To Reduce Inequality, Should We Tax the Capital Income?

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* Should we tax the capital income to reduce inequality? and, How to do better when we use the capital income taxation?
* Evidences from Vietnam: Inequality and economic growth (source: GSO)
  - The gap between the 20% of richest population and 20% of poorest population: 1993: 5 times; 2002: 8.1 times; 2012: 9.4 times
  - Gini coefficients of income inequality: 0.35 at 1998 but increases to 0.43 at 2010
Introduction
Motivation...continued

Literature bigskip
- Kuznet (1955): link between the economic growth and Inequality
- Judd (1985) and Chamley (1986): optimal capital income tax is zero in the long-run
- Alesina and Rodrik (1993): Find a negative empirical link between inequality and growth
- Benabou (1995): there is a negative relationship between inequality and redistribution if inequality is small
- Roemer and Lee (1998): a high taxe rate is not necessarily detrimental to economic growth
- Acemoglu, Golosov and Tsyvinski (2011): positive effect of tax on capital, even in the long-run, if politicians are more short-sighted than citizens
- Persson and Tabellini (2012): Inequality is harmful for growth
- Dalgaard and Hansen (2013): increasing taxes slower growth income per capita
Consider a production function
\[ F(K, L_A, L_B) = K^\kappa L_A^\alpha L_B^\beta, \ (\kappa, \alpha, \beta) >> 0, \ \kappa + \alpha + \beta = 1 \]
be a production function using capital \( K \), two types of labour \( L_A, L_B \) to produce a consumption good \( y = F(K, L_A, L_B) \).
Assume the price of the consumption good is 1, the price of the capital is \( r \), the wages are \( w^A, w^B \). Suppose we maximize the profit:

\[
\max \{ F(K, L_A, L_B) - rK - w^A L_A - w^B L_B : (K, L_A, L_B) \geq 0 \}
\]

We then have, if \((K, L_A, L_B)\) solve the problem,

\[
rK = \kappa F(K, L_A, L_B), \ w^A L_A = \alpha F(K, L_A, L_B), \ w^B L_B = \beta F(K, L_A, L_B)
\]
The economy

We have two classes. Class A owns the capital and works with a supply $L_A$. Class A consumes with a propensity $\gamma_A < 1$. Its saving will be invested in physical capital. Class B only works with a supply $L_B$ consumes all its income (does not save).

There is a firm which maximizes the profit

$$\max \{ F(K, L_A, L_B) - rK - w^A L_A - w^B L_B : (K, L_A, L_B) \geq 0 \}$$

The agents live TWO periods: period 0 and period 1.

Class A owns a physical capital $K_0$ at the beginning of period 0.

The supplies of labor $L_A, L_B$ are given at each period.
Without Taxation

**period 0**

Firm maximizes the profit. The prices and wages are obtained through the markets clearing. We obtain:

\[ r_0 = \kappa K^{\kappa-1} L_A^\alpha L_B^\beta \]
\[ w_0^A = \alpha K^{\kappa} L_A^{\alpha-1} L_B^\beta \]
\[ w_0^B = \beta K^{\kappa} L_A^\alpha L_B^{\beta-1} \]

Market clearing \( K = K_0 \) and the demands for labour must equal the supplies \( L_A, L_B \). From that, we obtain the price of the capital \( r_0 \), wages \( w_0^A, w_0^B \).
Without Taxation

period 0

Hence

\[ r_0 = \kappa K_0^{\kappa-1} L_A^\alpha L_B^\beta \]
\[ w_0^A = \alpha K_0^\kappa L_A^{\alpha-1} L_B^\beta \]
\[ w_0^B = \beta K_0^\kappa L_A^\alpha L_B^{\beta-1} \]
Without Taxation

**period 0**

Income of Class $A$, \( Y_0^A \) = \( r_0 K_0 + w_0^A L_A \)

Consumption \( c_0^A \) = \( \gamma_A (r_0 K_0 + w_0^A L_A) \)

Saving = Investment \( I_0 \) = \( (1 - \gamma_A)(r_0 K_0 + w_0^A L_A) \)

The supply of capital for period 1 is

\[ K_1 = K_0 (1 - \delta) + I_0 \]
Without Taxation

period 0

Income of Class $B$, $Y_0^B = w_0^B L_B$
Consumption $c_0^B = w_0^B L_B$

INEQUALITY INDEX:

$$\frac{Y_0^A / L_A}{Y_0^B / L_B} = \frac{\kappa + \alpha}{L_A} \times \frac{L_B}{\beta}$$

ASSUME

$$\frac{Y_0^A / L_A}{Y_0^B / L_B} = \frac{\kappa + \alpha}{L_A} \times \frac{L_B}{\beta} > 1$$
period 1

Again, Firm maximizes the profit. The prices and wages are obtained through the markets clearing. We obtain:

\begin{align*}
  r_1 &= \kappa K_1^{\kappa-1} L_A^\alpha L_B^\beta \\
  w_1^A &= K_1^\kappa L_A^\alpha - 1 L_B^\beta \\
  w_1^B &= K_1^\kappa L_A^\alpha \beta L_B^{\beta-1}
\end{align*}

Income of Class $A$, $Y_1^A = r_1 K_1 + w_1^A L_A$

Consumption $c_0^A = (r_1 K_1 + w_1^A L_A)$

Saving = 0
period 1

Income of Class $B$, $Y^B_1 = w^B_1 L_B$

Consumption $c^B_1 = w^B_1 L_B$

INEQUALITY INDEX:

$$\frac{Y^A_1}{L_A} = \frac{\kappa + \alpha}{L_A} \times \frac{L_B}{\beta}$$
period 0

The government taxes, ONLY in period 0, Household $A$, on her capital income with a tax rate $\tau$. The collected tax is transferred to Household $B$. The new income of $A$, investment and capital for period 1 will be:

\[
Y_0^A = r_0(1 - \tau)K_0 + w^A_0L_A
\]
\[
l_0' = (1 - \gamma_A) \left[ r_0(1 - \tau)K_0 + w^A_0L_A \right] < l_0
\]
\[
'K'_1 = (1 - \gamma_A) \left[ r_0(1 - \tau)K_0 + w^A_0L_A \right] + (1 - \delta)K_0
\]
\[
= K_1 - (1 - \gamma_A)\tau r_0 K_0 < K_1
\]
period 0

Income of Class $B$, $Y_0^B = w_0^B L_B + \tau r_0 K_0$

Consumption $c_0^B = w_0^B L_B + \tau r_0 K_0$

INEQUALITY INDEX:

\[
\frac{Y_0^A}{L_A} = \frac{(\kappa + \alpha) Y_0 - \tau r_0 K_0}{L_A} \times \frac{L_B}{\beta Y_0 + \tau r_0 K_0}
\]

where $Y_0 = r_0 K_0 + w_0^A L_A + w_0^B L_B$.

Explicitly:

\[
\frac{Y_0^A}{L_A} = \frac{L_B}{L_A} \times \frac{\kappa(1 - \tau) + \alpha}{\beta + \tau \kappa} < \frac{\kappa + \alpha}{L_A} \times \frac{L_B}{\beta} = \frac{Y_0^A}{L_A}
\]

The INEQUALITY INDEX IS REDUCED.
Assume moreover: \( \alpha > \frac{L_A}{L_A + L_B} \). In this case the inequality index \( \frac{Y'^A_0 / L_A}{Y'^B_0 / L_B} \) is always greater than one. We do not reverse the situation by taxation: the poor class becomes the rich class.
period 1

The new output

\[ Y'_1 = F(K'_1, L_A, L_B) < F(K_1, L_A, L_B) = Y_1 \]

The new prices and wages are

\[
\begin{align*}
    r'_1 &= \kappa K'_1 L_A^{\alpha-1} L_B^\beta > r_1 \\
    w'_A &= K'_1 \alpha L_A^{\alpha-1} L_B^\beta < w_A \\
    w'_B &= K'_1 L_A^\alpha \beta L_B^{\beta-1} < w_B 
\end{align*}
\]

Inequality index is the same as before \( \frac{\kappa + \alpha}{L_A} \times \frac{L_B}{\beta} \).

Observe \( Y'_1 < Y_1, \ w'_B < w_B \)
SUPPOSE WE TAX the CAPITAL INCOME in PERIOD 1 within a THREE-PERIOD MODEL.
In this case, we reduce the inequality index in period 1, but reduces the output in period 2, and so on.
We tax in period 0 capital income and invest in Human Capital for Labour $B$. If $h_B$ denotes the HC, the effective labor will be $h_B L_B$. Suppose

$$h_B = 1 + a_B \tau r_0 K_0$$

The output in period 1 is

$$Y_1 = \left[ K_1 - (1 - \gamma_A) \tau r_0 K_0 \right]^\kappa L_A^\alpha (1 + a_B \tau r_0 K_0) L_B^\beta$$

$$= (1 + a_B \tau r_0 K_0) \beta \left[ K_1 - (1 - \gamma_A) \tau r_0 K_0 \right]^\kappa L_A^\alpha L_B^\beta$$

We will find $\tau$ s.t. the output $Y_1$ is maximal.
It turns out to solve

$$\max \{ (1 + a_B \tau r_0 K_0)^\beta [K_1 - (1 - \gamma_A) \tau r_0 K_0]^\kappa : \tau \in [0, 1] \}$$

or equivalently

$$\max \{ (1 + a_B \tau r_0 K_0) [K_1 - (1 - \gamma_A) \tau r_0 K_0]^{\frac{\kappa}{\beta}} : \tau \in [0, 1] \}$$

If the solution is interior, we have the equation:

$$[K_1 - (1 - \gamma_A) \tau r_0 K_0] = (1 - \gamma_A)^{\frac{\kappa}{\beta}} \left( \frac{1}{a_B} + \tau r_0 K_0 \right)$$

We get

$$\tau = \frac{K_1 - (1 - \gamma_A)^{\frac{\kappa}{\beta}} \frac{1}{a_B}}{r_0 K_0 (1 - \gamma_A) \left[ 1 + \frac{\kappa}{\beta} \right]}$$
If
\[(1 - \gamma_A) \frac{\kappa}{\beta} \left( \frac{1}{a_B} + r_0 K_0 \right) > K_1 - (1 - \gamma_A) r_0 K_0\]
and
\[K_1 > (1 - \gamma_A) \frac{\kappa}{\beta} \left( \frac{1}{a_B} \right)\]
we have a unique optimal solution \(\tau \in (0, 1)\). The associated \(Y_1\) is strictly higher than the one without taxation. If
\[K_1 < (1 - \gamma_A) \frac{\kappa}{\beta} \left( \frac{1}{a_B} \right)\]
the optimal \(\tau\) is zero: No taxation is optimal. This case happens when \(a_B\) is small.
If
\[(1 - \gamma_A) \frac{\kappa}{\beta} \left( \frac{1}{a_B} + r_0 K_0 \right) < K_1 - (1 - \gamma_A) r_0 K_0\]
the optimal \(\tau\) is 1. This happens when \(a_B\) is high enough.
We can also tax the capital income and invest in New Technology, HC for $L_A$, HC for $L_B$. Formally, we suppose, given $\tau$, the investment in NT is $\tau_K r_0 K_0$, the one in HC for $L_A$ is $\tau_A r_0 K_0$, and for $L_B$, $\tau_B r_0 K_0$, with $\tau_K + \tau_A + \tau_B = \tau$. We suppose the investment in NT will give a productivity $(1 + a_K \tau_K r_0 K_0)$, the investments in HC will yield HC respectively $(1 + a_A \tau_A r_0 K_0)$, $(1 + a_B \tau_B r_0 K_0)$. The corresponding output is

$$Y_1 = [(1 + a_K \tau_K r_0 K_0) (K_1 - (1 - \gamma_A) \tau r_0 K_0)]^\kappa$$
$$\times [(1 + a_A \tau_A r_0 k_0) L_A]^\alpha$$
$$\times [(1 + a_B \tau_B r_0 K_0) L_B]^\beta$$

with $K_1 = K_0 (1 - \delta) + (1 - \gamma_A) (r_0 K_0 + w_0^A L_A)$

We solve the problem

$$\max\{(1 + a_K \tau_K r_0 K_0)^\kappa (1 + a_A \tau_A r_0 k_0)^\alpha (1 + a_B \tau_B r_0 K_0)^\beta : \tau_K + \tau_A + \tau_B = \tau\}$$

That is the first step. The second step is to maximize in $\tau$. 

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We obtain

\[ \tau_K = \tau \kappa + \frac{1}{r_0 k_0} \left[ \frac{\kappa - 1}{\alpha_K} + \frac{\kappa}{\alpha_A} + \frac{\kappa}{\alpha_B} \right] \]

\[ \tau_A = \tau \alpha + \frac{1}{r_0 k_0} \left[ \frac{\alpha - 1}{\alpha_A} + \frac{\alpha}{\alpha_K} + \frac{\alpha}{\alpha_B} \right] \]

\[ \tau_B = \tau \beta + \frac{1}{r_0 k_0} \left[ \frac{\beta - 1}{\alpha_B} + \frac{\beta}{\alpha_A} + \frac{\beta}{\alpha_K} \right] \]
Observe that if we do not tax the income in period 1, the inequality index will be again: $\frac{\kappa + \alpha}{L_A} \times \frac{L_B}{\beta}$. Should we tax ad infinitum the capital income and use it to invest in HC or NT? We will show that this policy is compatible with economic growth.
For simplicity we invest only in HC for labor $B$. 
First, we consider the infinite horizon economy without taxation. We obtain

\[ K_1 = K_0(1 - \delta) + (1 - \gamma_A)(r_0 K_0 + w_0^A L_A) \]
\[ = K_0(1 - \delta) + (1 - \gamma_A)(\kappa + \alpha) Y_0 \]
\[ = K_0(1 - \delta) + (1 - \gamma_A)(\kappa + \alpha) L_A^\alpha L_B^\beta K_0^\kappa \]

By induction

\[ K_{t+1} = K_t(1 - \delta) + (1 - \gamma_A)(\kappa + \alpha) L_A^\alpha L_B^\beta K_0^\kappa \]

Let \( \bar{K} > 0 \) satisfy

\[ \bar{K} = \bar{K}(1 - \delta) + (1 - \gamma_A)(\kappa + \alpha) L_A^\alpha L_B^\beta \bar{K}^\kappa \]

It can be easy shown that the sequence \( \{K_t\} \) is in \([K_0, \bar{K}]\) or \([\bar{K}, K_0]\). In this case the sequence of outputs \( Y_t \) will be bounded away from zero and bounded above.
We now tax the capital income in any period and use it for HC for labor $B$. Let $\{\tau_t\}$ be the sequence of optimal taxes, $K_t(\tau_1, \tau_2, \ldots, \tau_t)$ denote the optimal capital stock at period $t$, and $Y_t(\tau_1, \tau_2, \ldots, \tau_t)$ the corresponding outputs. We have

$$K_{t+1}(\tau_1, \ldots, \tau_{t+1}) = K_t(\tau_1, \ldots, \tau_t)(1 - \delta)$$
$$+ \pi (1 - \gamma_A) [(1 - \tau_{t+1})\kappa + \alpha] L_A^\alpha L_B^\beta K_t(\tau_1, \ldots, \tau_t)^\kappa$$
where $\pi = [1 + a_B \tau_t \kappa Y_{t-1}(\tau_1, \ldots, \tau_{t-1})]^\beta \times \ldots \times (1 + a_B \tau_1 \kappa Y_0)^\beta$.

The corresponding outputs are

$$Y_{t+1}(\tau_1, \ldots, \tau_{t+1}) = [1 + a_B \tau_{t+1} \kappa Y_t(\tau_1, \ldots, \tau_t)]^\beta \times \ldots \times (1 + a_B \tau_1 \kappa Y_0)^\beta \times L_A^\alpha L_B^\beta [K_{t+1}(\tau_1, \ldots, \tau_{t+1})]^\kappa$$
We can prove that there exist $K > 0$ s.t. for any $t$

$$K < K_{t+1} (\tau_1, \ldots, \tau_{t+1})$$

From there if we assume

$$K > (1 - \gamma A) \frac{\kappa}{\beta} \frac{1}{a_B}$$

then $\tau_t \geq \tau > 0$ for any $t$ and also

$$Y_{t+1}(\tau_1, \ldots, \tau_{t+1}) > \zeta \equiv L_A^\alpha L_B^\beta K^\kappa$$

for any $t$. 
It turns out that for any $t$:

$$Y_{t+1}(\tau_1, \ldots, \tau_{t+1}) > (1 + a_B \tau \kappa \zeta)^{t \beta} \zeta$$

We obtain $Y_{t+1}(\tau_1, \ldots, \tau_{t+1}) \rightarrow +\infty$

We have economic growth and inequality reduction.
* Usefulness of capital income tax if we use it to invest in human capital and New Technology
* Future research: Endogeneize rich and poor populations
Thank You