Endogenous Bourse Structures ◊

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Abstract. Using a club theory approach, this paper provides an equilibrium model in which traders must belong to at least one bourse in order to trade assets. We show, by means of examples, that: 1) traders' complementarities in preferences and endowments can determine the formation of *both* large bourses and bourses that are small dark pools of liquidity; 2) bourse formation costs explain the existence of bourses with incomplete markets. For this bourse economy equilibrium is shown to exist generically. We also analyze the welfare implications of considering instead a monopolist bourse that can or cannot exclude and distinguish among traders.

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1 Introduction

Bourses (or stock exchanges) have origins distant in history. Any group of agents who agree to trade their assets in fact constitutes a bourse. Over time bourses evolved and large trading organizations emmerged. By the 20th century, bourses were linked to their respective national countries, where outsiders were charged high commissions and trading fees in order to gain access to their liquidity.[1] But the evolution of bourses has never been as dramatic as in the last decade. In 2007 the European Commission enacted the Market in Financial Instruments Directive (Mifid) to facilitate competition across the region. The competitors are known as MTFs (Multilateral Trading Facilities). The change in regulation and the new electronic trading technologies are driving the old-style stock exchanges out of existence, and they are being progressively replaced by new global trading organizations. Any bourse can now be created, at convenient low formation costs, to trade any assets with self-picked traders and assets.[2] This possibility was a key element in the emergence of "dark pools of liquidity".[3] The following factors inspire the current research.

Fact Set 1 (Toward a unique global bourse?): In October 2006 the Chicago Mercantile Exchange (CME) and the Chicago Board of Trade (CBOT) created the world's largest futures exchange. In May 2006 the New York Stock Exchange (NYSE) (itself a product of the union between the New York Stock Exchange and American Stock Exchange) acquired Euronext (in turn the result of the merger between Paris, Amsterdam, and Brussels stock exchanges). In April 2007 Deutsche Börse acquired the International Securities Exchange (ISE). In February 2011 NYSE-Euronext and Deutsche Börse announced a friendly merger.

Fact Set 2 (Competition by small exchanges): Competition from small exchanges, offshore centers, banks trading consortia, and dark liquidity pools led to an increase in fragmentation in 2008 when the number of trading platforms increased. By 2009 MTFs accounted for about 20% of the trading in the largest shares across European main markets.

In response to the recent wave of demutualizations[4], and knowing the importance of

[1] See Hautcoer and Riva [17] for an analysis of State imposed regulations on the Paris Bourse during the 18th century.

[2] see "Derivatives trading platform bypasses intermediary banks", by Jeremy Grant, in Financial Times, January 17, 2011.

[3] Wealthy market participants trading large blocs of shares have smaller costs if the trading occurs in a "dark pool of liquidity" than in standard "lit" exchanges. Those costs are reduced in dark pools since there is no need of a broker (intermediation cost) and also participants are protected against adverse share price movements since the trades are privately negotiated.

[4] Demutualization is the process where traders move freely from their pre-assigned bourses (e.g., na-

such institutions to the market, we provide an equilibrium model of endogenous bourse formation. A central concept is our interpretation of a bourse. Precisely, a bourse is a public good that allows traders to diversify risk by trading its assets with the other members of the bourse. The attractiveness of a bourse is thus evaluated in the light of prices for its assets, which ultimately depend on the complementarities in preferences and endowments among the traders that form the bourse.

We propose an economy with three periods. Traders form bourses in period 0 and trade assets in their respective bourses in period 1. In period 2 there is uncertainty over states of nature, when the assets pay returns. Traders evaluate a bourse in period 0 by the risk sharing possibilities associated with periods 1 and 2. Our distinction among periods is also pertinent as it captures the fact that the acquisition of a bourse membership (in period 0) usually involves a commitment for trading in this bourse for a long period of time.[5] Thus, it is different from the short time activity of asset trading in period 1 - trading is usually achieved as a matter of seconds within a day. In such a framework, it is natural to assume that traders are price takers in their decision to enter a bourse - normally these decisions are not driven by a strategic motive of asset price manipulation.

The model proposed here has its foundations in both the cooperative theory of coalitions formation with local public goods (Ellickson [11] and Wooders [28, 29]) and the theory of general equilibrium with incomplete markets (GEI) (Arrow [4] and Hart [16]). Our model of bourse formation borrows from the existence result of Allouch and Wooders [2] (hereinafter AW) as it allows us to incorporate explicitly in our model important characteristics of the bourse industry: large number of traders, price taking behavior, increasing gains from trade in larger bourse sizes (with bourses being possibly unbounded in size[6]), multiple memberships, and competition from small dark pools of liquidity. In this setting,[7] ever-increasing gains from trade in larger bourses is in fact a possibility, but it is not self-imposed in the model.

The application of AW's existence proof to our bourse economy is not immediate, however, since some of AW's assumptions rely on the individual's utility function being

tional bourses) to their most preferred ones, without no restriction other than paying the corresponding membership fee.

[5]Usually, the exchange participant stays in the bourse since its year of accession. See, for example, the MICEX list of participants: http://www.micex.com/markets/stock/members/list.

[6]Other papers in the club / local public goods literature (for instance, Cole and Prescott [7] and Ellickson, Grodal, Scotchmer, and Zame [12]) do not allow clubs to be unbounded in size.

[7]There are other papers that do allow unbounded club sizes and also consider price-taking equilibrium – see Wooders [27, 25] and, more indirectly, Wooders [26]. Allouch, Conley, and Wooders [1] also allow unbounded club sizes, but require that all gains can be realized by coalitions strictly bounded in size.

evaluated on the club good. In our paper the utility that the public good (bourse) provides to the traders is endogenous, as it depends on the traders' evaluation of the risk sharing possibilities achieved in equilibrium in the different bourses. Thus, a crucial step to reconcile our model of bourse formation with that of AW is to show that there exists an open and dense set of economies where the trading equilibrium for any given bourse structure is continuous in traders' utility and endowment attributes (following Geanakoplos and Polemarchakis [15]). But our equilibrium model focuses more than equilibrium existence, by providing several examples that illustrate the different aspects of bourse formation, such as bourse size and composition, market incompleteness, multiple memberships, and monopoly power.

To the best of our knowledge, this paper is the first that seeks to analyze the specific market micro-structure of trading in bourses under the lens of club theory. In particular, we contribute with a different viewpoint to the market microstructure literature that analyzes the issues of concentration and fragmentation of trade across markets - see Pagano [20][8] and related works - and the impact of trading costs on trading behavior[9]. In our opinion, this club theory approach to finance is powerful for obtaining new insights on the functioning of financial markets, in the same way that the theory of networks and search theory contribute so much to the understanding of different issues in finance.

This novel approach provides a useful framework to analyze important issues, such as why do diverse financial market structures exist (included incomplete ones) and what are their welfare implications? More specifically, we provide a micro-founded justification of the emergence of large trading platforms (like the NYSE-Euronext-Deutsche Börse) and also show how, in some cases, large exchanges are ill-suited to certain types of situations. Also, we emphasize by means of examples the various trade-offs among the bourses' formation costs, trading complementarities, and the inherent asset structure of the bourses.

The outline is as follows. The model is presented in Section 2. Section 3 establishes an individual optimality condition that is compatible with inter-bourse arbitrage. Section 4 gives the equilibrium concept for our bourses economy and establishes the existence result. The key technical contribution relative to earlier literature is presented at the end

[8] By fully characterizing non-anonymous traders, we depart from Pagano [20] and related works in that we allow for every possible subset of traders to form a bourse, which contrasts with the two-bourse analysis of Pagano. We also depart from Pagano in that we do not limit our analysis to only one stock (see Pagano's fn. 3). Instead, we consider different asset structures, possibly incomplete, with more than one asset.

[9] See, for example, Lo, Mamaysky, and Wang [18] for an equilibrium model with fixed transaction costs.

of this section.

In Section 5 we provide several tractable numerical examples. Example 1 illustrates how the bourse structure affects welfare and trading volume through varying complementarities among different sets of traders. Motivated by Fact Set 1, the second example shows that large bourses form in equilibrium when bourse formation costs are proportional in size, as long as there exist good complementarities among traders. Example 3 provides the opposite case, pointed out in Fact Set 2, where small bourses provide more welfare given the bad complementarities in a larger bourse, with formation cost again proportional in size. Example 4 provides a case with multiple memberships, and compares different bourse structures for different technology scenarios.

Section 6 focuses on the endogeneity of the market incompleteness and the related inefficiency. In Example 5 we can see how technology, in the form of bourse formation costs, plays a crucial role in determining the incompleteness of the markets. There, a bourse with an incomplete asset structure may form in equilibrium even if another bourse with a complete asset structure is also available to the traders.

Section 7 presents an alternative economy with a monopolist bourse. We provide two examples. Example 6 asserts that a large (unique) bourse will not always extract all surplus from the traders, and argues that it is in the monopolist bourse's interest to exclude certain traders. Example 7 examines how an anonymous pricing set-up affects a bourse's profits, bourse structure, and traders' welfare. Also, in Section 7 we assert that a demutualizations process, where traders freely sort into bourses, recovers the efficiency of the bourse structure, constrained on the technology in the form of bourse formation costs. Section 8 concludes.

2 Bourse economies

2.1. *Trading*

The economy lasts for three periods, 0, 1, and 2. In period 0 traders form bourses; in period 1 asset trading occurs in each bourse[10]; and in period 2 assets pay returns. The set of states of uncertainty in the last period is $\boldsymbol{\Xi} \equiv \{1, ..., \Xi\}$, with representative element ξ. In each of the three periods all consumers trade commodities in a common market. Traders are assumed, for all effects, to be price takers in all periods.

[10]For simplicity we assume that trading occurs only once in period 1, although period 1 could have been modeled as a period that permits multiple trading rounds.

The set of commodities is $\mathbf{L} \equiv \{1, ..., L\}$, with representative element l. The number of state-time contingent goods is then $(2+\Xi)L$. Commodities are traded at prices $p = (p_0, p_1, p(1), ..., p(\Xi)) \in \mathbb{R}_+^L \times \mathbb{R}_+^L \times \mathbb{R}_+^{L\times\Xi}$, where p_0, p_1, and $p(\xi)$ are the price vectors at dates 0, 1, and 2 (state ξ), respectively. Similarly, p_{l0}, p_{l1}, and $p_l(\xi)$ are the good l prices at dates 0, 1, and 2 (state ξ), respectively.

An exchange participant or *trader* is a corporation that may trade on or through the exchange and is licensed under the ordinance of the corresponding exchange financial regulator to carry on asset trading activity. The set of traders is $\mathbf{I} \equiv \{1, ..., n\}$, with n assumed to be large but finite. Let $\mathbf{\Theta}$ be the set of traders' characteristics, endowed with a metric d. An element $\theta \in \mathbf{\Theta}$ describes the endowments and preferences of a trader of this type. We assume that traders' characteristics are observable. Let $\alpha : \mathbf{I} \to \mathbf{\Theta}$ be an attribute function, with $\alpha(i) = \theta$ describing trader i's endowments and preferences.[11] Then, an economy is represented by a pair $(\mathbf{I}, \alpha)$.

Trader i is endowed with a finite positive vector of private commodities $\omega^i = (\omega_0^i, \omega_1^i, (\omega^i(\xi), \xi = 1, ..., \Xi)) \in \mathbb{R}_{++}^L \times \mathbb{R}_{++}^L \times \mathbb{R}_{++}^{L\Xi}$. We assume that the total endowment of commodities is finite, that is, $\sum_{i\in\mathbf{I}} \omega^i < \infty$. We denote by $x^i = (x_0^i, x_1^i, (x^i(\xi), \xi = 1, ..., \Xi)) \in \mathbb{R}_+^L \times \mathbb{R}_+^L \times \mathbb{R}_+^{L\Xi}$ trader i's consumption bundle. Then, let $x_0^I \equiv (x_0^i \in \mathbb{R}_+^L : i \in \mathbf{I})$ denote traders' consumption bundles in period 0, and $x^I = (x^i \in \mathbb{R}_+^L \times \mathbb{R}_+^L \times \mathbb{R}_+^{L\Xi} : i \in \mathbf{I})$ denote traders' consumption bundles in the three periods.

Let $u^i(x)$ denote trader i's utility function defined on the consumption bundle $x \in \mathbb{R}_+^{(2+\Xi)L}$. In order to introduce a temporal distinction between period 0 (when traders choose bourses) and periods 1 and 2 (when traders already belong to bourses) we assume that the utility function is separable as follows[12]

$$u^i(x_0, x_1, x(1), ..., x(\Xi)) = u_0^i(x_0)u_1^i(x_1, x(1), ..., x(\Xi)). \tag{1}$$

Let us now impose the following assumptions.

(A1.i) For all $i \in \mathbf{I}$ and $l \in \mathbf{L}$, $\omega_{l0}^i > \tau$ with $\tau > 0$, and given $\varepsilon > 0$, there exists $\lambda > 0$ such that for any set $\mathbf{I}$ and pair of economies $(\mathbf{I}, \alpha)$ and $(\mathbf{I}, \beta)$, if $d(\alpha(i), \beta(i)) \leq \lambda$ for any

[11] Our fully non-anonymous analysis below will show how traders' attributes determine trading behavior. Thus, this paper departs from those models (e.g., Pagano [20] and related works) that assume traders' decisions to depend on the first and second order moments.

[12] This specific functional form is considered only for presentation purposes. We emphasize that other types of functional representations of u_0^i and u_1^i are also admissible. However, we point out that a form $u_0^i + u_1^i$ is not interesting, as the decision of choosing a bourse does not reflect the trading opportunities associated with a bourse.

i, then $\omega_0^{\alpha(i)} \leq \omega_0^{\beta(i)} + \varepsilon\bar{1}$, where $\bar{1} = (1, ..., 1) \in \mathbb{R}^L$. u_0^i is continuous, increasing, and strictly quasiconcave.

(A1.ii) u_1^i is twice continuous differentiable, increasing, and has the matrix of second derivatives $(D^2 u_1^i)$ negative semidefinite.

(A1.iii) $u_0^i(x_0) = 0$ if there exists a commodity $l \in \mathbf{L}$ with $x_{l0} = 0$.

(A1.iv) Given any attribute θ and any $\varepsilon > 0$, there is $\rho_\varepsilon^\theta > 0$ such that, for all $i \in \mathbf{I}$ with $\alpha(i) = \theta$ and all $x_0^i \in \mathbb{R}_+^L$, $u_0^i(x_0^i)\delta + \rho_\varepsilon^\theta < u_0^i(x_0^i + \varepsilon\bar{1})\delta$ holds, where $\delta = \min_{i \in \mathbf{I}} u_1^i(\omega_1^i, \omega^i(1), ..., \omega^i(\Xi)) > 0$.

(A1.v) Given $\varepsilon > 0$, there exist $\lambda_\varepsilon > 0$ and $\gamma > 0$, such that, for any set $\mathbf{I}$ and pair of economies $(\mathbf{I}, \alpha)$ and $(\mathbf{I}, \beta)$, if $d(\alpha(i), \beta(i)) \leq \gamma$, then $u_0^{\alpha(i)}(x_0^i) + \lambda_\varepsilon < u_0^{\beta(i)}(x_0^i + \varepsilon\bar{1})$, for any i and any $x_0^i \in \mathbb{R}^L$.

(A1.vi) Moreover, for $\varepsilon > 0$ and for all $\theta \in \mathbf{\Theta}$ with $\alpha(i) = \theta$, $u_0^{\alpha(i)}(\sum_i \omega_0^{\alpha(i)} + \varepsilon)$ is bounded above by some $\bar{u}_0$, and $u_1^{\alpha(i)}(\omega_1^{\alpha(i)}, \omega^{\alpha(i)}(1), ..., \omega^{\alpha(i)}(\Xi))$ is uniformly bounded below from some $\underline{u}_1 > 0$.

Assumption (A1.i), which characterizes endowments and utility in period 0, already appeared in AW [2]. However, the remaining assumptions in (A1) differ slightly. To see this, notice that in our economy the club good associated with each bourse, interpreted here as the assets trading facility, is endogenous. This implies that the trader's utility attained in each bourse structure is endogenous to the model, whereas in AW the trader's utility attained in each coalition structure is pre-defined. Thus, we must impose assumptions on our primitives, given the utility functional form specified in (1), in such a way that AW's assumptions hold. We refer the reader to the proof of our Lemma 3 in the Appendix, for the relationship between our assumptions and those of AW.

2.2. *Bourse structures*

The set of assets available in the economy is finite and denoted by $\mathbf{J} \equiv \{1, .., J\}$. $\mathcal{J}$ denotes the set of all possible nonempty subsets of $\mathbf{J}$. For our purposes, we associate a member of $\mathcal{J}$ with a bourse. A bourse is thus a club good that allows traders to diversify their risks by offering the specific activity of asset trading. As such, a bourse becomes a source of liquidity.

Each bourse has associated some assets for trading. To each coalition $S \subseteq \mathbf{I}$, we assign the set $J(S) \subseteq \mathcal{J}$, which consists of the different types of assets that are available for trade in bourse S.[13] Then, there is a mapping $S \to A(S)$, where $A(S) = [a_j(\xi)]_{\Xi \times J(S)}$

[13] This set of assets is exogenously given for bourse S, and can be thought as the assets that traders in

is the payoff matrix describing the returns at each state of nature of the different assets in $J(S)$. We assume that assets pay in the *numeráire* commodity, taken here to be good L. The returns matrix $A(S)$ is assumed to have full-column rank for each bourse S. We denote a bourse by the pair $(S, A(S))$.

A *bourse structure* is given by $F(\mathbf{I}) = \{(S_1, A(S_1)), ..., (S_k, A(S_k)), ..., (S_K, A(S_K))\}$. We denote by $\mathbf{F}(\mathbf{I})$ the set of all possible bourse structures. Traders can belong to several bourses, and therefore, a bourse structure is not a partition; it is possible that $S_k \cap S_{k'} \neq \emptyset$. We require the number of bourse memberships of every trader to be bounded (as in AW [2]) - naturally justified if bourse formation is expensive-. Also, we require that every bourse has always a trader with no memberships in other bourses (to guarantee that we can always find a trader for whom the assets are not colinear among different bourses).

Given an economy $(\mathbf{I}, \alpha)$ and a bourse structure $F(\mathbf{I})$, let us denote by $F[i; \mathbf{I}] = \{S_k \in F(\mathbf{I}) : i \in S_k\}$ the set of all bourses in $F(\mathbf{I})$ that contain trader i. Trader i can only trade assets with those traders in $F[i; \mathbf{I}]$. This implies that a trader not only chooses a bourse because of its assets available for trade, but also because of the wealth of its traders. We denote by y_j^i the trader i's position on asset $j \in J(S)$ in period 1, with $S \in F[i; \mathbf{I}]$. As usual, $y_j^i > 0$ denotes a purchase of asset j and $y_j^i < 0$ denotes the sale of this asset. Let us denote $y^I = (y^i \in \mathbb{R}^{J(S_k)} : S_k \in F[i; \mathbf{I}], i \in \mathbf{I})$. The prices associated with these assets in S are denoted by $q(S) \in \mathbb{R}_+^{J(S)}$.[14]

2.3. *Bourse formation costs, communication costs, and the transaction fees*

In order to adopt the technology of asset trading and build the trading platform, the bourse S faces (fixed) formation costs $z(S) \in \mathbb{R}_+^L$ (in terms of inputs of the private goods, e.g., hardware costs, software costs, or installation charges)[15]. Asset market characteristics of bourse S are given by the pair $(A, z)(S)$. Denoting by $|S|$ the cardinality of S, we say that two bourses S^1 and S^2 with $|S^1| < |S^2|$ have bourse formation costs: proportional in size if $\frac{z(S^1)}{|S^1|} = \frac{z(S^2)}{|S^2|}$, more than proportional in size if $\frac{z(S^1)}{|S^1|} < \frac{z(S^2)}{|S^2|}$, and less than proportional in size if $\frac{z(S^1)}{|S^1|} > \frac{z(S^2)}{|S^2|}$.

Let us now impose the following continuity condition on z with respect to attributes,

bourse S agree to issue. We are thus describing primary markets.

[14] Notice that our notion of a bourse is substantially different from an over-the-counter (OTC) market. In OTC markets, trading occurs bilaterally (usually between a trader and an intermediary) and an asset can have a different price depending on the bargaining of each couple of traders. Instead, here we model a bourse as a central market where the asset prices are common to all traders in that bourse (like the Tradingpoint Stock Exchange, that provides direct access to investors, without the need of intermediaries).

[15] See the HKEx security trading infraestructure at

http://www.hkex.com.hk/eng/market/sec_tradinfra/CMTradInfra.htm needed for our existence result below. There, S^α and S^β are the bourses comprising the same traders, but characterized by attributes α and β, respectively.

(A2) Given $\varepsilon > 0$, there exists $\lambda > 0$ such that, for any bourse S and any attribute functions α and β, if $d(\alpha(i), \beta(i)) \leq \lambda$ for every $i \in S$, then $z^\alpha_{S^\alpha} \leq z^\beta_{S^\beta} + \varepsilon\bar{1}$.

To fulfill the participantship requirement, all exchange participants are required to hold a bourse membership (or trading right) of the respective exchanges[16]. The membership fee that a trader i pays to participate in a bourse S is denoted by $\pi^i(S)$, and is dependent on trader's own type $\alpha(i)$ (e.g., broker-dealer and retail investor participants). A participation price system is a set $\Pi = \{\pi^i(S) \in \mathbb{R} : i \in S \text{ and } S \subset \mathbf{I}\}$.

For simplicity we do not model trading fees, whose importance has declined substantially since the implementation of the Mifid regulation.[17] We consider transaction fees instead, in the form of a transaction levy and a stamp duty, currently at the center of the debate among the leaders in the European Union. We denote the transaction fee for trading an amount y_j of asset j by $g_j(y_j)$. Transaction fees are denominated in units of the numeraire commodity L, and are paid by both sides (buyers and sellers) to a (unmodeled) financial regulator.[18] We assume that $g_j : \mathbb{R} \to \mathbb{R}_+$ is twice continuous differentiable (C^2) on y_j, increasing in $|y_j|$, and convex in y_j (i.e., $g'' > 0$). Thus, g has a U-shaped form.

Indeed, transaction fees can be non-linear. Typically, most traders pay a constant price per order, but some traders doing a high volume can sign up for a different offer involving a fixed part and a lower unit price. The debate has focused on the linear part, however. But with the increase in competition among exchanges, non-linear trasaction fees are more important. From a theoretical point of view, we observe here that the non-linearity of the transaction fee guarantees that the traders' demand functions are smooth in a context with colinear assets among different bourses (for this result, trading fees do not need to be large).

Finally, we assume that traders face communication costs in period 0 if they wish to move to different bourses. These communication costs for a bourse of size $|S|$ are given

[16] As stated in the Hong Kong Exchange (HKEx) rules, any broker-dealer intending to operate a brokerage business for products available on HKEx, using the trading facilities of the Stock Exchange and/or Futures Exchange, must be admitted and registered as an Exchange Participant of that Exchange. This membership fee is HK$500,000 (US$64,100).

[17] See Colliard [8] for a survey on this literature and for an analysis of the trading fees on the efficiency of the markets.

[18] For the HKEx the transaction levy is 0.003% of the consideration of the transaction, whereas the stamp duty is 0.001% on the value of stock transactions. These fees are paid to the Securities and Futures Commission and government, respectively.

by $c(\varepsilon_0) = \varepsilon_0 |S| \bar{1}$, where $\varepsilon_0 > 0$.

2.4. *Budget constraints*

Given prices p_0 and π^i, trader i's budget constraint at period 0, when he chooses a set of bourse memberships, is

$$p_0(x_0 - \omega_0^i) + \sum_{S_k \in F[i;\mathbf{I}]} \pi^i(S_k) \leq 0. \tag{2}$$

Observe that the communication costs do not enter in the trader's period 0 budget constraint since these are paid only if the trader decides to move to another bourse (see also AW [2]). The profits of bourse S are given by

$$\sum_{i \in S} \pi^i(S) - p_0 z(S). \tag{3}$$

Given prices p_1 and q, the budget constraint in period 1, once trader i has chosen the bourses he wishes to belong to, is

$$p_1(x_1 - \omega_1^i) + \sum_{S_k \in F[i;\mathbf{I}]} \sum_{j \in J(S_k)} (q_j y_j + p_{L1} g_j(y_j)) \leq 0. \tag{4}$$

Given prices $p(\xi)$, trader i's budget constraint in period 2 and node $\xi \in \Xi$ is[19]

$$p(\xi)(x(\xi) - \omega^i(\xi)) \leq p_L(\xi) \sum_{S_k \in F[i;\mathbf{I}]} \sum_{j \in J(S_k)} a_j(\xi) y_j. \tag{5}$$

3 Asset pricing and inter-bourse arbitrage

4.1. Asset pricing

By ignoring the bourse formation process in period 0, the trader's optimization problem would give us the standard asset pricing formula, that is, the asset price must be equal to the discounted asset dividends plus some extra positive shadow price λ_j if there would exist a short sale restriction binding at equilibrium.[20] However, in our bourse economy, the trader's optimization problem must also take into account the cost of his bourse

[19] Observe that in period 2 traders' default is not allowed. See Santos and Scheinkman [22] for a leading model with default and two clearing-houses.

[20] Notice that for our existence result we do not need to impose bounded short sales. See Bottazzi, Luque, and Páscoa [6] for an equilibrium model with repo markets and binding short sales due to the security possession constraint (the "box").

memberships paid in period 0. To put in evidence this new term and avoid complexity of the forms derived below, let us omit transaction fees from this analysis and assume $g_j(y_j) = 0$, for all y_j.

Now, let us fix a bourse structure $F(\mathbf{I})$ and denote the Lagrangian function for this trader by $\mathcal{L}^i(x, y, \tilde{\beta}, \tilde{\lambda}; p, \Pi, q, F(\mathbf{I}))$, where $\tilde{\beta}_0$, $\tilde{\beta}_1$, and $\tilde{\beta}(\xi)$ denote the Lagrange multipliers of the budget constraints in period 0, period 1, and state ξ (period 2), respectively, and $\tilde{\lambda}_{jk}$ denotes the Lagrange multiplier of a short sale constraint $y^i_j(S_k) \geq -m$. Observe that the membership fee is the only variable in budget constraint (2) depending on the asset trading of period 1. Assuming differentiability of $\pi^i(S_k)$ with respect to $y_j(S_k)$ and taking the first order condition of $\mathcal{L}^i$ with respect to $y_j(S_k)$, we obtain

$$-\tilde{\beta}_0 \frac{\partial \pi^i(S_k)}{\partial y_j(S_k)} - \tilde{\beta}_1 q_j(S_k) + \tilde{\lambda}_{jk} + \sum_{\xi \in \Xi} \tilde{\beta}(\xi) p(\xi) a_j(\xi) = 0, \tag{6}$$

Assuming for simplicity that the trader discounts with the same factor periods 0 and 1 (i.e., $\tilde{\beta}_0 = \tilde{\beta}_1$) and writing $\lambda_{jk} = \tilde{\lambda}_{jk}/\tilde{\beta}_0$ and $\beta(\xi) = \tilde{\beta}(\xi)/\tilde{\beta}_0$, we obtain

$$q_j(S_k) + \frac{\partial \pi^i(S_k)}{\partial y_j(S_k)} = \sum_{\xi \in \Xi} \beta(\xi) p(\xi) a_j(\xi) + \lambda_{jk}. \tag{7}$$

In the pricing condition (7) we can see that the sum of discounted dividends plus the no-short sale shadow price is now equal to the asset price $q_j(S_k)$ at which asset j is transacted in bourse S_k plus the additional term $\partial \pi^i(S_k)/\partial y_j(S_k)$. This new term represents the marginal change of trader i's membership fee in bourse S_k when trader i increases by one unit his trade of asset $j \in J(S_k)$. The term on the left hand side of (7) can be interpreted as the marginal cost for trader i of buying one unit of asset j in bourse S_k.

4.2. Absence of inter-bourse arbitrage

In our bourses economy it seems possible that the same asset j can have a different price in two different bourses, S_1 and S_2 (if traders have different endowments and preferences in the two bourses). If this happens, a trader i belonging to these two bourses has an inter-bourse arbitrage opportunity if $q_j(S_2)y_j(S_2) + q_j(S_1)y_j(S_1) < 0$ (receives a net income in period 1) and $p(\xi)a_j(\xi)(y_j(S_1) + y_j(S_2)) \geq 0$ for all $\xi \in \Xi$ (portfolio $(y_j(S_1), y_j(S_2))$ yields non-negative returns). We find that this inter-bourse arbitrage possibility is compatible with trader's optimality.

Proposition 1: *Let us assume that there exists an asset price differential $q_j(S_1) > q_j(S_2)$ for an asset j that is common to both bourses S_1 and S_2. Then, the inter-bourse arbitrage opportunity of a trader i who belongs to both bourses is compatible with trader's* *optimality if the following condition is satisfied:*

$$\frac{\partial \pi^i(S_1)}{\partial y_j(S_1)} y_j^i(S_1) + \frac{\partial \pi^i(S_2)}{\partial y_j(S_2)} y_j^i(S_2) - \lambda_{j1} y_j^i(S_1) > 0. \tag{8}$$

Proof of Proposition 1: Writing condition (7) for both bourse S_1 and bourse S_2 (with $\lambda_{j2} = 0$ as $y_j(S_2) > 0$) and multiplying each of them by $y_j^i(S_1)$ and $y_j^i(S_2)$, respectively, and then summing up the two resulting conditions and rearranging terms, we obtain

$$q_j(S_2)y_j(S_2) + q_j(S_1)y_j(S_1) - \sum_{\xi \in \Xi} \beta(\xi)p(\xi)a_j(\xi)(y_j(S_1) + y_j(S_2)) =$$

$$= \lambda_{j1} y_j(S_1) - y_j^i(S_1) \left. \frac{\partial \pi^i(S_1)}{\partial y_j(S_1)} \right|_{y_j^i(S_1)} - y_j^i(S_2) \left. \frac{\partial \pi^i(S_2)}{\partial y_j(S_2)} \right|_{y_j^i(S_2)}.$$

Now, inter-bourse arbitrage opportunities ($q_j(S_2)y_j(S_2) + q_j(S_1)y_j(S_1) < 0$ and $p(\xi)a_j(\xi)(y_j(S_1) + y_j(S_2)) \geq 0$ for all $\xi \in \Xi$) imply that the left hand side of this equation is negative, and so the right hand side is also negative, which proves condition (8). ■

Condition (8) says that the costs associated with acquiring the memberships to trade $y_j^i(S_1) < 0$ in bourse S_1 and $y_j^i(S_2) > 0$ in bourse S_2, plus paying the shadow price λ_{j1} for shorting $y_j^i(S_1) < 0$ in bourse S_1, must be positive. Notice that condition (8) could be written in terms of the primitives of the economy[21], but at the cost of more complexity in the expressions.

4 Equilibrium

Consistent with our argument that the acquisition of bourse membership (in period 0) usually involves a commitment for trading in that bourse for a long period of time, whereas the asset trading activity occurs constantly (in a matter of seconds) in the bourse that the trader belongs to, we distinguish between the evaluation of bourses (for each bourse structure traders assess the risk sharing attained in their respective bourses) and the formation of bourses (bourses are formed given these evaluations).

4.1. *Bourse evaluation*

Definition 1 (*Asset trading equilibrium for a given bourse structure*): Given the bourse structure $F(\mathbf{I})$, a price taking asset trading equilibrium consists of a system $(x_1^I, x^I(1), ..., x^I(\Xi), y^I, p, q)(F(\mathbf{I}))$, such that,

[21] Expressions (11) and (12) below show that, for the case of two and three traders bourse, respectively, the membership fee is a function of the utilities $(u_1^i)_{i \in S}$ and the endowments $(\omega_0^i)_{i \in S}$.

(D1.i) given trader's bourse memberships $F[i;\mathbf{I}]$, the trader chooses optimally his commodities and assets positions, that is, $(x_1^i, x^i(1), ..., x^i(\Xi), y^i)(F(\mathbf{I})) \in \arg\max u_1^i(x_1, x(1), ..., x(\Xi))$, subject to constraints (4) and (5).

(D1.ii) commodity markets clear at periods 1 and 2, i.e.

- $\sum_{i\in\mathbf{I}}(x_{l1}^i - \omega_{l1}^i) = 0$, for all $l \neq L$.
- $\sum_{i\in\mathbf{I}}(x_{L1}^i + \sum_{S_k\in F[i;\mathbf{I}]} \sum_{j\in J(S_k)} g_j\left(y_j^i\right) - \omega_{L1}^i) = 0$.
- $\sum_{i\in\mathbf{I}}\left(x_l^i(\xi) - \omega_l^i(\xi)\right) = 0$, for all l and $\xi \in \boldsymbol{\Xi}$.

(D1.iii) the asset market clears *for each bourse*, i.e. $\sum_{i\in S_k} y_j^i = 0$, $\forall j \in J(S_k)$, $\forall S_k \in F(\mathbf{I})$.

We denote the set of asset trading equilibria, given a bourse structure $F(\mathbf{I})$, by $E(F(\mathbf{I}))$.

4.2. *Bourse formation*

First, observe that, given the bourse structure $F(\mathbf{I})$, our specification of the utility function (1) allows us to define the trader i's utility in period 0 via the equilibrium point $\tilde{x}(F[i;\mathbf{I}]) \equiv (x_1, x(1), ..., x(\Xi))(F[i;\mathbf{I}])$ as follows

$$V^i(x_0, F[i;\mathbf{I}]) \equiv u_0^i(x_0)U_1^i\left(F[i;\mathbf{I}]\right) \tag{9}$$

where

$$U_1^i\left(F[i;\mathbf{I}]\right) \equiv u_1^i(\tilde{x}(F[i;\mathbf{I}])) \tag{10}$$

denotes the trader i's indirect utility, the utility $u_1^i(\cdot)$ evaluated at the equilibrium point $\tilde{x}(F[i;\mathbf{I}])$. Observe that the evaluation of trader's bourse memberships enters indirectly into his utility u^i through the access to income and the risk sharing that he gains from trading the securities offered in those bourses he belongs to. The following proposition asserts that the utility evaluated at the assets trading equilibrium $\tilde{x}\left(F[i;\mathbf{I}]\right)$ is continuous in trader's attributes. There, we write $\mathbf{I}^\alpha$ to refer to an economy $(\mathbf{I}, \alpha)$, and call an open and dense set with null complement a generic set.

Proposition 2: *There exists a generic set of economies for which, given* $\lambda > 0$, *there is a* $\gamma > 0$ *such that, for any pair of economies* $(\mathbf{I}, \alpha)$ *and* $(\mathbf{I}, \beta)$, *if* $d(\alpha(i), \beta(i)) \leq \gamma$ *for any* i, *then* $|U_1^{\alpha(i)}\left(F[i;\mathbf{I}^\alpha]\right) - U_1^{\beta(i)}\left(F[i;\mathbf{I}^\beta]\right)| < \lambda$.

Condition (f) in AW [2] requires that agents who are similar in attribute space are near-substitutes in the economy. For our bourse economy this is not a trivial issue, as the public good in our paper is endogenous to the model, and this makes the utility $u_1^i(\tilde{x}(F[i;\mathbf{I}])$

dependent on the assets trading equilibrium. In Proposition 2 we demonstrate that there exists an open and dense set of traders' attributes (endowments and utilities) for which the assets trading equilibrium is continuous in traders' attributes. This proof is the most critical point for the application of AW's existence result to our economy. In this proof we follow the lines of Geanakoplos and Polemarchakis [15, Section 6], where the Transversality theorem is used. Thus, as in that paper, we also have to consider a finite dimensional manifold of utility functions.[22]

It is also important to note that $V^i(x_0, F[i;\mathbf{I}])$ may represent ever-increasing gains from trade in larger bourse sizes, depending on how $u_0^i(x_0)$ and $u_1^i\left(\tilde{x}(F[i;\mathbf{I}])\right)$ interrelate in the functional form dictated by $u^i(x_0, \tilde{x}(F[i;\mathbf{I}]))$. In order for this possibility to be compatible with equilibrium existence, we need to impose the following assumption:

(A3) There is a bundle of goods $x_0^* \in \mathbb{R}_+^L$ such that for any economy $(\mathbf{I}, \alpha)$, any consumer $i \in \mathbf{I}$, and any $x^i \in \mathbb{R}_+^L \times \mathbb{R}_{++}^L \times \mathbb{R}_{++}^{L\Xi}$, we have

$$u_0^i(x_0^i + x_0^*)u_1^i(x_1^i, x^i(1), ..., x^i(\Xi)) \geq u_0^i(x_0^i)u_1^i\left(\sum\nolimits_i \left(\omega_1^i, \omega^i(1), ..., \omega^i(\Xi)\right)\right).$$

Assumption A3 permits ever-increasing gains from larger and larger "bourses" while, at the same time, allows for small "bourses". The assumption says that, even in the worst scenario where trader i cannot diversify risk in any bourse and, as a consequence, may consume a inferior bundle $(x_1^i, x^i(1), ..., x^i(\Xi))$, the trader will prefer to consume a very large amount of private goods in period 0, $x_0^i + x_0^*$, rather than consume the total endowments in periods 1 and 2. Notice that the consumption $x_0^i + x_0^*$ can be very large and even unfeasible for the trader - we require only the existence of such a large bundle x_0^*. This assumption A3 assures that the "Desirability of wealth" assumption of AW [2] holds for our economy.

Definition 2 ($c(\varepsilon_0)$-*equilibrium for the bourse formation*): A price taking $c(\varepsilon_0)$-bourse structure equilibrium for period 0 is an ordered triple $((x_0^I, F(\mathbf{I})), p_0, \Pi)$ that consists of an allocation of commodities x_0^I, a bourse structure $F(\mathbf{I})$, a commodity price vector p_0, and a participation price system Π such that,

(D2.i) $\sum_{i\in\mathbf{I}}(x_0^i - \omega_0^i) + \sum_{S_k \in F(\mathbf{I})} z(S_k) \leq 0.$

(D2.ii) For each $S \subset \mathbf{I}$, profits are non positive, i.e., $\sum_{i\in S} \pi^i(S) - p_0 z(S) \leq 0.$

[22]Extending our framework to consider instead an infinite dimensional manifold of utility functions would complicate substantially the proofs, and thus is left for some future research.

(D2.iii) if $V^i(y_0^i, F[i;S]) > V^i(x_0^i, F[i;\mathbf{I}])$, then $p_0(y_0^i - \omega_0^i) + \sum_{S_k \in F[i;S]} \pi^i(S_k) > -\varepsilon_0 p_0 \bar{1}$.

The stated equilibrium concept above follows AW [2]. As these authors point out, the equilibrium condition (D2.iii), requires utility maximization given the costs of bourse formation and budget constraints. The existence of some small frictions in the economy, interpreted as communication costs, affect the opportunities to change bourse memberships.

An additional equilibrium condition in Definition 2 would be to require that most consumers cannot be very far outside their budget constraints. As AW remark, this condition can be derived from conditions of the model and other parts of the definition of an equilibrium for the bourse formation process. Therefore, we omit such condition in Definition 2.

It may also occur that, depending on the composition of the set of traders, some traders cannot be accommodated in their preferred bourses. AW show that if the economy is large, then these traders constitute only a small proportion of the total population. For that, AW accommodate the equilibrium notion by taking into account these reminders. We prefer to avoid further notation and refer to the original paper for such refinement.

4.3. *Equilibrium for the bourse economy*

Finally, we introduce the equilibrium concept associated with the bourse economy proposed here.

Definition 3 ($c(\varepsilon_0)$-*equilibrium for the bourse economy*): We say that the vector $(((x_0^I, F(\mathbf{I}),\ p_0, \Pi), (x_1^I, x^I(1), ..., x^I(\Xi),\ y^I, p, q)(F(\mathbf{I})))$ constitutes a price taking $c(\varepsilon_0)$-equilibrium for our bourse economy if

(D3.i) $(x_0^I, F(\mathbf{I}), p_0, \Pi)$ is a $c(\varepsilon_0)$-equilibrium for the bourse formation.

(D3.ii) $(x_1^I, x^I(1), ..., x^I(\Xi), y^I, p, q)(F(\mathbf{I}))$ is an asset trading equilibrium for $F(\mathbf{I})$.

Observe that for a given bourse structure $F(\mathbf{I})$ there can be more than one asset trading equilibrium. It is well known that different beliefs among traders on the equilibrium realizations may lead to a problem of non-existence. To avoid this possibility we impose the standard "rational expectations hypothesis" (see Dutta and Morris [10]); that is, traders agree on the realization of prices at each state (consensus) and simultaneously believe that there is a single possible price in each state (degenerate beliefs). No information problems are considered here. Thus, traders' beliefs about the realization of prices in each state are self-fulfilling.

Theorem 1: *Let us assume A1, A2, and A3. If there are sufficiently many traders with attributes represented in the economy, there exists a generic set of bourse economies for which there is a price taking $c(\varepsilon_0)$-equilibrium with possibly ever-increasing gains from larger bourses.*

The proof of Theorem 1 is left for the Appendix. In this proof we first show that a trading equilibrium exists given a bourse structure. Notice that we do not require short sales to be bounded. A subtlety in this part of the proof is that market clearing no longer occurs for the whole economy (as in previous GEI models), but occurs in each bourse.[23] In the second part of the proof we show that there exists a measurable selector of the trading equilibria (recall that there can exist more than one), at which traders evaluate their bourse memberships. Finally, we show that a bourse structure equilibrium exists, given the bourse evaluation at the selected trading equilibrium.

The existence result of a bourse structure equilibrium relies on AW [2, Theorem 2], which says that: *under appropriate assumptions (namely, (a)-(h) and "desirability of wealth" - see our proof of Lemma 3 below), there exists a ε_0-price taking equilibrium with communication costs if there exist sufficiently many players with attributes represented in the economy (that is, with attributes in the range of $\alpha(\cdot)$).* However, the application of AW's result to our economy is not immediate. We need to assure that all assumptions required in AW [2, Theorem 2] are satisfied. The most tricky one is their assumption (f) ("utility of a bourse structure is continuous on the attributes"; see proof of Lemma 3 in the Appendix). This cannot be an assumption in our model because the utility that a trader obtains in a bourse (public good) is endogenous in our setup. Proposition 2 proves continuity on traders' attributes for a generic set of economies. However, AW's result relies on the use of a compact set of attributes where this continuity property holds. Thus, for each economy in this generic set of attributes, we have to find a compact subset (this set exists because the generic set is open) and use this set as the compact set used by AW to extend replica economies. Therefore, existence of equilibrium of our bourse economy holds for an open and dense set of economies, with null complement.

[23] Neither does the related field of security design properly model bourses as the place where traders issue and trade securities.

5 Complementarities, technology, and optimal bourse structures: Examples

Next, we provide several examples that illustrate the various trade-offs among the bourses' formation costs, trading complementarities, and the inherent asset structure of the bourses. In these examples, for simplicity the transaction fee is made equal to zero, as the main insights of the examples remain valid. Our terminology should give no space to confusion since we always give a name to each set of numbers, indicating its nature. In this way, for example, the set $S^1 = (1,2)$ indicates a bourse with traders 1 and 2, the set $\Xi = \{1,2\}$ indicates that the states of nature under consideration are 1 and 2, and the set $A(S) = \{(1,1),(1,2)\}$ indicates that there are two assets, the first paying one unit in each of the two states of nature, whereas the second asset is paying one unit in the first state of nature and two units in the second state of nature.

Example 1 (*Bourse structure affects welfare*): Our objective in this first example is to compare three different economies, one with trader set $S^1 = (1,2)$, another $S^2 = (1,2,3)$, and another with $S^3 = (1,2,4)$. In this example we shall focus on the bourse evaluation process and leave the analysis of period 0 for following examples. We consider just one good for consumption at each node. Trading of assets and commodities occurs as described in the model above. The set of states of nature in period 2 is $\Xi = \{1,2\}$. The asset structure is assumed to be complete, with $A(S) = \{(1,1),(1,2)\}$, for all $S = S^1, S^2, S^3$.[24]

Traders' attributes for the different economies are defined as follows. Trader i's utility is $u_1^i(x_1, x(1), x(2)) = \alpha_1^i \ln x_1 + \alpha^i(1) \ln x(1) + \alpha^i(2) \ln x(2)$. Traders 1 and 2's endowments and preference parameters are $(\omega_1^1, \omega^1(1), \omega^1(2)) = (2,2,6)$, $(\alpha_1^1, \alpha^1(1), \alpha^1(2)) = (1,1,0)$, $(\omega_1^2, \omega^2(1), \omega^2(2)) = (2,6,2)$ and $(\alpha_1^2, \alpha^2(1), \alpha^2(2)) = (1,0,1)$, respectively. We perform comparative statics with the three economies. Trader 3 is rich today and prefers to consume today, i.e., $(\omega_1^3, \omega^3(1), \omega^3(2)) = (6,2,2)$ and $(\alpha_1^3, \alpha^3(1), \alpha^3(2)) = (1,1/2,1/2)$. Trader 4 is rich today and prefers to consume tomorrow, i.e., $(\omega_1^4, \omega^4(1), \omega^4(2)) = (6,2,2)$ and $(\alpha_1^4, \alpha^4(1), \alpha^4(2)) = (1/2,1,1)$. Observe that trader 4 allows traders 1 and 2 to transfer wealth to those nodes where consumption is more valued to them. Trader 3 is not bound to make such transfers, as trader 3 has high endowments in the node where he most values consumption (period 1). Thus, trader 4 has better complementarities (in endowments and preferences) with traders 1 and 2 than trader 3 has. Moreover, trader 4 allows traders 1 and 2 to better diversify their risk in bourse S^3 than if they were alone in bourse S^1.

[24] For instance, asset 2 pays 1 unit of the good in state 1 and 2 units in state 2.

Let us abbreviate notation and redefine $U_1^i(F[i;\mathbf{I}]) \equiv U_1^i(S)$, with $i \in S$ (as in this example each trader belongs to just one bourse). The following tables give the traders' indirect utilities and portfolios for different bourses.[25]

$\mathbf{S}^1 = (1,2)$	$\mathbf{S}^2 = (1,2,3)$	$\mathbf{S}^3 = (1,2,4)$
$U_1^1(S^1) = 2.7726$	$U_1^1(S^2) = 2.8904$	$U_1^1(S^3) = 3.0754$
$U_1^2(S^1) = 2.7726$	$U_1^2(S^2) = 2.8904$	$U_1^2(S^3) = 3.0754$
$n.a.$	$U_1^3(S^2) = 2.7726$	$U_1^4(S^3) = 3.3965$

Table 1: Indirect utilities for a given bourse structure

	$\mathbf{S}^1 = (1,2)$	$\mathbf{S}^2 = (1,2,3)$	$\mathbf{S}^3 = (1,2,4)$
$(y_1^1(S), y_2^1(S))$	$(18, -12)$	$(14, -10)$	$(12.5882, -9.2941)$
$(y_1^2(S), y_2^2(S))$	$(-18, 12)$	$(-16, 10)$	$(-15.2941, 9.2941)$
$(y_1^3(S), y_2^3(S))$	n.a.	$(2, 0)$	n.a.
$(y_1^4(S), y_2^4(S))$	n.a.	n.a.	$(2.7058, 0)$

Table 2: Asset trading

In Table 1 we can see that the indirect utilities of traders 1 and 2 are substantially greater when they trade in the bourse with trader 4 than when they trade in the bourse with trader 3. Table 2 shows that trading in different bourses results in different trading volumes.[26] These observations indicate that trading complementarities are, indeed, an important determinant of traders' welfare and asset trading. ♣

Example 2: (*Large bourses are optimal if trading complementarities are good*): Being inspired by Fact Set 1, we illustrate here how good trading complemetarities are enough for a large bourse to emerge. Let us add to the set-up presented in Example 1 an initial period 0 where bourses form. We assume no uncertainty between periods 0 and 1. Our framework is again characterized by non-anonymity and market completeness.

We restrict our attention to the set of traders $\mathbf{I} = \{1, 2, 4\}$. The possible bourses are $S^1 = (1,2)$, $S^3 = (1,2,4)$, $S^6 = (1,4)$, and $S^7 = (2,4)$. Let $u_0^i(x_0) = (1/2) \cdot \ln x_0$, where x_0 is the good consumption in period 0. The modified utility function is then $V^i(x_0, S) =$

[25] The numerical compution procedure for this example and the following ones can be found in the working paper Faias and Luque [14].

[26] We can define trading volume of an asset j in a bourse S by $\eta_j(S) \equiv \frac{1}{2}\sum_{i \in S} |y_j^i(S)|$, where the coefficient 1/2 corrects for the double counting when summing the trades over all traders. Then, using values in Table 2, we can order trading volumes as $\eta_j(S^1) > \eta_j(S^2) > \eta_j(S^3)$, for $j = 1, 2$. Observe that a higher volume of trade does not necessarily correlate with traders' welfare. In fact, in this example, trading volume is lower when traders' welfare is higher.

$u_0^i(x_0)U_1^i(S) = (1/2)\ln x_0[\alpha_1^i \ln x_1^i + \alpha^i(1)\ln x^i(1) + \alpha^i(2)\ln x^i(2)]$. Good endowments in period 0 are $\omega_0^1 = 7$, $\omega_0^2 = 7$ and $\omega_0^4 = 5.9$.

In period 0 traders must pay for the membership fee to have access to the bourse trading facility. Membership fees cover the bourse formation cost. Trader i's membership fee in bourse S is denoted by $\pi^i(S)$. We compute the non-anonymous membership fees by considering a welfarist agent that maximizes the weighted sum of indirect utilities subject to individual budget constraints in period 0. It can be shown that the optimal membership fee for a trader i in a two-traders bourse $S = (i, k)$ is given by

$$\pi^i(S) = \frac{U_1^k(S)\omega_0^i - U_1^i(S)\omega_0^k}{U_1^i(S) + U_1^k(S)} + \frac{U_1^i(S)}{U_1^i(S) + U_1^k(S)} z(S) \tag{11}$$

whereas if it is a three-traders bourse $S = (i, j, k)$ we would have

$$\pi^i(S) = \frac{\omega_0^i\left[U_1^j(S) + U_1^k(S)\right] - U_1^i(S)(\omega_0^j + \omega_0^k)}{U_1^i(S) + U_1^j(S) + U_1^k(S)} + \frac{U_1^i(S)}{U_1^i(S) + U_1^j(S) + U_1^k(S)} z(S) \tag{12}$$

These formulas give an efficient characterization of the non-anonymous bourse membership pricing.[27] Notice that in both cases the equilibrium membership fee equations consist of the sum of a pure transfer (first term on the right hand side) and a poll tax (second term). The poll tax is such that all traders share the bourse S's formation cost $z(S)$. The pure transfer reflects the trader's valuation of the trading opportunities in bourse S. Then, $\sum_{i\in S}\pi^i = z(S)$. We consider bourse formation costs of the form $z(S) = 3|S|$. These costs are proportional to the bourse size because we seek to emphasize the role of complementarities in traders' attributes (we wish to see the complementarities as the driving force that determine the bourse composition). The membership fee values are $\pi^1(S^1) = \pi^2(S^1) = 3$, $\pi^1(S^6) = \pi^2(S^7) = 4.0022$, $\pi^4(S^6) = \pi^4(S^7) = 1.9978$, $\pi^1(S^3) = \pi^2(S^3) = 3.4889$ and $\pi^4(S^3) = 2.0222$. The indirect utilities are[28]

	$\mathbf{S}^1 = (1,2)$	$\mathbf{S}^6 = (1,4)$	$\mathbf{S}^7 = (2,4)$	$\mathbf{S}^3 = (1,2,4)$
Trader 1	$V^1(S^1) = 1.9218$	$V^1(S^6) = 1.3418$	$u^1(\omega^1) = 1.3488$	$V^1(S^3) = 1.9312$
Trader 2	$V^2(S^1) = 1.9218$	$u^2(\omega^2) = 1.3488$	$V^2(S^7) = 1.3418$	$V^2(S^3) = 1.9312$
Trader 4	$u^4(\omega^4) = 2.0253$	$V^4(S^6) = 2.1662$	$V^4(S^7) = 2.1662$	$V^4(S^3) = 2.3016$

Table 3: Indirect utilities in period 0

[27] An anonymous pricing context would make better (worse) off those traders who value more (less) the bourse than if the context were non-anonymous (not all surplus can be substracted when pricing is anonymous).

[28] The indirect utility of a trader who does not belong to a bourse is given by his utility u^i evaluated in his good endowments (recall that there is only one good and u^i is strictly increasing in the consumption of the good).

In this example with complete markets and proportional bourse formation costs, we find that the three traders prefer to sort in the largest possible bourse, $S^3 = (1, 2, 4)$. This happens even if traders 1 and 2 have to pay a higher membership fee in the three-traders bourse, since the gain in utility in the bourse evaluation process more than offsets this extra cost. Thus, good complementarities in preferences and endowments are sufficient here to obtain a large bourse in equilibrium. ♣

Example 3: (*Size versus tailored efficiency*[29]) This example illustrates the case opposite to Example 2, that poor complementarities alone can lead a small bourse to form in equilibrium. We assume that markets are complete. For this example it is worth considering larger replica bourses with N traders of each type. For example, in the three-traders bourse case, the bourse $S_N^2 = (1, 2, 3)^N$ will denote a bourse composed by N traders of each type. Observe that for any given bourse S_N with m types of traders and N traders of each type, the indirect utility (given in Example 1) of a given trader i is such that $U^i(S) = U^i(S_N)$, for any $N \in \mathbb{N}$.[30] This remains true for all the examples in this paper.

Again, we assume that bourse formation costs are proportional to bourse size in order to isolate the role of complementarities on the equilibrium outcome. In particular, we assume $z(S_N) = 3N|S_N|$, for any bourse S_N. Trader 3's endowment in period 0 is $\omega_0^3 = 5.9$. Our objective here is to compare the small bourse $S^1 = (1, 2)$ with the large bourse $S_N^2 = (1, 2, 3)^N$, with N large, where the third trader has poor complementarities with traders 1 and 2. The indirect utilities in these bourses are

	$\mathbf{S^1} = (1, 2)$	$\mathbf{S_N^2} = (1, 2, 3)^N$
Trader 1	$V^1(S^1) = 1.9218$	$V^1(S_N^2) = 1.8843$
Trader 2	$V^2(S^1) = 1.9218$	$V^2(S_N^2) = 1.8843$
Trader 3	$u^3(\omega^3) = 2.0253$	$V^3(S_N^2) = 1.7498$

Table 4: Indirect utilities at period 0

In Table 4 we see that all traders have a greater indirect utility in bourse $S^1 = (1, 2)$ than in the larger bourse $S_N^2 = (1, 2, 3)^N$, for any N (possibly large). This result contrasts

[29] The properties of self-picked traders and assets motivate the expression *tailored-efficiency*.

[30] To see this, notice that commodities and assets market clearing equations in S_N are the same as in S. Also, the membership pricing expressions (11) and (12) hold true for any N-replica bourse with formation cost $z(S_N) = Nz(S)$. For example, in an N-replica two-traders bourse, the objective function would be $N\left(\ln x_0^i\right) U^i(S_N)+ N(\ln x_0^j)U^j(S_N)$, given the restriction $N\pi^i(S_N) + N\pi^j(S_N) = Nz(S)$. This maximization problem is equivalent to the same problem when $N = 1$. Thus, this result relates to Allouch, Conley, and Wooders [1], where a group strictly bounded in size can achieve all gains to club formation.

with the values given in Table 1, where traders 1 and 2 prefer to be in a larger bourse with trader 3 (there, only the bourse evaluation process was under consideration). However, in the bourse formation process, the benefit for traders 1 and 2 of enlarging the bourse with a third trader with poor complementarities (like trader 3) is not enough to compensate the cost of paying a higher membership fee - in bourse S^1 the membership fee for both traders 1 and 2 is 3, while in bourse S^2_N the membership fee is $\pi^1(S^2) = \pi^2(S^2) = 3.3166$ (and $\pi^3(S^2) = 2.3667$ for trader 3).

We conclude that poor complementarities between trader 3 and traders 1 and 2 leads to a situation where the small bourse S^1 is preferred in equilibrium by traders of types 1 and 2 to the larger bourse S^2_N, for any N-replica. This result identifies poor traders' complementarities (in preferences and endowments) as an important force against the tendency toward a unique bourse. This example is in accordance with Fact Set 2, which suggests that traditional large exchanges are ill-suited to certain types of institutions (e.g., dark liquidity pools). ♣

Remark 1: *What is the effect of the implementation of a Tobin tax on certain (but not all) bourses?* It can easily be shown that for two possible bourses - subject to legislation by regulators - with the same traders and asset structure, traders will participate in the bourse with lower execution rates for their trades. Higher execution rates in a bourse, due to a Tobin tax (see Tobin [24]), can be accommodated in the form of higher formation cost, which penalizes traders in that bourse through higher memberships. As traders are free to move to their most preferred bourse, we can infer that the bourse with such high execution rates (or equivalently, high formation costs) will not be formed. In other words, if traders are free to choose their preferred bourses, then a Tobin tax on the financial transactions in some but not all bourses will not be effective, in the sense that those bourses with a Tobin tax imposed by their respective jurisdictions will not be formed in equilibrium. This result adds to the current international debate on the imposition of an institutional Tobin tax on financial transactions (see Stiglitz [23]).

Example 4: (*Multiple memberships*): Let us now consider an example with multiple memberships. Traders' names are now 5, 6, and 7. Traders' endowments and preferences parameters are $(\omega^5_0, \omega^5_1, \omega^5(1),\ \omega^5(2)) = (15, 6, 2, 2)$, $(\alpha^5_0, \alpha^5_1, \alpha^5(1),\ \alpha^5(2)) = (1/2, 1/2, 1, 0)$, $(\omega^6_0, \omega^6_1, \omega^6(1), \omega^6(2)) = (9, 2, 6, 6)$, $(\alpha^6_0, \alpha^6_1,\ \alpha^6(1), \alpha^6(2)) = (1/2,\ 1,\ 0, 1/2)$, $(\omega^7_0, \omega^7_1, \omega^7(1), \omega^7(2)) = (9, 2, 6, 6)$, and $(\alpha^7_0,\ \alpha^7_1,\ \alpha^7(1), \alpha^7(2)) = (1/2,\ 1, 1/2,\ 0)$. We assume that every possible bourse has an incomplete asset structure with only one asset with payoffs $(1, 1)$. Traders 6 and 7 are similar in preferences and endowments and, therefore, the two interesting bourse structures to compare are $\{S^8, S^9\} = \{(5, 6), (5, 7)\}$ and

$\{S^{10}\} = \{(5,6,7)\}$, one with multiple memberships where trader 5 is common to both bourses and the other with all traders in a single bourse.

We consider that bourse formation costs are $z(S) = 3|S|$ if $S = S^8, S^9$, and $z(S^{10}) = 7|S^{10}|$ if $S = S^{10}$. One possible justification of these bourse formation costs follows a location argument. Trader 5 can be thought of as in between traders 6 and 7, and this location causes the cost of creating the bourse S^8 or S^9 to be smaller than $z(S^{10})$.[31] Indirect utilities are $V^5(\{S^8, S^9\}) = 2.1239$ and $V^6(S^8) = V^7(S^9) = 1.9406$ for structure $\{S^8, S^9\}$, while $V^5(S^{10}) = 1.9512$ and $V^6(S^{10}) = V^7(S^{10}) = 1.2171$ for the unique bourse. For these values, all traders prefer the bourse structure $\{(5,6),(5,7)\}$. Therefore, one insight of this example is that low bourse formation costs for bourses with traders located relatively "close" can explain the existence of multiple memberships in equilibrium.

Another insight is that it is not necessarily true that a multiple memberships scenario is Pareto superior to a unique bourse scenario, or *viceversa*. To see this, we can consider instead a smaller $z(S^{10})$, for instance, if $z(S) = 3|S|$ for $S = S^8, S^9, S^{10}$. Then traders 6 and 7 can be shown to be better off with multiple memberships, but trader 5 would be better off in the unique bourse.[32] ♣

6 Incomplete markets

The issue of the optimality of the equilibrium of the bourse evaluation is related to results reported in earlier literature of general equilibrium with incomplete markets (GEI). Arrow [5] showed that, in an economy with complete markets, any competitive equilibrium is Pareto optimal. Then, Geanakoplos and Polemarchakis [15] demonstrated that if there is market incompleteness and assets pay off in the numeraire commodity, the competitive equilibrium allocations are typically Pareto suboptimal in a strong sense (markets do not make efficient use of the existing assets). In other words, equilibrium efficiency is "constrained" to the market incompleteness. These results naturally extend to our asset trading equilibrium (where the bourse structure is fixed). In the next example we can see that, if the formation costs associated with a bourse with an incomplete asset structure are (appropriately) less than proportional to the bourse size, then a bourse with an incomplete asset structure may result in equilibrium.

[31] For these costs, memberships are $\pi^5(S^8) = \pi^5(S^9) = 4.7619$ and $\pi^6(S^8) = \pi^7(S^9) = 1.2380$ for the multiple membership scenario, whereas $\pi^5(S^{10}) = 10.2310$ and $\pi^6(S^{10}) = \pi^7(S^{10}) = 5.3846$ for the unique bourse.

[32] For these proportional bourse formation costs, indirect utilities for the unique bourse are $V^5(S^{10}) = 2.5456$ and $V^6(S^{10}) = V^7(S^{10}) = 1.8735$.

Example 5 (*Market incompleteness as a consequence of bourse formation costs and complementarities*): We consider an economy with new traders, named traders 8, 9, and 10. Traders' endowments and preferences parameters are $(\omega_0^8, \omega_1^8, \omega^8(1), \omega^8(2)) = (10, 6, 2, 1)$, $(\alpha_0^8, \alpha_1^8, \alpha^8(1), \alpha^8(2)) = (1/2, 1/2, 1, 0)$, $(\omega_0^9, \omega_1^9, \omega^9(1), \omega^9(2)) = (6, 2, 6, 1)$, $(\alpha_0^9, \alpha_1^9, \alpha^9(1), \alpha^9(2)) = (1/2, 1, 1/2, 0)$, $(\omega_0^{10}, \omega_1^{10}, \omega^{10}(1), \omega^{10}(2)) = (6, 2, 6, 1)$ and $(\alpha_0^{10}, \alpha_1^{10}, \alpha^{10}(1), \alpha^{10}(2)) = (1/2, 1/2, 1, 1/2)$.

The possible bourses are $S^{11} = (8, 9)$, $S^{12} = (8, 10)$, $S^{13} = (9, 10)$, and $S^{14} = (8, 9, 10)$. The asset structure is complete, with $A(S) = \{(1, 1), (1, 2)\}$, for bourses $S = S^{12}, S^{13}, S^{14}$, but incomplete for bourse S^{11}, $A(S^{11}) = \{(1, 1)\}$. We consider that bourses S^{12}, S^{13}, and S^{14} have formation costs proportional to their sizes, with $z(S) = 3|S|$, for $S = S^{12}, S^{13}, S^{14}$, but assume that bourse S^{11} has less than proportional formation cost of the form $z(S^{11}) = 2|S^{11}|$ to illustrate that it is cheaper to provide an incomplete asset structure than a complete one.

$\mathbf{S}^{11} = (8, 9)$	$\mathbf{S}^{12} = (8, 10)$	$\mathbf{S}^{13} = (9, 10)$	$\mathbf{S}^{14} = (8, 9, 10)$
$V^8(S^{11}) = 1.7416$	$V^8(S^{12}) = 1.3581$	$V^8(\{8\}) = 1.546$	$V^8(S^{14}) = 1.6734$
$V^9(S^{11}) = 1.7416$	$V^9(\{9\}) = 1.546$	$V^9(S^{13}) = 1.1638$	$V^9(S^{14}) = 1.4728$
$V^{10}(\{10\}) = 2.0805$	$V^{10}(S^{12}) = 1.7262$	$V^{10}(S^{13}) = 1.7741$	$V^{10}(S^{14}) = 1.6973$

Table 5: Indirect utilities at period 0

From the results in Table 5, we conclude that, if bourse S^{11} formation costs are (appropriately) less than proportional to bourse size, then traders 8 and 9 end up strictly preferring the incomplete asset structure associated with bourse S^{11}, even if a complete asset structure is available in another bourse. ♣

The sorting of traders into bourses provides light to new insights concerning markets' efficiency. By analogy with Geanakoplos and Polemarchakis [15], efficiency of the asset trading equilibrium in our bourse economy can only be attained if the bourse structure formed in period 0 is characterized by a complete asset structure in every bourse. If a bourse with an incomplete asset structure is formed, efficiency would be "constrained" to the incompleteness of the asset structure, given the existing technology.

In Example 5 we saw that a bourse with an incomplete asset structure is chosen in equilibrium if formation costs are low enough. Thus, we conclude that the incompleteness of the asset structure is endogenous to the model, and crucially depends on the technology associated with the bourse formation costs. This conclussion adds to the earlier literature on endogenous market incompleteness, which focused on other frictions in the economy, such as the issuing costs faced by intermediaries who offer securities in a context of imperfect competition, or some type of collateral requirements for the household borrowers.

7 Monopolist bourse and demutualization

7.1. *Monopolist bourse*

We now extend our theory of bourses to an alternative scenario (different from the previous competitive one) where a monopolist bourse is considered instead, which decides on the bourse composition. This scenario resembles a situation where a bourse is imposed on a jurisdiction by its regulators. Also, one may think of this monopolist bourse as arising from the anti-competitive behavior of a large trading organization, a matter of concern for financial regulators.[33]

To start, let us consider a monopolist bourse that selectively invites traders by showing a membership plan consisting of the identity of invited traders and their membership fees. Invited traders can then accept it or not. The monopolist bourse's maximization problem is

$$\max_{S\subset\mathbf{I}}\left[\max_{\{\pi^i\}_{i\in S}}\sum_{i\in S}\pi^i(S)-z(S)\text{ s.t. }V^i(S)\geq V^i(\{i\}),\text{ for all }i\in S\right].\tag{13}$$

Observe that if the fees are non-anonymous, then the bourse can extract all the surplus from the traders in the bourse.[34] We denote bourse S's surplus by $\Phi(S)\equiv\sum_{i\in S}\pi^i(S)-z(S)$. Since a higher membership fee decreases traders' indirect utility $V^i(S)$, we find that in equilibrium the monopolist bourse chooses a binding participation constraint, $V^i(S)=V^i(\{i\})$, $\forall i\in S$.

Another benchmark is to consider a monopolist bourse that cannot selectively invite traders (as among national bourses, which were restricted to their national traders). Formally,

$$\max_{\{\pi^i\}_{i\in\mathbf{I}}}\sum_{i\in\mathbf{I}}\pi^i(\mathbf{I})-z(\mathbf{I})\text{ s.t. }V^i(\mathbf{I})\geq V^i(\{i\}),\text{ for all }i\in\mathbf{I}.\tag{14}$$

[33]In January 2008, the US Department of Justice came into serious discussions concerning the potential merger between the Chicago Mercantile Exchange (CME) and the energy exchange Nymex (see "CME in $11bn move for Nymex", Financial Times, January 29, 2008.).

[34]This monoplist's problem contrasts with the following "social planner's problem" $\max_{F(\mathbf{I})\in\mathbf{F}(\mathbf{I})}\left[\sum_{S\in F(\mathbf{I})}\left[\max_{\{\pi^i\}_{i\in S}}\sum_{i\in S}\pi^i(S)-z(S)\ \text{ s.t. }V^i(S)\geq V^i(\{i\}),\ \forall i\in S\right]\right]$.

Notice that the surplus extracted by the social planner is always greater than or equal to the surplus of the monopolist.

The questions we would like to answer here are whether a single large bourse can maximize the surplus, and whether we should allow the bourse to exclude traders. These questions directly relate to the comparison between problems (13) and (14). We address these questions by means of an example.

Example 6 (*Monopolist bourse, non-anonymity, and exclusion*): Consider again traders 1, 2 and 3, examined in Example 3, and bourses $S^1 = (1,2)$, $S^2 = (1,2,3)$, $S^4 = (1,3)$, and $S^5 = (2,3)$. Here, again, formation costs are $z(S) = 3|S|$, for all S. The monopolist bourse's solution to the maximization problem has the participation constraint binding, and therefore, $V^1(S) = V^1(\{1\}) = 1.3488$, $V^2(S) = V^2(\{2\}) = 1.3488$, and $V^3(S) = V^3(\{3\}) = 2.2052$, for any $S = S^1$, S^2, S^4, S^5. Memberships are obtained by the expression $\pi^i(S) = \omega_0^i - \exp(V^i(\{i\})/0.5U^i(S))$. We obtain the following values for the memberships and the surplus of each bourse.

$\Phi(S^1) = 2.7086$	$\Phi(S^2) = 0.907$	$\Phi(S^4) = -1.5691$	$\Phi(S^5) = -1.5691$
$\pi^1(S^1) = 4.3543$	$\pi^1(S^2) = 4.4571$	$\pi^1(S^4) = 3.2463$	n.a.
$\pi^2(S^1) = 4.3543$	$\pi^2(S^2) = 4.4571$	n.a.	$\pi^2(S^5) = 3.2463$
n.a.	$\pi^3(S^2) = 0.9928$	$\pi^3(S^4) = 1.1846$	$\pi^3(S^5) = 1.1846$

Table 6: Bourses' surpluses and memberships

Bourses with a negative surplus are not profitable, and therefore will never be created by a for-profit organization. Given the values in Table 6, we can assert that a large bourse will not always extract the highest possible surplus. In fact, $\Phi(S^2) = 0.907 < 2.7086 = \Phi(S^1)$. This result also tells us that a monopolist bourse should be able to exclude traders in order to maximize surplus. It is important to notice that for this result neither negative externalities among traders' attributes (think of default) nor increasing average bourse formation costs are needed. ♣

Next, let us consider an anonymous pricing setting, more realistic in certain contexts. Then, the monopolist's problem is

$$\max_{S \subset \mathbf{I}} \left[\max_{\pi} \pi|S| - z(S) \text{ s.t. } V^i(S) \geq V^i(\{i\}), \text{ for all } i \in S \right]. \tag{15}$$

Given that $V^i(S)$ is decreasing in the membership fee, in order to include a trader with small gain from participation ($V^i(S) - V^i(\{i\})$ small), the fee must be low enough. Since the fee must be equal for all participants, inviting such traders may negatively affect bourse's profits. Thus, the monopolist bourse may not invite traders with high autarky values, even if they exert strong complementarities to other traders. The issue in this framework is to understand how the anonymous pricing affects the bourse structure and traders' welfare. We pose the following example.

Example 7 (*Monopolist bourse and anonymity*): Consider the bourses analyzed in Example 3, but now with traders being anonymous. For each bourse, the participation constraint is binding for that trader(s) with lowest bourse valuation $V^i(\{i\})$. Then, $\pi(S) = 1.3488$, for all $S = S^1_M, S^2_M, S^4_M, S^5_M$. For this anonymity context, the surpluses are $\Phi(S^1) = 2.7086$, $\Phi(S^2) = -6.279$, $\Phi(S^4) = -3.6308$, and $\Phi(S^5) = -3.6308$. Given these values, we find that there are now bourses that become unprofitable (with respect to Example 6 with non-anonymity) when discrimination among traders is not possible. This is the case of the large bourse $S^2 = (1, 2, 3)$, which would disappear in an anonymous context. Another insight of this example is obtained by looking at the traders' indirect utilities. We find that the traders with a higher bourse valuation improve with respect to the non-anonymous setting (Example 6). These are $V^1(S^4) = 1.7952$, $V^2(S^5) = 1.7952$, $V^1(S^2) = 2.5912$, and $V^2(S^2) = 2.5912$. ♣

7.2. *Recovering efficiency through demutualization*

Finally, we can compare the monopolist bourse scenario with the framework portrayed in previous sections, where traders freely sort into bourses. By looking at the payoffs in Example 3 and payoffs in Examples 6, we can see how allowing the traders to sort freely into bourses provides them a greater indirect utility than if a monopolist bourse were established. Now, by relying on the efficiency result of AW [2], which asserts that a ε_0-equilibrium is in the core for an economy with sufficiently many traders of each attribute, we can assert that the process of demutualizations is Pareto improving, as it recovers the efficiency of the equilibrium of the bourse formation process. Also, as a consequence, there is a membership price for each type of trader such that the sum of the membership prices covers the cost $z(S)$, and such that these membership prices internalize the externalities that some members may impose on the others.

Now, recall that the trading equilibria are constrained efficient if the bourse asset structures are incomplete, which may occur in equilibrium if the bourse formation costs associated with a complete asset structure is sufficiently high (as in Example 5). Thus, we can guarantee efficiency only if the equilibrium for the bourse formation is characterized by a complete asset structure at every bourse. Otherwise, the equilibrium of the bourse economy is constrained efficient, given the existing technology.

In sum, the general equilibrium model proposed in this paper for the process of formation of bourses is well behaved (in the sense that equilibrium exists). In general, inefficiencies may arise in these markets: 1) if the equilibrium bourse structure is characterized by some bourses with incomplete asset structures, which are endogenously determined in equilibrium given the existing technology of bourse formation; and 2) if there is a monopolist bourse. Demutualization policies may alleviate the second type of inefficiency.

8 Final remarks

This paper is pioneering in examining financial market structures and their welfare properties under the new perspective of club theory. This approach revealed interesting insights, such as the possibility of inter-bourse arbitrage, the effect of traders' complementarities on the optimal size of bourses (large bourses versus dark pools of liquidity), and the role of the technology of bourse formation in the structure of the bourses and the incompleteness of the markets.

This paper paves the way to study other important issues in future research. One aspect that captures our attention is to make endogenous the asset structure associated with each bourse. This could be done by distinguishing between issued securities (issued at some intermediate period between periods 0 and 1) and securities borrowed through repo. Also, it would be interesting to allow in our model for default on the asset promises. A model with default, clearing houses and multiple bourse memberships may give rise to a complicated structure with contagion effects, a key aspect in the recent financial crisis.

9 Appendix

Proof of Theorem 1: This Theorem follows by Lemmas 1, 2, and 3 below. For convenience of exposition we present the proof of Proposition 2 immediately after the proof of Lemma 1.

Lemma 1: *Let us assume (A1.ii). Then, for a fixed bourse structure, there exists an asset trading equilibrium.*

Proof of Lemma 1: Let us consider a generalized game where allocated consumption and portfolios are restricted to a closed cube $\mathbf{K} \subseteq \mathbb{R}^{(\Xi+1)L+\sum_{S\in F(\mathbf{I})} J(S)}$ with center at the origin and large enough to contain the double of the aggregate endowment. Since a separate budget constraint must be satisfied at every state ξ and the demand is homogenous of degree zero in spot prices, the prices can be chosen in the simplex. In period 1 the simplex is such that $\sum_{l\in\mathbf{L}} p_{l1} + \sum_{S\in F(\mathbf{I})} \sum_{j\in\mathbf{J}(S)} q_j(S) = 1$, whereas in state ξ of period 2 it is such that $\sum_{l\in\mathbf{L}} p_l(\xi)=1$.

The players of the generalized game are the traders and $(1+\Xi)$ additional auctioneers.

Given a bourse structure $F(\mathbf{I})$, each trader chooses a vector $(x_1^i, x^i(1),...,x^i(\Xi), y^i)(F(\mathbf{I}))$ on $\mathbf{K}$ to maximize $u_1^i(x_1, x(1),...,x(\Xi))$, subject to constraints (4) and (5).

The first auctioneer chooses period 1 commodities prices $p_1 \in \mathbb{R}_+^L$ and assets prices $q \in \mathbb{R}_+^{J(S_1)} \times ... \times \mathbb{R}_+^{J(S_k)}$ in order to maximize

$$B_1 \equiv \sum_{l\neq L} p_{l1} \sum_{i\in\mathbf{I}} (x_{l1}^i - \omega_{l1}^i) + p_{L1} \sum_{i\in\mathbf{I}} (x_{L1}^i + \sum_{S_k\in F[i;\mathbf{I}]} \sum_{j\in J(S_k)} g_j(y_j^i) - \omega_{L1}^i) + \sum_{S_k\in F(\mathbf{I})} \sum_{j\in J(S_k)} q_j \sum_{i\in S_k} y_j^i. \quad (16)$$

The last part of B_1 accounts for the fact that the asset market clearing is achieved for each bourse. In period 2 there is an auctioneer for each node $\xi \in \Xi$ that chooses the commodity prices $p(\xi) \in \mathbb{R}_+^L$ in order to maximize

$$B(\xi) \equiv p(\xi) \sum_{i\in\mathbf{I}} \left(x^i(\xi) - \omega^i(\xi)\right). \quad (17\text{-}\xi)$$

An equilibrium for this generalized game, parametrized in the bourse structure $F(\mathbf{I})$, is a vector $(x_1^I, x^I(1),...,x^I(\Xi), y^I, p, q)(F(\mathbf{I}))$ such that, for each player (the n traders and the $(1+\Xi)$ auctioneers), the respective action solves his optimization problem parameterized by the other players' actions. We have that the generalized game has an equilibrium since it satisfies all the assumptions of Debreu's [9] theorem. In fact, the auctioneers' objective functions (16) and $(17\text{-}\xi)_{\xi\in\mathbf{\Xi}}$ are linear in their respective price variables, and for each period and state, prices are in the simplex. Traders' utilities are continuous and strictly concave by (A1.ii), and their choice variables $(x_1, x(1),...,x(\Xi), y)$ belong to non-empty, convex and compact sets. We can show that for this equilibrium there is no excess of demand of commodities, that is, $\sum_{i\in\mathbf{I}}(x_1^i-\omega_1^i) \leq 0$ and $\sum_{i\in\mathbf{I}}(x^i(\xi)-\omega^i(\xi)) \leq 0$, for every state ξ. First, observe that $B_1 \leq 0$ and $B(\xi) \leq 0$ as B_1 and $B(\xi)$ represent the aggregation of their respective traders' budget constraints. Now we argue that $\sum_{i\in\mathbf{I}}(x_{l1}^i-\omega_{l1}^i) \leq 0$, $l \neq L$. Otherwise, there exists a commodity $l' \neq L$ at period 1 with $\sum_{i\in\mathbf{I}}(x_{l'1}^i-\omega_{l'1}^i) > 0$. But, then the auctioneer would choose $p_{l'1}=1$, a contradiction with $B_1 \leq 0$. Analogously, for good L, $\sum_{i\in\mathbf{I}}(x_{L1}^i+\sum_{S_k\in F[i;\mathbf{I}]}\sum_{j\in J(S_k)} g_j(y_j^i)-\omega_{L1}^i) \leq 0$, which implies $\sum_{i\in\mathbf{I}}(x_{L1}^i-\omega_{L1}^i) \leq 0$ since $g_j(y_j^i) \geq 0$, for all y_j^i. By the same arguments, it also holds that $\sum_{i\in S_k} y_j^i \leq 0$, $\forall j \in J(S_k), \forall S_k$. The proof that there is no excess of supply of commodities in each node

ξ of period 2 follows by similar arguments, taking into account the aggregation of budget constraints in every state and the fact that the aggregated portfolios are non-positive.

We now consider a sequence of increasing closed cubes, $\mathbf{K}^n$, with center at the origin. Then, for each cube in the sequence, fix an equilibrium of the generalized game, $(x_1^{In}, x^{In}(1), ..., x^{In}(\Xi), y^{In})(F(\mathbf{I}))$. Notice that, as shown above, for each n and every i, $(x_1^{in}, x^{in}(1), ..., x^{in}(\Xi))(F(\mathbf{I})) \in [0, \sum_{i\in\mathbf{I}} \omega_1^i] \times \prod_{\xi\in\Xi}[0, \sum_{i\in\mathbf{I}} \omega^i(\xi)]$. Moreover, the corresponding sequence of equilibrium prices belongs to the simplex. Thus, there exists a converging subsequence. Let $(\breve{x}_1^I, \breve{x}^I(1), ..., \breve{x}^I(\Xi), \breve{p}, \breve{q})$ denote the limit of this subsequence.

Let us assume that $\breve{p}_{L1} > 0$ (we will prove this later). We now show that the corresponding subsequence of portfolios also converges. Indeed, the first order necessary and sufficient condition for trader i relative to an asset j in $J(S_k)$, with $S_k \in F[i, \mathbf{I}]$, is $-\tilde{\beta}_1^{n_q}[q_j^{n_q} + p_{L1}^{n_q} D_{y_j^{n_q}} g_j] + \sum_{\xi\in\Xi} \tilde{\beta}^{n_q}(\xi) p_L^{n_q}(\xi) a_j(\xi) = 0$, that is, $\tilde{\beta}_1^{n_q} p_{L1}^{n_q} D_{y_j^{n_q}} g_j = \sum_{\xi\in\Xi} \tilde{\beta}^{n_q}(\xi) p_L^{n_q}(\xi) a_j(\xi) - \tilde{\beta}_1^{n_q} q_j^{n_q}$. Since the function g_j is nonlinear, $D_{y_j} g_j$ is continuous and $D^2_{y_j} g_j > 0$ we conclude that $y_j^{n_q}$ converges, provided that $\tilde{\beta}_1^{n_q}$ and $\tilde{\beta}^{n_q}(\xi)$ also converge and $\lim_{n_q} \tilde{\beta}_1^{n_q} > 0$. We prove that the sequence of Lagrange multipliers is bounded and, therefore, converges. For each truncated economy, we have $u_1^i(x_1^{in_q}, x^{in_q}(1), ..., x^{in_q}(\Xi)) \leq u_1^i(\breve{x}_1^i, \breve{x}^i(1), ..., \breve{x}^i(\Xi))$. Actually, there are non-negative multipliers $(\tilde{\beta}_1^{n_q}, \tilde{\beta}^{n_q}(1), ..., \tilde{\beta}^{n_q}(\Xi))$ such that, for each nonnegative bundle, the following saddle point property is satisfied (see Rockafellar [21, Theorem 28.3])

$$\mathcal{L}^i(x_1, x(1), ..., x(\Xi), y, \tilde{\beta}_1^{n_q}, \tilde{\beta}^{n_q}(1), ..., \tilde{\beta}^{n_q}(\Xi)) \leq u_1^i(\breve{x}_1^i, \breve{x}^i(1), ..., \breve{x}^i(\Xi)),$$

which turns into $\tilde{\beta}_1^{n_q} p_1^{n_q} \omega_1^i + \sum_{\xi\in\Xi} \tilde{\beta}^{n_q}(\xi) p^{n_q}(\xi) \omega^i(\xi) \leq u_1^i(\breve{x}_1^i, \breve{x}^i(1), ..., \breve{x}^i(\Xi))$ if we choose $(x_1, x(1), ..., x(\Xi), y) = 0$. This inequality implies that the sequence of multipliers is bounded since $\omega_1^i \gg 0$, $\omega^i(\xi) \gg 0$ for each state ξ, $p^{n_q}(\xi)$ belongs to the simplex for each state ξ and $\breve{p}_{L1} > 0$.

Let $\breve{y}^i$ be the portfolio limit. We prove now that $(\breve{x}_1^I, \breve{x}^I(1), ..., \breve{x}^I(\Xi), \breve{y}^I, \breve{p}, \breve{q})(F(\mathbf{I}))$ is an asset trading equilibrium. All budget constraints and inequalities $\sum_{i\in\mathbf{I}}(\breve{x}_1^i - \omega_1^i) \leq 0$, $\sum_{i\in\mathbf{I}}(\breve{x}^i(\xi) - \omega^i(\xi)) \leq 0$, $\sum_{i\in S_k} \breve{y}_j^i \leq 0$, $\forall j \in J(S_k)$ and $\forall S_k$, are satisfied, since all of them hold in each truncated economy and, therefore, still hold in the limit.

To obtain market clearing in all markets we look at the first order conditions of the optimization problem of the first period auctioneer, who chooses p_1 and q. Let $\breve{\mu}$ denote the Lagrange multiplier for the constraint $\sum_{l\in\mathbf{L}} \breve{p}_{l1} + \sum_{S\in F(\mathbf{I})} \sum_{j\in\mathbf{J}(S)} \breve{q}_j(S) = 1$. Then, $\sum_{i\in\mathbf{I}}(\breve{x}_{l1}^i - \omega_{l1}^i) = \breve{\mu}$ for $l \neq L$, $\sum_{i\in\mathbf{I}}(\breve{x}_{L1}^i + \sum_{S_k\in F[i;\mathbf{I}]} \sum_{j\in J(S_k)} g_j(\breve{y}_j^i) - \omega_{L1}^i) = \breve{\mu}$, and $\sum_{i\in S_k} \breve{y}_j^i = \breve{\mu}$, $\forall j \in J(S_k), \forall S_k$. But by the Walras' law we can write, $\sum_{l\neq L} \breve{p}_{l1} \sum_{i\in\mathbf{I}}(\breve{x}_{l1}^i -$

$\omega^i_{l1}) + \breve{p}_{L1} \sum_{i\in\mathbf{I}}(\breve{x}^i_{L1} + \sum_{S_k\in F[i;\mathbf{I}]} \sum_{j\in J(S_k)} g_j(\breve{y}^i_j) - \omega^i_{L1}) + \sum_{S_k\in F(\mathbf{I})} \sum_{j\in J(S_k)} \breve{q}_j \sum_{i\in S_k} \breve{y}^i_j =$ $\breve{\mu} \sum_{l\neq L} \breve{p}_{l1} + \breve{\mu}\breve{p}_{L1} + \breve{\mu} \sum_{S_k\in F(\mathbf{I})} \sum_{j\in J(S_k)} \breve{q}_j = \breve{\mu} \left(\sum_{l\in\mathbf{L}} \breve{p}_{l1} + \sum_{S\in F(\mathbf{I})} \sum_{j\in\mathbf{J}(S)} \breve{q}_j(S)\right) = 0.$ Since $\sum_{l\in\mathbf{L}} \breve{p}_{l1} + \sum_{S\in F(\mathbf{I})} \sum_{j\in\mathbf{J}(S)} \breve{q}_j(S) = 1$ then $\breve{\mu} = 0$ which implies market clearing in all markets of period 1. The proof of market clearing in each state of nature for period 2 follows the same argument. Notice that market clearing in the asset market simplifies the Walras' law in each state of nature to $\breve{p}(\xi) \sum_{i\in\mathbf{I}}(\breve{x}^i(\xi) - \omega^i(\xi)) = 0$.

Now, we show that, given the structure $F(\mathbf{I})$ and prices $(\breve{p}, \breve{q})(F(\mathbf{I}))$, the vector $(\breve{x}^i_1, \breve{x}^i(1), ..., \breve{x}^i(\Xi), \breve{y}^i)$ is an optimal solution for consumer i with utility $u^i_1(x_1, x(1), ..., x(\Xi))$. Suppose it were not, say $(\hat{x}^i_1, \hat{x}^i(1), ..., \hat{x}^i(\Xi), \hat{y}^i)$ is budget feasible at $(\breve{p}, \breve{q})(F(\mathbf{I}))$, and $u^i_1(\hat{x}^i_1, \hat{x}^i(1), ..., \hat{x}^i(\Xi)) > u^i_1(\breve{x}^i_1, \breve{x}^i(1), ..., \breve{x}^i(\Xi))$. For n large enough $(\breve{x}^i_1, \breve{x}^i(1), ..., \breve{x}^i(\Xi), \breve{y}^i)$ belongs to the interior of the cube $\mathbf{K}^n$, and for λ small enough, $\lambda(\hat{x}^i_1, \hat{x}^i(1), ..., \hat{x}^i(\Xi), \hat{y}^i) + (1-\lambda)(\breve{x}^i_1, \breve{x}^i(1), ..., \breve{x}^i(\Xi), \breve{y}^i)$ belongs to $\mathbf{K}^n$, is budget feasible at prices $(\breve{p}, \breve{q})(F(\mathbf{I}))$ and $u^i_1(\lambda(\hat{x}^i_1, \hat{x}^i(1), ..., \hat{x}^i(\Xi)) + (1-\lambda)(\breve{x}^i_1, \breve{x}^i(1), ..., \breve{x}^i(\Xi))) > u^i_1(\breve{x}^i_1, \breve{x}^i(1), ..., \breve{x}^i(\Xi))$ by strict concavity of the utility function. By continuity of preferences, $\lambda(\hat{x}^i_1, \hat{x}^i(1), ..., \hat{x}^i(\Xi), \hat{y}^i) + (1-\lambda)(x^{in_q}_1, x^{in_q}(1), ..., x^{in_q}(\Xi), y^{in_q})$ would be chosen instead of $(x^{in_q}_1, x^{in_q}(1), ..., x^{in_q}(\Xi), y^{in_q})$ in the truncated economy associated to $\mathbf{K}^{n_q}$ at prices $(p^{n_q}, q^{n_q})(F(\mathbf{I}))$, a contradiction.

Finally, let us prove that $\breve{p}_{L1} \neq 0$. Suppose $\breve{p}_{L1} = 0$. Let e_{L1} be the canonical vector in the direction of this commodity. Monotonicity allows to conclude that $u^i_1((\breve{x}^i_1, \breve{x}^i(1), ..., \breve{x}^i(\Xi)) + ke_{L1}) > u^i_1(\breve{x}^i_1, \breve{x}^i(1), ..., \breve{x}^i(\Xi))$. Let $k = \min_{l\in\mathbf{L}} \omega^i_{l1}$, then for n large enough, $p^{n_q}_{L1} < 1$ and $u^i_1((1-p^{n_q}_{L1})(x^{in_q}_1, x^{in_q}(1), ..., x^{in_q}(\Xi)) + ke_{L1}) > u^i_1(x^{in_q}_1, x^{in_q}(1), ..., x^{in_q}(\Xi))$. But the bundle $((1-p^{n_q}_{L1})(x^{in_q}_1, x^{in_q}(1), ..., x^{in_q}(\Xi)) + ke_{L1}, (1-p^{n_q}_{L1})y^{in_q})$ would be affordable at prices (p^{n_q}, q^{n_q}) which contradicts the fact that $(x^{in_q}_1, x^{in_q}(1), ..., x^{in_q}(\Xi), y^{in_q})$ is an equilibrium for the truncated economy $\mathbf{K}^{n_q}$. ■

Proof of Proposition 2: This proof is a consequence of the following four steps.

Step 1: *$\tilde{x}^i(F[i; I])$ is a C^1 function in prices p and q.*

For this, notice that this proof does not follow straightforward the proof of Geanakoplos and Polemarchakis [15, Section 3] since in our context with multiple bourse membership it is possible that there exists the same asset in two bourses to which a trader might belong. This introduces a linear dependence in the asset structure for this trader. That is the asset structure may not be full column rank. The non-linear function g allows us to solve this problem, in the same way as the bid-ask spread solves an analogous problem in Faias [13].

Let us fix $p_{L1} = 1$ and $p_L(\xi) = 1$, for every ξ. The first order necessary and sufficient conditions for trader i's problem are:

$$D_1 u_1^i - \tilde{\beta}_1 p_1 = 0$$
$$D_\xi u_1^i - \tilde{\beta}(\xi) p(\xi) = 0, \xi = 1, ..., \Xi$$
$$-p(\xi)(x(\xi) - \omega^i(\xi)) + A_\xi^i y = 0, \xi = 1, ..., \Xi$$
$$\tilde{\beta}^T A^i - \tilde{\beta}_1 [q + D_y g] = 0$$
$$-p_1(x_1 - \omega_1^i) - qy - g(y) = 0$$

where T refers to the transpose of a matrix and $g(y) = \sum_j g_j(y_j)$. The columns of the return matrix $A^i = |\cdots A(S) \cdots|$ are those $A(S)$ with $S \in F[i, \mathbf{I}]$. Then, the element A_ξ^i denotes the line ξ of the return matrix A^i.

The Jacobian matrix with the second order derivatives with respect to $(x_1, x(\xi), \tilde{\beta}(\xi), y, \tilde{\beta}_1)$, with $x(\xi)$ and $\tilde{\beta}(\xi)$ generic elements of the corresponding Ξ-vector, is:

$$\mathbf{J} = \begin{bmatrix} D_1^2 u_1^i & 0 & 0 & 0 & -p_1 \\ 0 & D_\xi^2 u_1^i & -p(\xi) & 0 & 0 \\ 0 & -p(\xi)^T & 0 & A^i & 0 \\ 0 & 0 & A^{iT} & -\tilde{\beta}_1 D_y^2 g & -q - D_y g \\ -p_1 & 0 & 0 & -q^T - D_y g^T & 0 \end{bmatrix}$$

It is easy to see that the matrix $\mathbf{J}$ is non-singular. In fact, let $z = (\breve{x}_1, \breve{x}(\xi), \breve{\beta}(\xi), \breve{y}, \breve{\beta}_1)$ such that $\mathbf{J}z = 0$, then $z^T \mathbf{J} z = 0$, and using $\mathbf{J}z = 0$, it reduces to $\breve{x}_1^T (D_1^2 u_1^i) \breve{x}_1 + \breve{x}(\xi)^T (D_\xi^2 u_1^i) \breve{x}(\xi) - \breve{y}^T (v_1 D_y^2 g) \breve{y}$. Notice that this last equality can be written as

$$\begin{bmatrix} \breve{x}_1^T & \breve{x}(\xi)^T & \breve{y}^T \end{bmatrix} \begin{bmatrix} D_1^2 u_1^i & 0 & 0 \\ 0 & D_\xi^2 u_1^i & 0 \\ 0 & 0 & -\breve{\beta}_1 D_y^2 g \end{bmatrix} \begin{bmatrix} \breve{x}_1 \\ \breve{x}(\xi) \\ \breve{y} \end{bmatrix}$$

which implies $\breve{x}_1 = 0$, $\breve{x}(\xi) = 0$, and $\breve{y} = 0$ by negative definiteness of $D^2 u_1^i$ and $\breve{\beta}_1 D_y^2 g$. Then, back to $\mathbf{J}z = 0$, we obtain $\breve{\beta}(\xi) = 0$. Finally, again with $\mathbf{J}z = 0$, $\breve{\beta}_1 = 0$. Therefore, by the implicit function theorem, we conclude that individual excess demand is a C^1 function.

Step 2: For any choice of utilities $U \equiv (u_1^i)_{i \in \mathbf{I}}$ in a given utility space $\mathbf{U}$, there exists a generic set $\mathbf{W}(U)$ of endowments, such that for every economy $(\mathbf{I}, \alpha)$, $\alpha(i)$ $(i \in \mathbf{I})$ characterized by u_1^i and $(\omega^i)_{i \in \mathbf{I}} \in \mathbf{W}(U)$, the set of asset trading equilibria is a continuously differentiable function of the endowments.

This proof follows the lines of Geanakoplos and Polemarchakis' [15] proof of generic regularity. Our framework is different however, as we must adapt their proof to an economy where: 1) the asset market clearing occurs in each bourse; 2) the trading period accounts for assets *and* commodities; and 3) trading of assets involve paying g, which is itself a function of the asset trading.

Denote the price domain in period 1 by $\mathbf{M}_1 = \mathbb{R}^{L-1}_{++} \times \mathbb{R}^{\sum_{S\in F(\mathbf{I})} J(S)}_{+}$. In state ξ of period 2 the price domain is $\mathbf{M}(\xi) = \mathbb{R}^{L-1}_{++}$. Then, let $\mathbf{M} = \mathbf{M}_1 \times \mathbf{M}(1) \times ... \times \mathbf{M}(\Xi)$. In every node we can normalize the price of the numeraire commodity to be 1. Denote by $f : \mathbf{U} \times \mathbf{W} \times \mathbf{M}_+ \rightarrow \mathbb{R}^{(L-1)(\Xi+1)} \times \mathbb{R}^{\sum_{S\in F(\mathbf{I})} J(S)}$ the aggregate excess demand funcion of commodities (other than the numeraire) and assets, given utilities, endowments, commodity prices, and asset prices. Let us fix the utilities to $\mathcal{U}$ and show that f restricted to $\mathcal{U}$, denoted by $f_{|\mathcal{U}}$, is transverse to 0 (see Geanakoplos and Polemarchakis [15, Section 5]. That is, if for all $(\omega, p, q) \in \mathbf{W} \times \mathbf{M}_+$ with $f_{|\mathcal{U}}(\omega, p, q) = 0$, the Jacobian matrix $D_{(\omega,p,q)} f_{|\mathcal{U}}$ has full rank. This amounts to showing that there exists a set of independent vectors of directional derivatives that has dimension $(L-1)(\Xi+1) + \sum_{S\in F(\mathbf{I})} J(S)$.

Let us fix an element $(\omega, p, q) \in f^{-1}_{|\mathcal{U}}(0)$ and a given trader i. Now, consider an increase of one unit in $\omega^i_l(\xi)$ with $l \in \mathbf{L}\backslash L$, and a decrease in the endowment of the numeraire good, $\omega^i_L(\xi)$, in $p_l(\xi)$ units. Trader i's demand of good l remains unchanged in ξ , but the total supply in the l-commodity market in state ξ has increased one unit. Thus, there is a net effect of aggregate excess demand of $(0, ..., -1, ..., 0)$. This same argument also holds in period 1.

Consider a bourse S and let trader k be the trader with the only membeship in this bourse S (guaranteed by assumption). Now, for each asset $j(S) = 1, ..., J(S)$ in this bourse, we can increase $\omega^k_L(\xi)$ by $a_{j(S)}(\xi)$, for all ξ, and decrease ω^k_{L1} by $q_{j(S)}+(g_{j(S)}(y^k_{j(S)})-g_{j(S)}(y^k_{j(S)}-1))$. The *only* effect on trader k's demand is a decrease in asset $j(S)$ by one unit. As a consequence, the aggregate excess supply of asset $j(S)$ is now $(0, ..., -1, ..., 0)$. This argument holds for any asset $j(S)$ in each bourse S. This proves Step 2.

Let us conclude this Step 2 with two observations. First, notice that in each bourse we need to work with the trader k chosen above, since in an environment of multiplicity of bourse memberships, a trader who belongs to two different bourses could have available the same asset in two different bourses. If this occurs, nothing guarantees that the previous argument holds. Second, the assumption of matrix $A(S)$ having full rank is needed in this proof to guarantee that the increase of $\omega^k_L(\xi)$ by $a_{j(S)}(\xi)$ is offset by a change in trader k's demand of asset $j(S)$, and is not offset by an equivalent change in the demand of the co-linear assets.

Step 3: There exists a generic set $\mathbf{U}'\times\mathbf{W}'$, *such that for every economy* $(\mathbf{I}, \alpha)$, $\alpha(i)$ $(i \in \mathbf{I})$ *with* $(u^i_1)_{i\in\mathbf{I}} \in \mathbf{U}'$ *and* $(\omega^i)_{i\in\mathbf{I}} \in \mathbf{W}'$, *the set of asset trading equilibria is a continuously differentiable function of both the endowment and the utility assignment.*[35]

[35] A set is a continuously differentiable function if all its elements are continuously differentiable functions.

The proof mimics Geanakoplos and Polemarchakis' [15] proof of generic strong regularity, since for that proof the asset positions are fixed, and hence our bourse economy does not pose any further complication to their arguments (observe that for this proof we need to show that $f_{|\mathcal{U}}$ is transverse to 0, which has already been proven in the previous Step 2).

Step 4: There exists a generic set of economies for which, given $\lambda > 0$, there is a $\gamma > 0$ such that for any set I and pair of economies (I, α) and (I, β), if $d(\alpha(i), \beta(i)) \leq \gamma$ for any i, then $|U_1^{\alpha(i)}(F[i; \mathbf{I}^{\alpha}]) - U_1^{\beta(i)}(F[i; \mathbf{I}^{\beta}])| < \lambda$.

The proof of this last step follows by the continuity of u_1^i (by A1.ii) and the continuity of asset trading equilibria (Step 3). ■

Now, at this part of the proof of Theorem 1, we must observe that, given a bourse structure, the asset trading equilibrium may not be unique. This would imply that there is an indirect utility $U_1^i(F[i; \mathbf{I}])$ for each equilibrium solution, and thus more than one function $V^i(x_0^i, F[i; \mathbf{I}])$. Existence of an equilibrium for the bourse economy would require choosing a measurable selector of the equilibrium correspondence $E(\cdot)$. Then, we have to consider that for each bourse structure $F(\mathbf{I})$, the utility U_1^i is evaluated at $\tilde{x}^i(F[i; \mathbf{I}])$, the respective equilibrium consumption bundle of trader i at the equilibrium selection. The next proposition asserts the existence of a measurable selection.

Lemma 2: *There exists a measurable selection $\tilde{x}^I(F(\mathbf{I})) = (\tilde{x}^i(F[i; \mathbf{I}]) : i \in \mathbf{I})$ for the equilibrium correspondence $E(F(\mathbf{I}))$.*

Proof of Lemma 2: The proof follows by the Kuratowski-Ryll-Nardzewski measurable selection theorem (a weak measurable correspondence with non-empty closed values into a separable metrizable space admits a measurable selection)[36]. In fact, we have that $\mathbf{F}(\mathbf{I})$ is a finite set, and therefore the equilibrium correspondence $E(\cdot)$ defined in $\mathbf{F}(\mathbf{I})$ is trivially a weak measurable correspondence (see Aliprantis and Border [3, p. 600]). The correspondence takes values in the positive coordinate subset of a finite dimensional space and therefore it follows immediately that it is a separable metrizable space.

The correspondence $E(\cdot)$ takes closed values, i.e., if $(x_1^{I,s}, x^{I,s}(1), ..., x^{I,s}(\Xi), y^{I,s}, p^s, q^s)$ is a sequence in $E(F(\mathbf{I}))$ that converges to $(x_1^I, x^I(1), ..., x^I(\Xi), y^I, p, q)$, then $(x_1^I, x^I(1), ..., x^I(\Xi), y^I, p, q)$ also belongs to $E(F(\mathbf{I}))$. Given an equilibrium sequence, if we consider the

[36] We remark that the equilibrium correspondence is defined in the finite set of bourse structures, and therefore, a continuous measurable selector is not needed. Continuous selectors are in general used to construct continuous objective functions. Thus, they only fit if the correspondence is defined in a continuum set. See for instance Mas-Collel and Nachbar [19].

budget constraints of each trader and pass to the limit, we obtain that in the limit the budget constraint of each trader is satisfied. The same reasoning allows us to prove that the market clearing also holds in the limit.

Finally, it remains to show that in the limit each trader is maximizing his utility. Suppose not, so for a trader i there exists another bundle $(\breve{x}^i, \breve{y}^i)$ which is budget feasible and such that $u_1^i(\breve{x}^i) > u_1^i(\tilde{x}^i)$. Now, let $(\check{x}^i, \check{y}^i) = (\lambda\tilde{x}^{i,s} + (1-\lambda)\breve{x}^i, \lambda y^{i,s} + (1-\lambda)\breve{y}^i)$ with $\lambda \in [0,1]$. Observe that $(\check{x}^i, \check{y}^i)$ is budget feasible for s large enough and for λ close to one. Moreover, by continuity we have that $u_1^i(\breve{x}^i) > u_1^i(\tilde{x}^{i,s})$, for s large enough. Then, the strict quasiconcavity implies that $u_1^i(\check{x}^i) = u_1^i(\lambda\tilde{x}^{i,s} + (1-\lambda)\breve{x}^i) > u_1^i(\tilde{x}^{i,s})$. This is a contradiction because $((\tilde{x}^{i,s})_{i\in\mathbf{I}}, y^{I,s}, p^s, q^s)$ was an equilibrium for the given bourse structure $F(\mathbf{I})$.■

An immediate consequence of Lemma 2 is that $V^i(x_0, F[i;\mathbf{I}])$ is well defined.

Lemma 3: *Let us assume that A1, A2, and A3 hold. Then there exists a generic set of bourse economies for which there is a $c(\varepsilon_0)$-equilibrium with possibly ever-increasing gains from larger bourses.*

Proof of Lemma 3: To prove Lemma 3 we need to assure that all assumptions required in AW [2008, Theorem 2] are satisfied. In the next items we rewrite AW's assumption in our notation, to make clear how our steps proceed in the proofs.

First we indicate that our assumption (A1.i) over utility $u_0^i(x_0)$ implies AW's assumptions (a) monotonicity, (b) continuity, and (c) convexity on $V^i(\cdot, F[i;\mathbf{I}])$.

AW's condition (d) "Desirability of endowment" can be rewritten with our notation as follows:

$$\text{if } u_0^i(\omega_0^i - \tau\bar{1})U_1^i(\{i\}) < u_0^i(x_0^i)U_1^i(F[i;\mathbf{I}]), \text{ then } x_0 \gg 0.$$

We show this by contradiction. Assume $x_{l0} = 0$ for some l. Then, $u_0^i(x_0^i)U_1^i(F[i;\mathbf{I}]) = 0$ by (A1.iii). Now, $u_0^i(\omega_0^i - \tau\bar{1})U_1^i(\{i\}) > 0$ since $\omega_0^i - \tau\bar{1} \gg 0$ and $u_0^i(x_0)$ is increasing (by (A1.i)). Finally, $U_1^i(\{i\}) \geq 0$ since $U_1^i(\{i\}) \geq u_1^i(\omega_1^i, \omega^i(1), ..., \omega^i(\Xi)) > 0$ by (A1.vi). Thus, we obtain an impossibility.

AW's condition (e) "Private goods are valuable" says that, given any attribute θ and any $\varepsilon > 0$, there is $\rho_\varepsilon^\theta > 0$ such that, for all $i \in \mathbf{I}$ with $\alpha(i) = \theta$ and all $x_0^i \in \mathbb{R}_+^L$, $V^i(x_0^i, F[i;\mathbf{I}]) + \rho_\varepsilon^\theta < V^i(x_0^i + \varepsilon\bar{1}, F[i;\mathbf{I}])$ holds. This assumption is implied by our assumption (A1.iv). Actually, given $\varepsilon > 0$ and given $\rho_\varepsilon^\theta > 0$ satisfying assumption (A1.iv), and for any x_0^i, we have $(u_0^i(x_0^i + \varepsilon\bar{1}) - u_0^i(x_0^i))\, U_1^i(F[i;\mathbf{I}]) \geq (u_0^i(x_0^i + \varepsilon\bar{1}) - u_0^i(x_0^i))\, \delta > \rho_\varepsilon^\theta$, where the first inequality follows by the optimality of the equilibrium $\tilde{x}(F[i;\mathbf{I}])$ and the definition of δ, whereas the last inequality follows by assumption (A1.iv).

AW's condition (g), "Continuity with respect to attributes 2", says that, given $\varepsilon > 0$, there exists $\lambda > 0$ such that for any set $\mathbf{I}$ and pair of economies $(\mathbf{I}, \alpha)$ and $(\mathbf{I}, \beta)$, if $d(\alpha(i), \beta(i)) \leq \lambda$ for any i, then $\omega_0^{\alpha(i)} \leq \omega_0^{\beta(i)} + \varepsilon\bar{1}$, where $\bar{1} = (1, ..., 1) \in \mathbb{R}^L$. This is precisely what was assumed in (A1.i).

AW's assumption (h), "Continuity with respect to attributes 3" is precisely our assumption (A2).

Let us prove AW's assumption (f), which says that given $\varepsilon > 0$, there exists $\gamma > 0$, such that, for any set $\mathbf{I}$ and pair of economies $(\mathbf{I}, \alpha)$ and $(\mathbf{I}, \beta)$, if $d(\alpha(i), \beta(i)) \leq \gamma$, then $V^{\alpha(i)}(x_0^i, F[i; \mathbf{I}^\alpha]) < V^{\beta(i)}(x_0^i + \varepsilon\bar{1}, F[i; \mathbf{I}^\beta])$, for any i and any $x_0^i \in \mathbb{R}^L$. Proposition 2 asserts that, for each bourse structure $F(\mathbf{I})$, there is a generic set $\mathbf{U}' \times \mathbf{W}'(F(\mathbf{I}))$ where the assets trading equilibrium is continuous in traders' attributes. Now, the finite intersection $\mathbf{U}'' \times \mathbf{W}'' \equiv \bigcap_{F(\mathbf{I}) \in \mathbf{F}(\mathbf{I})} (\mathbf{U}' \times \mathbf{W}'(F(\mathbf{I})))$ is a generic set, where the assets trading equilibrium is continuous in traders' attributes for every bourse structure. Given an economy $(\mathbf{I}, \alpha)$ belonging to the generic set $\mathbf{U}'' \times \mathbf{W}''$, we can find a compact subset of economies containing $(\mathbf{I}, \alpha)$ (since $\mathbf{U}'' \times \mathbf{W}''$ is open) where Proposition 2 holds. We now prove that assumption (f) of AW holds in this compact set of economies for $x_0^i \leq \sum \omega_0^i + \varepsilon$, with $\varepsilon > 0$.[37]. Now, given $\varepsilon > 0$, by assumption (A1.v), there exists $\lambda_\varepsilon > 0$ and $\gamma_1 > 0$ such that if $d(\alpha(i), \beta(i)) \leq \gamma_1$, then $u_0^{\beta(i)}(x_0^i + \varepsilon\bar{1}) - u_0^{\alpha(i)}(x_0^i) > \lambda_\varepsilon$. By Proposition 2, there exists $\gamma_2 > 0$ such that, if $d(\alpha(i), \beta(i)) \leq \gamma_2$, then $|U_1^{\beta(i)}(F[i; \mathbf{I}]) - U_1^{\alpha(i)}(F[i; \mathbf{I}])| < \frac{\underline{u}_1}{\bar{u}_0}\lambda_\varepsilon$, where $\bar{u}_0$ and $\underline{u}_1$ are the upper and lower bounds in assumption (A1.vi). Now, we prove that there is a $\gamma > 0$ such that, if $d(\alpha(i), \beta(i)) \leq \gamma$ for all i, the following inequality holds $V^{\alpha(i)}(x_0^i, F[i; \mathbf{I}^\alpha]) < V^{\beta(i)}(x_0^i + \varepsilon\bar{1}, F[i; \mathbf{I}^\beta])$, which can be written as

$$u^{\alpha(i)}(x_0^i) U_1^{\alpha(i)}(F[i; \mathbf{I}]) < u^{\beta(i)}(x_0^i + \varepsilon\bar{1}) U_1^{\beta(i)}(F[i; \mathbf{I}]).$$

This inequality is equivalent to

$$u^{\beta(i)}(x_0^i + \varepsilon\bar{1}) - u^{\alpha(i)}(x_0^i) > \frac{u^{\alpha(i)}(x_0^i)}{U_1^{\beta(i)}(F[i; \mathbf{I}])}(U_1^{\alpha(i)}(F[i; \mathbf{I}]) - U_1^{\beta(i)}(F[i; \mathbf{I}])).$$

If $\gamma = \min\{\gamma_1, \gamma_2\}$, then the previous inequality holds. To see this, notice that if $d(\alpha(i), \beta(i)) \leq \gamma$ for all i, then

$$\begin{aligned} u^{\beta(i)}(x_0^i + \varepsilon\bar{1}) - u^{\alpha(i)}(x_0^i) > \lambda_\varepsilon > (\bar{u}_0/\underline{u}_1)(U_1^{\alpha(i)}(F[i; \mathbf{I}]) - U_1^{\beta(i)}(F[i; \mathbf{I}])) \geq \\ \geq (u^{\alpha(i)}(x_0^i)/U_1^{\alpha(i)}(F[i; \mathbf{I}]))(U_1^{\alpha(i)}(F[i; \mathbf{I}]) - U_1^{\beta(i)}(F[i; \mathbf{I}])). \end{aligned}$$

[37] Observe that AW [2, p. 271-272] only require assumption f) to be satisfied for a consumption x_0^i bounded above by the aggregate endowments plus some $\varepsilon > 0$.

It remains to show that our economy satisfies AW's assumption "Desirability of wealth", which says that there is $x_0^* \in \mathbb{R}_+^L$ and an integer η such that for any economy $(\mathbf{I}, \alpha)$ and any $i \in \mathbf{I}$, there is a coalition $S \in \mathbf{I}$ with $|S| \leq \eta$ and a club structure $F(S)$ satisfying $V^i(x_0^i + x_0^*, F[i; S]) \geq V^i(x_0^i, F[i; \mathbf{I}])$, for any $F(\mathbf{I})$ and any $x_0^i \in \mathbb{R}_+^L$. Notice that by (A3), $\tilde{x}(\{i\})$ satisfies $u_0^i(x_0^i + x_0^*) u_1^i(\tilde{x}(\{i\})) \geq u_0^i(x_0^i)\ u_1^i(\sum_i (\omega_1^i, \omega^i(1), ..., \omega^i(\Xi)))$, and also observe that $u_0^i(x_0^i) u_1^i(\sum_i (\omega_1^i, \omega^i(1), ..., \omega^i(\Xi))) \geq u_0^i(x_0^i)\ u_1^i(F[i; \mathbf{I}])$. Then, transitivity implies that $u_0^i(x_0^i + x_0^*) u_1^i(\tilde{x}(\{i\})) \geq u_0^i(x_0^i) u_1^i(F[i; \mathbf{I}])$, which satisfies the "Desirability of wealth" assumption for a bourse $S = \{i\}$ (and therefore, for $\eta = 1$ in AW's terminology).■
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