Human Capital Investment, Signaling, and Wage Differentials*

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Abstract

This study considers how individuals determine the ratio of two kinds of educational investment. One kind contributes to labor skills and the other does not, corresponding to human capital investment and signaling, respectively. We formulate an overlapping generations economy in which the rich and poor invest in both types of education. We argue that the ratio of human capital investment to signaling investment is a U-shaped function of the wage differentials between the rich and poor. Moreover, we identify three patterns of stable steady states for these wage differentials, namely, no-inequality, high-inequality, and multiple steady states. Using these results, we conclude that exogenous factors, such as a rapid increase in skill-biased technology, may switch the steady state from no inequality to high inequality. This causes an increase in the ratio of signaling investment for at least some periods during the transition to the new steady state.

Keywords: human capital investment, signaling, wage differentials, multiple steady states, overlapping generations

JEL classification: D31, D80, I24

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1 Introduction

For several decades, many countries have experienced expanded access to higher education. One reason is an increase in demand for high skilled worker from skill-biased technological progress, such as the IT revolution. On the other hand, it is known that higher educational attainment serves as a signal and has a big impact on students’ postgraduation earnings. Using US data, Jaeger and Page (1996) and Park (1999) find evidence of diploma effects: earning gains of 9–10% associated with high school graduation, 11% with associate’s degrees, and 20–30% with bachelor’s degrees. Thus, higher education plays a dual role of educating and signaling.

However, from the perspective of credentialing, the major contribution of higher education is signaling and students or their parents are concerned not with learning, but with being certified as having learned. Therefore, from this perspective, higher education is individually valuable but contains a lot of socially wasteful investment. Some sociologists state that students and/or their parents are the root cause of this. Dore (1976) [pp. 8] states:

The effect of schooling, the way it alters a man’s capacity and will to do things, depends not only on what he learns, or the way he learns it, but also on why he learns it. That is at the basis of the distinction between schooling which is education, and schooling which is only qualification, a mere process of certificating - or ‘credentialing’.

In order to describe this concept, our model separates educational investment into schooling that is education, or human capital investment, and schooling that is only qualification, or signaling investment. Signaling investment in our model implies investment undertaken only to obtain an observable signal, which does not contribute to labor skills. Thus, signaling investment is related to (but slightly distinct from) the concept of signaling theory in Spence (1973). Conversely, in our model, human capital investment not only contributes to labor skills but is also helpful for obtaining the signal. Thus, the ratio of human capital investment to signaling investment captures the degree of credentialing: a lower ratio means that credentialing (to earn certification) is households’ major purpose of receiving higher education.

In this study, we introduce an overlapping generations economy, in which parents are heterogeneous in their wages, being either rich or poor. Each parent invests in the above-mentioned two types of education for their children. Moreover, uncertainty exists in the acquisition of the signal, which is determined endogenously by the differences in each type
of educational investment by the rich and poor. That is, as the gap in the education level between rich and poor increases, the probability increases that the child of rich parent obtains the signal, whereas the probability decreases that the child of poor parent obtains the signal. In other words, this model considers competition to obtain signals between rich and poor, and this competition becomes an important factor that affects the ratio of human capital to signaling investment. This well describes the race for higher qualifications and education in credential society.

We obtain the following results. First, in static analysis, we find that the ratio of human capital to signaling investment for both rich and poor becomes a U-shaped function of the wage differential between rich and poor. In other words, the ratio of human capital investment to signaling investment is high when the wage differential is small or large, while the ratio is low with medium levels of wage differentials. The ratio of the two types of investment well describes the degree of credentialing in higher education; a lower ratio of human capital to signaling investment can be associated with a more credential society, which contains a lot of socially wasteful educational investment. This result can be obtained by a distinguishing concept of this model, which includes both human capital investment that contributes to skill enhancement and signaling investment that is only to the signal. To my knowledge, no studies have modeled the two types of educational investment (see Section 2 for a detailed explanation).

Furthermore, our model contributes to explaining the dynamics of the wage differentials by using the abovementioned two types of investment. For the second part of the results, on dynamic analysis, we find that three patterns of stable steady states of wage differentials exist in our economy: a unique steady state with no inequality, a unique steady state with high inequality, and multiple steady states. These alternative steady states are distinguished by the parameter representing the level of skill-biased technology, and we can show that an increase in this technology level leads to larger wage differentials in the steady state. Therefore this result well describes the expansion of wage differentials during the last three decades in many advanced countries, as explained by skill-biased technical change (SBTC). Moreover, by combining the static and the dynamic results, we show that rapid progress of skill-biased technology switches the economy from the no-inequality steady state to the high-inequality steady state, and the ratio of human capital to signaling investment decreases temporarily or permanently during the transition to the high-inequality steady state. Thus, SBTC, such as the IT revolution from the 1970s to the 1990s, would enhance households’ signaling investment, and thus, we can see that the recent increase in higher education attainment may
contain substantial investment aimed at credentialing.

Our result on static analysis, the U-shaped relationship between the investment ratio and wage differentials, has important implications for government education policy. For example, if governments provide education vouchers in societies with medium levels of wage differentials, there is less incidence of increasing human capital accumulation because households waste it on signaling investment. Furthermore, we should note that tax and redistribution policies affect the investment ratio through a reduction of wage differentials. Therefore, in societies with large wage differentials, these policies may decrease the ratio of human capital investment to signaling investment and prevent human capital accumulation.

The remainder of the paper is structured as follows. Section 2 provides the background of the model and discusses the related literature. Section 3 describes the construction of the model. Section 4 illustrates the mechanism underlying the U-shaped relationship between, on one hand, the ratio of human capital to signaling investment and, on the other, wage differentials. In addition, we discuss the policy implications of the results. In Section 5, we analyze the three patterns of steady states for the wage differentials and discuss the pattern of educational investment in these states. Section 6 concludes.

2 Background and Previous Literature

Many empirical works, including those of Lang and Kropp (1986), Hungerford and Solon (1987), Belman and Heywood (1991), and Bedard (2001), argue that higher education serves as a signal of higher productivity in addition to increasing individual human capital, and that this signal is rewarded in the labor market. Following these results, theoretical studies focusing on signaling as private educational investment have been developed.

The educational standards model developed by Costrell (1994) and Betts (1998) is closely related to the signaling model in Spence (1974). They analyze how an achievement standard for graduation influences worker productivity through the skill-enhancing efforts of students. In the educational standard model, the standard for graduation is set by policymakers who maximize social welfare, and students choose whether to make efforts to meet the standard. These models well describe students’ credentialing actions, although they contain an unrealistic assumption: wages do not depend on worker productivity as productivity cannot be observed by firms. Blankenau and Camera (2006) and De Paola and Scoppa (2007) add individual incentives to acquire skills in the educational standards model by assuming that skills
are observable with a fixed probability. Thus, students’ efforts depend both on the standard level and the probability of skill observation. Although our model does not describe educational standards, human capital investment and signaling investment in our model are related to effort incentives for skill accumulation and the attainment of educational qualifications in their models.¹

More closely related to our study are Futagami and Ishiguro (2004), Hendel et al. (2005), Yuki (2009), and D’Amato and Mookherjee (2012), who study educational signaling and wage inequality in a dynamic model (using an overlapping generations model). Futagami and Ishiguro (2004) address macroeconomic issues by considering how signaling behavior of individuals affects physical capital accumulation. They show that, in separating equilibrium in which only high-ability agents invest, physical capital accumulation enhances large wage differentials in the process of convergence toward a steady state. On the other hand, the models of Hendel et al. (2005), Yuki (2009), and D’Amato and Mookherjee (2012) assume that parents altruistically pay for their children’s education, and thus, parental income is a key determinant of signaling and dynamics of wage differentials, as with our model. The most notable difference between our model and these four models is that we deal with individual productivity as that generated by educational investment, not as given at birth. Thus, in our model, human capital investment that determines individual productivity is a new concept in the signaling literature.

From this perspective, our model is related to those of Blankenau and Camera (2009), in which investment for obtaining qualifications is separated from skill-enhancing investment. Blankenau and Camera (2009) construct a model in which homogeneous students choose their educational investment as follows: skilled (with qualification and skill), schooled (with qualification but no skill), and unschooled (no qualification and no skill).²

In addition, our work contributes to the macroeconomic literature on wage inequality by emphasizing SBTC. Increases in the demand for highly educated workers driven by an exponential pace of SBTC have been offered as explanations for an increase in US wage

¹Aside from the educational standards model, several studies describe individual incentives of investment to acquire skills by applying the basic model of signaling theory. Hopkins (2012) and Bidner (2014) show that by assuming positive assortative matching of worker–firm or worker–worker pair production, a higher level of investment not only serves as a signal and leads to matches with higher productive partners but also increases productivity and leads to higher wages.

²Blankenau and Camera (2009) conclude that when firms recognize the skill level of each student with a low probability, this counteracts the incentive of students to earn skills. Therefore, government education policy in the form of education subsidies may fail to ensure additional human capital accumulation of students.
inequality between college and high school graduates from the 1980s (for seminal empirical and theoretical works, see Katz and Murphy, 1992; Acemoglu, 1998; Autor et al., 1998; and Galor and Moav, 2000). Some subsequent theoretical studies analyze the relationship between the emergence of skill-biased technologies and educational attainment. They show that higher skill premiums driven by SBTC increase educational attainment of individuals with higher ability or income (see Caselli, 1999; Miyake et al., 2009; and Restuccia and Vandenbroucke, 2013), while the attainments of individuals with lower ability or income decrease because of an increased cost of educational investment (see Crifo, 2008; Nakamura, 2013). These results explain the widening wage inequality between college and high school graduates and the contemporary increase in college enrolment in many advanced countries in the decades after SBTC. Our study, however, introduces educational investment as signaling in the literature, and shows that the ratio of human capital to signaling investment decreases soon after SBTC but may increase during the transition to a steady state (i.e., there is a U-shaped transition path).

3 The Model

Consider a discrete-time overlapping generations economy in which each agent lives three periods: young, middle, and old. At $t = 0$, there is one rich agent and one poor agent in each generation (here, “rich young” refers to a young agent born to a rich middle-aged parent and “poor young” refers to a young agent born to poor middle-aged parent). Moreover, we assume there is no population growth.

In the first period of life, the young agents receive education from their parents. We assume both young agents are born with the same ability. However, the ex post educational attainment levels differ because of heterogeneity in parental income. At the end of the young period, either the rich young agent or the poor young agent obtains a signal with uncertainty. Although the rich young agent receives more educational investment than the poor young agent in equilibrium, he or she does not always obtain the signal. That is, if he or she fails to obtain the signal, the poorly educated poor young agent obtains the signal instead. A detailed explanation for this uncertainty of signal is explained in Subsection 3.3.

Instead of this setting, we can assume that there exists one unit of rich agent and one unit of poor agent in each generation. If we adopt this, however, we need additional assumptions for the game theoretical part of Subsection 3.5. On the other hand, the two-agent economy in this model can sufficiently show all mechanisms obtained from the two-unit agent model.
In the second period of life, the middle-aged agents work and obtain wage incomes. Firms can observe which middle-aged worker has the signal but cannot observe the individual productivity of each middle-aged worker; then, the middle-aged worker receives wages according to whether he has the signal. Consequently, the middle-aged worker with the signal becomes rich and the middle-aged worker with no signal becomes poor. Furthermore, each middle-aged worker gives birth to one child and allocates income between own consumption and investment in education for the child. Here, for simplicity, we assume that saving and borrowing do not occur from middle age to old age.

In the final period of life, the old agents work and consume, but they neither bear children nor invest in education. Moreover, unlike the middle worker, firms can observe the individual productivity of each old worker.\(^4\) The firms then pay wages to each old worker according to their individual productivity, instead of the signal. Note that the rich middle-aged agent in period \(t+1\) is not always consistent with the rich old agent in period \(t+2\). This is because, if a highly educated rich young agent fails to obtain the signal, he becomes a poor middle-aged agent, but later becomes a rich old agent.

3.1 Production

Firms produce final goods using only labor inputs. For both middle-aged and old workers, the individual output of the highly educated high-skilled worker and poorly educated low-skilled worker are assumed to follow the linear technologies, \(\phi h_H, h_L\), where \(h_H\) and \(h_L\) are the human capital of the high- and low-skilled worker. \(\phi > 1\) represents the level of skill-biased technology that only the high-skilled worker can use.

As firms cannot observe the individual productivity of each middle-aged worker, they pay middle-aged workers according to whether they have the signal. Given the assumption of perfect competition, the wages of middle-aged workers need to be consistent with the expected productivity of each worker.\(^5\) Then, the wage of the middle-aged worker with the

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\(^4\)This is a plausible assumption. In general, firms have only limited information about the quality of workers in the early stages of their careers, and distinguish among workers on the basis of easily observable variables, such as years of education or degree. On the other hand, with each successive period, firms can observe more of workers’ performance. Over time, these observations make the signal redundant. See the empirical results of Altonji and Pierret (2001).

\(^5\)We assume more than two firms simultaneously make wage offers to both workers. If all firms make the same offers, one randomly selected firm obtains both workers and produce.
signal, $\hat{w}_{H,t+1}$, and the wage of the middle-aged worker with no signal, $\hat{w}_{L,t+1}$, in period $t+1$ are:

$$\hat{w}_{H,t+1} = \hat{\theta}_{t+1}\hat{h}_{H,t+1} + (1 - \hat{\theta}_{t+1})\hat{h}_{L,t+1}, \quad (1)$$

$$\hat{w}_{L,t+1} = \hat{\theta}_{t+1}\hat{h}_{L,t+1} + (1 - \hat{\theta}_{t+1})\phi\hat{h}_{H,t+1}, \quad (2)$$

where $h_{H,t+1}$ is the human capital of the high-skilled middle worker, $h_{L,t+1}$ is the human capital of the low-skilled middle worker, and $\theta_{t+1}$ represents the uncertainty of the signal. As described in Subsection 3.3, the highly educated young agent obtains the signal with probability $\theta_{t+1}$, while the poorly educated young agent obtains the signal with probability $1 - \theta_{t+1}$. $\hat{h}_{H,t+1}$, $\hat{h}_{L,t+1}$, and $\hat{\theta}_{t+1}$ represent the firms’ expectations of $h_{H,t+1}$, $h_{L,t+1}$, and $\theta_{t+1}$. Since $\theta_{t+1}$ is endogenously determined according to the unobservable education levels of the young agents, wages are paid according to these expectations.\(^6\)

The wages of old workers are paid according to their productivity because firms can observe these levels. Then, in period $t+1$, the wages of high- and low-skilled old workers (born in period $t-1$) are $\phi h_{H,t}$ and $h_{L,t}$, respectively. That is, the wages of old workers are paid regardless of their signal, and so, the uncertainty has no relevance for the old wage.

### 3.2 Educational investment

Parents invest in two types of education for their children: one is productive and the other is not. Here, we refer to the former as human capital investment and the latter as signaling\(^6\)

We can interpret this setting as a circumstance in which both agents work in the same workplace where only the high-skilled worker can use the technology. The firm’s owner, however, cannot observe which middle-aged worker is using this technology, and so, the firm pays wages according to the signal.

Furthermore, using a constant elasticity of substitution (CES) production function, $(\alpha h_{H,t}^\rho + (1 - \alpha) h_{L,t}^\rho)^{1/\rho}$, where $\alpha > 1/2$ leads to the same result as in the abovementioned setting. In this case, the wages of each middle-aged worker and the wage differentials are:

$$\dot{\hat{w}}_{H,t+1} = \hat{\theta}_{t+1}\alpha h_{H,t+1}^\rho (\alpha h_{H,t}^\rho + (1 - \alpha) h_{L,t}^\rho)^{\frac{1}{\rho} - 1} + (1 - \hat{\theta}_{t+1}) (1 - \hat{\theta}_{t+1})\alpha h_{H,t+1}^\rho (\alpha h_{H,t+1}^\rho + (1 - \alpha) h_{L,t}^\rho)^{\frac{1}{\rho} - 1},$$

$$\dot{\hat{w}}_{L,t+1} = \hat{\theta}_{t+1} (1 - \alpha) h_{L,t+1}^\rho (\alpha h_{H,t+1}^\rho + (1 - \alpha) h_{L,t+1}^\rho)^{\frac{1}{\rho} - 1} + (1 - \hat{\theta}_{t+1})\alpha h_{H,t+1}^\rho (\alpha h_{H,t+1}^\rho + (1 - \alpha) h_{L,t+1}^\rho)^{\frac{1}{\rho} - 1},$$

$$\frac{\dot{\hat{w}}_{H,t+1}}{\dot{\hat{w}}_{L,t+1}} = \frac{\hat{\theta}_{t+1}\alpha h_{H,t+1}^\rho (\alpha h_{H,t+1}^\rho + (1 - \alpha) h_{L,t+1}^\rho)^{\frac{1}{\rho} - 1}}{\hat{\theta}_{t+1} (1 - \alpha) h_{L,t+1}^\rho (\alpha h_{H,t+1}^\rho + (1 - \alpha) h_{L,t+1}^\rho)^{\frac{1}{\rho} - 1}}.$$  

$\alpha/(1 - \alpha)$ can be interpreted as the same as $\phi$, and parameter $\rho$ represents the elasticity of substitution. In this case, we can obtain the same results as in the earlier setting, if $\rho > 0$. Therefore, for simplicity, we employ a linear technology for the production function, as mentioned above.
investment.

The human capital of the middle-aged generation born in period $t$ is given by:

$$h_{i,t+1} = B(e_{i,t})^\mu,$$

where $e_{i,t}$ is the human capital investment made by the parents, $h_{i,t+1}$ is human capital level, $\mu \in (0, 1)$, and $B > 0$. In this model, given that the educational investment of the rich is always higher than that of the poor in equilibrium, the subscript $i = H, L$ of $h_{i,t+1}$, and $e_{i,t}$ indicate both investment level and income type of each household. Similarly, signaling investment is as follows:

$$S_{i,t+1} = B'(s_{i,t})^\nu,$$

where $s_{i,t}$ is the signaling investment made by the parents, $S_{i,t+1}$ is signaling ability, and $\nu \in (0, 1)$. By the same reasoning as earlier, we use the subscript $i$ to represent both income and investment levels. Note that $S_{i,t+1}$ is not the signal itself but rather the ability to obtain the signal. Therefore, signaling investment $s_{i,t}$ here can be interpreted as investment only for obtaining the signal, such as school expenses for offspring who are less interested in learning than in doing the minimum work required to obtain the qualification and graduate. We assume that firms cannot observe the signaling ability $S_{i,t+1}$, signaling investment $s_{i,t+1}$, and human capital investment $e_{i,t}$. In addition, as mentioned, firms cannot observe the human capital level $h_{i,t+1}$ of middle-aged workers but can observe that of old workers.

These two types of educational investment can describe the households’ two different incentives for schooling: learning and signaling. We can see that $e_{i,t} + s_{i,t}$ represents the parents’ total expenditure on higher education. Furthermore, the ratio of human capital investment and signaling investment $e_{i,t}/s_{i,t}$ represents the households’ attitudes toward education. If a large part of their objective is credentialing, then the offspring will do the minimum effort necessary to obtain the qualification. Hence, it can be interpreted that most of the total expenditure is invested in signaling, that is, there is a large ratio of signaling investment. On the other hand, if they desire learning rather than credentialing, then the offspring are passionate about learning, and so, the expenditure can be said to be human capital investment. Therefore, we should focus on the ratio $e_{i,t}/s_{i,t}$, although we are modeling $e_{i,t}$ and $s_{i,t}$ separately in order to shed light on households’ incentives for higher education. In the next subsection, we present the details of the incentives of human capital investment and signaling investment.
3.3 Uncertainty of the signal

Here, we provide the definition of the probability $\theta_{t+1}$ that describes the uncertainty of the signal. We assume that a higher education institution provides the signal to one of the two young agents who obtains the higher score on a test. Although the signal is provided for the purpose of proving a difference in educational level among the two young agents, the highly educated young agent may fail the test with probability $1 - \theta_{t+1}$, and in this case, the poorly educated young agent obtains the signal instead. That is, we assume the highly educated young agent obtains the signal with probability $\theta_{t+1}$, and the poorly educated young agent obtains it with probability $1 - \theta_{t+1}$. Then, we express the uncertainty of the signal as follows:

$$\theta_{t+1} = \Theta(h_{t+1}, S_{t+1}),$$

where $\tilde{h}_{t+1} = h_{H,t+1}/h_{L,t+1}$ and $\tilde{S}_{t+1} = S_{H,t+1}/S_{L,t+1}$. Furthermore, we assume $\Theta \in \left[\frac{1}{2}, 1\right)$, $\Theta_h > 0$, $\Theta_S^h < 0$, $\Theta_{SS} < 0$, $\Theta_{hS} < 0$, $\Theta(1, 1) = \frac{1}{2}$, and $\lim_{h \to \infty, S \to \infty} \Theta = 1$.

The most important assumptions here are $\Theta_h^h > 0$ and $\Theta_S^h > 0$. They mean that this probability is an increasing function of the educational gaps between rich and poor young agents, $\tilde{h}_{t+1}$ and $\tilde{S}_{t+1}$. That is, as the gaps in the education levels between rich and poor agents increase, the rich young agent can obtain the signal more easily, whereas the poor young agent finds it more difficult to obtain the signal.\(^7\) However, note that both human capital and signaling investment are effective for the wages of offspring in middle age because a worker with the signal can obtain a higher wage in middle age, while only human capital investment is effective for the wages of offspring in old age because these wages are paid according to workers’ true productivity.

Now, we can obtain the property of this model by comparing equation (5) with commonly

\(^7\)These assumptions are satisfied by using simple exponential functions, such as:

$$\Theta(h_{t+1}, S_{t+1}) = 1 - \frac{1}{2} \exp(C(1 - h_{t+1}S_{t+1})), $$

where $C$ is a parameter that represents the curvature of this function. We use this function to draw figures in later sections.

\(^8\)This assumption can be rationalized by a well-recognized property, that is, a negative relationship between income inequality and intergenerational social mobility. Andrews and Leigh (2009) show that sons who grew up in countries that were more unequal were less likely to have experienced social mobility. In addition, theoretical studies support this relationship from the perspective of both human capital and signaling theory (see Ferrer, 2005; Solon, 2004). In our model, the educational gaps of the young, $\tilde{h}_{t+1}$ and $\tilde{S}_{t+1}$, are positively related with the parental wage differentials (as shown in equation (14)). Moreover, an increase in $\theta$ can be interpreted as a decrease in intergenerational social mobility.
used signaling theory. The crucial assumption in the signaling model is that low-ability agents find signaling more costly than do high-ability agents, that is, the marginal cost of investment is higher for low-ability agents than high-ability agents. Our model also has this property by assuming \( \Theta_h > 0 \). On the other hand, we have two major differences from the signaling model. First, the most usual signaling model takes workers’ skills as exogenously fixed, while our model endogenously determines workers’ skills by human capital investment of households. Second, in our model, only one young agent can obtain the signal, and the uncertainty of the signal is a function of the relative investment between two young agents. This well describes competition for qualifications among students.\(^9\)

### 3.4 Household

The preferences of each individual are defined over consumption and the expected wage of their offspring. The lifetime utilities of the rich middle-aged worker with the signal and the poor middle-aged worker with no signal in period \( t \) are defined by the following function:

\[
U_H = \alpha \ln c_{H,t} + (1 - \alpha) \left[ \Theta(\tilde{h}_{t+1}, \tilde{S}_{t+1}) \ln \tilde{w}_{H,t+1} + (1 - \Theta(\tilde{h}_{t+1}, \tilde{S}_{t+1})) \ln \tilde{w}_{L,t+1} \right] + \beta \left[ \alpha \ln c_{k,t+1} + (1 - \alpha) \ln \phi H_{t+1} \right],
\]

\[
U_L = \alpha \ln c_{L,t} + (1 - \alpha) \left[ \Theta(\tilde{h}_{t+1}, \tilde{S}_{t+1}) \ln \tilde{w}_{L,t+1} + (1 - \Theta(\tilde{h}_{t+1}, \tilde{S}_{t+1})) \ln \tilde{w}_{H,t+1} \right] + \beta \left[ \alpha \ln c_{k,t+1} + (1 - \alpha) \ln h_{L,t+1} \right],
\]

where \( c_{i,t} \) is own consumption in middle age, the brackets on the first line express the expected middle-age wages of their offspring, \( \beta \in (0, 1) \) is the time discount rate, \( c_{k,t+1} \) is own consumption in old age, and \( \ln \phi H_{t+1} \) and \( \ln h_{L,t+1} \) represent the old-age wages of their offspring.\(^{10}\) Furthermore, note that the subscript for old-age consumption \( k = H \) or \( L \) differs from \( i = H \) or \( L \) of \( U_i \) because the rich middle-aged agent is not always a highly productive worker. That is, the middle-age consumption types \( c_{H,t} \) and \( c_{L,t} \) are not always consistent with the old-age consumption types \( c_{H,t+1} \) and \( c_{L,t+1} \).

Finally, the budget constraint for the middle-aged agent is expressed as follows:

\[
c_{H,t} + e_{H,t} + s_{H,t} = w_{H,t},
\]

\(^9\)This structure is similar to that of a tournament model.

\(^{10}\)Strictly speaking, we should add time discounting to the altruistic terms because offspring obtain middle- and old-age wages one period after their parents’ middle- and old-age consumption, respectively. For notational simplicity, we assume the altruistic parameter \( \alpha \) contains this time discounting.
The budget constraint for the old agent is simply that old-age consumption equaling the old-age wage because we assume there is no saving from middle age to old age.

\[ c_{L,t} + c_{L,t+1} + s_{L,t} = w_{L,t}. \]  

(9)

3.5 Optimization and equilibrium

As mentioned above, each middle-aged agent in period \( t \) chooses \( c_{i,t}, e_{i,t}, \) and \( s_{i,t} \) within the budget \( w_{i,t} \). Given that the optimal allocations in old age are simply \( c_{H,t+1} = \phi h_{H,t} \) and \( c_{L,t+1} = h_{L,t} \), we exclude these terms in (6) and (7) from the optimization. Then, the utility maximization problem of the middle aged in period \( t \) is described as follows:

\[
\begin{aligned}
\max_{c_{i,t}, e_{i,t}, s_{i,t}} & \quad U_i \\
\text{s.t.} & \quad c_{i,t} + e_{i,t} + s_{i,t} = w_{i,t} \\
\text{given} & \quad w_{i,t}, \; \hat{w}_{i,t+1}, \; \hat{w}_{j,t+1}, \; e_{j,t}, \text{and } s_{j,t}.
\end{aligned}
\]

Note that because firms cannot observe the individual productivity of each middle-aged worker, the offspring’s wages in middle age \( \hat{w}_{i,t+1} \) and \( \hat{w}_{j,t+1} \) depend only on the firms’ expectations \( \hat{\theta}_{t+1}, \; \hat{h}_{i,t+1}, \) and \( \hat{h}_{j,t+1} \). Therefore parents take \( \hat{w}_{i,t+1} \) and \( \hat{w}_{j,t+1} \) as given in the optimization problems. After some simple deformation, the first-order condition (FOC) of the optimization problem for the rich middle-aged worker with respect to \( e_{H,t} \) and \( s_{H,t} \) is rewritten as follows:

\[
e_{H,t} : \frac{e_{H,t}}{w_{H,t} - e_{H,t} - s_{H,t}} = \frac{(1 - \alpha)\mu}{\alpha} \left[ \beta + \frac{\partial \Theta(\tilde{h}_{t+1}, \tilde{S}_{t+1})}{\partial \tilde{h}_{t+1}} \ln \frac{\hat{w}_{H,t+1}}{\hat{w}_{L,t+1}} \right], \tag{10}\]

\[
s_{H,t} : \frac{s_{H,t}}{w_{H,t} - e_{H,t} - s_{H,t}} = \frac{(1 - \alpha)\nu}{\alpha} \frac{\partial \Theta(\tilde{h}_{t+1}, \tilde{S}_{t+1})}{\partial \tilde{S}_{t+1}} \ln \frac{\hat{w}_{H,t+1}}{\hat{w}_{L,t+1}}. \tag{11}\]

The FOC of the optimization problem for the poor middle-aged worker with respect to \( e_{L,t} \) and \( s_{L,t} \) can be rewritten in the same manner:
\[ e_{L,t} : \frac{e_{L,t}}{w_{L,t} - e_{L,t} - s_{L,t}} = \frac{(1 - \alpha)\mu}{\alpha} \left[ \beta + \frac{\partial \Theta(h_{t+1}, \hat{S}_{t+1})}{\partial h_{t+1}} \hat{h}_{t+1} \ln \frac{\hat{w}_{H,t+1}}{\hat{w}_{L,t+1}} \right], \quad (12) \]

\[ s_{L,t} : \frac{s_{L,t}}{w_{L,t} - e_{L,t} - s_{L,t}} = \frac{(1 - \alpha)\nu}{\alpha} \frac{\partial \Theta(h_{t+1}, \hat{S}_{t+1})}{\partial S_{t+1}} \hat{S}_{t+1} \ln \frac{\hat{w}_{H,t+1}}{\hat{w}_{L,t+1}}. \quad (13) \]

In equilibrium, \( \hat{h}_{t+1}, \hat{S}_{t+1}, \) and \( \hat{\theta}_{t+1} \) must be determined uniquely. Moreover, in equilibrium, the actual education levels of rich and poor children must coincide with the firms' expectations: \( h_{i,t+1} = \hat{h}_{i,t+1}, S_{i,t+1} = \hat{S}_{i,t+1}, \) and \( \Theta(h_{t+1}, S_{t+1}) = \hat{\theta}_{t+1}. \) Then, in equilibrium, from (1) and (2), firms' expectations \( \hat{w}_{H,t+1} \) and \( \hat{w}_{L,t+1} \) satisfy:

\[ \hat{w}_{H,t+1} = \Theta(h_{t+1}, S_{t+1}) \phi_{h,t+1} + (1 - \Theta(h_{t+1}, S_{t+1})) h_{L,t+1}, \]

\[ \hat{w}_{L,t+1} = \Theta(h_{t+1}, S_{t+1}) h_{L,t+1} + (1 - \Theta(h_{t+1}, S_{t+1})) \phi_{h,t+1}. \]

Now the right-hand sides of (10) and (12), (11) and (13) are given by the same formula. Then the left-hand side of (10) equals that of (12), and the left-hand side of (11) equals that of (13). Using these two equations, we obtain:

\[ \frac{e_{H,t}}{e_{L,t}} = \frac{s_{H,t}}{s_{L,t}} = \frac{w_{H,t}}{w_{L,t}} \equiv \tilde{w}_t, \quad (14) \]

where \( \tilde{w}_t \equiv w_{H,t}/w_{L,t} \) is the wage differentials in middle age between the rich and the poor. Equation (14) represents the important equilibrium property that the investment rates between rich and poor are consistent with the level of parental wage differentials. In other words, human capital and signaling investment of the rich are always greater than those of the poor.\(^ {11} \) In addition, this equation implies that the rich and poor always choose the same educational investment ratio, irrespective of the level of the wage differential \( \tilde{w}_t, \) that is:

\[^{11}\text{Equations (10)--(13) are results of a case in which both rich and poor parents take as given that human capital and signaling investment of the rich are higher than those of poor. That is, both parents take } e_R = e_H, e_P = e_L, s_R = s_H, \text{ and } s_P = s_L \text{ as given, where } R \text{ represents rich and } P \text{ represents poor. Although other cases lead to different formulations of (10)--(13), we nonetheless obtain the result that the right-hand sides of (10) and (12), (11) and (13) are given by the same formula. That is, we always obtain } e_R/e_P = s_R/s_P = w_R/w_P, \text{ as shown in (14). This means that the equilibrium with } e_R > e_P \text{ and } s_R > s_P \text{ is only equilibrium in this model. For this reason, we use subscripts } H \text{ and } L \text{ to represent rich and poor, for simplicity.} \]
By substituting (14) into $\tilde{h}_{t+1}$ and $\tilde{S}_{t+1}$, we have:

$$\tilde{h}_{t+1} = \left(\frac{e_{H,t}}{e_{L,t}}\right)^\mu = (\tilde{w}_t)^\mu, \quad \tilde{S}_{t+1} = \left(\frac{s_{H,t}}{s_{L,t}}\right)^\nu = (\tilde{w}_t)^\nu.$$  \hfill (16)

Then, $\theta_{t+1}$ is expressed as a function of $\tilde{w}_t$, namely, $\theta_{t+1} = \Theta(\tilde{w}_t)$, which satisfies $\Theta'(\tilde{w}_t) > 0, \quad \Theta''(\tilde{w}_t) < 0, \quad \Theta(1) = 1/2$, and $\lim_{\tilde{w}_t \to \infty} \Theta(\tilde{w}_t) = 1$. Now we have $\tilde{h}_{t+1}$, $\tilde{S}_{t+1}$ and $\Theta$ as a function of $\tilde{w}_t$, so we can represent the wage differentials of the next generation $\tilde{w}_{t+1}$ as a function of $\tilde{w}_t$ as follows:

$$\tilde{w}_{t+1} = \frac{\tilde{w}_{H,t+1}}{\tilde{w}_{L,t+1}} = \frac{\Theta(\tilde{w}_t)\phi(\tilde{w}_t)^\mu + (1 - \Theta(\tilde{w}_t))}{\Theta(\tilde{w}_t) + (1 - \Theta(\tilde{w}_t))\phi(\tilde{w}_t)^\mu}. \quad (17)$$

Thus, we can express the right-hand sides of (10)–(13) as a function of $\tilde{w}_t$. Note that, because the middle aged take $\tilde{w}_t$ as a given in the optimization, we can derive the equilibrium value of each endogenous variable, $e_{i,t}$, $e_{i,t}$, and $s_{i,t}$, if we specify the functional form of $\Theta$. However, we omit this work because it is not necessary for our subsequent analysis.

### 4 Ratio of Human Capital to Signaling Investment

This section proposes a static analysis to examine how the ratio of human capital to signaling investment is a function of $\tilde{w}_t \in [1, \infty)$. As shown in equation (15), rich and poor parents divide educational expenditure into human capital and signaling investment in the same ratio. Then, from (10)–(13), we obtain:

$$\frac{e_t}{s_t} = \frac{\mu}{\nu} \left[ \frac{\varepsilon_h}{\varepsilon_s} + \frac{\beta}{\delta_{i+1} \ln \tilde{w}_{i+1}} \right], \quad (18)$$
where
\[ \varepsilon_h = \frac{\partial \Theta(h_{t+1}, S_{t+1})}{\partial h_{t+1}} / \Theta(h_{t+1}, S_{t+1}) \] and \[ \varepsilon_s = \frac{\partial \Theta(h_{t+1}, S_{t+1})}{\partial S_{t+1}} / \Theta(h_{t+1}, S_{t+1}) \).

For simplicity, we assume that the ratio of the elasticity of substitution \( \varepsilon_h / \varepsilon_s \) is constant (but \( \varepsilon_h \) and \( \varepsilon_s \) do not have to be constant).\(^{12}\) Given this assumption, we focus only on the denominator of the second term in (18).

\[ \partial \Theta(\tilde{h}_{t+1}, \tilde{S}_{t+1})/\partial \tilde{S}_{t+1} \] decreases and converges to zero with \( \tilde{w}_t \), whereas \( \tilde{S}_{t+1} \ln \tilde{w}_{t+1} \) increases and diverges to infinity with \( \tilde{w}_t \). Furthermore, we have \( \partial \Theta(\tilde{h}_{t+1}, \tilde{S}_{t+1})/\partial \tilde{S}_{t+1} \ln \tilde{w}_{t+1} = 0 \) when \( \tilde{w}_t = 1 \) from \( \ln \tilde{w}_{t+1} = 0 \). Then, as the next step, we examine the value of \( \partial \Theta(\tilde{h}_{t+1}, \tilde{S}_{t+1})/\partial \tilde{S}_{t+1} \ln \tilde{w}_{t+1} \) when \( \tilde{w}_t \to \infty \).

**Lemma 1.** \( \frac{\partial \Theta(\tilde{h}_{t+1}, \tilde{S}_{t+1})}{\partial \tilde{S}_{t+1}} \ln \tilde{w}_{t+1} \to 0 \) when \( \tilde{w}_t \to \infty \) (see the Appendix for details).

From this lemma, we find that \( \partial \Theta(\tilde{h}_{t+1}, \tilde{S}_{t+1})/\partial \tilde{S}_{t+1} \ln \tilde{w}_{t+1} \) is an inverse U-shaped function of \( \tilde{w}_t \) because it is equal to zero at \( \tilde{w}_t = 1 \) and \( \tilde{w}_t \to \infty \) but it takes some positive value for \( \tilde{w}_t \in (1, \infty) \). Strictly speaking, the function is not always single peaked because the denominator of the second term in (18) consists of an increasing and decreasing term. However, we can regard its external form as inverse U-shaped because the denominator equals zero at \( \tilde{w}_t = 1 \) and \( \tilde{w}_t \to \infty \).

**Proposition 1.** Finite value \( w' > 1 \) exists, such that \( e_t/s_t \) is decreasing in \( \tilde{w}_t \) for all \( \tilde{w}_t < w' \), and finite value \( w'' (> w') \) exists, such that \( e_t/s_t \) is increasing in \( \tilde{w}_t \) for all \( \tilde{w}_t > w'' \).

Figure 1 shows a numerical example of \( e_t/s_t \) that is depicted as a function of \( \tilde{w}_t \). \( w' \) in Proposition 1 is the smallest \( \tilde{w}_t \), which satisfies \( \partial(e_t/s_t)/\partial \tilde{w}_t = 0 \), and \( w'' \) is the largest \( \tilde{w}_t \), which satisfies \( \partial(e_t/s_t)/\partial \tilde{w}_t = 0 \). A numerical example in Figure 1 shows a case of \( w' = w'' \). That is, in this case, \( e_t/s_t \) is a single-peaked U-shaped form.

\(^{12}\)Although the variation of this term depends heavily on the functional form of \( \Theta \), it is difficult for us to specify a realistic form of this function. However, if we adopt an exponential function for a fully analytical solution, this term is more likely to become constant, as we assume. Of course, the simple exponential function shown in footnote 7 satisfies this property.
Notes: Figure 1 visualizes equation (18) by using \( \Theta(h_{t+1}, S_{t+1}) = 1 - \frac{1}{2} \exp \left( C(1 - h_{t+1} S_{t+1}) \right) \), \( C = 0.6 \), \( \mu = 0.8 \), \( \nu = 0.9 \), \( \phi = 1.5 \), and \( \beta = 0.2 \). Of course, this functional form for \( \Theta \) satisfies the following assumptions thus far: \( \Theta \in \left[ \frac{1}{2}, 1 \right) \), \( \Theta_{h} > 0 \), \( \Theta_{hh} < 0 \), \( \Theta_{s} > 0 \), \( \Theta_{ss} < 0 \), \( \Theta_{hS} < 0 \), \( \Theta(1, 1) = \frac{1}{2} \), \( \lim_{h \to \infty, S \to \infty} \Theta = 1 \), and \( \varepsilon_{h}/\varepsilon_{s} \) is constant. In this case, the denominator of the second term in (18) is single peaked.

We can intuitively interpret this U-shaped relationship by focusing on the following two opposite effects of wage differentials on households’ educational investment. The first effect is uncertainty of the signal. From equations (5) and (16), we can observe that the uncertainty of the signal decreases as the wage differential increases, and so, households have weak incentive for signaling investment when the wage differential is large. That is, large inequality in human capital investment sufficiently reduces the uncertainty of obtaining the signal. Therefore, if we focus only on the effect of the uncertainty, a lower wage differential strengthens signaling investment for both rich and poor parents. The second effect comes from value of the signal. Obviously, the signal becomes valuable as the wage differential increases because a large wage differential here means a large wage gap between the worker with the signal and the worker with no signal. In other words, a larger wage differential provides a stronger incentive for households to obtain the signal. Therefore, if we focus only on the effect of the value of the signal, a higher wage differential strengthens signaling investment. Thus, these two opposite
effects of wage differentials creates the inverse U-shape of the signaling investment ratio, that is, the U-shape of $e_t/s_t$.

From the societal perspective, Proposition 1 can be interpreted to mean that private investment in higher education is effective in societies with high wage differentials (highly unequal economies) as well as societies with low wage differentials (equitable economies), while it may contain much wasteful investment in economies with medium-sized wage differentials. Furthermore, the same logic can be applied to some public policies on higher education. If households have strong incentive for credentialing, part of public expenditure may be used wastefully as signaling investment by, for example, using publicly funded education vouchers. As shown theoretically in Blankenau and Camera (2009), this result also relates to the mechanism of weak connection between public education expenditure and accumulation of human capital. Furthermore, we argue that substantial redistribution policies affect household educational investment decisions. For instance, because an increase of public education expenditure decreases wage differentials, it may also decrease the ratio of human capital to signaling investment in countries with large wage differentials. This effect prevents public expenditure from encouraging human capital accumulation in highly unequal societies. This is consistent with the empirical results in Hanushek (1986, 2003), which state that expansion of public education expenditure in the US has not guaranteed human capital accumulation.

5 The Dynamics of Wage Differentials

In this section we investigate the steady state of the dynamic system of middle-aged wage differentials $\tilde{w}_t$. In our analysis, the wage differentials of the middle-aged generation are of intrinsic importance because this generation invests in education and determines the next generation’s wage differentials.\(^\text{13}\) To derive the steady states for $\tilde{w}_t$, we compare a large and small relationship between $\tilde{w}_{t+1}$ and $\tilde{w}_t$ in equation (17). The relationship $\tilde{w}_{t+1} > \tilde{w}_t$ can be rewritten as follows:

$$
\frac{1}{\Theta(\tilde{w}_t)} < \frac{\tilde{w}_t - \phi(\tilde{w}_t)^\mu}{1 - \phi(\tilde{w}_t)^\mu + 1} + 1 \equiv X(\tilde{w}_t).
$$

\(^{13}\)On the other hand, the equilibrium wage differential of the old generation, $\phi\tilde{h}$, is always larger than that of the middle-aged generation $\tilde{w}$.\(^{17}\)
Henceforth, we examine the ranges of $w_t$ that satisfy $1/\Theta(w_t) < X(w_t)$. From the definition of $\Theta$, we can see that $1/\Theta(w_t)$ is a convex and decreasing function of $w_t$. Furthermore, $1/\Theta(w_t) = 2$ when $w_t = 1$, and $1/\Theta(w_t) = 1$ when $w_t \to \infty$. Next, we examine $X(w_t)$. Note that because $1/\Theta(w_t)$ is always greater than one, we can confirm the analysis of $X(w_t)$ to the region of $w_t$ where $X(w_t) > 1$ holds. The following lemma is obtained.

Lemma 2. $X(w_t)$ is a convex and decreasing function in the region of $w_t$ where $X(w_t) > 1$ holds. Moreover, we have $X(1) = 2$ (see the Appendix for details).

From Lemma 2, we find that the shape of $X(w_t)$ is quite similar to that of $1/\Theta(w_t)$. What is different from $1/\Theta(w_t)$ is that $X(w_t)$ falls below one at a certain finite value of $w_t$, and then, $X(w_t) < 1$ always holds for any larger $w_t$, as shown in Lemma 2. Hence, our task here is simply to compare two convex and decreasing curves according to the change in the parameters.

Again, $1/\Theta = X = 2$ always holds when $w_t = 1$. As is clear from equation (19), we obtain $\partial X/\partial \phi > 0$, $\partial X/\partial \mu > 0$ and $\partial X/\partial \nu = 0$ for all $w_t \in (1, \infty)$. That is, increases in $\phi$ and $\mu$ shift up $X(w_t)$ for all $w_t$, except for $w_t = 1$. On the other hand, we have $\partial \Theta/\partial \phi = 0$, $\partial \Theta/\partial \mu > 0$, and $\partial \Theta/\partial \nu > 0$ from the assumption $\Theta_h > 0$ and $\Theta_S > 0$ for all $w_t \in (1, \infty)$. That is, increases in $\mu$ and $\nu$ shift down $1/\Theta(w_t)$ for all $w_t$, except for $w_t = 1$. Then, the following three cases are possible: (i) a no-inequality steady state at $w = 1$, denoted by $w_1^*$, (ii) a high-inequality steady state at $w > 1$, denoted by $w_2^*$, and (iii) multiple steady states. Therefore, we have the following proposition.

Proposition 2. Three patterns of stable steady states exist with respect to the wage differentials, according to the level of skill-biased technology $\phi$, the efficiency of human capital investment $\mu$, and the efficiency of signaling investment $\nu$. The steady state wage differentials $w^*$ satisfy:

(i) If the values of $\phi$, $\mu$, and $\nu$ are sufficiently low, a unique steady state exists with no inequality, $w_1^*$.

(ii) If values of $\phi$, $\mu$, and $\nu$ are sufficiently high, a unique steady state exists with high inequality, $w_2^*$.

(iii) Multiple steady states exist.
(i) No-inequality steady state $\bar{w}_1^*$

The steady state for $\Theta(h_{t+1}, S_{t+1}) = 1 - \frac{1}{2} \exp (C(1 - h_{t+1}S_{t+1}))$, $C = 1.5$, $\phi = 1.1$, $\mu = 0.7$, and $\nu = 0.8$.

(ii) High-inequality steady state $\bar{w}_2^*$

The steady state for $\Theta(h_{t+1}, S_{t+1}) = 1 - \frac{1}{2} \exp (C(1 - h_{t+1}S_{t+1}))$, $C = 1.5$, $\phi = 1.6$, $\mu = 0.7$, and $\nu = 0.8$.

(iii) Multiple steady states

Steady states for $\Theta(h_{t+1}, S_{t+1}) = 1 - \frac{1}{2} \exp (C(1 - h_{t+1}S_{t+1}))$, $C = 0.5$, $\phi = 1.6$, $\mu = 0.7$, and $\nu = 0.9$.

Figure 2
Figure 2 shows numerical examples of (i)~(iii). Of course, ranges of \( \tilde{w}_t \) that satisfy \( X(\tilde{w}_t) > 1/\Theta(\tilde{w}_t) \) on the left-hand side of the figures are consistent with ranges of \( \tilde{w}_t \) that satisfy \( \tilde{w}_{t+1} > \tilde{w}_t \) on the right-hand side of the figures. The most notable parameter here is \( \phi \), which represents the level of skill-biased technology. We can see from (i)~(iii) in Figure 2 that a high-inequality steady state tends to be realized for higher values of \( \phi \), as analytically proved above. Therefore, it can be said that if an economy with an initial steady state of no inequality experiences rapid increases of skill-biased technologies, its steady state may switch from no inequality to high inequality, and the economy suffers a large increase in wage differentials during the transition to the high-inequality steady state. This is consistent with a commonly asserted notion that SBTC from the 1970s to the 1990s caused large increases of wage inequality between highly educated and poorly educated workers. In turn, the effects of \( \mu \) and \( \nu \) are also intuitive. These parameters represent the efficiency of human capital and signaling investment, respectively. Therefore, in addition, Proposition 2 states that more efficient education leads to a high-inequality steady state.

Furthermore, it would be interesting to examine the relationship between change of the investment ratio and the switch in the steady states due to the rapid increase of \( \phi \). We suppose that the economy is initially close to the no-inequality steady state, and a rapid increase in \( \phi \) switches the steady state from \( \tilde{w}_1^* \) to \( \tilde{w}_2^* \). Then, the ratio of human capital to signaling investment changes during the transition to the steady state. Note that almost all education expenditure is invested in human capital investment in the neighborhood of the no-inequality steady state \( \tilde{w}_1^* \). Hence, the signaling investment ratio increases for at least a couple of periods from the moment the steady state switches to \( \tilde{w}_2^* \). For the next proposition, we once more use the notations \( \tilde{w}' \) to represent the lowest value of \( \tilde{w} \) satisfying \( \partial(e_t/s_t)/\partial \tilde{w}_t = 0 \) and \( \tilde{w}'' \) to represent the highest value of \( \tilde{w} \) satisfying \( \partial(e_t/s_t)/\partial \tilde{w}_t = 0 \).

**Proposition 3.** Suppose that the economy is initially in the neighborhood of the no-inequality steady state \( \tilde{w}_1^* \) and then switches to the high-inequality steady state \( \tilde{w}_2^* \). If \( \tilde{w}_2^* < \tilde{w}' \), \( e/s \) decreases for every period until the economy converges to a steady state \( \tilde{w}_2^* \). On the other hand, if \( \tilde{w}_2^* > \tilde{w}'' \), the economy experiences a U-shaped change of \( e/s \) during the transition to \( \tilde{w}_2^* \).

A numerical example of Proposition 3 is shown in Figure 3. The most definitive and important part of this proposition is that the ratio of signaling investment increases for some
period after the switch of steady state. This is because an increase in $\tilde{w}$ from $\tilde{w} = 1$ necessarily causes $e/s$ to decrease for any functional form of $\Theta$.

We can interpret Proposition 3 to mean that the rapid technological progress from the 1980s may have enhanced educational investment for the purpose of signaling. That is, the recent increase in higher educational attainment experienced in many countries may stem from an increase in socially wasteful educational expenditure caused by SBTC. If this is the case, the increase in educational attainment would make a slight contribution to human capital accumulation.

We draw a U-shaped function $e/s$ and transition path to the high-inequality steady state $\tilde{w}^2$. Here, we assume that the paths start from around the no-inequality steady state. The figure on the left is drawn by using $C = 3$, $\phi = 2$, $\mu = 0.4$, $\nu = 0.5$, and $\beta = 0.5$. In this case, $\tilde{w}^* \approx 0$ is larger than $\tilde{w}'' (= \tilde{w}')$, and so, $e/s$ changes in a U-shaped form during the transition to $\tilde{w}^2$. On the other hand, there are cases in which $\tilde{w}^* \approx 0$ is smaller than $\tilde{w}'$. The figure on the right shows this case by using $C = 3$, $\phi = 2$, $\mu = 0.25$, $\nu = 0.3$, and $\beta = 0.5$, and we have $\tilde{w}^* \approx 1.48$ and $\tilde{w}' = \tilde{w}'' \approx 1.68$.

Figure 3
6 Conclusions and Remarks

In this study, we presented a simple dynamic model that contains both human capital investment and signaling investment. By introducing two opposing types of education, human capital investment and signaling investment, into the model, we described the degree of credentialing from the investment ratio. Furthermore, we focused on the wage differentials between the rich and poor. Because parents care not only about the productivity of their offspring but also about the signal, the rich and poor compete with one another to obtain the signal. Therefore, the investment ratio of both the rich and poor depends on the wage differentials.

Our static analysis proved that the ratio of human capital investment to signaling investment becomes a U-shaped function of the wage differentials. On the other hand, from the dynamic analysis, we showed that three patterns of stable steady states exist with respect to the wage differentials: no-inequality, high-inequality, and multiple steady states. Under a higher level of skill-biased technology (\(\phi\)), an economy tends to have a high-inequality steady state. Furthermore, we showed that if the economy were initially around the no-inequality steady state, rapid progress of skill-biased technology would switch the steady state from no inequality to high inequality, and the ratio of signaling investment would increase, at least for some periods. From this result, we can see that the cause of the increase in higher educational attainment following the IT revolution not only is an increase in investment of human capital accumulation, but also may contain a large increase in investment aimed at credentialing.

Clearly, it may be hard to distinguish between human capital investment and signaling investment in existing educational investment. In reality, no educational investment would contribute nothing to human capital accumulation, such as signaling investment, in this model. However, the ratio of human capital investment to signaling investment in our model well describes whether households’ primary objectives for educational attainment are learning or credentialing. Because private decisions between learning and credentialing are less observable, our theoretical analysis makes a significant contribution to explaining households’ decisions.
Appendix

Proof of Lemma 1

To simplify the exposition, we use $\Theta_{t+1}$ instead of $\Theta(h_{t+1}, \tilde{S}_{t+1})$. Since we have $\ln \tilde{w}_{t+1} < \ln \phi h_{t+1} < \phi \ln \tilde{h}_{t+1}$ when $\tilde{w}_t \to \infty$, the denominator of the second term in (18) can be rewritten as follows:

$$\frac{\partial \Theta_{t+1}}{\partial \tilde{S}_{t+1}} \tilde{S}_{t+1} \ln \tilde{w}_{t+1} < \phi \frac{\partial \Theta_{t+1}}{\partial \tilde{S}_{t+1}} \tilde{S}_{t+1} \ln \tilde{h}_{t+1}.$$ 

In addition, we have $\tilde{h}_{t+1} = (\tilde{S}_{t+1})^\frac{\mu}{\nu}$, and then, from the right-hand side of the above equation, we have:

$$\phi \frac{\partial \Theta_{t+1}}{\partial \tilde{S}_{t+1}} \tilde{S}_{t+1} \ln \tilde{h}_{t+1} = \frac{\mu \phi}{\nu} \frac{\partial \Theta_{t+1}}{\partial \ln (\ln \tilde{S}_{t+1})} \to 0 \text{ (when } \tilde{w}_t \to \infty).$$

The last term of limit to zero is obtained by properties that $\Theta \to 1$ when $\tilde{w}_t \to \infty$, while $\ln(\ln \tilde{S}_{t+1}) \to \infty$ when $\tilde{w}_t \to \infty$. Then, we obtain:

$$\lim_{\tilde{w}_t \to \infty} \frac{\partial \Theta(h_{t+1}, \tilde{S}_{t+1})}{\partial \tilde{S}_{t+1}} \tilde{S}_{t+1} \ln \tilde{w}_{t+1} = 0.$$

Proof of Lemma 2

First, we check the sign of the first-order derivative:

$$\frac{dX_t}{d\tilde{w}_t} = \frac{1}{(1 - \phi(\tilde{w}_t)^{\mu+1})^2} \left[ 1 - \mu \phi(\tilde{w}_t)^{\mu-1} - \phi^2(\tilde{w}_t)^{2\mu} + \mu \phi(\tilde{w}_t)^{\mu+1} \right]$$

$$= \frac{1}{(1 - \phi(\tilde{w}_t)^{\mu+1})^2} \left[ 1 - \mu \phi(\tilde{w}_t)^{\mu-1} - (1 - \mu)\phi^2(\tilde{w}_t)^{2\mu} - \mu \phi^2(\tilde{w}_t)^{2\mu} + \mu \phi(\tilde{w}_t)^{\mu+1} \right].$$

The third and fourth terms in the second line resolve the third term in the first line. Note that we now confine the analysis to the region $X_t > 1$, so that our analytical range of $\tilde{w}_t$ becomes $1 < (\tilde{w}_t)^{1-\mu} < \phi$. Then, we have
\[ 1 - \mu \phi(\tilde{w}_t)^{\mu-1} - (1 - \mu)\phi^2(\tilde{w}_t)^{2\mu} < 1 - \mu - (1 - \mu) = 0, \]
\[ -\mu \phi^2(\tilde{w}_t)^{2\mu} + \mu \phi(\tilde{w}_t)^{\mu+1} = \mu \phi(\tilde{w}_t)^{\mu+1}(1 - \phi(\tilde{w}_t)^{\mu-1}) < 0. \]

Therefore, for all ranges that satisfy \( X_t > 1 \), we have \( dX_t/d\tilde{w}_t < 0 \). Next, we check the sign of the second-order derivative:

\[
\frac{d^2X_t}{d\tilde{w}_t^2} = \frac{1}{(1 - \phi(\tilde{w}_t)^{\mu+1})^3} \left[ (\mu^2 + 3\mu + 2)\phi(\tilde{w}_t)^{\mu} - (\mu^2 + 5\mu)\phi^2(\tilde{w}_t)^{2\mu-1} + \mu(\mu + 1)\phi^2(\tilde{w}_t)^{2\mu+1} + \mu(1 - \mu)\phi(\tilde{w}_t)^{\mu-2} - 2\phi^3(\tilde{w}_t)^{3\mu} \right]
\]

In the same manner as described above:

\[
(\mu^2 + 5\mu)\phi(\tilde{w}_t)^{\mu} - (\mu^2 + 5\mu)\phi^2(\tilde{w}_t)^{2\mu-1} = (\mu^2 + 5\mu)\phi(\tilde{w}_t)^{\mu}(1 - \phi(\tilde{w}_t)^{\mu-1}) < 0
\]
\[
2(1 - \mu)\phi(\tilde{w}_t)^{\mu} - 2(1 - \mu)\phi^3(\tilde{w}_t)^{3\mu} = 2(1 - \mu)\phi(\tilde{w}_t)^{\mu}(1 - \phi(\tilde{w}_t)^{\mu-1}) < 0
\]
\[
\mu(\mu + 1)\phi^2(\tilde{w}_t)^{2\mu+1} - \mu(\mu + 1)\phi^3(\tilde{w}_t)^{3\mu} = \mu(\mu + 1)\phi^2(\tilde{w}_t)^{2\mu+1}(1 - \phi(\tilde{w}_t)^{\mu-1}) < 0
\]
\[
\mu(1 - \mu)\phi(\tilde{w}_t)^{\mu-2} - \mu(1 - \mu)\phi^3(\tilde{w}_t)^{3\mu} = \mu(1 - \mu)\phi(\tilde{w}_t)^{\mu-2}(1 - \phi(\tilde{w}_t)^{\mu+1})(1 + \phi(\tilde{w}_t)^{\mu+1}) < 0.
\]

Therefore, for all ranges that satisfy \( X_t > 1 \), we have \( d^2X_t/d\tilde{w}_t^2 > 0 \).

\[ \square \]

References


