of the Turnovsky et al. logic for quantity uncertainty provides this extension. As we
noted above, Turnovksy et al. outline a direct approach to assessing the merits of price stabilization
first using a model with a single good and then consider the case where multiple goods are affected.
Here our focus is on using the insights from the single price case for one non-market good. Let
\( V(P_1, P_2, ..., P_k, ..., P_n, m) \) designate the indirect utility function in terms of private commodities with
prices \( P_i \), \( i = 1, ..., n \) and income, \( m \). In this context, a consumer gains (or loses) from stabilization
of good one’s price (\( P_1 \)) according to whether \((\partial^2 V) / \partial P_1^2 < (>) 0\). These authors develop the
features of preferences that are sufficient for price stabilization to reduce a consumer’s expected
utility. When we add one (or more) non-market good(s), \( q \), as quasi-fixed element(s) in \( V(·) \),
the issues are comparable. More specifically, we can define the gain from policies that replace a
random variable for \( \bar{q} \) designated as \( q \) with a constant value which is maintained at the expected
value amount of \( q \). One way to define this measure is in terms of the Hicksian willingness to pay
for replacing an uncertain \( q \) with a constant value of \( E(q) = \bar{q} \) be represented as \( B \). The formal
definition is given in equation (1). A local approximation of \( B \) can be derived using a Taylor series approximation of both sides of equation (1). The sign of \( B \) is seen to be dependent on the second derivative properties of \( V(\cdot) \) in terms of \( q \).

\[
E(V(P_1, \ldots P_n, q, m)) = V(P_1, \ldots P_n, \bar{q}, m - B)
\]

and

\[
B > 0 \text{ as } V_{qq} < 0
\]

This logic assumes \( q \) is the only stochastic term, and that \( q \) is a random variable with a finite variance. Thus, stabilizing the variations in \( q \) so it has a value corresponding to the population mean, \( \bar{q} \) would be a gain to a consumer if, given prices and income, \( V(\cdot) \) is concave in \( q \). For private goods, it is reasonable to assume consumption takes place where this condition is generally satisfied. For non-market goods, the decision mechanism that establishes the level of \( q \) available to each individual may not reflect each individual consumer’s desires. For public goods, allowing for heterogeneous agents, the outcome might be consistent with this property being satisfied for some individuals’ preferences but not others. One sufficient condition for the concavity of \( V \) in \( q \) is that the vN-M utility is \textit{jointly} concave in the vector of consumption of private goods and the public good: the consumer is risk averse with respect to lotteries with outcomes \((x_1, \ldots, x_n, q)\).

We return to a discussion of the relevance of our extensions below. Our adaptation proceeds in two steps. First, using the logic that yields equation (2) we establish a simple relationship between specific features of individual preferences, including: measures of risk aversion for shocks to wealth, aversion related to non-market services, and the properties of the demand for the non-market services. To our knowledge, the commodity specific risk aversion measure for non-market services has not been measured. Thus, efforts to link it to the other features of preferences, where we have some empirical experience, offer a basis for gauging its importance to the tradeoff measures we define in the second step of our extension. Under certainty, the virtual price or Marshallian marginal willingness to pay for \( q \), \( \pi \), can be defined using the indirect utility function as:

\[
\pi = V_q/V_m
\]

Differentiating (3) with respect to \( q \) we have:

\[
V_{qq} = \pi V_m + \pi \cdot V_{mq}
\]

Using the properties of the income elasticity of the virtual price of \( q \) we can re-write (4) in terms

\footnote{Fix prices \( P \) and income \( m \), let \( q'' \) and \( q' \) be two different quality levels and \( x'', \ x' \), the private consumption vector demand at \((P, m, q'')\) and \((P, m, q')\). Let \( u \) be the vN-M utility consumption-quality lotteries. For \( \lambda \in [0, 1] \) it follows that \( \lambda V(P, m, q'') + (1 - \lambda)V(P, m, q') = \lambda u(x'', q'') + (1 - \lambda)u(x', q') \leq u(\lambda(x'', q'') + (1 - \lambda)(x', q')) \leq V(P, m, \lambda q'' + (1 - \lambda)q'), \) where the first inequality is from the concavity of \( u \), and the second since private consumption \( \lambda x'' + (1 - \lambda)x' \) at quality \( \lambda q'' + (1 - \lambda)q' \) (since the budget constraint does not change with \( q \)).}
of preference features where there is empirical experience.

\[
\frac{m \pi}{\partial m} \frac{\partial \pi}{\partial m} = \frac{m V_{qm}}{\pi V_m} - m \frac{V_{mmV_q}}{V_m^2} \tag{5}
\]

or more compactly

\[
\frac{m \pi}{\partial m} \frac{\partial \pi}{\partial m} = -\frac{m V_{mm} m V_{qm}}{V_m V_q} \tag{6}
\]

Now re-writing (4) in terms of \(V_{mq}\) we can express the relative risk aversion coefficient for \(q, r_q = \frac{-(qV_qq)}{V_q}\) in terms of the income elasticity for \(\pi\), the relative risk aversion coefficient for risks to income \(r = \frac{-(mV_m m/V_m)}\), and the inverse “price” elasticity for \(q\) as

\[
\begin{align*}
\frac{r_q = s \cdot (r - n) + \theta}{4}
\end{align*}
\]

where \(s\) is the “income” share for \(q\) \((s = \pi q / m)\),

\[
\begin{align*}
\frac{n = \partial \pi}{\partial m} \cdot \frac{m}{\pi},
\end{align*}
\]

the income elasticity of demand for \(\pi\), and

\[
\begin{align*}
\frac{\theta = \partial \pi}{\partial q} \cdot \frac{q}{\pi}.
\end{align*}
\]

Weak complementarity and the Willig [1978] condition imply that \(n\) equals the income elasticity of the weak complement (Palmquist [2005]). As a result (7) can be re-written using that income elasticity, \(\varepsilon\), as

\[
\begin{align*}
\begin{align*}
\frac{r_q = s \cdot (r - \varepsilon) + \theta}{8}
\end{align*}
\end{align*}
\]

where \(s\) is the “income” share for \(q\) \((s = \pi q / m)\),

\[
\begin{align*}
\frac{n = \partial \pi}{\partial m} \cdot \frac{m}{\pi},
\end{align*}
\]

the income elasticity of demand for \(\pi\), and

\[
\begin{align*}
\frac{\theta = \partial \pi}{\partial q} \cdot \frac{q}{\pi}.
\end{align*}
\]

We have reasonable intuition for some of these measures. This connection is important because it helps to operationalize our measure of the compensation required for an individual to be willing to accept an uncertain level of environmental services. The second step in our analysis begins with the definition of that compensation measure \(C\). It would correspond to an ex ante payment that would hold expected utility constant in the presence of a policy that replaces a certain level of \(q\) with random values for \(q\), as defined in equation (8).

\[
\begin{align*}
\frac{E(V(\tilde{q}, p, m + C)) = V(\tilde{q}, p, m)}{9}
\end{align*}
\]

The notation \(\tilde{q}\) is the the random quantity of the environmental service induced by the policy, with \(E(\tilde{q}) = \bar{q}\). It could in principle be another level of \(q\) that could be specified in terms of the distribution assumed for \(q\). Replacing the left side of (9) with a second order Taylor series expansion,
treated the partial derivatives evaluated at \( \bar{q}, p \) and \( m \) as constants gives us

\[
V(\bar{q}, p, m) = V(\bar{q}, p, m + C) + \frac{\partial V}{\partial q} E((\bar{q} - \bar{q})^2) + \frac{1}{2} \frac{\partial^2 V}{\partial q^2} E[\bar{q} - \bar{q}]^2 + \frac{\partial V}{\partial m} C + \frac{1}{2} \frac{\partial^2 V}{\partial m^2} C^2 + C \frac{\partial^2 V}{\partial m \partial q} E[\bar{q} - \bar{q}] (10)
\]

Rearrange terms and use the fact that \( E(q) = \bar{q} \) to find that

\[
\frac{V(\bar{q}, p, m + C) - V(\bar{q}, p, m)}{V_m} - \frac{1}{2} \pi r_q \left( \frac{\sigma_q^2}{\bar{q}} \right) + C - \frac{1}{2} \frac{C^2}{m} m = 0 (11)
\]

where \( \sigma_q^2 = E((q - \bar{q})^2) \), the variance in environmental services with the policy that creates the uncertainty. We can further simplify equation (11) if we consider a second order Taylor series approximation for \( V(\bar{q}, p, m + C) \) to approximate \( V(\bar{q}, p, m) \). Since we are approximating \( V \) for a lower value of \( m \), the Taylor series terms are reversed in order. We are moving from the point of approximation \( m + C \) to \( m \). This strategy allows further simplification. Both substitutions imply we are approximating \( C \). We discuss the implications further in a closing section of the paper. The expression for our Taylor series is

\[
V(\bar{q}, p, m) = V(\bar{q}, p, m + C) + \frac{\partial V}{\partial m} (m - (m + C)) + \frac{1}{2} \frac{\partial^2 V}{\partial m^2} (m - (m + C))^2 (12)
\]

Simplify and solve for \( V(\bar{q}, p, m + C) \) to find

\[
V(\bar{q}, p, m + C) = V(\bar{q}, p, m) + \partial V/\partial m C - 1/2(\partial^2 V)/\partial m^2 C^2 (13)
\]

Substitute into equation (11) (recall \( r = -(m(\partial^2 V)/\partial m^2)/(\partial V/\partial m) \)) to conclude that

\[
2C - 1/2\pi r_q \frac{\sigma_q^2}{\bar{q}} = 0. (14)
\]

Or, rearranging terms, using our expression for \( r_q \) together with the assumption of weak complementarity and the Willig condition, \( C \) can be approximated with parameter estimates from the literature and an assumption about the uncertain in \( q \) (namely \( \sigma_q^2 \)) using

\[
C \approx \frac{1}{4} \pi (s(r - \varepsilon) + \theta) \frac{\sigma_q^2}{\bar{q}} (15)
\]

Notice this formulation is a variation on the conventional expression for the risk premium for shocks to income or wealth. Table 1 computes the values of \( C/\pi \) for selected values of the parameters. Setting income at $43,000, we selected four values for \( r (0.5, 1.5, 3.0, 11) \), three for \( \varepsilon (0.5, 0.75, 1.5) \), three for \( \theta (0.2, 1.0, 2.0) \) and five for \( \sigma_q^2 (5, 100, 150, 250, \text{ and } 500) \). For each combination of these parameters we evaluate the approximate compensation using three values for each of \( \pi (1, 3, 10) \) and \( q (60, 120, \text{ and } 1200) \). These values lead to virtual expenditures as shares of income that range from 0.001 to 0.28 with a mean of .05 across the scenarios we considered. As expected, higher levels
of the coefficient of relative risk aversion and the variance in $q$ lead to higher values of $C/\pi$. The inverse price elasticity for environmental services induces greater variability in $C/\pi$ than the income elasticity. While this could have been expected from the relationship for $r_q$ in equation (7), the effect was larger than we expected. Three qualifications to these simple calculations are important. First, our expression for $C/\pi$ is a local approximation. Ideally the logic underlining the relationship in equation (10) would be solved using a root finding approximation, over a wider range of parameter values consistently linked to a specific preference function. Second, our approximation ignores the different remainder terms to the two Taylor series expansions we used to derive equation (13). The larger the variance in $q$ relative to $\overline{q}$, the more important we expect these terms to be. Finally, and perhaps most important from the perspective of the economic interpretation of our definition for $C$, the policy assumed to give rise to uncertainty in $q$ must do something else! Our analysis effectively assumes that “thing” has a separable effect. As our examples suggested, it might be intended to reduce energy prices or affect supplies of other private goods. These effects should be included in the definition of $C$ in equation (8). That is, policy might change $p$ from $p^0$ to $p^1$ and replace $\overline{q}$ with $q$ simultaneously, so the relevant welfare measure would be given by the equation (14).

$$E(V(q, p^1, m + C)) = V(\overline{q}, p^0, m)$$ (16)

With a specific definition for the policy it is possible to extend the logic we outlined adapting the Turnovsky et al. logic in a parallel fashion to their treatment of the multiple price case. In this case the definition of a risk aversion would need to generalize the logic used in Turnovsky et. al. adaptation of Duncan [1977]. Detailed results on how this composite definition would change $C$ require more assumptions about how the policy affects marketed goods and how they interact with the environmental services that become uncertain.

### 3.2 Expected Surplus Maximization and Quality Stabilization

Now suppose that $q$ is possibly random but for now that changes in $q$ do not affect equilibrium prices (perhaps because the production of private goods is done under a Leontief technology—constant returns, no joint production, single nonproduced input). We want to compare how stabilization of $q$ affects welfare, and in particular if the choice of whether to stabilize or not under the criterion of expected consumer’s surplus agrees with expected utility. In particular we compare a distribution of $q$ according to a cdf $F$ with a completely certain $q$ equal to $Eq = \int q dF(q)$. The expected utility

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For example, the Herriges, Kling, and Phaneuf [2004] variation on the Stone-Geary specification that was used to evaluate the role of use and nonuse values might be adapted for this relationship. The approach assures mutual consistency in the parameter values being evaluated. Our approximate calculations implied 5 negative values of the 1620 computed for Table 1. These are inconsistent with the logic that define the C. No doubt there are other possible inconsistencies in the combinations of values we selected for that would arise once we recognize there would have more specific bounds on each set of values that would vary across different assumed preference functions. This follows because each of the “parameters” in this relationship is a function of the exogenous variables in the optimization problem that we adopted to characterize individual choice. Our numerical examples are intended to offer some perspective on the relative size of compensation compared to the virtual prices assumed for $q$. 

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8
gain from stabilizing $q$ is

$$\int [V(p, m, Eq) - V(p, m, q)]dF(q) = \int \left[ \int_q^{Eq} V_q(p, m, r)dr \right] dF(q)$$

$$= \int \left[ \int_q^{Eq} V_m \left( \frac{V_q}{V_m} \right) dr \right] dF(q)$$

(17)

where the integrand is evaluated at $(p, m, r)$. Now impose the Willig condition to find that the last expression equals ($D$ is the demand for the weak complement)

$$\int \left[ \int_q^{Eq} \left( \frac{V_m(p, m, r)}{D(p, m, r)} \int_p^\theta D_q(\theta, m, r)d\theta \right) dr \right] dF(q).$$

(19)

The integral wrt $q$ switches signs at $q = Eq$. The derivative of consumer’s surplus is by assumption positive (an increase in $q$ increases demand—strong complementarity.) If we can determine conditions under which $V_m/D$ is monotone in $q$, then we can compare expected utility and expected consumer’s surplus along the lines of Schlee (2013). We have

$$\frac{\partial V_m}{\partial q} D = V_m \left( \frac{V_{mq}}{V_m} \frac{D_q}{D} \right)$$

Assume that $D_q \geq 0$ (strong complementarity). Then if $V_{mq} \leq 0$, $V_m/D$ is decreasing in $q$. When will $V_{qm} \leq 0$? Suppose that $V(p, m, q) = T(f(x, q) + y)$ where $y$ is a numeraire, $x$ is the consumption of the weak complement, $f$ satisfies strictly increasing differences (so $x$ is a strong complement of $q$), and $T$ is strictly concave. Then $V_{mq} < 0$. If $T$ is affine then $V_{qm} = 0$. Moreover submodularity of $V$ in $q$ and $m$ is preserved under concave transformations: If $V$ is submodular in $q$ and $m$ and $\tilde{V} = T(V)$ where $T$ is concave, then $\tilde{V}$ is submodular. Of course $V_{mq} < 0$ is compatible with any sign for $V_{qq}$. Using arguments similar to Schlee (2013) one can show that if $V_m/D$ is decreasing in $q$, then for some number $K > 0$,

$$\int [V(p, m, Eq) - V(p, m, q)]dF(q) \geq K \left[ \int_q^{Eq} \left( \int_p^\theta D_q(\theta, m, r)d\theta \right) dr dF(q) \right]$$

(18)

(19)

(where indeed $K = V_m/D$ evaluated at $(p, m, Eq)$); or more succinctly,

$$\Delta EV \geq K \times \Delta ECS$$

were $\Delta EV$ is the change in expected utility from stabilizing $q$ at its mean. Intuitively, when $V_m/D$ is decreasing in $q$, that term puts more weight on $\Delta CS$ on the left side of the inequality when $q$ is low (the integrand is positive) than when it is high. It follows that if expected consumer’s surplus rises when $q$ is stabilized at its mean, then expected utility rises. In other words, under the assumptions made, expected consumer’s surplus if anything is biased against quality stabilization. In still other words, if stabilization maximizes aggregate expected consumers’ surplus and the preference
assumptions hold for every consumer, then stabilization is *ex ante* Pareto Optimal. This result holds even if no consumer is (globally) averse to risk in $q$.

**Remark 1.** 1. It’s obvious that the assumptions that (i) the mean is preserved and that (ii) the all uncertainty is removed are not needed: we can replace those with a “single crossing condition” as in Schlee (2013). We can strengthen the result to say that if the expected-surplus-maximizing distribution of quality is less variable than any feasible distribution of quality, and the stated assumptions hold for every consumer, then that distribution is *ex ante* Pareto Optimal (on the set of feasible quality distributions).

2. We can also extend the result to the case in which price is affected by quality (and since quality is uncertain, so is the price). This extension is straightforward.

4 Uncertainty as an Equilibrium Outcome for Price and Welfare

Our second case begins from the results in a recent paper (Schlee [2013]) by the first author. That analysis provides a sufficient condition for expected surplus maximizing policies to identify Pareto optimal decisions. The result is characterized as establishing interim Pareto optimal policies. In the context of this analysis, the term interim was intended to capture the notion that after consumers learn “their types” (in the sense defined below) there is no other policy that they weakly prefer to the interim Pareto optimal policy, and this preference is strict for at least one consumer type. As we explain below, this uncertainty about consumer types is what results in uncertainty about the equilibrium price of a private good. Four assumptions underlie Schlee’s result: (1) the policy affects only one price; (2) if there are policy induced income changes, they do not influence the demand for the good whose price is affected; (3) the marginal utility of income is increasing in the price of the affected good; and (4) the expected surplus maximizing policy generates a random price for this one good that is less variable than any other policy. In this section we use the framework developed in Schlee to consider a special case of his key theorem involving policies that are intended to increase the amount of a non-market good. Following common practice, we assume there is a private good that serves as a weak complement to $q$. To simplify notation from here on, the distinction between random values of non-market services and non-random is not considered. So we use $q$ to designate non-random services. Now we assume the price of the good serving as the weak complement becomes uncertain in the sense implied in Schlee’s earlier analysis. The reason for this uncertainty follows from equilibrium adjustment. Consumers respond to the change in $q$. Thus, even when we assume the cost of improving $q$ has no effect on the price of the weak complement, the behavior of other consumers adjusting to the change in $q$ does! Thus, the consumer type is treated as a feature of preferences. Types can be described as pairs of the preference parameter and income. We can assume that a type depends on the state of the world and that individuals privately

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10Schlee [2013] defines this less variable condition that the random price at any expected surplus maximizing policy is less variable than a random price at any other if the price function corresponding to that policy crosses the other price function (for the other policy) at most once from above.
know their own type. Uncertainty arises because a consumer doesn’t know the type realizations of all other consumers. Thus in the absence of that knowledge the equilibrium price will be uncertain. What we label as the type parameter that together with \( q \) determines the nature of adjustment and thus the equilibrium price. This feature is the reason we label the uncertainty in price as a policy-induced, uncertain, equilibrium price. With this background we re-write the indirect utility function in equation (15) explicitly recognizing the link between price, \( q \) and \( \phi \) as, \( p(\tilde{p}(q, \phi), \mathcal{P}, m(q), q) \). This function is intended to reflect the fact the price outcome is uncertain as an equilibrium outcome. The individual recognizes other individuals’ values for \( \phi \) affect the realized equilibrium price but when the policy decision to change \( q \) is evaluated it is treated as leading to a random price.

\[
v = \tilde{V}(q, m, \mathcal{P}, \phi) = V(\tilde{p}(q, \phi), \mathcal{P}, m(q), q)
\]

(20)

\( \mathcal{P} \) now corresponds to the prices of other goods that we will not consider further and drop from \( \tilde{V} \) and \( V(\cdot) \) in our subsequent development of the argument. The dependence of income on quality reflects changes in tax payments a consumer makes for different levels of \( q \). Differentiate \( \tilde{V} \) with respect to \( q \) to find

\[
\tilde{V}_q = V_m \tilde{p}_q + V_q + V_m m'(q)
\]

(21)

Using Roy’s identity to re-write this equation as

\[
\tilde{V}_q = V_m [-X_1 \tilde{p}_q + V_q/V_m + m'(q)]
\]

(22)

where \( X_1 \) is the Marshallian demand for the weak complement to \( q \). With weak complementarity, we can assume the marginal value (Marshallian virtual price) for \( q \) can be recovered from the demand for the weak complement \( X_1 \). Assuming that \( P^c < \infty \) is the choke price for \( X_1 \), such that \( X_1(P^c, q, m) = 0 \), then we can write the Marshallian virtual price of \( q \), denoted by \( \pi \), as

\[
\pi = \frac{V_q}{V_m} = \int_{\tilde{p}(q, \phi)}^{P^c} X_{1q}(t, q, m) dt
\]

(23)

Substitute (\ref{eq:virtual_price}) into (\ref{eq:marshallian_surplus}) to find that the virtual price to be the derivative of the Marshallian surplus, \( S(q, \phi) \) for a change in \( q \), measured using the demand for \( X_1 \)

\[
S_q = \tilde{V}_q/V_m = \int_{\tilde{p}(q, \phi)}^{P^c} X_{1q}(t, q, m) dt
\]

(24)

To use the argument in Schlee [2013] we need to establish that \( V_m \) and \( S_q \) have either positive covariance or negative covariance due to their relationship to the random variable \( \phi \). Consider the case where \( V_m \) is increasing in the price of the private good serving as the weak complement. Assume that \( \tilde{p} \) and \( \tilde{p}_q \) are differentiable in \( \phi \). This requirement implies \( V_m \tilde{p}_\phi \) will have the same
(for positive covariance) or the opposite sign (for negative covariance) as $S_{q\phi}$. Express $S_{q\phi}$ as\(^{11}\)

$$S_{q\phi} = -X_1p\tilde{p}\frac{p}{p_{\phi}} - X_1\tilde{p}_{q\phi} - X_1q\tilde{p}_{\phi}$$  \hspace{1cm} (25)$$

The last two terms on the right side describe the effect of a change in $q$ for the demand for the weak complement, $X_1$. These terms consider direct and indirect (through price) effects of changes in $q$. Given complementarity, it is natural to assume $X_1q > 0$ or $X_1p\tilde{p} + X_1q > 0$. To establish negative correlation between $V_m$ and $S_q$, we need to demonstrate that $\tilde{p}_{q\phi}/\tilde{p}_{\phi} \geq 0$. This result follows, assuming $\tilde{p}_{q\phi} > 0$ and $V_{mp} > 0$ as seen in the next equation.

$$S_{q\phi} = \tilde{p}_{\phi} \left[-X_1p\tilde{p}_q - X_1q - X_1\left(\frac{\tilde{p}_{q\phi}}{\tilde{p}_{\phi}}\right)\right]$$  \hspace{1cm} (26)$$

If the bracketed term on the right side is non-positive, then $V_m$ and $S_q$ will have negative covariance. This condition establishes the primary implication of the Schlee (2013) framework once it has been adapted for uncertainty due to equilibrium adjustments to environmental policy. When the change in expected surplus is negative due to a change in $q$, the change in expected utility from $q$ is also negative: in short if $V_m$ and $S_q$ have negative covariance then

$$\Delta EV \geq \Delta EV_m \times \Delta ECS.$$  \hspace{1cm} (27)$$

This result is important because it asserts that the required adjustment to consumer surplus as a result of uncertainty about the equilibrium responses to the quantity (or quality) change will accurately represent individual expected utilities from that change. What is the bracketed expression in equation (26) negative? If, first, the equilibrium effect of an increase in the public good on consumption of the private good is positive: $X_1p\tilde{p} + X_1q > 0$. And, second if $p_{q\phi}/\tilde{p}_{\phi} \geq 0$. Establishing the required properties of the equilibrium price response is straightforward. Suppose that the equilibrium price is determined implicitly by $Z(p, q, \phi) = 0$, and that the function $Z$ is differentiable with $Z_p < 0$. (A natural interpretation is that $Z$ is the excess demand for good 1.) In the appendix we confirm that $p_{q\phi}/\tilde{p}_{\phi} \geq 0$ if $-Z_{\phi}/Z_p$ is increasing in $q$ and decreasing in $p$. In the appendix we also give examples of economies satisfying these conditions.\(^{12}\) One argument to demonstrate the condition is satisfied follows from properties of the excess demand for $X_1$. Assuming the percentage response of the rate change in excess demand with $q$ as $q$ changes exceeds percentage response in the rate of change of excess demand with $p$ as $q$ changes then the price response required for negative covariance will be satisfied.\(^{13}\) One loose interpretation of these properties of excess demand functions considers the equilibrium adjustment we attribute to $\phi$. It is about differences in the types of consumers of $X_1$ and their reactions to a change in $q$. It would suggest that changes in $q$ induce a larger response in excess demand due to the ways different economic agents respond than in the rate of price responsiveness of the quantity demanded to $q$. There are a number of potential

\(^{11}\)This result assumes that the Marshallian choke price does not change with $q$ or $\phi$.

\(^{12}\)As a simple example, it is satisfied if the supply of good 1 is constant, and the demand for good 1 is given by $\phi(a - p/q)$.

\(^{13}\)A formal statement of the argument is given in the Appendix along with examples.
examples of these type responses. Sorting models, especially the pure characteristics framework, maintain a relationship between price and quality that implies adjacent substitutes. Environmental quality is assumed in these models to be a weak complement to housing in a specific community. All of the estimates of the parameters for these models find results consistent with the negative covariance required for the expected consumer surplus measure to consistently reflect changes in expected utility. These models require a single crossing condition to assure the ascending bundles condition that describes the consistent ranking of community price indexes and the values of the index for local public goods across communities. To reconcile this property with the observed pattern of community choices by households differentiated by income, applications have consistently found the joint distribution of income and latent taste for local public goods has a negative covariance. Thus, the empirical evidence in these applications seems consistent with the requirement for the joint implication of weak complementarity, an equilibrium stochastic price, and a consistent relationship between expected surplus and expected utility for exogenous quality changes that lead heterogeneous agents to adjust. This adjustment takes the form of sorting among communities in the model. Analytical sorting models have also allowed non-market mechanisms to determine the level of \( q \) as a part of the definition of an equilibrium allocation of households among communities. For example, in a median voter model the tax/local public good combination is determined by the median agent in each community and these outcomes together with equilibrium prices for housing provide incentives for households to sort. Given heterogeneous preferences for public goods we can expect endogenous determination of all three variables as households sort with knowledge of community housing prices and the processes determining tax rates and local public goods (see Epple and Romano [1994,1996] as examples). Our result is relevant to these cases as well. They are captured in the interpretation of the \( p(q, \cdot) \) function as an equilibrium outcome of an adjustment process. Households can be assumed to sort, \( q \) can be determined by a voting process, with each responding to an external policy change. What is important is that the individual does not know the ultimate effects of the distribution of the \( \theta \)'s for other agents at the time we consider evaluating the policy. As a result, the price outcome represented by this function is treated as random and we maintain the assumptions for preferences that underlie Schlee’s basic result. The structural sorting models describe situations where the households involved in the adjustment thru sorting are assumed to know the outcomes. Our argument raises an implicit question. Namely, in the absence of empirical models to describe these situations, can a simple decision rule assist in making interim policy judgments? Our answer would be that it can! Epple and his co-authors have explored a number of model structures with these forms of adjustment. As a rule, the assump-

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14 For a more specific discussion and a summary of these applications see Kuminoff et al [2013].

15 The single-crossing condition in this case relaxes the Willig [1978] condition that we used in our earlier expression for \( rq \) and requires that the relationship between the change in price and with a change quantity be signed as income changes. More specifically \((\partial / \partial m)(dp/dq) > 0\).

16 Further support for the negative covariance has been derived using a deduced form strategy that combines survey data and a widely recognized index of individual attitudes toward environmental (the New Ecological Paradigm) along with consistent measures of local neighborhood specific price indexes, without any structural assumptions about preferences. Fishman and Smith [2013] report strong support for the negative covariance using neighborhoods in metropolitan Phoenix.

17 See Kuminoff et al. [2013] for more specific discussion of these cases.
tions necessary to characterize an equilibrium are less restrictive than the preference restrictions for Schlee’s result. Thus, by treating the equilibrium price outcome as random from the individual’s perspective we are able to include a wide range of cases in the literature on policies that affect local public goods or non-market environmental services. Discussion There are numerous examples that fit the opening motivation for our analysis that policies can be important sources for uncertainty inside and outside markets. Discussion of a couple will help to characterize when our results are relevant to policy evaluation and when they are not. Part of the debate concerning the Keystone Pipeline is about the time profile of risks to environmental resources such as key aquifers along its route that will be implicitly accepted for the duration of the pipeline’s working life. Similarly the short lived proposals to make decisions about federally subsidized home mortgages conditional on environmental reviews of properties that have oil or gas leases, reflect concerns about the effects of risks of contamination on land values. Indeed, Muehlenbachs, Spiller and Timmins [2014] found in a detailed hedonic property value analysis fusing home sales in New York and Pennsylvania that the disamenity effects of the perceived risk of groundwater contamination due to fracking on housing values were large and significant ranging from -10% to -22%. The estimated effects exceeded the potential royalty income that homes with piped water supplies might expect because they were proximate to gas supplies. With a piped water supply these homes would not face the risk of a contaminated well. The differences in state policies toward shale development no doubt partially reflect concerns about these risks. Our compensation measure for policy-induced quantity uncertainty is a local approximation for sources of risk that cause local variation in environmental services. That is, the likely variation in services induced by the uncertainty is relatively small around the expected service level. In many applications this mean level of services may not be a well-defined concept because the services themselves are not well-defined. Nonetheless, we would suggest our approach can be treated as is a starting point for discussions of how the concept of compensation should be defined for environmental bonds that could help in securitizing some sources of environmental risk. Policy-induced price uncertainty can arise with policies intended to address long term environmental issues. So our discussion of the case of price uncertainty need not require large disruption to an economy, as our general equilibrium characterization may have implied. As Borenstein et al. [2014] document, the complex array of regulations in California’s greenhouse gas reduction program and the diversity of outcomes in how firms respond to them implies considerable uncertainty in the business as usual emissions. A key element in the program is a cap and trade permit system. Thus they conclude price volatility is likely to be an inherent feature of the policy’s design. Since the components in California’s program are also present to some degree in the EU’s permit system they suggest price volatility would be likely for this system. Both of these


19This proposal was made in Smith [2012]. This approach to securitizing environmental risks is related to the catastrophe bonds (CATs). Kunreuther and Michel-Kerjan [2009] explain the concept by noting that insurers may wish to access funds if a severe disaster produces large losses. In their example they suggest the terms can imply forgiveness of the interest, principal or both if a trigger set of conditions is reached. These triggers are generally related to the size of the disaster.
outcomes stem from policy design and fit our story. The basic message is that welfare measures used to evaluate the policies involved need to consider how their implementation affects the set of exogenous changes consumers are assumed to experience when we define welfare measures for the policies. Our analysis is a start for a more detailed analysis of a range of characterizations of how policy design and implementation can influence uncertainty inside and outside markets. Summary Applied welfare models have a subtle but potentially important difference in the strategy used for price uncertainty as compared to uncertainty in the amount a non-market good that is available to an individual. Two contributions to the literature on the implications of price uncertainty for policy judgments are used to illustrate how the same machinery can yield new insights when used to address quasi-fixed non-market goods. The first of these is a straight forward adaptation of the Turnovsky et al. analysis of the limits to Waugh’s early conclusion that price instability can be welfare enhancing as compared to a fixed price (at the mean) in a partial equilibrium context. We show that the same logic provides some traction for the task of defining a compensation measure for policies that create uncertainty in environmental services. Using the Turnovsky et al. logic we develop an approximation for compensation. The logic we use is similar to that developed with a willingness to pay (as opposed to compensation) logic for defining the risk premium that would be defined for a certainty equivalent formulation of the individual choice problem. It is also possible to characterize the commodity specific coefficient of relative risk aversion for the environmental services in terms of parameters where we have greater experience with the range of plausible estimates. Our second exercise has direct relevance to several types of policies including recent literature using equilibrium sorting models to describe household responses to policies changing non-market goods. We show that partial equilibrium expected consumer surplus measures that treat equilibrium adjustments in response to a policy induced change in non-market good as a source of price uncertainty from an individual’s perspective, would consistently reflect changes in expected utility. This conclusion would suggest simple expected surplus measures defined based on an assumption that prices change are random due to equilibrium adjustments in response to policy may well offer the best strategy for quick evaluation exercises. In this context, the analyst is unlikely to have the time or resources to develop a full equilibrium model to assess policies that could induce heterogeneous households to adapt and, as a result, lead to price changes in linked private goods. Instead, characterizing the adjustment as leading to uncertain prices and using the change in expected consumer surplus would be a practical alternative with desirable properties.

5 Appendix

5.1 Conditions for $\tilde{p}_{pq}/\tilde{p}_q \geq 0$

Suppose that the equilibrium price is determined implicitly by the equation $Z(p, q, \phi) = 0$. The canonical case is $Z$ is the excess demand for good 1 in an economy with non-market good equal to $q$ and taste or technology parameter $\phi$: $Z(p, q, \phi) = X^D_1(p, q, \phi) - X^s_1(p, q, \phi)$, where $X^D_1$ is aggregate demand for good 1 and $X^s_1$ is aggregate supply. (In the case of a public good, $S$ should not depend on $q$.) Suppose we are able to choose the units of the parameter $\phi$ so that $Z_\phi > 0$ and assume
that $Z_p < 0$ always. Then of course $p_\phi > 0$ always. We want conditions to ensure that $\ln p_\phi$ is non-decreasing in $q$. The next result is an immediate consequence of the implicit function theorem.

**Lemma 1.** If $\ln(Z_\phi) - \ln(-Z_p)$ is increasing in $q$, then $\ln p_\phi$ is increasing in $q$.

**Proposition.** If the condition in the Lemma holds, weak complementarity holds, $V_m$ is increasing in $p$ for a consumer, and equilibrium consumption of good 1 increases in $q$ for a consumer, the that consumer is interim expected utility falls when $q$ increases if interim expected surplus falls when $q$ increases.

### 5.2 Some examples

Some formal examples may help in understanding the implications of the Lemma and the Proposition.

**Example 1.** Suppose that $Z$ is a competitive economy’s excess demand for the weak complement. Let the aggregate demand for good 1 be $D(p, q, \theta) = g(q^{-1}, \theta)$, where $g_1 \leq 0$ with a strict inequality whenever $D > 0$. If

$$y \frac{g_{11}(y, \theta)}{g_1(y, \theta)} \leq 1 \quad \text{for } y > 0,$$

then $D_{pq} \geq 0$, implying that $-\ln (-Z_p)$ is increasing in $q$. If $g_{12} \leq 0$, $\theta$ does not enter supply, and $q$ does not affect supply, then the condition in the Lemma holds. If (28) holds, $\theta$ does not affect demand, and $q$ does not affect supply, then the condition in the Lemma holds. (If income enters demand—$D(p, m, q, \theta) = g(q^{-1}, \theta, m)$—then the Willig condition holds if $g_1 g_3 = g_3 g_{13}$, which is equivalent to the condition that the income elasticity of demand does not depend on price. The last in turn follows if the indirect utility function is additively separable in price and income, for example homothetic preferences.) And interpretation of this form for $D$: an increase in $q$ increases each consumer’s valuation for the weak complement, and it increases the valuation of high valuation consumers more than it does for low valuation consumers (the inverse demands indexed by $q$ become more spread out, the smaller is total consumption). If $\theta$ affects demand it could represent the “size” of the market (the inverse demands indexed by $\theta$ becomes more spread out, the larger is total consumption). A special case is the linear demand $D = \theta \left(1 - \frac{p}{q}\right)$.

**Example 2.** Suppose that the market for the private good is competitive with a fixed supply $X_1^s$ (housing services in a narrow valley). Suppose that there are two consumer types: one type does not care for the private good at all and the other type has a demand for good 1 equal to $f(p/q)$, where $f$ is a decreasing function. Let $\phi$ equal the number of consumers of the second type. Then the aggregate demand is $X_1^p(p, q, \phi) = \phi f(p/q)$. Since $X_1^s$ is a constant, the equilibrium aggregate quantity is constant; and because the demand for the second type of consumer is proportional to aggregate demand, each consumer’s equilibrium consumption is constant. ($X_{1pp\phi} + X_1q = 0$). The equilibrium price is given by $p = qg(X_1^s/\phi)$, where $g = f^1 - 1$. It follows that $\tilde{p}_{q\phi}/\tilde{p}_\phi = 1/q > 0$. So the conclusion of Proposition 1 holds here.

References


