Duopolistic Investment Opportunities under Agency Risk: Designing Contracts for Optimal Investment Decisions

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Abstract

Real options theory frequently assumes that investment decisions are directly taken by the owners or that, if there is a manager, he is fully aligned with them. However, empirical literature demonstrates that managers may reveal misaligned interests, which may deteriorate the value of the firm, consequently having a major impact on value maximizing decisions, namely, on the optimal timing to invest. This paper provides a contribute to the existing real options by dropping this assumption in the non-exclusive option case, specifically providing a duopolistic leader-follower framework where an agency problem between the owners of the option and their respective managers, is embodied and solved by an optimal labor contract scheme that aims to eliminate inadequate actions from the managers. In this model, both firms shareholders need not to follow the future evolution of project value drivers in order to guarantee optimal behavior. An analytical application is provided, being shown that even small deviations from the optimal compensation scheme may lead to highly sub-optimal decisions.

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1 Introduction

Real options theory emerged in capital budgeting field as an answer to the limitations of classic approaches, such as the Net Present Value, embodying growth opportunities until then ignored. Firstly alluded by Myers (1977) which considered growth opportunities on corporate assets as a necessary component of the firms market value, it was founded on the original work of Black and Scholes (1973), but rapidly evolved and was expanded to diverse areas. Nonetheless, differently from financial options, real-world investment decisions concerns non-exclusive opportunities, usually taken in a competitive environment (Zingales (2000)). Consequently, the necessary interaction of real options theory, industrial organization and game theory emerged, encompassing the problematic of multiplayer investment opportunities.

A diverse and complex set of models has flown, presenting different characteristics, approaches and applications to general cases and also specific industries, such as research and development (Reiss (1998), Garlappi (2004), Weeds (2002)), real estate development (Grenadier (1996)), financial sector Baba (2001), agriculture (Odening et al. (2007)), airplane production sector (Shackleton et al. (2004)) and much others, showing that the applicability of real options game theory is far from being exhausted. We highlight Azevedo and Paxson (2011) which provide a complete review and supply a complete set of characteristics to categorize the type of Real Options Game models.

The founding work on real options game theory was developed by Smets (1993). Although it was originally focused on international finance, presenting a framework where firms can expand their profit flows through investment, it was further generalized and simplified on Dixit and Pindyck (1994), where a duopolistic real options game is presented in a new market context. Preemptive actions, in consequence of symmetric strategies between homogeneous players, leads to an earlier entry trigger for the leading firm and a later trigger for the follower, comparatively to the exclusive real option scenario.

Several developments came after, where we highlight Thijssen et al. (2002), which proposes the use of symmetric mixed strategies in a stochastic environment in order to deal with the problem of investment coordination failure[2]. This solution avoids the necessity of previous assumptions regarding the investment decisions and firms roles. Huisman et al. (2003) provides some developments, namely the case of asymmetric firms (lower investment costs for one of the players), the impact of different possible technologies and the effect that new information have on the reduction of overall uncertainty. Nonetheless,

[2] Although this is an important step stone, for simplicity sake we rely on Dixit and Pindyck (1994) results and assumptions in this paper.

this field is not exclusive of duopolistic interaction as seen, for example, on Bouis et al. (2009) which provides a multiple firms framework, generalizing to an n-dimensional market a three-firm oligopolistic model. Asymmetrical market shares and variable market size hypotheses were further developed by Pereira and Rodrigues (2010). Nonetheless, in all these papers it is ignored that investment decisions are often taken by managers that may not be fully aligned with owners' value maximization target.

Value maximization goal has been considered, for long, the prevailing long-term goal of firms. As Jensen (2001) explains, this means that all management decisions should be taken with a purpose of increasing the value of all financial claims on a firm. Although, this has been a source of debate, it has been accepted in the last 200 years as the more suitable solution. Nonetheless, the attribution of decision taking role to an agent, without the appropriate and effective mechanisms of incentives and control, may lead to suboptimal decisions and value misappropriation. Some circumstances of necessary shift of the decision role are, for example, high ownership dispersion (Berle and Means (1932)) or owners lack of expertise (Shleifer and Vishny (1997)). Although we can trace agency issues to Smith (1776)[3], the formalization of agency theory and agency issues is acknowledged to Jensen and Meckling (1976). Agency risk arises when in a contractual relation between utility maximizing parties, the principal delegates decision authority in a context of asymmetrical information, which may lead to agency costs if there is interest misaligning between the parties.

Financial literature proposes internal and external mechanisms in order to mitigate agency risk and thus reduce agency costs. The first branch contains incentive contracts (Jensen and Smith Jr (1985)), insider ownership (Jensen and Meckling (1976)), existence of large investors (Shleifer and Vishny (1986)), board of administration (Fama and Jensen (1983)), the existence of debt and dividend policy, which reduces free cash-flows (Jensen (1986)). External mechanisms encompasses, for instance, managers reputation (Shleifer and Vishny (1997)), managers market competition (Fama (1980)), output market competition (Hart (1983)), takeovers market (Easterbrook and Fischel (1981)), monitoring by investment professionals (Chung and Jo (1996)) or legal framework (Shleifer and Vishny (1997)). Despite its relevance, these mechanisms have limitations, which several works have shown. We highlight Holmstrom (1982), Shleifer and Vishny (1997), Grossman and Hart (1980), Burkart (1995), Jensen (1993).

Investment timing decisions, when studied under a real options approach, usually tend to assume perfect aligning of interests between managers and shareholders or full control

[3]"The directors of companies, being managers of others people money, cannot be expected to watch over it with the same vigilance with which they watch over their own."

of investment decisions by the owners, ignoring though the impact of agency conflicts. This has been, for long time, a usual assumption which we can track to Myers (1977) who explicitly assumes that firms managers act in the shareholders interests.

Considering a firm as a nexus of relations bonded by a contractual structure (Jensen and Meckling (1976)) and not conditioned by inside inefficiencies[4] and outside inefficiencies[5], it is theoretically possible to design an optimal contract so that the risk bearing and risk premium are shared between the principal and the agent, eliminating value maximization deviations and misappropriations.

The context in which we present our work differs from closed related literature, since we analyze the agency problem under a duopolistic investment timing context. Consequently, we do not find similar works on literature that approach the agency relation in non-exclusive investment opportunities. Nonetheless, we observe some rapid developments on real options agency theory in the past few years, concerning the problematic of agency relations in investment timing decisions contexts.

A relevant contribution to the present work is found on Cardoso and Pereira (2011), which embodied the agency issues in an exclusive investment opportunity model. The stablished contractual relation between the owners and the manager of the option may lead to value misappropriation under a sub-optimal contractual relation, although it may be avoided ex-ante through an optimal contract structure, conducing to the non-agency optimal investment timing decision.

We highlight the major advances of Grenadier and Wang (2005), which examines investment timing decision for a single project, where the owner delegates the investment decision to the manager. Managerial behavior accounts for asymmetric information and moral hazard, generating sub-optimal decisions that can be corrected through an optimal contract, aligning the incentives of owners and managers. Nishihara and Shibata (2008) extends this model incorporating a relationship between an audit mechanism and bonus-incentives sensitive to managers deviated actions. Shibata and Nishihara (2010) extends these papers by incorporating debt financing on investment expenditure.

Moreover, Hori and Osano (2010) presents an agency model under a real options framework where managerial compensation is designed endogenously including a contingent claim on firms cash-flows using stock options. Kanagaretnam and Sarkar (2011), in order to mitigate the underinvestment problem of Mauer and Ott (2000), considers an agency compensation with a fixed component and a equity share aligning the interests of manager with both bondholders and shareholders.

[4]Self-dealing contract negotiation as noted by Shleifer and Vishny (1997).

[5]Legal distortions, for example.

We aim to relax the assumption that managers are perfectly aligned with owners (as assumed in the standard real options literature, e.g. Dixit and Pindyck (1994)) and frequently ignored in non-exclusive investment opportunities. Moreover, we provide a perceptive but yet meaningful framework where two firms share an option to enter in a new market, but for plausible reasons (i.e., incapacity, opportunity cost or control difficulty) both need to hire a manager entity to supervise the option, to follow market conditions, to perceive the rivals preemptive behavior and, consequently, to take the investment decision. Although, since managers are utility maximizers, some interests misaligning may lead to suboptimal investment timing decision, which demands ex-ante optimal contract design to mitigate the risk and eliminate eventual costs.

The rest of the paper is organized as follows. Section 2 presents the competition framework under a necessary agency relationship established in both firms, where subsection 2.1 concerns the results for the follower firm and subsection 2.2 concerns the solution for the leader firm. Section 3 presents the impact of agency contract definition on the resulting equilibria of both firms, where subsection 3.1 sets the equilibrium solution that aligns the managers interests with those of both firms shareholders, and section 3.2 analyzes the ex-ante incentives of principal and agents regarding the consequences of contract design. Section 4 presents a comparative statics analysis and a numerical example.

2 Competition Framework

In this section, we will present the problem of agency interaction between shareholders and a management entity considering that the opportunity to invest is simultaneously shared with another symmetric firm. We consider that shareholders lack the ability, knowledge or time to take the appropriate optimal choices in search of equity value maximization. So, similarly to our monopoly agency solution, each firm will need to outsource their optimal investment decisions, delegating power to an agent. Firms owners will negotiate with each respective manager and design a contract that pays a management fee w_i while the project is idle and, after exercising the option to invest, a mix of timely continuous fixed wage w_a plus a value-sharing bonus fee ϕ. As we will see, and similarly to the monopolistic case, both firms' agency contracts are arranged initially, without further intervention of the shareholders on the investment decisions. Furthermore, if the proper contract is accorded, shareholders guarantee managers optimal behavior, and interests alignment.

Both firms are risk-neutral and fully aware of the rivals behavior. They can produce a unit of output after entering the market, which is totally absorbed by consumers due to their infinitely elastic demand nature. The price of each of these units behaves stochastically over time, according to equation (1), which follows the Dixit and Pindyck (1994) Chapter 9 notation:

$$P_t = Y_t D(Q_t) \tag{1}$$

and

$$dY_t = \alpha Y_t dt + \sigma Y_t dz \tag{2}$$

The market value is represented by Y_t, consisting on an exogenous shock following a geometric Brownian motion according to equation (2). D_t is the inverse demand function - non-stochastic decreasing function - that depends on the market supply output (Q_t) which has three states, the first relating the absence of players (Q_t=0), the second concerning the monopoly state (Q_t=1) and the last one representing the duopoly state (Q_t=2). For simplification we will further consider Y_t as Y. In relation to equation (2), dz is the increment of the Wiener process, α is the instantaneous conditional expected relative change in V, also known as drift. $\alpha = r - \delta$ ($r > \delta$), where r is the risk-free rate and δ (> 0) represents the opportunity cost from deferring, and σ is the instantaneous conditional standard deviation.

We will further present a leader-follower framework, extending the duopolistic model of Dixit and Pindyck (1994), which provides a simplifying but meaningful adaption of Smets (1993). Initially, only one of the firms invests a fixed cost K_s, earning a temporarily monopoly pay-off, and after the second firm achieves its optimal exercising moment, also investing K_s, both firms will equally share the market (the firms are assumed to be symmetric ex-post). Nonetheless, both Dixit and Pindyck (1994) and Smets (1993) consider that investment timing decisions in duopoly context are free of agency issues, which we will further embody.

In order to provide the adequate competitive incentives in a symmetrical strategies environment, we need to ensure that, while managing the investment option, both agents are in equal positions and have the same incentives. This leads to the following assumptions:

Assumption 1: *Owners can't administer directly the option to invest. Also they are unable of properly observe some key value drivers (namely, Y, σ, α), so the option becomes useless without a manager.*

Assumption 2: *We assume the presence of ex-ante perfect information between firms which, additionally to symmetrical characteristic of firms, implies equal composition of fixed wage w_a and value-sharing factor ϕ.*

Assumption 3: *Managers do not have time restrictions. We assume managers as an abstract entity so that there are no lifetime restrictions.*

Assumption 4: *Managers and options to invest aren't scarce, so that owners can always find a manager for running his projects, and managers can always find another investment opportunity needing to be managed.*

Assumption 5: *The parameter w_i represents the managers market price for running an idle project (meaning that the owners can't find a less expensive manager). Also, we assume that the fixed wage to manage the active project, w_a, is lower than w_i, so that manager's utility function integrates awaiting value. Note, however, that the lower fixed salary will be compensated by an appropriate value-sharing bonus.*

Assumption 6: *Manager can only broke contract before the option exercise and this only happens if the option becomes worthless. In this case manager will earn a fixed compensation $\frac{w_i}{r}$.*

Assumption 7: *For the owners, value-sharing bonus are less expensive than monitoring costs.*

Assumption 8: *Managers are not wealth enough neither can gather the necessary resources (leverage, for instance) to buy the project.*

Assumption 9: *Both firms and managers are risk-neutral.*

Assumption 10: *Firms are assumed to be all-equity.*

2.1 Follower Firm

Solving this problem backwardly, therefore, assuming that the leader already invested, we will find firstly the optimal decision for the follower firm, specifically for both the follower firm shareholders and the follower manager. During the period which is not yet optimal to invest, the follower shareholders' value function $S^F(Y)$ must satisfy the following ordinary differential equation:

$$\frac{1}{2}\sigma^2 Y^2 S^{F''}(Y) + (r-\delta)YS^{F'}(Y) - rS^F(Y) - w_i = 0 \tag{3}$$

solved by a second-order Cauchy-Euler equation that, after embodying the absorption barrier stated on equation (7), results on the following general equation:

$$S^F(Y) = BY^{\beta_1} - \frac{w_i}{r} \tag{4}$$

subject to the boundary conditions:

$$
\begin{aligned}
S^F(Y_s^F) &= (1-\phi)\frac{Y_s^F D(2)}{r-\alpha} - \frac{w_a}{r} - K_s &(5)\\
S^{F'}(Y_s^F) &= (1-\phi)\frac{D(2)}{r-\alpha} &(6)\\
S^F(0) &= -\frac{w_i}{r} &(7)
\end{aligned}
$$

where equation (5) and equation (6) concerns, respectively, the value matching condition and the smooth pasting condition at the optimal investment trigger Y_s^F.

Therefore, accounting conditions (5) and (6), we have the value function for the shareholders of the follower firm $S^F(Y)$:

$$
S^F(Y) = \begin{cases} \left(\frac{Y}{Y_s^F}\right)^{\beta_1} \frac{1}{\beta_1 - 1}\left(K_s - \frac{w_i - w_a}{r}\right) - \frac{w_i}{r} & \text{for } Y < Y_s^F \\ (1-\phi)\frac{D(2)Y}{r-\alpha} - K_s - \frac{w_a}{r} & \text{for } Y \geq Y_s^F \end{cases} \tag{8}
$$

being the optimal investment trigger Y_s^F:

$$
Y_s^F = \frac{\beta_1}{\beta_1 - 1}\frac{1}{1-\phi}\frac{\delta}{D(2)}\left(K_s - \frac{w_i - w_a}{r}\right) \tag{9}
$$

where β_1 is the positive root of the fundamental quadratic equation:

$$
\frac{1}{2}\sigma^2\beta_1(\beta_1 - 1) + (r-\delta)\beta_1 - r = 0 \tag{10}
$$

so that

$$
\beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \tag{11}
$$

Similarly to the proprietary option scenario presented earlier, the effective optimal investment moment will be choose by the manager. Therefore, we need to estimate his value function $M^F(Y)$ and preferred investment trigger Y_m^F, solving the following o.d.e.:

$$
\frac{1}{2}\sigma^2 Y^2 M^{F''}(Y) + (r-\delta)YM^{F'}(Y) - rM^F(Y) + w_i = 0 \tag{12}
$$

The general solution of equation (12) aggregates an homogenous component and a particular solution that, satisfying the absorption barrier stated in equation (16), will take the form:

$$M^F(Y) = CY^{\beta_1} + \frac{w_i}{r} \tag{13}$$

conditioned by the restrictions:

$$M^F(Y_m^F) = \phi\frac{Y_m^F D(2)}{r-\alpha} + \frac{w_a}{r} \tag{14}$$

$$M^{F'}(Y_m^F) = \phi\frac{D(2)}{r-\alpha} \tag{15}$$

$$M^F(0) = \frac{w_i}{r} \tag{16}$$

The first two conditions are, respectively, the value matching and the smooth pasting conditions, which aim to ensure optimal decisions. As noted, the latter restriction is an absorption barrier related to Y's stochastic characteristic.

After some arithmetics concerning the remaining boundary conditions (equation (14) and (15)), we obtain the specific value function for the manager entity of the follower company ($M^F(Y)$) and its optimal investment trigger (Y_m^F):

$$M^F(Y) = \begin{cases} \left(\frac{Y}{Y_m^F}\right)^{\beta_1} \frac{1}{\beta_1 - 1}\left(\frac{w_i - w_a}{r}\right) + \frac{w_i}{r} & \text{for } Y < Y_m^F \\ \phi\frac{D(2)Y}{\delta} + \frac{w_a}{r} & \text{for } Y \geq Y_m^F \end{cases} \tag{17}$$

$$Y_m^F = \frac{\beta_1}{\beta_1 - 1}\frac{1}{\phi}\frac{\delta}{D(2)}\frac{w_i - w_a}{r} \tag{18}$$

2.2 Leader Firm

Using backward calculation, we will now derive the payoff for the leader firm, assuming that the follower firm will behave optimally. After entering the market, paying the sunk cost K_s, the manager of the leader firm has no further decisions to make. Nonetheless, the payoff of both the principal and agent of the leading company will be affected by their rivals' subsequent actions.

We will firstly solve the leader firm shareholders' problem. Therefore, we must solve the following o.d.e. to achieve their value function $S^L(Y)$:

$$\frac{1}{2}\sigma^2 Y^2 S^{L''}(Y) + (r-\delta)YS^{L'}(Y) - rS^L(Y) + (1-\phi)D(1)Y - w_a = 0 \tag{19}$$

The last two terms, which is the non-homogeneous component, represents the cashflow that shareholders earn during the monopolistic period. The general solution of this o.d.e.

is given by the following equation:

$$S^L(Y) = GY^{\beta_1} + (1-\phi)\frac{YD(2)}{r-\alpha} - \frac{w_a}{r} \tag{20}$$

The unknown constant G is calculated using only a boundary restriction, which is the value matching condition at the optimal trigger of the follower shareholders, since the firms are symmetric ex-post, both value functions must meet at $Y_s^L = Y_s^F$. So, at Y_s^F, the leader shareholders value during their monopoly state must equal the simultaneous investment value. This boundary condition is represented on equation (21), where β_1 is as previously stated on equation (11):

$$S^L(Y_s^F) = (1-\phi)\frac{Y_s^F D(2)}{r-\alpha} - \frac{w_a}{r} \tag{21}$$

This yields the following value function for the leader firm shareholders:

$$S^L(Y) = \begin{cases} (1-\phi)\dfrac{YD(1)}{r-\alpha} + \left(\dfrac{Y}{Y_s^F}\right)^{\beta_1} \dfrac{\beta_1}{\beta_1 - 1}\left(1 - \dfrac{D(1)}{D(2)}\right)\left(K_s - \dfrac{w_i - w_a}{r}\right) - \dfrac{w_a}{r} & \text{for } Y < Y_s^F \\ (1-\phi)\dfrac{YD(2)}{r-\alpha} - \dfrac{w_a}{r} & \text{for } Y \geq Y_s^F \end{cases} \tag{22}$$

The optimal trigger $Y_s^L < Y_s^F$ for the leader shareholder must be such that it will be indifferent for both firms' shareholders to become a leader or a follower. This happens when the value for the leader shareholder excluding the sunk cost K_s equals the value for the follower shareholders. Therefore, through this rent equalization principle we obtain equation (64) on Appendix, allowing the derivation of Y_s^L. This equation allow us to prove the existence of a root, other than and strictly below the root Y_s^F:

Lemma 2.1. *There exists a unique point* $Y_s^L \in \left(0, Y_s^F\right)$ *such that:*

$$S^L(Y_s^L) - K_s = S^F(Y_s^L) \tag{23}$$

$$S^L(Y) - K_s < S^F(Y), \ \textit{for } Y < Y_s^L \tag{24}$$

$$S^L(Y) - K_s \geq S^F(Y), \ \textit{for } Y > Y_s^L \tag{25}$$

Proof 2.1: See appendix.

This results lead us to the following expression concerning the stopping time of the leader shareholders:

$$T_s^L = inf\{t \geq 0 : Y \in \left[Y_s^L, Y_s^F\right]\} \tag{26}$$

The shareholders framework is important in order to identify the firm's optimal decisions to implement. Nevertheless, the prevailing investment decisions are taken by the manager, and so we will now present the leader manager framework, using the same backwards calculation presented earlier in this section. In order to achieve that, we must solve the following o.d.e:

$$\frac{1}{2}\sigma^2 Y^2 M^{L''}(Y) + (r-\delta)YM^{L'}(Y) - rM^L(Y) + \phi D(1)Y + w_a = 0 \tag{27}$$

Symmetrically to the shareholders, the last two terms concerns the cash-flow earned by the manager after entering the market but before the rival's entry. The general solution of the o.d.e is given by the following equation:

$$M^L(Y) = HY^{\beta_1} + \phi\frac{YD(1)}{r-\alpha} + \frac{w_a}{r} \tag{28}$$

In order to find the solution we will use the following value matching condition at the follower manager's entry point (Y_m^F):

$$M^L(Y_m^F) = \phi\frac{Y_m^F D(2)}{r-\alpha} + \frac{w_a}{r} \tag{29}$$

This results in the following value function for the leader company manager:

$$M^L(Y) = \begin{cases} \phi\frac{YD(1)}{r-\alpha} + \left(\frac{Y}{Y_m^F}\right)^{\beta_1}\frac{\beta_1}{\beta_1-1}\left(1-\frac{D(1)}{D(2)}\right)\left(\frac{w_i-w_a}{r}\right) + \frac{w_a}{r} & \text{for } Y < Y_m^F \\ \phi\frac{YD(2)}{r-\alpha} + \frac{w_a}{r} & \text{for } Y \geq Y_m^F \end{cases} \tag{30}$$

Manager of the leading company will choose its optimal trigger $Y_m^L < Y_m^F$, at the moment that it is indifferent for both managers to become leader or follower. The expression that yields Y_m^L is represented in equation (70), from the Appendix. We must define and prove that this point exists and it is unique. Therefore:

Lemma 2.2. *There exists a unique point $Y_m^L \in \left(0, Y_m^F\right)$ such that:*

$$M^L(Y_m^L) - K_s = S^F(Y_s^L) \tag{31}$$

$$M^L(Y) < M^F(Y), \ for \ Y < Y_m^L \tag{32}$$

$$M^L(Y) \geq M^F(Y), \ for \ Y > Y_m^L \tag{33}$$

Proof 2.2: See appendix.

The stopping time of the leader manager will be:

$$T_m^L = inf\{t \geq 0 : Y \in \left[Y_m^L, Y_m^F\right]\} \tag{34}$$

3 Contract Design and Agency Issues

This section explains the relationship between agency problems and contract design, presenting the impact of agency contract definition on the equilibria of both of the follower and leader equity holders and their respective managers. As mentioned earlier, this compensation scheme is defined by combining the three components - the exogenous idle project's fixed wage (w_i), the active project's fixed wage (w_a) and value-sharing component (ϕ) - which, if adequate, can assure aligned managerial behavior. This section will also present the impact of the contractual framework in agency costs mitigation.

3.1 Optimal Contract Solution

In order to align shareholders and manager's interests at the follower firm, an optimal mix of w_a and ϕ must be provided. This ensures that the agent chooses Y_s^j ($j \in \{L, F\}$ where L stands for the Leader and F for the Follower) as its respective optimal investment trigger. Equaling both players' optimal triggers:

$$Y_s^F = Y_m^F \tag{35}$$

Shareholders and managers of the follower firm get the following optimal combination:

$$\phi^* = \frac{K_m}{K_s} \tag{36}$$

where:

$$K_m = \frac{w_i - w_a}{r} \tag{37}$$

Similarly to the follower firm optimal contract, the leader shareholders also ensure optimal behavior of its manager, through an equal compensation package, mixing the fixed wage w_a and the value-sharing component ϕ concerning the relative opportunity cost between the leader firm's players, therefore guaranteeing that manager takes the optimal decision, investing at Y_s^L.

Therefore, considering $\Omega(Y,\phi)$ as the aggregate value function of the leader shareholders ($S^L(Y)$ as stated on Equation (22)) and the leader manager ($M^L(Y)$ as stated on Equation (30)), restricted by the leader managers prevailing entry triggers Y_m^L and Y_m^F, both dependent variables of ϕ:

$$\Omega(Y,\phi) = \begin{cases} \frac{YD(1)}{r-\alpha} + \frac{\beta_1}{\beta_1 - 1}\left(1 - \frac{D(1)}{D(2)}\right)\left[\left(\frac{Y}{Y_s^F(\phi)}\right)^{\beta_1} K_s - \left(\left(\frac{Y}{Y_s^F(\phi)}\right)^{\beta_1} - \left(\frac{Y}{Y_m^F(\phi)}\right)^{\beta_1}\right)\frac{w_i - w_a}{r}\right] & \text{for } Y < Y_m^F \\ \frac{YD(2)}{r-\alpha} & \text{for } Y \geq Y_m^F \end{cases} \tag{38}$$

Consequently, we have:

Lemma 3.1. *The optimal contract mix $\phi^*(w_a, w_i, Ks)$, presented on equation (36) guarantees that manager chooses the leader shareholders' optimal trigger:*

$$Y_s^L|_{\phi=\phi^*} = Y_m^L|_{\phi=\phi^*} \tag{39}$$

Implied from:

$$\arg\max_{\phi\in(0,1)} \Omega(Y,\phi) = \frac{K_m}{K_s} \tag{40}$$

Proof 3.1: See appendix.

These results show, first of all, that shareholders of both the leader and follower firm can determine an optimal combination of fixed wage and a value-sharing component ensuring that managers optimally choose their entry triggers, avoiding value misappropriation. Secondly, this ex-ante contract definition transfer to managers the preemption behavior, ensuring that they will dispute the role of market leader. Note that interests are maintained aligned even if a firm is preempted by the rival, ensuring that the follower manager will wait until the followers' optimal trigger is achieved. Thirdly, by defining this optimal contract, shareholders do not need to know the behavior of some variables, such as Y, σ or α, being free to pursuit other activities.

3.2 Ex-ante Sub-optimal Contract Definition and Agency Risk

In this subsection we analyze the ex-ante incentives of managers concerning the impact of a sub-optimal contract design on firms value losses in consequence of agency issues. In order to do so, we compare the aggregated value of manager and shareholders with the agency-free model of Dixit and Pindyck (1994).

Using the Pareto criterion, the multiplicity of pre-game contract mixes will be reduced to one optimal combination $\phi^*(w_a, w_i, Ks)$, since its resulting equilibria guarantees that the best possible result is undoubtably achieved. In most of the circumstances (see Lemma. (3.2)) this labor contract structure Pareto-dominates all other combinations of the aggregate value of shareholders and manager of the leader firm, implying that no deviation will be profitable. In some specific circumstances, this contract is not the only possible contract combination that achieves the non-agency value result. Although, we may observe, in some sub-optimal cases, the absence of value deterioration comparing with the non-agency solution, these situations are not optimal combinations since the resulting triggers are not aligned, promoting value misappropriation between players.

Ex-ante, both firms will try to achieve the leader role so, the expected value of successful preemptive strategy will overcome at $Y \in \left[Y_s^L, Y_m^F\right)$ the follower strategy, which becomes a non credible strategy. Consequently, ex-ante, each firm's principal and agent will embody the leader's value function as the expected value when choosing the value-sharing factor ϕ which will be the same for both firms, due to their symmetric nature and perfect information.

Comparing the non-agency leader value function $L(Y)$ of Dixit and Pindyck (1994) with the aggregate value function $\Omega(Y, \phi)$ (stated on equation (38)), we want to study the incentives of both firms shareholders and managers when defining the labor contract under the expectation of gaining the leader role. Therefore:

Lemma 3.2. *There exists a consistent tangible point at $\phi^*(w_a, w_i, Ks)$, such that:*

$$\Omega(Y, \phi^*) = L(Y), \; for \; \phi = \phi^* \tag{41}$$

Other contract combinations may result in value loss, such that:

$$\Omega(Y, \phi) < L(Y), \; for \; \phi \neq \phi^* \tag{42}$$

Proof 3.2: See appendix.

Outside the optimal contract mix ($\phi \neq \phi^*$) there are value misappropriation between firms' shareholders and respective managers in consequence of triggers misaligning, although, in some specific circumstances, we may not see aggregated value destruction compared with the non-agency. These results implies that the optimal labor contract is the only one that fully mitigates all agency costs guaranteeing that the aggregate value of shareholders and managers always equals the non-agency result in the leader firm, which corresponds to the expected role for both firms.

If, for some unexpected reason, a sub-optimal contract combination is chosen, then both firms' shareholders and managers face ex-ante agency risk resulting from the pre-emptive behavior distortion, and the consequent potential value misappropriation and value destruction that may result in sub-optimal contexts. This expectation distortion, produces an ex-ante incentive for both firms to chose this optimal contract design which always provides the best possible solution for the shareholders and manager, specifically, the Pareto optimal equilibrium. We prove that ϕ^* provides the first best solution for the aggregate leader firm - ex-ante expected result - corresponding to a Pareto equilibrium and also that this solution fully eliminates all agency costs.

4 Comparative Statics and Analytical Application

In this section we will study some analytical relations among the fundamental variables. The first subsection concerns the study of the sensitiveness of those variables, in both leader and follower firms. The second subsection will be responsible for the presentation of an empirical application, accompanied with a visual presentation and a respective characterization. Whenever necessary, the data presented on Table (1) will be used to illustrate the descriptions.

Parameter	Value	Description
$D(1)$	100	Monopolistic inverse demand function
$D(2)$	50	Duopolistic inverse demand function
K_s	\$1,000	Investment cost
r	0.05	Risk-free interest rate
σ	0.20	Instantaneous volatility
δ	0.03	Dividend-yield
w_i	\$5	Fixed wage for managing the idle project
w_a	\$4	Fixed wage for managing the active project
ϕ	−	Value-sharing rate

Table 1: The base case parameters.

4.1 Comparative Statics

Recalling equation (36), the higher the fixed idle wage w_i, the higher the necessary ϕ^*, representing a higher opportunity cost for the manager weighting on the investment timing decision. The inverse relation is valid for the remaining variables (w_a, r and K_s), meaning that the higher w_a and r, the lower the opportunity cost of the investment decision. Similarly, a superior K_s promotes a higher value that shareholders must abdicate in order to obtain their share on the firm's value.

Concerning the follower firm, we observe similar properties to the monopolistic case, namely the effect on optimal trigger for the follower shareholder and manager. Analyzing the leader firm, in which we study each of the optimal trigger through an iterative process based on Table 1, we find a similar pattern to the followers result. Therefore:

$$\frac{\partial Y_s^F(\phi)}{\partial \phi} > 0 \tag{43}$$

$$\frac{\partial Y_m^F(\phi)}{\partial \phi} < 0 \tag{44}$$

$$\frac{\partial Y_s^L(\phi)}{\partial \phi} > 0 \tag{45}$$

$$\frac{\partial Y_m^L(\phi)}{\partial \phi} < 0 \tag{46}$$

In the shareholder's case, a higher (lower) value-sharing factor generates a higher (lower) shareholders optimal trigger, representing an increase on the value of the option to defer the project implementation, while the opposite stands for the manager. Since the manager chooses the real project implementation trigger then, any deviation from the optimal value-sharing factor will generate interests misaligning and agency issues.

Concretely, whenever ϕ changes, the $Z(Y)$ function represented on equation (64), will present a positively correlated change, affecting the optimal triggers Y_s^L and Y_s^F (Figure (1)). The opposite is valid for the N(Y) function (stated on equation (70)) which presents an opposite relation with ϕ (Figure (2)), resulting in inverse movements of Y_m^L and Y_m^F.

Considering Y^{L*} and Y^{F*} as the leader firm and follower firm optimal investment triggers in the non-agency model and considering a fixed w_i and w_a ($w_i > w_a$), we observe that:

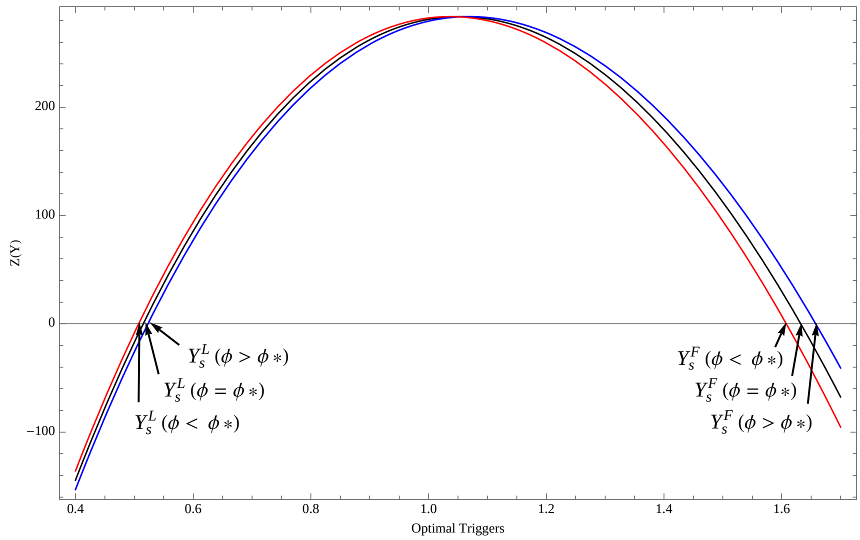


Figure 1: The triggers Y_s^L and Y_s^F for different levels of ϕ. Optimal ϕ^* and $Z(Y)$ calculated according to Table 1.

$$Y_s^F \to \frac{\beta}{\beta-1}\frac{\delta}{D(2)}\left(K_s + \frac{w_a - w_i}{r}\right) < Y^{F*} \quad \text{as} \quad \phi \to 0 \tag{47}$$

$$Y_m^F \to +\infty \quad \text{as} \quad \phi \to 0 \tag{48}$$

$$Y_s^F \to +\infty \quad \text{as} \quad \phi \to 1 \tag{49}$$

$$Y_m^F \to \frac{\beta}{\beta-1}\frac{\delta}{D(2)}\frac{w_i - w_a}{r} \quad \text{as} \quad \phi \to 1 \tag{50}$$

$$\tag{51}$$

$$Y_s^L < Y^{L*} \quad \text{as} \quad \phi \to 0 \tag{52}$$

$$Y_m^L \to +\infty \quad \text{as} \quad \phi \to 0 \tag{53}$$

$$Y_s^L \to +\infty \quad \text{as} \quad \phi \to 1 \tag{54}$$

$$Y_m^L < Y^{L*} \quad \text{as} \quad \phi \to 1 \tag{55}$$

$$\tag{56}$$

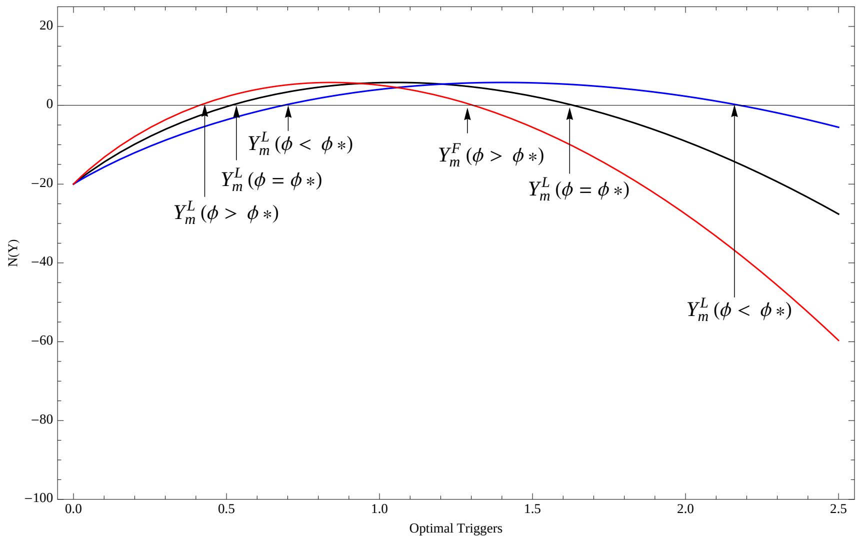


Figure 2: The triggers Y_m^L and Y_m^F for different levels of ϕ. Optimal ϕ^* and $N(Y)$ calculated according to Table 1.

and by fixing ϕ, and remember that $\phi \in (0, 1)$, we see that:

$$Y_s^F \to \frac{\beta}{\beta - 1} \frac{\delta}{D(2)} \frac{1}{1-\phi} K_s > Y^{F*} \quad \text{as} \quad w_a \to w_i \tag{57}$$

$$Y_m^F \to 0 \quad \text{as} \quad w_a \to w_i \tag{58}$$

$$Y_s^L > Y^{L*} \quad \text{as} \quad w_a \to w_i \tag{59}$$

$$Y_m^F \to 0 \quad \text{as} \quad w_a \to w_i \tag{60}$$

These results (figures (1) and (2)) mean that in a suboptimal contract $\phi \neq \phi^*$, both firm's managers will present sub-optimal behavior, choosing triggers that do not fulfill the expectations of the shareholders. This leads to sooner investments, if a higher-than-optimal share of the value is given or later investments if the value-sharing factor is not enough.

The results of equations (47) and (52) show that, in the absence of a value-sharing compensation, the entry moment of both firms' shareholders will be lower than the non-agency model but, since the ex-post investment fixed wage does not surpass the idle project fixed wage ($w_a < w_i$), the managers will choose not to invest, maintaining a higher fixed salary (equations (48) and (53)). In opposition, as represented in equations (49) and (54), $\phi = 1$ translates the hypothetical scenario where shareholders give the total project value to their managers. In this case, shareholders will incur in a cost K_s but, since they do not get any cash-flows, they prefer not to invest. Nevertheless, in this case, the firms' real entry points (Y_m^L and Y_m^F) will only depend on the managers investment cost, $\frac{w_i - w_a}{r}$, tending to its minimal values.

Equations (57) and (59) shows that if w_a tend to w_i, shareholders lose wage savings, so that the option to delay the investment is still valuable at the optimal entry points Y^{L*} and Y^{F*}. Although the investment cost is higher for the shareholder, if $w_i > w_a$, managers will have a lower opportunity cost to invest so take the decision sooner (equations (58) and (60)).

4.2 Analytical Application

We will now present a numerical example, considering data on Table (1). We consider a scenario where two firms intend to invest in a new market, being both symmetric under perfect information. Both firms shareholders contract a manager entity to monitor the investment opportunity and its key value-drivers, choosing the investment entrance when adequate. Players will design an optimal contract, using a fixed component and a contingent element based on projects cash-flows, in order to maximize the expected aggregate value of shareholders and manager. This contract framework transfers to managers the incentive to preempt the rival firm, implying that managers' optimal trigger will equal the desired shareholders investment trigger. The resulting leader, determined exogenously

We will analyze the analytical result, considering equation (36) for the optimal ϕ, equations (9) and (18) to determine the optimal trigger point for the follower firms shareholders and manager respectively, and equation (64) and (70) from the Appendix to iteratively determine the leader firms shareholder and manager optimal entry triggers. We find the following output results:

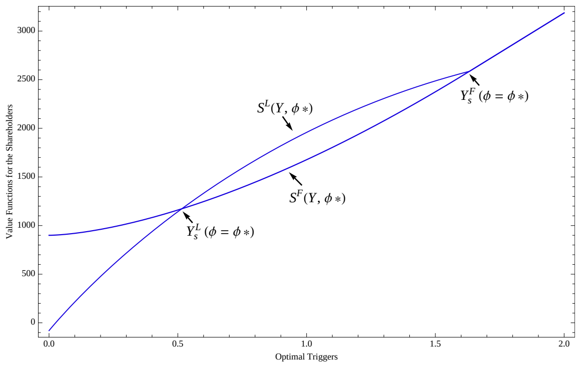
Output	Value
ϕ^*	0.02
$Y_s^F = Y_m^F = Y^{F*}$	\$1.63
$Y_s^L = Y_m^L = Y^{L*}$	\$0.52

Table 2: The output values for the parameters presented in Table 1.

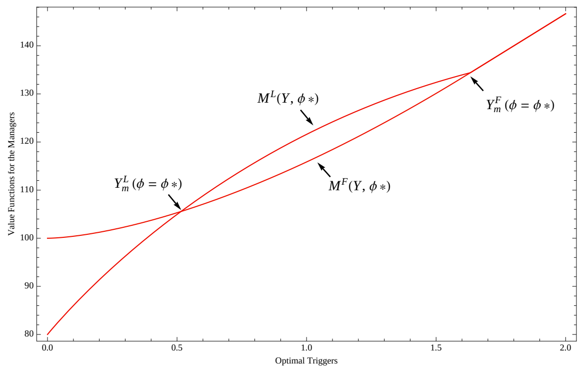
In both firms, negotiation between shareholders and respective managers will lead to the value-sharing rate of 0.02, the price that ensures ex-ante preemptive optimal behavior of both managers. This optimal contract will guarantee the alignment of managers and shareholders, leading to the leader entry trigger when Y hits \$0.52 and to the subsequent follower entry moment, when a price of \$1.63 is achieved.

Figure (4) shows the ex-ante incentive of both firms' shareholders and managers have to negotiate the optimal contract. In this specific numerical example, the optimal contract $\phi^*(w_a, w_i, K_s)$ is the only combination that ensures value preservation, since all other possible value sharing components ϕ result in a lower expected aggregated value. Nonetheless, we reiterate that this may not always happen, so that other sub-optimal solutions may preserve the aggregate firm value, although it fails in aligning triggers.

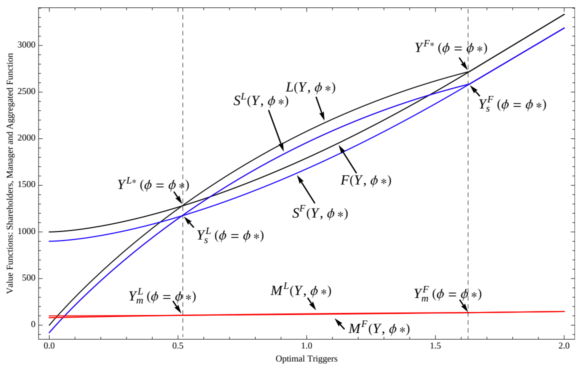
Figures 3(a) and 3(b) concern, respectively, the value functions of both firms shareholders and the value function of both firms managers. Since both firms shareholders and respective managers sign the optimal contract mix $\phi^*(w_a, w_i, K_s)$, then the entry moment chosen by managers will fulfill shareholders requirements, equaling the optimal non-agency equilibria (Y^{F*} and Y^{L*}). This is graphically visible on figure 3(c) which also shows that, in the presence of optimal contractual relations, the aggregated shareholders and managers value equals the non-agency value solution, confirming figure (4) result.



(a) Shareholders position.



(b) Manager position.



(c) Both positions.

Figure 3: The value functions for the shareholders and for manager of both leader and follower firms. The parameters are according to Table 1, and $\phi = \phi^* = 0.02$.

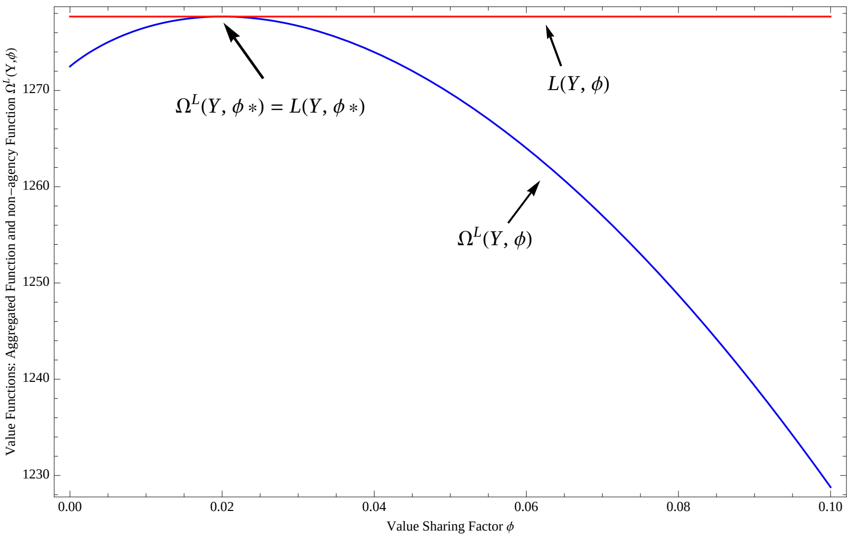


Figure 4: The aggregated value function $\Omega(Y, \phi)$ compared with the non-agency leader function $L(Y)$ for different levels of ϕcalculated according to Table 1.

5 Conclusion

This paper analyzes the impact of agency relations in a non-exclusive investment opportunity, overcoming the usual assumption that managers are fully aligned with owners or that the investment opportunity is managed by the shareholders. In this model the interaction between two firms, that aim to achieve a leader role in this market, is transferred to their respective managers which will maximize their own utility when taking the investment decision. Consequently, shareholders delegate the decision role to managers, which may lead to interest misaligning under sub-optimal contract design due to the information asymmetry that arises between shareholders and managers when the game begins. Considering that each firms' shareholders and respective managers have risk bearing asymmetry, the agency contract design plays a crucial role in interest aligning while taking the investment timing decision.

We propose an optimal contract mix, through fixed wages and a value-sharing component, that enforces optimal aligned behavior of both firms' managers. This contract, determined ex-ante, immediately ensures optimal preemptive behavior when determining the leader role, but also ensures that the exogenously determined follower firm manager will take the optimal investment decision. Since interest aligning is ensured ex-ante, shareholders of both firms need not to follow the evolution of the investment opportunity value drivers in order to guarantee optimal behavior.

Deviations from the optimal contract lead to sub-optimal investment triggers, which may lead in most of the circumstances to shareholders and managers aggregated value destruction. This situation incentives the contractual negotiation to tend to the optimal contract, reflecting the relative opportunity cost of manager and shareholders.

Appendix

Lemma 2.1: There exists a unique point $Y_s^L \in \left(0, Y_s^F\right)$ such that:

$$S^L(Y_s^L) - K_s = S^F(Y_s^L) \tag{61}$$

$$S^L(Y) - K_s < S^F(Y), \text{ for } Y < Y_s^L \tag{62}$$

$$S^L(Y) - K_s \geq S^F(Y), \text{ for } Y > Y_s^L \tag{63}$$

Proof. 2.1 We define the function $Z(Y) = S^L(Y) - K_s - S^F(Y)$ which represents the gain of preempting the rival paying the investment cost against being preempted. Rearranging using equations (22) and (8) we have:

$$Z(Y) = (1-\phi)\frac{YD(1)}{r-\alpha} + \left(\frac{Y}{Y_s^F}\right)^{\beta_1} \frac{1}{\beta_1 - 1}\left[\beta_1\left(1 - \frac{D(1)}{D(2)}\right) - 1\right]\left(K_s - \frac{w_i - w_a}{r}\right) - K_s + \frac{w_i - w_a}{r} \tag{64}$$

Calculating $Z(Y)$ at $Y = 0$ we get $Z(0) = -K_s + \frac{w_i - w_a}{r}$, and $Z(Y)$ at $Y = Y_s^F$ we have $Z(Y_s^F) = 0$. Deriving $Z(Y)$ at Y_s^F results in:

$$Z'(Y)|_{Y=Y_s^F} = -(1-\phi)(\beta_1 - 1)\frac{D(1) - D(2)}{r-\alpha} < 0 \tag{65}$$

since $\beta_1 > 1$, $\phi < 1$, $r > \alpha$ and $D(1) > D(2)$.

We prove uniqueness of Y_s^L demonstrating strict concavity of $Z(Y)$ over the interval $\left(0, Y_s^F\right)$. The second derivative of $Z(Y)$ is:

$$Z''(Y) = \beta_1\left(\frac{Y}{Y_s^F}\right)^{\beta_1} \frac{(K_s - \frac{w_i - w_a}{r})}{Y^2}\left[\beta_1\left(1 - \frac{D(1)}{D(2)}\right) - 1\right] < 0 \tag{66}$$

These results guarantee that Y_s^L is unique over the interval $\left(0, Y_s^F\right)$.

□

Lemma 2.2: There exists a unique point $Y_m^L \in \left(0, Y_m^F\right)$ such that:

$$M^L(Y_m^L) - K_s = S^F(Y_s^L) \tag{67}$$

$$M^L(Y) < M^F(Y), \text{ for } Y < Y_m^L \tag{68}$$

$$M^L(Y) \geq M^F(Y), \text{ for } Y > Y_m^L \tag{69}$$

Proof. 2.2 Defining the function $N(Y) = M^L(Y) - M^F(Y)$ we intend to comprehend the managerial benefit of preempting its rival . Rearranging using equations (30) and (17) we have:

$$N(Y) = \phi\frac{YD(1)}{r-\alpha} + \left(\frac{Y}{Y_s^F}\right)^{\beta_1} \frac{1}{\beta_1 - 1}\left[\beta_1\left(1 - \frac{D(1)}{D(2)}\right) - 1\right]\left(\frac{w_i - w_a}{r}\right) - \frac{w_i - w_a}{r} \tag{70}$$

Calculating $N(Y)$ at $Y = 0$ and at $Y = Y_m^F$ we get $N(0) = -\frac{w_i - w_a}{r}$ and $N(Y_m^F) = 0$. Deriving $N(Y)$ at Y_s^F results in:

$$N'(Y)|_{Y=Y_m^F} = -\phi(\beta_1 - 1)\frac{D(1) - D(2)}{r - \alpha} < 0 \tag{71}$$

Since equation (71) is negative, we prove that condition N(Y) have at least on root in the interval $(0, Y_m^F)$. Additionally, it is necessary to prove that Y_m^L is unique by demonstrating strict concavity of $N(Y)$ over this interval:

$$N''(Y) = \beta_1 \left(\frac{Y}{Y_s^F}\right)^{\beta_1} \frac{\frac{w_i - w_a}{r}}{Y^2} \left[\beta_1 \left(1 - \frac{D(1)}{D(2)}\right) - 1\right] < 0 \tag{72}$$

The second derivative of $N(Y)$ is represented on equation and guarantees that Y_m^L is the only root of $N(Y)$ in interval $\left(0, Y_m^F\right)$.

□

Lemma 3.1: The optimal contract mix $\phi^*(w_a, w_i, Ks)$, presented on equation (36) guarantees that manager chooses the leader shareholders' optimal trigger:

$$Y_s^L|_{\phi=\phi^*} = Y_m^L|_{\phi=\phi^*} \tag{73}$$

Implied from:

$$\arg\max_{\phi \in (0,1)} \Omega(Y, \phi) = \frac{K_m}{K_s} \tag{74}$$

Proof. 3.1 The optimal contract mix $(\phi^*(w_a, w_i, Ks))$ of the leader firm is obtained by finding the maximum value possible for the aggregated leader firm value $\Omega(Y, \phi)$, as stated on equation (38). Therefore, we have:

$$\frac{\partial \Omega(Y, \phi)}{\partial \phi} = 0 \Rightarrow \phi = \phi^* = \frac{K_m}{K_s} \tag{75}$$

$$\frac{\partial^2 \Omega(Y, \phi))}{\partial \phi^2} < 0 \tag{76}$$

□

Lemma 3.2: There exists a consistent tangible point at $\phi^*(w_a, w_i, Ks)$, such that:

$$\Omega(Y, \phi^*) = L(Y),\ for\ \phi = \phi^* \tag{77}$$

Other contract combinations may result in value loss, such that:

$$\Omega(Y, \phi) < L(Y),\ for\ \phi \neq \phi^* \tag{78}$$

Proof. 3.2 Considering:

$$\Theta(Y,\phi) = L(Y) - \Omega(Y,\phi) \tag{79}$$

Our aim is to demonstrate that the optimal contract mix $\phi^*(w_a, w_i, Ks)$ totally mitigates all agency costs. Substituting $\phi = \phi^*$ in equation (79), we verify that $\Theta(Y,\phi^*) = 0$, implying that the aggregate value of shareholders and manager equals the non-agency solution.

We also want to understand if there are agency costs if $\phi \neq \phi^*$ and in what circumstances, comparatively to the non-agency solution. Nonetheless, equation will have different breaking points, meaning that when $\phi \neq \phi^*$ there will be a mismatch between the triggers Y_m^F and Y^F, breaking the aggregate function $\Theta(Y,\phi)$ in three branches. Consequently, we must study three different situations (where Y^F is the optimal investment trigger of follower firm in the agency-free model). So, rearranging equation (79):

$$\Theta(Y,\phi) = \begin{cases} \frac{\beta_1}{\beta_1 - 1}\left(1 - \frac{D(1)}{D(2)}\right)\left[K_s\left(\left(\frac{Y}{Y^F}\right)^{\beta_1} - \left(\frac{Y}{Y_s^F}\right)^{\beta_1}\right) + \frac{w_i - w_a}{r}\left(\left(\frac{Y}{Y_s^F}\right)^{\beta_1} - \left(\frac{Y}{Y_m^F}\right)^{\beta_1}\right)\right] & \text{for } Y < Y_m^F \ \wedge \ \phi > \phi^* \\ \frac{\beta_1}{\beta_1 - 1}\left(1 - \frac{D(1)}{D(2)}\right)\left[K_s\left(\left(\frac{Y}{Y^F}\right)^{\beta_1} - \left(\frac{Y}{Y_s^F}\right)^{\beta_1}\right) + \frac{w_i - w_a}{r}\left(\left(\frac{Y}{Y_s^F}\right)^{\beta_1} - \left(\frac{Y}{Y_m^F}\right)^{\beta_1}\right)\right] & \text{for } Y < Y^F \ \wedge \ \phi < \phi^* \\ \frac{Y\,(D(2) - D(1))}{\delta} - \frac{\beta_1}{\beta_1 - 1}\left(1 - \frac{D(1)}{D(2)}\right)\left[\left(\frac{Y}{Y_s^F}\right)^{\beta_1} K_s - \left(\left(\frac{Y}{Y_s^F}\right)^{\beta_1} - \left(\frac{Y}{Y_m^F}\right)^{\beta_1}\right)\frac{w_i - w_a}{r}\right] & \text{for } Y^F < Y < Y_m^F \ \wedge \ \phi < \phi^* \end{cases} \tag{80}$$

$$sgn\{\Theta(Y,\phi)\} = \begin{cases} \Theta(Y,\phi) > 0 \quad \text{if } \frac{\left(\frac{1}{Y^F}\right)^{\beta_1} - \left(\frac{1}{Y_s^F}\right)^{\beta_1}}{\left(\frac{1}{Y_m^F}\right)^{\beta_1} - \left(\frac{1}{Y_s^F}\right)^{\beta_1}} > \phi^* \text{ , for } Y < Y_m^F \ \wedge \ \phi > \phi^* \\ \Theta(Y,\phi) > 0 \quad \text{if } \frac{\left(\frac{1}{Y^F}\right)^{\beta_1} - \left(\frac{1}{Y_s^F}\right)^{\beta_1}}{\left(\frac{1}{Y_m^F}\right)^{\beta_1} - \left(\frac{1}{Y_s^F}\right)^{\beta_1}} > \phi^* \text{ , for } Y < Y^F \ \wedge \ \phi < \phi^* \\ \Theta(Y,\phi) > 0 \quad \text{if } K_s\left(\frac{Y}{Y_s^F}\right)^{\beta_1} > \frac{YD(2)}{\delta}\frac{\beta_1 - 1}{\beta_1} + \frac{w_i - w_a}{r}\left[\left(\frac{Y}{Y_s^F}\right)^{\beta_1} - \left(\frac{Y}{Y_m^F}\right)^{\beta_1}\right] \text{ , for } Y^F < Y < Y_m^F \ \wedge \ \phi < \phi^* \end{cases} \tag{81}$$

This result proves that, in most of the circumstances that $\phi \neq \phi^*$, there are agency costs and, consequently, the non-agency value $L(Y)$ is higher than the aggregated result $\Omega(Y,\phi)$. Nonetheless, the most important observation is that the optimal labor contract $\phi^*(w_a, w_i, Ks)$ always mitigates all agency costs, providing the first best solution.

□

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