Indirect Taxation in a Vertical Differentiated Market with Positional Externalities

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Abstract: Social factors are usually important when consumers purchase vertically differentiated products. The purchase of such goods usually offers utility over and above their characteristics; it may cause “status” and “envy” effects. This paper generalizes the standard vertical differentiation model to incorporate both these effects. It examines the effects of commodity taxation on profits and welfare, in a multiproduct monopoly which produces products of different quality. Although in the classical vertical differentiation model the optimal choice of the government is to subsidize the products, when the positional effects are strong enough, the government should tax these goods to restore welfare. We also explore, in the case of discriminatory taxation, which variant should be taxed more. It is shown that the optimal tax policy is to tax less the more luxurious variant.

\textit{Keywords: Commodity tax, vertically differentiated market, positional externalities}

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1 Introduction

Is it possible that social factors related with the purchasing decisions of consumers decrease welfare? How could the government intervene and restore such inefficiencies? The following example by Frank (2008) shows how such factors can cause harm. If some job candidates begin wearing expensive custom-tailored suits, then the other candidates become less likely to make favorable impressions on interviewers. Therefore, every candidate will start spend more on suits but since all spend more, the probability of success remains the same. In such a framework in which what matters is the relative and not the absolute consumption, a tax on the positional goods can be welfare improving.

Although such social factors are particularly present in vertically differentiated markets, as they are related with the differences in quality for the same product, in the standard vertical differentiation model (Gabszewicz and Thisse (1979), Mussa and Rosen (1978), Shaked and Sutton (1982)) they are completely ignored. In these models consumers care only about the characteristics of the product they buy. However, higher quality products such as designer clothing, sunglasses, tablets and handbags are a few of the many products that consumers buy not only for their quality but also for the social status these confer to them.

A “brand name” product may act as an indication of wealth to those who purchase it. On the other hand, the purchase of a product may affect negatively those who haven’t made a similar purchase. In our framework, we allow for the presence of both effects which we model as network externalities. Each consumer’s utility is affected positively by the number of people who have bought a lower quality model of the good or have made no purchase at all (status effect). In a similar way, each consumer’s utility is negatively affected by the number of consumers who have bought a higher quality model (envy effect). 1

The objective of this paper is, to find the optimal tax (or subsidy), of a multiproduct monopoly which produces products of different quality when the purchase of these products is associated with such positional externalities. Since the goods are produced by a monopoly, when such social factors do not exist, there is an incentive for the government to subsidize the products to correct for the monopoly pricing. However, when such factors are present, the increase in utility of those who initially could not afford to buy the good but now buy it at the lower (subsidized) price is not the only effect. As more consumers afford to buy the product, the status effect reduces the utility of those who initially purchase it. Finally as more people buy the product, the envy effect makes those who still do not buy the product worse off. The level of the optimal tax or subsidy depend on how strong these effects are.

It is shown that, although in the classical model of vertically differentiated markets

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1 The literature of the effects of the social factors on consumption originates from Veblen (1899). Leibenstein (1950) shows that the demand for a good may be a decreasing function of the number of people purchasing it. This “snob” effect is attributed to peoples’ need for exclusiveness. If, on the other hand, the demand increases as the number of people who purchase it increase, we have the “bandwagon” effect. Basu (1987) illustrates a model where aggregate excess demand for a good or a service is used as an index of status and increases consumers’ valuation of the particular commodity. The demand of a product may increase, when a purchase of such product acts as a signal of the consumers’ wealth (Bagwell and Bernheim (1996)) or improves their “image” (Eaton and White (2002)). Grilo et al. (2001), use the term “vanity” to describe “snob” effects and examine a horizontally differentiated duopoly showing that its presence results in higher prices and relaxes price competition.
the optimal choice of the government is to subsidize the product, when consumers also care about the status the product confers, a unit tax is welfare improving if the positional externality is sufficiently high.

When the monopoly produces two variants of the status good, and the positional effects are strong enough, the government should tax these goods to restore welfare. We also explore, in the case of discriminatory taxation, which variant should be taxed more. It is shown that the optimal tax policy is to tax less the most luxurious variant.

The consequences of taxation in models with status considerations has not been neglected in the literature. Ireland (1994), studies the impacts of a unit tax policy when consumers, in their efforts to impress, consume more of the status good trying to send a distorted signal of being of higher type. Spectators understand this and consumers are left with a choice of suboptimal consumption. He finds that a corrective tax could lead to a Pareto improvement.

Truyts (2012), investigates optimal indirect taxation when consumers choose consumption not only for intrinsic reasons but also for signaling. The utility function of consumers depends not only on their consumption but also on the spectators’ estimate of the income of the consumer according to his visible consumption. In this case consumers tend to spend too much on visible goods and optimal taxes deal with distortions resulting from signaling. In the benchmark case of pure costly signaling, in which goods are used only for signaling and do not have any intrinsic utility, goods can be taxed without burden.

The paper is structured as follows. Section 2 presents the model. In Section 3, we study the optimal tax when the monopoly produces one variant of the product. Section 4 deals with the optimal taxation when the monopoly produces two variants of the status good. Section 5 summarizes and concludes.

# 2 The model

There is a continuum of consumers who differ in their willingness to pay $\theta$ for quality. We assume that $\theta$ is distributed according to $F(\cdot)$ on the interval $[0, 1]$. There is a monopoly in the market that can produce one or two variants of the product: $S_1$ denotes the low quality and $S_2$ the high quality product with $S_1 < S_2$. Consumers buy at most one unit of the vertically differentiated good. Let the consumer of type $\theta_2$ to be indifferent between consuming the high or the low quality version of the product. Also, let the consumer of type $\theta_1$ to be indifferent between consuming the high or the low quality version of the product. As a result, there is a mass of $F(1 - \theta_2)$ consumers who buy the high end product, a mass of $F(\theta_2 - \theta_1)$ consumers who buy the less advanced product and a mass of $F(\theta_1)$ consumers who do not buy at all. As a result, $F(\theta_2)$ also denotes the mass of consumers who either buy $S_1$ or do not buy at all. Furthermore, $\alpha_i$ where $i = 1, 2$ are a non-negative parameters that show the intensity of the positional effects. These are modeled as network effects as follows: $\alpha_1$ denotes the status effect from the purchase of a variant of higher quality. The utility of those who purchase increases, when the mass of those who buy less advanced variants increases (note that as the demand remains constant this is equivalent to less consumers buying a variant of the same or higher quality). Similarly, $\alpha_2$ denotes the envy effect from either not purchasing at all or purchasing a variant of lower quality. The utility of these consumers increases, the less
consumers buy a product of higher quality than theirs.\footnote{Here we allow $\alpha_1$ to be different from $\alpha_2$. This approach is more general than Deltas and Zacharias (2012) who assume $\alpha_1 = \alpha_2$. Furthermore, they do not examine the optimal taxation of the monopoly as we do here.} Consumers’ utility who buy $S_2$ rises as $F(\theta_2)$ increases.

The indirect utility that a consumer of type $\theta_i$ derives from the consumption of the highest possible variant $S_2$ at price $P_2$ is given by:

$$U_i = \theta_i S_2 + \alpha_1 F(\theta_2) - P_2$$

Consumers who buy $S_1$ have positive utility as $F(\theta_1)$ increases but negative utility as $F(1 - \theta_2)$ increases. The indirect utility that a consumer of type $\theta_i$ derives from the consumption of $S_1$ at price $P_1$ is given by:

$$U_i = \theta_i S_1 + \alpha_1 F(\theta_1) - \alpha_2 F(1 - \theta_2) - P_1.$$  \hfill (1)

The indirect utility of a consumer who does not buy the product is affected negatively as the number of those who buy any variant of the product $F(1 - \theta_1)$ increases:

$$U_0 = -\alpha_2 F(1 - \theta_1).$$  \hfill (2)

The demand functions are defined by

$$Q_1 = F(\theta_2 - \theta_1) \text{ and } Q_2 = F(1 - \theta_2).$$  \hfill (3,4)

The monopoly produces $S_i$ with $i = 1, 2$ at a constant marginal cost $c_i$ with $c_i < S_i$. \hfill (5)

The government imposes a tax $t_i$ per unit of variant $i$. The profits $\pi$ of the monopoly are:

$$\pi = (P_2 - t_2 - c_2) F(1 - \theta_2) + (P_1 - t_1 - c_1) F(\theta_2 - \theta_1).$$  \hfill (6)

Consumer surplus is the sum in the surplus of the consumers that consume the product and of those that do not make a purchase, and is given by

$$CS = \int_0^{\theta_2} (-\alpha_2 F(1 - \theta_1)) \ dF(\theta) + \int_{\theta_1}^{\theta_2} (\theta S_1 + \alpha_1 F(\theta_1) - \alpha_2 F(1 - \theta_2) - P_1) \ dF(\theta) +$$

$$+ \int_{\theta_2}^1 (\theta S_2 + \alpha_1 F(\theta_2) - P_2) \ dF(\theta).$$  \hfill (7)

Tax revenues are the sum of revenues from taxing the low and the high quality variant of the product

$$R = t_1 F(\theta_2 - \theta_1) + t_2 F(1 - \theta_2).$$  \hfill (8)

The welfare of the country is given by the sum of the consumer surplus, the profits and the government’s tax revenues

$$W = CS + \pi + R.$$  \hfill (9)

We first explore the case in which the monopoly produces only one variant. We examine the effects of the social planner’s intervention under various ways that the positional parameters may affect the various subsets of consumers. We then assume that the government can impose an optimal tax and compare these two ways of intervention. Finally, we do the same assuming that the monopoly produces two variants.
3 The monopoly produces only one variant of the status good

To make the analysis easier, in what follows we will assume that $F(\cdot)$ is the uniform distribution. In this section we assume that the monopoly produces only one variant of the good. As a result, consumers can either buy the variant of the status good, or not buy at all. The indirect utility that a consumer of type $\theta_i$ derives from the consumption of the product with quality $S$ at price $P$ is given by

$$U_i = \theta_i S + \alpha_1 \theta_1 - P$$  \hspace{1cm} (10)$$

while the indirect utility of a consumer who does not buy the product is

$$U_0 = -\alpha_2 (1 - \theta_1).$$  \hspace{1cm} (11)$$

The consumer who is indifferent between buying the product or not is of theta

$$\theta_1 = \frac{P - \alpha_2}{S + \alpha_1 - \alpha_2},$$  \hspace{1cm} (12)$$

and therefore, the demand function is given by

$$Q = 1 - \theta_1 = \frac{S + \alpha_1 - P}{S + \alpha_1 - \alpha_2}.$$  \hspace{1cm} (13)$$

Consumer surplus is the sum in the surplus of the consumers that consume the product and of those that do not make a purchase, and is given by

$$CS = \int_0^{\theta_1} (-\alpha_2 (1 - \theta_1)) \, d\theta + \int_{\theta_1}^1 (\theta S + \alpha_1 \theta_1 - P) \, d\theta.$$  \hspace{1cm} (14)$$

Finally, the profits of the monopoly are:

$$\pi = (P - t - c)(1 - \theta_1).$$  \hspace{1cm} (15)$$

We first characterize the equilibrium in the absence of government intervention (i.e., when government does not impose a tax), which we refer to as the unregulated equilibrium. The optimal values of the variables in the unregulated equilibrium are presented in Deltas and Zacharias (2013) for $a_1 = a_2$. For $a_1 \neq a_2$ one can find them by setting $t = 0$ in Lemma 1 and are the following

$$P^{t=0} = \frac{\alpha_1 + c + S}{2}, Q_i^{t=0} = \frac{S + \alpha_1 - c}{2S + \alpha_1 - \alpha_2}, \pi^{t=0} = \frac{1}{4} \frac{(S + \alpha_1 - c)^2}{S + \alpha_1 - \alpha_2}.$$  \hspace{1cm} (16)$$

Note that, from (5), in the absence of status and envy effects the quantity is always positive. The additional requirements in the general case is that the envy effect, is not very strong i.e.,

$$S + \alpha_1 - \alpha_2 > 0$$  \hspace{1cm} (17)$$

so as the profits and quantity to be positive.

The welfare level in the unregulated equilibrium is

$$W = \frac{(S + \alpha_1 - c) (3S^2 - 3Sc + \alpha_1 (5S - 6\alpha_2 + 2\alpha_1 - 2c) + 2\alpha_2 (2\alpha_2 - 3S + c))}{8(\alpha_2 - \alpha_1 - S)^2}.$$
3.1 The first best optimum

We start by characterizing the first best optimum, when the monopoly produces one variant of the status good, so as to use it as a benchmark. In the first best optimum it is the social planner and not the monopoly that chooses the price of the product. The social planner chooses the appropriate price that maximizes country’s welfare. In this way, she removes the distortion in the market that comes from the monopoly pricing.

We analyze the general case in which the product not only confers utility above its characteristics to those who buy them (due to status considerations), but also creates disutility to those who don’t buy the product (due to the envy effect). In the presence of status and envy effects, we have the following Proposition.

Proposition 1 The social planner sets price

\[ P^{sp} = \frac{\alpha_1 S + Sc + (c + \alpha_1 + \alpha_2)(\alpha_1 - \alpha_2)}{S + 2(\alpha_1 - \alpha_2)}. \]  \hspace{1cm} (18)

The quantity of the monopoly is

\[ Q^{sp} = \frac{S - c + \alpha_1 - \alpha_2}{S + 2(\alpha_1 - \alpha_2)}, \]  \hspace{1cm} (19)

its profits are

\[ \pi^{sp} = \frac{(S - c + \alpha_1 - \alpha_2)[\alpha_1 (S - c) + \alpha_1^2 - \alpha_2^2 + \alpha_2 c]}{[S + 2(\alpha_1 - \alpha_2)]^2} \]  \hspace{1cm} (20)

and the welfare is

\[ W^{sp} = \frac{1}{2} \frac{(S - c + \alpha_1 - \alpha_2)^2}{S + 2(\alpha_1 - \alpha_2)}. \]  \hspace{1cm} (21)

Note for later use that since \( \frac{\partial^2 W^{sp}}{\partial P^2} = -\frac{S + 2(\alpha_1 - \alpha_2)}{(S + \alpha_1 - \alpha_2)^2} \), the welfare function is concave when

\[ S + 2(\alpha_1 - \alpha_2) > 0. \]  \hspace{1cm} (22)

Using this restriction and the fact that from (12), after using (18)

\[ \theta_1^{sp} = \frac{c + \alpha_1 - \alpha_2}{S + 2(\alpha_1 - \alpha_2)}, \]  \hspace{1cm} (23)

we have

\[ c + \alpha_1 - \alpha_2 > 0 \]  \hspace{1cm} (24)

so as \( \theta_1^{sp} > 0 \). Moreover, for the quantity to be positive it should be

\[ S - c + \alpha_1 - \alpha_2 > 0. \]  \hspace{1cm} (25)

These inequalities will be of use later on.

By setting \( \alpha_2 = \alpha_1 = 0 \) in (18) and (20), we can examine the first best outcome in the standard vertical differentiation model with fixed qualities. One can see that in the absence of status and envy effects, the price that the social planner chooses is equal to the marginal cost and the profits of the monopoly are zero. The social planner removes the distortion caused by the monopoly.
We now examine the effects of each social factor separately. In the literature, the purchase of a status good is considered a signal of wealth (Ireland 1994). As a result, the purchase of such good affects positively only those who buy the product. In that framework, those who do not buy, do not suffer any disutility from not purchasing the product. It is interesting to explore the first best optimum when the purchase of the good has only this positional feature. The signal approach corresponds to $\alpha_2 = 0$ in the present model, as in such case those who do not buy the product do not suffer any disutility.\footnote{In the vertical differentiation literature, this model has been explored by Lambertini and Orsini (2002). However, they do not examine tax implications.} In this case, the price a social planner would choose is

$$P_{\alpha_2=0}^{sp} = \frac{(\alpha_1 + c) (\alpha_1 + S)}{S + 2\alpha_1}, \quad (26)$$

It is easy to show that $P_{\alpha_2=0}^{sp} > c$, i.e., the social planner sets a price which is higher than the marginal cost (which is the optimal price in the absence of social factors). As those who buy gain more utility the less they are, it is optimal for the social planner to set a higher price than the marginal cost, so as to reduce the quantity sold.

Furthermore, we can observe that the price a social planner would choose increases as the intensity of the status effect increases:

$$\frac{\partial P_{\alpha_2=0}^{sp}}{\partial \alpha_1} = \frac{S (S - c) + 2\alpha_1 (S + \alpha_1)}{(S + 2\alpha_1)^2} > 0.$$  

From above, as $\alpha_1$ increases, the price increases. A rise in the price on the one hand reduces consumers utility, but on the other hand results to less consumers buying the product which rises the utility of those who still buy. An increase in $a_1$ intensifies the rise in the utility of those who purchase the product. This beneficial effect overcomes the negative effect in welfare from the consumers that have to pay a higher price and of those that cannot afford to buy the good anymore.

In the presence of the envy effect, those who don’t buy the product experience disutility since they envy those who purchase the good. However, the buyers do not gain additional utility from status considerations i.e., $\alpha_1 = 0$. In this case we have:

$$P_{\alpha_2=0}^{sp} = \frac{c (S -\alpha_2) - \alpha_2^2}{S - 2\alpha_2}, \quad (27)$$

The social planner’s price is increasing in $\alpha_2$ since

$$\frac{\partial P_{\alpha_1=0}^{sp}}{\partial \alpha_2} = \frac{S (c - 2\alpha_2) + 2\alpha_2^2}{(S - 2\alpha_2)^2},$$

The nominator has two roots: $\frac{1}{2} \left( S \pm \sqrt{S(S - 2c)} \right)$. Note that if $S < 2c$, a rise in $\alpha_2$, rises the price and decreases the quantity sold. In that case in which $S - c < c$, i.e., the quality of the product is not so high comparing to the cost, and the damage from increasing the price is not very high since the utility the consumers derived was low due to the low quality of the product. Therefore the social planner reacts to the rise of $\alpha_2$ by rising the price since in that case the lower quantity sold rises the utility of those who don’t buy without reducing much the utility of those who cannot afford to buy it anymore.
If on the other hand \( S > 2c \), the price is increasing in the envy parameter when the envy effect is small\(^4\), i.e., \( \alpha_2 < \frac{1}{2} \left( S - \sqrt{S(S - 2c)} \right) \). However, when the difference between the quality of the product and the cost is high enough \((S - c > c)\), there are circumstances under which a rise in \( \alpha_2 \) results in a price reduction. This happens because the consumers now derive high utility due to the high quality of the product and therefore the beneficial effect from reducing the price and make the good available to more people exceeds the negative effect that the increased purchases have on the utility of those who don’t buy.

Let first assume that the two effects are of the same magnitude: When \( \alpha_2 = \alpha_1 > 0 \) from (18) we see that the price the social planner sets exceeds the marginal cost. On the other hand, from (19) we see that the quantity is not affected by the presence of the status and envy effects as the two social factors cancel each other. Although profits increase, welfare is not affected.

### 3.2 Optimal taxation in the presence of the status and envy effects

Now we turn to the case in which the government imposes a tax on the status good. Here, the government can not choose directly the price that the monopoly sets as in the social planner’s solution, but can affect it indirectly through the tax rate. It is useful to consider first the general case with both status and envy effects. Then we will isolate each of these characteristics in our analysis.

There are two stages in the model: first the government imposes a unit commodity tax and then the monopoly chooses the appropriate price that maximizes its profits. We solve the model using backwards induction. First we solve the second stage of the game, in which the monopoly chooses the price of the product given the tax set by the government. Then we find the optimal tax that the government will choose.

Starting from the last stage of the game, we express the solution of the monopoly as a function of the optimal tax. We have:

**Lemma 1** Given the tax rate imposed by the government, the monopoly that produces only one variant of the status good sets price and quantity

\[
P = \frac{\alpha_1 + c + S + t}{2}, \quad Q = \frac{1}{2} S + \frac{\alpha_1 - c - t}{\alpha_1 - \alpha_2}
\]

and its profits are:

\[
\pi = \frac{(S - c + \alpha_1 - t)^2}{4 [S + \alpha_1 - \alpha_2]},
\]

Furthermore, the tax revenues and consumer’s surplus are

\[
R = \frac{t(S - c + \alpha_1 - t)}{2 [S + \alpha_1 - \alpha_2]},
\]

\[
CS = \frac{[4\alpha_2 (\alpha_2 - \alpha_1) + S (\alpha_1 - 4\alpha_2 + S - c - t)] (S - c - t + \alpha_1)}{8 [S + \alpha_1 - \alpha_2]^2},
\]

\(^4\)We know that \( a_2 \) cannot be higher than \( \frac{1}{2} \left( S + \sqrt{S(S - 2c)} \right) \), since this is equivalent to \( 2\alpha_2 - S > \sqrt{S(S - 2c)} \), which is a contradiction since from (22) \( 2\alpha_2 - S < 0 \).
and the welfare is

\[ W = \frac{(S - c - t + \alpha_1) [t(S + 2(\alpha_1 - \alpha_2)) + S(5\alpha_1 - 3c + 3S - 6\alpha_2) + 2(\alpha_2 - \alpha_1)(2\alpha_2 + \alpha_1 + c)]}{8(S + \alpha_1 - \alpha_2)^2}. \]  

(32)

One can see that a higher tax increases the equilibrium price while reduces the quantity. Then we turn to the first stage, where the government chooses the tax rate that maximizes country’s welfare. The optimal tax rate can be found by maximizing (32) with respect to \( t \) and is given by:

**Proposition 2** When the monopoly produces one variant of the status good the optimal tax rate is

\[ t = \frac{2\alpha_2(S + \alpha_1 - \alpha_2) - S(S - c + \alpha_1)}{S + 2(\alpha_1 - \alpha_2)} = \alpha_2 - SQ. \]  

(33)

where \( Q = \frac{S-c+\alpha_1-\alpha_2}{S+2(\alpha_1-\alpha_2)} \).

\( W \) is concave in \( t \) when \( S > 2(\alpha_2 - \alpha_1) \) since \( \frac{\partial W^2}{\partial t^2} = -\frac{S+2(\alpha_1-\alpha_2)}{4(S+\alpha_1-\alpha_2)^2} \). This is a useful condition for later on.

Substituting the optimal tax in (28),(29),(30),(31) and (32), one obtains the following Proposition

**Proposition 3** The monopoly that produces only one variant of the status good sets price and quantity

\[ P = \frac{(\alpha_1 - \alpha_2)(c + a_2 + a_1) + S(a_1 + c)}{S + 2(\alpha_1 - \alpha_2)}, Q = \frac{S-c+\alpha_1-\alpha_2}{S+2(\alpha_1-\alpha_2)} \]  

(34)

and its profits are:

\[ \pi = \frac{(S + \alpha_1 - \alpha_2)(S - c + \alpha_1 - \alpha_2)^2}{[S + 2(\alpha_1 - \alpha_2)]^2}. \]  

(35)

Furthermore, the tax revenues and consumer’s surplus are

\[ R = \frac{[(\alpha_1 - \alpha_2)(2a_2 - S) - S(-a_2 + S - c)](S - c + \alpha_1 - \alpha_2)}{[S + 2(\alpha_1 - \alpha_2)]^2}, \]  

(36)

\[ CS = \frac{[4a_2(\alpha_2 - \alpha_1) + S(\alpha_1 - 3a_2 + S - c)](S - c + \alpha_1 - \alpha_2)}{2[S + 2(\alpha_1 - \alpha_2)]^2}, \]  

(37)

and the welfare is

\[ W = \frac{1}{2} \frac{(S - c + \alpha_1 - \alpha_2)^2}{S + 2(\alpha_1 - \alpha_2)}. \]  

(38)

In the unregulated equilibrium (that is without any government intervention) the monopoly charges different price from the socially optimal.\(^5\) Under regulation, a social planner corrects the distortion that is created from monopoly pricing.

\(^5\)It is easy to show that when the behavior of consumers is not affected by social factors or when there is only envy effect, the monopoly sets higher than the socially optimal price.
With the tax, the government achieves the same price, quantity, consumer surplus and welfare as in the first best. If the tax rate is positive, i.e. \( t > 0 \), the profits are lower than in the social planner’s solution since from (29) and (20), after using (33) and (28), we get
\[
\pi - \pi^{sp} = -tQ.
\]
Moreover, the tax revenues are positive. Therefore, by imposing a tax the government achieves the same welfare as in the first best, but takes some of the monopoly gains as tax revenues. The opposite occurs when the tax is negative: the government subsidizes the product and increases the firm’s profits. Notice that when the two effects are of the same size, i.e. \( a_1 = a_2 > 0 \), their effects on welfare are cancelled out. The welfare is the same as in the case without the positional effects.

**Corollary 1** By imposing a unit tax the government achieves the same price, quantity and welfare as in the first best for all values of \( a_1 \) and \( a_2 \).

We now examine how the social factors affect the pricing strategy and the quantity sold by the monopoly.

The effects of the two positional effects on the price and the quantity sold are:
\[
\frac{\partial P}{\partial \alpha_1} = \frac{S [S - c + 2 (\alpha_1 - \alpha_2)] + 2 (\alpha_1 - \alpha_2)^2}{[S + 2 (\alpha_1 - \alpha_2)]^2},
\]
\[
\frac{\partial P}{\partial \alpha_2} = \frac{S [c + 2 (\alpha_1 - \alpha_2)] + 2 (\alpha_1 - \alpha_2)^2}{[S + 2 (\alpha_1 - \alpha_2)]^2} = \frac{\partial P}{\partial \alpha_1} + S \frac{\partial Q}{\partial \alpha_1}
\]
and
\[
\frac{\partial Q}{\partial \alpha_1} = - \frac{\partial Q}{\partial \alpha_2} = \frac{2c - S}{[S + 2 (\alpha_1 - \alpha_2)]^2}.
\]

When \( \alpha_1 > \alpha_2 \), an increase either in \( \alpha_1 \) or in \( \alpha_2 \), increases price. When the status effect dominates the envy effect, the price increases as the increase in utility of those who continue to buy (due to the status effect) is higher than the decrease in utility of those who continue not to buy (due to the envy effect). The result may be reversed when the status effect \( \alpha_1 \) is sufficiently smaller than the envy effect \( \alpha_2 \).

Notice that the effect on the quantity sold is opposite when the two effects increase: if \( S > 2c \), \( Q \) decreases as \( \alpha_1 \) increases and increases when \( \alpha_2 \) increases. The opposite occurs when \( S < 2c \).

When the government does not impose a tax i.e., in the unregulated equilibrium, an increase in \( a_2 \) always results in higher quantity (\( \frac{\partial Q}{\partial a_2} = \frac{S + a_1 - c}{(2S + a_1 - a_2)^{3/2}} > 0 \)) while the effect of \( a_1 \) on the quantity depends on the sign of \( S + c - a_2 \) since \( \frac{\partial Q}{\partial a_1} = \frac{S + c - a_2}{(2S + a_1 - a_2)^{3/2}} \). It is evident that the tax changes the effect that these social factors have on the quantity sold. Summarizing

**Proposition 4** The equilibrium quantity of the monopoly is increasing in the status parameter and decreasing in the envy parameter when \( S < 2c \), is invariant to both of them when \( S = 2c \) and decreasing in the status parameter and increasing in the envy parameter otherwise.
In order to have a better understanding of the forces at work and be able to isolate the effect of each social factor, in what follows we will analyze three simplified cases of the model. We begin with the optimal tax in the absence of envy effect. Then, we find how the optimal tax changes in the absence of status effects. Finally, we find the optimal tax when the two effects are of the same magnitude.

It is interesting to discuss shortly about the case in which there are no status or envy effects. The optimal values in the absence of such social factors can be easily found by setting \( a_1 = a_2 = 0 \). The optimal tax in the standard model in which there is no status and envy effects is negative\(^6\) and equal to \( t_{a_2 = a_1 = 0} = -(S - c) \). Since the price that the monopoly sets in the unregulated equilibrium is always higher than the price that a social planner would choose, the optimal strategy for the government is to impose a negative tax (subsidy) such as to reduce the price the monopoly sets. In this case, the price and the quantity are the same as the one that the social planner would choose. Therefore, with the tax the government can correct the distortion from monopoly pricing (note that, after the tax, the price that the monopoly sets is equal to the marginal cost) and achieve the same welfare as in the first best optimum.

Another interesting observation in this case in which there are no social effects is that the profits of the monopoly are only due to the subsidies. In order to achieve lower price, the government transfers funds to the monopoly by subsidizing the product, by an amount equal to the monopoly profits.

In what follows we isolate each social factor and we analyze how it affects the equilibrium.

### 3.2.1 Optimal taxation in the presence of the status effect

We now turn to the case in which the purchase of the good offers an additional status to those who purchase it (i.e., \( a_1 > 0 \)). However, there is not an envy effect. Those who do not buy the product have zero utility and are not affected negatively when the number of consumers that buy the product increases (i.e. \( a_2 = 0 \)). We have the following Corollary

**Corollary 2** The price and quantity the monopoly would choose in the presence of the status effect are

\[
Q_{a_2 = 0} = \frac{a_1 + S - c}{S + 2a_1}, P_{a_2 = 0} = \frac{(\alpha_1 + c)(S + a_1)}{S + 2a_1},
\]

its profits and the government revenues are

\[
\pi_{a_1 = 0} = \frac{(S + \alpha_1)(S - c + \alpha_1)^2}{(S + 2\alpha_1)^2}, \quad R = -\frac{S(S - c + \alpha_1)^2}{(S + 2\alpha_1)^2}
\]

the optimal tax rate is

\[
t_{a_2 = 0} = -\frac{S(a_1 + S - c)}{S + 2a_1} = -SQ_{a_2 = 0}
\]

and the welfare is

\[
W_{a_2 = 0} = \frac{(S - c + a_1)^2}{2(S + 2a_1)}
\]

\(^6\) The quantity \( Q_{a_2 = a_1 = 0} \) is positive only when \( S - c > 0 \), and therefore, the tax is negative.
As it is evident from (41), the tax rate is negative. When the government does not intervene, that is in the unregulated equilibrium in which \( t = 0 \), the price that the monopoly sets is always higher than the price that a social planner would choose. Therefore, the optimal strategy for the government is to impose a negative tax (subsidy) in order to reduce the price the monopoly sets.

The lower price makes the good available to more people: This increases the utility of those who can afford to buy now from zero to positive. On the other hand, the increased number of purchases reduces the utility of the consumers who initially could afford to buy due to the status effect. The beneficial effects of a price reduction are stronger than those which reduce welfare, and therefore, a subsidy that reduces the price is welfare increasing.

Furthermore, from
\[
\frac{\partial a_2}{\partial a_1} = \frac{S(S - 2c)}{(S + 2a_1)^2} = -S \frac{\partial Q_{a_2=0}}{\partial a_1}
\]
as \( a_1 \) increases, when the number of people who purchase the product increases (i.e. when \( \frac{\partial Q_{a_2=0}}{\partial a_1} > 0 \)), the optimal tax decreases. A rise in \( a_1 \) not only increases directly the price, but also affects it through the tax, as it is evident from equation (28) in Lemma 1. If the rise in \( a_1 \) results in more people buying the product, the utility of those who could buy the good before the rise in \( a_1 \) will decrease a lot. The lower utility comes from, not only the higher price, but also the fact that the higher number of consumers, combined with the stronger status effect, reduces their utility since the product they buy is not so scarce any more. This negative effect dominates the positive effect of the increased utility of the new buyers. Therefore, it is optimal for the government to decrease the tax, so as to put a downward pressure in the price and boost consumers’ utility. If, on the other hand, the rise in \( a_1 \) leads to a reduction in the number of people that buy the product, consumers’ utility will increase a lot, not only because less people buy the good but also due to the higher \( a_1 \). In that case, the government will increase the tax so as to increase the price.

### 3.2.2 Optimal taxation in the presence of the envy effect

Here we explore the case in which, although the purchase of the good does not give any additional utility to those who buy it (\( a_1 = 0 \)), it affects negatively those who do not buy it. That is, we have the envy effect (\( a_2 > 0 \)). In this case we have

**Corollary 3** The monopoly that produces only one variant of the good, in the presence of the envy effect sets price and quantity
\[
P_{a_1=0} = \frac{-a_2 (\alpha_2 + c) + Sc}{S - 2\alpha_2}, Q_{a_1=0} = \frac{S - c - \alpha_2}{S - 2\alpha_2},
\]
its profits are
\[
\pi_{a_1=0} = \frac{(S - \alpha_2) (S - c - \alpha_2)^2}{(S - 2\alpha_2)^2}
\]
and the welfare is
\[
W_{a_1=0} = \frac{1}{2} \frac{(S - c - \alpha_2)^2}{S - 2\alpha_2}.
\]
Moreover, the optimal tax rate is
\[ t_{a_1=0} = \frac{-2a_2^2 + 2Sa_2 - S(S-c)}{S - 2a_2} = \alpha_2 - SQ_{a_1=0}. \] (45)

Note that the welfare in (32) is concave in this case with only envy effect when \( S > 2a_2 \). For \( Q_{a_1=0} > 0 \) we require that \( S > c + \alpha_2 \). Furthermore, for \( P_{a_1=0} > 0 \), we also require that \( Sc > a_2(a_2 + c) \). The optimal tax may be positive or negative. Thus, contrary to the case where there are not social factors or where there is only status effect (in which the optimal tax is negative), in the presence of envy effect, there are circumstances under which the optimal strategy for the government is to tax and not subsidize the commodity the monopoly produces. For example, for \( c \) close to \( S \) or for high \( a_2 \), \( t_{a_1=0} > 0 \).

3.2.3 Optimal taxation with two positional effects of the same magnitude

To avoid complications that arise when we assume that the two social effects are of different magnitude, here we assume that the magnitude of the status and envy effects are the same i.e., \( \alpha_1 = \alpha_2 = \alpha \). This allow us to derive the effects of a tax in the model by Deltas and Zacharias (2013).

By imposing this restriction in (34) and (35) one can find that the monopoly that produces only one variant of the status good sets price and quantity
\[ P = a + c \text{ and } Q = \frac{S - c}{S}, \]
its profits are \( \pi = \frac{(S-c)^2}{S} \) and the welfare is \( W = \frac{1}{2} \frac{(S-c)^2}{S} \).

Although the quantity sold is the same with the case that there are no social factors associated with the purchase of the good the price increases. As the number of consumers who buy does not change, the envy effect remains the same. On the other hand, the increase in price allows the monopoly to extract surplus from those who buy and enjoy the status effect.

At this point, it is interesting to see how the values in the equilibrium change when there is a tax. The results of Deltas and Zacharias (2013) can be found in our framework by setting \( \alpha_1 = \alpha_2 = \alpha \) in the unregulated equilibrium in (16). One can see that with the introduction of the tax can increase or decrease the price and quantity, depending on whether the optimal tax is negative or positive, since
\[ P^{t=0} - P^t = -\frac{t}{2}, \quad Q^{t=0} - Q^t = \frac{t}{2S}. \]

Additionally, by setting \( \alpha_1 = \alpha_2 = \alpha \) in (33), the optimal tax is given by

\[ t = \alpha + c - S. \tag{46} \]

We have the following Corollary:

**Corollary 4** When the monopoly produces one variant of the status good and the status effect is the same with the envy effect, the optimal strategy for the government is to impose a tax when the positional externality is sufficiently high.

From (46) we have that the optimal tax is positive if the difference between the quality and the cost is lower than the positional externality i.e., \( a > S - c \). Moreover, observe also that, since \( \frac{\partial^2 W}{\partial a^2} > 0 \), as the positional externality increases, the optimal tax increases as well.

Although in the classical model of vertically differentiated markets the optimal choice of the government is to subsidize the product, when the social considerations from purchasing a product increase, a unit tax is always welfare improving over the no tax equilibrium.

In general, a tax increase leads to a higher price, and a lower quality sold. When the price rises, consumers' utility on the one hand tend to decrease (since they buy the product at a higher price), but on the other hand the lower quantity sold puts an upward pressure on consumers utility due to the status and envy factors: Here we have three effects: The consumers who still purchase the product although they pay a higher price, they enjoy a higher utility due to the status effect. Those who wouldn't have bought the product also enjoy a higher utility as less consumers buy the product due to the envy effect. Similarly, those who stop purchasing the product after the price increase suffer less as now less people buy it due to the envy effect. As a result, when the positional effect is strong enough (i.e. \( a > S - c \)), the optimal strategy for the government is to increase the price by imposing a tax.

### 3.2.4 Effects of social factors on welfare and tax

So far we have shown that by imposing the appropriate tax, the government can achieve the first best outcome.

The explicit solution of the tax equilibrium allows to derive insights about comparative statics with respect to the envy and status parameters. We start with the effects of the positional parameters on welfare. As the monopoly, by its pricing policy, may affect welfare we focus on the levels of welfare that can be achieved after the government’s intervention.

From (38) we find that the effect of the two network effects on welfare is of equal but opposite magnitude

\[
\frac{\partial W}{\partial \alpha_1} = -\frac{\partial W}{\partial \alpha_2} = \frac{\partial W}{\partial (\alpha_1 - \alpha_2)} = \frac{(c + \alpha_1 - \alpha_2)(S - c + \alpha_1 - \alpha_2)}{(S + 2(\alpha_1 - \alpha_2))^2} = \theta_1 Q > 0.
\]

Changes in \( \alpha_1 \) and \( \alpha_2 \) have opposite effects on welfare. The welfare is increasing in \( \alpha_1 \) and decreasing in \( \alpha_2 \). An increase in the status parameter, increases welfare as those
who buy the product enjoy a higher utility. Similarly, as the envy parameter increases, the welfare decreases, as those who do not buy suffer more from not buying. As the (positive) difference $\alpha_1 - \alpha_2$, increases, welfare increases.

What is interesting is how the optimal tax changes when social issues start being important. We have analyzed the comparative statics when only status or envy effects exist and now we focus on the general case. The optimal tax increases as $a_1$ increases when $S > 2c$, and decreases when the opposite occurs as:

$$\frac{\partial t}{\partial a_1} = \frac{S(S - 2c)}{(S + 2(\alpha_1 - \alpha_2))^2} = -S\frac{\partial Q}{\partial a_1}. \quad (47)$$

One can see from (47) that, the optimal tax and the quantity sold move in opposite directions as $a_1$ increases. When an increase in $a_1$ results in an increase in $Q$ (that is when $S < 2c$) the optimal tax decreases. The optimal tax increases when the opposite occurs.

We also have that the optimal tax and the price move the same way as $a_2$ increases. For small (positive) differences in $\alpha_1 - \alpha_2$ an increase in $\alpha_2$ increases both the optimal tax and the price of the good.

$$\frac{\partial t}{\partial a_2} = \frac{2(2(\alpha_1 - 2\alpha_2)\alpha_1 + 2\alpha_2^2 + 2S(\alpha_1 - \alpha_2) + cS)}{(S + 2(\alpha_1 - \alpha_2))^2} = 2\frac{\partial P}{\partial a_2}. \quad (48)$$

One can see from that for low $a_2$, $\frac{\partial t}{\partial a_2} > 0$ and therefore the optimal tax rises as $a_2$ increases. In general, when the envy effect increases the price, a higher tax will be imposed.

The effect of the product quality on the equilibrium tax is also of interest. The partial derivative of equilibrium tax with respect to $S$ is given by

$$\frac{\partial t}{\partial S} = \frac{-(2(\alpha_1 - \alpha_2)^2 - S(S + 2(\alpha_1 - \alpha_2)) - 2\alpha_1(S - c) + 2\alpha_2(S + a_1 - c)}{(S + 2(\alpha_1 - \alpha_2))^2}. \quad (49)$$

The numerator has two roots: $a_2 = S + \alpha_1 - \frac{c}{2} \pm \frac{1}{2}\sqrt{2S(S - 2c) + c^2}$. Note that $a_2$ cannot be greater than the lower root. (See the proof in the Appendix). Therefore

$$a_2 < \left(S + \alpha_1 - \frac{c}{2} - \frac{1}{2}\sqrt{2S(S - 2c) + c^2}\right) \quad (49)$$

Thus the numerator is always negative, which implies that $\frac{\partial t}{\partial S} < 0$ i.e., the better the quality of the product the higher the tax. A rise in the product quality on the one hand increases consumers’ utility but on the other hand tend to increase the price (when the tax rate remains constant) which leads to lower utility. Since the consumers that buy the product benefit more now (from the better quality), the government now has an incentive to impose a lower tax so as to reduce the price and make the good available to more people.

Summarizing

**Proposition 5**  The optimal tax is a) decreasing in the quality of the product, b) increasing in the status parameter when the quantity is increasing in the status parameter and c) increasing in the envy parameter when the price is increasing in the status parameter.
4 The monopoly produces two variants of the status good

In the previous section, we have specified the equilibrium and the optimal tax when the monopoly produces one variant of the product. In this section we analyze how the status and envy factors affect the pricing decisions and welfare of a multiproduct monopoly. More particular, in this section we assume that the monopoly produces two variants of the same good. We start by characterizing the unregulated equilibrium and the first best optimum. Then we explore the case in which the government can impose a uniform unit tax rate to both variants that the monopoly produces. Finally, we allow the government to discriminate and impose different tax rates in each variant of the product. Due to the complexity of the results, our attention is focus on the case when \( a_1 = a_2 = a \).

We first derive the unregulated equilibrium when the monopoly produces two variants of the status good. The monopoly chooses prices that maximize its profits. Maximization of (6), after setting \( t_1 = t_2 = 0 \) and \( a_1 = a_2 = a \), gives

\[
\begin{align*}
P_1^{t=0} &= \frac{S_1 + c_1}{2} + \frac{a (c_2 - c_1)}{2 (S_2 - S_1)}, \\
P_2^{t=0} &= \frac{S_2 + c_2}{2} + \frac{a (-S_2 c_1 + S_1 c_2 + (S_2 - S_1)(a + S_1))}{2S_1 (S_2 - S_1)} = \frac{S_2 + c_2}{2} + a Q_1^{t=0} + \frac{a}{2}.
\end{align*}
\]

(50)

Substituting the optimal prices, one can find the equilibrium quantities

\[
\begin{align*}
Q_1^{t=0} &= \frac{S_1 c_2 + S_2 c_1 + a (S_2 - S_1)}{2 (S_2 - S_1) S_1}, \\
Q_2^{t=0} &= \frac{S_2 - S_1 - c_2 + c_1}{(S_2 - S_1) 2}.
\end{align*}
\]

(52)

4.1 The first best optimum

To achieve the first best optimum, a social planner will choose the price for each variant of the product that maximizes the welfare.

**Proposition 6** The social planner sets prices

\[
\begin{align*}
P_1^{sp} &= c_1 + \frac{a c_2 - c_1}{S_2 - S_1}, \\
P_2^{sp} &= c_2 + \frac{a (S_1 (S_2 - S_1) + c_2 S_1 - c_1 S_2)}{(S_2 - S_1)} = c_2 + a S_1 \left(1 + Q_1^{sp} \right).
\end{align*}
\]

(51)

and the quantities and the profits of the monopoly are

\[
\begin{align*}
Q_1^{sp} &= \frac{S_1 c_2 - S_2 c_1}{S_1 (S_2 - S_1)}, \\
Q_2^{sp} &= \frac{S_2 - S_1 - c_2 + c_1}{S_2 - S_1}, \\
\pi^{sp} &= \frac{a S_1 - c_1}{S_1}
\end{align*}
\]

(52) (53)

and the welfare

\[
W_2^{sp} = \frac{1}{2} - \frac{S_2 S_1^2 + 2 c_2 S_1^2 - 2 S_2 c_1^2 - S_1 (S_2 - c_2)^2 + S_2 c_1^2}{S_1 (S_2 - S_1)}
\]

(54)
Note that, as it is evident from (52), the socially optimal quantities are not affected by the social factors, the status and envy effects change only the prices. Inspection of (51) reveals that the optimal prices are equal to the marginal cost of producing each variant when there is no status and envy effects. The optimal prices are increased when status or envy effect are added in the model. This has some straightforward intuition. In the presence of status and envy effects, the social planner has an incentive to increase the prices. In that case, less people buy both variants of the products, and as a consequence not only the utility of the consumers who buy the product increases (due to the status effect) but also of those who do not buy increases (due to the envy effect). Of course due to the higher price, the utility of those who cannot buy the good or the high quality variant anymore decreases. However the latter effect is not strong enough to overcome the former.

Therefore, from (52) we have the following Corollary

**Corollary 5** When the monopoly produces two variant of the status good, we have:

\[ P_{i}^{sp} > P_{i}^{sp \mid a_2 = a_3 = 0}, \quad P_{i}^{sp} > P_{i}^{sp \mid a_2 = a_3 = 0}, \quad \text{for } i = 1, 2. \]

Furthermore we can compare the optimal price here with the unregulated equilibrium in (66) we have

\[
P_{1}^{sp} - P_{1}^{\pi} = \frac{S_1 - c_1}{2} - a \frac{(c_2 - c_1)}{2(S_2 - S_1)}, \quad P_{2}^{sp} - P_{2}^{\pi} = \frac{S_2 - c_2}{2} - a \frac{(c_2 - c_1)}{2(S_2 - S_1)} - \frac{S_1 - c_1 - a}{2S_1}.
\]

### 4.2 Optimal taxation: uniform tax \( t_1 = t_2 = t^u \)

In this case we assume that the government cannot discriminate and imposes the same tax on both variants of the product. Our objective is to investigate the welfare effects of a uniform tax.

**Proposition 7** When the monopoly produces two variants of the status good, the optimal uniform tax rate is

\[ t^u = a - S_1 + c_1. \]

The equilibrium prices and quantities are

\[
P_1 = P_1^{sp} - aQ_2, \quad P_2 = P_2^{sp} + Q_2 (S_2 - S_1 + a).
\]

\[
Q_1 = \frac{S_2S_1 - S_1^2 + S_1c_1 + S_1c_2 - 2S_2c_1}{2 (S_2 - S_1) S_1}, \quad Q_2 = \frac{S_2 - S_1 - c_2 + c_1}{(S_2 - S_1) 2}.
\]

The above Proposition shows that the price in variant 1 is lower than the social optimum and in variant two higher.

Moreover, the optimal strategy for the government is to impose a tax when the positional externality is sufficiently high. In that case in which \( a > S_1 - c_1 \), the price of variant 1 is higher than the unregulated equilibrium while the price of variant 2 is higher than the unregulated equilibrium when \( S > a > S_1 - c_1 \).

\[\text{Note that } P_1^{\pi} - P_1 = -\frac{1}{2} t \text{ and } P_2^{\pi} - P_2 = -\frac{1}{2} t \frac{(S-a)}{S}.\]
The intuition is easy to be understood when there is no status effect (i.e., \( a = 0 \)). In the unregulated equilibrium the price that the monopoly sets for both variants is always higher than the price that a social planner would choose (i.e. equal to the cost), as one can see from (66) and (51). With the tax the government achieves the price of variant 1 to be equal to the cost, as in the first best optimum, and the price of the variant 2 to be lower than the unregulated equilibrium but not as low as in the first best optimum. Therefore a uniform tax corrects the distortion due to the monopoly pricing in one variant. The reason for this is that there are two distortions in the market, due to the monopoly pricing of the two goods, and one policy instrument. Therefore the tax can correct the distortion in the price of only one good.

**Corollary 6** With a uniform tax and in the absence of social factors the government corrects the distortion due to the monopoly pricing in only one variant. This cannot be achieved in the presence of these effects.

Apart from the distortion from monopoly pricing of both variants, there is also the distortion from the status effect. With one uniform tax, the government cannot correct totally any of these distortions.

Comparing the welfare that can be achieved from optimal taxation with the welfare in the unregulated equilibrium and the social optimum, one can find that

\[
W_t^u - W_{2}^{t=0} = \frac{1}{8} \left( a + c_1 - S_1 \right)^2 > 0, \\
W_2^{sp} - W_t^{u} = \frac{1}{8} \left( -S_2 + S_1 + c_2 - c_1 \right)^2 > 0.
\]

Therefore with the tax the welfare is higher than with the case of unregulated equilibrium, but cannot achieve the welfare of the first best optimum.

Moreover, the welfare achieved with uniform taxation in the case of two variants is higher than the welfare with the one variant since

\[
W_t^u - W_1 = \frac{3}{8} \left( -S_2 + S_1 + c_2 - c_1 \right)^2 > 0.
\]

The previous discussion results in the following ranking of welfare under the different cases

\[
W_2^{sp} > W_t^u > W_2^{t=0}.
\]

### 4.3 Optimal taxation: Nonuniform tax \( t_1 \neq t_2 \)

Suppose now that the tax rate that is levied in each variant is different. We start solving the game from the last stage. Maximization of (6) with respect to \( P_1 \) and \( P_2 \), after using (67) and (68), gives the solution to the last stage of the game, that is the optimal prices that the monopoly will choose. Then the government chooses the tax rates so as to maximize the welfare. The optimal taxes rates can be found by maximizing (9), after substituting the optimal prices of the first stage and (6), (7), and (8).

The optimal taxes are given by

\[
t_1 = -S_1 + a + c_1, \quad (55) \\
t_2 = -S_2 + a + c_2. \quad (56)
\]
The optimal taxes are positive if \( a > S_1 - c_1 \) and \( a > S_2 - c_2 \). Otherwise the government will subsidize the product. Interestingly, the optimal tax policy is to tax less the most luxurious variant. Although in the absence of status effect the optimal choice of the government is to subsidize the product, when consumers also care about the status the product confers, the government should impose a tax when the positional externality is sufficiently high.

The equilibrium prices and quantities are

\[
P_{1,1}^{t_1, t_2} = c_1 + \frac{a(c_2 - c_1)}{(S_2 - S_1)}, \quad P_{2,2}^{t_1, t_2} = c_2 + \frac{a(S_1(S_2 - S_1) - S_2c_1 + S_1c_2)}{S_1(S_2 - S_1)}. \tag{57}
\]

\[
Q_{1,1}^{t_1, t_2} = \frac{S_1c_2 - S_2c_1}{(S_2 - S_1) S_1}, \quad Q_{2,2}^{t_1, t_2} = \frac{S_2 - S_1 - c_2 + c_1}{(S_2 - S_1)}. \]

The difference between the two taxes are

\[
t_1 - t_2 = (S_2 - S_1) Q_{2,2}^{t_1, t_2} > 0 \tag{58}
\]

From (58) one can see that \( t_1 > t_2 \) if \( Q_{2,2}^{t_1, t_2} > 0 \) i.e., if the variant of the higher quality is produced. In that case, the optimal tax policy is to tax less the most luxurious variant.

**Proposition 8** The optimal tax policy is to tax less the most luxurious variant.

With discriminatory taxation, the optimal prices, the optimal quantities, the consumer surplus and the welfare is the same as in the social planner solution. The monopoly’s profits are lower since the government takes part of the profits as tax revenues. Therefore, the welfare when the government has two tax rates at its disposal is the same as in the first best, something that is not true in the case of uniform taxation. With uniform taxation the welfare is lower than the social optimum.

It is interesting to find out whether the welfare is higher when two variants of the good are produced, with discriminatory taxation \( (W_{t_1, t_2}^{t_1, t_2}) \), rather than one. To see that we use (21) and (38), and we have

\[
W_{t_1, t_2}^{t_1, t_2} - W^1 = \frac{1}{2} \frac{(-S_2 + S_1 + c_2 - c_1)^2}{S_2 - S_1} > 0. \tag{59}
\]

It is evident that the welfare when two variants are produced is higher. Therefore

\[
W_{2,sp}^{sp} = W_{t_1, t_2}^{t_1, t_2} > W^1 \tag{60}
\]

**Proposition 9** The welfare achieved when the monopoly produces two variants is the same as in the social optimum and higher than when it produces one variant.

### 5 Conclusions

This paper has introduced social factors that affect consumers behavior such as status and envy effects, in a framework in which the market is vertically differentiated in terms of product quality. The main objective of this paper has been to identify the optimal tax
or subsidy when a monopoly produces one or two variants of the product. It is shown when the positional effects are strong enough, the optimal policy for the government is to tax these goods. In the case of discriminatory taxation, when two variants are produced, it is optimal to tax less the most luxurious variant.

The limitations of the paper suggest avenues for future research. It remains, for instance, an open question whether the results would be the same if the quality of the variants was not fixed but endogenously determined. However it would be a difficult task, given the fact that the analytics of this case are extremely cumbersome.
References


6 Appendix

Proof of Proposition 1

Consumer surplus and profits from (14) and (15), after using (12) become

\[
CS = \frac{(S - P + \alpha_1) \left( S\alpha_1 + 2\alpha_2 \left( -a_1 + a_2 - S \right) + S^2 - SP \right)}{2 \left( S + \alpha_1 - \alpha_2 \right)^2}, \tag{61}
\]

\[
\pi = \frac{(P_1 - c_1) \left( S - P + a_1 \right)}{\left( S + \alpha_1 - \alpha_2 \right)}. \tag{62}
\]
Note that in the first best optimum there is no tax and consequently tax revenues. Taking this into account and after substituting the above equations in (9), we can find

$$W^{sp} = \frac{1}{2} \frac{(S + \alpha_1 - P) [P(S + 2\alpha_1 - 2\alpha_2) + 2\alpha_2 (\alpha_2 - \alpha_1 - S + c) + S(\alpha_1 + S - 2c) - 2\alpha_1]}{(S + \alpha_1 - \alpha_2)^2}.$$  \hspace{1cm} (63)

The solution of the first order condition $\frac{\partial W}{\partial P} = 0$ gives (18). After substituting the optimal price in (13), (62) and (63), we obtain the consumer surplus

$$CS = \frac{[4\alpha_2(\alpha_2 - \alpha_1) + S(\alpha_1 - 3a_0 + S - c)](S_i - c + \alpha_1 - \alpha_2)}{2[S + 2(\alpha_1 - \alpha_2)^2]},$$

and also (19), (20) and (21).

**Proof of Lemma 1**

The marginal consumer who is indifferent from buying and not buying at all has $\theta_1$:

$$\theta_1 S + \alpha_1 \theta_1 - P = -\alpha_2(1 - \theta_1) \Rightarrow \theta_1 = \frac{P - \alpha_2}{S + \alpha_1 - \alpha_2}.$$  \hspace{1cm} (64)

The monopoly’s profits are given by

$$\pi = (P - t - c)(1 - \theta_1) = (P - t - c) \left(1 - \frac{P - \alpha_2}{S + \alpha_1 - \alpha_2}\right).$$  \hspace{1cm} (65)

By taking the First Order Conditions we have:

$$\frac{\partial \pi}{\partial P} = 0 \Leftrightarrow \frac{S + a_1 + c - 2P + t}{S + \alpha_1 - \alpha_2}$$

which gives the optimal price of the monopoly

$$P = \frac{\alpha_1 + c + S + t}{2}.$$  \hspace{1cm} (65)

If we insert the price from above in (13), we have (28) By substituting (28) in (64) we find the monopoly’s profits (29).

The consumer surplus, can be found by substituting (65) in (61). We find (30) after using (13) in $R = tQ$. Finally, by substituting (29),(30)and (31)) in (9), one obtains (32).

**Proof of Proposition 2**

By setting $\alpha_2 = \alpha_1 = 0$ in (34), (35), (36), (38) and (33) we have the results of Proposition 2.

**Proof of Proposition 3**

By setting $\alpha_2 = 0$ in (34), (35), (36), (38) and (33) we have the results of Proposition 3.

**Proof of Proposition 4**

By setting $\alpha_1 = 0$ in (34), (35), (36), (38) and (33) we have the results of Proposition 4.

The unregulated equilibrium when the monopoly produces two variants of the status good.
The monopoly chooses prices that maximize its profits. Maximization of (6), after setting \( t_1 = t_2 = 0 \) and \( a_1 = a_2 = a \), gives

\[
P_{1t=0} = \frac{S_1 + c_1}{2} + \frac{a(c_2 - c_1)}{2(S_2 - S_1)},
\]
\[
P_{2t=0} = \frac{S_2 + c_2}{2} + \frac{a(-S_2c_1 + S_1c_2 + (S_2 - S_1)(a + S_1))}{2S_1(S_2 - S_1)}.
\]

(66)

Substituting the optimal prices, one can find the equilibrium quantities

\[
Q_{1t=0} = \frac{S_1c_2 - S_2c_1 + a(S_2 - S_1)}{2(S_2 - S_1)S_1}, \quad Q_{2t=0} = \frac{S_2 - S_1 - c_2 + c_1}{(S_2 - S_1)2}.
\]

Proof of (49)

Proof: Assume, for a proof by contradiction, that \( \alpha_2 > \left(S + \alpha_1 - \frac{c}{2} - \frac{1}{2}\sqrt{2S(S - 2c) + c^2}\right) \). Multiplying both sides and we get \( 2S(S - 2c) + c^2 > (S - c + S + 2\alpha_1 - 2\alpha_2)^2 \). We square both sides and we get \( -2S(c + \alpha_1 - \alpha_2) - 2(S - c + \alpha_1 - \alpha_2)(S + 2\alpha_1 - 2\alpha_2) > 0 \), which is a contradiction since we know that \( S + 2\alpha_1 - 2\alpha_2 > 0, S - c + \alpha_1 - \alpha_2 > 0, c + \alpha_1 - \alpha_2 \). QED

Proof of Proposition 8

The consumer of type \( \theta_2 \) is indifferent between consuming the high or the low quality version of the product. Her utility if she buys the high quality version is \( \theta_2S_2 + \alpha_1F(\theta_2) - P_2 \), while with the low quality version is \( \theta_2S_1 + \alpha_1F(\theta_1) - \alpha_2F(1 - \theta_2) - P_1 \). The equalization of these two give

\[
\theta_2 = \frac{P_1(-S_1 + a_2) + (a_2 - P_2)(a_2 - a_1 - S_1)}{S_2(-a_2 + S_1 + a_1) - S_1^2 + a_1^2 + a_2^2 - a_1a_2}.
\]

(67)

Similarly the consumer of type \( \theta_1 \) is indifferent between consuming the low quality version or not buying the good,

\[
\theta_1 = \frac{P_1(S_2 + a_1 - S_1) - P_2a_2 + a_2^2}{S_2(-a_2 + S_1 + a_1) - S_1^2 + a_1^2 + a_2^2 - a_1a_2}.
\]

(68)

Setting \( a_1 = a_2 = a \) and substituting the above equations in (7) and (6), one can find the welfare. Maximizing welfare with respect to \( P_1 \) and \( P_2 \) gives (51). Substituting the optimal price in (3) and (6) gives (52) and (53).