Strategic Formation of Customer Relationship Networks

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We consider two intermediaries who compete in terms of quantities for customers on an output market and for resources on an input market. Additionally, the intermediaries strategically choose which customer relationships they are willing to establish for given costs of link formation and a parameter that indicates the substitutability between their products from the customers’ point of view. Our main goal is to investigate how costs of link formation and the products’ substitutability influence the stability of customer relationship networks. For networks with a monopolistic input market and three types of customers, we determine upper and lower bounds for the costs of link formation that induce stability for a given network of customer relationships. In our analysis, we allow for varying degrees of the products’ substitutability. Moreover, we compare the regions of stability for different networks of customer relationships.

The basic model of a supply-chain we use has been introduced in Häckner (2000, 2003). Intermediaries compete in quantities or prices for customers and resources. One main assumption is that all market participants interact with each other. The strategic formation of networks for research and development collaborations has already been intensively studied departing from the model of Goyal and Joshi (2003), for instance, and has been analyzed dynamically by Dawid and Hellmann (2014). The network is used to model the externalities
that are induced by transfer of knowledge through a cooperation. Forming a link reduces the production costs and, hence, endogenously introduce asymmetries. Kranton and Minehart (2000, 2001) introduce “buyer-seller networks”. Kranton and Minehart (2001) use simultaneous ascending-bid auctions to model competition and analyze efficiency as well as pairwise stability within this framework. More recently, contracts along supply-chains have also been investigated in matching theory with contracts, as in Ostrovsky (2008), Westkamp (2010) and Hatfield and Kominers (2012), for instance. Interestingly, Hatfield and Kominers (2013) directly study firm exit within supply-chain networks based on the model of Ostrovsky (2008). There is a different effect between exit of end customer or producer and exit of intermediaries. Besides these economic contributions, trade in Internet transactions along supply-chains of service and network providers has also been analyzed in Nagurney and Wolf (2013). Using variational inequalities, Nagurney and Wolf (2013) investigate a model, in which service providers compete in quantities and network providers acting as intermediaries compete in prices. Our analysis complements the existing literature on oligopoly competition by explicitly considering the strategic formation of customer relationship networks. In comparison to buyer-seller networks we analyze trade along supply-chains. Preference orderings over different networks result from strategic interaction of two intermediaries taking the customers’ purchasing decisions into account. This distinguishes our approach from matching theory, where preference orderings are assumed to be exogeneously given over the set of available contracts. Compared to the literature on network formation, we interpret links in a different way. Links are customer relationships that enable trade and, therefore, directly have an impact on the equilibrium prices and quantities on the output as well as on the input market.

For a network with two intermediaries there may exist three different types of customers that have at least one link. A customer may only be linked to exactly one of the intermediaries or may be linked to both of them. Suppose there are \( n_1 \) customers linked to intermediary 1, \( n_2 \) customers linked to intermediary 1 and 2, and \( n_3 \) customers linked to intermediary 2 with \( n_1, n_2, n_3 \geq 0 \). Customers of type 1 or 3 are exclusive customers for one of the intermediaries and customers of type 2 are joint customers. Networks are defined by the number of exclusive and joint customers of the two intermediaries \((n_1, n_2, n_3)\). The input market is supposed to be monopolistic with a single input supplier who always delivers resources to both intermediaries. Hence, the network for the input market is assumed to be fixed to focus on the strategic formation of customer relationships (in a first step). Some examples for those networks can be found in Figure 1. Figure 1(a) shows a network of customer relationships where all customers are linked to both intermediaries. Thus, all customers are joint customers. In contrast, in Figure 1(c) all customers are linked to either intermediary 1 or to intermediary 2. Figure 1(b) illustrates a symmetric network where both intermediaries have exactly one exclusive customer, and, in addition, there is one joint customer. The network of customer relationships in Figure 1(d) is asymmetric in the sense that only intermediary 1 has some exclusive customers.
To describe the customers’ preferences we use the same standard utility function as H"{a}ckner (2000, 2003) and model customers according to the three different types as described above. Let $\gamma \in [-1,1]$ be the degree of horizontal product differentiation or the degree of substitutability (including possible complementarities). Customers’ utilities are as follows:

\[
\begin{align*}
  u_1(q_1, I, g) &= aq_{11} - \frac{1}{2}q_{11}^2 + I, \\
  u_2(q_2, I, g) &= aq_{12} + aq_{22} - \frac{1}{2}(q_{12}^2 + q_{22}^2 + 2\gamma q_{12}q_{22}) + I, \\
  u_3(q_3, I, g) &= aq_{23} - \frac{1}{2}q_{23}^2 + I,
\end{align*}
\]

where $a > 0$ denotes the products’ common quality level and $I$ denotes consumption of other goods. We suppose that a customer’s income is sufficiently large such that utility maximization leads to an interior solution. For $\gamma = -1$ a joint customer considers the two products (of the two intermediaries) to be complements, while for $\gamma = 1$ the products are considered to be substitutes. Moreover, for $\gamma = 0$ a joint customer’s utility function is additive separable and thus the two intermediaries can be considered as monopolists for their products. Note that a joint customer’s utility function in (2) reduces to an exclusive customer’s utility function as in (1) and (3) if the respective quantities are set to zero. We first compute equilibrium prices and quantities the two intermediaries choose for given input prices and network of customer relationships $(n_1, n_2, n_3) \in \mathbb{N}_+^3$ such that $n_1 + n_2 + n_3 \leq n$. 

![Figure 1: Examples with Two Intermediaries and One Input Supplier](image-url)
After having determined the intermediaries’ total demand on the input market and the input price a monopolistic input supplier charges, we examine the strategic network formation on the customer market. More precisely, we look at the incentives for the two intermediaries to strategically form costly relationships to customers. Figure 2 shows all networks of customer relationships with $n_1 + n_2 + n_3 \leq 3$ and $n_1, n_2, n_3 \geq 0$ and indicates the possible transitions between the networks by adding or deleting single links.

Figure 2: Networks with $n_1 + n_2 + n_3 \leq 3$ and $n_1, n_2, n_3 \geq 0$

The substitutability of the intermediaries’ products, $\gamma$, and the costs of link formation, $c$, influence the intermediaries’ equilibrium profits and, thus, have an impact on the incentives to strategically form relationships to customers.

We introduce a notion of local stability and identify for each network with a monopolistic input market and three customers (see Figure 2) regions of $(\gamma, c)$, in which the given network is stable. Additionally, for networks with $n$ customers we analyze stability regions for selected networks and determine the limit regions of stability as $n$ goes to infinity. It turns out that the shape of the stability regions does not significantly change compared to a setting with relatively small $n$. 
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References


