Asymmetric collusion with growing demand*

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Abstract. We characterize collusion sustainability in markets where demand is growing and may trigger entry by new firms whose efficiency may be different from that of the incumbent firms. More precisely, we obtain the critical discount factor for collusion before and after the entry. We find that the way how firms split the monopoly profit along the collusive path is crucial in the incentives for collusion. In particular, collusion may be sustainable if firms divide the monopoly profit according to their bargaining power, but it may not be sustainable if firms get a share of the monopoly profit equal to their share in the industry capital.

Keywords: Collusion; Growing demand; Nash Bargaining.

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1 Introduction

In 1992, the European Commission approved the merger of Nestlé with the Source Perrier SA (hereafter, simply Perrier).[1] After this merger, the groups Nestlé-Perrier and BSN-Volvic became the biggest suppliers in the French bottled water market, with approximately the same capacity of water.[2] The demand for bottled water grew in 1989 and in 1990 and it was expected "a realistic growth rate of 5%" over the following years.[3] As a result, it was very likely to enter new firms in the market.[4] However, the entry in this market requires a new firm to make a large initial investment: it has not only to the purchase of rights to explore one source but also to invest in advertising to familiarize consumers with a new brand.

Inspired by this case, we study the sustainability of collusion in a market where the demand is growing over the time and may trigger the entry of a new firm. More precisely, we build a model with two incumbents and one potential entrant. The total capital available to the industry fixed over the time. A firm with more capital can produce the same quantity of output at a lower cost (Perry and Porter, 1985). The two incumbents are assumed to be identical, but the entrant may differ in the stock of capital.[5]

We start by studying the sustainability of collusion between the two incumbents before the entry of a third firm. Notice that the forecast of the entry of a new firm has implications on the incentives for collusion even before the entry. Regarding the periods that follow the entry, we analyze the possibility of the entrant being accommodated in the collusive agreement (full collusion) and the possibility of being excluded (partial collusion). As in Patinkin (1947), we assume that the individual collusive quantities are chosen in order to maximize the firms' joint profit (or, equivalently, to minimize the total cost to supply their joint demand). An additional difficulty when dealing with the case of full collusion concerns

[1]European Commission Decision of 22 July 1992, Case n. IV/M.190 - Nestlé/Perrier.

[2]The remaining firms were small local producers with significantly lower market shares.

[3]See the recitals 44 and 50 of the European Commission Decision.

[4]The European Commission have ensured that there were available sources of water to be explored. The merger was only approved since that Nestlé committed itself to sell some brands and capacity of water to a competitor (with no connections to Nestlé or to BSN), such that this competitor would have, at least, 3000 million litres of capacity per year.

[5]These assumptions fit very well to the French bottled water industry, immediately after the Nestlé-Perrier merger. In addition to the groups Nestlé-Perrier and BSN-Volvic, there are also small local producers. However, as Compte *et al.* (2002), they can be ignored to simplify the analysis. Notice that we are assuming that the sources of water in France have all been discovered and that there is no substantial improvement in transports that could make imports profitable. In this case, the assumption of limited total capacity is reasonable.

the allocation of the joint profit. If the incumbents and the entrant have equal shares of capital, their joint profit is naturally divided in equal parts. The doubt arises when their shares of capital are different. In this case, we consider two possible rules: (*i*) the solution of the correspondent Nash bargaining problem; (*ii*) each firm receives the profit corresponding to the quantity it produces under joint profit maximization. Interestingly, according to the second rule, the firm's output quota is proportional to its capital. For this reason, we designate this rule by Proportional rule.[6] We compare the individual profits with the two allocation rules and study the implications of choosing one or the other on the incentives for collusion.

The two rules only give the same allocation of the monopoly profit when the three firms are symmetric. Otherwise, the Proportional rule is preferred by the firm(s) owning more capital, while the Nash bargaining rule is preferred by the firm(s) with less capital. According to the Nash bargaining rule, a firm may receive a profit that is different from that correspondent to the quantity it produces (determined so as to maximize the joint profit). In this case, there must exist side-payments between firms.

If firms adopt the Nash bargaining rule, the incentives for collusion do not qualitatively depend on how the industry capital is split between the incumbents and the entrant. If firms give little value to future profits (i.e., the discount factor is low) and: (*i*) the demand grows slowly, collusion is not sustainable either before or after the entry of the third firm; (*ii*) the demand growth is moderate, collusion could be sustainable after the entry, but the cartel breaks down before the entry; (*iii*) the demand grows extremely fast, collusion can be sustained either before or after the entry. When the discount factor is high, collusion between the three firms is always sustainable. Moreover, a faster demand growth increases the possibilities of collusion between the two incumbents before the entry. These results are very similar to those obtained by Vasconcelos (2008) for the case in which the three firms have equal shares of the capital.

Interestingly, if the monopoly profit is allocated according to the Nash bargaining rule, the higher the share of capital owned by a firm, the higher the incentives for the firm to disrupt the collusive agreement. This finding is exactly the opposite from that commonly found in the literature.[7] This difference may result from the assumption of this particular allocation

[6] In this setting, the Proportional rule conforms with the joint profit maximization outcome. For some motivation for this rule, see Bos and Harrington (2010) and the references cited therein.

[7] See, for example, Motta(2004) or Vasconcelos (2005).

rule. In fact, if firms adopt the Proportional rule (as in Vasconcelos(2005)), the firm with a lower share of the industry capital has higher incentives to deviate. However, with this rule, the incentives for collusion depend on how the industry capital is distributed between the incumbents and the entrant. In particular, a large discrepancy in firms capacities may hinder collusion. Independently of the rule used by firms to allocate output, the overall message is that asymmetries hurt collusion. Our results suggest, however, that the mapping between firms' assets and their incentives to disrupt the collusive agreement depends very much on the rule chosen by firms to allocate the collusive output.

We assume that, in the case of partial collusion, the cartel formed by the two incumbents behaves like a Stackelberg leader, choosing its quantity before the entrant. We find that if the firms have a reasonable discount factor, collusion before the entry is always sustainable. After the entry, collusion is also possible, but only for few values of the discount factor and demand growth.

Some other papers have discussed the impacts of the demand growth on the sustainability of collusion. However, the work of Vasconcelos (2008) is the closest to ours. Actually, we extend his model by considering that the incumbents may differ from the entrant in the stock of capital.[8] We also explore in more depth the possibility of partial collusion, since it is possible in our context but it was not with his assumptions.[9] The model of Capuano (2002) also analyzes collusion in a market with growing demand when entry is possible. In fact, we borrow from him the analytical expression for the aggregate demand.

Due to possible asymmetries across firms, the division rule of the collusive profit does not follow immediately in our model. Following Osborne and Pitchik (1983), we start by assuming that the firms split the monopoly profit according to their bargaining power. Although the specification of cost function is different in the two models, both reach a similar conclusion: the smallest firm is the one that benefits more from such a division rule (since its profit per unit of capacity is higher than that obtained by the biggest firm).[10]

[8]Our assumption is more suitable when attempting to study the French market of bottled water after the Nestlé-Perrier merger. Despite the incumbent groups (Nestlé-Perrier and BSN-Volvic) have similar capacities, the same is not true regarding a potential entrant.

[9]We restrict the analysis to the case of perfect collusion, while Vasconcelos (2008) derives the maximal degree of collusion that can be sustained in equilibrium. Such a simplification is made because the asymmetry between firms complicates the expressions for profits and it would be very hard to derive the maximal degree of collusion.

[10]We assume that the firm with more capital has a lower unit cost. Osborne and Pitchik (1983) assume that firms have different capacities but produce at a constant unit cost up to their capacities.

Osborne and Pitchik (1983) consider that side-payments between the firms may be feasible. On the contrary, in his model of price competition, Harrington (1991) claims that it is not reasonable to assume the existence of side-payments. He also argues that to consider firms maximizing joint profits is an *ad hoc* assumption. Instead of specifying an optimal collusive price, the author considers that firms choose prices and market shares according to the Nash bargaining solution. These assumptions are quite different from ours. We obtain the (individual) quantities that maximize the joint profit and derive the Nash bargaining solution to divide it between the firms. In our opinion, both assumptions are legitimate.[11]

Compte *et al.* (2002) study the impacts of asymmetric capacity constraints on collusion in a general setting and analyze the Nestlé-Perrier merger in the light of their findings. One of their objectives is to examine whether the conditions imposed by the European Commission to allow for the merger of Nestlé with Perrier ensured that collusion would not easily be sustainable after the merger.[12] The differences between their article and ours are evident. First, we take the market structure (after the merger) as given and analyze how the entry of a new firm would affect the incentives for collusion between the two incumbent groups (Nestlé-Perrier and BSN-Volvic). Second, they consider price competition with capacity constraints, while we consider quantity competition with asymmetric production costs. Third, in their model the demand is stable over the time, while demand is growing in our model, which is actually more suitable to analyze the case of Nestlé-Perrier.[13] Finally, they find that the division rule of the collusive profit that is most favorable to collusion is the one in which a firm receive a share proportional to its capacity. In our model, collusion may not be sustainable in equilibrium with the Proportional rule. Thus, at least for these cases, the Nash bargaining rule is preferable.

The contribution of Brock and Scheinkman (1985) is a pioneer in analyzing the impacts of capacity constraints on the ability to collusion. These authors assume that the N firms are symmetric regarding their capacities and study the impact of changing the (individual) capacity on the critical discount factor. Fabra (2006) also considers that firms are symmetric

[11]Miklós-Thal (2009) characterizes the optimal collusion in the presence of cost asymmetry, without restricting to any class of strategies. Curiously, she finds that, if side-payments are allowed, cost asymmetry facilitates collusion.

[12]In a recent contribution, Olczak (2009) incorporates demand uncertainty in the model of Compte *et al.* (2002).

[13]Compte *et. al* characterize optimal penal codes, while we assume that all firms obtain the Cournot profit in the punishment phase (trigger strategies).

with respect to their capacities. In her model, firms support zero marginal costs up to their capacities and an infinite cost above the capacity. We assume that production costs are not constant and that they depend on the share of capital owned by the firm. More precisely, we use a simplified version of the cost function considered by Vasconcelos (2005).[14]

The remainder of the paper is organized as follows. Section 2 sets up the basic model. Section 3 determines conditions for full collusion to be sustainable after and before the entry of the third firm. Section 5 presents the results obtained from numerical simulation. Section 6 concludes. Appendix A analyzes the sustainability of collusion if the incumbents do not include the entrant in their collusive agreement (partial collusion). Appendix B derives the expressions for profits in the different competitive scenarios. Appendix C contains the proofs of most propositions.

2 Model

We consider a supergame model of quantity competition between two incumbents (firm 1 and firm 2) and one potential entrant (firm 3). The three firms produce homogeneous goods. Following Capuano (2002), the demand is a linear function with deterministic growth, measured by a parameter $\mu > 1$. More precisely, in period t, the (aggregate) demand is given by:

$$Q_t = \mu^t - p_t, \tag{1}$$

where p_t denotes the price in period t. The inverse demand is, therefore, given by: $p_t = \mu^t - Q_t$.

As in the model of Perry and Porter (1985), a firm has to own a fraction of the industry capital to produce units of output. The stock of capital available for the industry is fixed and normalized to one. We consider that the two incumbents are symmetric, owning equal shares of capital: $k_1 = k_2 = k$ with $k < \frac{1}{2}$.[15]

[14]The differences between Vasconcelos (2005) and our paper are numerous. Vasconcelos (2005) only looks for equilibria in which firms get a share of the market that equals their share in the industry capital, for all equilibrium paths. We do not restrict our attention to any specific kind of equilibria. Moreover, the demand in his model is stable. Finally, he considers simple penal codes strategies, which punish the deviant in a more severe way than the trigger strategies that we consider.

[15]There only exits the possibility of a new firm to enter in the market if each incumbent owns less than half of the total capital.

The firms' cost function is assumed to depend negatively on the stock of capital and the marginal production costs are increasing. Moreover, there are no fixed production costs.[16] More precisely, the cost of the firm i, owning a share k_i of the industry capital, to produce q_{it} units of output is given by:

$$C(q_{it}, k_i) = \frac{q_{it}^2}{2k_i}. \tag{2}$$

To enter in the market, the firm 3 has to invest in capital. The entry cost is assumed to be fixed, $F > 0$. In particular, it does not depend on the amount of capital acquired. As a result, it is on the interest of an entrant to get all the available capital in the market, that is, $k_3 = 1 - 2k$. The entry occurs when the present value of the firm is maximal.

Firms play an infinitely repeated game. In each period t, for $t \in \{1, 2, ...\}$, the active firms simultaneously choose the quantity to produce. Thus, the firm's payoff is the discounted sum of its profit in each period. We assume that the three firms have the same discount factor, δ. For technical reasons, we restrict the variation of the demand growth parameter, μ, such that: $\mu^2\delta < 1$.

In the first period, each (active) firm produces the quantity established by the collusive agreement. The firm keeps producing the collusive quantity as long as there is no defections. If one firm disrupts the collusive agreement, all the firms start producing the Nash equilibrium quantity in all the following periods. In short, the firms use trigger strategies.

In the competitive path, the firm i chooses the quantity, q_{it}, that maximizes its own profit:

$$\Pi_{it}^c(q_{it}) = (\mu^t - Q_t)q_{it} - \frac{q_{it}^2}{2k_i}.$$

In the case of competition between the two incumbents, the Cournot equilibrium profit of each firm is equal to:[17]

$$\Pi_{1t}^c(2) = \Pi_{2t}^c(2) = \frac{k(1+2k)}{2(1+3k)^2}\mu^{2t} \equiv \alpha_2\mu^{2t}. \tag{3}$$

If the firm 3 has already entered in the market, the equilibrium profit of the incumbent i,

[16] This is a simplified version of the cost function considered by Vasconcelos (2005).

[17] See Appendix B for details.

for $i \in \{1, 2\}$, is equal to:

$$\Pi_{it}^c(3) = \frac{2k(1+2k)(1-k)^2}{(3+3k-8k^2)^2}\mu^{2t} \equiv \alpha_{3i}\mu^{2t}, \tag{4}$$

while the equilibrium profit of the entrant is:

$$\Pi_{3t}^c(3) = \frac{(1+k)^2(3-10k+8k^2)}{2(3+3k-8k^2)^2}\mu^{2t} \equiv \alpha_{33}\mu^{2t}. \tag{5}$$

3 Collusion

When the third firm decides the moment to enter in the market, it takes into account the demand growth and the entry cost, F. The greater the value of F, the later the entry. If F is extremely high, the firm may even decide not to enter. We assume that F is not prohibitive.

If the two incumbents are colluding when the firm enters in the market, they may: (*i*) accommodate the entry of the new firm; or (*ii*) exclude the third firm from their collusive agreement. In this section, we focus our attention on the sustainability of full collusion. In Appendix A, we analyze a scenario of partial collusion.

3.1 Collusive quantities

The cartel is assumed to choose the quantity to be produced by each firm that maximizes their joint profit. Thus, before the entry, the cartel chooses (q_{1t}, q_{2t}) such to maximize:

$$\Pi_t^m(q_{1t}, q_{2t}) = \left[\mu^t - (q_{1t}+q_{2t})\right](q_{1t}+q_{2t}) - \left(\frac{{q_{1t}}^2}{2k} + \frac{{q_{2t}}^2}{2k}\right).$$

The individual collusive output and the cartel profit in period t are given by:[18]

$$q_{1t}^m(2) = q_{2t}^m(2) = \frac{k}{1+4k}\mu^t$$

and

$$\Pi_t^m(2) = \frac{k}{1+4k}\mu^{2t}.$$

[18] See Appendix B for details.

As the two incumbents are symmetric, they divide the monopoly profit in equal parts. Thus, the collusive profit of each incumbent is:

$$\Pi_{1t}^m(2) = \Pi_{2t}^m(2) = \frac{k}{2(1+4k)}\mu^{2t} \equiv \beta_2 \mu^{2t}. \tag{6}$$

After the entry, the cartel chooses (q_{1t}, q_{2t}, q_{3t}) such to maximize:

$$\Pi_t^m(q_{1t}, q_{2t}, q_{3t}) = \left[\mu^t - (q_{1t} + q_{2t} + q_{3t})\right](q_{1t} + q_{2t} + q_{3t}) - \left[\frac{{q_{1t}}^2}{2k} + \frac{{q_{2t}}^2}{2k} + \frac{{q_{3t}}^2}{2(1-2k)}\right].$$

In this case, the aggregate output is equal to $Q_t^m(3) = \frac{\mu^t}{3}$ and the firm i produces $q_{it}^m(3) = k_i Q_t^m(3)$. The correspondent joint profit is:

$$\Pi_t^m(3) = \frac{\mu^{2t}}{6}.$$

The three firms probably differ in their stock of capital $\left(\text{unless } k = \frac{1}{3}\right)$. To consider that the three firms divide the monopoly profit in equal parts may not be reasonable. Therefore, we need to specify how this profit is split between the three firms.

3.2 Collusive agreements

When the colluding firms are asymmetric, there are several admissible rules to divide the collusive (aggregate) profit. We consider the two following rules:

a) the *Nash bargaining rule* - each firm receives a share of the joint profit correspondent to the Nash bargaining solution;

b) the *Proportional rule* - each firm receives the profit corresponding to the quantity it produces under joint profit maximization, $q_{it}^m(3)$.

Let us start by determining the individual profits, if firms adopt the Nash bargaining rule. Below, $\Pi_{it}^{m,N}(3)$, for $i \in \{1,2,3\}$, denotes the collusive profit of firm i in period t.

Lemma 1. *If firms allocate the monopoly profit according to the Nash bargaining rule, the individual profit of the incumbent i, for $i \in \{1,2\}$, is:*

$$\Pi_{it}^{m,N}(3) = \frac{k\left(21 - 6k - 51k^2 + 32k^3\right)}{9\left(3 + 3k - 8k^2\right)^2}\mu^{2t} \equiv \beta_{3i}\mu^{2t}, \tag{7}$$

while the collusive profit of the entrant is:

$$\Pi_{3t}^{m,N}(3) = \frac{27 - 30k - 93k^2 + 60k^3 + 64k^4}{18\left(3 + 3k - 8k^2\right)^2}\mu^{2t} \equiv \beta_{33}\mu^{2t}. \tag{8}$$

Proof. Let λ_{it}, for $i \in \{1, 2, 3\}$, be the share of firm i in the monopoly profit of period t, that is, $\Pi_{it}^m(3) = \lambda_{it}\Pi_t^m(3)$. These weights must solve the following problem:

$$max_{\lambda_{1t},\lambda_{2t},\lambda_{3t}} \ [\lambda_{1t}\Pi_t^m(3) - \Pi_{1t}^c(3)] \left[\lambda_{2t}\Pi_t^m(3) - \Pi_{2t}^c(3)\right] \left[\lambda_{3t}\Pi_t^m(3) - \Pi_{3t}^c(3))\right]$$

s.t.

$$\lambda_{1t} + \lambda_{2t} + \lambda_{3t} = 1 \quad \text{and} \quad \lambda_{it}\Pi_t^m(3) - \Pi_{it}^c(3) \geq 0.$$

Substituting $\lambda_{3t} = 1 - \lambda_{1t} - \lambda_{2t}$ into the objective function, the correspondent first-order conditions are:

$$\begin{cases} \left[\left[1 - \lambda_{1t} - \lambda_{2t}\right]\Pi_t^m(3) - \Pi_{3t}^c(3)\right] - \left[\lambda_{1t}\Pi_t^m(3) - \Pi_{1t}^c(3)\right] = 0 \\ \left[\left[1 - \lambda_{1t} - \lambda_{2t}\right]\Pi_t^m(3) - \Pi_{3t}^c(3)\right] - \left[\lambda_{2t}\Pi_t^m(3) - \Pi_{2t}^c(3)\right] = 0 \end{cases}$$

As $\Pi_{1t}^c(3) = \Pi_{2t}^c(3)$, it follows that $\lambda_{1t} = \lambda_{2t} \equiv \lambda_t$, where:

$$\lambda_t = \frac{\Pi_t^m(3) + \Pi_{1t}^c(3) - \Pi_{3t}^c(3)}{3\Pi_t^m(3)}. \tag{9}$$

Substituting the expressions for profits, we conclude that λ_t is constant over the time:

$$\lambda_t = \frac{1 + 6\alpha_{31} - 6\alpha_{33}}{3} \equiv \lambda \ , \ \forall t.$$

As a result, the collusive profit of the incumbent i, for $i \in \{1, 2\}$, is:

$$\Pi_{it}^m(3) = \frac{1 + 6\alpha_{3i} - 6\alpha_{33}}{18}\mu^{2t} = \frac{k\left(21 - 6k - 51k^2 + 32k^3\right)}{9\left(3 + 3k - 8k^2\right)^2}\mu^{2t},$$

while:

$$\Pi_{3t}^m(3) = \frac{1 - 12\alpha_{3i} + 12\alpha_{33}}{18}\mu^{2t} = \frac{27 - 30k - 93k^2 + 60k^3 + 64k^4}{18\left(3 + 3k - 8k^2\right)^2}\mu^{2t}.$$

□

If firms adopt the Proportional rule, the collusive profit of firm i in period t is given by:

$$\Pi_{it}^{m,P}(3) = \left(\mu^t - \frac{\mu^t}{3}\right)\frac{k_i\mu^t}{3} - \frac{1}{2k_i}\left(\frac{k_i\mu^t}{3}\right)^2 = k_i\frac{\mu^{2t}}{6} = k_i\Pi_t^m(3). \tag{10}$$

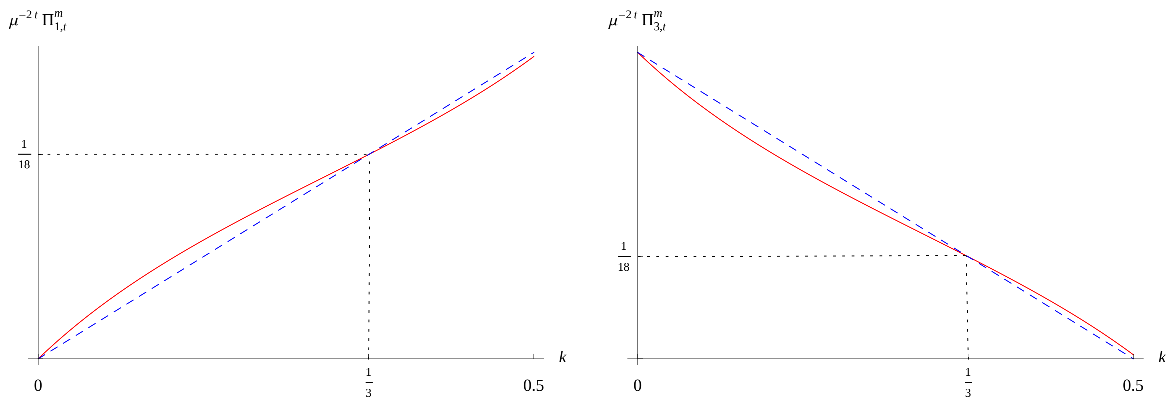
In this case, the share of each firm in the monopoly profit is equal to its share in the industry capital.

When $k = \frac{1}{3}$, the three firms have the same bargaining power and split the monopoly profit in equal parts. As a result, the two rules coincide in this particular case. The next proposition compares the share of each firm in the cartel's profit when the incumbents and the entrant are asymmetric.

Proposition 1. *When $k \neq \frac{1}{3}$, the large firm(s) prefer the Proportional rule, whereas a division of profits based on the Nash bargaining solution would be preferred by the smallest firm(s).*

Proof. See Appendix C. □

More precisely, if $k < \frac{1}{3}$, the entrant prefers the Proportional rule and the incumbents prefer the Nash bargaining rule. If $k > \frac{1}{3}$, the entrant prefers the Nash bargaining rule, while the incumbents prefer the Proportional rule. These results are shown in Figure 1.



(a) Incumbent. (b) Entrant.

Figure 1: Collusive profits with the Nash bargaining rule (solid line) and with the Proportional rule (dashed line).

With the Nash Bargaining rule, the firms divide the monopoly profit according to their relative bargaining power. The bargaining power of one firm is measured by its payoff if the bargain is not reached, the called *threat*. In our model, the firms' threats correspond to the

Cournot profits. The higher the share of capital owned by one firm, the higher its Cournot profit. Thus, a firm with more capital has more bargaining power.

However, the three firms can equally well break the agreement and force the reversion to the threat-point. As a result, they equally divide the excess of the joint profit over the sum of the Cournot profits. Notice that, using the equality (9) and that $\Pi^c_{1t}(3) = \Pi^c_{2t}(3)$, we can write the collusive profit of the incumbent 1 as follows:

$$\begin{aligned}\Pi^{m,N}_{1t}(3) &= \lambda_t \Pi^m_t(3) = \frac{\Pi^m_t(3) + \Pi^c_{1t}(3) - \Pi^c_{3t}(3)}{3} \\ &= \Pi^c_{1t}(3) + \frac{\Pi^m_t(3) - [\Pi^c_{1t}(3) + \Pi^c_{1t}(3) + \Pi^c_{3t}(3)]}{3}.\end{aligned}$$

By analogy, we could write the collusive profit of the entrant as the sum of its Cournot profit with the third part of the excess of the monopoly profit over the sum of the Cournot profit of the three firms. Thus, it is clear that a larger firm receives a higher share of the monopoly profit. However, this is the firm for which the difference between the collusive profit and the Cournot profit is lower.[19]

With the Proportional rule, each firm receives the profit corresponding to the quantity it produces under joint profit maximization. As a larger firm is more efficient, it produces a higher share of the cartel's output. Thus, when the monopoly profit is allocated according to the output quota, the larger firms are the most benefited.

However, along the collusive path, the firm i always produces the quantity $q^m_{it}(3)$, regardless the rule adopted to allocate the monopoly profit. With the Proportional rule, each firm receives the profit corresponding to the quantity it produces. Thus, if firms adopt the Nash bargaining rule, there must exist side-payments between the firms. More precisely, when $k < \frac{1}{3}$, the incumbents must receive a higher profit than that corresponding to the quantity they produce. This implies that the entrant has to pay the following amount to the incumbent i, for $i \in \{1, 2\}$:

$$SP_{3\to i} = \Pi^{m,N}_{it} - \Pi^{m,P}_{it} = \frac{k(1-3k)\left(15 - 21k - 48k^2 + 64k^3\right)}{18\left(3 + 3k - 8k^2\right)^2}\mu^{2t}.$$

[19] In a quite different setting, Osborne and Pitchik (1983) also found that when the firms use the Nash bargaining solution to divide up the monopoly profit, "the balance of forces is in favor to the small firm" (p. 60).

Conversely, when $k > \frac{1}{3}$, each incumbent i has to pay the following amount to the entrant:

$$SP_{i\to 3} = \frac{\Pi_{3t}^{m,N} - \Pi_{3t}^{m,P}}{2} = \frac{k(3k-1)\left(15-21k-48k^2+64k^3\right)}{18\left(3+3k-8k^2\right)^2}\mu^{2t} = -SP_{3\to i}.$$

3.3 Sustainability of collusion after the entry

Recall that, in this section, we are considering that the incumbents include the entrant in their collusive agreement (*full collusion*). Regarding the rule to allocate the monopoly profit, we start by assuming that firms adopt the Nash bargaining rule and then we study the sustainability of collusion using the Proportional rule.

3.3.1 Nash bargaining rule

Suppose that the firm 3 enters in the market along the collusive path. Consider also that firms have a high enough discount factor for collusion to be sustainable after the entry. As a result, the entrant receives the collusive profit in all the periods that follow its entry. Thus, if the entry occurs in period t, the present discounted value of the profit of firm 3 is:[20]

$$V^m(t) = \sum_{s=t}^{\infty} \Pi_{3s}^{m,N}(3)\delta^s - \delta^t F = \beta_{33}\sum_{s=t}^{\infty}\mu^{2s}\delta^s - \delta^t F = \beta_{33}\frac{\left(\mu^2\delta\right)^t}{1-\mu^2\delta} - \delta^t F,$$

where β_{33} is given by (8). The optimal entry period is that for which V^m is maximum. If t was a continuous variable, the optimal entry period must verify the following first-order condition:

$$\beta_{33}\frac{\mu^{2t}\delta^t}{1-\mu^2\delta}ln(\mu^2\delta) - F\delta^t ln(\delta) = 0.$$

Solving this equation in order to t, we obtain:

$$t_1(\mu,\delta,F,k) = \frac{1}{2ln(\mu)}ln\left[\frac{ln(\delta)}{ln(\mu^2\delta)}\frac{F(1-\mu^2\delta)}{\beta_{33}}\right]. \tag{11}$$

[20] We are assuming that, if the firm supports the entry cost in period t, it already makes the profit of this period.

To ensure that $t_1(\mu, \delta, F, k) > 1$, it is necessary that:

$$F \geq \frac{ln(\mu^2\delta)}{ln(\delta)} \frac{\mu^2 \beta_{33}}{1-\mu^2\delta}.$$

However, in our model, the time is discrete and the expression obtained for t_1 may not be an integer. If this happens, the firm 3 must compare the value of V^m in the largest previous integer of t_1 with the value of V^m in the smallest following integer of t_1. The optimal discrete entry time is, therefore, given by:

$$\tilde{t}_1 = \begin{cases} \lceil t_1 \rceil & \text{if} \quad V^m(\lceil t_1 \rceil) > V^m(\lfloor t_1 \rfloor) \\ \lfloor t_1 \rfloor & \text{if} \quad V^m(\lceil t_1 \rceil) \leq V^m(\lfloor t_1 \rfloor) \end{cases}, \tag{12}$$

where:

$$\lceil t \rceil = \max \{n \in \mathbb{N} : n \leq t\} \quad \text{and} \quad \lfloor t \rfloor = \min \{n \in \mathbb{N} : n \geq t\}.$$

It is straightforward to see that the higher the share of firm 3 in the industry capital, the earlier is its entry in the market. This is a very natural result, since a higher share of capital corresponds to a higher collusive profit.

Let us write the incentive compatibility constraint (hereafter, ICC) that must be satisfied for collusion to be sustainable after the entry. We have already determined the profit of each firm in the collusive and in the punishment paths. Thus, to write the ICC, it is only missing to compute the deviating profits.

If the firm i, for $i \in \{1, 2, 3\}$, decides to deviate in period t, it assumes that the rivals are producing the collusive output and chooses the quantity, $q^d_{it}(3)$, that maximizes its individual profit:

$$\Pi^d_{it}(q_{it}) = \left[\mu^t - \left(q_{it} + \frac{k_j}{3}\mu^t + \frac{1-k_i-k_j}{3}\mu^t\right)\right] q_{it} - \frac{q^2_{it}}{2k_i},$$

for $j \in \{1, 2, 3\}$ and $j \neq i$. It is straightforward to obtain the deviating profit of firm i:[21]

$$\Pi^d_{it}(3) = \frac{k_i (2+k_i)^2}{18(1+2k_i)} \mu^{2t}.$$

[21]For details, see Appendix B.

Thus, if the deviating firm is the incumbent i, for $i \in \{1,2\}$, its profit in period t:

$$\Pi_{it}^{d}(3) = \frac{k\,(2+k)^2}{18(1+2k)}\mu^{2t} \equiv \gamma_{3i}\mu^{2t}. \tag{13}$$

If the deviating firm is the entrant, we obtain that:

$$\Pi_{3t}^{d}(3) = \frac{(3-2k)^2(1-2k)}{18(3-4k)}\mu^{2t} \equiv \gamma_{33}\mu^{2t}. \tag{14}$$

In each period $t \geq t_1$ that follows the entry, the incumbent i prefers to be in collusion than to deviate, if the following incentive compatibility constraint holds:

$$\sum_{s=t}^{\infty} \Pi_{is}^{m,N}(3)\delta^{s-t} \geq \Pi_{it}^{d}(3) + \sum_{s=t+1}^{\infty} \Pi_{is}^{c}(3)\delta^{s-t} \;\Leftrightarrow\; (\beta_{3i}-\alpha_{3i})\,\delta^{-t}\sum_{s=t+1}^{\infty}(\mu^2\delta)^s \geq (\gamma_{3i}-\beta_{3i})\,\mu^{2t}$$

$$\Leftrightarrow\; \mu^2\delta \geq \frac{\gamma_{3i}-\beta_{3i}}{\gamma_{3i}-\alpha_{3i}} \equiv \mu^2\tilde{\delta}_{iN}. \tag{15}$$

Substituting the expressions for α_{3i}, β_{3i} and γ_{3i} and with some simple algebra, we obtain:

$$\mu^2\tilde{\delta}_{1N}(k) = \mu^2\tilde{\delta}_{2N}(k) = \frac{-6+36k+51k^2-190k^3-103k^4+208k^5+64k^6}{k\,(36+33k-186k^2-119k^3+208k^4+64k^5)}$$

and

$$\mu^2\tilde{\delta}_{3N}(k) = \frac{45-240k+309k^2+414k^3-1348k^4+1088k^5-256k^6}{54-270k+342k^2+390k^3-1332k^4+1088k^5-256k^6}.$$

It is straightforward to verify that, $\tilde{\delta}_{1N}\left(\frac{1}{3}\right) = \tilde{\delta}_{2N}\left(\frac{1}{3}\right) = \tilde{\delta}_{3N}\left(\frac{1}{3}\right)$. It is also possible to check that $\frac{d\tilde{\delta}_{1N}(k)}{dk} > 0$, for $k \in \left(0,\frac{1}{2}\right)$, which means that the higher the value of k, the more difficult is for the incumbents to comply with the collusive agreement. On the contrary, in the considered domain, we have that $\frac{d\tilde{\delta}_{3N}(k)}{dk} < 0$, meaning that a higher value for k makes collusion more appealing to the entrant.

Moreover, if $\mu^2\delta < \frac{49}{94} \approx 0.521$, there is no possibility of collusion, regardless of the value of k. If $\mu^2\delta \geq \frac{45}{54} \approx 0.833$, perfect collusion is always sustainable.

As the three firms have the same discount factor, the critical discount factor is given by:

$$\tilde{\delta}_N(k) = \max\left\{\tilde{\delta}_{1N}(k), \tilde{\delta}_{3N}(k)\right\} = \begin{cases} \tilde{\delta}_{3N}(k) & \text{if} \quad k < \frac{1}{3} \\ \tilde{\delta}_{1N}(k) & \text{if} \quad k \geq \frac{1}{3} \end{cases}. \tag{16}$$

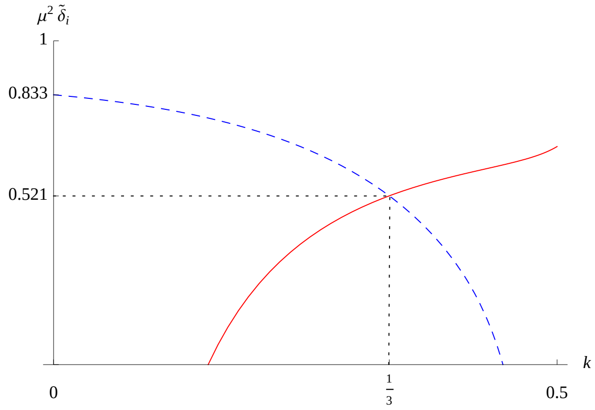


Figure 2: Critical adjusted discount factor for the incumbents (solid line) and for the entrant (dashed line), if firms adopt the Nash bargaining rule.

There are several works in the literature showing that the existence of asymmetry in capacities hinders collusion.[22] Our findings reinforce this result, inasmuch as the minimum value for the critical discount factor is obtained when firms have equal shares of capital, $k = \frac{1}{3}$.

What is actually a surprising result of our model is that the firm owning the highest share of capital is the one for which the ICC is binding. For $k < \frac{1}{3}$, it is the entrant that has more incentives to defect, while for $k > \frac{1}{3}$ the incumbents have more incentives to disrupt the collusive agreement. This finding is exactly the opposite of that in the model of Vasconcelos (2005).[23] However, he considers that, in the collusive scenario, firms obtain the profit corresponding to quantity they produce (Proportional rule). Let us analyze whether a change in the allocation of the monopoly profit might be responsible for this difference in results.

3.3.2 Proportional rule

The profit that a firm obtains if it deviates from the collusive agreement is determined by assuming that the other firms are producing the quantities that maximize the joint profit. The profit along the punishment path (i.e., the Cournot profit) does not also depend on how the monopoly profit is divided between firms. As a result, a different allocation rule of the monopoly profit only changes the profits along the collusive phase. Thus, to write the ICC if the firms adopt the Proportional rule, it is only necessary to substitute β_{3i} in (15)

[22]See, for example, Lambson (1995), Davidson and Deneckere (1984), Compte *et al.* (2002).

[23]Vasconcelos (2005) assumes that the demand is stable. This is a limit case of our model, in which $\mu \to 1$. Such an assumption would naturally change the magnitude of $\tilde{\delta}$, but not the nature of the results.

by $\frac{k_i}{6}$. By doing so, the critical discount factor for the incumbents not to deviate from de collusive agreement is:

$$\mu^2\tilde{\delta}_{1P}(k) = \mu^2\tilde{\delta}_{2P}(k) = \frac{\left(3 - 11k^2 + 8k^3\right)^2}{k\left(36 + 33k - 186k^2 - 119k^3 + 208k^4 + 64k^5\right)},$$

and, for the entrant, it is:

$$\mu^2\tilde{\delta}_{3P}(k) = \frac{k\left(3 + 3k - 8k^2\right)^2}{27 - 81k + 9k^2 + 213k^3 - 240k^4 + 64k^5}.$$

It is straightforward to check that $\tilde{\delta}_{iN}\left(\frac{1}{3}\right) = \tilde{\delta}_{iP}\left(\frac{1}{3}\right)$, $\forall i \in \{1, 2, 3\}$. This was expected because, when the firms have equal shares of capital, the Nash bargaining rule and the Proportional rule coincide.

As the three firms were assumed to have the same discount factor, the critical value is:

$$\tilde{\delta}_P(k) = \max\left\{\tilde{\delta}_{1P}(k), \tilde{\delta}_{3P}(k)\right\} = \begin{cases} \tilde{\delta}_{1P}(k) & \text{if} \quad k < \frac{1}{3} \\ \tilde{\delta}_{3P}(k) & \text{if} \quad k \geq \frac{1}{3} \end{cases}.$$

Proposition 2. *If $k \in (0, 0.199) \cup \left(0.436, \frac{1}{2}\right)$ and the firms adopt the Proportional rule to allocate the monopoly profit, perfect collusion is* **not** *sustainable after the entry.*

Proof. See Appendix C. □

In Figure 3, we represent the critical (adjusted) discount value for the incumbents and for the entrant, if they adopt the Proportional rule.

If the firms divide the monopoly profit according to the Proportional rule, the binding ICC is that of the small firm(s). This is the result found by Vasconcelos (2005) and it is exactly the converse of that we have obtained with the Nash bargaining rule. This finding alerts us to the importance of the allocation rule of the monopoly profit in the firms' incentives to sustain collusion.

If $k < 0.199$, the incumbents are considerably smaller than the entrant, which owns more than 50% of industry capital. If the firms adopt the Proportional rule, the incumbents get a small share of the monopoly profit. The Cournot profit of the incumbents is not

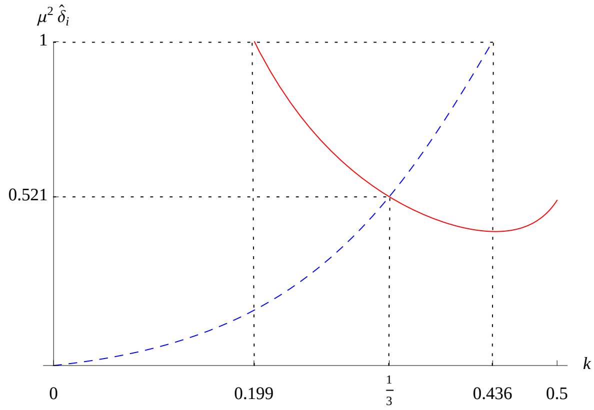


Figure 3: Critical adjusted discount factor for the incumbents (solid line) and for the entrant (dashed line), if firms adopt the Proportional rule.

significantly higher than their collusive profit. As a result, the incumbents may be better off if they deviate.

If $k > 0.436$, there is also an imbalance in the distribution of the industry capital, but now in favor of the incumbents. The entrant has, at most, 12.8% of the industry capital. Thus, for the reasons mentioned above, the entrant has no incentives to comply with the collusive agreement.

3.4 Sustainability of collusion before the entry

3.4.1 Optimal entry time

Suppose that entry occurs when the incumbents are in the punishment path. If firms play as Cournot oligopolists in all stages that follow the entry, the present discount value of the entrant's profits, if it enters in period t, is:

$$V^c(t) = \sum_{s=t}^{\infty} \Pi^c_{3s}(3)\delta^s - \delta^t F = \alpha_{33}\frac{(\mu^2\delta)^t}{1-\mu^2\delta} - \delta^t F,$$

where α_{33} is given in (5). If t was a continuous variable, the maximum value for V^c would be achieved at:[24]

$$t_2(\mu, \delta, F, k) = \frac{1}{2ln(\mu)} ln\left[\frac{ln(\delta)}{ln(\mu^2\delta)}\frac{F(1-\mu^2\delta)}{\alpha_{33}}\right]. \quad (17)$$

[24]To obtain the expression for t_2, we followed the same steps as those to obtain t_1.

The entry is later if it occurs along the punishment path than along the collusive path (i.e., $t_2 > t_1$). This occurs because the entrant has more profits if it is in collusion with the incumbents than if it is competing with them (i.e., $\beta_{33} > \alpha_{33}$).

To ensure that the expression obtained for t_2 represents a value greater than 1, the entry cost, F, must be sufficiently high:

$$F \geq \frac{\mu^2 \alpha_{33} ln(\mu^2 \delta)}{ln(\delta)(1-\mu^2\delta)}.$$

If the expression (17) is non integer, the optimal (discrete) entry time of firm 3 is:

$$\tilde{t}_2 = \begin{cases} \lceil t_2 \rceil & \text{if} \quad V^c(\lceil t_2 \rceil) > V^c(\lfloor t_2 \rfloor) \\ \lfloor t_2 \rfloor & \text{if} \quad V^c(\lceil t_2 \rceil) \leq V^c(\lfloor t_2 \rfloor) \end{cases}.$$

3.4.2 Critical discount factor

Now, we determine the critical discount factor for collusion to be sustainable (by the incumbents) before the entry. Below, we assume that, after the entry, firms divide the monopoly profit according to the Nash bargaining rule. By doing so, we are ensuring that there always exist a value for δ such that collusion is sustainable after the entry, regardless of the value of k. Moreover, we consider that $\delta(k) > \tilde{\delta}(k)$, $\forall k \in \left(0, \frac{1}{2}\right)$, where $\tilde{\delta}_N(k)$ is given by (16).

Consider a period $t \in \left\{0, 1, ..., \tilde{t}_1 - 1\right\}$, where $\tilde{t}_1$ is given by (12). The incumbent i, for $i \in \{1, 2\}$, is willing to collude with the other incumbent before the entry of firm 3 if the following incentive compatibility constraint is satisfied:

$$\sum_{s=t}^{\tilde{t}_1 - 1} \delta^{s-t} \Pi_{is}^m(2) + \sum_{s=\tilde{t}_1}^{\infty} \delta^{s-t} \Pi_{is}^{m,N}(3) \geq \Pi_{it}^d(2) + \sum_{s=t+1}^{t+\tilde{t}_2-1} \delta^{s-t} \Pi_{is}^c(2) + \sum_{s=t+\tilde{t}_2}^{\infty} \delta^{s-t} \Pi_{is}^c(3). \qquad (18)$$

Notice that we have not yet determined the collusive profit of the incumbent, $\Pi_{is}^m(2)$, nor its deviating profit, $\Pi_{is}^d(2)$, before the entry. By maximizing the incumbents' joint profit and dividing the monopoly profit in two equal parts, we obtain that:[25][26]

$$\Pi_{it}^{m}(2) = \frac{k}{2(1+4k)}\mu^{2t} \equiv \alpha_2 \mu^{2t}. \tag{19}$$

If the incumbent i deviates in period t, while the other incumbent is producing the collusive output, it obtains the following profit:[27]

$$\Pi_{it}^{d}(2) = \frac{k(1+3k)^2}{2(1+2k)(1+4k)^2}\mu^{2t} \equiv \gamma_2 \mu^{2t}. \tag{20}$$

Substituting the expressions for profits, the ICC (18) can be rewritten as follows:

$$\beta_2 \sum_{s=t}^{\tilde{t}_1-1}(\mu^2\delta)^s + \beta_{3i}\sum_{s=\tilde{t}_1}^{\infty}(\mu^2\delta)^s \geq \gamma_2(\mu^2\delta)^t + \alpha_2 \sum_{s=t+1}^{t+\tilde{t}_2-1}(\mu^2\delta)^s + \alpha_{3i}\sum_{s=t+\tilde{t}_2}^{\infty}(\mu^2\delta)^s,$$

which is equivalent to:

$$\beta_2 \frac{(\mu^2\delta)^t - (\mu^2\delta)^{\tilde{t}_1}}{1-\mu^2\delta} + \beta_{3i}\frac{(\mu^2\delta)^{\tilde{t}_1}}{1-\mu^2\delta} \geq \gamma_2(\mu^2\delta)^t + \alpha_2\frac{(\mu^2\delta)^{t+1} - (\mu^2\delta)^{\tilde{t}_2+t}}{1-\mu^2\delta} + \alpha_{3i}\frac{(\mu^2\delta)^{\tilde{t}_2+t}}{1-\mu^2\delta} \tag{21}$$

Lemma 2. *If the incentive compatibility constraint* (21) *is satisfied for* $t = \tilde{t}_1 - 1$*, then it is satisfied for all* $t \in \{0, 1, ..., \tilde{t}_1 - 1\}$.

Proof. See Appendix C. □

The Lemma states that if the incentive compatibility constraint is satisfied for the period that immediately precedes the entry of firm 3, it is verified for all the previous periods. Substituting $t = \tilde{t}_1 - 1$ in the inequality (21), we obtain that:

$$\beta_2 \frac{(\mu^2\delta)^{\tilde{t}_1-1} - (\mu^2\delta)^{\tilde{t}_1}}{1-\mu^2\delta} + \beta_{3i}\frac{(\mu^2\delta)^{\tilde{t}_1}}{1-\mu^2\delta} \geq \gamma_2(\mu^2\delta)^{\tilde{t}_1-1} + \alpha_2\frac{(\mu^2\delta)^{\tilde{t}_1} - (\mu^2\delta)^{\tilde{t}_2+\tilde{t}_1-1}}{1-\mu^2\delta} + \alpha_{3i}\frac{(\mu^2\delta)^{\tilde{t}_2+\tilde{t}_1-1}}{1-\mu^2\delta} \tag{22}$$

[25] For details, see Appendix B.

[26] As the two incumbents own equal shares of capital, it is indifferent if they divide the monopoly profit according to the Nash bargaining rule or to the Proportional rule. In both cases, each incumbent gets half of the joint profit.

[27] For details, see Appendix B.

Now, we find a sufficient condition for collusion to not be an equilibrium of the game. In other words, we look for values of δ such that the condition (22) does not hold.

Proposition 3. **(No collusion)** *Given $\mu > 1$ and $k \in \left(0, \frac{1}{2}\right)$, the incumbent i is not willing to collude if the discount factor, δ, satisfies the following inequality:*

$$\left(\mu^2\delta\right)^{t_2-1}(\alpha_2 - \alpha_{3i}) + \mu^2\delta(-\beta_2 + \beta_{3i} + \gamma_2 - \alpha_2) + (\beta_2 - \gamma_2) < 0. \tag{23}$$

Proof. See Appendix C. □

Therefore, it may happen that firms are patient so that collusion would be sustainable after the entry, $\delta(k) > \tilde{\delta}_N(k)$, but not the sufficient for collusion to be sustainable (by the two incumbents) before the entry. In the next proposition, we present a sufficient condition for perfect collusion to be sustainable before the entry.

Proposition 4. **(Perfect collusion)** *Given $\mu > 1$ and $k \in \left(0, \frac{1}{2}\right)$, the incumbent i is willing to (perfectly) collude before the entry if the discount factor, δ, satisfies the following inequality:*

$$\left(\mu^2\delta\right)^{t_2+1}(\alpha_2 - \alpha_{3i}) + \mu^2\delta(-\beta_2 + \beta_{3i} + \gamma_2 - \alpha_2) + (\beta_2 - \gamma_2) \geq 0. \tag{24}$$

Proof. See Appendix C. □

4 Numerical examples

In this section, we make graphical representation of the sufficient conditions obtained in Propositions 3 and 4. We restrict our attention to the cases in which collusion can be sustained after the entry (by the two incumbents and by the entrant). Recall that the critical discount factor depends on how the three firms divide the monopoly profit. The admissible regions of parameters (μ, δ) are, therefore, such that:

- $\mu^2\tilde{\delta}_{3N}(k) \leq \mu^2\delta(k) < 1$ if $k < \frac{1}{3}$ and $\mu^2\tilde{\delta}_{1N}(k) \leq \mu^2\delta(k) < 1$ if $k \geq \frac{1}{3}$, if firms adopt the Nash bargaining rule;

- $\mu^2\tilde{\delta}_{1P}(k) \leq \mu^2\delta(k) < 1$ if $k < \frac{1}{3}$ and $\mu^2\tilde{\delta}_{3P}(k) \leq \mu^2\delta(k) < 1$ if $k \geq \frac{1}{3}$, if firms adopt the Proportional rule.

In the figures below, these admissible regions for the parameters correspond to the areas in between the two dashed lines. More precisely, the dotted line represents the critical (adjusted) discount factor in each scenario, while the dashed line corresponds the condition $\mu^2\delta = 1$.

The thin solid line represents the sufficient condition identified in the proof of Proposition 3.[28] For the pairs (δ, μ^2) below this line, no collusion can be sustained as an equilibrium before the entry. The thick solid line represents the sufficient condition identified in the proof of Proposition 4.[29] For the pairs (δ, μ^2) above this line, perfect collusion can be sustained before (and after) the entry.

To build the graphics we need to assign a value to the (fixed) entry cost, F, and to the share of capital owned by each incumbent, k. With regard to F, we allow for two possibilities: one in which F is low and another in which F is relatively high. We consider that F is low if the entry occurs up to the 20^{th} period and relatively high if the entry occurs between the 20^{th} and the 60^{th} periods.[30] Regarding k, we consider three possibilities: (i) $k = 0.1$, corresponding to the case in which the incumbents are small (when compared with the entrant); (ii) $k = 1/3$, corresponding to the case in which the incumbents and the entrant are identical; (iii) $k = 0.4$, corresponding to the case in which the incumbents are large (when compared with the entrant).

4.1 Nash bargaining rule

Let us start by considering that the (fixed) entry cost is low. The graphics obtained are presented in Figure 4.

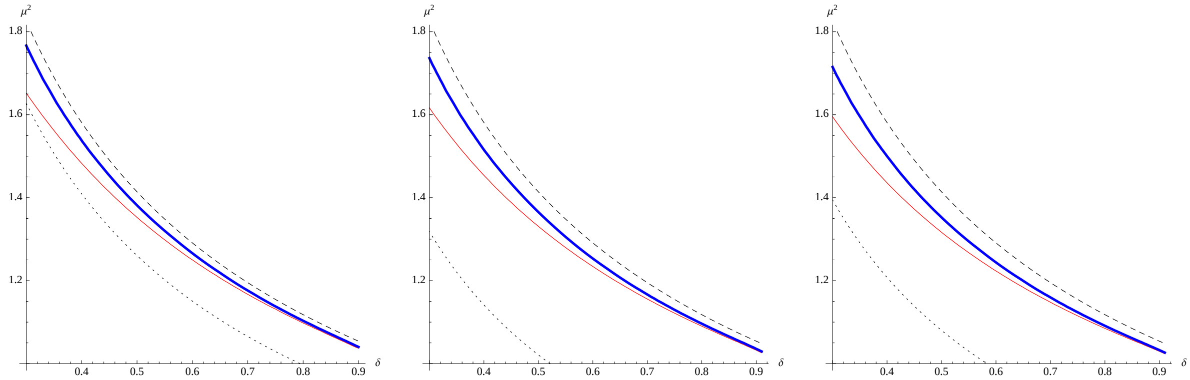
Summarizing, for the pairs of parameters (μ^2, δ):

- below the dotted line, no collusion can be sustainable neither before nor after the entry;

[28] This line is the boundary of the region defined by condition (23).

[29] This line corresponds to the boundary of the region defined by condition (24).

[30] Notice that the limits for F depend on μ, δ and on k. For simplicity, we set reasonable values for $\mu(= 1.2)$, $\delta(= 0.6)$ and determine, for each value of k, the correspondent values for F.

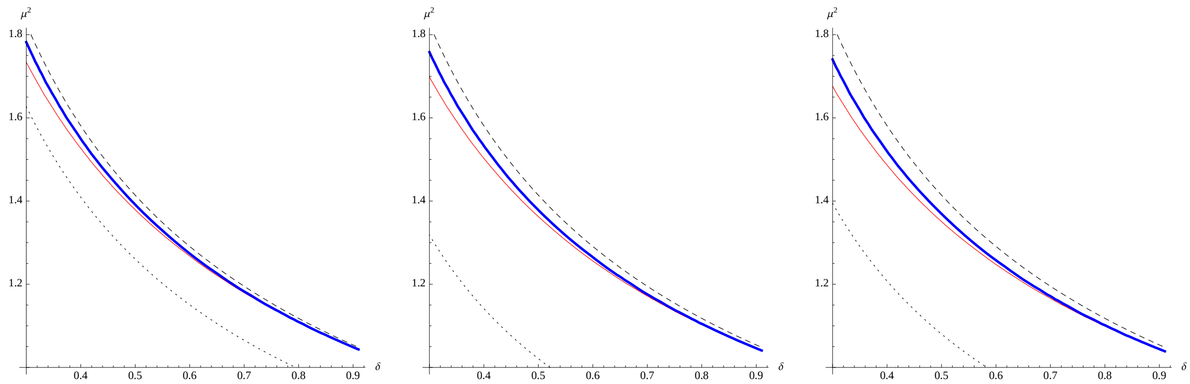


(a) $k = 0.1$; $F = 1.9$. (b) $k = 1/3$; $F = 0.8$. (c) $k = 0.4$; $F = 0.5$.

Figure 4: Sustainability of collusion with the Nash bargaining rule, when F is low.

- in the area in between the dotted line and the thin solid line, collusion would be sustainable after the entry, but it is not before the entry;
- in the area in between the thick solid line and the dashed line, full collusion is sustainable before and after the entry;
- in the area in between the two solid lines, collusion can be both sustainable after the entry as not. For some pairs, it may even exist the two types of equilibria.

In Figure 6, we present the graphics obtained if the entry cost is relatively high.

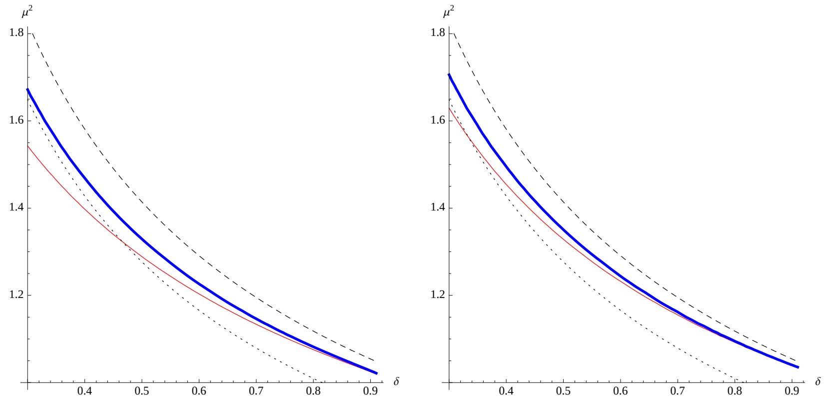


(a) $k = 0.1$; $F = 5.7$. (b) $k = 1/3$; $F = 2.5$. (c) $k = 0.4$; $F = 1.6$.

Figure 5: Sustainability of collusion with the Nash bargaining rule, when F is relatively high.

4.2 Proportional rule

As we saw in Proposition 2, if $k = 0.1 < 0.199$ and firms adopt the Proportional rule, (full) collusion is not sustainable after the entry. Moreover, if $k = \frac{1}{3}$, the three firms have equal shares of capital. As a result, the Nash Bargaining rule and the Proportional rule coincide. Thus, it only makes sense to consider the case $k = 0.4$.



(a) $F = 0.5$.   (b) $F = 1.6$.

Figure 6: Sustainability of collusion with the Proportional rule, when $k = 0.4$.

Comparing these figures with the correspondents if firms adopt the Nash bargaining rule, the major difference we find is with respect to the position of the dotted lines. Graphically, we confirm that the Proportional rule establishes a higher critical discount factor for collusion to be sustainable after the entry, that is, collusion is less likely to occur.

5 Conclusions

Inspired by the case of Nestlé-Perrier merger, we analyzed how asymmetry in the stock of capital owned by the incumbent firms and the entrant may affect the sustainability of collusion. Curiously, the market entry of a new firm changes the incentives for collusion even before the entry. On the one hand, the prospect of the entry reduces the expected cost of a deviation, without changing its short-run benefit. On the other hand, the incumbents are aware that by disrupting the collusive agreement they can delay the entry. After the entry, the new firm can either be included in the collusive agreement or be excluded from it. In the case of full collusion, the existence of asymmetry between firms makes the distribution of the monopoly profit nontrivial. We considered that firms could choose the Nash bargaining rule or the Proportional rule. We found that, regardless the allocation rule, the overall message is that asymmetries hurt collusion. Our results, however, suggest that the mapping between firms' size and their corresponding incentives to abide by the collusive agreement depend very much on the rule chosen by the collusion partners to allocate the collusive profit.

Either when the collusive agreement is all-inclusive or it is not, collusion is easier to sustain after the entry of the new firm than before. More precisely, there are many combinations of parameters (discount factor and demand growth), for which collusion would be sustainable after the entry, but it is not before the entry (despite the lower number of active firms in the industry). This phenomenon is even more pronounced when the incumbents decide not to include the entrant in their collusive agreement (partial collusion). Actually, if the incumbents are very small (when compared with the entrant), their collusion before the entry is almost impossible.

If the (fixed) entry cost is not prohibitively high, the entrant will ultimately become active in the market. It is obvious that, the higher the entry cost, the later the entry. We found, however, that the magnitude of this cost has no significant impact on the sustainability of collusion.

In our model, the share of capital owned by each incumbent is exogenously given. As the entry cost does not depend on the capital acquired and the industry capital is limited, the share of capital of the entrant is automatically determined so as. This is a strong assumption but fits very well to the French industry of bottled water industry, after the Nestlé-Perrier merger. It would, however, be interesting to introduce an initial stage, in which the incumbents choose their capacities, as in the models of Benoit and Krisnha (1987) and Knittel and Lepore (2010). We leave this to future work since it is out of the scope of this article.

Appendix A: Partial collusion

Up to this moment, we have considered that collusion was all-inclusive. More precisely, we have assumed that if the incumbents were colluding, they would include the entrant in a more inclusive agreement. Consider now that the incumbents form a cartel, but they do not include the entrant in their agreement. The entrant chooses, therefore, the output level that maximizes its individual profit in each period.

We assume that the cartel acts as a Stackelberg quantity leader, while the entrant is a follower, playing *a la* Cournot. Thus, in each period, there is the following two-stage game:

> 1^{st} *stage:* The incumbents choose the quantities that maximize their joint profit;
>
> 2^{nd} *stage:* The entrant observes the quantity chosen by each incumbent and chooses the quantity that maximizes its individual profit.

The equilibrium profit of the incumbent i, for $i \in \{1, 2\}$, in period t is given by:[31]

$$\Pi_{it}^{pm}(3) = \frac{2k(1-k)^2}{(3-4k)(3+4k-8k^2)}\mu^{2t} \equiv \zeta_{3i}\mu^{2t}, \tag{25}$$

while the profit of the entrant is:

$$\Pi_{3t}^{pm}(3) = \frac{(1-2k)(3-4k^2)^2}{2(3-4k)(3+4k-8k^2)^2}\mu^{2t} \equiv \zeta_{33}\mu^{2t}. \tag{26}$$

Proposition 5. *If $k \in (0, k^*)$, for $k^* \approx 0.342$, the entrant profits more than each incumbent. If $k \in \left(k^*, \frac{1}{2}\right)$, each incumbent profits more than the entrant.*

Proof. See Appendix B. □

In Figure 7, it is represented the individual profit of each firm in the scenario of partial collusion.

Curiously, if the three firms have equal shares of capital, that is, $k = \frac{1}{3} < k^*$, the entrant profits more than each incumbent. There exists *second-mover advantage.* This is a surprising

[31]See Appendix A for details.

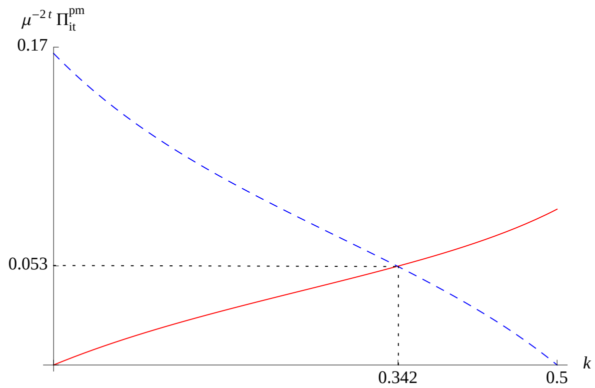


Figure 7: Adjusted profit of each incumbent (solid line) and of the entrant (dashed line), in the scenario of partial collusion.

result, since that, in the Stackelberg model, the leader uses to have advantage in playing first.

A.1 Sustainability of collusion after the entry

A.1.1 Optimal entry time

The firm 3 enters in the market when its present discounted value is maximal. In this case, it is given by:

$$V^{pm}(t) = \zeta_{33}\frac{(\mu^2\delta)^t}{1-\mu^2\delta} - \delta^t F.$$

Following the same steps as in the case of full collusion (Section 3.3.1), we find that the optimal (discrete) entry time is given by:

$$\tilde{t}_3 = \begin{cases} \lceil t_3 \rceil & \text{if} \quad V^{pm}(\lceil t_3 \rceil) > V^{pm}(\lfloor t_3 \rfloor) \\ \lfloor t_3 \rfloor & \text{if} \quad V^{pm}(\lceil t_3 \rceil) \leq V^{pm}(\lfloor t_3 \rfloor) \end{cases},$$

where

$$t_3(\mu,\delta,F,k) = \frac{1}{2ln(\mu)} ln\left[\frac{ln(\delta)}{ln(\mu^2\delta)}\frac{F(1-\mu^2\delta)}{\zeta_{33}}\right]. \tag{27}$$

To ensure that $t_3(\mu,\delta,F,k) \geq 1$, the entry cost, F, must be sufficiently high:

$$F \geq \frac{ln(\mu^2\delta)}{ln(\delta)}\frac{\mu^2\zeta_{33}}{1-\mu^2\delta}.$$

A.1.2 Critical discount factor

Let us write the incentive compatibility constraint that must be satisfied for each incumbent to be willing to collude with the other incumbent after the entry of firm 3. To do so, we need the expression for the deviating profit. If the incumbent i, $i \in \{1,2\}$, decides to deviate in period t, its profit is equal to:

$$\Pi_{it}^{pd}(3) = \frac{2k(1-k)^2\left(3+3k-8k^2\right)^2}{(1+2k)(3-4k)^2\left(3+4k-8k^2\right)^2}\mu^{2t} \equiv \eta_{3i}\mu^{2t}. \tag{28}$$

As a result, the incumbent i, for $i \in \{1,2\}$, is willing to (partially) collude after the entry if the following ICC is satisfied:

$$\sum_{s=t}^{\infty} \Pi_{is}^{pm}(3)\delta^{s-t} \geq \Pi_{it}^{pd}(3) + \sum_{s=t+1}^{\infty} \Pi_{is}^{c}(3)\delta^{s-t},$$

where $\Pi_{is}^{c}(3)$ is the Cournot profit of one incumbent, if the three firms are active in the market. The expression for $\Pi_{is}^{c}(3)$ is given in (4). Substituting the expressions for profits in the last inequality, we obtain that:

$$\mu^2\delta \geq \frac{\eta_{3i}-\zeta_{3i}}{\eta_{3i}-\alpha_{3i}} \equiv \mu^2\bar{\delta}_i. \tag{29}$$

It follows that $\bar{\delta}_1 = \bar{\delta}_2 \equiv \bar{\delta}$, which was expected, since the incumbents are symmetric. After some algebraic manipulation, we can write the critical (adjusted) discount factor as follows:

$$\mu^2\bar{\delta}(k) = \frac{\left(3+3k-8k^2\right)^2}{18+36k-79k^2-96k^3+128k^4}. \tag{30}$$

Deriving this expression in order to k, we obtain that:

$$\frac{d(\mu^2\bar{\delta})}{dk} = \frac{2k\left(3+8k^2\right)\left(3+3k-8k^2\right)}{\left(18+36k-79k^2-96k^3+128k^4\right)^2}.$$

As $3+3k-8k^2$ is positive, for all $k \in \left(0,\frac{1}{2}\right)$, the critical discount factor, $\bar{\delta}$, is (strictly) increasing in k. Thus, a high value of k enhances the possibilities of collusion (after the entry). Notice that, if the incumbents are very small (when compared with the entrant), the difference between their collusive profit and their Cournot profit is not very significative. This provides an incentive for the incumbents to break the collusive agreement, in order to receive the deviating profit. The only chance for collusion to be sustainable is if the incumbents greatly value their future profits. Notice that when $k \to 0$, we have that $\bar{\delta} \to 0$.

When $k \to \frac{1}{2}$, we have that $\mu^2\bar{\delta} \to 0.51$. Thus, if the incumbents own high shares of capital, collusion is almost certain (for reasonable values of δ).

A.2 Sustainability of collusion before the entry

If the firm 3 enters in the market along the punishment phase (the incumbents are competing *a la* Cournot), its optimal entry time is given by the expression (17).

The incumbents are willing collude before the entry if the following incentive compatibility constraint:

$$\sum_{s=t}^{\tilde{t}_3-1} \delta^{s-t}\Pi_{is}^{m}(2) + \sum_{s=\tilde{t}_3}^{\infty} \delta^{s-t}\Pi_{is}^{pm}(3) \geq \Pi_{it}^{d}(2) + \sum_{s=t+1}^{t+\tilde{t}_2-1} \delta^{s-t}\Pi_{is}^{c}(2) + \sum_{s=t+\tilde{t}_2}^{\infty} \delta^{s-t}\Pi_{is}^{c}(3) \tag{31}$$

is satisfied for all periods t, $t \in \{0, 1, ..., \tilde{t}_3 - 1\}$. Substituting the expressions for profits and using the fact that if the ICC is verified for $t = \tilde{t}_3 - 1$, then it is verified for all $t \in \{0, 1, ..., \tilde{t}_3 - 1\}$, we can write it as:

$$(\mu^2\delta)^{\tilde{t}_2}(\alpha_2 - \alpha_{3i}) + \mu^2\delta(-\beta_2 + \zeta_{3i} + \gamma_2 - \alpha_2) + (\beta_2 - \gamma_2) \geq 0.$$

Similarly to the case of full collusion, it is possible to show that the incumbents are not willing to collude in any period before the entry if:

$$(\mu^2\delta)^{t_2-1}(\alpha_2 - \alpha_{3i}) + \mu^2\delta(-\beta_2 + \zeta_{3i} + \gamma_2 - \alpha_2) + (\beta_2 - \gamma_2) < 0. \tag{32}$$

On the contrary, if the inequality above is satisfied:

$$(\mu^2\delta)^{t_2+1}(\alpha_2 - \alpha_{3i}) + \mu^2\delta(-\beta_2 + \zeta_{3i} + \gamma_2 - \alpha_2) + (\beta_2 - \gamma_2) \geq 0, \tag{33}$$

the incumbents are willing to collude in all periods before the entry.

Appendix B: Profits

B.1 Two firms

B.1.1 Cournot competition

In the case of competition between the two incumbents, each firm chooses the quantity that maximizes its own profit (taking the quantity produced by the rival firm as given). In period t, the firm i,$i \in \{1,2\}$, chooses q_{it} that maximizes:

$$\Pi_{it}(q_{it}, q_{jt}) = p_t q_{it} - C(q_{it}, k) = \left[\mu^t - (q_{it} + q_{jt})\right] q_{it} - \frac{q_{it}^2}{2k},$$

for $j \in \{1,2\}, j \neq i$. The associated first-oder condition (henceforward, FOC) is:

$$-2q_{it} - \frac{q_{it}}{k} - q_{jt} + \mu^t = 0. \tag{34}$$

By analogy, the FOC of the maximization problem of firm j is:

$$-2q_{jt} - \frac{q_{jt}}{k} - q_{it} + \mu^t = 0. \tag{35}$$

Combining (34) and (35), we obtain the equilibrium output of each firm in period t:

$$q_{1t}^c(2) = q_{2t}^c(2) = \frac{k}{1+3k}\mu^t.$$

The correspondent individual profit is equal to:

$$\Pi_{1t}^c(2) = \Pi_{2t}^c(2) = \frac{k(1+2k)}{2(1+3k)^2}\mu^{2t}.$$

B.1.2 Collusion

If the two incumbents decide to form a cartel, their joint share of the industry capital is equal to $2k$. In period t, firms produce the quantity, $q_{1t}^m(2)$ and $q_{2t}^m(2)$, that maximize their joint profit:

$$\Pi_t^m(q_{1t}, q_{2t}) = \left[\mu^t - (q_{1t} + q_{2t})\right](q_{1t} + q_{2t}) - \left(\frac{{q_{1t}}^2}{2k} + \frac{{q_{2t}}^2}{2k}\right),$$

The correspondent first-order conditions are:

$$\begin{cases} \mu^t - 2q_{1t} - 2q_{2t} - \frac{q_{1t}}{k} = 0 \\ \mu^t - 2q_{1t} - 2q_{2t} - \frac{q_{2t}}{k} = 0 \end{cases}.$$

Solving this system, we obtain that:

$$q_{1t}^m(2) = q_{2t}^m(2) = \frac{k}{1+4k}\mu^t.$$

Substituting these quantities in the expression of joint profit, we obtain that:

$$\Pi_t^m(2) = \frac{k}{1+4k}\mu^{2t}.$$

As the two incumbents are symmetric, they divide the monopoly profit in equal parts:

$$\Pi_{1t}^m(2) = \Pi_{2t}^m(2) = \frac{k}{2(1+4k)}\mu^{2t}.$$

B.1.3 Deviation

If the incumbent i, $i \in \{1, 2\}$, decides to deviate in period t, it produces the quantity, $q_{it}^d(2)$, that maximizes the following function:

$$\Pi_{it}^d(q_{it}) = \left[\mu^t - \left(q_{it} + \frac{k}{1+4k}\mu^t\right)\right] q_{it} - \frac{q_{it}^2}{2k}.$$

The associated first-order condition is:

$$-2q_{it} - \frac{q_{it}}{k} + \mu^t - \frac{k}{1+4k}\mu^t = 0,$$

whose solution is

$$q_{it}^d(2) = \frac{k(1+3k)}{1+6k+8k^2}\mu^t.$$

Substituting this quantity in $\Pi^d_{it}(q_{it})$, we obtain the deviating profit of firm i:

$$\Pi^d_{it}(2) = \frac{k(1+3k)^2}{2(1+2k)(1+4k)^2}\mu^{2t}.$$

B.2 Three firms

B.2.1 Cournot competition

If the firm 3 is active in the market and the three firms are in competition, the firm 1 chooses $q^c_{1t}(3)$ that maximizes the following function:

$$\Pi_{1t}(q_{1t}, q_{2t}, q_{3t}) = \left[\mu^t - (q_{1t} + q_{2t} + q_{3t})\right] q_{1t} - \frac{q^2_{1t}}{2k}.$$

The correspondent first-order condition is:

$$-2q_{1t} - \frac{q_{1t}}{k} - q_{2t} - q_{3t} + \mu^t = 0. \tag{36}$$

By analogy, the FOC correspondent to the profit maximization problem of firm 2 is:

$$-q_{1t} - 2q_{2t} - \frac{q_{2t}}{k} - q_{3t} + \mu^t = 0. \tag{37}$$

The profit function of the firm 3 is given by:

$$\Pi_{3t}(3) = \left[\mu^t - (q_{1t} + q_{2t} + q_{3t})\right] q_{3t} - \frac{q^2_{3t}}{2(1-2k)}.$$

Thus, the associated FOC is:

$$-q_{1t} - q_{2t} - 2q_{3t} - \frac{q_{3t}}{1-2k} + \mu^t = 0. \tag{38}$$

Combining (36), (37) and (38), we obtain the individual quantities in equilibrium:

$$q^c_{1t}(3) = q^c_{2t}(3) = \frac{2(1-k)k}{3+3k-8k^2}\mu^t \quad \text{and} \quad q^c_{3t}(3) = \frac{(1+k)(1-2k)}{3+3k-8k^2}\mu^t. \tag{39}$$

The correspondent (individual) profits are:

$$\Pi_{1t}^{c}(3) = \Pi_{2t}^{c}(3) = \frac{2k(1+2k)(1-k)^2}{(3+3k-8k^2)^2}\mu^{2t} \quad \text{and} \quad \Pi_{3t}^{c}(3) = \frac{(1+k)^2(3-10k+8k^2)}{2(3+3k-8k^2)^2}\mu^{2t}.$$

B.2.2 Collusion

If the three firms are in collusion in period t, the firms produce the quantities $(q_{1t}^m, q_{2t}^m, q_{3t}^m)$ that maximize their joint profit:

$$\Pi_t^m(q_{1t}, q_{2t}, q_{3t}) = \left[\mu^t - (q_{1t}+q_{2t}+q_{3t})\right](q_{1t}+q_{2t}+q_{3t}) - \left(\frac{{q_{1t}}^2}{2k} + \frac{{q_{2t}}^2}{2k} + \frac{{q_{3t}}^2}{2(1-2k)}\right),$$

The correspondent first-order conditions are:

$$\begin{cases} -2q_{1t} - \frac{q_{1t}}{k} - 2q_{2t} - 2q_{3t} + \mu^t = 0 \\ -2q_{1t} - 2q_{2t} - \frac{q_{2t}}{k} - 2q_{3t} + \mu^t = 0 \\ -2q_{1t} - 2q_{2t} - 2q_{3t} - \frac{q_{3t}}{1-2k} + \mu^t = 0 \end{cases}.$$

Solving this system, we obtain that:

$$q_{1t}^m(3) = q_{2t}^m(3) = \frac{k}{3}\mu^t \quad \text{and} \quad q_{3t}^m(3) = \frac{1-2k}{3}\mu^t.$$

Substituting them in the expression for Π_t^m, we obtain that:

$$\Pi_t^m(3) = \frac{\mu^{2t}}{6}.$$

B.2.3 Deviation

Consider that the three firms are colluding and the firm i, $i \in \{1,2,3\}$, decides to deviate in period t. In this case, it produces the quantity that maximizes the following function:

$$\Pi_{it}^d(q_{it}) = \left[\mu^t - \left(q_{it} + \frac{k_j}{3}\mu^t + \frac{1-k_i-k_j}{3}\mu^t\right)\right] q_{it} - \frac{q_{it}^2}{2k_i},$$

for $j \in \{1,2,3\}$ and $j \neq i$. The associated first-order condition is:

$$-2q_{it} + \mu^t - \frac{q_{it}}{k_i} - \frac{1-k_i-k_j}{3}\mu^t - \frac{k_j}{3}\mu^t = 0.$$

Solving this equation in order to q_{it}, we obtain:

$$q_{it}^d(3) = \frac{k_i\left(2+k_i\right)}{3\left(1+2k_i\right)}\mu^t.$$

The correspondent profit is:

$$\Pi_{it}^d(3) = \frac{k_i\left(2+k_i\right)^2}{18(1+2k_i)}\mu^{2t}.$$

B.3 Partial collusion

B.3.1 Collusion

Consider that the incumbents form a cartel, but they do not include the entrant in their agreement. Moreover, assume that the entrant becomes a follower, while the cartel behaves as a Stackelberg leader. Therefore, the entrant chooses the quantity, $q_{3t}^{pm}(3)$, that maximizes its individual profit:

$$\Pi_{3t}^{pm}(q_{1t}, q_{2t}, q_{3t}) = [\mu^t - (q_{1t} + q_{2t} + q_{3t})]q_{3t} - \frac{q_{3t}^2}{2(1-2k)}.$$

Thus, its best-response function to the incumbents' output is given by:

$$q_{3t}(q_{1t}, q_{2t}) = \frac{1-2k}{3-4k}\left(\mu^t - q_{1t} - q_{2t}\right). \tag{40}$$

As the cartel acts as a Stackelberg leader, it chooses the quantities $q_{1t}^{pm}(3)$ and $q_{2t}^{pm}(3)$ that maximize the incumbents' joint profit, given the best-response function of the firm 3. That is, the cartel maximizes the following function:

$$\begin{aligned}\Pi_t^{pm}(q_{1t}, q_{2t}) &= \left[\mu^t - q_{1t} - q_{2t} - \frac{1-2k}{3-4k}\left(\mu^t - q_{1t} - q_{2t}\right)\right](q_{1t} + q_{2t}) - \left(\frac{q_{1t}^2}{2k} + \frac{q_{2t}^2}{2k}\right) \\ &= \frac{2(1-k)}{3-4k}\left(\mu^t - q_{1t} - q_{2t}\right)(q_{1t} + q_{2t}) - \left(\frac{q_{1t}^2}{2k} + \frac{q_{2t}^2}{2k}\right).\end{aligned}$$

The correspondent first-order conditions are:

$$\begin{cases} -q_{1t} - \frac{q_{1t}}{k} - q_{2t} + \left(-1 + \frac{1-2k}{3-4k}\right)(q_{1t} + q_{2t}) + \mu^t - \frac{1-2k}{3-4k}\left(-q_{1t} - q_{2t} + \mu^t\right) = 0 \\ -q_{1t} - q_{2t} - \frac{q_{2t}}{k} + \left(-1 + \frac{1-2k}{3-4k}\right)(q_{1t} + q_{2t}) + \mu^t - \frac{1-2k}{3-4k}\left(-q_{1t} - q_{2t} + \mu^t\right) = 0 \end{cases}.$$

Solving the system, we obtain that:

$$q_{1t}^{pm}(3) = q_{2t}^{pm}(3) = \frac{2k(1-k)}{3+4k-8k^2}\mu^t.$$

Substituting these quantities in the best-response function of the entrant, we obtain:

$$q_{3t}^{pm}(3) = \frac{(1-2k)\left(3-4k^2\right)}{(3-4k)\left(3+4k-8k^2\right)}\mu^t$$

As a result, the profit of the incumbent i, for $i \in \{1,2\}$, in period t is given by:

$$\Pi_{it}^{pm}(3) = \frac{\Pi_t^{pm}(q_{1t}^{pm}, q_{2t}^{pm})}{2} = \frac{2k(1-k)^2}{9-40k^2+32k^3}\mu^{2t},$$

while the profit of the entrant is:

$$\Pi_{3t}^{pm}(3) = \frac{(1-2k)\left(3-4k^2\right)^2}{2(3-4k)\left(3+4k-8k^2\right)^2}\mu^{2t}.$$

B.2.3 Deviation

If the incumbent i, for $i \in \{1,2\}$, deviates in period t, it produces the quantity, $q_{it}^{pd}(3)$, that maximizes the following function:

$$\Pi_{it}^{pd}(q_{it}) = \left[\mu^t - \left(q_{it} + \frac{2k(1-k)}{3+4k-8k^2}\mu^t + \frac{(1-2k)\left(3-4k^2\right)}{(3-4k)\left(3+4k-8k^2\right)}\mu^t\right)\right] q_{it} - \frac{q_{it}^2}{2k}.$$

The first-order condition of this maximization problem is:

$$-2q_{it} - \frac{q_{it}}{k} + \mu^t - \frac{2(1-k)k\mu^t}{3+4k-8k^2} - \frac{(1-2k)\left(3-4k^2\right)\mu^t}{(3-4k)\left(3+4k-8k^2\right)} = 0.$$

Solving the equation in order to q_{it}, we obtain that:

$$q_{it}^{pd}(3) = \frac{2k(1-k)\left(-3-3k+8k^2\right)}{(1+2k)(3-4k)\left(-3-4k+8k^2\right)}\mu^t.$$

The correspondent profit is:

$$\Pi_{it}^{pd}(3) = \frac{2k(1-k)^2\left(3+3k-8k^2\right)^2}{(1+2k)(3-4k)^2\left(3+4k-8k^2\right)^2}\mu^{2t}.$$

Appendix C: Proofs

Proof of Proposition 1

An incumbent i, for $i \in \{1,2\}$,has higher profits with the Nash bargaining rule than with the Proportional rule if:

$$\begin{aligned} \Pi_{it}^{m,N}(3) \geq \Pi_{it}^{m,P}(3) \quad &\Leftrightarrow \quad k\left(15 - 66k + 15k^2 + 208k^3 - 192k^4\right) \geq 0 \\ &\Leftrightarrow \quad -k\left(k - \frac{1}{3}\right)\left(45 - 63k - 144k^2 + 192k^3\right) \geq 0 \end{aligned}$$

The polynomial $45-63k-144k^2+192k^3$ is always positive, for $k \in \left(\frac{1}{2}\right)$. Thus, the incumbent prefers the Nash bargaining rule if $k < \frac{1}{3}$. Otherwise, it is better off with the Proportional rule.

Let us now analyze how the allocation rule affects the profit of the entrant. This firm has a higher share of the monopoly profit with the Nash bargaining rule than with the Proportional rule if:

$$\Pi_{3t}^{m,N}(3) \geq \Pi_{3t}^{m,P}(3) \Leftrightarrow 2k\left(k - \frac{1}{3}\right)\left(45 - 63k - 144k^2 + 192k^3\right) \geq 0$$

Consequently, the entrant prefers the Nash bargaining rule if $k > \frac{1}{3}$ and prefers the Proportional rule otherwise. □

Proof of Proposition 2

Perfect collusion is not sustainable if the (adjusted) discount factor of some firm is greater than one. We analyze, in separate, the the critical value of the incumbents and of the entrant:

(i) The critical (adjusted) discount factor of the firm 1 (analogous to firm 2) is greater than one if:

$$\mu^2\delta_{1,P}(k) > 1 \Leftrightarrow \frac{\gamma_{31} - \frac{k}{6}}{\gamma_{31} - \alpha_{31}} > 1 \Leftrightarrow \frac{k\left(-3 + 18k - 3k^2 - 72k^3 + 64k^4\right)}{6\left(3 + 3k - 8k^2\right)^2} < 0.$$

For $k \in \left(0, \frac{1}{2}\right)$, the last inequality is equivalent to:

$$f(k) = -3 + 18k - 3k^2 - 72k^3 + 64k^4 < 0.$$

The first-order derivative of f is given by:

$$f'(k) = 18 - 6k - 216k^2 + 256k^3,$$

whose roots are:

$$k_1 = \frac{3 - \sqrt{393}}{64} < 0 \quad \vee \quad k_2 = \frac{3 + \sqrt{393}}{64} \in \left(0, \frac{1}{2}\right) \quad \vee \quad k_3 = \frac{3}{4} > \frac{1}{2}.$$

Thus, f' is positive for $k \in (0, k_2)$ and it is negative for $k \in (k_2, 1/2)$. Therefore, f is (strictly) increasing in $(0, k_2)$ and it (strictly) decreasing in $(k_2, 1/2)$. Moreover,

$$f(0) < 0 \quad ; \quad f(k_2) > 0 \quad \text{and} \quad f\left(\frac{1}{2}\right) > 0.$$

As f is continuous, by the intermediate value theorem, there exists $k^* \in (0, k_2)$ such that $f(k^*) = 0$. As f is increasing in this domain, k^* is the unique root of f in $(0, k_2)$. Finally, as $f(k_2) > f(1/2) > 0$ and f is (strictly) decreasing in $(k_2, 1/2)$, we conclude f has no roots in this interval. As a result,

$$f(k) < 0 \ \wedge \ k \in \left(0, \frac{1}{2}\right) \Leftrightarrow k \in (0, k^*).$$

Using, for example, the bisection method we can find that $k^* \approx 0.199$.

(ii) Consider now the case of the entrant:

$$\mu^2 \hat{\delta}_3(k) > 1 \Leftrightarrow \frac{\gamma_{33} - \frac{1-2k}{6}}{\gamma_{33} - \alpha_{33}} > 1 \Leftrightarrow \frac{2k(1-2k)\left(3 - 6k - 9k^2 + 16k^3\right)}{3\left(3 + 3k - 8k^2\right)^2} < 0.$$

For $k \in \left(0, \frac{1}{2}\right)$, the last inequality is equivalent to:

$$g(k) = 3 - 6k - 9k^2 + 16k^3 < 0.$$

As g is continuous, $g(0) > 0$ and $g(1/2) < 0$, there is $k^{**} \in (0, 1/2)$, such that $g(k^{**}) = 0$.

Let us now prove that k^{**} is unique. The first-order derivative of g is:

$$g'(k) = -6 - 18k + 48k^2,$$

whose roots are:

$$k_4 = \frac{3-\sqrt{41}}{16} < 0 \qquad \vee \qquad k_5 = \frac{3+\sqrt{41}}{16} > \frac{1}{2}.$$

Therefore, the function g' is negative for $k \in \left(0, \frac{1}{2}\right)$, which implies that g is (strictly) decreasing in this domain. As a result, k^{**} is unique and

$$g(k) < 0 \ \wedge \ k \in \left(0, \frac{1}{2}\right) \Leftrightarrow k \in \left(k^{**}, \frac{1}{2}\right).$$

Once again, using the bisection method, we find that $k^{**} \approx 0.436$. □

Proof of Lemma 2

Dividing the inequality (21) by δ^t, we obtain that:

$$\beta_2 \frac{(\mu^2\delta)^t - (\mu^2\delta)^{\tilde{t}_1}}{\delta^t(1-\mu^2\delta)} + \beta_{3i} \frac{\mu^{2\tilde{t}_1}\delta^{\tilde{t}_1 - t}}{1-\mu^2\delta} \geq \gamma_2 \mu^{2t} + \alpha_2 \frac{(\mu^2\delta)^{t+1} - (\mu^2\delta)^{\tilde{t}_2+t}}{\delta^t(1-\mu^2\delta)} + \alpha_{3i} \frac{\mu^{2(\tilde{t}_2+t)}\delta^{\tilde{t}_2}}{1-\mu^2\delta} \tag{41}$$

Evaluating the last inequality at $t = \tilde{t}_1 - 1$, the ICC can be written as follows:

$$(\gamma_2 - \beta_2)\mu^{2(\tilde{t}_1-1)} \leq \beta_{3i} \frac{\mu^{2\tilde{t}_1}\delta}{1-\mu^2\delta} - \alpha_2 \frac{(\mu^2\delta)^{\tilde{t}_1} - (\mu^2\delta)^{\tilde{t}_1-1+\tilde{t}_2}}{\delta^{\tilde{t}_1-1}(1-\mu^2\delta)} - \alpha_{3i} \frac{\mu^{2(\tilde{t}_1-1+\tilde{t}_2)}\delta^{\tilde{t}_2}}{1-\mu^2\delta}$$

Multiplying both sides by $\delta^{\tilde{t}_1}(1-\mu^2\delta)$ and after some rearranging, we obtain:

$$(\mu^2\delta)^{\tilde{t}_1-1}\left[\gamma_2 - \beta_2 + (\beta_2 - \gamma_2 - \beta_{3i} + \alpha_2)\mu^2\delta + (\alpha_{3i} - \alpha_2)(\mu^2\delta)^{\tilde{t}_2}\right] \leq 0.$$

As F is assumed to be sufficiently high to guarantee that $\tilde{t}_1 > 1$ and $\mu^2\delta < 1$, a sufficient condition for the last ICC to be satisfied is:

$$A(\mu, \delta, k) \equiv \gamma_2 - \beta_2 + (\beta_2 - \gamma_2 - \beta_{3i} + \alpha_2)\mu^2\delta + (\alpha_{3i} - \alpha_2)(\mu^2\delta)^{\tilde{t}_2} \leq 0. \tag{42}$$

Consider now a period $t = \tilde{t}_1 - \tau$ for $1 \leq \tau \leq \tilde{t}_1$. Substituting in 41 and rearranging the terms of the inequality, it becomes:

$$\gamma_2 \mu^{2(\tilde{t}_1-\tau)} \quad - \quad \beta_2 \frac{(\mu^2\delta)^{\tilde{t}_1-\tau} - (\mu^2\delta)^{\tilde{t}_1}}{\delta^{\tilde{t}_1-\tau}(1-\mu^2\delta)} \leq$$
$$\beta_{3i}\frac{\mu^{2\tilde{t}_1}\delta^{\tau}}{1-\mu^2\delta} - \alpha_2 \frac{(\mu^2\delta)^{\tilde{t}_1-\tau+1} - (\mu^2\delta)^{\tilde{t}_1-\tau+\tilde{t}_2}}{\delta^{\tilde{t}_1-\tau}(1-\mu^2\delta)} - \alpha_{3i}\frac{\mu^{2(\tilde{t}_1-\tau+\tilde{t}_2)}\delta^{\tilde{t}_2}}{1-\mu^2\delta}.$$

Multiplying both sides of the inequality by $\delta^{\tilde{t}_1-\tau}(1-\mu^2\delta)$, we obtain:

$$\gamma_2(\mu^2\delta)^{\tilde{t}_1-\tau}(1-\mu^2\delta) \quad - \quad \beta_2(\mu^2\delta)^{\tilde{t}_1-\tau} + \beta_2(\mu^2\delta)^{\tilde{t}_1} \leq$$
$$\beta_{3i}(\mu^2\delta)^{\tilde{t}_1} - \alpha_2(\mu^2\delta)^{\tilde{t}_1-\tau+1} + \alpha_2(\mu^2\delta)^{\tilde{t}_1-\tau+\tilde{t}_2} - \alpha_{3i}(\mu^2\delta)^{\tilde{t}_1-\tau+\tilde{t}_2}$$

$$\Leftrightarrow (\mu^2\delta)^{\tilde{t}_1-\tau}\left[\gamma_2 - \beta_2 + (\alpha_2-\gamma_2)\mu^2\delta + (\beta_2-\beta_{3i})(\mu^2\delta)^{\tau} + (\alpha_{3i}-\alpha_2)\mu^{\tilde{t}_2}\right] \leq 0$$

Notice that we can write the last inequality as follows:

$$A(\mu,\delta,k) - (\beta_2-\beta_{3i})\mu^2\delta\left[1-(\mu^2\delta)^{\tau-1}\right] \leq 0. \tag{43}$$

Therefore, if condition (42) holds and $\beta_2 - \beta_{3i} > 0$, the condition (43) is, *a fortiori*, verified (recall that $\mu^2\delta < 1$ and $\tau \geq 1$, implying that $1-(\mu^2\delta)^{\tau-1} > 1$). Let us show that $\beta_2-\beta_{3i} > 0$. The expressions for β_2 and for β_{3i} are given in (6) and (7), respectively. Therefore,

$$\beta_2 - \beta_{3i} = \frac{k\left(39+6k-201k^2-88k^3+320k^4\right)}{18(1+4k)\left(3+3k-8k^2\right)^2}.$$

As $k > 0$, the inequality above is equivalent to:

$$p(k) \equiv 39+6k-201k^2-88k^3+320k^4 > 0.$$

The first-order derivative of p is given by:

$$p'(k) = 6-402k-264k^2+1280k^3.$$

Moreover, as p' is continuous and:

$$\left.\begin{array}{r} p'(-0.5) < 0 \\ p'(-0.4) > 0 \end{array}\right\} \Rightarrow \exists k_1 \in (-0.5, -0.4) : p'(k_1) = 0$$

$$\left.\begin{array}{r} p'(0) < 0 \\ p'(0.1) > 0 \end{array}\right\} \Rightarrow \exists k_2 \in (0, 0.1) : p'(k_2) = 0$$

$$\left.\begin{array}{r} p'(0.6) < 0 \\ p'(0.7) > 0 \end{array}\right\} \Rightarrow \exists k_3 \in (0.6, 0.7) : p'(k_3) = 0.$$

As p' is a polynomial of third degree, its only zeros are k_1, k_2 and k_3. We conclude, therefore, that p is increasing in $(0, k_2)$ and decreasing in $\left(k_2, \frac{1}{2}\right)$. Thus, the minimum of p in the interval $(0, 0.5)$ must be achieved at one limit of this interval. As $p(0) > p\left(\frac{1}{2}\right) > 0$, we conclude that $p(k) > 0$, $\forall k \in \left(0, \frac{1}{2}\right)$. This ends the proof. □

Proof of Proposition 3

Multiplying both sides of the inequality (22) by $(1-\mu^2\delta)(\mu^2\delta)^{1-\tilde{t}_1}$ and rearranging the terms, this ICC can be rewritten as follows:

$$(\mu^2\delta)^{\tilde{t}_2}(\alpha_2 - \alpha_{3i}) + \mu^2\delta(-\beta_2 + \beta_{3i} + \gamma_2 - \alpha_2) + (\beta_2 - \gamma_2) \geq 0. \tag{44}$$

Let us now look at the incentive compatibility constraint (18). It is straightforward to see that the ICC is more easily satisfied the smaller the value of $\tilde{t}_2$. When $\tilde{t}_2$ decreases, the right hand side of the inequality decreases, while the left hand side does not change. Recall, however, that $\tilde{t}_2$ was defined as being an integer. In order to avoid integer problems and because we are looking for a sufficient condition for collusion not be sustainable, we can focus on the extreme value of $\tilde{t}_2$ that most facilitates collusion, that is, $\tilde{t}_2 = t_2 - 1$.

Thus, for any δ that verifies the following inequality:

$$(\mu^2\delta)^{t_2-1}(\alpha_2 - \alpha_{3i}) + \mu^2\delta(-\beta_2 + \beta_{3i} + \gamma_2 - \alpha_2) + (\beta_2 - \gamma_2) < 0,$$

where $i \in \{1, 2\}$, collusion is not sustainable before the entry. □

Proof of Proposition 4

To avoid the integer problems of the inequality (44), we may substitute $\tilde{t}_2$ for an integer. Looking at the incentive compatibility constraint (18), we conclude that a higher value for $\tilde{t}_2$ hinders collusion, since it shortens the punishment phase. As we are looking for a sufficient condition for collusion to be sustainable, we can focus on the value of $\tilde{t}_2$ that most hurt collusion. Thus, substituting $\tilde{t}_2$ for $t_2 + 1$ in the inequality (22), we come to the following inequality:

$$(\mu^2\delta)^{t_2+1}(\alpha_2 - \alpha_{3i}) + \mu^2\delta(-\beta_2 + \beta_{3i} + \gamma_2 - \alpha_2) + (\beta_2 - \gamma_2) \geq 0.$$

□

Proof of Proposition 5

Let us compare the profit of each incumbent with the profit of the entrant when the incumbents are colluding and the entrant is competing *a la* Cournot:

$$\Pi_{it}^{pm}(3) \geq \Pi_{3t}^{pm}(3) \Leftrightarrow -\frac{9 - 30k - 16k^2 + 100k^3 - 64k^4}{2(3-4k)\left(3+4k-8k^2\right)^2} \geq 0$$

Restricting to the domain $\left(0, \frac{1}{2}\right)$, the last inequality is equivalent to:

$$p(k) \equiv 9 - 30k - 16k^2 + 100k^3 - 64k^4 \leq 0.$$

The first-order derivative of p is given by:

$$p'(k) = -30 - 32k + 300k^2 - 256k^3.$$

As p' is continuous and:

$$\left.\begin{array}{r} p'(-0.3) > 0 \\ p'(-0.2) < 0 \end{array}\right\} \Rightarrow \exists k_1 \in (-0.3, -0.2) : p'(k_1) = 0$$

$$\left.\begin{array}{r} p'(0.5) < 0 \\ p'(0.6) > 0 \end{array}\right\} \Rightarrow \exists k_2 \in (0.5, 0.6) : p'(k_2) = 0$$

$$\left.\begin{array}{r} p'(0.8) > 0 \\ p'(0.9) < 0 \end{array}\right\} \Rightarrow \exists k_3 \in (0.8, 0.9) : p'(k_3) = 0.$$

As p' is a polynomial of third degree, its only zeros are k_1, k_2 and k_3. Thus, p' has no zeros in the interval $\left(0, \frac{1}{2}\right)$, which means that p is (strictly) decreasing in this interval. Therefore, the polynomial p can have, at most, one zero in the interval $\left(0, \frac{1}{2}\right)$. Moreover, as:

$$p(0.3415) > 0 \quad \text{and} \quad p(0.3416) < 0$$

we conclude that there exists $k^* \in (0.3415, 0.3416)$ such that $p(k^*) = 0$. Thus, if $k \in \left(k^*, \frac{1}{2}\right)$, then $p(k) \leq 0$. $\square$

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