The sources of sharing externalities: specialization vs competition

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Since Adam Smith (1776), it has been widely acknowledged that an efficient division of labor is a key driver of increasing returns and gains from specialization.

Nevertheless, explicit ways of modeling these gains have been developed solely in the last few decades.

In the wake of the path-breaking papers by Dixit and Stiglitz (1977), Krugman (1979, 1981) and Ethier (1982), a huge number of models has been put forward which combine:

- free entry,
- product differentiation, and
- various forms of specialization.
• Ethier (1982) has developed a model of monopolistic competition with intermediate goods

• That model has become a workhorse in
  ▶ endogenous growth theory with horizontal innovations
  ▶ economics of agglomeration

• The key feature of this model is the presence of sharing externalities (also known as Marshallian externalities), which generate external increasing returns to scale (EIRS)

• However…
...in the Ethier’s model and its applications, the only key-factor of EIRS is the specialization economies: a wider differentiation of the intermediate good results in deeper specialization and higher TFP.

Meanwhile, an increase in the degree of intermediate inputs’ differentiation may trigger at least two other important effects:

- **Complexity diseconomies**: using technologies with a larger number of production tasks and/or more varieties of an input may hinder the manufacturing activity (Kremer, 1993).
- **Competition effect**: the proliferation of varieties implies more firms ⇒ toughness of competition in the market for intermediate goods changes. This may have a non-trivial impact on the aggregate output and other key market variables.
The aim of the paper

- The interaction between these forces and its role in generating EIRS is definitely understudied in the literature.

- The main reason: technology in the final sector has typically been assumed to display constant elasticity of substitution (CES):
  - horizontal innovation paradigm in endogenous growth theory (Grossman and Helpman, 1990; Krugman, 1990, Ch. 11; Romer, 1990; and Rivera-Batiz and Romer (1991))
  - Marshallian externalities approach which studies agglomeration economies in cities (Abdel-Rahman and Fujita, 1990; Duranton and Puga, 2004; Fujita and Thisse, 2013)

- But!: In the CES world, competition effect is washed out.

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We

- develop an extension of the Ethier’s model to the case of a non-specified CRS technology in the final-good sector
- provide a decomposition of the whole impact of sharing externalities into
  - a competition effect, which stems from the market interactions between producers of intermediate inputs, and
  - a specialization/complexity effect, which we model as a supply-side counterpart of the notion of “love/aversion for variety”
- obtain a full characterization of the impact of horizontal innovation (entry) on prices, markups, and wages
Moreover, we

- make a distinction between price-decreasing/increasing and markup-decreasing/increasing competition, and provide necessary and sufficient conditions for each type of competition to occur

- discriminate also between wage-increasing and wage-decreasing competition

- obtain a necessary and sufficient condition for EIRS to occur

- find that the competition effect may either reinforce or weaken the impact of the specialization effect on aggregate output

Our main results hold for any well-behaved technology which satisfies symmetry and constant returns to scale
The model and preliminary results
Basic setup

- The economy is composed by two sectors:
  - The intermediate inputs sector (\(I\)-sector), in which a differentiated intermediate good is produced under monopolistic competition
  - The final good sector (\(F\)-sector), in which perfectly competitive firms share the same CRS technology, which uses varieties of the intermediate good as inputs, to produce the final good

- The only production factor used in the \(I\)-sector is labor
- The labor market is perfectly competitive

- The main departure of our modeling strategy from Ethier (1982) lies in working with a non-specified production function instead of the CES
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The $F$-sector: production function

- All firms operating in the $F$-sector are endowed with the same production function $F$:

  \[ Y = F(q), \]  

- Here $q = (q_i)_{i \in [0, n]}$ is the vector of intermediate inputs
- Assumptions about $F(q)$:
  - **CRS**: $F(q)$ is positive homogenous of degree 1
  - **Strict quasi-concavity**: $F(q)$ is strictly quasi-concave in $q$, which implies that each input exhibits a diminishing marginal product
  - **Symmetry**: any permutation of intermediates does not change the final output
CES: variations on a theme

- **CES** (Dixit and Stiglitz, 1977):

  \[ F(q) = \left( \int_0^n q_i^\rho \, di \right)^{1/\rho}, \quad 0 < \rho < 1. \]  

  \[ (2) \]

- **Augmented CES** (Ethier, 1982; Benassy, 1998):

  \[ F(q) = n^\nu \left( \int_0^n q_i^\rho \, di \right)^{1/\rho}, \quad 0 < \rho < 1. \]

  \[ (3) \]

- When sufficiently negative, \( \nu \) is a measure of the magnitude of the complexity effect
- Complexity diseconomies are said to occur if and only if \( \nu < 1 - 1/\rho \)
- Otherwise (\( \nu > 1 - 1/\rho \)), specialization economies take place
Translog technologies

- Translog production function (Kim, 1992):

\[
\ln F(q) = \frac{1}{n} \int_0^n \ln q_i \, di - \frac{\alpha}{2n} \left[ \int_0^n (\ln q_i)^2 \, di - \frac{1}{n} \left( \int_0^n \ln q_i \, di \right)^2 \right]
\]

- Translog price index (Feenstra, 2003):

\[
\ln P(p) = \frac{1}{n} \int_0^n \ln p_i \, di - \frac{\beta}{2n} \left[ \int_0^n (\ln p_i)^2 \, di - \frac{1}{n} \left( \int_0^n \ln p_i \, di \right)^2 \right]
\]
Kimball-type production functions

- Kimball (1995) defined the production function \( Y = F(q) \) as a solution to the following equation:

\[
\int_0^n \phi\left( \frac{q_i}{Y} \right) \, di = 1
\]

- Here \( \phi(\cdot) \) is some function, which is assumed to be increasing and concave.
Specialization economies vs complexity diseconomies

- Denote by $\varphi(n)$ the level of output that can be produced when a firm uses one unit of each input.
- Assume that firm’s total expenditure $E$ on inputs is fixed.
- **Specialization economies** take place if and only if
  
  $$\frac{E}{n} \varphi(n) > \frac{E}{k} \varphi(k), \quad k < n$$

- Equivalently, specialization economies occur if and only if
  
  $$\frac{n\varphi'(n)}{\varphi(n)} > 1$$

- Otherwise, **complexity diseconomies** occur.
Proposition 1.

- The augmented CES, the translog technologies, and Kimball-type technologies are all different (except the CES that can be obtained as a special case of Kimball).

- Kimball-type technologies with \( \phi(\cdot) \) such that \( \phi(0) = 0 \) exhibit specialization economies.

- The translog production function generates complexity diseconomies, while under the translog price index neither specialization economies nor complexity diseconomies take place.
Consider a special case of Kimball-type technology with
\[ \phi(q/Y) \equiv a(q/Y)^\rho - b, \] where \( a, b > 0, \) \( 0 < \rho < 1 \)

This yields the following modification of the “augmented CES” technology:

\[ F(q) = A(n) \left( \int_0^n q_i^\rho \, di \right)^{1/\rho}, \quad A(n) \equiv \left( \frac{a}{1 + bn} \right)^{1/\rho} \]

The interaction between specialization and complexity is captured by

\[ \frac{\varphi(n)}{n} = \frac{1}{n} \left( \frac{an}{1 + bn} \right)^{1/\rho} \quad (4) \]

Hence, \( \varphi(n)/n \) is bell-shaped
Each $F$-firm seeks to minimize production costs, treating $Y$ as a given:

$$\min_{\mathbf{q}} \int_0^n p_i q_i \, di \quad \text{s.t.} \quad F(\mathbf{q}) \geq Y$$

The first-order condition for cost minimization is given by

$$p_i = \lambda \Phi(q_i, \mathbf{q})$$

Here $\Phi(q_i, \mathbf{q}) \equiv \partial F / \partial q_i$ is the marginal product of input $i$, while $\lambda$ is the Lagrange multiplier.

For the CES case, the marginal products are given by

$$\Phi(q_i, \mathbf{q}) = q_i^{\rho - 1} \mathcal{A}(\mathbf{q}), \quad \mathcal{A}(\mathbf{q}) \equiv \left( \int_0^n q_j^\rho \, dj \right)^{(1-\rho)/\rho}$$
Inverse demands

- Using the envelope theorem, we obtain the inverse demand schedule for input $i$:

$$\frac{p_i}{P(p)} = \Phi(q_i, q)$$

- Here $P(p)$ is the price index:

$$P(p) \equiv \min_q \int_0^n p_i q_i d_i \quad \text{s.t.} \quad F(q) \geq 1$$
Specialization economies (complexity diseconomies) take place if and only if the price index decreases (increases) with the variety $n$ of inputs at a symmetric outcome, i.e. when all prices are fixed at the same level: $p_i = p$
There is a continuum of $J$-firms sharing the same technology, which exhibits increasing returns to scale.

Firm $i$’s labor requirement for producing output $q_i$ is given by $f + cq_i$, where

- $f > 0$ is the fixed cost, and
- $c > 0$ is the marginal production cost.

Profit $\pi_i$ of firm $i$ is defined by

$$\pi_i \equiv (p_i - cw)q_i - f$$

Here $w$ stands for the wage rate.
First-order conditions

- The **first-order condition** for profit maximization is given by

\[
\Phi(q_i, q) + q_i \frac{\partial \Phi}{\partial q_i} = \frac{cw}{P(p)}
\]

- The left-hand side is positive homogenous of degree zero \(\Rightarrow\) the solution **cannot be unique**

- The “proper” equilibrium is pinned down by the labor balance condition:

\[
c \int_{0}^{n} q_i \, di + fn = L
\]
Second-order conditions

- To rule out multiple asymmetric equilibria, we impose a second-order condition:

\[(A) \quad \text{The left-hand side of the first-order condition is decreasing in } q_i \text{ for any } q\]

- Imposing (A) is equivalent to assuming that the operating profit of each firm is strictly concave in its output
- For Kimball-type production functions, (A) boils down to

\[-\frac{\phi'''(q/Y)}{\phi''(q/Y)} \frac{q}{Y} < 2 \quad \text{for all } q/Y > 0\]
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Equilibrium for a given number of firms
Equilibrium prices: are firms real price-makers?

- Each $\mathcal{F}$-firm’s profits are given by $(1 - P)Y$
- Free entry implies

$$P = 1, \quad p^*(n) = \frac{\phi(n)}{n}$$

- Hence, when the number of firms is given, market interactions in the $\mathcal{I}$-sector are fully irrelevant for equilibrium prices of inputs!
- Why? – because the $\mathcal{I}$-firms accurately anticipate the equilibrium value of the price index, which is driven to $P = 1$ by
  - perfect competition in sector $\mathcal{F}$, and
  - correctness of the intermediate firms’ expectations
- To sum up: when $n$ is given, things work as if $\mathcal{I}$-firms were price-takers, even though they are actually price makers
Under a given $n$, the equilibrium prices are

$$p^*(n) = \frac{\phi(n)}{n}$$

Hence, the inputs’ price at a symmetric equilibrium increases (decreases) with the number of firms $n$ in sector $\mathcal{I}$ when specialization economies (complexity diseconomies) take place.
The profit-maximizing markup

- The first order condition for profit maximization may be recast as

\[ \frac{p_i - cw}{p_i} = r(n), \]

- Here \( r(n) \) is defined by

\[ r(n) \equiv \eta(q_i, q)|_{q_j=q_i \forall j \in [0,n]}, \]

\[ \eta(q_i, q) \equiv -\frac{\partial \Phi}{\partial q_i} \frac{q_i}{\Phi(q_i, q)}. \]
The threefold nature of $r(n)$

- First, $r(n)$ the profit-maximizing markup, which may serve as an inverse measure of the degree of product market competition
- Second, $r(n)$ is also the marginal product elasticity
- Finally, $r(n)$ also reflects the degree of product differentiation:

$$r(n) = \frac{1}{\sigma(n)},$$

where $\sigma(n)$ is the elasticity of technological substitution across inputs
How $r(n)$ behaves under different technologies?

- **CES:**
  \[ r(n) = 1 - \rho \]

- **Translog production function:**
  \[ r(n) = 1 - \alpha n \]

- **Translog price index:**
  \[ r(n) = \frac{1}{1 + \beta n} \]

- **Kimball-type production function:**
  \[ r(n) = -\frac{\xi \phi''(\xi)}{\phi'(\xi)} \bigg|_{\xi = \phi^{-1}(1/n)} \]
Equilibrium

Equilibrium output of the final good:

\[ Y^*(n) = \frac{L}{c} [1 - r(n)] \frac{\varphi(n)}{n} \]

Equilibrium wage:

\[ w^*(n) = \frac{1}{c} [1 - r(n)] \frac{\varphi(n)}{n} \]

Thus, we have decomposed both \( Y^*(n) \) and \( w^*(n) \) into the product of

- the competition effect \( 1 - r(n) \), and
- the specialization/complexity effect \( \varphi(n)/n \)

The former increases with \( n \) if and only if \( r'(n) < 0 \), while the latter increases (decreases) if specialization economies/complexity diseconomies occur.
The impact of entry on prices, wages, and markups

We say that competition is

- **Price-decreasing** if $\frac{\partial p^*}{\partial n} < 0$ (and price-increasing otherwise)
- **Markup-decreasing** if $\frac{\partial [(p^* - cw^*)/p^*]}{\partial n} < 0$ (and markup-increasing otherwise)
- **Wage-decreasing** if $\frac{\partial w^*}{\partial n} < 0$ (and wage-decreasing otherwise)

**Proposition 2.** Competition is

- **Price-increasing** (price-decreasing) if and only if the $\mathcal{F}$-firms enjoy specialization economies (suffer from complexity diseconomies);
- **Markup-decreasing** (markup-increasing) if and only if $r'(n) < 0$ ($r'(n) > 0$); and
- **Wage-increasing** (wage-decreasing) if and only if the following inequality holds (does not hold):

$$\frac{\varphi'(n)n}{\varphi(n)} > 1 + \frac{r'(n)n}{1 - r(n)}$$
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The impact of entry: CES and the translogs

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External increasing returns to scale
Free entry equilibrium

- The number of $\mathcal{I}$-firms $n^*(L)$:

$$r(n) = \frac{f}{L}n$$

- The aggregate production function:

$$Y^*(L) = \frac{L}{c} \frac{\varphi[n^*(L)]}{n^*(L)} [1 - r(n^*(L))]$$

- The equilibrium wage:

$$w^* = \frac{1}{c} \frac{\varphi[n^*(L)]}{n^*(L)} [1 - r(n^*(L))]$$
Why competition?

- How (and why) does toughness of competition in the $\mathcal{I}$-sector shape the $\mathcal{F}$-sector's aggregate production function?
- To understand this, restate the aggregate production function as follows:

$$Y^*(L) = \frac{\varphi [n^*(L)]}{n^*(L)} Q[L, n^*(L)]$$

- Here $Q[L, n^*(L)]$ is the aggregate output of the $\mathcal{I}$-sector:

$$Q[L, n^*(L)] \equiv q^*(L)n^*(L) = \frac{L}{c} [1 - r(n^*(L))]$$

- Thus, competition among input-producing firms affects total output of the final good through the aggregate output of the intermediate good.
- More precisely, $Q[L, n^*(L)]$ is more (less) than proportional to $L$ when competition is markup-decreasing (-increasing).
Proposition 3. EIRS take place if and only if competition is wage-increasing.

Specialization economies: \[ \frac{\varphi'(n)n}{\varphi(n)} > 1 \]

EIRS: \[ \frac{\varphi'(n)n}{\varphi(n)} > 1 + \frac{r'(n)n}{1 - r(n)} \]

NB: These two conditions coincide only in the CES case!!!
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Proposition 4. *Compared to the CES case, markup-decreasing competition damps the specialization effect, but simultaneously triggers a positive competition effect. Under markup-increasing competition, the situation is reversed.*

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How do the specialization and competition effects interact?

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How does all this work? Examples

- CES:

\[ Y^*(L) = AL^{1/(1-\rho)}, \quad A \equiv \frac{\rho}{c} \left( \frac{1-\rho}{f} \right)^{\rho/(1-\rho)} \]

- Translog price index:

\[ Y^*(L) = \frac{f}{4\beta c} \left( \sqrt{1+4L/f} - 1 \right)^2 \]

- Translog production function:

\[ Y^*(L) = \frac{\alpha}{c} L \]
Dessert: a micro-foundation for an S-shaped production function

- Consider again the Kimball-type production function with
  \[ \phi(q/Y) = a(q/Y)\rho - b \]
- The resulting aggregate production function \( Y^*(L) \) reads as

\[
Y^*(L) = \frac{f}{1-\rho} \left( \frac{L}{L + af/(1-\rho)} \right)^{1/\rho}
\]

- Increasing returns to scale arise when \( L \) is sufficiently small; otherwise, decreasing returns to scale occur.
- This provides a simple micro-foundation for an S-shaped aggregate production function (Shapley and Shubik, 1967) used in the analysis of poverty traps (Skiba, 1978; Azariadis and Stachurski, 2005; Banerjee and Duflo, 2005)
What have we learned?

- We have disentangled the interplay between two effects:
  - the specialization/complexity effect, arising from the final output sector, and
  - the competition effect, stemming from the market interactions among firms within the intermediate input sector.

- The latter effect is driven by the presence of a variable elasticity of technological substitution and vanishes in the CES case.
Potential future work

- Accounting for heterogeneity of firms in productivity, and studying the role of selection among firms in generating sharing externalities
- More realistic frameworks in which more than two productive sectors are simultaneously active in the economy
- Considering market structures different from monopolistic competition in the input-producing sector
Thank you for your attention!