Information Sharing Rules and the Veto Mechanism

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Abstract. In a scenario with a continuum of asymmetrically informed agents, we analyze how the initial information of a trader may be altered when she becomes a member of a coalition. We elaborate on the informational feasibility constraints that the veto mechanism which underlies the core solutions requires.

In a perfect competition frame, either arbitrarily small coalitions (Schmeidler, 1972) or, alternatively, large coalitions (Vind, 1972), are enough to block any allocation which is not in the core. We set examples that show that these results may fail in markets with asymmetric information. However, under mild assumptions, we extend the characterizations of the core provided by Vind and Schmeidler to economies with asymmetrically informed traders. We then focus on information sharing rules based on the coalitions' size.

Assuming the existence of coalitions to which the sharing rule associates an information finer than all the others, we show that the corresponding cores coincide with the one defined by this finest information. Finally, characterizations for the weak fine, the fine and the private core are obtained as particular cases of this equivalence theorem.

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1 Introduction

This paper focuses on the veto mechanism which underlies the core solutions for economies with a continuum of asymmetrically informed agents that trade a finite number of commodities.

Asymmetries in information create market imperfections which are particularly relevant to the rules that regulate the information sharing within coalitions. In this framework the information that each trader initially possesses may vary when she becomes a member of a group. For instance, this variation can occur as a result of an information sharing process among traders belonging to the same coalition or, on the contrary, it could be a consequence of some rules that prevent the use of information other than the common one.

This aspect has been widely acknowledged by the literature and several alternative notions of core have been proposed such as the fine, the coarse, and the private core. In the fine core, introduced by Wilson (1978), traders in a coalition pool their initial private information whereas in the coarse core, also introduced by Wilson (1978), traders within each coalition only use their common information. In the private core (Yannelis, 1991), the information of an individual is not modified when a coalition is formed, that is, each member maintains her private information independently of the coalition she belongs to.

In this paper we go a step further by focusing on the information changes that may occur when a trader becomes member of a coalition. To this end, we consider the notion of information sharing rule introduced by Allen (2006) that allows for a great variety of possibilities on trader's information inside a coalition.

We look at the measure of blocking coalitions when traders are asymmetrically informed and analyze the results which Schmeidler (1972) and Vind (1972) provided for complete information economies. These results gave a sharper interpretation to Aumann's core equivalence theorem (Aumann, 1964) as a characterization of perfect competition. Schmeidler showed that in an atomless economy, where finitely many commodities are traded, any allocation that is not blocked by "small" coalitions is competitive. Vind's result is symmetric; any non-competitive allocation can be blocked by a "big" coalition. We state examples which show that, in general, these results do not hold in the framework of asymmetric information economies. This emphasizes that asymmetries in the information may produce market imperfections. Our first main results provide, under mild assumptions on the information sharing rule, extensions of both Schmeidler and Vind's theorems for atomless economies with asymmetrically informed agents.

In the classical core notions which have been addressed in the literature on economies with asymmetric information (fine, coarse and private core), the rule that associates information to agents in a blocking coalition is fixed a priori and is independent of any characteristic of the coalition itself. To overcome this limitation, we specify rules that assign an information structure to each member of a coalition depending on the size of the coalition itself.

This idea is initially developed in a general framework; we assume that in the market there are exogenous thresholds for the sizes of coalitions. When a trader takes part in a coalition, she has access to a specific sharing rule according to the size of the coalition itself. Assuming that the rule assigns to a class of coalitions, defined by a threshold, an information which is finer than the information assigned to any other class of coalitions, we show that the resulting core only depends on the finest information and coincides with the one associated with this finest information sharing rule. We prove this equivalence by using our extension of the Vind's theorem.

We then detail the veto mechanism by analyzing a particular and more explicit sharing rule, following the traditional ways of modelling information within coalitions (common, pooled and private information). In the mechanism we consider, the process of information sharing may only take place within coalitions with a size smaller than an exogenous threshold. It has a natural interpretation if the transmission of information is costly; larger the size of the coalition more difficult the communication among its members. Thus, only small coalitions pool information. Under the conditions which guarantee Vind's result in an asymmetric information framework, our main result states that the core solutions associated with this mechanism is equivalent to the core associated with an opposite mechanism, where traders pool information only in coalitions whose measure is larger than an exogenous threshold. Further, we prove that, depending on the informational requirement for an allocation to be feasible, both these cores coincide with either the weak fine core or the fine core.

The paper proceeds in the following order. In Section 2, we present the economic model. In Section 3, we extend the core characterization by Schmeidler and Vind to the framework of asymmetric information. In Section 4, the process of information sharing within a coalition is regulated according to its measure. We then obtain an equivalence result for the veto mechanisms depending on the information that coalitions of a certain measure may acquire. Finally, we analyze information sharing rules that use common, private or pooled information. We obtain the coincidence of the associated cores as particular cases of the previous equivalence result.

2 General set-up

2.1 Preliminary notations

Let Ω be the (non–empty) set of states of nature. The set of subsets of Ω is $\mathcal{P}(\Omega) := \{A; A \subseteq \Omega\}$. A partition of Ω is a family of non–empty subsets of Ω, $P \subset \mathcal{P}(\Omega) \setminus \{\emptyset\}$, such that *(i)* $\bigcup_{A \in P} A = \Omega$, and *(ii)* if $A, B \in P$ and $A \neq B$ then $A \cap B = \emptyset$.

A binary relation $\succeq$ can be defined on the set $\mathcal{P}$ of all partitions of Ω as follows. Given $P, Q \in \mathcal{P}$, P is finer than Q (or, equivalently, Q is coarser than P), denoted $P \succeq Q$, if for every $A \in P$ there is $B \in Q$ such that $A \subseteq B$.
The binary relation $\succeq$ is easily shown to be reflexive, transitive and antisymmetric, so that $(\mathcal{P}, \succeq)$ is a partially ordered set.
Moreover, $(\mathcal{P}, \succeq)$ is a lattice where, for every $P, Q \in \mathcal{P}$, the supremum $P \vee Q$ and the infimum $P \wedge Q$ are defined as follows:

$$\begin{array}{ll} P \vee Q & \text{is the coarsest partition finer than both } P \text{ and } Q \\ P \wedge Q & \text{is the finest partition coarser than both } P \text{ and } Q. \end{array}$$

By denoting $A(\omega)$ the block in the partition A of Ω which contains ω, it holds that:

$$(P \vee Q)(\omega) = P(\omega) \cap Q(\omega)$$

$$(P \wedge Q)(\omega) = \{z \in \Omega : z \in R(z_1), z_1 \in R(z_2), ..., z_k \in R(\omega)\},$$

where $z_1, \ldots, z_k \in \Omega$ and $R(\omega)$ is either $P(\omega)$ or $Q(\omega)$.

The definitions above can be easily extended to a finite number of partitions.

A signal on Ω with values on Z is a mapping $f : \Omega \to Z$.

The signal f induces a partition of Ω, defined by $P_f = \{f^{-1}(s), s \in Z\}$. Reciprocally, any partition P can be seen as the partition induced by a signal. To see this, define the equivalence relation

$$\omega \sim \omega' \text{ if and only if } P(\omega) = P(\omega'),$$

where $P(z)$ denotes the unique element (block) of P containing z. Let $\Omega/\!\sim$ be the quotient set of the equivalence relation $\sim$ and define

$$f : \Omega \to \Omega/\!\sim$$

as the natural projection $f(\omega) = P(\omega)$. It is clear that the partition induced by the signal f is, precisely P, that is, $P_f = P$.

2.2 The economic model

Let $\mathcal{E}$ be an exchange economy with asymmetric information and a continuum of traders modeled by the finite measure space $(I, \mathcal{A}, \mu)$, where I denotes the set of agents, $\mathcal{A}$ is the Lebesgue σ-algebra on I and μ is the Lebesgue measure on $\mathcal{A}$.

The economy extends over two time periods $\tau = 0, 1$ and consumption takes place at $\tau = 1$. At $\tau = 0$ there is uncertainty over the states of nature and agents make contracts that may be contingent on the realized state of nature at period $\tau = 1$, that is, trading is characterized by ex ante contract arrangements. The exogenous uncertainty is described by a measurable space $(\Omega, \mathcal{F})$, where Ω is a finite set of k states of nature and $\mathcal{F}$ denotes the algebra of all events.

Agents are partially and asymmetrically informed with respect to states of nature. Specifically, the initial information of trader $t \in I$ is described by a measurable signal Π_t defined on Ω. If state $\bar{\omega}$ occurs, trader t observes the image $\Pi_t(\bar{\omega})$ and she is not able to distinguish $\bar{\omega}$ from the states of nature contained in $\Pi_t^{-1}(\Pi_t(\bar{\omega}))$. We will use the symbol Π_t to denote the signal or the partition on Ω defined by the signal.

There is a finite number ℓ of commodities in each state and therefore $(\mathbb{R}_+^\ell)^k$ is the commodity space. Each agent t is characterized by her endowments $e(t, \omega) \in \mathbb{R}_+^\ell$ for each $\omega \in \Omega$ and has a preference relation over the consumption set represented by the utility function $U_t : (\mathbb{R}_+^\ell)^k \to \mathbb{R}_+$. The utility function U_t is said to be monotone if for every $x, y \in (\mathbb{R}_+^\ell)^k$ such that $x \geq y$, it holds that $U_t(x) \geq U_t(y)$.

An allocation $f : I \times \Omega \longrightarrow \mathbb{R}_+^\ell$ is physically feasible if $\int_I f(t, \omega) d\mu(t) \leq \int_I e(t, \omega) d\mu(t)$ for every $\omega \in \Omega$. In order to define the informational feasibility condition of an allocation, let us associate to each consumer t a partition $\mathcal{P}_t$ of Ω which may differ from the initial information Π_t. The interpretation is that during the trading process the information that each agent initially possesses may vary; as a consequence, the constraint that information imposes over allocations is expressed with reference to the partition $\mathcal{P}_t$ rather than Π_t.

The allocation f is feasible if it is physically feasible and $f(t, \cdot)$ is $\mathcal{P}_t$-measurable for almost all $t \in I$.

For the sequel, it is also worth emphasizing the following points on how uncertainty and information are modeled.

- As a consequence of Ω being finite, in spite of the fact that it could exist infinitely many signals, only a finite number of different partitions of Ω exist. Let

$\hat{\Pi}_1, \hat{\Pi}_2, \ldots, \hat{\Pi}_n$ denote the n distinct information partitions of traders.

- Given a coalition $S \subseteq I$, $\bigvee_{t \in S} \Pi_t$ denotes the maximum information that coalition S can dispose of, whenever each member of S opts for sharing her initial information with everybody else within the coalition. Formally, this is the coarsest refinement of each partition Π_t, that is:

$$\bigvee_{t \in S} \Pi_t = \left\{ \bigcap_{t \in S} \Pi_t(\omega) : \omega \in \Omega \right\}$$

  where, following the notation stated before, $\Pi_t(\omega)$ is the block in the partition Π_t which contains ω.

- $\bigwedge_{t \in S} \Pi_t$ is the common information partition associated to the coalition S. Thus, $A \in \bigwedge_{t \in S} \Pi_t$ if and only if A is the smallest subset of Ω such that for every $\omega \in A$, $\Pi_t(\omega) \subseteq A$, for every $t \in S$.

The first point is of particular interest for our purposes. In fact, regarding information, we can identify only a finite number n of agents' types, which are defined by:

$$I_i = \{t \in I \text{ such that } \Pi_t = \hat{\Pi}_i\}, \qquad i \in \{1, 2, \ldots, n\}.$$

We assume that, for every $i \in \{1, \ldots, n\}$, the set I_i is measurable and $\mu(I_i) > 0$.

For each coalition $S \subseteq I$, we can focus on the information types which are actually present within such a coalition; they are represented by the following set:

$$I_S = \{i \in \{1, 2, \ldots, n\} : \mu(S \cap I_i) > 0\}$$

Note that for every coalition S we have $S = \bigcup_{i \in I_S} S \cap I_i$.

Using this notation, we have that $\hat{\Pi}_i(\omega) \in \left(\bigwedge_{t \in S} \Pi_t\right) \bigcap \left(\bigvee_{t \in S} \Pi_t\right)$ for some $i \in I_S$ if and only if $\hat{\Pi}_i(\omega) = \Pi_t(\omega)$ for every $t \in S$.

2.3 Information and coalitions

We remark that a cooperative solution of an economy (in particular, the core) requires to state specifically the allocations that every coalition is able to obtain. In order to define a core solution for markets with asymmetries in information, it becomes crucial how information is assigned when several agents agree to join together to block a proposed allocation of commodities.

In this Section, we deal with the information that agents can dispose of when they become members of a coalition and the associated blocking mechanism. In order to model how the initial information of a trader changes, we use the notion of the information sharing rule as introduced by Allen (2006).

Given a coalition S, an information sharing rule for S is a function $\Upsilon(S)$ which associates a partition $\Upsilon_t(S)$ to each member $t \in S$; the partition $\Upsilon_t(S)$ is intended as the information that agent t can dispose of once the coalition S has been formed. Thus, an information sharing rule Υ for the economy $\mathcal{E}$ is a collection $(\Upsilon(S))_{S\in\mathcal{A}}$.

Consider two information sharing rules Υ^1 and Υ^2; we say that Υ^1 is finer than Υ^2, denoted by $\Upsilon^1 \succeq \Upsilon^2$, if for every $S \subseteq I$ we have $\Upsilon^1_t(S) \succeq \Upsilon^2_t(S)$, for every $t \in S$.

Given an information sharing rule Υ, a coalition S Υ–blocks an allocation f via g if:

(i) For every $t \in S$, $g(t, \cdot)$ is $\Upsilon_t(S)$-measurable,

(ii) $\int_S g(t, \omega) d\mu(t) \leq \int_S e(t, \omega) d\mu(t)$ for every $\omega \in \Omega$ and

(iii) $U_t(g(t, \cdot)) > U_t(f(t, \cdot))$ for every $t \in S$.

The core of the economy $\mathcal{E}$ under the information sharing rule Υ, denoted by $C^{\Upsilon}(\mathcal{E})$, is the set of feasible allocations that are not Υ-blocked by any coalition.

It is worth noting that different specifications of both, partitions $\mathcal{P}_t$ which appear in the notion of feasibility, and the information sharing rule Υ, lead to well-known distinct notions of core for asymmetric information economies (see Wilson, 1978, Yannelis, 1991 and Allen, 2006).

3 Asymmetric information and market imperfections

In every economic situation where agents behave cooperatively, the formation of coalitions could involve costs depending on the size or measure of the coalition itself. In this respect, it becomes a topic of interest to characterize cooperative solutions in terms of the blocking power of only those coalitions whose formation costs are not too high. In an asymmetric information framework, this aspect becomes more appealing, since the feasibility of trades involve not only physical resources but also information constraints. Then, the cost related to the transmission of information could also be taken into account.

By addressing exchange economies with a finite number of commodities and a continuum of traders with complete information, Schmeidler (1972) shows that it is enough to consider arbitrarily small coalitions in order to obtain the core. In the same issue of Econometrica, Grodal (1972) and Vind (1972) complement the previous result characterizing other families of coalitions which are able to block any allocation that does not belong to the core. In the light of the core-Walras equivalence (see Aumann, 1964), the results by Schmeidler, Grodal and Vind have been interpreted as a characterization of perfect competition. In other words, under perfect competition we can assume that arbitrarily small coalitions (Schmeidler and Grodal) or, symmetrically, arbitrarily big coalitions (Vind), are enough to block any non competitive outcome.

Extensions of these results more general settings, where perfect competition is not guaranteed, may require additional assumptions. For instance, Hervés-Beloso et al., (2000) have extended Schmeidler's, Grodal's and Vind's results to an infinite dimensional setting by requiring a kind of myopic behavior of the agents (see also Evren and Hüsseinov, 2008). In the case of economies with asymmetric information, the characterizations of Schmeidler and Vind have been attained by Hervés-Beloso et al. (2005), by Evren and Hüsseinov (2008) and by Pesce (2010). However, in all these results traders maintain their initial information independently of the coalition they belong to. In a general asymmetric information framework, without extra assumptions on the information sharing rule, imperfections of the market may arise and, consequently, it should not be surprising that the above mentioned characterizations of perfect competition do not hold. To show this point, we first state two examples which prove that Schmeidler's and Grodal's results and Vind's result, respectively, may fail without extra assumptions on the information sharing rule[1].

Example 1. Consider an economy with asymmetric information, two possible states of nature, a and b, and one commodity in each state. Every agent in $[0, 1]$ has an initial endowment given by $(1, 1)$, that is, one unit of every commodity in each state. Let x and y denote the consumption in a and b, respectively. Preferences are given by the following utility functions:

$$U_t(x, y) = \begin{cases} x^2 y & \text{if } t \in A = [0, 1/2) \\ \\ xy^2 & \text{if } t \in B = [1/2, 1] \end{cases}$$

[1]The examples we state are in accordance with the work by Serrano et al. (2001) where it is shown that the core convergence to competitive allocations, which is another perfect competition test, may fail when the coarse blocking mechanism is considered.

Agents in A have complete information whereas agents in B are not able to distinguish the two states a and b, that is:

$$\Pi_t = \begin{cases} \{\{a\},\{b\}\} & \text{if } t \in A \\ \\ \{a,b\} & \text{if } t \in B \end{cases}$$

We consider the information sharing rule Υ defined as follows:

$$\Upsilon_t(S) = \begin{cases} \bigvee_{t\in S} \Pi_t & \text{if } \mu(S) > \alpha \\ \\ \bigwedge_{t\in S} \Pi_t & \text{if } \mu(S) \leq \alpha \end{cases}$$

Then, the initial endowment allocation is Υ–blocked by every coalition S with measure greater than α such that $\mu(S \cap A) > 0$ and $\mu(S \cap B) > 0$. However, it cannot be Υ–blocked by any coalition S with $\mu(S) \leq \alpha$.

Example 2. Consider the same economy as before except that preferences are given by the following utility functions:

$$U_t(x,y) = \begin{cases} x^2 y & \text{if } t \in [0,1/4) \\ \\ xy^2 & \text{if } t \in [1/4,1/2) \\ \\ xy & \text{if } t \in [1/2,1] \end{cases}$$

Then, the initial allocation is coarse blocked by the coalition $[0,1/2)$ via the allocation which assigns $(4/3,2/3)$ to every agent in $[0,1/4)$ and $(2/3,4/3)$ to agents in $[1/4,1/2)$.

Note that the common information associated with any coalition $S \subset [0,1]$ such that $\mu(S \bigcap [1/2,1]) > 0$ does not distinguish between a and b. Therefore, the initial allocation cannot be blocked by any coalition with measure larger than $1/2$ whenever the sharing information rule is given by the coarse blocking structures.

In order to obtain a general version of both Schmeidler's and Vind's results in the framework of asymmetric information economies, we will state several properties for the information sharing rule Υ which we will use throughout this paper.

(P1) For every $S, S' \in \mathcal{A}$ such that $S' \subseteq S$ and $I_S = I_{S'}$, it holds that:

$$\Upsilon_t(S') = \Upsilon_t(S) \text{ for every } t \in S'$$

(P2) For every $S, S' \in \mathcal{A}$ such that $S' \subseteq S$ and for every $t \in S'$:

$$\Upsilon_t(S) \succeq \Upsilon_t(S')$$

Property (P1) states that when trader t becomes part of two coalitions, one contained in the other, the information she can dispose of in the smaller one is the same as the information she can dispose of in the larger one whenever the information types included in these two coalitions are the same. Property (P2) requires that if we consider an initial coalition and additional members join this group, then, membership in the original coalition cannot become worse off from an informational point of view. The information sharing rules that satisfy (P2) are referred to as nested by Allen (2006) and, under this property, she shows the non-emptiness of the core for NTU games with a finite number of players and asymmetric information.

It is worth noting that, in general, there is any relation between the measure of a coalition S and the number of informational types included in S. As a consequence, properties (P1) and (P2) cannot be compared. The next example illustrates this point.

Example 3. Consider as in Example 1 two informational types, $A = [0, 1/2)$ and $B = [1/2, 1]$, whose initial information is given by:

$$\Pi_t = \begin{cases} \{\{a\}, \{b\}\} & \text{if } t \in A \\ \{a, b\} & \text{if } t \in B \end{cases}$$

We consider the coarse information sharing rule Υ, defined as follows:

$$\Upsilon_t(S) = \bigwedge_{t \in S} \Pi_t\,, \text{ for every } t \in S\,.$$

Note that $\Upsilon_t([0, 2/3]) = \{a, b\}$, for every $t \in [0, 2/3]$ whereas $\Upsilon_t([0, 1/3]) = \{\{a\}, \{b\}\}$, for every $t \in [0, 1/3]$. Thus, property (P1) holds but property (P2) is not satisfied.

In order to show that property (P2) does not imply property (P1), it is enough to notice that the sharing rule defined in the Example 1 is nested but it does not satisfy property (P1). Indeed, if we consider the coalitions $S = (1/5, 3/4)$ and $S' = (2/5, 7/10)$ then $I_S = I_{S'}$. However, agents in S have full information but every agent in S' has no information when the coalitions are formed.

We remark that properties (P1) and (P2) together can be written as follows:

(P3) For every $S, S' \in \mathcal{A}$ with $S' \subseteq S$ and for every $t \in S'$, it holds that:

$$\begin{aligned} \Upsilon_t(S) \succeq \Upsilon_t(S') \quad & \text{if } I_{S'} \subset I_S; \\ \Upsilon_t(S) = \Upsilon_t(S') \quad & \text{if } I_S = I_{S'} \end{aligned}$$

Note that properties (P1) and (P2) trivially hold for the private information sharing rule Υ_t^p, given by $\Upsilon_t^p(S) = \Pi_t$, for every $S \in \mathcal{A}$ and for every $t \in S$.

Furthermore, (P1) holds for any sharing rule Υ with $\Upsilon(S)$ depending only on the informational types I_S, which are actually present in coalition S. This is particularly the case for the fine information sharing rule, where agents share information within every coalition. It is also the case for the coarse information sharing rule, where agents within a group are restricted to use the common information. In addition, if the information that any agent can dispose of within a coalition does not decrease when the number of types increases[2], then property (P3) is satisfied.

We also consider the next condition which states that the requirements for an allocation f to be informationally feasible imply that, for every t, the bundle $f(t, \cdot)$ is compatible with the information that the individual t disposes of when the large coalition is formed.

(P4) For every $t \in I$, $\Upsilon_t(I) \succeq \mathcal{P}_t$.

Next we state an extension of Schmeidler's result (1972) for asymmetric information economies under an information sharing rule Υ which satisfies the property (P1). Starting from a blocking coalition S, a crucial point in the proof is that an arbitrarily small blocking coalition can be built in such a way that it contains the same informational types as S. This fact, together with property (P1), makes it possible to overcome the informational constraints over the blocking allocation.

Theorem 3.1 *Consider the asymmetric information economy $\mathcal{E}$ and an information sharing rule Υ with property (P1). Let f be an allocation which is Υ-blocked via g by a coalition S. Then, for any $\varepsilon \in (0, 1]$ there exists a coalition $S_\varepsilon \subseteq S$ with $I_{S_\varepsilon} = I_S$ and $\mu(S_\varepsilon) = \varepsilon\mu(S)$ which also Υ-blocks f via the same g.*

[2] This occurs, for example, when agents share their information; on the contrary, it no longer occurs when agents forming a coalition are restricted to use their common information (see Example 3).

Proof. Let S be a coalition that Υ-blocks f under the information sharing rule Υ. That is, there exists an allocation g such that:

(i) $g(t,\cdot)$ is $\Upsilon_t(S)-$ measurable $\mu-$ a.e. in S,

(ii) $\int_S g(t,\cdot)d\mu(t) \leq \int_S e(t,\cdot)d\mu(t)$ and

(iii) $U_t(g(t,\cdot)) > U_t(f(t,\cdot))\,\mu-$ a.e. in S.

For each $S_i = \{t \in S : \Upsilon_t(S) = \hat{\Pi}_i\}$ let us define the vectorial measure m by $m(A) = \left(\int_A (g(t,\cdot) - e(t,\cdot))\, d\mu(t), \mu(A)\right)$ for every $A \subseteq S_i$. Applying Lyapunov's theorem, for any $\varepsilon \in (0,1)$ there exists $S'_i \subseteq S_i$ such that $m(S'_i) = \left(\varepsilon \int_{S_i} (g(t,\cdot) - e(t,\cdot))d\mu(t), \varepsilon\mu(S_i)\right)$. Let us consider the coalition $S_\varepsilon = \bigcup_{i \in I_S} S'_i$. Since the subcoalitions S'_i are disjoint we have $m(S_\varepsilon) = \sum_{i \in J} m(S'_i)$. Therefore, we can conclude that:

- $\int_{S_\varepsilon} (g(t,\cdot) - e(t,\cdot))d\mu(t) = \varepsilon \sum_{i \in I_S} \int_{S_i} (g(t,\cdot) - e(t,\cdot))d\mu(t) \leq 0;$
- $\mu(S_\varepsilon) = \sum_{i \in I_S} \mu(S'_i) = \varepsilon\mu(S);$
- $g(t,\cdot)$ is $\Upsilon_t(S)$ - measurable, by virtue of property (P1).

That is, coalition S_ε Υ-blocks f via g.

Q.E.D.

The previous remarks regarding property (P1) allows us to apply the extension of Schmeidler's result to allocations which do not belong to either private, fine or coarse core.

We now turn to the blocking power of large coalitions by providing an extension of the above mentioned Vind's result to asymmetric information economies.

Theorem 3.2 *Consider the asymmetric information economy $\mathcal{E}$ and an information sharing rule Υ with properties (P3) and (P4). Suppose that utility functions U_t are continuous and monotone. Let f be an allocation which is Υ-blocked via g by a coalition S. Then, for any $\alpha \in (0, \mu(I))$ there exists a coalition S_α with $\mu(S_\alpha) = \alpha$ which also Υ-blocks f.*

Proof. Let S be a coalition that blocks f under the information sharing rule Υ. That is, there exists an allocation g such that:

(i) $g(t,\cdot)$ is $\Upsilon_t(S)$ - measurable $\mu-$ a.e. in S,

(ii) $\displaystyle\int_S g(t,\cdot)d\mu(t) \leq \int_S e(t,\cdot)d\mu(t)$ and

(iii) $U_t(g(t,\cdot)) > U_t(f(t,\cdot))\, \mu-$ a.e. in S.

Since (P3) implies (P1), we can apply Theorem 3.1, and therefore consider that S is small enough so that $\mu(S) < \min_i \mu(I_i)$.

Let us define $H(t) = U_t(g(t,\cdot) - U_t(f(t,\cdot)$. Lusin's theorem guarantees the existence of a compact set $K \subset S$, with $\mu(K) > 0$, such that H, f and g are continuous functions on K. Consider the sequence of functions g_n given by $g_n(t,\cdot) = \frac{n}{n+1} g(t,\cdot)$ and let $H_n(t) = U_t(g_n(t,\cdot)) - U_t(f(t,\cdot))$. Note that $g_n(t,\cdot) \leq g_{n+1}(t,\cdot)$ for all n and t. Then by monotonicity of preferences we have that H_n is a monotone increasing sequence of continuous functions defined on K with pointwise limit H. Applying Dini's theorem, we know that there exists $\overline{n}$ such that $U_t(g_n(t,\cdot)) > U_t(f(t,\cdot))$ for every $n \geq \overline{n}$ and every $t \in K$. Take any $N > \overline{n}$ and consider the allocation $\widehat{g}$ given by

$$\widehat{g}(t,\cdot) = \begin{cases} \frac{N}{N+1}\, g(t,\cdot) \text{ if } t \in K \\ g(t,\cdot) \text{ if } t \in S \setminus K \end{cases}$$

By construction we have $\displaystyle\int_S \widehat{g}(t,\cdot)d\mu(t) + \delta \leq \int_S e(t,\cdot)d\mu(t)$, for some $\delta \gg 0$.
Let $\varepsilon \in (0,1)$. Applying Lyapunov Theorem we obtain that there exists $S_\varepsilon \subset S$ such that $\displaystyle\int_{S_\varepsilon} \widehat{g}(t,\cdot)d\mu(t) = \varepsilon \int_S \widehat{g}(t,\cdot)d\mu(t)$ and $\displaystyle\int_{S_\varepsilon} f(t,\cdot)d\mu(t) = \varepsilon \int_S f(t,\cdot)d\mu(t)$. Let h be the allocation given by

$$h(t,\cdot) = \begin{cases} \widehat{g}(t,\cdot) \text{ if } \ t \in S_\varepsilon \\ f(t,\cdot) + \frac{\varepsilon\delta}{2\mu(S\setminus S_\varepsilon)} \text{ if } \ t \in S \setminus S_\varepsilon \end{cases}$$

Note that $U_t(h(t,\cdot)) > U_t(f(t,\cdot))$ for every $t \in S$. Moreover:

$$\int_S h(t,\cdot)d\mu(t) = \int_S (\varepsilon\widehat{g}(t,\cdot) + (1-\varepsilon)f(t,\cdot))\, d\mu(t) + \frac{\varepsilon\delta}{2}$$

By applying Lyapunov's Theorem once more, there exists $A \subset I \setminus S$ such that $\mu(A) = (1-\varepsilon)\mu(I \setminus S)$ and $\displaystyle\int_A (f(t,\cdot) - e(t,\cdot))\, d\mu(t) = (1-\varepsilon)\int_{I\setminus S} (f(t,\cdot) - e(t,\cdot))\, d\mu(t)$.

Now, let ϵ be small enough so that $\mu(S \bigcup A) > \mu(I) - \min_i \mu(I_i)$. This guarantees that the coalition $B = S \bigcup A$ verifies that $\mu(B \bigcap I_i) = \mu(B_i) > 0$ for every $i = 1, \ldots, n$ which implies $I_B = \{1, \ldots, n\} = I_I$. Then, since (P1) is implied by (P3), we have that $\Upsilon_t(B) = \Upsilon_t(I)$ for every $t \in B$.

Consider the allocation z defined by

$$z(t,\cdot) = \begin{cases} h(t,\cdot) \text{ if } \ t \in S \\ f(t,\cdot) + \frac{\varepsilon\delta}{2\mu(B)} \text{ if } \ t \in A \end{cases}$$

As mentioned above, by monotonicity of preferences $U_t(z(t,\cdot)) > U_t(f(t,\cdot))$ for every $t \in B$. Additionally, properties (P3) and (P4) allow us to ensure that $z(t,\cdot)$ is $\Upsilon_t(S)$ - measurable for every $t \in B$. More precisely, condition (P2), which is implied by (P3), guarantees that $z(t,\cdot)$ is $\Upsilon_t(S)$ - measurable for every $t \in S_\epsilon$, whereas (P4) leads us to confirm that $z(t,\cdot)$ is $\Upsilon_t(S)$ - measurable for every $t \in B \setminus S_\epsilon$. Finally, $\int_B (z(t,\cdot) - e(t,\cdot))\, d\mu(t) \leq 0$.

Therefore, we have constructed an arbitrarily large coalition B. By using Theorem 3.1 once again, we conclude the proof.

Q.E.D.

4 Information and coalitions' size

In this Section our aim is to define a general rule where the information sharing among traders is regulated on the size of the coalition they belong to. Thus, the information that an agent t is allowed to use when she joins a coalition S is specified as a signal $\Upsilon_t(S)$ which depends not only on her own information and the information of the other members of S but also on the size of the coalition S.

For this purpose, we consider a partition $(M_j, j \in J)$ of the interval $M = [0, \mu(I)]$, where I is the set of agents. Observe that the partition $(M_j, j \in J,)$ which could be finite or infinite, defines a family of thresholds in such a way that for each coalition S, there is just an index $j \in J$ such that $\mu(S) \in M_j$.

For every index $j \in J$ there is an information sharing rule Υ^j, formally:

Given a coalition $S \subseteq I$, let us consider the index $j \in J$ such that $\mu(S) \in M_j$ and define

$$\Upsilon_t(S) = \Upsilon_t^j(S) \text{ , for each } t \in S. \tag{1}$$

The intuition behind this setup is the following: in the market there are some relevant thresholds for the coalitions' size identified by the sets $(M_j)_{j\in J}$. When trader t takes part in coalition S, she has access to some specific information given by a sharing rule according to the size of the coalition S itself.

Schmeidler (1972) argued that the size of a coalition can be interpreted as a measure of the amount of (or cost of) information and communication needed in order to form such a coalition. For atomless economies with a finite number of commodities and complete information, we have already mentioned that Schmeidler (1972) showed that in order to get the core, and consequently the competitive outcomes, it is enough to consider the veto power of arbitrarily small coalitions. Grodal (1972) and Vind (1972) completed this result by specifying other families of coalitions that are able to block any non-competitive allocation. Consequently, the measure of coalitions matters and becomes even more important when we consider cooperative behavior and asymmetries of information are present. In fact, the set of coalitions that can be actually formed and the type of information that is available to their members may be restricted by the lack of communication. Thus, following Schmeidler, we could argue that, because of the cost of communication, only small coalitions are formed free and spontaneously and, in addition, their member are able to pool information.

In our definition, the number of the thresholds associated to the sharing rules could be either just one, to define small and big coalitions, or two, in order to precise small, middle size and big coalitions. However the definition contemplates a general situation that allows that the thresholds could be any finite number or even infinitely many.

On the other hand, and in order to formalize the idea behind the previous argument that the members of a given class of coalitions defined by some specific size (small coalition for example), are able to collaborate getting a better knowledge of the state of nature, we will set the following property,

(F) There exists an index $o \in J$ such that $\Upsilon^o \succeq \Upsilon^j$ for every $j \in J$ and Υ^o satisfies properties (P3) and (P4).

Now we are in conditions to state our main equivalence result. The proof exploits the veto power of both small and big coalitions under asymmetries of information.

Theorem 4.1 *Consider the asymmetric information economy $\mathcal{E}$ and suppose that the information sharing rule Υ defined by (1) satisfies assumption (F). Then, the Υ-core coincides with the Υ^o-core, that is, $\mathcal{C}^{\Upsilon}(\mathcal{E}) = \mathcal{C}^{\Upsilon^o}(\mathcal{E})$.*

Proof. It is easy to see that $\mathcal{C}^{\Upsilon^o}(\mathcal{E}) \subseteq \mathcal{C}^{\Upsilon}(\mathcal{E})$. To prove this, let f be Υ-blocked by a coalition $S \subseteq I$ via allocation g. Since Υ^o is finer than any Υ^j in the collection $\mathcal{F}$, it holds that $g(t, \cdot)$ is $\Upsilon^o_t(S)$-measurable, for every $t \in S$. Then f is Υ^o-blocked by the coalition S via the same allocation g.

To prove that $\mathcal{C}^{\Upsilon}(\mathcal{E}) \subseteq \mathcal{C}^{\Upsilon^o}(\mathcal{E})$ assume that f does not belong to $\mathcal{C}^{\Upsilon^o}(\mathcal{E})$. Hence, there exists a coalition S and an allocation $g : S \times \Omega \longrightarrow \mathbb{R}^{\ell}_{+}$ such that:

(i) for every $t \in S,\ g(t,\cdot)$ is $\Upsilon^o_t(S)$-measurable,

(ii) $\displaystyle\int_S g(t,\omega)d\mu(t) \leq \int_S e(t,\omega)d\mu(t)$ for every $\omega \in \Omega$ and

(iii) $U_t(g(t,\cdot)) > U_t(f(t,\cdot))$ for every $t \in S$.

Let $k \in J$ such that $\mu(S) \in M_k$. If $k = o$, the proof ends. If $k \neq o$, by making use of Theorem 3.2, we can find a coalition $\tilde{S}$, with $\mu(\tilde{S}) \in M_o$, which Υ^o-blocks f.

Q.E.D.

This equivalence result establishes that the core which comes from a sharing rule that accomplishes the property (F) only depends on the finest information which is associated with a set of coalitions of a certain measure. We remark that the result depends neither on the precise thresholds specifying the sizes of these coalitions nor on the number of thresholds. What is relevant is the existence of a set of coalitions whose members obtain a finest information when they join together.

Next we apply this equivalence result to some concrete situations which allow us to obtain characterizations of the main core solutions that have been already analyzed in the literature on economies with asymmetric information. For this, we specify a blocking system in which the family $(\Upsilon^j, j \in J)$ are the traditional common, pool and private information sharing rules. Precisely, the process of information sharing can only take place within coalitions with sizes smaller than an exogenous threshold, whereas traders in the remainder coalitions, either keep their initial information or are restricted to use the common information. This is in accordance with the idea that the information transmission within groups of individuals is costly; the larger the coalition is, the more difficult the communication among its components will be.

In order to formalize the mechanism, consider the partition of the set $M = [0, \mu(I)]$ formed by the intervals $M_1 = [0, s)$, $M_2 = [s, b]$, $M_3 = (b, \mu(I)]$, where $s, b \in [0, \mu(I)]$, and $s < b$. That is, there are two thresholds that separate small, middle and big coalitions. The information sharing rule $\widehat{\Upsilon}$ is defined by:

$$\widehat{\Upsilon}_t(S) = \begin{cases} \bigvee_{t \in S} \Pi_t & \text{if } \mu(S) < s \\ \Pi_t & \text{if } \mu(S) \in [s, b] \\ \bigwedge_{t \in S} \Pi_t & \text{if } \mu(S) > b \end{cases} \tag{2}$$

That is, we consider the fine, private and coarse information sharing rule, respectively, depending on the measure of coalitions. Note that assumption (F) holds for $\widehat{\Upsilon}$.

We recall that the fine core, denoted by $\mathcal{C}_f$, is the set of allocations f such that $f(t,\cdot)$ is Π_t-measurable and is not blocked by any coalition whose members pool their information. The weak fine core, denoted by $\mathcal{C}_{wf}$, is the set of allocations f such that $f(t,\cdot)$ is $\bigvee_{t\in I}\Pi_t$-measurable and is not fine blocked by any coalition. That is, the difference between both core solutions is the requirement for an allocation to be informationally feasible. Since any bundle that is Π_t-measurable is also $\bigvee_{t\in I}\Pi_t$-measurable, we have $\mathcal{C}_f \subset \mathcal{C}_{wf}$.

Therefore, as an immediate consequence of Theorem 4.1 and the definitions of fine and weak fine core we have the following

Corollary 4.1 *Consider the asymmetric information economy $\mathcal{E}$ and the information sharing rule $\widehat{\Upsilon}$ defined by (2). It holds that: if $\mathcal{P}_t = \Pi_t$ ($\mathcal{P}_t = \bigvee_{t\in I}\Pi_t$, respectively) for almost every $t \in I$,*

$$\mathcal{C}^{\widehat{\Upsilon}}(\mathcal{E}) = \mathcal{C}_f(\mathcal{E}), \quad (\mathcal{C}^{\widehat{\Upsilon}}(\mathcal{E}) = \mathcal{C}_{wf}(\mathcal{E}), \text{ respectively}).$$

The corollary states that in order to obtain the fine core it is enough that traders within small coalitions pool their information.

Observe that if instead of the rule $\widehat{\Upsilon}$ we consider $\overline{\Upsilon}$ defined by

$$\overline{\Upsilon}_t(S) = \begin{cases} \bigwedge_{t\in S}\Pi_t & \text{if } \mu(S) < s \\ \Pi_t & \text{if } \mu(S) \in [s,b] \\ \bigvee_{t\in S}\Pi_t & \text{if } \mu(S) > b \end{cases} \tag{3}$$

we obtain the following result

Corollary 4.2 *Consider the asymmetric information economy $\mathcal{E}$ and the information sharing rule $\overline{\Upsilon}$ defined by (3). It holds that if $\mathcal{P}_t = \Pi_t$ ($\mathcal{P}_t = \bigvee_{t\in I}\Pi_t$, respectively) for almost every $t \in I$,*

$$\mathcal{C}^{\overline{\Upsilon}}(\mathcal{E}) = \mathcal{C}_f(\mathcal{E}), \quad (\mathcal{C}^{\overline{\Upsilon}}(\mathcal{E}) = \mathcal{C}_{wf}(\mathcal{E}), \text{ respectively}).$$

Observe that $\overline{\Upsilon}$ is symmetric to $\widehat{\Upsilon}$ in the sense that the information is pooled in the big coalitions instead of the small coalitions. Nevertheless the resulting core is the same.

These results could be interpreted as follows. In the spirit of Schmeidler, one can easily argue that individuals pool information when they form small coalitions. Although it may be surprising, when coalitions are large enough one could provide symmetric arguments saying that agents also pool their information. When an individual joins a large coalition, she makes her signal public assuming that the contribution of her private information is negligible. Our last equivalence result sustains this assertion and shows that the allocations we get via the corresponding core solution are the same, independently of the size of the coalitions whose members pool information.

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