A Model of Market and Political Power Interactions for Southern Europe

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\textbf{Abstract:} In recent years the growth pattern of most Southern European countries has been disturbed, as these countries are suffering from economic crises that go beyond the usual business cycle. In this paper, we develop a dynamic general equilibrium model of market and political power interactions that explains this growth reversal. Moreover, the model is consistent with several stylized facts of those countries, where wages in the public sector relative to the private sector are high and these wage differentials correlate negatively with public sector employment over total employment, total factor productivity, and output growth. The model is a synthesis of the insiders-outsiders labor market structure and the concept of an elite government. Outsiders form a group of workers that supply labor to a competitive private sector. And, insiders form a group of workers that enjoy market power in supplying labor to the public sector and influence the policy decisions of government, including those that affect the development and maintenance of public sector infrastructures.

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1. INTRODUCTION

In this paper we develop a dynamic general equilibrium model of market and political power interactions, based on a synthesis of the insiders-outsiders labor market structure of Assar Lindbeck and Dennis Snower (1986) and the concept of an elite government of Daron Acemoglu (2006), as this elite coincides with the group of insiders. In particular, we identify insiders as a group of workers that enjoy market power in supplying labor to the public sector and control/influence the policy decisions of government, including those that affect the development and maintenance of public sector infrastructures. And, we identify outsiders as a group of workers that supply labor to a competitive private sector. This “insiders-outsiders society” model is capable of explaining the growth disruption of recent years experienced by the Southern European countries, while it is consistent with several stylized facts of their respective economies. Namely, that wages in the public sector relative to the private sector are high and these wage differentials correlate negatively with public sector employment over total employment, total factor productivity, and output growth.

As there is no obvious economic reason why the average compensation rate in the public sector relative to the average compensation rate in the private sector (henceforth, the “public sector wage premium”) should be very different from country to country, or why it should vary considerably across time in the same country, Figure 1, below, should come as a big surprise. First, as the bars representing averages indicate, public sector wage premia are quite different across a large sample of OECD countries. Second, as the comparison between the bars representing the maximum and the minimum for the same country indicates, in some of those economies, the public sector wage premium exhibits significant variability over time. Further, the scatter plots, in Figure 2, show a relatively strong and statistically significant negative correlation between the public sector wage premium on the one side and on the other side, the ratio of public sector employment over total employment, total factor productivity growth, and the output per capita growth rate, respectively. These results might have ominous implications for the countries with relatively high wage premia in the public sector, like the four Southern European countries (Greece, Portugal, Spain, and Portugal) where the public sector wage premium is consistently higher than in the other countries.

1 The standard measure for comparing wages across sectors, is the average compensation rate. OECD defines the compensation rate in a sector as the ratio of the compensation of employees and the number of employees in that sector. Compensation of employees is defined as the total remuneration in cash or in kind, payable by enterprises to employees in return for work done by the latter, during the accounting period. It includes wages and salaries and employers’ social security contributions. The number of employees refers to dependent employment and thus excludes the self employed.

2 This sample is defined by data availability alone with the exception of the non inclusion of Switzerland. The latter has an unusually small number of public employees that makes comparisons with the other countries, inappropriate.

3 The years that correspond to the minimum and maximum of the wage premium in the public sector in the countries with the highest average wage premium in the public sector in Figure 1 are as follows: Greece (1978, 2009), Portugal (1983, 1991), Netherlands (1970, 1983), Spain (1997, 1970) Australia (2007, 1983), Italy (1976, 2009) and Ireland (1978, 2007).

4 The correlation between the output per capita growth rate and the public sector wage premium is a conditional correlation between the five year average wage premium and the respective five year average growth rate after been filtered for the estimated effects of explanatory variables, thought to be important determinants of growth. Formally, these are residuals computed by running a standard Barro regression. See the Appendix for details.
Spain and Italy) that are in the top of the list in Figure 1. Moreover, these stylized facts raise some obvious questions for both theory and policy. Why do they exist, in the first place? What are their implications for the behavior of the aggregate level of economic activity, public finances and other key macro variables? What is the role of labor market institutions, if any, in shaping this picture?

This paper seeks to explain the growth disruption of recent years in Southern European countries, like Greece, Italy, Portugal and Spain, while being consistent with the above mentioned stylized facts. The organization of the economic and political systems of these countries is characterized by relatively large public sectors, with basic networks and utility services provided by government and more importantly by agencies or firms that, on the one hand, are heavily regulated and, on the other hand, labor therein is organized in powerful independent unions. Moreover, there

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5 Recently, there have been studies that show empirically that Greece, Portugal, Spain, Italy, and Ireland exhibit higher public sector wage premia than other Euro Area countries. None of these papers, however, relate the wage premium in the public sector to other macro variables and, most importantly, they do not relate public sector wage premia differentials to employment ratios, total factor productivity and growth rate differentials across countries, or for a given country, over time. Kollintzas et al. (2014), confirming the bulk of studies mentioned above, provide evidence that, in countries with a relatively high public sector wage premium, like the Southern European countries, only a small fraction of public sector wage premia can be explained by the usual variables thought to be important determinants of wage differentials, like education, tenure, sex, etc.

6 Over the last eight years (2007-2014), the cumulative percentage GDP change in these four Southern European countries and in the other seventeen countries in our sample has been -11.30 and 6.34, respectively. While, the same statistic over the period 1994-2007 was 39.63 and 46.46, respectively.

7 Chapter 1 of the “Industrial Relations in Europe 2012” extensive report of the European Commission (2013) identifies the Southern European countries as a cluster sharing a similar industrial relations system, referred to as “state-centered.” And, in Chapter 3, the same cluster of countries is identified when it comes to public sector industrial relations. Similar classifications have been made in Visser (2013) with respect to wage bargaining institutions.
Figure 2: Public sector wage premium vs public sector employees over total employees

Figure 3: TFP growth rate vs public sector wage premium

Figure 4: GDP Growth rate vs the public sector wage premium
are important strategic interactions between these unions and the government.\textsuperscript{8} Thus, this paper develops an otherwise simple neoclassical growth model, which takes into account these particular features shared by Southern European economies. According to this model, the deep and sustained growth reversal occurred in these countries is a consequence of the organizational structure of the labor market that has important implications on the workings of the economic and political systems.

In particular, this model incorporates and extends the idea of the insiders – outsiders labor market of Lindbeck and Snower (1986), with wages differing across identical labor services due to the particular organization of the labor market.\textsuperscript{9} Although insiders and outsiders are identical, the wages of insiders are higher than those of the outsiders, creating, what we call the “labor misallocation effect,” that lowers output and output growth towards the steady state. More specifically, outsiders work on the production of a final good, while insiders work on the production of intermediate goods, produced by monopolies controlled by government. For that reason, intermediate goods enter the final goods production function through a Dixit-Stiglitz aggregator that incorporates the so called “variety” effect, whereby an increase in the number of intermediate goods increases output. Further, this aggregator allows for intermediate goods to be gross complements, as one should think of the services of various network infrastructures, provided by the State (e.g., power, water, phone, roads, railways, harbors, airports, etc.). The wage rate of outsiders is determined competitively. Each intermediate good producer prices its output satisfying a zero profit condition, taking the wage rate offered by the corresponding insiders’ union as given. This determines each intermediate good producer’s employment and output. Then, the corresponding wage rate is determined by the respective union that takes the demand for labor it faces, as given.

This is the well known Monopoly-Union model of McDonald and Solow (1981) and Oswald (1983). Since there are as many independent unions of insiders as there are intermediate good producers, overall equilibrium in the market for insiders’ labor is characterized by a Nash equilibrium among all insiders’ unions. This modeling choice is, again, consistent with labor market institutions of Southern European countries, where the wage setting process in the public sector is characterized by trade union fragmentation and, at the same time, lack of co-ordination.\textsuperscript{10} This is quite different from other typically identified country clusters. For example, in Anglo-Saxon

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\textsuperscript{8} This interaction has been recognized in the political science literature since the late seventies (Schmitter (1977), Sargent (1985), Cawson (1986)).

\textsuperscript{9} In the insiders-outsiders theory of Lindbeck and Snower (1986), some worker participants (“insiders”) have privileged positions relative to others (“outsiders”). Insiders get market power by resisting competition in a variety of ways, including harassing firms and outsiders that try to hire/be hired, by underbidding the wages of insiders and by influencing pertinent legislation (Saint-Paul (1996)). There has been no association of the wage premium in the public sector and insiders-outsiders labor market, to our knowledge, in the literature. However, the importance of insiders-outsiders labor markets for providing the microeconomic foundations for justifying the strength of unions has been at the core of this literature (see, e.g., the survey by Lindbeck and Snower (2001)). As already mentioned, in the previous footnote, the strength of the unions in the public sector in the South European countries has been noticed in the political science literature.

\textsuperscript{10} See Sections 3.5.2 and 3.9 in European Commission (2013).
countries wage bargaining is thought, in general, to be competitive and labor unions are thought to play a relatively small role in wage setting. On the other hand, in the Nordic countries, labor unions in all sectors are thought to be powerful but cooperative, thereby internalizing the externalities associated with a high wage premium of one industry/sector on the rest.11

In the symmetric equilibrium case, given reasonable parameter restrictions, the ratio of the wage rate of insiders over that of the outsiders (i.e., the public sector wage premium) is greater than one and increasing in the degree intermediate goods are gross complements, as well as in the number of publicly provided intermediate goods. Moreover, the wage premium and the ratio of employment in the public sector over total employment are inversely related, giving rise to the “labor misallocation effect”. For a fixed number of insiders’ unions, this model is formally equivalent to a standard Cass-Koopmans neoclassical growth model, where Total Factor Productivity (TFP) declines with the wage premium, but increases with the number of intermediate goods, as the “variety” effect dominates over the “labor misallocation” effect. However, the overall effects on steady state capital, output and growth towards the steady state, depend on the after-tax labor productivity. For it is assumed that the underlying infrastructure, associated with the publicly provided intermediate goods, is financed by a distortionary income tax. Then, it is shown that the effect of an increase in the number of publicly provided intermediate goods on steady state output and growth towards this steady state is negative (positive), depending on the existing number of publicly provided intermediate goods. If this number is higher or lower than a certain threshold, the combination of the “labor misallocation” and the tax distortion effects dominates over (is dominated by) the “variety” effect. All this being quite plausible, as the “variety” effect (“labor misallocation” and tax distortion effects) decreases (both increase) with the existing number of publicly provided intermediate goods.

Further, if the number of publicly provided intermediate goods is allowed to vary, each insiders’ union realizes that it has a common interest with all other insiders’ unions in controlling/influencing the number of publicly provided intermediate goods. Hence, it is to the interest of all insiders’ unions to cooperate so as to control/influence government and its budget. Thus, we consider a politicoeconomic equilibrium defined as the solution to the problem of a government, seeking to maximize an objective function, that to some degree is influenced by representative household preferences and is likewise influenced by insiders’ unions preferences. This maximization is subject to the underlying economic equilibrium and the government budget constraint. Under plausible restrictions, we prove that such a politicoeconomic equilibrium exists and is characterized by a steady state which is globally asymptotically stable. Moreover, it is shown that such politicoeconomic equilibrium will be characterized by a number of publicly provided intermediate goods.

intermediate goods that is greater the greater is the influence of insiders. This, in turn, implies that such a politicoeconomic equilibrium will be supported by a higher (distortionary) income tax rate and/or debt level, the greater is the influence of insiders. This is the “political effect” that, depending on the number of publicly provided intermediate goods, may further reduce steady state capital, output, and output growth towards the steady state. It follows, therefore, that, to the degree that the political and economic system of a country is like the insiders-outsiders society of this model, it would exhibit a relatively high wage premium in the public sector, low public to total employment ratio, and would exhibit labor misallocation and tax distortion effects that reduce steady state after-tax total factor productivity, capital, output and growth towards this steady state.12

It is the combination of the above two results, namely that a government influenced by insiders will choose a higher number of publicly provided intermediate goods and that after-tax total factor productivity rises or falls depending on whether this number is lower or higher than a certain threshold, that leads to the model’s prediction that countries which behave as an insiders – outsiders society, although may enjoy relatively high growth early on, will eventually suffer from a growth reversal.

The results of this paper relate to several different strands of the literature on political economy, public finance, growth and European integration. First, it relates to the rent seeking / special interests political economy literature. In particular, it incorporates two basic ideas of that literature. First, that insiders seek rents from the political system for their own benefit and that the agents of the political system accommodate these demands in pursuit of their economic and political goals. Second, that, once the political system allows it, rent seekers are formed in groups, so as to take advantage of their common interests in rent seeking, by controlling/influencing government.13 Also, it shares with the recent political economy and economic growth literature, the idea that resources devoted to rent seeking are ultimately detrimental to growth.14

In his unifying theory Acemoglu (2006) develops a general framework for analyzing the growth implications of politicoeconomic equilibria, when there are three groups of agents: workers, “elite” producers and “middle class” producers. Elite producers control the government and tax middle class producers through a distorting income tax and distribute the proceeds among themselves via a lump-sum transfer. The motives for increasing the distorting tax are three: (a) “Revenue extraction”: the provision of resources for the benefit of the elite; (b) “Factor price

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12 Fernandez-de-Cordoba et al. (2012), consider a dynamic general equilibrium model that, as in our case, emphasizes the role of political economy issues in public-private wage developments. However, they focus on the behavior of public and private sector wages over the business cycle within the context of a non-cooperative game between unions and the government for the determination of wages in the public sector.

13 The idea that the various beneficiaries of government policies are more likely to get politically organized, whereas the interests of the un-organized general public are neglected is found in the pioneering works of Schattschneider (1935), Tullock (2010), Olson (1965), Weingast et al. (1981) and Becker (1983, 1985).

manipulation”: the lowering of factor prices used in the elite’s production process; and (c) “Political consolidation”: the impoverishment of middle class producers, so as to prevent them from acquiring the resources necessary to achieve political power. To anticipate the workings of the model, we may think of insiders as acting according to Acemoglu’s three motives. The first and the third of these motives for increasing the distorting income tax, are captured by the need for the maintenance of the existing (old) and the creation of the new publicly provided infrastructure that ensures the funding for their employers businesses. The second motive is captured by the fact that an increase in the income tax rate, increases the user cost of capital and the wage rate of outsiders, lowering the demand for these factors and increasing the demand for services of intermediate good products. This, in turn, increases the demand for insiders’ labor, in such a way so as to increase the wage premium in the public sector. Like in Acemoglu, it is this effect that seems to be the most damaging for the economy. However, there are important differences between Acemoglu’s framework and the one developed herein. First, the roles of “elite entrepreneurs” and “middle class entrepreneurs” are taken, here, by “insiders” and “outsiders”, that they are both workers. Second, since insiders are organized in unions, that set the wage rate, there is an additional distortion in our model’s economy over and above the tax distortion. This additional distortion strengthens the “factor price manipulation” effect. Third, there is a fundamental nonlinearity, as an increase in the distorting tax rate, so as to increase the number of publicly provided intermediate goods, may be beneficial for the economy, if the number of existing publicly provided intermediate goods is relatively low and the opposite may be true, if the existing number of those goods is relatively high.

Furthermore, it relates to the “varieties of capitalism” literature of political science, pioneered by Hall and Soskice (2001), as well as Esping-Andersen’s (1990) “three worlds of welfare capitalism” social model analysis. In this literature, it has been suggested (Molina and Rhodes (2007)) that the Southern European countries have their own “variety” of capitalism, where the state plays a major role. In a sense, the insiders-outsiders society idea is based on the institutional complementarity between market organizations, where the wage premium favors individual groups of society (“insiders”) and the political system, where these groups control or influence government, for insiders’ collective benefit. As already noted, this interaction has been emphasized on another strand of the political science literature, namely, that on “neo-corporatism” (Schmitter (1977), Sargent (1985), Cawson (1986)).

Finally, our results should be of interest to the European integration question, as countries that have gone beyond a certain point toward the insiders-outsiders society (e.g., South European countries) will have a difficulty following the others in after-tax TFP growth, as already suggested by several policy influential economists (see, e.g., Blanchard (2004), Alesina and Giavazzi (2004)).
The rest of the paper is organized as follows: Section 2 develops the model. Section 3 establishes the main results of the paper. Section 4 discusses the model’s explanation of the growth reversal and the stylized facts mentioned above and Section 5 concludes.

2. MODEL

Time is discrete and there is no uncertainty. The economy consists of a large number of identical households whose members supply labor and capital services and consume a final good. This final good can either be consumed or invested and is produced by means of physical capital and labor services, as well as, the services of a number of intermediate goods provided by government. Household labor consists of two kinds: Labor supplied to the final good producers in a competitive market and labor supplied to the publicly provided intermediate goods through independent monopolistic labor unions. Moreover, these monopolistic unions cooperate in controlling/influencing all government policies. Household members supplying their labor competitively will be referred to as “outsiders” and household members supplying their labor through labor unions will be referred to as “insiders.” And, the model economy will be referred to as the “insiders-outsiders society.”

2.1. Households and Firms

Household preferences are characterized by a standard time separable lifetime utility function of the form

\[ U^h = \sum_{t=0}^{\infty} \beta^t c_t^{1-\gamma} \left( -\frac{1}{1-\gamma} \right) \]

where: \( \beta \in (0,1) \) is the household discount factor, \( c_t \) is consumption per capita in period \( t \), and \( 1/\gamma \in (0,\infty) \) is the constant elasticity of intertemporal substitution. Households own physical capital, that depreciates according to a fixed geometric depreciation rate, \( \delta \in (0,1) \) and evolves according to

\[ k_t = (1-\delta)k_{t-1} + i_t \]

where \( k_t \) is capital stock at the beginning of period \( t \) and \( i_t \) is gross investment in period \( t \). In every period \( t \), each household has available a fixed amount of labor time, \( \bar{h} \in (0,\infty) \), that can be allocated to the production of the final good, \( h^o_t \), and the production of services from a continuum of intermediate goods, \([0,N]\), provided by government. Thus, the time constraint of each household, in every period \( t \), is given by:

\[ h^o_t + \int_0^{N} h^i_t(z) \, dz \leq \bar{h} \]

where: \( h^i_t(z) \) is labor time devoted by each household to the production of services from the \( z \) intermediate good, in period \( t \).

Although our formulation of households allocating time among final and intermediate good sectors is admittedly a highly schematic one, as already mentioned, the reader may think of
households having many members, where some are “insiders” and others are “outsiders.” As will be made apparent below, the numbers of insiders and outsiders in our model are determined by the demand side (firms and unions), exclusively. Allowing insiders’ layoffs, as in Cole and Ohanian (2004), could determine the numbers of insiders and outsiders by the supply side (households), as well. For tractability purposes, we have chosen not to pursue this extension here. At any rate, in our model, as well as in Cole and Ohanian (2004), there is perfect household insurance among household members, whether insiders or outsiders. Hence, the profoundly important income distribution effects of the insiders–outsiders society are, consequently, ignored. And, as is, of course, the important question of who chooses to become an insider and who ends up as an outsider, in the presence of these income distribution effects.

The budget constraint facing each household, in any given period \( t \), is given by:

\[
c_t + k_{t+1} - (1 - \delta)k_t = (1 - \tau_t) \left[ r_t k_t + w^o_i h^o_i + \int_0^{N_t} w^i(z) h^i(z) dz \right]
\]

where: \( \tau_t \) is the income tax rate in period \( t \), \( r_t \) is the rental rate of capital services in period \( t \), \( w^o_i \) is the wage rate for labor time devoted to the production of the final good, \( w^i(z) \) is the wage rate for labor time devoted to the production of services from the \( z \) intermediate good, in period \( t \).

The representative household seeks a consumption, capital accumulation and time allocation plan so as to maximize lifetime utility, subject to the time and budget constraints, (1) and (2), respectively. In so doing, the representative household takes all prices, income tax rates, and numbers of intermediate goods, as given.\(^{15} \)

In view of the specific functional form of the temporal utility function, specified above, necessary and sufficient conditions for a solution to the problem of the representative household are the standard side conditions along with the Euler condition:

\[
\frac{C_{t+1}}{C_t} = \beta [(1 - \delta) + (1 - \tau_{t+1}) r_{t+1}] \frac{1}{\beta} \forall t \in \mathbb{N}_+
\]

Production in the final good sector takes place in a large number of identical firms. The production technology of the representative firm in this sector is:

\[
Y_t = K^o_t (A L^o_t)^a \left[ \int_0^{N_t} x_t(z) \xi^{1-a} dz \right]^{1-b} \zeta^{a+b-1} ; \quad a, b > 0, a + b < 1 & \zeta \in (0, 1]
\]

\(^{15} \)Obviously, if \( w^i(z) \) is different from \( w^o \), the household will not be indifferent between \( h^o \) and \( h^i(z) \). If, for example, \( w^i(z') \), for some \( z' \), is greater than \( w^o \), the household would prefer \( h^i(z') \) over \( h^o \). Likewise, if \( w^i(z') \) is greater than \( w^i(z^*) \), for any given \( z' \) and \( z^* \), the household would prefer \( h^i(z') \) over \( h^i(z^*) \). In any case, in the solution to the household’s problem, (1) will hold with equality. Later, \( h^o \) and \( \{ h^i(z) \}_{z=0..h_i} \) will be set following demand conditions and institutional constraints, without violating household incentives.
where: $Y_t$ is output supplied in period $t$, $K_t$ is physical capital services used in period $t$, $L_t^0$ is labor services used in period $t$, $A_t$ is a parameter that designates the level of (Harrod–neutral) technology at the beginning of period $t$ and grows according to: $A_{t+1} = (1 + g_A) A_t$, $g_A \in [0, \infty)$; and $x_t(z)$ is the services from the $z$ intermediate good used in period $t$. The RHS of (4) is a constant returns to scale production function. The Dixit-Stiglitz aggregator is used to model the composite of all intermediate good inputs, $$\left[ \int_0^{N_t} x_t(z)^{\zeta} \, dz \right]^{(1/\zeta)},$$ in a tractable manner. Thus, we assume that there is a continuum of intermediate good products and that $N_t$ is a positive real number. Restricting $\zeta \in (0,1]$ ensures that output increases with the number of intermediate goods, so as to capture the so called “variety” effect, introduced by Romer (1990).

Although we think of intermediate goods as goods provided by government, we do not think of these goods as pure public goods. In particular, we think of intermediate goods as being excludable, in the sense that only those final good producers that pay for using the services from a given intermediate good can use those services. Moreover, these goods are not necessarily non-rival, in the sense that a final good producer that uses the services from a given intermediate good may or may not limit the amount of services used by other final good producers. Actually, most publicly provided services are excludable and to a great extend rival. For example, in many countries basic utilities (electrical power, water and sewage, garbage and waste collection and disposal, stationary telephony and natural gas), transportation networks (railroads, harbors, airports), and various licenses (foods and drugs, fire and flood safety) are provided to their users for a price.

Cole and Ohanian (2004), in their seminal paper, where they examine the effects of New Deal policies on the recovery from the Great Depression in the United States, consider two kinds of intermediate goods sectors. In their model, labor is supplied to two sectors: the noncompetitive “cartel” sector and the “competitive” sector, much like in our formulation, but in their model government policies are exogenous. Further, the number of intermediate good products (varieties) in their formulation, is taken to be fixed and only the distribution of those products between the two sectors is endogenously determined. We have opted to consider publicly provided intermediate

16 It can be easily verified that this aggregator belongs to the CES family of production functions, in that it exhibits constant elasticity of substitution across intermediate goods. That is, the elasticity of substitution between any two intermediate goods is $\sigma = \frac{1}{1-\zeta}$. Thus, for $\zeta \to 1, \sigma \to +\infty$, in which case intermediate goods are perfect substitutes and for $\zeta \to -\infty, \sigma \to 0$, in which case, intermediate goods are perfect complements. For $\zeta < 1-a-b$ ($\zeta > 1-a-b$ ) intermediate goods are gross complements (gross substitutes). For $\zeta = 1-a-b$, the aggregator becomes as in Romer (1990), where the marginal productivity of any given intermediate good is not affected by the input of any other intermediate good. That is, $[\partial^2 Y_t / \partial x_t(z') \partial x_t(z^*)]_{z^* = 0} \Rightarrow 0$ as $\zeta = 1-a-b; \forall z', z^* \in [0, N_t] \ni z' \neq z^*$. 
goods only, for as already said, we are motivated by Southern European macroeconomic and political structures, where the state plays a major role and the number (varieties) of publicly provided intermediate goods products may have been an important contributor to growth. And, obviously, the way the variety effect is modeled, here, gives an incentive for expanding the public sector via the increase of the number of publicly provided intermediate goods, $N_t$.\(^{17}\)

Let $p_t(z)$ be the price for the services of the $z$ intermediate good in period $t$. At the beginning of any given period $t$, the representative final good producer, maximizes profits:

$$
\pi^*_t = K_t^a (A_t L_t^0)^b \left[ \int_0^{N_t} x_t(z) \frac{1-a-b}{\zeta} dz \right]^{\frac{1-a-b}{\zeta}} - r_t K_t - w_t^0 L_t^0 - \int_0^{N_t} p_t(z) x_t(z) dz ,
$$

(5)

taking all input prices and the number of intermediate good producers as given.

The (inverse) demand for the services from the $z$ intermediate good is:

$$
(1-a-b) K_t^a (A_t L_t^0)^b \left[ \int_0^{N_t} x_t(z') \frac{1}{\zeta} dz' \right]^{\frac{1-a-b}{\zeta}} x_t(z)^{\zeta-1} = p_t(z); \forall z \in [0, N_t].
$$

(6)

Demand increases (decreases) with the composite of all intermediate good inputs, $\left[ \int_0^{N_t} x_t(z') \frac{1}{\zeta} dz' \right]^{(1/\zeta)}$ or, for that matter, with any given intermediate good $z' \in [0, N_t]$, if and only if $\zeta < 1-a-b$ ($\zeta > 1-a-b$). That is, if and only if intermediate goods are gross complements (gross substitutes). Clearly, however, gross complementarity is more compatible with the idea of public intermediate goods being basic utilities, transportation networks, licenses, etc. Therefore, throughout, we shall maintain the assumption that intermediate goods are gross complements:

**Assumption 1:** $1-a-b-\zeta > 0$

As the infrastructure associated with each intermediate good is provided by government, the services of intermediate goods are produced by using labor only:

$$
X_t(z) = \Phi(z) A_t L_t^r(z); \Phi(z) \in (0, \infty), \forall z \in [0, N_t] & t \in \mathbb{N},
$$

(7)

where: $X_t(z)$ is output supplied in period $t$ and $L_t^r(z)$ is labor services used in period $t$.

\(^{17}\) A generalization of the model that includes privately provided intermediate goods, as well, is straightforward. This, however, comes at a cost of increasing considerably the model’s complexity and, consequently, preventing qualitative analysis.
In any given period $t$, the representative producer of services from the $z$ intermediate good chooses labor input, so as to achieve zero profits:

$$\pi^i_t(z) = p_i(z) X_i(z) - w_i(z)L_i(z) = 0,$$

(8)

taking the production technology constraint (7), the demand for its services (6), the number of intermediate good producers, the labor input choices of all other intermediate good producers and wages as given. This gives the following (inverse) aggregate demand for labor in the production of services from the $z$ intermediate good:

$$(1 - a - b)A_k^{1-a}K_t^{\alpha z}L_t^b \left\{ \int_0^\infty [\Phi(z')L_t'(z')]^{\frac{1-a-b-\zeta}{\zeta}} \Phi(z')L_t'(z')^{-1} dz' \right\}.$$

(9)

Clearly, given Assumption 1, this demand increases with the weighted average of the labor input in the production of services of all intermediate goods, $\left\{ \int_0^\infty [\Phi(z')L_t'(z')]^{\frac{1-a-b-\zeta}{\zeta}} \Phi(z')L_t'(z')^{-1} dz' \right\}$. This formulation is consistent with the Southern European model, where public utilities, transportation networks, and other publicly provided services are supplied by a single agency/firm that has a monopoly, but is heavily regulated. However, these agencies/firms end up behaving like unregulated monopolist, due to the behavior of the union that controls their labor input. And this is the model feature we turn next.

### 2.2 Insiders’ Unions

Labor used in the production of services from each intermediate good $z$ is organized in a union. That is, there is a separate union $z$ for each intermediate good $z$, for all $z$. We refer to these unions as “insiders’ unions.” Following the standard union literature, we assume that the preferences of the $z$ union of insiders are characterized by the utility function

$$U^i(z) = \sum_{t=0}^{\infty} \beta^t \ln \left[ w_i^\lambda(z) - w_i^\lambda(z) \right] L_t(z),$$

where $\lambda(z) \in (0, 1)$, $\forall z \in [0, N_t]$ and $t \in \mathbb{N}_+$. This form of union preferences corresponds to the “utilitarian” model of McDonald and Solow (1981) and Oswald (1982), where the representative union member has a constant relative rate of risk aversion, provided that union membership is fixed. Here, union membership is determined by the union and

---

18. This is not a crucial assumption and the propositions of this paper would go through with publicly provided intermediate good service producers having some other objective, like regulated profits. For simplicity purposes, this is not pursued in this paper.

19. The number of final good producers is irrelevant, in this model, due to the CRS production function in (4) and perfect competition. Moreover, the number of $z$ intermediate good service producers is also irrelevant due to the CRS production function in (7) and the zero profit restriction. Thus, without loss of generality, (9) has been expressed in representative final good producers units.

20. A classic example is the Greek Power Company (ΔΕΗ), which although a de facto monopoly, has more or less zero profits, but its labor union (ΤΕΝΟΠ-ΔΕΗ) has substantial market and political power, that results in substantial wage premiums and other benefits for its members (See, e.g., Michas (2011), for a narrative).
is fixed and equal to employment in the production of services of the corresponding intermediate
good sector. Further, \( w_i^0 \) is the “alternative wage” for insiders, in the sense that, \( w_i'(z) - w_i^0 \) is the
wage premium of insiders over outsiders and at the same time the wage premium in the public
sector. The latter, as already noted, are all those that work in the final good sector of the economy.
And, finally, \( \lambda(z) \) is a parameter that measures the relative preference of the wage premium over
employment for the \( z \) union of insiders. As usual, we take \( \lambda(z) \) to stand for a measure of the
union’s relative bargaining power.

At the beginning of any given period \( t \), the \( z \) union of insiders seeks a wage and employment
plan so as to maximize its utility, subject to the aggregate demand for labor in the production of
services from the \( z \) intermediate good (9); and, the institutional constraint: \( L_i'(z) > 0 \), if and only if
\( w_i'(z) > w_o' \); \( \forall z \in [0,N_i] \) \& \( \forall t \in \mathbb{N}_+ \). In so doing, the \( z \) union of insiders takes the aggregate capital,
the aggregate employment of outsiders, the wage and employment choices of all other unions of
insiders and the number of intermediate good producers, as given.

Let \( \eta_i(z) = -\frac{\partial L_i'(z)}{\partial w_i'(z)} \frac{w_i'(z)}{L_i'(z)} \) be the elasticity of the demand for labor facing the \( z \) union of
insiders. Then, provided that \( \eta_i(z) > \lambda(z) \), as we shall ensure below, there exists a unique solution
to the problem of the \( z \) union of insiders, which is interior (i.e., \( w_i'(z) > w_o' \), \( L_i'(z) > 0 \)) and such that
there is a wage premium given by:

\[
\nu_i(z) = \frac{w_i'(z)}{w_o'} = \frac{1}{1 - \frac{\lambda(z)}{\eta_i(z)}}
\]

(10)

This is the well known tangency condition of the union indifference curve and the demand for
labor facing that union. In this solution \( L_i'(z) \) is less than the employment level that corresponds to
a situation where \( w_i'(z) = w_o' \).\(^{21}\) Although all union members are employed, the union restricts
employment, and hence union membership, in order to raise the wage rate enjoyed by its members.
This, of course, implies an important “misallocation” effect of the insiders-outsiders society. This

\[^{21}\] Observe that, \( 1/\lambda(z) = \frac{d[l_i'(z) - w_i']}{dL_i'(z)} \frac{L_i'(z)}{w_i'(z) - w_o'} \) is the elasticity along the indifference curves of the \( z \)-union of
insiders in the \( [L_i'(z), w_i'(z) - w_o'] \) space and \( 1/\eta_i(z) = \frac{d[w_i'(z) L_i'(z)]}{dL_i'(z) w_i'(z)} \), is the elasticity of the inverse demand curve
for labor faced by the \( z \)-union of insiders. Thus, if in the solution to the problem of the \( z \)-union of insiders the slopes of
these two curves must be the same, we must have: \( w_i'(z) = \frac{[1/\lambda(z)]/[1/\lambda(z)] - [1/\eta_i(z)]}{[1/\lambda(z)]/[1/\lambda(z)] - [1/\eta_i(z)]} \). Hence, \( \eta_i(z) > \lambda(z) \)
implies that \( w_i'(z) > w_o' \). Now, the last fact and the fact that \( \eta_i(z) > 0 \), implies that the employment that corresponds
to \( w_o' \), \( L_i'(z) \), is greater than \( L_i'(z) \).
friction has profound implications for both output and growth. It will be more convenient, however, to examine the important implications of this effect, as well as, the restrictions imposed upon the model’s parameters by the condition \( \eta_i(z) > \lambda(z) \), after the model’s structure has been completed. Again, however, this is consistent with the Southern European economic model, where the workers of publicly provided intermediate goods are organized in powerful and independent labor unions, while the corresponding intermediate good producers are heavily regulated.

2.3 Government Budget

The government’s budget constraint, expressed in representative household units, in any given period \( t \), is given by:

\[
\int_{N_i} \hat{\Psi}_i(z)dz + \int_{0}^{N_i} \hat{\Psi}_i(z)dz = \tau_i \left[ r_k + w_i^0 + \int_{0}^{N_i} w_i(z)h_i(z)dz \right]
\]

where \( \hat{\Psi}_i(z) \) is the cost of setting up (dismantling) new (old) \( z \) intermediate good infrastructure in period \( t \) and \( \hat{\Psi}_i(z) \) is the cost of administering and maintaining the existing \( z \) intermediate good infrastructure in period \( t \). That is, the first term in the LHS of (11) should be thought of as the investment cost of new infrastructure and the second term in the LHS of (11) as the cost of maintaining the existing infrastructure.22 \( \hat{\Psi}_i(z) \) and \( \hat{\Psi}_i(z) \) will be further specified, shortly.

2.4 Symmetric Equilibrium

For tractability purposes, in what follows we shall characterize the equilibrium in the symmetric case, where there are no differences across intermediate good service producers, the corresponding insiders’ unions, and the distributions of \( \Phi(z) \), \( \hat{\Psi}_i(z) \) and \( \hat{\Psi}_i(z) \) are uniform.

More specifically, we assume: \( \Phi(z) = \Phi; \Phi > 0, \lambda(z) = \lambda; \lambda \in (0,1), \hat{\Psi}_i(z) = \bar{\psi}y_i; \)

---

22 The introduction of public capital would make equation (11) much less abstract. For example, investment in new publicly provided capital infrastructure could take the form \( \int_{N_i} \hat{\Psi}_i(z)i^0_i(z)dz \), where \( \hat{\Psi}_i(z) \) and \( i^0_i(z) \) stand for the unit cost and investment quantity of the new \( z \) intermediate good infrastructure in period \( t \), respectively. And, maintenance of existing publicly provided capital infrastructure could take the form \( \int_{0}^{N_i} \hat{\Psi}_i(z)k^0_i(z)dz \), where \( \hat{\Psi}_i(z) \) and \( k^0_i(z) \) stand for the unit cost and capital stock of the old \( z \)-intermediate good infrastructure in period \( t \), respectively. Depending on where one wants to focus, \( \hat{\Psi}_i(z) \) and \( \hat{\Psi}_i(z) \) could be specified accordingly. For example, in order to capture adjustment costs in investment quantity, \( \hat{\Psi}_i(z) \) could be made to depend on \( i^0_i(z) \) and to capture adjustment costs in varieties, \( \hat{\Psi}_i(z) \) could be made to depend on \( z \), etc. Likewise, to capture vintage capital, \( \hat{\Psi}_i(z) \) could be made to depend on \( u \), such that \( u \in (N_i,N_{i+1}) \) for all \( u \leq t \). And, to capture depreciation, \( \hat{\Psi}_i(z) \) could be made to depend on \( t-u \). We have avoided these complications here, to focus on the essence of the insiders–outsiders society.
\( \dot{\Psi}(x) = \dot{\psi} y_i; \dot{\psi} > \ddot{\psi}; \forall \in [0, N_i] \) & \( t \in \mathbb{N} \). The last two restrictions make investment in new infrastructure and maintenance of existing infrastructure, fixed functions of output per efficient household. Obviously, these are strong restrictions for analyzing business cycle effects. But, herebelow, they are not so restrictive, as we limit our attention in steady states and convergence towards these steady states. Also, the restriction \( \dot{\psi} > \ddot{\psi} \) incorporates the notion that it is more expensive to develop than to maintain one unit of public sector infrastructure.

Then, the equilibrium of this economy, where all agents solve their respective problems and all markets clear, is characterized by the following set of equations:\(^{23}\)

\[
\begin{align*}
    h^i_t &= \frac{b \nu(N_i)}{b \nu(N_i) + (1-a-b)} h \\
    N_i h^i_t &= \frac{1 - \alpha - b}{b \nu(N_i) + (1-a-b)} h \\
    y_i &= \xi(N_i) k^a_i \\
    \frac{c_{t+1}}{c_t} &= \left[ \beta (1 + g_A)^{-1} \left\{ (1-\delta) + \alpha \left[ 1 - \dot{\psi} (N_{t+2} - N_{t+1}) - \ddot{\psi} N_{t+1} \right] \xi(N_{t+1}) k_{t+1}^{a-1} \right\} \right]^{1} \\
    \frac{k_{t+1}}{k_t} &= (1 + g_A)^{-1} \left\{ (1-\delta) + \left[ 1 - \dot{\psi} (N_{t+1} - N_i) - \ddot{\psi} N_i \right] \xi(N_i) k_t^{a-1} - c_t k_t^{-1} \right\}
\end{align*}
\]

where

\[
\begin{align*}
    \nu(N_i) &= \frac{w_t^f}{[1 - \lambda (1-\zeta)]N_i + \lambda (1-a-b-\zeta)} \\
    \xi(N_i) &= b^b (1-a-b)^{(1-a-b)/2} \Phi_{(1-a-b)/2}^{(1-a-b)/2} \left[ \frac{v(N_i)^b}{[1-a-b+b \nu(N_i)]^{1-a}} \right]
\end{align*}
\]

Equations (12) and (13) give the allocation of total household time between final good and intermediate public good production and for that matter between insiders and outsiders, respectively. Equation (14) gives output in the neoclassical growth model format, so that \( \xi(N_i) \), defined in Equation (18), is total factor productivity, in period \( t \). Equations (15) and (16) are the laws of motion of consumption and capital. The former incorporates the government’s budget constraint and the latter incorporates the resource constraint of the economy. Equation (17) specifies the public sector wage premium, \( \nu(N_i) \), which is tantamount to the wage premium of insiders over outsiders. Clearly, the wage premium affects the economy’s resource allocation through total factor

\(^{23}\) To simplify notation \( c_i, k_i, y_i \) in (14)-(16) and henceforth, are equal to the previously defined \( c_i, k_i, y_i \) divided by \( A_i h \).
productivity. The number of publicly provided intermediate goods, \( N_i \), affects total factor productivity both directly and indirectly through the wage premium. The former is associated to the variety effect and the latter to the misallocation effect, discussed in the Introduction. Hence, the number of publicly provided intermediate goods affects the economy’s resource allocation, via after-tax total factor productivity, \[ 1 - \psi(N_{t+1}) - \psi(N_t) \xi(N_t) \], threefold: First, through the wage premium, second through the variety effect, and third through taxation. The latter is associated with what we shall refer to as the “political effect.”

2.4.1 The Insiders’ Wage Premium

To ensure that the wage premium of insiders over outsiders is greater than one, we need the following parameter restriction:

**Assumption 2:**

\[
(1 - \zeta) - \frac{1 - \alpha - b - \zeta}{N_t} > 0
\]

This simply implies that the demand for insiders’ labor is downward slopping and puts a lower bound on \( N_t \). That is, \( N_t \geq \frac{1 - \alpha - b - \zeta}{1 - \zeta} \). Also, given \( \lambda \in (0,1) \), Assumptions 1 and 2 ensure that \( 0 < \frac{\lambda}{\eta} = \lambda \left[ (1 - \zeta) - \frac{1 - \alpha - b - \zeta}{N_t} \right] < 1 \), and it follows from (17) that the wage premium is greater than one. Moreover, in view of Assumption 1:

\[
\nu'(N_t) = \lambda \frac{(1 - a - b - \zeta)}{N_t^2} \nu(N_t)^2 > 0 \quad \text{and} \quad \nu''(N_t) = -2\lambda \frac{(1 - a - b - \zeta)}{N_t^3} \left[ 1 - \lambda (1 - \zeta) \right] \nu(N_t)^3 < 0. 
\]

Observe, then, that a necessary and sufficient condition for \( \nu'(N_t) \) to be positive (negative) is that intermediate goods are gross complements (substitutes). Hence, Assumption 1 (gross complementarity) ensures that the wage premium increases with the number of intermediate goods. Summarizing results, we have shown the following:

**Proposition 1 (Properties of Insiders’ Wage Premium):** Given Assumptions 1 and 2, \( \nu(N_t) : \left[ \frac{1 - \alpha - b - \zeta}{1 - \zeta} , +\infty \right) \rightarrow \left( 1, \frac{1}{1 - \lambda (1 - \zeta)} \right) \) is strictly increasing and strictly concave in \( N_t \) and approaches asymptotically \( \frac{1}{1 - \lambda (1 - \zeta)} \). Also, \( \nu(N_t) \) is greater: (i) the greater the relative
bargaining power of unions, \(\lambda\); (ii) the lower the elasticity of labor demand facing intermediate good service producers, \(\eta = \left[1 - \zeta - \frac{1 - \alpha - b - \zeta}{N_t}\right]^{-1}\); and (iii) the greater the degree intermediate goods are gross complements, \(1 - \alpha - b - \zeta\).

The economic rationale behind the results of Proposition 1 is straightforward. The wage premium is a consequence of the organization of the labor market. And, in particular, of the market power enjoyed by insiders’ unions. Suppose, that labor input in the production of services of intermediate goods is supplied competitively. Then, since labor services are identical, equilibrium in the labor market implies that \(h^o\) and \(N_t h^i\) are set so that the marginal products of labor in the final good sector and the services of the intermediate goods sector are equal to the common (real) wage rate. And, there is no wage premium (i.e., \(v(N_t) = 1\)). Alternatively, the latter will hold in this model, under two possibilities. First, when \(\lambda = 0\), that is when the union does not care about the wage premium. And second, when \(\eta = +\infty\), that is when the union faces an horizontal demand for labor.

In view of (12) and (13), an immediate implication of Proposition 1 is the following:

**Corollary 1:** Given Assumptions 1 and 2, the ratio of employment in the publicly provided intermediate goods sector (i.e., public employment, \(N_t h^i\)) over total employment, and employment in the final good sector (i.e., private employment, \(h^o\)) over total employment decrease and increase, respectively, with the public sector wage premium and reach their maximum and minimum values, respectively when there is no wage premium.

When \(v(N_t) > 1\), the monopolistic unions restrict labor input, so as to receive a higher wage rate. This result relates to what we refer to as the “labor misallocation” effect, to whose implications we turn next.24

### 2.4.2 Total Factor Productivity

Given Proposition 1, total factor productivity, \(\xi(N_t)\), is positive. As already mentioned, \(N_t\) affects \(\xi(N_t)\) both directly, through the middle term in the RHS of (18) and, indirectly, through the

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24 Much like the standard insiders-outsiders labor market theory suggests, this model can easily account for outsiders’ unemployment, by introducing a minimum wage rate which is greater than \(w^0\) and increases the reservation wage of insiders. In fact, the higher the wage premium in the public sector, the stronger the “misallocation” effect and the lower the demand for outsiders labor, implying greater unemployment amongst outsiders, for any given minimum wage rate.
relative wage premium, \( v(N_t) \). The direct effect of \( N_t \) on \( \xi(N_t) \) is positive and relates to the production technology assumed. And, in particular, the property of the production function that, as long as intermediate goods are not perfect substitutes (i.e., \( 0 < \zeta < 1 \)), an increase in the number of intermediate goods, increases TFP and output. For, each intermediate good input is subject to diminishing returns to scale and, therefore, for any given amount of the aggregate input, \( N_t x_t \), more output is produced if there are more intermediate goods, \( N_t \), composing this aggregate input. This is what is referred to as the “love-for-variety” effect or simply “variety” effect in the growth literature. The indirect effect relates to the wage premium being greater one, for if the wage premium is one, the last term in the RHS of (18) becomes unity. This effect is negative. To check this, we look at the change in \( \xi(N_t) \) brought about by a change in the relative wage premium that does not emanate from a change in \( N_t \) (i.e., \( \frac{\partial \xi}{\partial V[N_t, fixed]} \)) and the change in \( \xi(N_t) \) brought about a change in \( N_t \) (i.e., \( \xi'(N_t) \)). It follows from (18) that \( \frac{\partial \xi}{\partial V[N_t, fixed]} > 0 \) as \( 1+\frac{b}{1-a}(v-1)\xi < v \). Given Assumptions 1 and 2, \( v > 1 \). But, for \( v > 1, 1+\frac{b}{1-a}(v-1) < v \). Therefore, given Assumptions 1 and 2, \( \frac{\partial \xi}{\partial V[N_t, fixed]} < 0 \). The latter defines the “labor misallocation effect.” Hence, the overall effect on \( \xi(N_t) \) of a change in \( N_t \) is not obvious. Herebelow, we summarize results and we show that the overall effect on \( \xi(N_t) \) of a change in \( N_t \) is positive.

**Proposition 2 (Properties of Total Factor Productivity):** Given Assumptions 1 and 2,

\[
\xi(N_t) : \left\{ \frac{1-a-b-\zeta}{1-\zeta}, \infty \right\} \rightarrow \left\{ \frac{b^b(1-a-b)^{(1-a-b)}}{(1-a)^{(1-a)}} \frac{1-a-b-\zeta}{1-\zeta} \left\{ \frac{(1-a-b)(1-\zeta)}{\xi}, \infty \right\}, \text{ such that: (a)} \right. \\
\frac{\partial \xi}{\partial V[N_t, fixed]} < 0, \text{ and (b) } \xi'(N_t) > 0, \forall N_t \in (0, \infty).
\]

**Proof:** In the Appendix.

That is, given gross complementarity (i.e., Assumption 1) and unions facing downward sloping labor demand (i.e., Assumption 2), the “variety” effect dominates over the “labor misallocation” effect. To further illustrate the implications of this “labor misallocation” effect, associated with the equilibrium considered in the previous subsections, it is instructive to consider the Second Best associated with this equilibrium. In this model, there are two reasons that the equilibrium is not Pareto Optimum: Proportional income taxes and the market power of insiders’ unions. Thus, we
shall focus our attention to characterizing efficiency losses with respect to a “Second Best”
outcome. That is, when there is no insiders-outsiders organization of society, but there is a “tax
distortion” effect. In this case, of course, there are no insiders’ unions and there is no relative wage
premium, nor a “labor misallocation” effect. Formally, we define as a “Second Best” outcome for
this economy an equilibrium, where the relative wage premium \( \nu^\text{SB}(N) = 1 \), for all \( t \in \mathbb{N}_+ \). The
Second Best is also characterized by (12) – (17), with TFP given by:

\[
\xi^\text{SB}(N) = b^b (1 - a - b)^{(1 - a - b)} \Phi^{(1 - a - b)} \frac{1}{(1 - a)^{(1 - a)}} \frac{1}{N} \left(1 - \frac{(1 - a)^{(1 - a)} v(N)^b}{[1 - a - b + b v(N)]^a}\right).
\]

We may think of \( \pi(N) \) as the TFP gap due to the “labor misallocation” effect. Clearly, this TFP
gap is proportional to the corresponding output gap, \( y^\text{SB} - y = \pi(N) k^a \). This is a measure of the
equilibrium efficiency losses relative to the Second Best, where there is no insiders-outsiders
organization of society. First, we characterize the sign of \( \pi(N) \) and second, the change of \( \pi(N) \).

As in the case of \( \xi(N) \), it is useful to distinguish between two effects: The change in \( \pi(N) \) brought about by a change in the relative wage premium that does not emanate from a change in
\( N \) (i.e., \( \frac{\partial \pi}{\partial \nu} \bigg|_{N, \text{fixed}} \)) and the change in \( \pi(N) \) brought about by a change in \( N \) (i.e., \( \pi'(N) \)). Then
it is a straightforward application of the results in Propositions 1 and 2 that:

**Corrolary 2 (Second Best):** Given Assumptions 1 and 2, \( \pi(N) : (0, \infty) \rightarrow (0, \infty), \frac{\partial \pi}{\partial \nu} \bigg|_{N, \text{fixed}} > 0, \) and

\( \pi'(N) > 0, \forall N \in (0, \infty). \)

**Proof:** In the Appendix.

As a consequence of the labor misallocation effect, the TFP gap increases with both the public
sector wage premium and the number of publicly provided intermediate goods.

### 2.4.3 Growth with a Fixed Number of Publicly Provided Intermediate Goods

Next, we turn to the question of how the number of publicly provided intermediate goods
affects capital, output and growth. There are two ways to look into the answer to this question: with
a fixed and a variable number of publicly provided intermediate goods. It is instructive to start the
analysis with a fixed (given) number of publicly provided intermediate goods. In the case where \( N \),

is fixed, say, $N_i = \bar{N}$, such that $\hat{\psi}\bar{N} \in (0,1)$, $\forall t \in \mathbb{N}_+$, the transitional dynamics of the equilibrium are now characterized by:

$$\frac{c_{i+1}}{c_t} = \left\{ \beta (1 + g_i)^{-1} \left[ (1 - \delta) + \alpha(1 - \hat{\psi}\bar{N})\xi(\bar{N})k_{i+1}^{\alpha-1} \right] \right\}^{\frac{1}{\gamma}}$$

$$\frac{k_{i+1}}{k_t} = (1 + g_i)^{-1} \left[ (1 - \delta) + (1 - \hat{\psi}\bar{N})\xi(\bar{N})k_i^{\alpha-1} - c_tk_t^{-1} \right]$$

**Figure 5:** Steady state and transitional dynamics with fixed number of publicly provided intermediate goods, $\bar{N}$

Thus, any equilibrium steady state, say $(k^*, c^*) \in (0, \infty)^2$, must satisfy the conditions $\frac{k_{i+1}}{k_t} = \frac{c_{i+1}}{c_t} = 1$, $\forall t \in \mathbb{N}_+$. It follows from the above two equations that the locus $\frac{c_{i+1}}{c_t} = 1$ is given by the vertical line in Figure 5. And, the locus $\frac{k_{i+1}}{k_t} = 1$ is given by the inverse U-shaped curve in Figure 5. The intersection of these two lines (point A) defines the equilibrium steady state $(k^*, c^*) = \left( \frac{\alpha(1 - \hat{\psi}\bar{N})\xi(\bar{N})}{\beta^{-1}(1 + g_i) - (1 - \delta)} \right)^{\frac{1}{\gamma-1}}, (1 - \hat{\psi}\bar{N})\xi(\bar{N})k^{\alpha-1} - (g_i + \delta)k^* \right)$. Also, it follows by the above two equations that the transitional dynamics around this steady state are as indicated by the directions of the arrows in Figure 5. Following standard arguments, it can be shown that there exists a unique stable local trajectory to the steady state, to which the economy converges, monotonically.
Given any initial value of $k_0$, consumption “jumps” to the value that corresponds to this stable local trajectory. Clearly, $(k^*, c^*)$ differs from the steady state of the Second Best (point B, say $(k^{SB}, c^{SB})$), which lies to the north east of point A, by virtue of Corollary 2. And, for any given initial value of $k_0$, transitional dynamics (monotone convergence) will imply higher growth rates towards the steady state of the Second Best, versus that of point A.

Now, we are interested in the steady state and the transitional dynamics for different values of $\bar{N}$. Consider first an increase in the relative wage premium $\nu(.)$ that does not come from a change in $\bar{N}$. Clearly, in this case, following Proposition 2, $\xi(\bar{N})$ will decrease. The $\frac{k_{t+1} - k_t}{k_t} = 1$ locus will drop and the $\frac{c_{t+1} - c_t}{c_t} = 1$ locus will move left. The new steady state (illustrated by point C, in Figure 5) will lie to the south west of $(k^*, c^*)$. And, convergence to this steady state will imply slower growth.

Finally, if $\bar{N}$ increases, both loci will move in the direction $(1-\hat{\psi}\bar{N})\xi(\bar{N})$ moves. Where the new steady state is going to be, is now ambiguous and depends on the way $(1-\hat{\psi}\bar{N})\xi(\bar{N})$ changes with $\bar{N}$. As the following proposition makes clear, for $\bar{N}$ sufficiently high, $(1-\hat{\psi}\bar{N})\xi(\bar{N})$ will decrease with $\bar{N}$. But, for $\bar{N}$ sufficiently low the opposite might be true.

**Proposition 3 (Variety and Labor Misallocation Effects vs Tax Distortion Effect):** Given Assumptions 1 and 2, $\frac{d(1-\hat{\psi}\bar{N})\xi(\bar{N})}{d\bar{N}} < 0$ for all $\bar{N} \in \left(1 - \frac{a - b - \zeta}{1 - \zeta}, \frac{1}{\hat{\psi}}\right)$, such that:

$$\bar{N} > \frac{(1-a-b)(1-\zeta)}{\hat{\psi}[(1-a-b)(1-\zeta) + \zeta]}, \text{ And if, } \frac{1-a-b-\zeta}{1-\zeta} < \frac{(1-a-b)(1-\zeta)}{\hat{\psi}[(1-a-b)(1-\zeta) + \zeta]}, \text{ there exists a sub-interval } \left(\frac{1-a-b-\zeta}{1-\zeta}, \bar{N}^\prime\right) \text{ of } \left(\frac{1-a-b-\zeta}{1-\zeta}, \frac{(1-a-b)(1-\zeta)}{\hat{\psi}[(1-a-b)(1-\zeta) + \zeta]}\right) \text{ such that } \frac{d(1-\hat{\psi}\bar{N})\xi(\bar{N})}{d\bar{N}} > 0, \text{ for all } \bar{N} \text{ in this sub-interval.}$$

**Proof:** In the Appendix.

Since, $\hat{\psi}$ should be a relatively small number, the condition $\frac{1-a-b-\zeta}{1-\zeta} < \frac{(1-a-b)(1-\zeta)}{\hat{\psi}[(1-a-b)(1-\zeta) + \zeta]}$, puts an upper bound on $\hat{\psi}$, that seems reasonable for all
practical purposes. For that matter, we shall refer to 
\[
\frac{(1-a-b)(1-\zeta)}{\tilde{\psi}[(1-a-b)(1-\zeta) + \zeta]}
\] as the threshold value of \( N \).

Proposition 3 can be illustrated in Figure 5, also. In this case, an increase in \( \tilde{N} \) that decreases
(increases) \((1-\tilde{\psi}\tilde{N})\tilde{\zeta}(\tilde{N})\) corresponds to a movement northeast (southwest) of point A, like point
C (B). Hence, in the case of a fixed number of publicly provided intermediate goods, an increase in
the number of these goods will have ambiguous effects on steady state output and growth towards
this steady state, as these effects will depend on the existing number of publicly provided
intermediate goods. However, the rationale for this nonlinearity is straightforward. For a relatively
low \( N \), an increase in this number is associated with the dominance of the “variety” effect over the
combination of the “labor misallocation” and “tax distortion” effects. On the contrary, for a
relatively high \( N \), an increase in this number is associated with the dominance of the combination of
the “labor misallocation” and “tax distortion” effects over the “variety” effect. For, as it can be
easily verified, the “variety” effect (“labor misallocation” and “tax distortion” effects) is decreasing
(are increasing) with \( N \). The important implication of this result for the stylized facts of the
Introduction, will be discussed in the next section.

2.5 Government’s Objective Function

The stage has, now, been set to investigate the case of an endogenous income tax rate or an
endogenous number of publicly provided intermediate goods, \( N_i \). This income tax rate or number
of publicly provided intermediate goods, of course, must be decided by government. To do this, we
must specify the government’s objective function. Once the government objective function is
specified, the problem of government is a straightforward social planner’s problem. That is,
government decides on the income tax rate or the number of publicly provided intermediate goods,
so as to maximize its objective function, subject to the equilibrium laws of motion of the previous
section and the government budget constraint. The solution to this problem is the so called
politicoeconomic equilibrium.

First, we consider the case of the Median Voter Government, where the objective function of
government is the objective function of the representative household. Moreover in order to simplify,
henceforth, we consider the case where \( \gamma = \delta = 1 \). This is the case of logarithmic household
preferences and full capital depreciation. Then, the equilibrium laws of motion (15) and (16),
reduce to:\(^{25}\)

\[
c_i = (1-\alpha\beta)[1-\tilde{\psi}(N_{i+1}-N_i)-\tilde{\psi}N_i]\tilde{\zeta}(N_i)k_i^n
\]  

\(^{25}\)To verify this, observe that (20) comes as an implication of the resource constraint (16) and that (19) satisfies the
Euler condition (15).
\[ k_{r+1} = a \beta (1 + g_x)^{-1} \left[ 1 - \psi (N_r - N) - \psi N_r \right] \xi(N_r)k_r^a \quad (20) \]

And, the temporal utility function of the representative household becomes logarithmic, so that the objective function of the representative household and the so called Median Voter Government is given by:

\[
W^{MV}\left[ \{k_{r+1}, N_{r+1}\}_{r=0}^{\infty}; (k_0, N_0) \right] = \sum_{t=0}^{\infty} \beta^t \ln c_t \\
= \sum_{t=0}^{\infty} \beta^t \ln \left( (1 - a \beta) \left[ 1 - \psi (N_{r+1} - N_r) - \psi N_r \right] \xi(N_r)k_r^a \right) 
\quad (21)\]

The problem of the Median Voter Government is to find a plan of the form \( \{k_{r+1}, N_{r+1}\}_{r=0}^{\infty} \) so as to maximize (21), subject to (20). We shall refer to the solution of this problem as the Median Voter politicoeconomic equilibrium. It should be mentioned that the government budget constraint, is such that choosing the number of intermediate goods in the beginning of period \( t+1 \) completely determines the income tax rate. Thus, this politicoeconomic equilibrium assumes that there is a commitment technology with respect to the income tax rate.26

Second, motivated by the paradigm of South European countries, where political parties and governments have been dominated by unions and especially those of the greater public sector, we wish to consider a situation where insiders’ unions are controlling government. We shall refer to this type of government as Government of Insiders. But since in the equilibrium considered in the previous subsection and, in particular, in the Nash equilibrium characterizing the outcome of the insiders’ unions strategic interaction, we assumed that each union takes the number of publicly provided intermediate goods as given and beyond their control, it seems contradictory to argue that unions cooperate to control/influence government.27 However, there is no such contradiction. Unions “play” non-cooperatively vis-à-vis each other with respect to the wage rate, as an increase in the wage rate set by each union affects positively its own utility but negatively each other union. This is because, such an increase, due to the assumed gross complementarity, lowers labor demand facing all other unions. But, have an incentive to cooperate with each other with respect to the income tax rate / the number of publicly provided intermediate goods. This is because a higher, say, income tax rate, increases the number of publicly provided intermediate goods and increases the demand for labor facing each union, also due to gross complementarity. Hence, all insiders’ unions have an incentive to increase this tax rate (financing of the underlying infrastructure). For that matter, unions’ interests are simultaneously to compete for wage premiums and cooperate for the

26 Admittedly, here we avoid all problems that arise due to the lack of such commitment. See, e.g., Acemoglu (2009, Ch. 22), for what is referred to as the “hold up” problem.
27 This is what is referred to as “political elite” (see e.g., Acemoglu 2009, ch. 22). Elites are taken to make the political decisions and possibly engage in economic activities. In our case, the political elite consists of the members of insiders’ unions. Or, again, in Acemoglu’s terminology, we assume insiders’ unions to enjoy de facto political power.
number of publicly provided intermediate goods. On the contrary, however, in a world of no
insiders, there is no need for such cooperation. We consider then the objective function of Insiders
Government to be a function of the sum of utilities of all insiders’ unions,
\[
\sum_{i=0}^{\infty} \beta^i \ln \int_0^1 \left[ w_i'(z) - w_i^o \right]^{\lambda(z)} L_i(z) dz
\]
which in the symmetric case reduces to:
\[
W^{GI}\left[ \{k_{t+1}, N_{t+1}\}_{t=0}^{\infty}; (k_0, N_0) \right] = \sum_{i=0}^{\infty} \beta^i \ln \left[ \nu(N_i) \xi(N_i) k^o_i \right]^{\lambda}
\]
\[
= \sum_{i=0}^{\infty} \beta^i \left\{ \lambda \ln \left[ \frac{\nu(N_i)}{(1-\alpha)\nu(N_{t+1})} \right] + \lambda \ln c_i \right\}
\]
where
\[
\nu(N_i) = \frac{\nu(N_i) - 1}{\nu(N_i) \left[ 1 - a - b + \nu(N_i) \right]^{-1}}
\]

The problem of the Government of Insiders is to find a plan of the form \( \{k_{t+1}, N_{t+1}\}_{t=0}^{\infty} \) so as to
maximize (22), subject to (20). Clearly, then, Median Voter Government preferences depend on
consumption of the representative household only. But, Government of Insiders preferences depend
on a fraction, \( \lambda \), of the representative household preferences; the function \( \nu\nu(N_i) \), which, as will
be shown in the proof of the next proposition, is an increasing and concave of the public sector
wage premium, \( \nu(N_i) \); and government’s share of output, \( [1 - \psi(N_{t+1} - N_t) - \psi N_t] \). Interestingly,
as can be seen from (23), this last dependence occurs in such a way so as to offset the effect of the
government’s share of output incorporated in the consumption of the representative household.

Further, since we are interested in comparing societies with different politicoeconomic
structures, we wish to consider a hybrid of the government objective functions introduced above.
That is, following the political economy literature (see, e.g., Persson and Tabellini (2002), Ch. 7),
we consider a government that to some degree is influenced by representative household
preferences and is influenced, likewise, by insiders’ unions preferences. Thus, to avoid scale
problems, we consider a government that seeks to minimize a weighted average of the percentage
deviations of: (a) the welfare of the representative household from the welfare achieved under the
solution of the Median Voter; and (b) the welfare of all insiders’ unions from the welfare achieved
under the solution of the Government of Insiders:\[28\]
\[
W^\rho = \rho \left\{ \bar{W}^{G^I} - W^{G^I}\left[ \{k_{t+1}, N_{t+1}\}_{t=0}^{\infty}; (k_0, N_0) \right] \right\} + (1 - \rho) \left\{ \bar{W}^{G^I} - W^{G^I}\left[ \{k_{t+1}, N_{t+1}\}_{t=0}^{\infty}; (k_0, N_0) \right] \right\}
\]

\[28\] We prove below that the solutions to the Median Voter government and the Government of Insiders exist, so that (24)
is well defined.
subject to the capital law of motion (20), where \(1 - \rho \in (0,1)\) is the relative influence of insiders’ unions on government. We shall refer to this problem as the problem of the Hybrid Government and to the solution of this problem as the Hybrid politicoeconomic equilibrium. Clearly, for \(\rho = 1 (\rho = 0)\), the Hybrid politicoeconomic equilibrium collapses to the Median Voter (Government of Insiders) one. And, for an appropriate choice of \(\rho \in (0,1)\), the Hybrid equilibrium represents a politicoeconomic equilibrium with any given degree of insiders influence over the representative household in government decisions.

3. POLITICOECONOMIC EQUILIBRIUM

In this section we characterize the basic properties of the Hybrid politicoeconomic equilibrium. The following is the main result of this paper.

**Proposition 4 (Politicoeconomic equilibrium):** Suppose \(\rho \in [0,1)\) or \(\rho = 1\) and

**Assumption 3**

\[
\frac{(1-a-b)(1-\xi)[1-\xi-\psi(1-a-b-\xi)]}{(1-a-b-\xi)\xi} > \left(\beta^{-1} - 1\right)\bar{\psi} + \bar{\psi},
\]

Then, given Assumptions 1 and 2:

(a) The Hybrid politicoeconomic equilibrium is characterized by (20) and

\[
\varphi(N_{r+1}) = \frac{\beta^{-1}\bar{\psi}}{1-\psi(N_{r+1}-N_i)-\psi N_i} - \frac{\bar{\psi} - \psi}{1-\psi(N_{r+2}-N_{r+1})-\psi N_{r+1}},
\]

where

\[
\varphi(N_{r+1}) = A \frac{\xi'(N_{r+1})}{\xi(N_{r+1})} + B \frac{\psi'(N_{r+1})}{\nu(N_{r+1})},
\]

with \((A, B) = \left(\frac{\rho + \lambda(1-\rho)}{\rho + \alpha \beta \lambda(1-\rho)}, \frac{(1-\alpha \beta) \lambda (1-\rho)}{\rho + \alpha \beta \lambda (1-\rho)}\right)\).

(b) There exists a unique steady state, \((k^{\rho}, N^{\rho}) \in (0,\infty) \times \left(\frac{1-\alpha - b - \xi}{1-\xi}, \frac{1}{\bar{\psi}}\right)\), associated with this equilibrium, such that:

\[
k^{\rho} = \left[\frac{a \beta (1-\psi N^{\rho}) \xi(N^{\rho})}{1+g_d} \right]^{1-\alpha}
\]

and \(N^{\rho}\) is the unique solution to:

\[(1-\psi N)\varphi(N) = (\beta^{-1} - 1)\bar{\psi} + \bar{\psi} .\]

(c) Moreover, an increase in the relative influence of insiders’ unions, \(1 - \rho\), would lead to a higher steady state value, \(N^{\rho}\).
Proof: In the Appendix.

Difference equations (20) and (25) describe the transitional dynamics of the politicoeconomic equilibrium. Condition (25) is the key condition characterizing the transition of publicly provided intermediate goods. These costs (benefits) are measured in terms of discounted utility decreases (increases) associated with decreases (increases) of an equivalent amount in consumption, as the amount of resources used to increase (decrease) the number of publicly provided intermediate goods. It is instructive to consider, first, these costs and benefits in the simpler case of the Median Voter politicoeconomic equilibrium. In this case, $\rho = 1$, so that $A = 1$ and $B = 0$; and therefore,

$$\varphi(N_{t+1}) = \frac{\xi'(N_{t+1})}{\xi(N_{t+1})}.$$ Then, observe that an increase in the number of publicly provided intermediate goods in any given period $t$ on private capital will involve the following four effects: (i) A decrease at the beginning of period $t+1$ at a rate equal to $a\beta\psi[1-\psi(N_{t+1}-N_i)-\psi N_i]\xi(N_i)k^n_i$, due to the diversion of resources from private capital to the construction of the underlying infrastructure associated with the increase in the number of publicly provided intermediate goods (i.e., new infrastructure). (ii) A decrease at the beginning of period $t+2$ at a rate equal to $a\beta\psi[1-\psi(N_{t+2}-N_{t+1})-\psi N_{t+1}]\xi(N_{t+1})k^n_{t+1}$, due to the diversion of resources from private capital to the maintenance of the new public sector infrastructure. (iii) An increase at the beginning of period $t+1$ at a rate equal to $a\beta\psi[1-\psi(N_{t+2}-N_{t+1})-\psi N_{t+1}]\xi(N_{t+1})k^n_{t+1}$, due to the ensuing non-diversion of resources to the construction of this infrastructure, since it is already in place. (iv) An increase at the beginning of period $t+1$ at a rate equal to $a\beta[1-\psi(N_{t+2}-N_{t+1})-\psi N_{t+1}]\frac{\xi'(N_{t+1})}{\xi(N_{t+1})}\xi(N_{t+1})k^n_{t+1}$, due to the increase in TFP, brought about by the new infrastructure. In terms of discounted utility, changes in consumption in period $t$ are valued at

$$\beta'c^{-1}_t = \beta'(1-a\beta)(1-\psi(N_{t+1}-N_i)-\psi N_i)\xi(N_i)k^n_i$$

and, changes in consumption in period $t+1$ are valued at

$$\beta'c^{-1}_{t+1}.$$ Hence, in terms of discounted utility, the costs and benefits associated with a marginal increase in the number of publicly provided intermediate goods are

$$\beta'c^{-1}_t a\beta\psi[1-\psi(N_{t+1}-N_i)-\psi N_i]\xi(N_i)k^n_i + \beta'c^{-1}_{t+1}a\beta\psi[1-\psi(N_{t+2}-N_{t+1})-\psi N_{t+1}]\xi(N_{t+1})k^n_{t+1}$$

and

$$\beta'c^{-1}_t a\beta\psi[1-\psi(N_{t+2}-N_{t+1})-\psi N_{t+1}]\xi(N_{t+1})k^n_{t+1} + a\beta[1-\psi(N_{t+2}-N_{t+1})-\psi N_{t+1}]\frac{\xi'(N_{t+1})}{\xi(N_{t+1})}\xi(N_{t+1})k^n_{t+1},$$

respectively. Simplifying terms, it follows that (25), simply requires that these costs and benefits should be equal at the margin. In the opposite case of the Government of Insiders politicoeconomic
equilibrium, where \( \rho = 0 \), so that \( A = \frac{1}{a \beta} \) and \( B = \frac{1-a \beta}{a \beta} \), \( \varphi(N_{t+1}) = \frac{1}{a \beta} \left[ \frac{\xi'(N_{t+1}) - (1-a \beta) \psi'(N_{t+1})}{\psi'(N_{t+1})} \right] \).

Then, observe that an increase in the number of publicly provided intermediate goods, in this case, will involve exactly the four effects (i) – (iv) on private capital as in the case of the Median Voter politicoeconomic equilibrium, but now these changes are valued differently. That is, in terms of discounted utility, changes in consumption in period \( t \) and period \( t+1 \) are, now, valued at \( \lambda \beta e_t^{-1} \) and \( \lambda \beta e_{t+1}^{-1} \), respectively. Moreover, discounted utility is, now, directly (i.e., not through consumption) affected by four additional terms. First, in period \( t+1 \), by \( \lambda \beta e_{t+1}^{-1} \frac{\psi'(N_{t+1})}{\psi'(N_{t+1})} \), as the increase in the number of publicly provided intermediate goods raises the public sector wage premium, increasing the utility of insiders. Finally, the other three terms are opposite and proportional (i.e., multiplied by \( 1-a \beta \)) as the effects (i)-(iii) on private capital. This, as already mentioned, is due to the fact that insiders preferences depend directly, positively, on governments, share of output (i.e., through \( \{1-a \beta\}[1-\psi(N_{t+1} - N_t) - \psi N_t]^{-1} \)). Again, equating these costs and benefits gives (25), for \( \rho = 0 \). Clearly then, for \( \rho \in (0,1) \), (25) is a linear combination of its above two special versions. Interestingly, the weights in this linear combination (i.e., \( A \) and \( B \)) depend not only on \( \rho \), but on insiders union power, \( \lambda \), and the technological parameters \( a \) and \( b \).

Now, the stage has been set to look into the steady state of the politicoeconomic equilibrium. This steady state is characterized by (27) and (28). Condition (28) that characterizes the steady state number of publicly provided intermediate goods is crucial. First, recall that Proposition 2 ensures that \( \frac{\xi'(N)}{\xi(N)} \) is strictly positive. Likewise, it is shown that, given Assumptions 1 and 2, \( \frac{\psi'(N)}{\psi(N)} \) is strictly positive. Moreover, it is shown that \( \frac{\psi'(N)}{\psi(N)} \) approaches \( +\infty \), as \( N \) approaches \( \frac{1-a-b-\zeta}{1-\zeta} \) (i.e., \( \psi(N) \) approaches 1) and approaches 0, as \( N \) approaches \( \frac{1}{\psi} \) (i.e., the endpoint of its range).

Further, it is shown that both \( (1-\psi N)\frac{\xi'(N)}{\xi(N)} \) and \( (1-\psi N)\frac{\psi'(N)}{\psi(N)} \) are strictly decreasing and strictly concave in \( N \). Clearly then, \( (1-\psi N)\varphi(N) \) is positive, strictly decreasing, strictly concave and approaches 0, as \( N \) approaches \( \frac{1}{\psi} \). Further, when \( \rho \in [0,1) \), it approaches \( +\infty \), as \( N \) approaches \( \frac{1-a-b-\zeta}{1-\zeta} \). These facts, imply that when \( \rho \in [0,1) \), (28) has a unique interior solution, \( N^\rho \). This
fact is illustrated in Figure 6, below, where the \((1 - \psi N)\varphi(N)\) locus intersects the \((\beta^{-1} - 1)\tilde{\psi} + \tilde{\psi}\) locus at a unique point \(N^\rho = \left(\frac{1 - \alpha - b - \zeta}{1 - \zeta}, \frac{1}{\psi}\right)\).

Figure 6. Illustration of existence and uniqueness of the steady state politicoeconomic equilibria.

When \(\rho = 1\) (i.e., in the Median Voter politicoeconomic equilibrium), \(A = 1\) and \(B = 0\). Then, \((1 - \psi N)\varphi(N) = (1 - \psi N)\frac{\xi'(N)}{\xi(N)}\) and Assumption 3 in needed to ensure that the \((1 - \psi N)\varphi(N)\) locus takes a value higher than \((\beta^{-1} - 1)\tilde{\psi} + \tilde{\psi}\), as \(N\) approaches \(\frac{1 - a - b - \zeta}{1 - \zeta}\). In Figure 6, \((1 - \psi N)\varphi(N)\) for the Median Voter and Government of Insiders politicoeconomic equilibria are denoted by \((1 - \psi N)\varphi^{MV}(N)\) and \((1 - \psi N)\varphi^{GI}(N)\), respectively. Obviously, since \((1 - \psi N)\varphi^{MV}(N) < (1 - \psi N)\varphi^{GI}(N)\), the steady state number of publicly provided intermediate goods in the Median Voter politicoeconomic equilibrium, \(N^{MV}\), is less than the corresponding number of the Government of Insiders politicoeconomic equilibrium, \(N^{MV}\). The ordering between \(N^{GI}\) and \(N^{MV}\) is a manifestation of the “political effect” mentioned in the Introduction. Recall that the Median Voter solution incorporates the “labor misallocation” effect. So, steady state capital per efficient household in the Median Voter solution is already lower than the Second Best (i.e., the no
wage premium but with distorting taxation Median Voter politicoeconomic equilibrium). Thus, while the Median Voter social planner chooses the number of publicly provided intermediate goods balancing (at the margin) the “variety” effect with the combination of the “labor misallocation” and the “tax distortion” effects, the Government of Insiders chooses that number so as to balance the same effects but with greater values for these effects, due to the utility gains from the public sector wage premium and government’s share of output. For that matter, the Government of Insiders chooses a greater number of intermediate goods than the Median Voter social planner. In fact, as Part c of Proposition 4 makes clear, generally, an increase in insiders influence over government, \((1 - \rho)\), would imply a higher steady state number of publicly provided intermediate goods.

Next, we turn on steady state capital, given by (27). Combining Propositions 3 and 4, it is apparent that there is no direct answer to the question whether there will be a higher or a lower steady state capital in the Median Voter social planner solution or the Government of Insiders solution. Or, in general in the Hybrid politicoeconomic equilibrium, how steady state capital will vary with an increase in the influence of insiders over government. In particular, for relatively low numbers of steady state publicly provided intermediate goods, \(N^\rho\), an increase in insiders influence over government, leading to a higher number of those goods in the steady state, may entail higher steady state capital and faster growth (i.e., growth along the convergence to the steady state). But, for a relatively higher number of steady state publicly provided intermediate goods, \(N^{\rho'}\), a higher number for these goods leads to lower steady state output and growth. We summarize this important result in the following corollary.

**Corollary 3:** Consider two values of \((1 - \rho), (1 - \rho')\) and \((1 - \rho^*)\), such that \((1 - \rho') > (1 - \rho^*)\).

Suppose that \((k^\rho, N^\rho)\) and \((k^{\rho'}, N^{\rho'})\) are the steady states defined in Proposition 4 that correspond to \((1 - \rho')\) and \((1 - \rho^*)\), respectively. If \(N^{\rho'}, N^{\rho''} \in \left[\frac{(1 - a - b)(1 - \zeta)}{\tilde{\psi}((1 - a - b)(1 - \zeta) + \zeta)} \frac{1}{\tilde{\psi}}\right]\), so that \(\frac{d(1 - \tilde{\psi} N)\dot{\xi}(N)}{dN} < 0\), \(k^{\rho'} > k^\rho\). And, if \(N^{\rho''}, N^{\rho'''} \in \left[\frac{1 - a - b - \zeta}{1 - \zeta}, \frac{(1 - a - b)(1 - \zeta)}{\tilde{\psi}((1 - a - b)(1 - \zeta) + \zeta)}\right]\) so that \(\frac{d(1 - \tilde{\psi} N)\dot{\xi}(N)}{dN} > 0\), then \(k^{\rho'''} < k^{\rho''}\).

Unfortunately, no clear cut answer can be obtained if \(N^{\rho'}\) is in the sub-interval of relatively large \(N\) and \(N^{\rho''}\) is in the sub-interval of relatively low \(N\).

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29 In terms of Figure 6, the Second best corresponds to a solution with a \((1 - \tilde{\psi} N)\dot{\xi}(N)\) locus that declines faster than the one depicted in this figure.
Proposition 4 is helpful in explaining the stylized facts of the Introduction. For, if countries differ with respect to $1 - \rho$ (i.e., the relative weight of insiders in influencing the government), countries with high $1 - \rho$ will eventually have a high number of publicly provided intermediate goods and these countries will be more likely to have a number of publicly provided intermediate goods which is higher than the threshold of Proposition 3. Then these countries will have lower steady state capital and output, than countries with relatively low $1 - \rho$.

We conclude this section with establishing that the Hybrid politicoeconomic equilibrium is at least asymptotically stable around the steady state. In particular, it is characterized by a sequence of numbers of publicly provided intermediate goods, $\{N_t\}_{t=0}^\infty$, converging monotonically to the steady state, for any $N_0$, sufficiently close to $N$.

**Proposition 5** (a) Let $\{N_t\}_{t=0}^\infty$ be the solution to (25) for any given $N_0$ and let $N$ be the unique steady state defined by (28). Around this steady state, the first order approximation of the solution to (25), $N_{t+1} = g(N_t), \quad N = g(N)$, for some function $g(\cdot) : \left(\frac{1-\alpha-b-\zeta}{1-\psi}, \frac{1}{\psi}\right) \rightarrow \left(\frac{1-\alpha-b-\zeta}{1-\psi}, \frac{1}{\psi}\right)$, is given by:

$N_{t+1} = g(N_t) = (1 - \sigma)N + \sigma N_t$, \quad (29)

where,

$$\sigma = \frac{1}{2} \left[ \frac{b\tilde{\psi} - \tilde{\psi}}{\tilde{\psi} - \psi} - \frac{(1-\psi)\tilde{\psi}}{(\psi - \tilde{\psi})\phi'(N)} - \frac{\tilde{\psi}}{\psi - \tilde{\psi}} \left( \frac{1-\psi N}{\psi - \tilde{\psi}} \phi'(N) \right)^2 - 4\beta^{-1} \right]^{1/2}.$$

$$\in \left(0, \frac{\psi - \tilde{\psi}}{\psi}\right) \subset (0,1)$$

Proof: In the Appendix.

Since, $\sigma \in \left(0, \frac{\psi - \tilde{\psi}}{\psi}\right)$, it follows that the convergence of the number of publicly provided intermediate goods to its steady state, $N$, is monotonic. Moreover, it follows from (20) and (29) that around the steady state $(k, N)$ of the politicoeconomic equilibrium, the law of motion of capital can be approximated by:

$$k_{t+1} = a\beta \left(1 + g_a\right)^{-1} \left\{1 - (1 - \sigma)\tilde{\psi}N + [(1 - \sigma)\tilde{\psi} - \tilde{\psi}]N_t \right\} \xi(N_t)k_t^a$$

(30)

And, since $\sigma \in \left(0, \frac{\psi - \tilde{\psi}}{\psi}\right)$, $(1 - \sigma)\tilde{\psi} - \tilde{\psi} > 0$ and therefore the convergence of capital is also monotonic.
4. STYLIZED FACTS EXPLANATIONS AND THE CASE OF A GROWTH REVERSAL

Recall from Proposition 4, that an increase in insiders influence over government (i.e., $1 - \rho$) will lead to an increase in the steady state number of publicly provided intermediate goods, $N$. Further, recall from Proposition 3 that steady state TFP, $\left(1 - \psi N\right)\xi(N)$, does not change monotonically with the steady state number of publicly provided intermediate goods, $N$. That is, for a relatively low $N$, an increase in this number is associated with the dominance of the “variety” effect over the combination of the “labor misallocation” and the “political effect,” while the opposite is true after a certain threshold, $\bar{N}$. These results, along with the monotonic convergence of $(k, N)_{t=0}^{\infty}$ towards $(k, N)$, established in Proposition 5, allow for the possibility of a growth reversal, brought about by an increase in insiders influence over government. Despite the fact that here we have two state variables, this possibility can be illustrated with the help of the standard neoclassical growth phase diagram. In Figure 7, the horizontal axis measures capital in the current period and the vertical axis measures capital in the next period. For any given $(k, N)$ in the current period, capital in the next period is given by

$$\frac{a\beta}{1 + g_A}(1 - \psi N)\xi(N)k^a$$

where $N = g(N)$. Suppose, now, that $N \leq \bar{N}$, where $\bar{N}$ is the threshold value (i.e., $\frac{d(1 - \psi \bar{N})\xi(\bar{N})}{d\bar{N}} = 0$ and for $N$ below (above) $\bar{N}$, $\frac{d(1 - \psi N)\xi(N)}{dN} > 0 (< 0)$). Clearly, the locus

$$\frac{a\beta}{1 + g_A}(1 - \psi N)\xi(N)k^a$$

as long as $N_i < \bar{N}$, lies above any transition locus

$$\frac{a\beta}{1 + g_A}(1 - \psi \bar{N})\xi(\bar{N})k^a$$

It also follows, as it can be readily seen from (30), that as long as $N_i < N_{t+1}$, next period capital will be given by a higher transition locus than that giving the current period capital. It follows that capital is moving along a rising trajectory, like the dotted line in Figure 7. Suppose, now, that while capital is at $C$, there is an increase in insiders influence, so that the new steady state is $\bar{N}$, where $\bar{N} < \bar{N}$. Clearly, since the steady state locus

$$\frac{a\beta}{1 + g_A}(1 - \psi \bar{N})\xi(\bar{N})k^a$$

is also an upper bound to all steady state loci, the new steady state locus, $\frac{a\beta}{1 + g_A}(1 - \psi \bar{N})\xi(\bar{N})k^a$ is below the first. For a sufficient increase in insiders’ power over government, point $C$ maybe above the new steady state locus, $\frac{a\beta}{1 + g_A}(1 - \psi \bar{N})\xi(\bar{N})k^a$. For
precisely the same reasons like the ones used to establish the dotted trajectory, now capital will follow a falling trajectory, like the dashed trajectory from C to D, establishing the growth reversal in the case of a rising influence of insiders over government.

**Figure 7: A Growth Reversal**

This growth reversal possibility serves as an explanation of what may have occurred in the Southern European economies. An indication of the increase in the influence of insiders over government, in the Southern European countries since the early eighties, is the very high public sector wage premia reported in Figure 1. In the model’s framework, one may think of South European countries, as countries with a low initial level of $N$, but with a progressively higher $1 - \rho$, as insiders’ influence over government grew stronger. Thus, about thirty years ago, the advent of the insiders-outsiders society in Southern European countries, when these countries were at a lower stage of development and, to a varying degree, they were lacking adequate infrastructures, may have helped them develop and grow. Precisely because, it led to the development of that infrastructure, when private provision of this infrastructure was poor or non-existing. But, eventually, the insiders-outsiders society may have exceeded its usefulness and insiders’ unions enjoyed substantial wage premia, leading to labor misallocation and tax distortion and/or high debt, that caused the growth problems these countries are experiencing at the present.

Credence to the validity of the model are the stylized facts of Figures 1-4. First, the wage premium in the public sector is justified and related to union power, production technology, especially the degree of complementarity among publicly provided intermediate goods and the
degree of government involvement in the economy, with the number of publicly provided intermediate goods thought to be a proxy of the latter (Proposition 1). Second, it provides for an explanation for the negative correlation between public sector wage premium and the ratio of public over total employment (Corollary 1). Third, it provides for an explanation for the negative correlation between public sector wage premium and after tax TFP (Proposition 3). Finally, the negative correlation between public sector wage premium and output growth can be decomposed into two parts: for a given country over time, it can be explained by growth reversal arguments similar to the ones discussed above, brought about by the advent of the insiders-outsiders society. And, for different countries, this negative correlation can be attributed to the degree their economies are characterized by the insiders-outsiders society. For example, one may think of the South European countries having very high \( \lambda \) and very high \( 1 - \rho \), so that the threshold of \( N \), in Proposition 3, is exceeded, while countries with very low or non-existent wage premia in the public sector, the Anglo-Saxon countries (except Australia before the millennium), for example, having very low \( \lambda \) and very low \( 1 - \rho \), so that steady state \( N \) is below this threshold. For some other countries the model’s structure may be altogether inappropriate. For example, the Nordic countries, where wage premia in the public sector are practically negligible, have very strong unions in both public and private sectors, but their unions co-operate to internalize the cost to the economy associated with a high wage premium in one industry or sector.\(^{30}\) In our model’s jargon this, practically means that outsiders behave like insiders and the Government of Insiders behaves like the Median Voter. All of these cases, however, could be addressed by an extension of our model, that incorporates a sector of privately provided intermediate goods and their corresponding unions, along the lines of Cole and Ohanian (2004).

5. CONCLUSIONS

In a synthesis of the insiders-outsiders labor market structure and the concept of an elite government, we constructed a dynamic general equilibrium model of market and political power interactions that can explain the growth reversal characterizing, in recent years, most Southern European countries. In these countries public sector unions act independently in their respective markets, but co-operate to influence government policies, including those that affect public sector infrastructures. Moreover, the model is consistent with several stylized facts pertaining to the wage premium in the public sector, such as: the negative correlation between the public sector wage premium, on the one hand, and the ratio of public sector employment over total employment, total factor productivity, and output growth, on the other hand.

\(^{30}\) See European Commission (2013) and Kollintzas et al. (2014), for related country clusterings.
ACKNOWLEDGEMENTS

We are grateful to Assar Lindbeck and Fabrice Collard for valuable comments. Dimitris Papageorgiou and Vanghelis Vassilatos gratefully acknowledge co-financing of this research by the European Union (European Social Fund — ESF) and Greek national funds through the Operational Program “Education and Lifelong Learning” of the National Strategic Reference Framework (NSRF) — Research Funding Program ARISTEIA II_Public Sector Reform, Research Grant, No 5328.
REFERENCES


APPENDIX

Barro Regressions

We estimate Barro regressions adding the public sector wage premium to a list of standard explanatory variables employed in such regressions. Our sample consists of a panel of twenty-one OECD countries (Austria, Australia, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Israel, Italy, Japan, Korea, Netherlands, Norway, Portugal, Spain, Sweden, UK and US) from 1975 to 2009. Data points for the regressions are five year average. Table A.1 presents the results.

Table A.1: Barro Regressions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Dependent variable: Real per capita GDP growth rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.186**</td>
</tr>
<tr>
<td></td>
<td>(2.295)</td>
</tr>
<tr>
<td>Log(per capita GDP)</td>
<td>-0.026**</td>
</tr>
<tr>
<td></td>
<td>(-2.094)</td>
</tr>
<tr>
<td>Log(per capita GDP) squared</td>
<td>0.001**</td>
</tr>
<tr>
<td></td>
<td>(2.336)</td>
</tr>
<tr>
<td>Government non-wage consumption-to-GDP ratio</td>
<td>-0.228***</td>
</tr>
<tr>
<td></td>
<td>(-4.342)</td>
</tr>
<tr>
<td>Investment-to-GDP ratio</td>
<td>0.035</td>
</tr>
<tr>
<td></td>
<td>(0.751)</td>
</tr>
<tr>
<td>Exports-to-GDP ratio</td>
<td>0.021*</td>
</tr>
<tr>
<td></td>
<td>(1.876)</td>
</tr>
<tr>
<td>Wage premium</td>
<td>-0.012***</td>
</tr>
<tr>
<td></td>
<td>(-3.399)</td>
</tr>
<tr>
<td>Log of years of schooling</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>(-0.526)</td>
</tr>
<tr>
<td>Population growth</td>
<td>0.183</td>
</tr>
<tr>
<td></td>
<td>(1.475)</td>
</tr>
<tr>
<td>Number of observations</td>
<td>140</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.29</td>
</tr>
</tbody>
</table>

Notes: (i) Estimates are based on pooled OLS with robust standard errors; (ii) t-statistics in parenthesis; (iii) * indicates statistical significance at the 10% level of significance, ** at the 5% level, and *** at the 1% level; (iii) Columns (4)-(6) correspond to the case without the Netherlands. The latter is a significant outlier. Its exclusion changes the GDP growth - public sector wage premium correlation coefficient, as well as its t-statistic from −0.20 to −0.24 and from 2.43 to 2.88, respectively.
**Data sources and definitions**

Our main data source is the OECD Economic Outlook 90. Data is annual and the time span is 1970-2010. Missing values and or variables have been added from OECD Economic Outlook no. 85 and 88, OECD Aggregate National Accounts and AMECO.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Gross Domestic Product</td>
<td>OECD Economic Outlook and AMECO</td>
</tr>
<tr>
<td>Real Gross Domestic Product</td>
<td>OECD Economic Outlook and AMECO</td>
</tr>
<tr>
<td>Total compensation of employees</td>
<td>OECD Economic Outlook and OECD Aggregate National Accounts</td>
</tr>
<tr>
<td>Government final consumption expenditure</td>
<td>OECD Economic Outlook</td>
</tr>
<tr>
<td>Government final non-wage consumption expenditure</td>
<td>OECD Economic Outlook and AMECO</td>
</tr>
<tr>
<td>Government final wage consumption expenditure¹</td>
<td>OECD Economic Outlook and AMECO</td>
</tr>
<tr>
<td>Exports of goods and services</td>
<td>OECD Economic Outlook</td>
</tr>
<tr>
<td>Dependent employment - Total economy (Total employees)²</td>
<td>OECD Economic Outlook</td>
</tr>
<tr>
<td>Dependent employment in the private sector (Private sector employees)²</td>
<td>OECD Economic Outlook</td>
</tr>
<tr>
<td>General government employment³</td>
<td>OECD Economic Outlook</td>
</tr>
<tr>
<td>Working age population 15-64</td>
<td>OECD Economic Outlook</td>
</tr>
<tr>
<td>Gross fixed capital formation</td>
<td>OECD Economic Outlook</td>
</tr>
<tr>
<td>Total compensation of employees in the private sector</td>
<td>Own calculations (Total compensation of employees minus government final wage consumption expenditure)</td>
</tr>
<tr>
<td>Compensation rate in the private sector</td>
<td>Own calculations (Total compensation of employees in the private sector divided by private sector employees)</td>
</tr>
<tr>
<td>Compensation rate in the public sector</td>
<td>Own calculations (Government final wage consumption expenditure divided by government employment)</td>
</tr>
<tr>
<td>Public sector wage premium</td>
<td>Own calculations (Compensation rate in the public sector divided by the compensation rate in the private sector)</td>
</tr>
<tr>
<td>Total Factor Productivity</td>
<td>AMECO</td>
</tr>
</tbody>
</table>

Notes:
1. For Australia, government final wage consumption is computed as $CGW = WSSS − WSSE * EEP$, where $WSSS$ is total compensation of employees, $WSSE$ is the compensation rate in the private sector and $EEP$ is dependent employment in the private sector. Then, government final wage consumption expenditure is computed as $CGNW = CG − CGW$, where $CG$ is total government consumption.
2. For Germany and Korea, total dependent employment, $EE$, and dependent employment in the private sector, $EEP$, are respectively computed from the following relationships: $WSST = WSSS / EE$ and $WSSE = (WSSS − GCW) / EEP$, where $WSST$ is the compensation rate of the total economy, $WSSE$ is the compensation rate in the private sector, $WSSS$ is total compensation of employees, and $GCW$ is government final wage consumption expenditure. For Israel, $EE$ is computed as $EE = ET − ES$, where $ET$ is total employment and $ES$ is total self-employment. Then, $EEP$ is computed as $EEP = EE − GE$, where $GE$ is general government employment.
3. For Australia, Austria, Germany, Greece and Korea, general government employment is computed as $GE = EE − EEP$, where $EE$ is total dependent employment and $EEP$ is dependent employment in the private sector.
Proof of Proposition 2

Part a

Given Assumptions 1 and 2, \( v(N) > 1, \forall N \in \left( \frac{1 - \alpha - b - \zeta}{1 - \zeta}, \infty \right) \). Therefore, it follows from definition (18), that \( \xi(N) > 0, \forall N \in \left( \frac{1 - \alpha - b - \zeta}{1 - \zeta}, \infty \right) \). Fix \( N = N = \left( \frac{1 - \alpha - b - \zeta}{1 - \zeta}, \infty \right) \) and consider \( \xi \) as a function of \( \overline{v} = v(N) > 1 \). That is, let:

\[
\xi(\overline{N}) = \xi\left[ v^{-1}(\overline{N}) \right] = \overline{\xi}(\overline{v}) \equiv b'(1-a-b)^{1-a-b} \Phi^{1-a-b} \frac{N^{(1-a-b)(1-\zeta)}}{\zeta} \left[ 1-a-b+b\overline{v} \right]^{-a}.
\]

And, therefore, given Assumptions 1 and 2,

\[
\frac{\partial \xi}{\partial v}\bigg|_{N=\overline{N}} = \overline{\xi}'(\overline{v}) = b'(1-a-b)^{1-a-b} \Phi^{1-a-b} \frac{N^{(1-a-b)(1-\zeta)}}{\zeta} \left[ 1-a-b+b\overline{v} \right]^{-a} < 0.
\]

Part b

Differentiate \( \xi(N) \) with respect to \( N \), to get:

\[
\xi'(N) = \xi(N) \frac{1-a-b}{N} \left( 1 - \zeta - \frac{bv'(N)N[v(N)-1]}{v(N)[1-a-b+bv(N)]} \right),
\]

where:

\[
\chi(N) = \frac{1-a-b}{\zeta} \frac{\lambda b(1-a-b-\zeta)v(N)[v(N)-1]}{N[1-a-b+bv(N)]}.
\]

Clearly, then, given Assumptions 1 and 2,

\[
\xi'(N) < 0 \text{ as } \chi(N) < 0, \text{ or as } \varphi(N) \equiv 0, \text{ where: } \varphi(N) \equiv v(N)^{2} - [1 + \hat{v}(N)]v(N) - \frac{1-a-b}{b} \hat{v}(N),
\]

and, \( \hat{v}(N) \equiv \frac{(1-\zeta)N}{\lambda \zeta (1-a-b-\zeta)} > 0 \). But, given Assumptions 1 and 2, \( [1 + \hat{v}(N)] > v(N) \), \( \forall N \in \left( \frac{1-a-b-\zeta}{1-\zeta}, \infty \right) \). To see this, note that \( v(N) = 1, \) for \( N = \left( \frac{1-a-b-\zeta}{1-\zeta}, \infty \right) \). And, by Proposition 1, it is strictly increasing, strictly concave, and approaches asymptotically \( \frac{1}{1-\lambda(1-\zeta)} > 1 \), as \( N \to \infty \). On the other hand, \( [1 + \hat{v}(N)] \) is \( 1 + \frac{1}{\lambda \zeta} > \frac{1}{1-\lambda(1-\zeta)} \) for \( N = \left( \frac{1-a-b-\zeta}{1-\zeta}, \infty \right) \). Therefore, given
Assumptions 1 and 2, $\varphi(N) < 0$, $\forall N \in \left( \frac{1-\alpha-b-\zeta}{1-\zeta}, \infty \right)$ and hence $\xi'(N) > 0$, $\forall N \in \left( \frac{1-\alpha-b-\zeta}{1-\zeta}, \infty \right)$.

Q.E.D.

Proof of Proposition 3

Note that:

$$\frac{d(1-\psi N)\xi(N)}{dN} > 0 \text{ as } \frac{\xi'(N)\xi(N)}{\xi(N)} \leq 1-\psi N$$

Therefore, in view of (A.2.1) and (A.2.2), we have:

$$\frac{d(1-\psi N)\xi(N)}{dN} < 0 \text{ as } \Re[\nu(N), N] < 0$$

where

$$\Re[\nu(N), N] \equiv \nu(N)^2 - [1+\bar{\nu}(N)]v(N) - \frac{1-a-b}{b}\bar{\nu}(N)$$

and

$$\bar{\nu}(N) \equiv \left[\frac{1-\zeta}{\zeta} - \frac{\psi N}{(1-a-b)(1-\psi N)}\right] \frac{\psi N}{\lambda \psi (1-a-b-\zeta)}$$

Given Assumptions 1 and 2, $0 < \frac{1-\alpha-b-\zeta}{1-\zeta} < 1$ and in view of the facts that $\tau = \psi N$ and $0 < \tau \leq 1$, $N$ is restricted to be in the interval $N_i \equiv \left(N_i, \bar{N}_i\right) \equiv \left(\frac{1-\alpha-b-\zeta}{1-\zeta}, \frac{1}{\psi}\right)$. Note, then, that if $N \in N_2 \equiv \left[N_2, \bar{N}_2\right] \equiv \left[\frac{(1-a-b)(1-\zeta)}{\psi (1-a-b)(1-\zeta)+\zeta}, 1\right]$, $\bar{\nu}(N) \leq 0$. And, since Assumptions 1 and 2 imply that $\nu(N) \geq 1, \forall N \in N_i$, it follows that $\Re[\nu(N), N] > 0, \forall N \in N_i \cap N_2 = \left[\max\left\{N_i, \bar{N}_2\right\}, \bar{N}_2\right]$. Hence, $\frac{d(1-\psi N)\xi(N)}{dN} < 0, \forall N \in N_i \cap N_2$. Further, observe that if $N_2 \leq N, N_i \cap N_2 = N_i$ and there is no other case left to consider. Thus, suppose that $N_2 > N_i$ and consider the case where $N \in N_3 \equiv \left(N_3, \bar{N}_3\right) = \left(N_i, \bar{N}_2\right)$. Clearly, in this case $\bar{\nu}(N) > 0, \forall N \in N_3$. It follows that $\Re[\nu(N), N]$ may be factored as follows:

$$\Re[\nu(N), N] = [\nu(N) - \bar{\nu}(N)][\nu(N) - \nu(N)]$$
where $v(N), v(N) : N \rightarrow \mathbb{R}$, such that:

$$-v(N), v(N) > 0$$

$$v(N)\overline{v}(N) = \frac{1 - \alpha - b}{b} \overline{v}(N) < 0$$

Therefore, $\Re[v(N), N] < 0$, for all $N \in N_3$, if and only if $1 \leq v(N) < \overline{v}(N)$, for all $N \in N_3$.

In this case, of course, $\frac{d(1 - \dot{v}N)\xi(N)}{dN} > 0$, $\forall N \in N_3$. To complete the proof, it suffices to show that there exists a non-empty sub-interval of $N_1$ such that $1 \leq v(N) < \overline{v}(N)$. To show this, first observe that $1 \leq v(N) < \overline{v}(N)$, $\forall N \in N_3$, if and only if $1 \leq v(N) < 1 + \dot{v}(N) + \varepsilon$, $\forall N \in N_3$, where $0 < \varepsilon < \sup_{N \in N_3}[-v(N)]$. Recall that $v(N) : N_1 \rightarrow (1, v(N))$ is strictly increasing throughout its domain and $v(N_2) = v(N_3) = 1$. Consider, now, any sufficiently small $\varepsilon \in \left(0, \sup_{N \in N_3}[-v(N)]\right)$ and observe that there exists a $N_4(\varepsilon) \in N_3$ and $N_4(\varepsilon) > N_3$ such that $v(N_4(\varepsilon)) = 1 + \varepsilon$. Clearly, then, $1 \leq v(N) < 1 + \dot{v}(N) + \varepsilon$ for all $N \in N_4(\varepsilon) \equiv (N_4(\varepsilon), N_4(\varepsilon)) = (N_3, v^{-1}(1 + \varepsilon)) \subseteq N_3$ (See Figure A.1).

Q.E.D.

**Proof of Proposition 4**

**Part a**

The Euler – Lagrange conditions associated with the Hybrid politicoeconomic equilibrium are given by:
\[
[\rho \alpha \beta + (1-\rho)\alpha \beta \lambda]k_{t+1}^+ - \mu_t (1+g_a) + \\
\alpha \beta \mu_{t+1} \left[1-\bar{\psi}(N_{t+2} - N_{t+1}) - \bar{\psi}N_{t+1} \right] \xi(N_{t+1})k_{t+1}^a = 0
\]  
(A.4.1)

and

\[
-\beta^{-1} \rho \bar{\psi} \left[1-\bar{\psi}(N_{t+1} - N_t) - \bar{\psi}N_t \right] + \\
\rho \left\{ (\bar{\psi} - \bar{\psi}) + [1-\bar{\psi}(N_{t+2} - N_{t+1}) - \bar{\psi}N_{t+1}] \xi'(N_{t+1}) \right\} \left[1-\bar{\psi}(N_{t+2} - N_{t+1}) - \bar{\psi}N_{t+1} \right]^{-1} + \\
\lambda(1-\rho) \left[ \frac{\nu'(N_{t+1})}{\nu(N_{t+1})} + \frac{\xi'(N_{t+1})}{\xi(N_{t+1})} \right] - \beta^{-1} \mu_t \alpha \beta \bar{\psi} \xi(N_t)k_t^a + \\
\mu_{t+1} \alpha \beta \left\{ (\bar{\psi} - \bar{\psi}) + [1-\bar{\psi}(N_{t+2} - N_{t+1}) - \bar{\psi}N_{t+1}] \xi'(N_{t+1}) \right\} \xi(N_{t+1})k_{t+1}^a = 0
\]  
(A.4.2)

where \( \mu_t \) is the Lagrange multiplier associated with (20).

In view of (20), (A.5.1) can be rewritten as:

\[
\mu_t (1+g_a)k_{t+1} = [\rho \alpha \beta + (1-\rho)\alpha \beta \lambda] + \alpha \beta \mu_{t+1} (1+g_a)k_{t+2} = 0
\]  
(A.4.3)

Solving (A.5.3) forward and provided that \( \lim_{u \to \infty} (\alpha \beta)^u \mu_{t+u} (1+g_a)k_{t+u+1} = 0 \) gives:

\[
\mu_t (1+g_a)k_{t+1} = \frac{\rho \alpha \beta + (1-\rho)\alpha \beta \lambda}{1-\alpha \beta}
\]  
(A.4.4)

We verify that (A.5.4) satisfies \( \lim_{u \to \infty} (\alpha \beta)^u \mu_{t+u} (1+g_a)k_{t+u+1} = \frac{\rho \alpha \beta + (1-\rho)\alpha \beta \lambda}{1-\alpha \beta}\lim_{u \to \infty} (\alpha \beta)^u = 0 \), since \( a, \beta \in (0,1) \).

Therefore,

\[
\mu_t = \frac{\rho \alpha \beta + (1-\rho)\alpha \beta \lambda}{(1-\alpha \beta)(1+g_a)k_{t+1}}
\]  
(A.4.5)

Then, in view of (A.5.5), (A.5.2) gives:

\[
\phi(N_{t+1}) = \frac{\beta^{-1} \bar{\psi}}{1-\bar{\psi}(N_{t+1} - N_t) - \bar{\psi}N_t} - \frac{\bar{\psi} - \bar{\psi}}{1-\bar{\psi}(N_{t+2} - N_{t+1}) - \bar{\psi}N_{t+1}};
\]  
(A.4.6)

where

\[
\varphi(N_{t+1}) = A \frac{\xi'(N_{t+1})}{\xi(N_{t+1})} + B \frac{\nu'(N_{t+1})}{\nu(N_{t+1})};
\]  
(A.4.7)

with

\[
(A, B) = \left( \frac{\rho + \lambda (1-\rho)}{\rho + \alpha \beta \lambda (1-\rho)}, \frac{(1-\alpha \beta) \lambda (1-\rho)}{\rho + \alpha \beta \lambda (1-\rho)} \right).
\]  
(A.4.8)

**Part b**

Let any steady state defined by:
\[ \cdots = k_{i-1} = k_i = k_{i+1} = \cdots = k^\rho; \]
\[ \cdots = N_{i-1} = N_i = N_{i+1} = \cdots = N^\rho. \]

It follows from (20) and (A.5.6) that any steady state must satisfy:
\[ k^\rho = \left[ \frac{a\beta(1-\psi N^\rho)\xi(N^\rho)}{1+g_d} \right]^{1/a} \]  
(A.4.9)
and
\[ (1-\psi N^\rho)\varphi^\rho(N^\rho) = (\beta^{-1}-1)\bar{\psi} + \bar{\psi} \]  
(A.4.10)
To prove the existence and uniqueness of the steady state, first note that the RHS of (A.4.10) is a positive constant, \((\beta^{-1}-1)\bar{\psi} + \bar{\psi}\) and the LHS in this equality is \((1-\psi N)\varphi(N) = A(1-\psi N)\frac{\varphi^\rho(N)}{\xi(N)} + B(1-\psi N)\frac{\nu'(N)}{\nu(N)}\), where \(A > 0\) and \(B \geq 0\), with \(B = 0\) if and only if \(\rho = 1\). Consider first the function \((1-\psi N)\frac{\varphi^\rho(N)}{\xi(N)}\). It was shown in the proof of Proposition 2, that, given Assumptions 1 and 2,
\[ (1-\psi N)\frac{\varphi^\rho(N)}{\xi(N)} = (1-a-b)(1-\psi N)\left[1-\frac{b\nu'(N)-\nu(N)-1}{\nu(N)[1-a-b+b\nu(N)]}\right] > 0, \forall N \in \mathbb{N}_i \]  
(A.4.11)
Further, recall from Proposition 1 that, given Assumptions 1 and 2:
\[ \nu'(N_i) = \lambda(1-a-b-\zeta)[\nu(N_i)]^2 > 0, \quad \nu''(N_i) = -2\lambda(1-a-b-\zeta)[1-\lambda(1-\zeta)][\nu(N_i)]^3 < 0, \quad \text{and} \]
\[ \nu(N') = 6\lambda(1-a-b-\zeta)[1-\lambda(1-\zeta)]^2\nu(N_i)^4 > 0. \]
Also, recall from Proposition 2, that given Assumptions 1 and 2,
\[ (1-\psi N)\frac{\varphi^\rho(N)}{\xi(N)} \rightarrow (1-a-b)(1-\zeta)^2 \left(1-\psi \frac{1-a-b-\zeta}{1-\zeta}\right) > 0, \quad \text{as} \quad N \rightarrow \mathbb{N}_i \text{ and} \quad (1-\psi N)\frac{\varphi^\rho(N)}{\xi(N)} \rightarrow 0 \]
as \(N \rightarrow \mathbb{N}_i\). Next, consider, the function \((1-\psi N)\frac{\nu'(N)}{\nu(N)}\), where
\[ \nu(N) = \frac{[\nu(N)-1]}{\nu(N_i)[1-a-b+b\nu(N)]^{(1-\lambda)^2}}. \]
It follows that:
\[ (1-\psi N)\frac{\nu'(N)}{\nu(N)} = \frac{\lambda(1-a-b-\zeta)v(N)^2}{N^2} \left(1-\psi N\right) \left[1-\frac{1}{\nu(N)[\nu(N)-1]} - \frac{(1-\lambda)b}{\lambda[1-a-b+b\nu(N)]}\right] \]  
(A.4.12)
Using the above stated properties of \(\nu(N)\), it follows by tedious but otherwise straightforward algebra, that given Assumptions 1 and 2,
\[ \frac{d[\xi'(N)/\xi(N)]}{dN} < 0, \quad \frac{d^2[\xi''(N)/\xi(N)]}{dN^2} > 0, \quad \frac{\nu'(N)}{\nu(N)} > 0, \]
\[
\frac{d\left[\frac{\psi(N)}{\psi(N)}\right]}{dN} < 0, \quad \frac{d^2\left[\frac{\psi(N)}{\psi(N)}\right]}{dN^2} > 0 \quad \forall N \in \left(\overline{N}, \overline{N}\right). \]
Furthermore, it follows from (A.5.12) that \( (1 - \psi)N \frac{\psi(N)}{\psi(N)} \to +\infty \) as \( N \to \overline{N} \) and \( (1 - \psi)N \frac{\psi(N)}{\psi(N)} \to 0 \) as \( N \to \overline{N} \). Combining results, it follows that: \( (1 - \psi)N \varphi(N) > 0, \quad \frac{d[(1 - \psi)N \varphi(N)]}{dN} < 0, \quad \frac{d^2[(1 - \psi)N \varphi(N)]}{dN^2} > 0 \quad \forall N \in \left(\overline{N}, \overline{N}\right). \)
Moreover, \( (1 - \psi)N \varphi(N) \to 0 \) as \( N \to \overline{N} \). Furthermore, as long as \( \rho < 1 \), \( (1 - \psi)N \varphi(N) \to +\infty \) as \( N \to \overline{N} \); and when \( \rho = 1 \), in which case \( (1 - \psi)N \varphi(N) = (1 - \psi)N \frac{\xi'(N)}{\xi(N)} \), Assumption 3 implies that \( (1 - \psi)N \varphi(N) \) is continuous, positive, strictly decreasing and strictly concave, \( \forall N \in \left(\overline{N}, \overline{N}\right) \). Further it takes a value greater than \( (1 - \psi)(N - N^0) \) as \( N \to \overline{N} \); and takes the value 0, as \( N \to \overline{N} \). Therefore, \( (1 - \psi)N \varphi(N) \) takes the value \( (1 - \psi)(N^0 - \overline{N}) \) at a unique point in the open interval \( \left(\overline{N}, \overline{N}\right), N^0 \). And, therefore, \( k^0 \in (0, \infty) \), defined by (A.4.9), is also unique.

**Part c**

Note that \( \rho \) affects \( (1 - \psi)N \varphi(N) \) only through \( A \) and \( B \). Then, since both of these constants are strictly decreasing functions of \( \rho \) and \( (1 - \psi)N \varphi(N) \) is strictly increasing in both \( A \) and \( B \), it is immediate from Part b, that an increase in \( 1 - \rho \) increases the steady state value of value of \( N, N^0 \).

Q.E.D.

---

**Proof of Proposition 5**

Let:

\[
h\left(N^0, N^1, N^2\right) \equiv \varphi\left(N^1\right) - \frac{\beta^{-1}\psi}{1 - \psi\left(N^1 - N^0\right) - \psi N^0} \quad \frac{\psi - \psi}{1 - \psi\left(N^1 - N^0\right) - \psi N^0} \quad \text{(A.5.1)}
\]

Then,

\[
h\left(N^0, N^1, N^2\right) = 0 \quad \text{(A.5.2)}
\]

is equivalent to (A.4.6). And,

\[
h\left(N, N, N\right) = 0 \quad \text{(A.5.3)}
\]

is equivalent to (A.4.10).
Taking a first order approximation of \( h(N_i, N_{i+1}, N_{i+2}) \) around \( h(N, N, N) \), we have:

\[
h(N_i, N_{i+1}, N_{i+2}) \approx h(N, N, N) + h_3(N, N, N)(N_{i+2} - N) + h_2(N, N, N)(N_{i+1} - N) + h_1(N, N, N)(N_i - N) = 0
\]  
(A.5.4)

where \( h_i(N, N, N), \ i = (1, 2, 3) \) denote the partial derivatives of \( h \) with respect to its first, second and third argument, respectively, evaluated at \((N, N, N)\). In view of (A.5.3), (A.5.4) yields:

\[
\frac{\psi'(\psi - \psi)}{(1 - \psi N)^2} (N_{i+2} - N) - \left[ \frac{\beta^{-1} \psi^2}{(1 - \psi N)^2} + \frac{(\psi - \psi)^2}{(1 - \psi N)^3} - \phi'(N) \right] (N_{i+1} - N) + \frac{\beta^{-1} \psi' (\psi - \psi)}{(1 - \psi N)^2} (N_i - N) = 0
\]  
(A.5.5)

It follows that the characteristic equation associated with (A.5.5) is:

\[
\sigma^2 - \left[ \frac{\beta^{-1} \psi'}{\psi - \psi} + \frac{\psi - \psi}{\psi} \left( \frac{1 - \psi N}{\psi - \psi} \right) \phi'(N) \right] \sigma + \beta^{-1} = 0
\]

This equation has two roots, \( \sigma \in \left( 0, \frac{\psi - \psi}{\psi} \right) \subset (0,1) \) and \( (\sigma\beta)^{-1} > 1 \). Then, since \( N_i \) is restricted to be in the interval \( \left( \frac{1 - \alpha - b - \zeta}{1 - \zeta}, \frac{1}{\psi} \right) \), the unique solution of (A.5.5) is given by:

\[
N_{i+1} - N = \sigma (N_i - N)
\]  
(A.5.6)

where:

\[
\sigma = \frac{1}{2} \left[ \left\{ \frac{\beta^{-1} \psi'}{\psi - \psi} + \frac{\psi - \psi}{\psi} \left( \frac{1 - \psi N}{\psi - \psi} \right) \phi'(N) \right\} - \left\{ \left[ \frac{\beta^{-1} \psi'}{\psi - \psi} + \frac{\psi - \psi}{\psi} \left( \frac{1 - \psi N}{\psi - \psi} \right) \phi'(N) \right]^2 - 4\beta^{-1} \right\}^{1/2} \right]
\]

Q.E.D.