House Prices and the Taylor Rule

Paulo Brito*, Giancarlo Marini† and Alessandro Piergallini‡

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Abstract

Deviations from the Taylor rule occurred between 2002 and 2006 are argued to be a key source of the 2007-2009 financial crisis, for they triggered the boom and bust in house prices. This paper investigates global dynamics in an optimizing monetary model with housing-wealth effects. It is shown that even the Taylor rule is unable to avoid possible house-price instabilities. In particular, it is demonstrated that under the Taylor rule there exist self-fulfilling paths of house prices and inflation along a heteroclinic orbit converging to a liquidity trap equilibrium. Bifurcation analysis reveals that the degree of activism of monetary policy plays a crucial role in the determination of global house-price dynamics along the decelerating-inflation path. In addition, it is proved that liquidity traps cannot be ruled out by interest-rate feedback rules responding to both inflation and house prices. Reacting to housing inflation even complicates equilibrium dynamics by introducing the possibility of local indeterminacy.

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*Department of Economics, Universidade Técnica de Lisboa, Rua Miguel Lupi, 20, 1249-078 Lisboa, Portugal. Phone: + 35121 3925981. Fax: + 351213922808. E-mail: pbrito@iseg.utl.pt.

†Department of Economics, Tor Vergata University, Via Columbia 2, 00133 Rome, Italy. Phone: +390672595723. Fax: +39062020500. E-mail: marini@uniroma2.it.

‡Department of Economics, Tor Vergata University, Via Columbia 2, 00133 Rome, Italy. Phone: +390672595431. Fax: +39062020500. E-mail: alessandro.piergallini@uniroma2.it.

1 Introduction

From 2002 to 2006 the Federal Funds Rate was 2-3 percentage points below the path prescribed by the Taylor rule for any period since 1980s (Poole, 2007; Taylor, 2007). Significant downward interest-rate gaps from the Taylor rule were also visible over the same period in the OECD countries as a group (Ahrend, Cournède and Price, 2008).

Leamer (2007) and Taylor (2007, 2010, 2011) contend that such a "Great Deviation", resulting in a too accommodating monetary policy, was a key source of the economic and financial crisis erupted in 2007, for it triggered boom-bust dynamics in house prices.

This paper shows that even the Taylor rule is unable to avoid possible house-price instabilities combined with inflation dynamics divergent from the target rate. Specifically, we investigate global dynamics in an optimizing monetary model with housing-wealth effects and demonstrate that following Taylor's prescriptions does not eliminate the possibility of off-target paths whereby house prices and inflation follow an heteroclinic orbit converging to a liquidity trap equilibrium. From the bifurcation analysis, we find that the degree of activism of monetary policy may critically alter global house-price dynamics along the decelerating-inflation path. We also show that global indeterminacy cannot be ruled out by interest-rate feedback rules responding to both inflation and house prices. Reacting to housing inflation may even exacerbate macroeconomic instability by introducing the possibility of local indeterminacy.

The paper is organized as follows. Section 2 lays out the paper's connections with the literature. Section 3 presents the model. Section 4 analyzes the issue of global equilibrium dynamics under a conventional Taylor rule. Section 5 extends the analysis to the case of interest rate policy rules reacting to house prices. Section 6 concludes.

2 Links to the Literature

The paper is linked to both empirical and theoretical literature. Empirical evidence by Muellbauer (2007), Campbell and Cocco (2007), and Carroll, Otsuka and Slacalek (2011) finds highly significant housing-wealth effects on consumption. However, the link between house prices and consumption dynamics is typically overlooked in standard frameworks for monetary policy analysis (e.g., Taylor, 1999; Woodford, 2003; Galí, 2008). In this paper we intend to consider an optimizing framework in which housing-wealth effects do affect the monetary policy transmission mechanism.

Theoretical works by Benhabib, Schmitt-Grohé and Uribe (2001, 2002, 2003) and Schmitt-Grohé and Uribe (2009) show that once global dynamics are taken into account, the usual local stabilizing properties of Taylor (1993,1999)-type interest-rate feedback rules disappear. In particular, Taylor rules give rise to multiple self-fulfilling decelerating inflation paths converging to a long-run equilibrium around which the monetary authority is no longer able to ensure aggregate stability. Their model is based on the standard infinite-horizon representative agent setup. Therefore, aggregate demand dynamics only depends on real interest rates. To incorporate housing-wealth effects, we must relax this paradigm. Specifically, we shall use an overlapping generations framework à la Yaari (1965)-Blanchard (1985)-Weil (1989). In this way, aggregate demand dynamics will depend not only on real interest rates, but also on housing wealth. As a consequence, monetary policy decisions will affect aggregate demand and inflation through their effects on both the real interest rate and house prices. The model with housing-wealth effects derived in this paper also constitutes a useful theoretical benchmark to investigate the dynamic properties of monetary policies feedback rules whereby the nominal interest rate reacts not only to inflation but also to house prices.

Using a New Keynesian model with households' borrowing constraints, Iacoviello (2005) examines the role of house prices for the business cycle and the design of optimal monetary policy. Consistently with the business cycle literature, the approach relies on local dynamics, thereby abstracting from possible multiplicities of steady-state equilibria. This paper is different in two respects. First, we present an alternative way to analyze the implications of house prices for monetary policy design, since we employ an overlapping generations optimizing model. Second, a central focus of this paper is to depart from local analysis. As recently advocated by Cochrane (2011), we use the criterion of global determinacy to evaluate the connection between monetary policy rules and macroeconomic stability.

The analytical results derived in our model with housing-wealth effects show that the Taylor rule, even in the case in which responds to house prices, cannot rule out the possibility of house-price instabilities interacted with inflation paths divergent from the target rate. The results thus appear to provide additional theoretical support to the literature proposing other paradigms for stabilizing policy making. Ensuring global equilibrium uniqueness may require a Non-Ricardian fiscal policy regime (Benhabib, Schmitt-Grohé and Uribe, 2002; Cochrane, 2011), an interest rate policy guaranteeing a sufficiently high real return on financial assets (Benassy, 2008, 2009), or a price-level targeting rule (Adao, Correia and Teles, 2011).

3 The Model

Consider the following monetary version of the Yaari (1965)-Blanchard (1985)-Weil (1989) overlapping generations setup, extended to incorporate housing in the agents' asset menu. Each individual faces a common and constant instantaneous probability of death, $\mu > 0$. Population grows at a constant rate n. At each instant t a new generation is born. The birth rate is $\beta = n + \mu$. Let $N(t)$ denote population at time t, with $N(0) = 1$. So the size of the generation born at time t is $\beta N(t) = \beta e^{nt}$, and the size of the surviving cohort born at time $s \leq t$ is $\beta N(s) e^{-\mu(t-s)} = \beta e^{-\mu t} e^{\beta s}$. Total population at time t is given by $N(t) = \beta e^{-\mu t} \int_{-\infty}^{t} e^{\beta s} ds$. As in Blanchard (1985), there is no dynastic altruism. Financial wealth of newly born individuals is therefore zero. Agents supply one unit of labor inelastically, which is transformed one-for-one into output.

The representative agent of the generation born at time $s \leq 0$ chooses the time path of consumption, $\overline{c}(s,t)$, real money balances, $\overline{m}(s,t)$, and housing $\overline{h}(s,t)$ in order to maximize the expected lifetime utility function given by

$$E_0 \int_0^{\infty} \left[\alpha \log \Lambda \left(\overline{c}(s,t), \overline{m}(s,t)\right) + (1-\alpha) \log \overline{h}(s,t)\right] e^{-\rho t} dt, \tag{1}$$

where E_0 is the expectation operator conditional on period 0 information, $\rho > 0$ is the pure rate of time preference, and $\Lambda(\cdot)$ is a strictly increasing, strictly concave and linearly homogenous function. Consumption and real money balances are Edgeworth complements, that is $\Lambda_{cm} > 0$, and the elasticity of substitution between the two is lower than unity. Because the probability at time 0 of surviving at time $t \geq 0$ is $e^{-\mu t}$, the expected lifetime utility function (1) is

$$\int_0^\infty \left[\alpha \log \Lambda\left(\overline{c}(s,t), \overline{m}(s,t)\right) + (1-\alpha)\log \overline{h}(s,t)\right] e^{-(\mu+\rho)t} dt. \tag{2}$$

Individuals accumulate their financial wealth, $\overline{a}(s,t)$, in the form of real money balances, interest bearing public bonds, $\overline{b}(s,t)$, and housing $q(t)\overline{h}(s,t)$, where $q(t)$ is the relative house price. Therefore, $\overline{a}(s,t) = \overline{b}(s,t) + \overline{m}(s,t) + q(t)\overline{h}(s,t)$. The instantaneous budget constraint is given by

$$\begin{aligned} \dot{\overline{a}}(s,t) &= \left(R(t) - \pi(t) + \mu\right)\overline{a}(s,t) + \overline{y}(s,t) - \overline{\tau}(s,t) - \overline{c}(s,t) - \\ &\quad - R(t)\overline{m}(s,t) + \left[\frac{\dot{q}(t)}{q(t)} - \left(R(t) - \pi(t)\right)\right] q(t)\overline{h}(s,t), \end{aligned} \tag{3}$$

where $R(t)$ is the nominal interest rate, $\pi(t)$ is the inflation rate, $\overline{\tau}(s,t)$ are real lump-sum taxes, and $\mu\overline{a}(s,t)$ is an actuarial fair payment that individuals receive from a perfectly competitive life insurance company in exchange for their financial wealth at the time of death, in the spirit of Yaari (1965).[1] Since the asset menu includes housing equity, a reverse-mortgage mechanism à la Eschtruth and Tran (2001) is operative.

Agents are prevented from engaging in Ponzi's games, so that

$$\lim_{t\to\infty} \overline{a}(s,t) e^{-\int_0^t (R(j)-\pi(j)+\mu)dj} \geq 0. \tag{4}$$

Letting $\overline{z}(s,t)$ denote total consumption at time t for the agent born at time s, defined

[1] Insurance companies collect financial assets from deceased individuals and pay fair premia to current generations. The presence of the life insurance market precludes the possibility for individuals of passing away leaving unintended bequests to their heirs. See Blanchard (1985).

as physical consumption plus the interest forgone on real money holdings,

$$\overline{z}(s,t) = \overline{c}(s,t) + R(t)\overline{m}(s,t), \tag{5}$$

the individual optimizing problem can thus be solved using a two-stage procedure.[2]

In the first stage, consumers solve an intratemporal problem of choosing the efficient allocation between consumption, $\overline{c}(s,t)$, and real money balances, $\overline{m}(s,t)$, in order to maximize function $\Lambda(\cdot)$, for a given level of total consumption, $\overline{z}(s,t)$. The marginal rate of substitution between consumption and real money balances must equal to the nominal interest rate, $\Lambda_m(\overline{c}(s,t), \overline{m}(s,t)) / \Lambda_c(\overline{c}(s,t), \overline{m}(s,t)) = R(t)$. Because preferences are linearly homogenous, this optimality condition assumes the following form:

$$\overline{c}(s,t) = \Gamma(R(t))\overline{m}(s,t), \tag{6}$$

where $\Gamma'(R) > 0$.

In the second stage, individuals solve an intertemporal problem of choosing the optimal time paths of total consumption, $\overline{z}(s,t)$, and housing, $\overline{h}(s,t)$, in order to maximize their lifetime utility function (2), given the constraints (3), (4) and the optimal condition (6).[3] Optimality yields

$$\dot{\overline{z}}(s,t) = (R(t) - \pi(t) - \rho)\,\overline{z}(s,t), \tag{7}$$

$$\frac{(1-\alpha)}{\alpha}\frac{\overline{z}(s,t)}{q(t)\overline{h}(s,t)} = (R(t) - \pi(t)) - \frac{\dot{q}(t)}{q(t)}, \tag{8}$$

$$\lim_{t\to\infty} \overline{a}(s,t)e^{-\int_0^t (R(j)-\pi(j)+\mu)dj} = 0. \tag{9}$$

Substituting the optimality condition (8) in the instantaneous budget constraint (3), integrating forward, applying the transversality condition (9), and using the law of motion of total consumption (7), total consumption can be expressed as a linear function of total

[2] See Deaton and Muellbauer (1980). In the context of the Yaari–Blanchard framework, see Marini and van der Ploeg (1988).

[3] See Appendix A for analytical details.

wealth:

$$\overline{z}(s,t) = \alpha(\mu+\rho)\left(\overline{a}(s,t)+\overline{k}(s,t)\right), \tag{10}$$

where $\overline{k}(s,t) \equiv \int_t^{\infty}\left(\overline{y}(s,t)-\overline{\tau}(s,t)\right)e^{-\int_t^v (R(j)-\pi(j)+\mu)dj}dv$ denotes human wealth, defined as the present discounted value of after-tax labor income. From (5), (6), and (10), it also follows that

$$\overline{c}(s,t) = \frac{\alpha(\mu+\rho)}{L(R(t))}\left(\overline{a}(s,t)+\overline{k}(s,t)\right). \tag{11}$$

Combining next (5), (6), and (7), we obtain the optimal time path of individual consumption:

$$\dot{\overline{c}}(s,t) = \left(R(t)-\pi(t)-\rho\right)\overline{c}(s,t) - \frac{L'(R(t))}{L(R(t))}\dot{R}(t)\overline{c}(s,t), \tag{12}$$

where $L(R) \equiv 1 + R/\Gamma(R)$ and $L'(R) > 0$. According to (12), the optimal consumption growth rate is identical across all generations. Function $L(R)$ satisfies $L(0) = 1$, $L(\infty) = +\infty$, $L'(0) = \infty$ and $L'(\infty) = 0$. The latter properties are verified, for example, if we assume $\Lambda(\bar{c}, \bar{m})$ is a CES function.

3.1 Aggregation and Fiscal Policy

Let now derive the evolution of aggregate variables. Let define the population aggregate for a generic economic variable at individual level, $\overline{x}(s,t)$, as $X(t) \equiv \beta e^{-\mu t}\int_{-\infty}^{t}\overline{x}(s,t)e^{\beta s}ds$, and corresponding quantity in per capita terms as $x(t) \equiv X(t)e^{-nt} = \beta\int_{-\infty}^{t}\overline{x}(s,t)e^{\beta(s-t)}ds$.

Suppose that each agent faces identical age-independent income and tax flows, so that $\overline{y}(s,t) = \overline{y}(t)$ and $\overline{\tau}(s,t) = \overline{\tau}(t)$, as in Blanchard (1985). Using $\overline{a}(t,t) = 0$ and $\overline{c}(t,t) = [\alpha(\mu+\rho)/L(R(t))]\,\overline{k}(t,t)$, the budget constraint, the optimal time path of consumption, the optimal time path of house prices, and the transversality condition expressed in per capita terms are of the form[4]

$$\begin{aligned} \dot{a}(t) &= (R(t) - \pi(t) - n)\, a(t) + y(t) - \tau(t) - c(t) - \\ &\quad - R(t)m(t) + \left[\frac{\dot{q}(t)}{q(t)} - (R(t) - \pi(t))\right] q(t)h(t), \end{aligned} \tag{13}$$

$$\dot{c}(t) = (R(t) - \pi(t) - \rho)\, c(t) - \frac{L'(R(t))\dot{R}(t)}{L(R(t))} c(t) - \frac{\alpha\beta(\rho + \mu)}{L(R(t))} a(t), \tag{14}$$

$$\frac{\dot{q}(t)}{q(t)} = (R(t) - \pi(t)) - \frac{(1-\alpha)}{\alpha} \frac{L(R(t))c(t)}{q(t)h(t)}. \tag{15}$$

$$\lim_{t \to \infty} a(t) e^{-\int_0^t (R(j) - \pi(j) + \mu) dj} = 0. \tag{16}$$

From (14), the rate of change of per capita consumption depends on the level of financial wealth $a(t)$ since future cohorts' consumption is not valued by agents currently alive. In particular, older generations are wealthier than younger generations, and so consume more and save less. Only in the limiting case in which the birth rate β is equal to zero, per capita consumption dynamics follows the standard Euler equation prevailing in the infinitely-lived representative agent paradigm.

The flow budget constraint of the government in per capita terms is given by

$$\dot{b}(t) + \dot{m}(t) = (R(t) - \pi(t) - n)\, b(t) - \tau(t) - \pi(t)m(t), \tag{17}$$

To concentrate on the implications of housing-wealth effects, the government is assumed to adopt a tax policy consisting in balancing the budget at all times. It follows taxes are such that

$$\tau(t) + \pi(t)m(t) = (R(t) - \pi(t) - n)\, b(t). \tag{18}$$

[4]See Appendix B for analytical details.

3.2 Monetary Policy Rules

We assume that the monetary authority follows a conventional Taylor rule, controlling $R(t)$ according to a feedback rule of the form

$$R(t) = T(\pi(t)), \tag{19}$$

where $R(\cdot)$ is a continuous, strictly increasing and strictly positive function. Monetary policy is active when $T'(\pi(t)) > 1$ and passive when $T'(\pi(t)) < 1$.

In particular, we may assume as advocated by Taylor (1993, 1999) a linear rule such as

$$T(\pi) = \tilde{r} + \pi + \gamma(\pi - \tilde{\pi}) \tag{20}$$

where $\tilde{r}$ and $\tilde{\pi}$ are the central bank's targets for the real interest rate and the inflation rate, and $\gamma > 0$ is the policy parameter featuring an active monetary policy.

We could assume instead one of the following alternative feedback policy rules which also controls for the prices of housing. The first controls for the level of house prices,

$$T(\pi, q) = \tilde{r} + \pi + \gamma(\pi - \tilde{\pi}) + \epsilon(q - \tilde{q}) \tag{21}$$

where $\tilde{q}, \epsilon > 0$, and the second controls for the inflation in housing prices

$$T\left(\pi, \frac{\dot{q}}{q}\right) = \tilde{r} + \pi + \gamma(\pi - \tilde{\pi}) + \delta\frac{\dot{q}}{q} \tag{22}$$

where $\delta > 0$.

3.3 Equilibrium

Total output $\overline{y}(t)$ and housing supply $\bar{h}(t)^s$ are assumed to grow at the constant rate n, without loss of generality. It follows that per capita output and housing are constant and can be normalized to one, $y(t) = y = h^s = 1$, for analytical convenience. Equilibrium in the goods market requires that $c(t) = y = 1$. Equilibrium in the housing market requires that $h(t) = h^s = 1$. The balanced budget rule implies $\dot{b}(t) + \dot{m}(t) = 0$, so that total government liabilities are constant over time, $b(t) + m(t) = l$, where l is a constant. For analytical convenience and without loss of generality, we can study the dynamic properties of the model normalizing the constant l to zero.

Using the law of motion of per capita consumption (12), the equilibrium real interest rate is given by

$$R(t) - \pi(t) = \rho + \frac{L'(R(t))\dot{R}(t)}{L(R(t))} + \frac{\alpha\beta(\rho+\mu)}{L(R(t))}q(t). \tag{23}$$

From (23), the nominal interest rate dynamics are given by

$$\dot{R}(t) = \frac{1}{L'(R(t))} \left[\, (R(t) - \pi(t) - \rho)\, L(R(t)) - \alpha\beta(\rho+\mu)q(t) \right]. \tag{24}$$

From (15), house price dynamics are given by

$$\dot{q}(t) = \left[R(\pi(t)) - \pi(t)\right] q(t) - \frac{(1-\alpha)}{\alpha} L\left[R(\pi(t))\right]. \tag{25}$$

The equilibrium dynamic system is completed by introducing a monetary policy rule. For now, we consider a generic rule as

$$R(t) = T\left(\pi(t), q(t), \dot{q}(t)/q(t)\right), \tag{26}$$

where we assume that $T(\pi, q, \dot{q}/q)$ is increasing and additively separable in all its components. Then we can solve equation (26) for π to get

$$\pi(t) = P(R(t), q(t), \dot{q}(t)/q(t)). \tag{27}$$

Again an active policy is such that $\partial T/\partial \pi > 1$, which implies that $0 < \partial P/\partial R < 1$ for any $(R, q, \dot{q}/q)$. To be an equilibrium, the paths $(R(t), q(t), \pi(t))_{t\in[0,\infty)}$, solving equations (24), (25) and (27) should verify the no-Ponzi game and the transversality conditions.

4 Conventional Taylor Rule

4.1 Steady-State and Local Dynamics

Assuming a generic conventional Taylor rule implies $\pi = P(R) = T^{-1}(R)$, where $0 < P^{'}(R) < 1$. Then we can write the system (24)-(25) as

$$\dot{q} = (R(t) - P(R(t)))\,(q(t) - \Psi(R(t)))\,, \tag{28}$$

$$\dot{R} = \frac{\alpha\beta(\rho+\mu)}{L'(R(t))}\,(\Phi(R(t)) - q(t))\,, \tag{29}$$

where

$$\Psi \equiv \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{L(R)}{R-P(R)}\right), \tag{30}$$

$$\Phi \equiv \left(\frac{R-P(R)-\rho}{\alpha\beta(\rho+\mu)}\right)L(R). \tag{31}$$

Equilibrium steady states are bounded values for R and q, such that the transversality condition (16) holds. Therefore, asymptotically $\lim_{t\to\infty}(R(t) - P(R(t)) + \mu) > 0$.

Let us define

$$r_+ \equiv \frac{\rho + \sqrt{\rho^2 + 4\beta\,(1-\alpha)\,(\rho+\mu)}}{2},\ r_- \equiv \frac{\rho - \sqrt{\rho^2 + 4\beta\,(1-\alpha)\,(\rho+\mu)}}{2} \tag{32}$$

and observe that $r_+ > \rho > 0 > r_-$. The following proposition applies.

Proposition 1 *Assume a conventional Taylor rule in which the monetary policy is globally active. Then: (a) if $P(0) < -r_+$, there is a unique steady state equilibrium $(R^*, q^*) = (0, q_0^*)$, where*

$$q_0^* = -\left(\frac{1-\alpha}{\alpha}\right)\frac{1}{P(0)} > 0; \tag{33}$$

(b) if $P(0) \geq 0$, there is a unique steady state equilibrium is $(R^*, q^*) = (R_1^*, q_1^*)$, where

$$q_1^* = \left(\frac{1-\alpha}{\alpha}\right)\frac{L(R_1^*)}{r^*} > 0, \tag{34}$$

where $r^* = r_+ > \rho$, *and* $R_1^* = \{R : \ R - P(R) = r^*\}$ *is the unique element; and (c) if* $0 > P(0) \geq -r_+$, *there are two equilibrium steady states* $(0, q_0^*)$ *and* (R_1^*, q_1^*).

Proof. See Appendix C.

As a result, interest-rate feedback rules of the Taylor-type may give rise to multiple steady-state values for house prices. From Proposition 1, the steady-state level of house prices q_0^*, associated with $R^* = 0$, depends upon the specification of the policy rule. Both the steady-state levels of the nominal interest rate R_1^*, and of house prices q_1^* also depend on the policy rule.

The long-run inflation rates associated to both steady states are $\pi_0^* = P(0) < 0$ and $\pi_1^* = P(R_1^*) = R_1^* - r^* > \pi_0^*$.[5] For the case of a linear rule à la Taylor (1993, 1999), we can determine R_1^* explicitly. Specifically, substituting equation (20) yields $R_1^* = (1+\gamma)(P(0)+r^*)/\gamma = r^* + \tilde{\pi} + (r^* - \tilde{r})/\gamma$, because $P(0) = (\gamma\tilde{\pi} - \tilde{r})/(1+\gamma)$. In this case, a necessary condition for the existence of two steady states is $(1+\gamma)r^* - \gamma\tilde{\pi} > \tilde{r} > \gamma\tilde{\pi}$.

Let now compare the steady-state value of house prices associated with a zero nominal interest rate with the one associated a positive interest rate. From the expressions in Proposition 1, we get $q_0^* > q_1^*$ if and only if $r^* + P(0)L(R_1^*) > 0$. Therefore, the relationship between the house prices for the two possible long-run nominal interest rates depends upon the long-run real interest rate, the substitutability between real money balances and consumption, and the characteristics of the Taylor rule. The latter condition is sufficient for the existence of two steady states, since it satisfies $r^* + P(0) > 0$. However, if $r^* + P(0) > 0 \geq r^* + P(0)L(R_1^*)$, we have two steady states and $q_0^* \leq q_1^*$. If we assume a linear conventional Taylor rule, the case in which $q_0^* > q_1^*$ occurs if

$$\Gamma\left(\frac{(1+\gamma)r^* + \gamma\tilde{\pi} - \tilde{r}}{\gamma}\right) > \frac{\tilde{r} - \gamma\tilde{\pi}}{\gamma}, \tag{35}$$

which tends to be the case the long-run real interest rate r^*, the inflation target $\tilde{\pi}$, the

[5] We could assume a more general monetary rule such that it would be active for very high interest rates (e.g., $\lim_{R\to\infty} P'(R) < 1$). In this case, there could be other steady states in which the monetary policy would be locally passive such that $P'(R) > 1$.

monetary policy feedback parameter γ, and/or the elasticity of substitution between real money balances and consumption are sufficiently large.

Next we explore local equilibrium dynamics. Linearizing the equations (28) and (29) in the neighborhood of any point (R, q), we get the Jacobian

$$J = \begin{pmatrix} R - P(R) & (1 - P'(R))(q - \Psi(R)) - (R - P(R))\Psi'(R) \\ -\frac{\alpha\beta(\rho+\mu)}{L'(R)} & \frac{\alpha\beta(\rho+\mu)}{(L'(R))^2}\left(\Phi'(R)L'(R) - (\Phi(R) - q)L''(R)\right) \end{pmatrix}. \tag{36}$$

The trace and the determinant of the Jacobian matrix are

$$\mathrm{tr}J = R - P(R) + \frac{\alpha\beta(\rho+\mu)}{L'(R)}\left(\Phi'(R) - (\Phi(R) - q)\frac{L''(R)}{L'(R)}\right)$$

and

$$\det J = \frac{\alpha\beta(\rho+\mu)}{L'(R)}\left[\begin{array}{c} (R - P(R))\left(\Phi'(R) - \Psi'(R) + (q - \Phi(R))\frac{L''(R)}{L'(R)}\right) + \\ +(1 - P'(R))(q - \Psi(R)) \end{array}\right].$$

We readily observe that

$$\Psi'(R) = \Psi(R)\left(\frac{L'(R)}{L(R)} - \frac{1 - P'(R)}{R - P(R)}\right)$$

and

$$\Phi'(R) = \Phi(R)\left(\frac{1 - P'(R)}{R - P(R) - \rho} - \frac{L'(R)}{L(R)}\right).$$

Hence the following proposition holds.

Proposition 2 *Let the assumptions in Proposition 1 hold, such that there are two steady state equilibria. Then, the steady state* $(0, q_0^*)$ *is a singular saddle point and the steady state* (R_1^*, q_1^*) *is a source.*

Proof. See Appendix D.

From Proposition 2, since both $R(t)$ and $q(t)$ are jump variables, the steady state in which $R^* = 0$ is locally indeterminate and the steady state $R^* = R_1^* > 0$ is locally determinate.

Therefore, in the neighborhood of (R_1^*, q_1^*) there exists a unique equilibrium converging asymptotically to the steady state. In the absence of exogenous fundamental shocks, the only path of $R(t)$ and $q(t)$ converging asymptotically to (R_1^*, q_1^*) is the steady state itself. As a result, even in the presence of housing-wealth effects, an active monetary policy stance in the spirit of Taylor (1993, 1999) exhibits local stabilizing properties. Suppose that the occurrence of an exogenous shock brings about an upward deviation of inflation from the target. If monetary policy is active, the real interest rate increases and house prices decrease. Aggregate consumption declines because of both the increase in the real interest rate and the decrease in house prices which generates a negative wealth effect. The associated fall in aggregate demand causes prices to decrease, hence dampening the initial inflationary pressure. Suppose next that an exogenous shock leads to high level of house prices. Aggregate demand and thus inflation are stimulated via the positive wealth effect on aggregate consumption. If monetary policy is active, the real interest rate increases and house prices decrease, thereby inducing aggregate stability.

Nevertheless, in a small neighborhood around the steady state in which $R^* = 0$, there is local indeterminacy. That is, there exist infinite equilibrium paths of $R(t)$ and $q(t)$ converging asymptotically to the steady state. In particular, for any initial $R(0)$ there exists a $q(0)$ such that the time paths of $R(t)$ and $q(t)$ satisfying the system (28)-(29) will converge asymptotically to the steady state. As we shall analyze in the next section, local indeterminacy at $R^* = 0$ means there is a liquidity trap. However, it is also a singular steady state, in the sense that the eigenvalues for system (R, q) are infinite. Singular steady states appear in economic theory from the existence of static constraints in some macroeconomic models, as in Leeper and Sims (1994). However, in our case the singularity has a different nature: it is related to the properties of function $\Gamma(R)$, because when R tends to zero the relationship between consumption and money demand becomes locally insensitive to the nominal interest rate. This type of singularity seems to not have been uncovered by the previous papers.

4.2 Global Dynamics

Benhabib, Schmitt-Grohé and Uribe (2001) and Schmitt-Grohé and Uribe (2009) considered the perils of the Taylor rule by conducting a global dynamics analysis of the effect of the policy. Here we examine global dynamics for our extension. Next we want to see whether changing the Taylor rule by incorporating the housing prices significantly modifies the dynamics.

Proposition 3 *Let the assumptions in proposition 1 hold, such that there are two steady state equilibria. Then, there is a heteroclinic orbit joining steady states* (R_1^*, q_1^*) *and* $(0, q_0^*)$*. The orbit has a positive slope near steady state* (R_1^*, q_1^*) *and has a zero slope near* $(0, q_0^*)$*.*

Proof. See Appendix E.

Figures 1 and 2 present the phase diagrams for the cases in which $q_0^* > q_1^*$ and $q_0^* < q_1^*$. In both cases, there is an heteroclinic trajectory joining the two steady states, and this trajectory is positively sloped at the neighborhood of (R_1^*, q_1^*), but while in the first case house prices increase, they decrease in the second. This implies that the nominal interest rate follows a non-monotonous trajectory in the first case.

As a consequence of Proposition 3, there is not only local indeterminacy at the steady state $(q_0^*, 0)$, but also global indeterminacy: any point along the heteroclinic is an equilibrium and if it is not located at any of the two steady states, there is a transition dynamics which converges asymptotically to the the liquidity trap. Next we set out the rationale for our results.

The isocline $\dot{q} = 0$ has just one branch, $q = \Psi(R)$, while the isocline $\dot{R} = 0$ has two branches, $R = 0$ and $q = \Phi(R)$. The slopes of the last two equations are, respectively

$$\left.\frac{dq}{dR}\right|_{\dot{q}=0} = \Psi'(R) = \Psi(R)\left(\frac{L'(R)}{L(R)} - \frac{1-P'(R)}{R-P(R)}\right) \tag{37}$$

and

$$\left.\frac{dq}{dR}\right|_{\dot{R}=0} = \Phi'(R) = \Phi(R)\left(\frac{1-P'(R)}{R-P(R)-\rho} + \frac{L'(R)}{L(R)}\right). \tag{38}$$

With the assumptions made at Proposition 1, the isocline $\dot{q} = 0$ has a positive value at $R = 0$ and an asymptote at this point, because $q = \Psi(0) > 0$ if $P(0) < 0$ and $\Psi'(0) = \Psi(0)(L'(0) + (1 - P'(0))/P(0)) = +\infty$. On the other side, $\Psi'(\infty) = 0$. For positive nominal interest rates, the slope depends upon the relationship between the wealth effect on consumption and the monetary rule. The schedule has a global negative slope if $\Psi^{'}(R) < 0$ globally, that is, if $L'(R)/L(R) < (1 - P'(R)/(R - P(R))$. This is the case depicted in Figure 1. However, if this condition does not hold, that is locally $L'(R)/L(R) > (1 - P'(R)/(R - P(R))$, the isoclines are locally increasing. This is the case depicted in Figure 2.

An example for this is the following: if the monetary rule is linear and $\Lambda(c, m)$ is a CES function,

$$\Lambda(\bar{c}, \bar{m}) = \left[\eta \bar{c}^{\frac{\zeta}{\zeta-1}} + (1-\eta)\,\bar{m}^{\frac{\zeta}{\zeta-1}}\right]^{\frac{\zeta-1}{\zeta}}, \tag{39}$$

with $0 < \eta, \zeta < 1$, the first case occurs if

$$-\left(\frac{1+\gamma}{\gamma}\right)P(0) = \frac{\tilde{r} - \gamma\tilde{\pi}}{\gamma} < \left[\frac{1}{\xi}\left(\frac{\zeta}{1-\zeta-\gamma}\right)^{\zeta}\right]^{1/(1-\zeta)},$$

where $\xi \equiv [\eta/(1-\eta)]^{1-\zeta}$. Then, if $\gamma > 1 - \zeta$ the isocline $\dot{q} = 0$ is locally increasing. If $\gamma < 1 - \zeta$ the isocline $\dot{q} = 0$ may (not) be locally increasing, for an active Taylor rule, if the target for the real interest rate is high (low), the target for the inflation rate is low (high), and/or the degree of reactiveness of the nominal interest rate to inflation is low (high).

If the first case occurs, then we always have $q_1^* < q_0^*$. In the second case, we may have $q_1^* > q_0^*$, depending upon the deep parameters for the consumer demand and the monetary rule. A necessary condition for $q_1^* > q_0^*$ is that locally $\Psi'(R) > 0$.

The equilibrium point $R^* = 0$ always exists and is, geometrically, in the the intersect of isocline $\dot{q} = 0$ with the first branch of isocline $\dot{R} = 0$. The second equilibrium point, which exists under the conditions of Proposition 1, is in the intersection of the isocline $\dot{q} = 0$ with the second branch of the isocline $\dot{R} = 0$, whose slope is given by equation (38).

For the range in which $q > 0$, we have $\Phi(R) > 0$, which means that the branch of the isocline, in which $q = \Phi(R)$, is globally increasing. Because

$$\Psi(0) - \Phi(0) = \frac{(P(0) + r_+)(P(0) + r_-)}{\alpha\beta(\rho + \mu)P(0)}$$

and $P(0) < 0$, a necessary condition for the existence of steady state (R_1^*, q_1^*), which was introduced in Proposition 1, is $P(0) + r_+ > 0$, implying that $\Psi(0) - \Phi(0) > 0$, which means that the isocline $\dot{q} = 0$ cuts the $q-$axis above the isocline $\dot{R} = 0$. In Proposition 2, we proved that this steady state is locally a source.

Furthermore, we may conjecture that the unstable manifold associated to the equilibrium point (R_1^*, q_1^*) and the stable manifold associated to the equilibrium point $(0, q_0^*)$ should intersect. The next proposition proves that this conjecture is right, which means, that a heteroclinic orbit defined as

$$\Omega \equiv \left\{ (R, q) : \lim_{t \to \infty} (R(t), q(t)) = (0, q_0^*), \lim_{t \to -\infty} (R(t), q(t)) = (R_1^*, q_1^*) \right\}$$

exists. In our case, Ω corresponds to the set of all the equilibrium values for the nominal interest rate and the house prices. All the other points lead to a violation of the transversality and/or the no-Ponzi game condition.

This means that there is global indeterminacy: any initial point $(R(0), q(0) \in \Omega$ is an equilibrium point and converges asymptotically to $(0, q_0^*)$. This is an extension of the "peril of the Taylor rule" argument made by Benhabib, Schmitt-Grohé and Uribe (2001). In our case, condition (35) implies that if the monetary policy is more (less) active, then the reduction of the nominal interest rate is correlated with an increase (decrease) in the prices of houses. Along a liquidity-trap path, in fact, the central bank adopting the Taylor rule tries to stimulate aggregate demand and thus inflation by decreasing the nominal interest rate more than proportionally with the decline in inflation. This policy hence triggers a fall in the real interest rate. As a result, if the interest rate cuts are sufficiently aggressive, house prices are likely to increase along the deflationary trajectory,

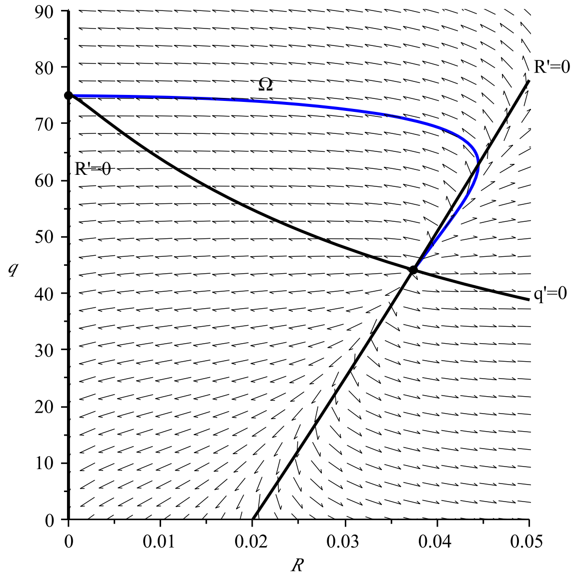

Figure 1: Phase diagram for a conventional Taylor rule: $r^* + P(0)L(R_1^*) > 0$. The figure is built by assuming $L(R) = 1 + \xi R^{1-\zeta}$ and a linear rule $R = \tilde{r} + \pi + \gamma(\pi - \tilde{\pi})$ and the values of the parameters $\alpha = 0.5$, $\beta = 0.01$, $\xi = 1$, $\zeta = 0.4$, $\mu = 0.01$, $\rho = 0.02$, $\gamma = 0.5$, $\tilde{r} = 0.03$ and $\tilde{\pi} = 0$.

as it emerges from the bifurcation diagram in Figure 3.

5 Alternative Taylor Rules

We now examine local and global equilibrium dynamics for the case of Taylor rules which incorporate house prices. We start with rule (21) and next we study rule (22).

5.1 Taylor Rule Depending on the Level Housing Prices

For the case of rule (21), we have $\pi = P(R, q)$, where

$$P(R, q) = \frac{1}{1+\gamma} R - \frac{\epsilon}{1+\gamma}\tilde{q} + P(0, 0), \tag{40}$$

with $P(0, 0) \equiv (\gamma\tilde{\pi} - \tilde{r} - \epsilon\tilde{q})/(1 + \gamma)$.

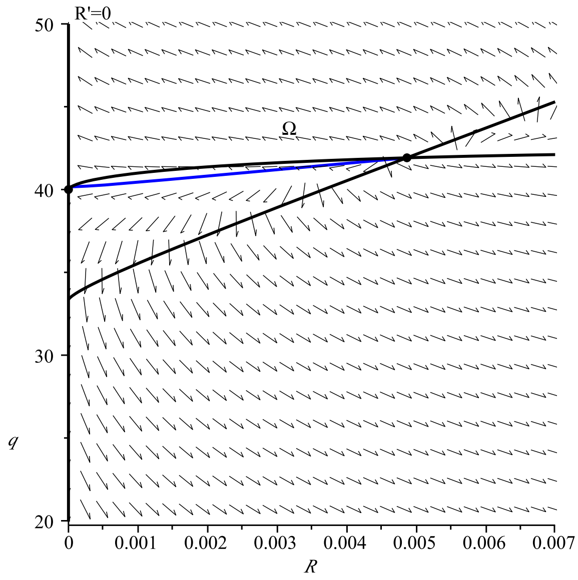

Figure 2: Phase diagram for a conventional Taylor rule: $r^* + P(0)L(R_1^*) < 0$. We use the same functional forms and parameters as in Figure 1 with $\gamma = 0.2$ instead.

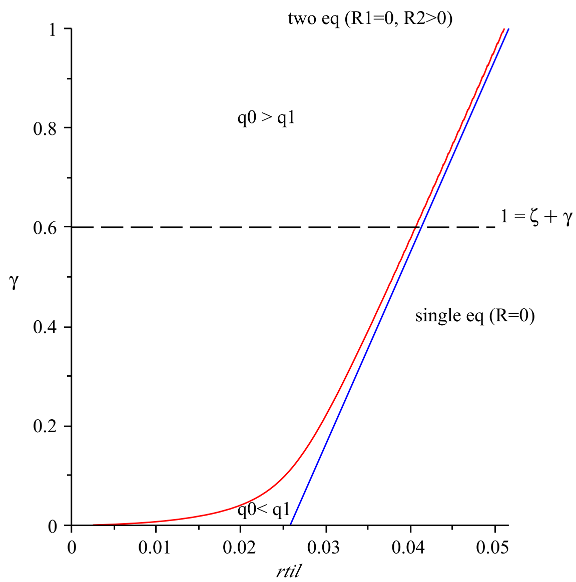

Figure 3: Bifurcation diagram for the conventional Taylor rule in the space $(\tilde{r}, \gamma)$. The values for the other parameters are as in Figure 1.

Hence, the dynamic system assumes the form

$$\dot{q} = (R(t) - P(R(t), q(t)))\,(q(t) - \Psi(R(t), q(t)))\,, \tag{41}$$

$$\dot{R} = \frac{\alpha\beta(\rho+\mu)}{L'(R(t))}\;(\Phi(R(t), q(t)) - q(t)\;)\,, \tag{42}$$

where $\Psi(R,q)$ and $\Phi(R,q)$ are as in equations (30) and (31), in which $P(R)$ is substituted by $P(R,q)$ as in equation (40).

This case does not present substantial changes as regards the conventional Taylor rule, as we shall see in the following Proposition.

Proposition 4 *Assume a modified Taylor rule depending on the level of housing price as in equation* (21) *in which the monetary policy is active, meaning that* $0 < \partial P/\partial R < 1$, *and define*

$$BP(0,0) \equiv \frac{P(0,0)}{2} + \left[\;\left(\frac{P(0,0)}{2}\right)^2 + \frac{\epsilon(1-\alpha)}{\alpha(1+\gamma)}\right]^{1/2} > 0.$$

Hence:

(a) if $r_+ + P(0,0) \leq B(P(0,0)$, *then there is an unique equilibrium steady state* $(R^*, q^*) = (0, q_0^*)$ *where*

$$q_0^* = \frac{1+\gamma}{\epsilon} BP(0,0); \tag{43}$$

(b) if $r_+ + P(0,0) > B(P(0,0)$ *then there are two equilibrium steady states* $(R^*, q^*) = (0, q_0^*)$ *and* $(R^*, q^*) = (R_1^*, q_1^*)$ *where and*

$$q_1^* = \frac{(1+\gamma)(P(0,0)+r_+) - \gamma R_1^*}{\epsilon} \tag{44}$$

and

$$R_1^* = \left\{ R:\; L(R) = \left(\frac{\alpha(1+\gamma)r_+}{(1-\alpha)\epsilon}\;\right)\left(P(0,0) + r_+ - \frac{\gamma}{1+\gamma}\;R\right)\;\right\},$$

and there is a heteroclinic trajectory connecting those two equilibrium steady states, *starting from equilibrium* (R_1^*, q_1^*)*, which is locally a source, and converging asymptotically to equilibrium* $(0, q_0^*)$*.*

Proof. See Appendix F.

The equilibrium $(0, q_0^*)$ is again singular and it behaves globally as a saddle point: for $R^* = 0$ we have $\dot{q} \lesseqqgtr 0$ if and only if $q \lesseqqgtr q_0^*$, and the vector field is horizontal on the space (R, q). The equilibrium steady state (R_1^*, q_1^*) also displays local determinacy, because the local Jacobian has positive eigenvalues for the admissible values of the parameters.

The global dynamics is as in the model with a conventional Taylor rule (see Figure 4): there is a heteroclinic orbit connecting equilibria (R_1^*, q_1^*) to $(0, q_0^*)$; these are the only combinations of equilibrium nominal interest rates and housing prices, which means that there is global indeterminacy.

Therefore, this Taylor rule would not change qualitatively the dynamics, only quantitatively. However, because

$$P(0,0) = \frac{\gamma\tilde{\pi} + \epsilon\tilde{q} - \tilde{r}}{1+\gamma}$$

can have any sign, although it has all parameters positive, the definition of the target values is less stringent than in the case of the conventional Taylor rule. Furthermore, we can exclude the case in which there is an unique steady state equilibrium (R_1^*, q_1^*). That is the equilibrium with a zero nominal interest rate always exists.

5.2 Taylor Rule Depending on the Rate of Growth of Housing Prices

For the case of rule (22), we have

$$\pi = \frac{R + \gamma\tilde{\pi} - \tilde{r} - \delta\dot{q}/q}{1+\gamma}$$

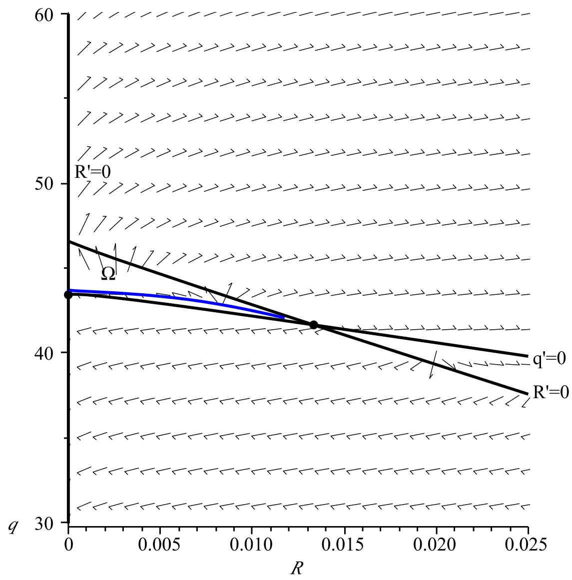

Figure 4: Phase diagram for Taylor rule (21). The figure is built by assuming the same functional forms and the same parameters as Figure 1 and $\epsilon = 0.002$ and $\tilde{q} = 50$.

and the dynamic system becomes

$$\dot{q} = \frac{\gamma R(t) + \tilde{r} - \gamma\tilde{\pi}}{1+\gamma-\delta}\left(q(t) - \Psi(R(t))\right) \tag{45}$$

$$\dot{R} = \frac{\alpha\beta(\rho+\mu)}{L'(R(t))}\left(\Phi(R(t)) - q(t)\ \right) + \frac{\delta}{1+\gamma}\frac{L(R(t))}{L'(R(t))}\frac{\dot{q}(t)}{q(t)}, \tag{46}$$

where $\Psi(R)$ and $\Phi(R)$ are as in equations (30) and (31), which in this case become

$$\Psi \equiv \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{(1+\gamma)}{\gamma R + \tilde{r} - \gamma\tilde{\pi}}\right)L(R), \tag{47}$$

$$\Phi \equiv \left(\frac{\gamma R + \tilde{r} - \gamma\tilde{\pi} - (1+\gamma)\rho}{\alpha\beta(\rho+\mu)(1+\gamma)}\right)L(R). \tag{48}$$

Therefore, we can state the following proposition.

Proposition 5 *Let the same assumptions as in Proposition 1 hold. Then the existence, multiplicity and magnitudes of the steady-state equilibria are as in Proposition 1. In addition:*

(a) if $1+\gamma > \delta$, *then the steady state* $(0, q_0^*)$ *(if it exists) is singular and is a generalized*

saddle point and the steady state (R_1^*, q_1^*) *(if it exists) is a source, and there is a heteroclinic orbit joining them.*

(b) if $1+\gamma < \delta$*, then the steady state* $(0, q_0^*)$ *(if it exists) is singular and is a generalized sink and the steady state* (R_1^*, q_1^*) *(if it exists) is a saddle point. If there are two steady state equilibria, the saddle manifold associated to* (R_1^*, q_1^*) *is the boundary of the basin of attraction of* $(0, q_0^*)$*.*

As in the case of the conventional Taylor rule (see Proposition 1), the steady-state equilibria of housing prices are given in equations (33) and (33). From now on, consider the cases in which there are two steady state equilibria, with phase diagrams given in Figure 5, for the case in which $1 + \gamma > \delta$, and in Figure 6, for the case in which $1 + \gamma < \delta$.

In the case $\delta < 1+\gamma$, whereby the central bank reacts more to consumer-price inflation than to house-price inflation, the global dynamics is similar as in the case of the conventional Taylor rule: there will be global indeterminacy in the sense that there is an interval for initial values of R, $R(0) \in (0, R_1^*)$ which are equilibrium values and the nominal interest rate tends (possibly in a non-monotonous way) to the steady state $R^* = 0$. For a given initial value of the nominal interest rate, there will by a single value for equilibrium house prices. The pair $(R(t), q(t))$ will follow along the heteroclinic orbit Ω: see Figure 5.

In the case $\delta > 1 + \gamma$, whereby the central bank reacts more to house-price inflation than to consumer-price inflation, the global dynamics change substantially: the two steady-state equilibria are both locally and globally indeterminate; there will be global indeterminacy of a different type. First, the initial value for the nominal interest rate can be unbounded, $R(0) \in (0, +\infty)$. Second, for any initial value of the nominal interest rate $R(0)$, there will be an infinite number of equilibrium values for housing prices, with different equilibrium trajectories, one converging to the steady-state equilibrium $(0, q_0^*)$, and the other converging to the steady-state equilibrium (R_1^*, q_1^*), if the path is located at $\mathcal{W}_1^s$, which is the stable manifold associated to the latter equilibrium steady state: see Figure 6.

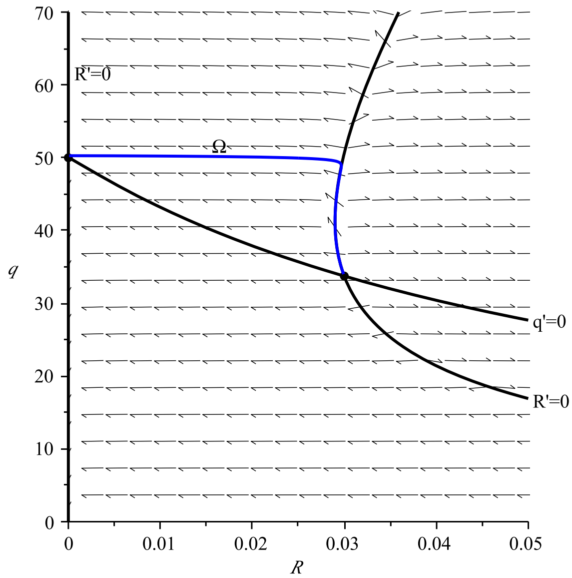

Figure 5: Phase diagram for Taylor rule (22). The figure is built by assuming the same functional forms and the same parameters as in Figure 1 and $\delta = 0.5 < 1 + \gamma$.

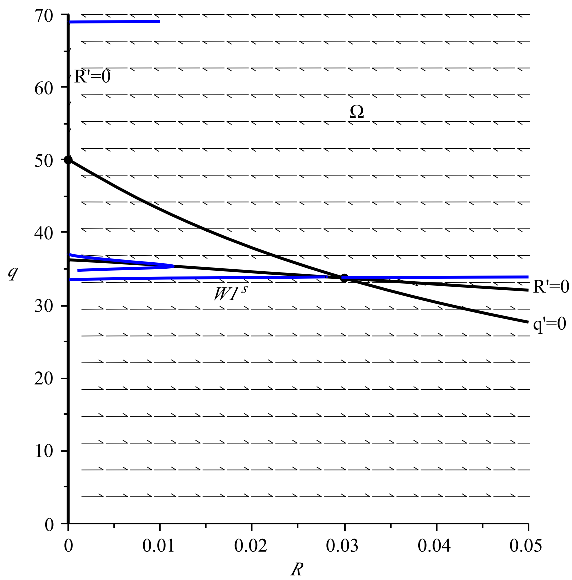

Figure 6: Phase diagram for Taylor rule (22). The figure is built by assuming the same functional forms and the same parameters as in Figure 1 and $\delta = 5 > 1 + \gamma$.

6 Conclusions

The Federal Reserve's accommodating monetary policy of 2002-2006 - the so-called Great Deviation from the Taylor rule - is argued to be responsible for the global crisis started in 2007, because it generated boom-bust patterns in house prices. In this paper we have presented an overlapping generations economy with housing-wealth effects and demonstrated that house-price instabilities can occur even if the central bank follows the Taylor rule. In particular, the Taylor rule can well open the door to an off-target trajectory in which house prices and inflation converge to a liquidity trap. Ruling out global indeterminacy would thus require alternative paradigms for monetary policy making.

References

Adao, B., Correia, I. and P. Teles (2011), "Unique Monetary Equilibria with Interest Rate Rules", *Review of Economic Dynamics* 14, 432-442.

Ahrend R, B. Cournède and R. Price (2008), "Monetary Policy, Market Excesses and Financial Turmoil", *OECD Economics Department Working Papers* 597.

Benassy, J. P. (2008), "The Fiscal Theory of the Price Level Puzzle: A Non-Ricardian View", *Macroeconomic Dynamics* 12, 31-44.

Benassy, J. P. (2009), "Interest Rate Rules and Global Determinacy: An Alternative to the Taylor Principle", *International Journal of Economic Theory* 5, 359-374.

Benhabib J., S. Schmitt-Grohé and M. Uribe (2001), "The Perils of Taylor Rules", *Journal of Economic Theory* 96, 40-69.

Benhabib J., S. Schmitt-Grohé and M. Uribe (2002), "Avoiding Liquidity Traps", *Journal of Political Economy* 110, 535-563.

Benhabib J., S. Schmitt-Grohé and M. Uribe (2003), "Backward-Looking Interest-Rate Rules, Interest-Rate Smoothing, And Macroeconomic Instability", *Journal of Money, Credit and Banking* 35, 1379-1412.

Blanchard, O. J. (1985), "Debt, Deficits, and Finite Horizons", *Journal of Political Economy* 93, 223-247.

Campbell, J. Y. and J. F. Cocco (2007), "How do House Prices Affect Consumption? Evidence from Micro Data", *Journal of Monetary Economics* 54, 591-621.

Carroll, C. D., M. Otsuka and J. Slacalek (2011), "How Large Are Housing and Financial Wealth Effects? A New Approach", *Journal of Money, Credit and Banking* 43, 55-79.

Cochrane, J. (2011), "Determinacy and Identification with Taylor Rules", *Journal of Political Economy* 119, 565-615.

Deaton, A. and J. Muellbauer (1980), *Economics and Consumer Behavior*, Oxford: Oxford University Press.

Eschtruth, A. and L. Tran (2001), "A Primer on Reverse Mortgages", *Center for*

Retirement Research, Boston College.

Galí, J. (2008), *Monetary Policy, Inflation and the Business Cycle*, Princeton: Princeton University Press.

Iacoviello, M. (2005), "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle", *American Economic Review* 95, 739-764.

Leamer, E. E. (2007), "Housing is the Business Cycle", *Proceedings*, Federal Reserve Bank of Kansas City, 149-233.

Leeper E. and C. Sims (1994), "Toward a Modern Macro Model Usable for Policy Analysis", *NBER Macroeconomics Annual 1994*, 81-117.

Marini, G. and F. Van der Ploeg (1988), "Monetary and Fiscal Policy in an Optimizing Model with Capital Accumulation and Finite Lives", *Economic Journal* 98, 772-786.

Muellbauer, J. N. (2007), "Housing, Credit and Consumer Expenditure", *Proceedings*, Federal Reserve Bank of Kansas City, 267- 334.

Poole, W. (2007), "Understanding the Fed", *Federal Reserve Bank of St. Louis Review*, January/February 89, 3-13.

Schmitt-Grohé, S. and M. Uribe (2009), "Liquidity Traps with Global Taylor Rules", *International Journal of Economic Theory* 5, 85-106.

Taylor, J. B. (1993), "Discretion Versus Policy Rules in Practice", *Carnegie-Rochester Conference Series on Public Policy* 39, 195-214.

Taylor, J. B. (1999), "A Historical Analysis of Monetary Policy Rules", in J. B. Taylor (Ed.), *Monetary Policy Rules*, Chicago and London: University of Chicago Press, 319-347.

Taylor, J. B. (2007), "Housing and Monetary Policy," in *Housing, Housing Finance, and Monetary Policy*, Federal Reserve Bank of Kansas City.

Taylor, J. B. (2010), "Does the Crisis Experience Call for a New Paradigm in Monetary Policy?" Warsaw School of Economics, (June 23), *CASE Network Studies and Analyses* No. 402.

Taylor, J. B. (2011),"Macroeconomic Lessons from the Great Deviation", in D. Acemoglu and M. Woodford, *NBER Macroeconomics Annual 2010*, 25.

Weil, P. (1989), "Overlapping Families of Infinitely-Lived Agents", *Journal of Public*

Economics 38, 183-198.

Woodford, M. (2003), *Interest and Prices*, Princeton and Oxford: Princeton University Press.

Yaari, M. E. (1965), “Uncertain Lifetime, Life Insurance, and the Theory of the Consumer”, *The Review of Economic Studies* 32, 137-150.

A Solution of the Representative Consumer Problem

In the intertemporal optimization problem, the representative consumer born at time s chooses the optimal time path of total consumption, $\overline{z}(s,t)$, to maximize the lifetime utility function (2), given (6) and the constraints (3) and (4). Using the definition of total consumption, $\overline{z}(s,t) \equiv \overline{c}(s,t) + R(t)\overline{m}(s,t)$, and the optimal intratemporal condition (6), we can write

$$\log \Lambda\left(\overline{c}(s,t), \overline{m}(s,t)\right) = \log v(t) + \log \overline{z}(s,t), \tag{49}$$

where $v(t) \equiv \Lambda\left(\frac{\Gamma(R(t))}{\Gamma(R(t))+R(t)}, \frac{1}{\Gamma(R(t))+R(t)}\right)$ is the same for all generations. Therefore, the intertemporal optimization problem can be formalized in the following terms:

$$\max_{\{\overline{z}(s,t)\}} \int_0^\infty \left[\alpha\left(\log v(t) + \log \overline{z}(s,t)\right) + (1-\alpha)\log \overline{h}(s,t)\right] e^{-(\mu+\rho)t} dt, \tag{50}$$

subject to

$$\begin{aligned} \dot{\overline{a}}(s,t) &= \left(R(t) - \pi(t) + \mu\right)\overline{a}(s,t) + \overline{y}(s,t) - \overline{\tau}(s,t) - \overline{z}(s,t) + \\ &\quad + \left[\frac{\dot{q}(t)}{q(t)} - \left(R(t) - \pi(t)\right)\right] q(t)\overline{h}(s,t), \end{aligned} \tag{51}$$

and given $\overline{a}(s,0)$. The optimality conditions are

$$\dot{\overline{z}}(s,t) = (R(t) - \pi(t) - \rho)\overline{z}(s,t). \tag{52}$$

$$\frac{1-\alpha}{\alpha}\overline{z}(s,t) = \left[\left(R(t) - \pi(t)\right) - \frac{\dot{q}(t)}{q(t)}\right] q(t)\overline{h}(s,t), \tag{53}$$

Therefore, the individual budget constraint (51) can be expressed as

$$\begin{aligned} \dot{\overline{a}}(s,t) &= \left(R(t) - \pi(t) + \mu\right)\overline{a}(s,t) + \overline{y}(s,t) - \overline{\tau}(s,t) - \\ &\quad -\overline{z}(s,t) + \left[\frac{\dot{q}(t)}{q(t)} - \left(R(t) - \pi(t)\right)\right] q(t)\overline{h}(s,t) \\ &= \left(R(t) - \pi(t) + \mu\right)\overline{a}(s,t) + \overline{y}(s,t) - \overline{z}(s,t) - \frac{1-\alpha}{\alpha}\overline{z}(s,t) \\ &= \left(R(t) - \pi(t) + \mu\right)\overline{a}(s,t) + \overline{y}(s,t) - \frac{1}{\alpha}\overline{z}(s,t). \end{aligned} \tag{54}$$

Integrating forward (54), using the transversality condition (9) and (52), total consumption turns out to be a linear function of total wealth:

$$\overline{z}(s,t) = \alpha(\mu+\rho)\left(\overline{a}(s,t) + \overline{k}(s,t)\right), \tag{55}$$

where $\overline{k}(s,t)$ is human wealth, defined as the present discounted value of after-tax labor income, $\overline{k}(s,t) \equiv \int_t^\infty \left(\overline{y}(s,t) - \overline{\tau}(s,t)\right) e^{-\int_t^v (R(j)-\pi(j)+\mu)dj} dv$. From (6),

$$\overline{z}(s,t) = L(R(t))\overline{c}(s,t), \tag{56}$$

where $L(R(t)) \equiv 1 + R(t)/\Gamma\left(R(t)\right)$. Time-differentiating (56) yields

$$\dot{\overline{z}}(s,t) = L'(R(t))\overline{c}(s,t)\dot{R}(t) + L(R(t))\dot{\overline{c}}(s,t). \tag{57}$$

Therefore, the dynamic equation for individual consumption is

$$\dot{\overline{c}}(s,t) = \left(R(t) - \pi(t) - \rho\right)\overline{c}(s,t) - \frac{L'(R(t))\dot{R}(t)}{L(R(t))}\overline{c}(s,t). \tag{58}$$

B Aggregation

The per capita aggregate financial wealth is given by

$$a(t) = \beta \int_{-\infty}^{t} \overline{a}(s,t)e^{\beta(s-t)}ds. \tag{59}$$

Differentiating with respect to time yields

$$\dot{a(t)} = \beta\overline{a}(t,t) - \beta a(t) + \beta \int_{-\infty}^{t} \dot{\overline{a}}(s,t)e^{\beta(s-t)}ds. \tag{60}$$

$\overline{a}(t,t)$ is equal to zero by assumption. Using (3) yields

$$\begin{aligned}
\dot{a(t)} &= -\beta a(t) + \mu a(t) + (R(t) - \pi(t))\, a(t) + y(t) - \tau(t) - c(t) - \\
&\quad -R(t)m(t) + \left[\frac{\dot{q}(t)}{q(t)} - (R(t) - \pi(t))\right] q(t)h(t) \\
&= (R(t) - \pi(t) - n)\, a(t) + y(t) - \tau(t) - c(t) - \\
&\quad -R(t)m(t) + \left[\frac{\dot{q}(t)}{q(t)} - (R(t) - \pi(t))\right] q(t)h(t). \qquad (61)
\end{aligned}$$

Using (11), the per capita aggregate consumption is given by

$$c(t) = \frac{\alpha(\mu+\rho)}{L(R(t))}\left(a(t) + k(t)\right), \qquad (62)$$

where $k(t) = \int_t^\infty (y(t) - \tau(t))\, e^{-\int_t^v (R(j)-\pi(j)+\mu)dj} dv$ is the per capita aggregate human wealth. Next differentiate with respect to time the definition of per capita aggregate consumption, to obtain

$$\dot{c(t)} = \beta\overline{c}(t,t) - \beta c(t) + \beta \int_{-\infty}^{t} \dot{\overline{c}}(s,t) e^{\beta(s-t)} ds. \qquad (63)$$

Note that $\overline{c}(t,t)$ denotes consumption of the newborn generation. Since $\overline{a}(t,t) = 0$, from (11) we have

$$\overline{c}(t,t) = \frac{\alpha(\mu+\rho)}{L(R(t))}\left(\overline{k}(t,t)\right). \qquad (64)$$

Using (12), (62) and (64) into (63) yields the time path of per capita aggregate consumption:

$$\dot{c}(t) = (R(t) - \pi(t) - \rho)\, c(t) - \frac{L'(R(t))\dot{R}(t)}{L(R(t))} c(t) - \frac{\alpha\beta(\rho+\mu)}{L(R(t))} a(t). \qquad (65)$$

C Proof of Proposition 1

Any equilibrium steady states, (R^*, q^*), are bounded points $(R,q) \in (\mathbb{R}_+, \mathbb{R}_{++})$ such that $\dot{q} = \dot{R} = 0$ and the transversality condition holds if $R^* - P(R^*) > -\mu$. From equation (28), $\dot{q} = 0$ if and only if $q = \Psi(R)$. From equation (29), $\dot{R} = 0$ if and only if $R = 0$, because

$L'(0) = \infty$, or $q = \Phi(R)$. Therefore, there are two candidates for equilibrium steady states. First, we have $R = 0$ and $q = q_0^* \equiv \Psi(0) = -(1-\alpha)/(\alpha P(0))$ which constitute indeed an equilibrium point only if $P(0) < 0$. In this case, the transversality condition is verified, because the condition $-P(0) > -\mu$ always holds. Second, from the condition $q = \Psi(R) = \Phi(R)$, there is a second candidate if the set $\{R \geq 0 : \ \Psi(R) = \Phi(R) > 0\}$ is non-empty. We have

$$\Psi(R) = \Phi(R) \Leftrightarrow (R - P(R) - \rho)(R - P(R)) - \alpha\beta(\rho + \mu) = (r - r_+)(r - r_-) = 0,$$

where $r = R - P(R) > -\mu$ from the transversality condition. Because $\alpha\beta(\rho + \mu) > 0$, then $r_+ > \rho > 0 > r_-$. As $L(R) > 1$, then the condition $r > \rho$ must hold, which implies that the transversality condition is always met. Therefore, an equivalent condition for the existence of a second steady state is $R_1^* = \{R \geq 0 : \ R - P(R) = r_+\}$. From the assumption that the monetary policy is globally active, $R - P(R)$ is monotonically increasing in R, which means that $R - P(R) \in [-P(0), +\infty)$. Therefore, the steady state (R_1^*, q_1^*) exists if $P(0) + r_+ \geq 0$, where $q_1^* = \Psi(R_1^*) = \Phi(R_1^*)$. As a result, there is multiplicity if $0 > P(0) \geq -r_+$, there is only one steady state (R_1^*, q_1^*) if $P(0) > 0$, and there is only steady state $(0, q_0^*)$ if $P(0) + r_+ < 0$.

D Proof of Proposition 2

For steady state equilibrium $(R^*, q^*) = (R_1^*, q_1^*)$, we have the trace and determinant of the jacobian given by

$$\mathrm{tr} J(R_1^*) = r^* + \alpha\beta(\rho + \mu)\frac{\Phi'(R_1^*)}{L'(R_1^*)} = 2r^* - \rho + \frac{(1 - P'(R_1^*)L(R_1^*)}{L'(R_1^*)}$$

and

$$\det J(R_1^*) = r^*\alpha\beta(\rho + \mu)\left(\frac{\Phi'(R_1^*) - \Psi'(R_1^*)}{L'(R_1^*)}\right) = \frac{(2r^* - \rho)(1 - P'(R_1^*))L(R_1^*)}{(r^* - \rho)L'(R_1^*)} > 0,$$

because

$$\Phi'(R_1^*) = q_1^* \left(\frac{1 - P'(R_1^*)}{r^* - \rho} + \frac{L'(R_1^*)}{L(R_1^*)} \right) > 0,$$

yielding the eigenvalues $\lambda_1 = 2r^* - \rho > 0$ and $\lambda_2 = (1 - P'(R_1^*))L(R_1^*)/L'(R_1^*) > 0$ if the monetary policy is locally active, $1 - P'(R_1^*) > 0$. Then the steady state (R_1^*, q_1^*) is a source.

Evaluating the trace and the determinant for equilibrium $(R^*, q^*) = (0, \Psi(0))$, we get

$$\text{tr}J(0) = -P(0) + \alpha\beta(\rho + \mu) \left(\frac{\Phi'(0)}{L'(0)} + (\Psi(0) - \Phi(0)) \frac{L''(0)}{(L'(0))^2} \right)$$

and

$$\det J(0) = -P(0) \frac{\alpha\beta(\rho + \mu)}{L'(0)} \left(\Phi'(0) - \Psi'(0) + (\Psi(0) - \Phi(0)) \frac{L''(0)}{L'(0)} \right).$$

As

$$\frac{\Psi'(0) - \Phi'(0)}{L'(0)} = \Psi(0) - \Phi(0) = \frac{(P(0) + r_+)(P(0) + r_-)}{\alpha\beta(\rho + \mu)P(0)} > 0,$$

if $P(0) + r_+ > 0$, and as $L'(0) = \infty$ and $L''(0)/(L'(0))^2 = -\infty$, then

$$\begin{aligned} \text{tr}J(0) &= -(2P(0) + \rho) + \frac{(P(0) + r_+)(P(0) + r_-)}{P(0)} \frac{L''(0)}{(L'(0)^2} = -\infty, \\ \det J(0) &= (P(0) + r_+)(P(0) + r_-) \left(1 - \frac{L''(0)}{(L'(0))^2} \right) = -\infty, \end{aligned}$$

which means that the eigenvalues of the Jacobian are infinite and the steady state is singular. In order to determine the local dynamics, we need o de-singularize it, using a similar method as in singular dynamical systems theory. In the case of our model, the natural way to remove the singularity introduced by $L'(R)$ at $R = 0$, would be to recast the system in variables (L, q),

$$\begin{aligned} \dot{q} &= q\,(R - P(R)) - \left(\frac{1 - \alpha}{\alpha} \right) L, \\ \dot{L} &= L'(R)\dot{R} = L\,(R - P(R) - \rho) - \alpha\beta(\rho + \mu)q, \end{aligned}$$

where $L \geq 1$, $R = R(L)$ is increasing and $R(1) = 0$, $R'(1) = 0$. However, in this case the single steady state is associated with (R_1^*, q_1^*) which does not solve our problem. In the proof of Proposition 3 we show that the singular steady state $(0, q_0^*)$ behaves as a generalized saddle point.

E Proof of Proposition 3

As the system (28)-(29) does not have an explicit solution, we must employ qualitative methods in order to study global dynamics. One possibility is to find a first integral, i.e., a Lyapunov function $V(R, q) =$ constant, which is a first integral of system (28)-(29). Maybe does not exist, either. Another method is to determine a trapping area for the heteroclinic orbit. The rationale is the following: as the steady state (R_1^*, q_1^*) is a source, the unstable manifold is all the set $\mathbb{R}_+$; as the steady state $(0, q_0^*)$ is a saddle point, the stable manifold is, locally, composed by a single trajectory belonging to $\mathbb{R}_+$; therefore there is an intersection of the unstable manifold of the first point and of the stable manifold of the second. In order to prove that it exists, and to characterize it, we consider a trapping area for the heteroclinic orbit. In order to prove this, we start by determining the slopes of the heteroclinic orbit in the neighborhoods of the two equilibria, we build a trapping area enclosing the heteroclinic, and prove that all the trajectories escape from it, which the exception of the heteroclinic orbit.

E.1 Slopes of the Eigenspaces Associated to the Two Equilibria

The unstable eigenspace $\mathcal{E}_1^u$ is the tangent space to the unstable manifold associated to equilibrium (R_1^*, q_1^*), in the space (R, q),

$$\mathcal{W}_1^u \equiv \left\{ (R, q) : \lim_{t \to -\infty} (R(t), q(t)) = (R_1^*, q_1^*) \right\}.$$

The unstable eigenspace $\mathcal{E}_1^u$ is spanned by the eigenvectors $(V_1, 1)^\top$ and $(V_2, 1)^\top$ associated to eigenvalues λ_1 and λ_2, where

$$V_1 = -\frac{r_+ \Psi'(R_1^*)}{r_+ - \rho}, \; V_2 = \frac{(r_+ - \rho) L'(R_1^*)}{L(R_1^*)}.$$

In general, $V_2 > 0$ and the sign of V_1 is ambiguous. Observe that the slope of $\frac{dq}{dR}\big|_{\mathcal{E}_1^u}$ has the opposite sign of the isocline $\dot{q} = 0$ locally. Let us call $\mathcal{E}_{1,+}^u$ ($\mathcal{E}_{1,-}^u$) the eigenspace related to the dominant (non-dominant) eigenvalue. The following conditions can be proved: (1) if $r_+ > (1 - P'(R_1^*))L(R_1^*)/L'(R_1^*)$, then $2r_+ - \rho > (1 - P'(R_1^*))L(R_1^*)/L'(R_1^*)$, which is equivalent to $\lambda_1 > \lambda_2$, and therefore $\mathcal{E}_{1,+}^u = \{ \; (R, q) : \;\; (q - q_1^*) = V_1(R - R_1^*)\}$ and $\mathcal{E}_{1,-}^u = \{ \; (R, q) : \;\; (q - q_1^*) = V_2(R - R_1^*)\}$. In this case, $V_1 < 0$ and $V_2 > 0$ and the slope associated to the dominant eigenvalue is negative and the slope associated to the non-dominant eigenvalue is positive; (2) if $\lambda_1 < \lambda_2$, which is equivalent to $2r_+ - \rho < (1 - P'(R_1^*))L(R_1^*)/L'(R_1^*)$, then $r_+ < (1 - P'(R_1^*))L(R_1^*)/L'(R_1^*)$, and $\mathcal{E}_{1,+}^u = \{ \; (R, q) : (q - q_1^*) = V_2(R - R_1^*)\}$ and $\mathcal{E}_{1,-}^u = \{ \; (R, q) : \;\; (q - q_1^*) = V_1(R - R_1^*)\}$. In this case, $V_1 > V_2 > 0$ and the slope associated to the both eigenvalues are both positive, but the one associated with $\mathcal{E}_{1,-}^u$ is steeper.

The stable manifold associated to steady state $(0, q_0^*)$, can be formally defined in space (q, R) as

$$\mathcal{W}_0^s \equiv \left\{ \; (R, q) : \; \lim_{t \to \infty} (R(t), q(t)) = (0, q_0^*) \right\}.$$

However, we saw that the projection of the steady state $(0, q_0^*)$ in space (R, q) is singular. This means that the solution approaches the singular steady state asymptotically with an infinite speed. In order to characterize the dynamics in the space (q, R) in the neighborhood of $(0, q_0^*)$, we have to take a different approach: observe that, as $R'(1) = 0$, then a naïve calculation for the slope of the stable manifold in the neighborhood of the singular equilibria could be $dq/dR = (r_+ - \rho)/(\alpha\beta(\rho + \mu)R'(1)) = \infty$.

Instead, observe that along the singular surface $R = 0$ we have $\dot{R} = 0$ and $\dot{q} = P(0)(\Psi(0) - q)$. Then, from any point along this surface where $q \neq q_0^* = \Psi(0)$ an unstable trajectory unfolds. This means that any trajectory coming from $R > 0$ will be deflected away from the equilibrium point on hitting the surface $R = 0$, with the exception of the one which converges to the equilibrium point $(0, q_0^*)$. The (global) direction of the vector field generated by equations (28)-(29) given by

$$\left.\frac{dq}{dR}\right|_{(\dot{q},\dot{R})} = \frac{L'(R)(R - P(R))(q - \Psi(R))}{\alpha\beta(\rho+\mu)(\Phi(R) - q)}. \tag{66}$$

In order to determine the slope of the trajectory which converges to the singular steady state, from equation (66) we determine the slope of the vector field hitting the surface $R = 0$,

$$\left.\frac{dq}{dR}\right|_{R=0} = \frac{-L'(0)P(0)(q - \Psi(0))}{\alpha\beta(\rho+\mu)(\Phi(0) - q)} = \begin{cases} \infty, & \text{if } q \neq q_0^* \\ 0, & \text{if } q = q_0^* \end{cases}$$

because $L'(0) = \infty$. Therefore, the stable manifold associated to the singular steady state $(0, q_0^*)$ is horizontal in the space (R, q).

The heteroclinic orbit, $\Omega = \mathcal{W}_0^s \cap \mathcal{W}_1^u$, is tangent to an horizontal line in the neighborhood of the equilibrium point $(0, q_0^*)$ and is positively slopped in the neighborhood of (R_1^*, q_1^*), because it is tangent to $\mathcal{E}_{1,-}^u$.

E.2 Trapping Area

Next we consider the case in which $\lambda_1 < \lambda_2$, which is depicted in Figure 1. Recall that Ω is tangent to a line $dq/dR = 0$ in the neighborhood of point $(0, q_0^*)$. Observe that in the neighborhood of point (R_1^*, q_1^*) the slope of the eigenspace associated to the non-dominant eigenvalue (λ_1) and of the isocline $\dot{R} = 0$ are both positive but the former is less steep that the later because (see Figure 7)

$$\left.\frac{dq}{dR}\right|_{\dot{R}} = \left.\frac{dq}{dR}\right|_{\mathcal{E}_{1,-}^u} = \Phi'(R_1^*) + \frac{r_+}{r_+ - \rho}\,\Psi'(R_1^*) = -\frac{2r_+ - \rho}{r_+ - \rho} q_1^* \left(\frac{L'(R_1^*)}{L(R_1^*)}\right) < 0.$$

As the heteroclinic Ω is tangent to that eigenspace in the heighborhood of that equilibrium point, it will start between the isocline $\dot{R} = 0$ and $\mathcal{E}_{1,-}^u$ and will never cross this line.

This allows to consider the trapping area whose sides are given by the segment of the isocline $\dot{q} = 0$ between the two equilibria (recall that the two equilibria lay along this isocline), by a line passing through to the steady state (R_1^*, q_1^*) whose slope is given by the eigenvector which is associated to the non-dominant eigenvalue and by the horizontal segment such that $q = q_0^*$, between the q axis and the previous eigenvector-line.

Formally, the trapping area $[A, B, C]$ is defined by the vertices $A \equiv (0, q_0^*)$, $B \equiv (R_1^*, q_1^*)$, and $C \equiv (R_C, q_0^*) = (R_1^* - (q_1^* - q_0^*)/V_1, q_0^*)$ and the sides $(A, B) = \{ (R, q) \in (0, R_1^*) \times (q_1^*, q_0^*) : \ \dot{q} = 0\}$, $(A, C) = \{ (R, q) : R \in (0, R_1^*), \ q = q_0^*\}$, and $(B, C) = \{ (R, q) \in (R_1^*, R_C) \times (q_1^*, q_0^*) : q = q_1^* + V_1(R - R_1^*)\}$.

Next we have to prove that the (global) direction of the vector field, given by equation (66), which is generated by equations (28)-(29), allows us to demonstrate that all the trajectories hitting the three boundaries of the trapping exit the trapping area.

At side (A, B) we have $\dot{q} = 0$ and $\dot{R} < 0$, because

$$\dot{R}\Big|_{(A,B)} = \frac{\alpha\beta(\rho + \mu)\,(\Phi(R) - \Psi(R))}{L'(R)} = \frac{L(R)(R - P(R) - r_+)(R - P(R) - r_-)}{L'(R)(R - P(R))} < 0,$$

because $-r_- < R - P(R) < r_+$ if $R \in [0, R_1^*)$. Then, as the slope of the vector field in the interval is

$$\frac{dq}{dR}\Bigg|_{(A,B)} = 0$$

then the vector field points globally out of the trapping area within (A, B) with a horizontal slope.

At side (A, C) the vector field corresponds to a horizontal line between point A and point C, which is in the intersection of a horizontal line passing through the q-axis and the direction defined by the eigenvector associated to the dominant eigenvalue from point B. These two lines meet at point C. Along this line we have $\dot{R} < 0$ for $R \in (0, \Phi^{-1}(q_0^*))$, $\dot{R} = 0$ at point $R = \Phi^{-1}(q_0^*)$ and $\dot{R} > 0$ for $R \in \Phi^{-1}(q_0^*), R_C$. In all the side (A, C) we have $\dot{q} > 0$. As the slope of the vector field is given by

$$\left.\frac{dq}{dR}\right|_{(A,C)} = \frac{L'(R)(R - P(R))(q_0^* - \Psi(R))}{\alpha\beta(\rho+\mu)(\Phi(R) - q_0^*)} \begin{cases} < 0, & \text{if } R \in (0, \Phi^{-1}(q_0^*) \\ \infty & \text{if } R = \Phi^{-1}(q_0^*) \\ > 0, & \text{if } R \in (\Phi^{-1}(q_0^*), R_C) \end{cases}$$

then the vector field points globally out of the trapping area within (A, C).

At side (B, C), which is a segment of the eigenspace $\mathcal{E}^u_{1,-}$ between points (R_1^*, q_1^*) and (R_C, q_0^*), the vector field has local time-variations given by $\dot{R} > 0$ and $\dot{q} > 0$. As this side has a slope given by V_1, then the trajectories exit the trapping are if the slope of the vector field is less steep than V_1, that is if and only if

$$\left.\frac{dq}{dR}\right|_{(B,C)} - \left.\frac{dq}{dR}\right|_{\mathcal{E}^u_{1,-}} = \frac{L'(R)(R - P(R))(q - \Psi(R))}{\alpha\beta(\rho+\mu)(\Phi(R) - q)} + \frac{r_+\Psi'(R_1^*)}{r_+ - \rho} < 0.$$

The denominator is equivalent to

$$\begin{aligned}
&L'(R)(R - P(R))(q - \Psi(R)) + (\Phi(R) - q)\left(r_+ L'(R_1^*) - L(R_1^*)(1 - P'(R_1^*))\right) < \\
&L'(R)(R - P(R))(q - \Psi(R)) + (\Phi(R) - q)\left((R - P(R))L'(R_1^*) - L(R_1^*)(1 - P'(R_1^*))\right) < \\
&L'(R_1^*)(R - P(R))(\Phi(R) - \Psi(R)) - (\Phi(R) - q)L(R_1^*)(1 - P'(R_1^*)) < \\
&(\Phi(R) - q)\left(L'(R_1^*)(R - P(R)) - L(R_1^*)(1 - P'(R_1^*))\right) < \\
&(\Phi(R) - q)\left(L'(R_1^*)r_+ - L(R_1^*)(1 - P'(R_1^*))\right) < 0
\end{aligned}$$

because, as $R > R_1^*$, then $L'(R) < L'(R_1^*)$, $R - P(R) > r_+$, $q > \Psi(R)$ and because we assume $L'(R_1^*)r_+ < L(R_1^*)(1 - P'(R_1^*)$ from $\lambda_1 < \lambda_2$. Then, all the trajectories reaching segment (B, C) will exit the trapping area.

Therefore, there is a single trajectory exiting point B $((R_1^*, q_1^*))$ and that does not hits the boundaries of the trapping area, but reaches point A $((0, q_0^*))$. And this is the heteroclinic trajectory Ω. This trajectory crosses the isocline $\dot{R} = 0$.

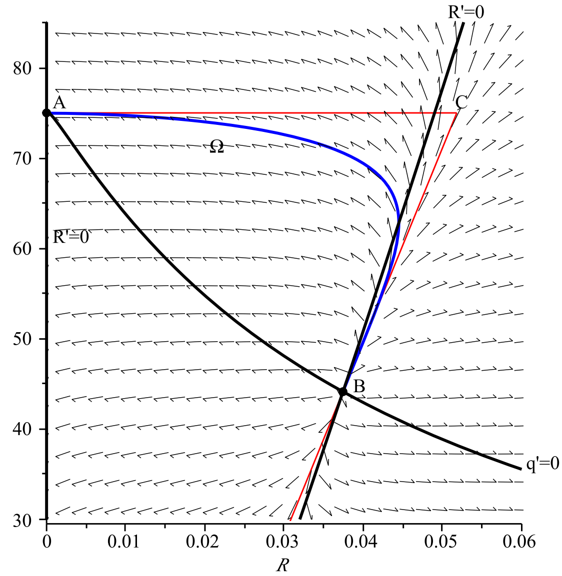

Figure 7: Proof of Proposition 3.

F Proof of Proposition 4

We use the same methods as for the proof of Propositions 1, 2 and 3.

First, the steady-state conditions are $q = \Psi(R, q)$ and $R = 0$ or $q = \Phi(R, q)$, and a steady state is an equilibrium if $R^* - P(R^*, q^*) + \mu > 0$. A steady state exists and is an equilibrium if there is a $q^* > 0$ such that $q^* = \Psi(0, q^*)$ and $-P(0, q^*) > -\mu$. Using the Taylor rule (21), the equilibrium condition is equivalent to $(q - q_+)(q - q_-) = 0$, where

$$q_{\pm} = \left(\frac{1+\gamma}{\epsilon}\right) \left\{ \frac{P(0,0)}{2} \pm \left[\left(\frac{P(0,0)}{2}\right)^2 + \frac{1-\alpha}{\alpha} \frac{\epsilon}{1+\gamma} \right]^{1/2} \right\}.$$

As $q_- < 0 < q_+$, then

$$q_0^* = q_+ = \left(\frac{1+\gamma}{\epsilon}\right) BP(0,0) > 0,$$

which holds for any $P(0, 0)$. As

$$\begin{aligned} -P(0, q_0^*) &= \frac{\epsilon}{1+\gamma} q_0^* - P(0,0) = BP(0,0) - P(0,0) = \\ &= -\frac{P(0,0)}{2} + \left[\left(\frac{P(0,0)}{2}\right)^2 + \frac{1-\alpha}{\alpha} \frac{\epsilon}{1+\gamma} \right]^{1/2} > 0, \end{aligned}$$

then the transversality condition holds without further conditions.

An interior steady state determined from the non-negative values of (R, q) such that $q = \Psi(R, q) = \Phi(R, q) > 0$. If we define $r(R, q) \equiv R - P(R, q)$, this condition is equivalent to $(r - r_+)(r - r_-) = 0$. As a necessary condition for a positive q is $r(R, q) > \rho$, then $r(R, q) = r_+ > -\mu$, which means that the transversality condition is automatically verified. This is equivalent to $\gamma R + \epsilon q = (1+\gamma)(P(0,0) + r_+)$. Substituting in $q = \Psi(R, q)$, we get the equation

$$L(R) = \frac{\alpha}{1-\alpha}\left[\ \left(\frac{1+\gamma}{\epsilon}\right)(P(0,0) + r_+) - \frac{\gamma}{\epsilon}R\ \right] r_+.$$

After some algebra, we can prove that this equation has a non-negative solution for R if and only if $r_+ + P(0,0) \geq BP(0,0) > 0$.

The local dynamics are determined in the same way as in the proofs of Propositions 2 and 3. But, in this case, the eigenvalues for steady state (R_1^*, q_1^*) are

$$\begin{aligned} \lambda_1 &= 2r_+ - \rho > 0, \\ \lambda_2 &= \frac{\epsilon}{1+\gamma} q_1^* + \frac{\gamma}{1+\gamma} \frac{L(R_1^*)}{L'(R_1^*)} > 0, \end{aligned}$$

for the values of the parameters such that there are two steady states.

G Proof of Proposition 5

It is easy to see that the steady state conditions are exactly the same as in the case of the conventional Taylor rule. The only thing that may change is related to the local and global dynamics of the model.

Applying the same methods as for the proof of Proposition 2, we find the eigenvalues for steady state (R_1^*, q_1^*):

$$\begin{aligned}\lambda_1 &= 2r_+ - \rho > 0\\ \lambda_2 &= \frac{\gamma}{1+\gamma-\delta}\frac{L(R_1^*)}{L'(R_1^*)},\end{aligned}$$

where $\text{sign}(\lambda_2) = \text{sign}(1+\gamma-\delta)$.